On the Number and Size of Jurisdictions within Large Metropolitan Areas

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Abstract

The paper studies how the institutional organization of the metro, the working of labor and land markets, and the process of tax competition between central and suburban jurisdictions shape the economic structure of the metro when consumers and firms are free to choose their locations. Contrary to general belief, we show that institutional fragmentation is socially desirable when suburban jurisdictions are not allowed to free ride on the public goods provided by the central city. Specifically, jobs are more agglomerated than the supply of public goods. We also show that the institutional and economic structure of the central city never coincide. Last, fiscal competition leads to an excessive decentralization of jobs, whereas corporate tax harmonization sustains the optimal employment distribution..

Keywords: metropolitan area, fiscal competition, local labor markets, suburbanization, metro fragmentation

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1 Introduction

According to Alain Juppé, a former prime minister of France and the mayor of the city of Bordeaux, within today's administrative limits "governments are too small to deal with the big problems and too big to deal with the small problems." Bruce Katz, a vice president at the Brookings Institution, goes one step further when he said: "metro governance is almost uniformly characterized by fragmentation and balkanisation, by culture of competition rather than one of collaboration." And, indeed, the institutional structure of a metro has a significant impact on the efficiency of local public services as well as on its residents' welfare through housing expenditures, commuting costs and the accessibility to jobs (Glaeser and Kahn, 2001; Cheshire and Magrini, 2009). The relative absence of a sound economic analysis of the working of metropolitan areas is, therefore, surprising since, for example, the estimated GDP of the metropolitan area of Tokyo or New York in 2006 is similar to those of Canada or Spain.

A metropolitan area is typically formed by a central city and several suburban jurisdictions. Policy-makers then stress the need for coordinating the actions of local governments as fiscal competition fosters a misallocation of public resources when the environment in which it operates is strategic. When consumers share the same preferences for local public policies, the basic trade-off is well known. In joining a small jurisdiction consumers enjoy a higher utility from the public good, but the cost of this good is shared among a smaller number of tax-payers. The problem may be tackled from a different, but equally important, angle by recognizing that workplaces and residences do not necessarily belong to the same jurisdiction. In practice, the central city attracts a large number of workers residing in adjacent but independent areas, thus generating a substantial amount of cross-border commuting. As a result, workers face second trade-off: either they earn a high wage in firms centrally located and bear high commuting costs, or they receive a lower pay in firms set up in secondary business centers whose access is less costly. We combine these two trade-offs within a unifying framework.

Rather than assuming that all jurisdictions are a priori identical, we acknowledge that the working of a metro is to a large extent governed by its internal hierarchy. The key-issue is then to understand how local governments interact with the labor and land markets to determine the economic structure of the metro. As a consequence, we are able to differentiate administrative borders and economic borders. Since jurisdictions compete in tax to finance local public goods, the institutional fragmentation of the metro affects the location of economic activities. Furthermore, the location of activities within cities also depends on the price of land, which capitalizes the costs and benefits associated with a particular location. In addition, since workers are free to commute within the whole metro, the share of suburbanites working in the central city is endogenous too. Evidently, a growing suburban population incentivizes firms to form secondary employment centers because land and labor are cheaper therein. However, when firms locate in such centers, they bear additional costs to acquire business services supplied in the central city (Schwartz, 1993; Porter, 1995). The drop in communication costs brought about by the new information technologies reduces these costs while allowing firms to benefit from the agglomeration economies characterizing the central city. In this event, the economic structure of the metro is polycentric.

Our purpose is to study how the institutional organization of the metro, the working of labor and land markets, and the process of tax competition between central and suburban jurisdictions interact to shape the economic structure of the metro when consumers and firms are free to choose their locations. To accomplish this task, we need a full-fledged general equilibrium model. Unlike the existing literature, our model encompasses all these effects by combining concepts borrowed from local public finances and urban economics. Evidently, the more relevant case for our purpose is that of a polycentric metro. This also concurs with the empirical evidence collected by Glaeser and Kahn (2001, p.6) for US metros, i.e. "25 percent of employees work within three miles of their metropolitan area center." However, given the prevalence of the monocentric city model in urban economics, we will also treat the monocentric configuration that we see as a limiting case. Moreover, the framework we propose displays enough versatility to study how a particular institutional context interacts with market forces to determine the morphology and economic performance of the metro.

Our main results may be organized in three distinct batches.

1. To start with, we study the first best outcome which we use as a benchmark. The planner, who aims to maximize welfare within the whole metro, chooses where consumers live and work as well as the number and size of jurisdictions. When the number of potential jurisdictions is fixed, it is always socially desirable to decentralize the provision of the public good among identical political entities. Moreover, *firms and jobs should be more agglomerated than the provision of public goods*. In other words, the administrative and economic borders of the central city never coincide. Such a result clashes with the general belief according to which such borders should be the same. This is because the planner chooses the size of jurisdictions to permit the best provision of public services, whereas the optimal sizes of the central and secondary business centers depends on the interplay between commuting and communication costs. In addition, whether the optimal metro is mono or polycentric depends on several parameters. When commuting costs are low, communication costs are high, and/or the total population is small, all jobs are located in the central business district. Otherwise, the suburban jurisdictions host firms and jobs.

2. We then proceed by studying the decentralized outcome when the number of jurisdictions and their borders are exogenous. There are three types of players, i.e. a continuum of

consumers, a continuum of firms, and a finite number of local governments. Consumers choose a residence and a workplace. Firms choose a location and the wages paid to their employees. Jurisdictions supply a local public good consumed by their residents only. To finance this good, local governments choose non-cooperatively a head tax levied on their residents and a business tax paid by the firms located in their jurisdiction.[1] We consider the following sequence of interaction between local governments and market factors: first, consumers choose where to live; then, local governments set non-cooperatively business and head taxes with the aim to maximize total welfare within their own community; last, firms choose their locations while labor and land markets clear. Once consumers are mobile, the specification of governments' objective is known to be a controversial issue (Scotchmer, 2002; Cremer and Pestieau, 2004). Our three-stage game obviates this difficulty because governments know who their residents are, and thus may determine the total welfare to maximize. In particular, our staging captures one fundamental fact highlighted by Glaeser and Khan (2001), that is, jobs have followed people in many US metros.

Fiscal competition yields a very contrasted taxation pattern. First, business tax competition is fierce: *suburban governments do not tax firms, whereas the central city government always taxes firms located in the central business district* (CBD). Once the level of commuting costs and/or communication costs decreases, the CBD looses some of its comparative advantage, which entices the central city to lower its business tax rate. By contrast, both types of jurisdictions tax their residents. Because the central city collect taxes from the CBD firms, its head tax is lower than the tax rate selected by the suburban jurisdictions. By adopting a very aggressive business tax policy, the suburban jurisdictions are able to attract a large number of firms. This concurs with Kaz for whom the culture of competition that prevails in many metros is damaging to the central city. Less expected, perhaps, *corporate tax harmonization delivers the efficient outcome*. This clashes with Kanbur and Keen (1993) who argued that fiscal harmonization is not desirable in a different, but related, context involving cross-shopping instead of cross-commuting. In addition, though the CBD is always bigger than any secondary employment center, the CBD labor force need not exceed the total suburban employment. Thus, our results agree with the empirical evidence provided by Glaeser and Kahn (2001) who observed that administrative borders have a significant impact on the economic structure of the metro.

Once it is recognized that suburbanites commuting to the CBD consume the public services supplied by the central city, business tax competition becomes harsher. Everything else being

[1] Hoyt (1992) assumes that the central city has more 'market power' than the suburban jurisdictions. Specifically, the central city government influences the land rent in the suburban jurisdictions, whereas the tax policy chosen by the government of a suburban jurisdiction has no impact on its land rent because its population share is negligible. Hoyt shows that the property tax is higher in the central city. However, unlike us, Hoyt does not allow households to choose their residential location and workplace within the metropolitan area.

equal, the consumption of the central city's public good makes the CBD more attractive to suburbanite workers, which incentivizes the suburban governments to subsidize firms. Simultaneously, the central city government increases its business tax rate to reduce the crowding of its public services. As a result, firms located in the CBD (SBDs) pay a lower (higher) wage to their employees. All in all, *central city's residents are hurt twice by the suburbanites' free-ridding behavior through a lower quality of public services and a lower wage*. However, it is worth stressing that spillovers do not generate the misallocation of firms and jobs within the metro, they worsen it.

3. Since the optimal metro is symmetric, it is reasonable to focus on the case where the central city and the suburban jurisdictions share the same border. This permits one a clear and suggestive comparison between the optimal and equilibrium outcomes. First, we show that the suburban jurisdictions undertax firms, and thus the size of the CBD is too small. This agrees with the idea that the "economic government" of the central city would be too small. Furthermore, when agglomeration economies are localized at the city center, business tax competition may be at the origin of additional efficiency losses. We show that corporate tax harmonization incentivizes firms to distribute themselves in a socially optimal way.

The symmetric setting is especially well suited to study the determination of the central city's border. We first assume that a benevolent planner chooses the central city border while anticipating the effects of his choice upon local governments' and private agents' behavior. This second best approach allows us showing that business tax competition creates a distortion not only in the allocation of jobs but also in the distribution of population between the central city and the edge cities. Because the central city collects more business tax revenues than the edge cities under fiscal competition, the population of the central city is too large relative to the first-best size.

We also consider the formation of jurisdiction through consumers' collective actions. Specifically, the central city's border is said to be *stable* if no coalition of suburbanites seeks joining the central city and no group of urbanites wants to live in a suburban jurisdiction. This mechanism yields the second-best solution when the metro is monocentric. The reason lies in the fact the inter-jurisdictional differences in local public policies are capitalized into land values. However, when the metro is polycentric, this need not hold. Indeed, a unilateral change in the administrative border between the central city and any edge city affects commuting flows with *all* edge cities, which generates a new external effect across jurisdictions. Specifically, the size of the central city chosen under border stability may be too large.

Related literature. Ever since Tiebout (1956), it is widely acknowledged that a wide portfolio of local jurisdictions allows consumers to live in the locale supplying the tax/service package that fits best their preferences. However, once it is recognized that the provision of public goods and services is often governed by increasing returns, political fragmentation may generate a substantial waste of resources. Indeed, decentralization implies that similar public goods are supplied in a large number of jurisdictions, and thus the fixed cost associated with the construction of public facilities is paid many times. This trade-off has been studied independently by Cremer et al. (1985) and Alesina and Spolaore (1997) in different, but related, contexts. These authors reach the same conclusion: there are too many jurisdictions and, therefore, excessive public expenditure. Though relevant when consumers are immobile, this framework is not suitable for studying metropolitan areas where consumers choose both where to live and where to work.

There is also a vast literature focusing on the number and sizes of clubs and jurisdictions, ranging from the pioneering contributions of Buchanan (1965) to the recent game-theoretic literature on coalition formation and network interactions (Jackson and Zenou, 2012). However, these contributions ignore the role played by labor markets and jobs' locations in consumers' residential choices, the importance of which is recognized in recent empirical works. For example, Rhode and Strumpf (2003) showed that Tiebout mechanisms are not a dominant factor in the long-run residential choices within the Boston Metropolitan Area, even though this metro is often presented as the archetype of the Tiebout model. By contrast, the interaction between land and labor markets is central to urban labor economics. However, this strand of literature does not account for tax competition and its effect on the economic structure of large cities (Zenou, 2009).

This paper is also related to the tax competition literature. The bulk of research on (international) tax competition builds on a setting in which capital is mobile and labor immobile (Keen and Konrad, 2012). Furthermore, jurisdictions are assumed to be asymmetric when they differ in size and/or factor endowment (Peralta and van Ypersele, 2005). Only a handful of papers have studied tax competition within a metro. When workers are mobile, they are assumed to live and work within the same jurisdiction (Braid, 1996), while the literature on fiscal policy within large metropolitan area does not account for the fact that jobs may be located outside the central city (Noiset and Oakland, 1995) or disregards tax competition (Bradford and Oates, 1974; Braid, 2002).

Thus, our paper may be viewed as an attempt to reconcile these strands of literature by providing a unifying framework. Indeed, in our setting, tax policies are determined non-cooperatively, as in the fiscal competition literature, while the number and structure of jurisdictions are endogenous, as in coalition formation theory. Two papers share the some of the main features of our approach. First, Perroni and Scharf (2001) studied the effects of capital tax competition when the number of jurisdictions is endogenous. However, individuals are residentially immobile. Second, Braid (2010) focussed on the distances between jurisdictions that choose to offer, or not to offer, a local public good that may be consumed by non-residents. His setting, however, does not capture the interactions between the provision of public goods and the labor and land markets.

2 The Model

We consider a large and, for simplicity, closed metropolitan area. Note, however, that our setting could easily be extended to deal with a metro trading the numéraire against goods produced in other cities. There are three goods, i.e. land available within the metro, a public good provided by jurisdictions, and the numéraire produced by profit-maximizing firms.

2.1 Jurisdictions and the provision of public goods

The metropolitan economy is formed by $m+1$ jurisdictions and L workers/consumers who are free to choose their residential location and workplace. The metro has a *star-shaped morphology*, meaning that $m \geq 2$ suburban jurisdictions are connected only to the central jurisdiction, while the central jurisdiction is connected directly to each suburban jurisdiction.[2] The central jurisdiction ($i = 0$) hosts the central business district (CBD) at $x = 0$, while each suburban jurisdiction ($i = 1, ..., m$) may, or may not, accommodate a secondary business district (SBD) at $x_i^s > 0$ along the ith radial axis.[3] Formally, the metro is described by m one-dimensional half-lines sharing the same initial point $x = 0$. Distances and locations are expressed by the same variable x measured from the CBD.

The land density is constant and consumers use the same fixed size lot. The units of land and labor are chosen for both parameters to be normalized to one. The border shared by the central city and the ith suburban jurisdiction is denoted by b_i. Moreover, since we focus on the case where suburbanites work in the CBD, the two residential areas are adjacent at b_i. Let B_i be the outer limit of the ith suburban jurisdiction defined by $[b_i, B_i]$. The border b_i is exogenous but B_i is determined by consumers' residential choices. The central city population (λ_0) and the suburban jurisdiction i population (λ_i) are, respectively, given by

[2] Alternatively, we could assume that each suburban jurisdiction is connected to its two adjacent jurisdictions via a circular belt. In this event, each suburban jurisdiction shares a common border with its two neighbors. Owing to the multiplicity of the possible routes connecting workers to their workplaces, such a setting makes the formal analysis more involved. Our main results hold true as long as the central city has more direct connections to the suburban jurisdictions than each jurisdiction taken separately.

[3] Our model does not explain why the CBD is formed. Doing this would require introducing various types of agglomeration economies that would make the formal analysis more complex. We have nothing new to add to what is known in this domain (Duranton and Puga, 2004). It is worth to stressing, however, that apart from the existence of the CBD, the internal economic structure of the metro is endogenous.

$$\lambda_0 = \sum_{i=1}^{m} b_i \qquad \lambda_i = B_i - b_i$$

where $\lambda_0 + \Sigma_i \lambda_i = L$. Hence, the entire space within the metro is used. Because the tax policy may differ across jurisdictions, their sizes need not be the same. By implication, the CBD and SBDs (if any) may accommodate different numbers of firms. In practice, the central city population is often larger than the population of a suburban population, i.e. $\Sigma_i b_i > B_i - b_i$.

Each jurisdiction supplies a public good of a fixed size $g > 0$ whose cost F is constant and the same across jurisdictions. To finance this good, the ith jurisdiction uses two instruments, that is, a *head* tax t_i levied on each resident living in the jurisdiction and a *business* tax T_i paid by the firms located therein.[4] To capture the idea that the utility of public services decreases with the number of users, we consider a *congestible* public good. Specifically, the utility of this good is equal to $g - \alpha\lambda$, where $\alpha > 0$ is a constant and λ the number of users.[5] In what follows, we assume that only the local residents use the public good supplied by their jurisdictions. Later on, we will account for the consumption of the central city's public good by cross-border commuters.

Instead of focusing on a congestible public good, we could assume that the public good is made available in a facility located at the center of the jurisdiction. In this case, consumers have to bear the travel cost to the public facility, which increases with the distance between their location and the facility location. Evidently, the average distance rises with the population size, and thus the average utility of the public good also decreases with the jurisdiction's population.

In this paper, cities and jurisdictions do not necessarily coincide. A metropolitan area endowed with a local government supplying the public good to its residents is called a *jurisdiction.* When a jurisdiction hosts firms, it is called a *city.* Unlike jurisdictions whose borders b_i are chosen by an upper-tier government, the economic border y_i of a city is endogenous and determined by the location of the consumer indifferent between working in the CBD and the ith SBD. The central jurisdiction is always a city because it hosts the CBD. By contrast, a suburban jurisdiction need not accommodate an SBD. If it does, we call it an *edge city* to differentiate it from the *central city.* In this case, the metro is *polycentric*; otherwise, it is *monocentric.* The most interesting case to study involves edge cities since job decentralization appears to be a powerful trend in most US metropolitan areas. However, we will also consider the case of suburban jurisdictions hosting residents only because this metro structure is common in many other countries.

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[4]Instead one could think of using the aggregate land rent to finance the local public good. The Henry George Theorem holds when each jurisdiction reaches its optimal size (..). This condition can hardly be satisfied here because the total population size and the number of jurisdictions are given. Furthermore, the land rent capitalizes several effects in our setting.

[5]The parameter α can be reinterpreted as the marginal cost to be paid for serving one more user.

2.2 Workers and land rents

Each consumer bears a unit commuting cost given by $\tau > 0$. Therefore, commuting costs are equal τx or $\tau |x - x_i^s|$ according to the location of her employment center. Because we want to account for the fact that there are more in-commuters than out-commuters, we consider the following three commuting patterns: (i) a consumer lives and works in the central city; (ii) a consumer lives in an edge-city but works in the central city; and (iii) a consumer resides and works in the same suburban jurisdiction.

When a consumer lives and works in the central city, her indirect utility is given by

$$V_0(x) = w_0 - R_0(x) - \tau x + g - \alpha\lambda_0 - t_0 + \frac{ALR_0}{\lambda_0}$$

where $R_0(x)$ is the land rent at a distance x from the CBD, while w_0 is the wage paid by the firms located in CBD, t_0 the head tax levied by the central city's government, while

$$ALR_0 = \sum_{i=1}^{m} \int_0^{b_i} R_0(x)\mathrm{d}x$$

is the aggregate land rent in the central city, which is evenly redistributed among the residents. When a consumer lives in the jurisdiction $i = 1, ..., n$ and works in the central city, her indirect utility becomes

$$V_i^0(x) = w_0 - R_i(x) - \tau x + g - \alpha\lambda_i - t_i + \frac{ALR_i}{\lambda_i}$$

where $R_i(x)$ and t_i are, respectively, the land rent and head tax in the suburban city i, while

$$ALR_i = \int_{b_i}^{B_i} R_i(x)\mathrm{d}x.$$

By contrast, when a consumer lives and works in the jurisdiction $i = 1, ..., n$, her indirect utility is

$$V_i^i(x) = w_i - R_i(x) - \tau |x - x_i^s| + g - \alpha\lambda_i - t_i + \frac{ALR_i}{\lambda_i}$$

where w_i is the wage rate paid in the corresponding SBD.

Without loss of generality, we assume that the opportunity cost of land is zero. The land rent at each location in the central city is as follows. Given $V_0(x)$, the equilibrium land rent in the central city must solve $\partial V(x)/\partial x = 0$ or, equivalently, $R'(x) + \tau = 0$ whose solution is

$$R_0(x) = r_0 - \tau x \tag{1}$$

where r_0 will be determined in the subsection 4.2. Furthermore, the land rent prevailing in the ith jurisdiction is given by

$$R_i(x) = \max\left\{\Phi_i^0(x), \Phi_i^i(x), 0\right\}$$

where $\Phi_i^0(x)$ ($\Phi_i^i(x)$) is the bid rent at x of a worker living in the ith jurisdiction and working in the central city (the edge city i). Given $V_i^0(x)$ and $V_i^i(x)$, the equilibrium land rent is such as $\partial V_i^0(x)/\partial x = \partial V_i^i(x)/\partial x = 0$. As a consequence, the bid rents are

$$\Phi_i^0(x) = r_i^0 - \tau x \qquad \Phi_i^i(x) = r_i^i - \tau \left| x - x_i^s \right|.$$

Both r_i^i and r_i^0 will be determined in the subsection 4.1.2.

2.3 Firms and wages

Firms produce a homogeneous good, which is used as the numéraire. Each firm operates under increasing returns and requires a fixed amount of labor units, while the marginal requirement is zero; labor is the only production factor. We choose the unit of labor for the fixed requirement to be equal to 1. By implication, the total number of firms established in the metropolitan area is given by $n = L$. Firms can locate either in the CBD or in one of the edge cities (if any), where they form a SBD. It is well documented that the CBD supplies specialized and nontradable business services that are not available in a SBD. As a consequence, when a firm chooses to set up in a SBD, it must incur an additional cost $K > 0$ for using such services. Though alternative interpretations are possible,[6] we find it convenient to call K the *communication cost* between the CBD and the SBD. Note that adding distance-sensitive costs makes the algebra heavier and does not affect our results.

Because the cost of shipping the consumption good within the metropolitan area is much lower than the commuting cost, we assume that a firm's revenue is independent of its location. Let Π_0 (Π_i) be the profits earned by a firm set up in the central city (the edge city i). When the firm is located in the CBD, we have

$$\Pi_0 = I - w_0 - T_0 \tag{2}$$

where I denotes the firm's revenue. When a firm sets up in the edge city i, its profit function becomes:

$$\Pi_i = I - w_i - K - T_i. \tag{3}$$

In each employment center, the equilibrium wages are determined by a bidding process in which firms compete for workers by offering them higher wages until no firm can earn positive profits. Thus, setting (2) and (3) equal to zero and solving, respectively, for w_0 and w_i, we get

$$w_0 = I - T_0 \qquad w_i = I - K - T_i. \tag{4}$$

[6] For example, one expects CBD firms to have a larger market share than SBD firms. In this case, K would stand for the revenue loss incurred by the firms located in SBDs.

Hence, business taxes are capitalized in local wages. Note that consumers residing within the same suburban jurisdiction do not earn the same wage when some of them work in different employment centers. As a consequence, the head tax paid in the jurisdiction where the consumer lives and the business tax paid in the jurisdiction where she works affect her utility level, whence her residence and workplace. By implication, both types of taxes affect the equilibrium pattern of activities.

How the equilibrium value of firms' revenue I is determined is immaterial for our analysis because I does not enter the profit/utility differential between any two places. Without loss of generality, we may treat I as a constant and assume that I stands for $I + g$. In this case, g is set equal to 0.

Observe that the analysis undertaken below holds true when firms produce a nontradable horizontally differentiated service. In this case, individuals working in larger business districts have access to more nontradable services. Jobs are decentralized when consumers are willing to trade fewer nontradable services against a lower price for land and a shorter commuting.

3 The Optimal Metropolitan Area

In this section, we study the optimal organization of the metropolitan area. To be precise, we assume that a social planner maximizes total welfare in the metro by choosing the administrative borders between the central city and the suburban jurisdictions, consumers' residential locations and workplaces, and firms' locations. Our setting being symmetric about the central city, the optimal administrative borders are such that $b_i = b$ while $B_i = B = L/m$ for all $i = 1, ..., m$.

It is well known in urban economics that using a utilitarian welfare function leads to the unequal treatment of equals (Mirrlees, 1972), whereas equals are equally treated in equilibrium. However, Wildasin (1986) has shown that this pseudo-paradox arises because the marginal utility of income is different across consumers at different locations. Using a utilitarian approach is therefore unjustified unless the individual utilities are quasi-linear.

Individual utilities being linear, the aggregate welfare W within the metropolitan area is given by

$$\begin{aligned} W &= m\int_0^b (I - \tau x - \alpha m b)\mathrm{d}x + m\int_b^y \left[I - \tau x - \alpha(B-b)\right]\mathrm{d}x \\ &\quad + m\int_y^B \left[I - K - \tau\left|x - \frac{B+y}{2}\right| - \alpha(B-b)\right]\mathrm{d}x - (m+1)F \qquad (5) \end{aligned}$$

where $y > b$ is the location of the individual indifferent between working in the CBD or the SBD located at the middle point $x^s = (B+y)/2$ of the segment $[y, B]$. When $b < y < B$ there are one CBD and m SBDs; when $b < y = B$ there are no SBDs but $m+1$ jurisdictions; and when $b = y = B$ there is a single city and a single jurisdiction.

Maximizing W with respect to b, y and m amounts to minimizing the social cost

$$C = CC + KC + GC + PC \tag{6}$$

where C is the sum of the following four types of costs: (i) the total commuting costs:

$$CC = m\int_0^y \tau x \mathrm{d}x + m\int_y^B \tau\left|x - \frac{B+y}{2}\right|\mathrm{d}x;$$

(ii) the total communications costs:

$$KC = (L - my)\,K;$$

(iii) the total congestion costs of the public good:

$$GC = \alpha b^2 m^2 + \alpha\,(L - mb)^2\,/m;$$

and (iv) the cost of providing a public good in all jurisdictions:

$$PC = (m + 1)F.$$

1. Assume first that the number m of jurisdictions is given. By choosing b, the planner determines the population size in each type of jurisdiction (λ_0 and λ). Evidently, a marginal expansion of the central city (a higher b) reduces the number of residents in all suburban jurisdictions. Consequently, the congestion cost decreases therein, whereas it rises in the central city.

Differentiating C with respect to b yields

$$\bar{b} = \frac{L}{m(m+1)} < B. \tag{7}$$

Thus, regardless of L and m it is always optimal to break up the metro into $m+1$ independent jurisdictions. The optimal size of a suburban jurisdiction is equal to

$$\bar{\lambda} = B - \bar{b} = \frac{L}{m+1} > 0.$$

The optimal size of the central city being equal to $m\bar{b} = \bar{\lambda}$, the central and suburban jurisdictions all have the same population size. In other words, *the optimal number of suburbanites is larger than the number of urbanites*. They both increase at the same pace with the population size of the metro (L) and decreases equally with its institutional fragmentation (m). In particular, rasing the number of jurisdictions leads to a smaller central city.

Furthermore, by choosing y, the planner determines the size of the CBD (my) and that of each SBD ($B - y$). Commuting costs CC reach their lowest value when the average traveled distance is minimized, i.e. $y = B/3$. Communication costs KC are minimized when all firms are located in the CBD, i.e. $y = B$. Because y does not affect directly the congestion costs GC, the optimal value of y is the outcome of the trade-off between commuting and communication costs. By implication, the optimal value of y must belong to the interval $(B/3, B)$.

Differentiating C with respect to y, we obtain

$$\bar{y} = \frac{B}{3} + \frac{2K}{3\tau} > \bar{b} \tag{8}$$

because $m \geq 2$. As a consequence, the CBD labor pool always encompasses the central city, while the SDBs are located in the suburban jurisdictions. As the level of communication cost decreases, the CBD shrinks at the benefit of the SBDs whose size rises. Likewise, when the total population of the metro gets larger, the labor pool of both types of cities expands; however, the employment share of the CBD decreases.

It remains to check under which condition the metro is polycentric ($\bar{y} < B$). This is so if and only if

$$K < \tau B. \tag{9}$$

In this event, the optimal metro involves $m + 1$ local labor markets. Thus, high commuting costs, low communication costs, or both generate the decentralization of jobs. In the same vein, owing to the higher commuting costs prevailing in the central city, a population rise fosters the emergence of SBDs. By contrast, a higher number of suburban jurisdictions promotes the concentration of jobs in the central city because the average commuting to the CBD is shorter. Moreover, as the CBD labor pool encompasses the central city, the employment rate in the central city always exceeds 1 whereas it is smaller than 1 in the suburban cities.

In addition, the size of the CBD is equal to

$$m\bar{y} = \frac{L}{3} + \frac{2mK}{3\tau} \tag{10}$$

which exceeds the size of a SBD. Put differently, *the CBD is always larger than a SBD*. Note, however, that the CBD employment level need not exceed the total number of suburban jobs. Indeed, the former is greater than the latter if and only if

$$K > \frac{\tau B}{4}.$$

As a result, for intermediate communication costs such as $\tau B/4 < K < \tau B$, the metro is polycentric, even though the CBD captures the majority of jobs.

If the condition (9) does not hold, communication costs are too high, commuting costs are too small, or the number of suburban jurisdictions is too large for SBDs to emerge: $\bar{y} = B$. Under these circumstances, the agglomeration of firms and jobs in the CBD, whence a monocentric metro, is socially desirable. Interestingly, the labor market is integrated though the metro is fragmented into several suburban jurisdictions. This means that, at the social optimum, the *institutional fragmentation* of the metro and the *agglomeration* of firms and jobs in the CBD do not necessarily conflict.

Furthermore, for any given $m \geq 2$, *the optimal borders of the central city and the limits of its labor pool never coincide.* To be precise, the planner always chooses for the central city a population size smaller than the population size of its labor pool. However, the suburbs may, or may not, host firms and workers. In other words, when a benevolent planner is empowered to choose both the residential location and workplace of individuals, merging all places within a single large city is never socially desirable.

Proposition 1 comprises a summary.

Proposition 1 *Consider a central planner maximizing total welfare within the metropolitan area. Then, for any given m the optimal metro always involves institutional fragmentation. Furthermore, the urban labor market may be segmented or integrated.*

2. The planner may also choose the structure of the central city and the degree of fragmentation of the metro through the variable m. Two cases must be distinguished. In the first one, it is optimal to concentrate firms in the CBD. Differentiating C with respect to m thus leads to the following equilibrium condition:

$$-F + \frac{\tau L^2}{2m^2} + \frac{\alpha L^2}{(m+1)^2} = 0 \tag{11}$$

which does not have a simple analytical solution for the optimal number $\bar{m}$ of jurisdictions. However, it is readily verified that $\mathrm{d}\bar{m}/\mathrm{d}\tau > 0$, $\mathrm{d}\bar{m}/\mathrm{d}L > 0$, and $\mathrm{d}\bar{m}/\mathrm{d}F < 0$. Lowering commuting costs and/or raising the fixed cost of the public good leads to a smaller number of suburban jurisdictions. As a consequence, the optimal fragmentation of the metro is governed by the trade-off between commuting costs and increasing returns. In particular, when the fixed cost F associated with a new jurisdiction is large and/or commuting costs are low, it is never optimal to fragment the metro ($\bar{m} = 0$). On the other hand, fragmentation is always desirable when the total population is sufficiently large.

In the second case, it is optimal to break up the metro into several cities. The optimal value of m is now implicitly given by

$$-F + \frac{\tau L^2}{6m^2} + \frac{\alpha L^2}{(m+1)^2} + \frac{K^2}{3\tau} = 0$$

which, unlike (11), depends on the level K of communication costs. The effects of the population size (L) and fixed costs (F) are qualitatively the same as in the first case. However, lowering the unit commuting cost (τ) now has an ambiguous impact on the optimal number of cities. Indeed, two opposing effects are at work.

On the one hand, for a given $\overline{y}$, decreasing the unit commuting cost reduces the total level of commuting costs within the metro. This incentivizes the planner to select a smaller value for $\bar{m}$ because total communication costs are lower. On the other hand, since $\overline{y}$ increases when the unit commuting cost falls, $\bar{m}$ should increase to reduce total commuting costs. The former effect dominates the latter one when communication costs are sufficiently low. To be precise, $\mathrm{d}\bar{m}/\mathrm{d}\tau > 0$ if and only if $\sqrt{2}K < \tau B$. In this event, decreasing commuting costs makes the central city more attractive ($m\bar{b}$ rises) but, as shown by (see.10), the size of the CBD may either grow or decline. Conversely, when $\tau B < \sqrt{2}K$, the number of suburban jurisdictions decreases with τ. The size of the CBD unambiguously increases with lower commuting costs, while the population of the central city shrinks. In brief, the trade-off between commuting costs and increasing returns is much more involved than in the first case.

To sum up, we present the next proposition.

Proposition 2 *Consider a central planner choosing b and m to maximize total welfare in the metro. Then, the optimal number of jurisdictions decreases with F. It increases with τ when the metro is monocentric but it may decrease when the metro is polycentric.*

When the optimum involves several cities, a larger population always generates a bigger CBD. By contrast, the impact of a larger L on the population of the central city is ambiguous. Indeed, for a given value of m, a larger population fosters the growth of each city. However, a larger L also leads to an hike in m, which in turn fosters the shrinking of the incumbent cities by redistributing the total population over a larger number of jurisdictions. On the contrary, the suburban population $\bar{m}\bar{\lambda}$ always increases with L. In addition, the relative population share of each city, $\bar{\lambda}/L$, decreases with L, which implies that the size of the incumbent jurisdictions either decreases or increases less than proportionally with the total population L.

The above discussion shows that *institutional, economic and demographic parameters interact in nontrivial ways to shape the optimal structure of the metro.*

4 Tax Competition and the Metro Structure

In this section, we consider a decentralized tax setting in which the institutional environment, i.e. the number m of suburban jurisdictions and the administrative borders, b_i, between these jurisdictions and the central city are given. Each jurisdiction owes its land and the aggregate land rent is evenly redistributed among the residents. Our purpose is to find how b_i and m affect the tax policies (t_i and T_i) and the location of jobs.

The spatial structure of the metro implies that competition among jurisdictions is strategic. The interactions between local governments and market forces is described by a three-stage game which blends atomic and non-atomic players. There are three groups of players: a continuum of consumers, a continuum of firms, and $m+1$ local governments. Consumers chooses where to live and where to work; firms choose where to locate and the wage to pay to their employees; and local governments choose a head tax and a business tax. In the first stage, consumers are free yo choose the jurisdiction they want to join and their location therein, anticipating the head tax they will pay and the wage they will earn. In second stage, the population in all jurisdictions is determined so that local governments can choose simultaneously and non-cooperatively a head tax and a business tax to maximize the total welfare of their residents. Last, firms choose their profit-maximizing locations and consumers their workplace, while land and labor markets clear.

We seek a subgame perfect Nash equilibrium. As usual, the game is solved by backward induction. Consumers being mobile and identical, they reach the same (indirect) utility level V^* at the end of the game. Recall that y_i is the location of the consumer indifferent between working in the CBD or in the ith SBD. Hence, we have $V^* = V_0(x)$ for $0 \leq x \leq b_i$, $V^* = V_i^0(x)$ for $b_i < x \leq y_i$, and $V^* = V_i^i(x)$ for $y_i < x \leq B_i$. In the remaining of this section, we assume that such an equilibrium exists. Since all consumers living in the central city earn the same wage, pay the same tax, and bear the same congestion cost, the land rent paid at the same distance from the CBD along the different radial axes must be equal. In particular, it must be that $R_0(x) = r_0 - \tau x$ for $x < \min\{b_1, ..., b_m\}$ where r_0 has the same expression regardless of the radial axis. In the setting above, the business tax affects the choice of a consumer's workplace while the head tax affects her residential choice.

4.1 Labor and land market equilibrium

In the third stage, firms and consumers observe the tax rates chosen by the local governments. Then, firms select a location as well as the wage they pay while consumers choose their job places. Though the whole space is used, consumers' mobility implies that the equilibrium land rent equalizes utility across consumers. Recall That, at this stage of the game, the tax rates T_i and t_i $(i = 0, 1, ..., m)$ are known to firms and consumers.

4.1.1 Job location

To determine the distribution of jobs within the metro, we must find the location y_i of the marginal worker. We assume throughout this section that y_i exceeds b_i. The location of the

SBD (x_i^s) is the middle point of the segment connecting y_i and B_i:

$$x_i^s = y_i + \frac{B_i - y_i}{2}. \tag{12}$$

The worker at y_i is indifferent between the CBD or the SBD if and only if $V_i^0(y_i) = V_i^i(y_i)$ or, equivalently,

$$w_0 - w_i = \tau y_i - \tau(x_i^s - y_i) = \tau \frac{3y_i - B_i}{2}. \tag{13}$$

Hence, the difference in the CBD and SBD wages must compensate the marginal worker for the difference in commuting costs along the ith radial axis. Plugging (4) and (12) into (13), we obtain the equilibrium *economic border* of the central city:

$$y_i^*(T_0, T_i) = \frac{B_i}{3} + \frac{2[K - (T_0 - T_i)]}{3\tau} \tag{14}$$

which generally differs from the institutional border b_i. Evidently, the economic border along the ith radial axis rises (shrinks) with T_i (T_0) because the central city becomes relatively more (less) attractive.

Furthermore, the equilibrium shares of firms located in the CBD and in the ith SBD are, respectively, given by

$$\theta_0 = \frac{\Sigma_i y_i^*}{L} \qquad \theta_i = \frac{B_i - y_i^*}{L}. \tag{15}$$

The launching of a new suburban jurisdiction leads a fraction of consumers living in the ith edge city to move into the new jurisdiction where the distance to the CBD is shorter and the price of land is zero. As a result, the ith SBD shrinks with m. However, (14) implies that y_i decreases by $\Delta/3$ when Δ consumers quit the ith edge city, which implies that the CBD also shrinks if at least $2\Delta/3$ consumers work in the SBD of the new jurisdiction. But this need not be the case because these consumers are now located closer to the CBD. In addition, the number of residents in the new jurisdictions depends on the tax policy implemented therein. To assess the impact of fragmentation on the size of CBD, it appears to be convenient to consider a symmetric setting in which $T_i = T$ and $b_i = b$. In this case, the size of the CBD is given by my^*. Using (14), it is readily verified that my^* increases with m if and only if K is greater than $T_0 - T$, that is, the wage paid in the CBD exceeds that paid in an edge city.

The following proposition is a summary.

Proposition 3 *Assume that the institutional and fiscal environments of the metro are exogenous. Then, a more fragmented metropolitan area has smaller SBDs. Furthermore, when the suburban jurisdictions set the same business tax, institutional fragmentation brings about a larger CBD if and only if the wage paid by the CBD firms exceeds that paid by the SBD firms.*

Lowering the unit commuting cost does not change the residential pattern. Therefore, (15) shows that, when τ falls, the size of the CBD increases if and only if the CBD firms pay higher wages. Note also that the metro has SBDs if and only if $\theta_0 < 1$, that is,

$$\tau > \bar{\tau} \equiv m\frac{K - (T_0 - \Sigma_i T_i/m)}{L}. \tag{16}$$

Indeed, when the unit commuting cost is smaller than $\bar{\tau}$, the wage gap (13) is narrow enough for all firms to be located within the CBD where they bear no communication costs. If the suburban jurisdictions charge on average lower business taxes than the central city, the condition (16) becomes more stringent, and thus the metro is more likely to be polycentric.

We now study the impact of the number of jurisdictions on the tax rates. The budget constraints faced by local governments are, respectively, given by:

$$F = \theta_0 L T_0 + t_0 \lambda_0 \qquad F = T_i \theta_i L + t_i \lambda_i. \tag{17}$$

The budget constraints above imply that the head taxes are, respectively, given by

$$t_0 = \frac{F}{\lambda_0} - \frac{\theta_0 L}{\lambda_0} T_0 \qquad t_i = \frac{F}{\lambda_i} - \frac{\theta_i L}{\lambda_i} T_i. \tag{18}$$

Thus, at given business taxes, the fragmentation of the metro affects both t_0 and t_i through different tax base effects. Consider first the impact on t_i. The launching of a new suburban jurisdiction leads a fraction of consumers leaving in the ith edge city to move therein. As a consequence, the cost of the public good is shared among a smaller number of residents. Additionally, the number of firms located in the corresponding SBD shrinks. Combining these two effects shows that the tax paid by the jurisdiction i's residents must rise for the budget constraint to hold. The impact on t_0 is is less clear-cut. For a given and fixed administrative border, more fragmentation expands the population living in the central city. As a result, the cost of the public good is shared among a higher number of residents. Nevertheless, Proposition 3 shows that the CBD may accommodate a greater or smaller number of firms. As a consequence, a more fragmented metropolitan area triggers a head tax hike in the suburban jurisdictions, but the head tax may increase or decrease in the central city depending on the CBD's attractiveness.

4.1.2 Land rent

We now turn to the determination of the equilibrium land rents. First, since T_0 and T_i are given, the utility reached in the central city is given by

$$V_0(x) = I - T_0 - r_0 - \alpha\lambda_0 + \frac{\theta_0 L T_0 - F}{\lambda_0} + \frac{A L R_0}{\lambda_0}$$

for $0 \le x \le b_i$. As for the suburbanites, we have

$$V_i^0(x) = I - T_0 - r_i^0 - \alpha\lambda_i + \frac{\theta_i LT_i - F}{\lambda_i} + \frac{ALR_i}{\lambda_i}$$

for $b_i < x \le y_i^*$ while

$$V_i^i(x) = I - T_i - K - r_i^i - \alpha\lambda_i + \frac{\theta_i LT_i - F}{\lambda_i} + \frac{ALR_i}{\lambda_i}$$

for $y_i^* < x \le B_i$.

It remains to determine the value of r_0 for the central city as well as the values of r_i^0 and r_i^i for each suburban jurisdiction, which satisfy the equilibrium conditions $V^0 = V_i^0 = V_i^i = V^*$. Note that the cost

$$C_i \equiv T_i + \alpha\lambda_i + (\theta_i LT_i - F)/\lambda_i - ALR_i/\lambda_i$$

differs across edge cities since the borders b_i are a priori different. Free mobility between edge cities implies that r_i^i takes its lowest value in the edge city, say city 1, which generates the highest cost C_i. Denote by R_1^1 the land rent prevailing in city 1. At the border with the central city, we have $R_1^1(y_1^*) = R_1^1(B_1) = 0$ since there is no competition for land at these two endpoints. This in turn implies

$$r_1^1 = \frac{\tau\,(B_1 - y_1^*)}{2}$$

which yields $R_1^1(x) = r_1^1 - \tau|x - x_1^s|$.

Let $R_1^0(x) = r_1^0 - \tau x$ be the land rent paid by the consumers living in city 1 and working in the CBD. Replicating the above argument where $R_1^0(y^*) = 0$, we obtain

$$r_1^0 = \tau y_1^*$$

and thus $R_1^0(x) = r_1^0 - \tau x$. Using the above expressions for the land rents, we get the aggregate land rate in city 1:

$$ALR_1 = \tau\left[\frac{(y_1^* - b_1)^2}{2} + \frac{(B_1 - y_1^*)^2}{4}\right].$$

Because $V_i^i = V^*$ holds in equilibrium, the land rent in any edge city $i \neq 1$ is such that

$$r_i^i = r_1^1 + \left(-T_i - \alpha\lambda_i + \frac{\theta_i LT_i - F}{\lambda_i} + \frac{ALR_i}{\lambda_i}\right) - \left(-T_1 - \alpha\lambda_1 + \frac{\theta_i LT_1 - F}{\lambda_1} + \frac{ALR_1}{\lambda_1}\right)$$

and

$$r_i^0 = r_1^0 + \left(\frac{T_i\theta_i L}{\lambda_i} - \alpha\lambda_i - \frac{F}{\lambda_i} + \frac{ALR_i}{\lambda_i}\right) - \left(\frac{T_1\theta_1 L}{\lambda_1} - \alpha\lambda_1 - \frac{F}{\lambda_1} + \frac{ALR_1}{\lambda_1}\right)$$

while

$$r^0 = r_1^0 + \tau\frac{3y_1 - B_1}{2} + \left(\frac{T_0\theta_0 L}{\lambda_0} - \alpha\lambda_0 - \frac{F}{\lambda_0} + \frac{ALR_0}{\lambda_0}\right) - \left(\frac{T_1\theta_1 L}{\lambda_1} - \alpha\lambda_1 - \frac{F}{\lambda_1} + \frac{ALR_1}{\lambda_1}\right)$$

since $V_0 = V_1^0$ in equilibrium.

4.2 Tax competition between the central and suburban jurisdictions

4.2.1 Business tax

As firms locate in the third stage, governments anticipate the location and size of the SBDs, x_i^s and $\theta_i^*(T_i, T_0)$, whence the values of y_i^*. At the tax competition stage, the welfare in the central city is given by

$$W_0 = \sum_{i=1}^{m} \int_0^{b_i} V_0(x)\mathrm{d}x$$

which is equivalent to

$$W_0 = \lambda_0(I - T_0 - t_0) - \sum_{i=1}^{m} \int_0^{b_i} \tau x \mathrm{d}x - \alpha\lambda_0^2. \tag{19}$$

Substituting the budget constraint of the central city

$$F = t_0\lambda_0 + T_0\theta_0 L$$

into (19), we obtain

$$W_0 = \lambda_0(I - T_0) + T_0\theta_0 L - F - \tau\frac{\sum_{i=1}^{m} b_i^2}{2} - \alpha\lambda_0^2 \tag{20}$$

where θ_0 is endogenous whereas λ_0 is exogenous.

When the central city government chooses T_0, it affects the local wage, whence the CBD size, as well as the cost of the public good incurred by its residents. However, it disregards the impact of its decision on the fiscal basis of the edge cities. What is new in our analysis is the fact that consumers can work outside their jurisdictions ($y_i > b_i$). As a consequence, a change in T_0 also affects the wage of the suburban workers who commute to the CBD as well as the level of total commuting costs within the entire metro. These two effects are not taken into account by the central city government when it chooses its business tax.

Regarding the ith suburban jurisdiction, it involves two types of workers, those who work in the CBD and those who work in their own SBD. After substitution of the jurisdictions i's budget constraint, the total welfare in the ith suburban jurisdiction is given by

$$W_i = \int_{b_i}^{y_i^*} w_0 \mathrm{d}x + \int_{y_i^*}^{B_i} w_i \mathrm{d}x + \int_{b_i}^{B_i} \left(\frac{T_i\theta_i L - F}{\lambda_i} - \alpha\lambda_i\right) \mathrm{d}x - \int_{b_i}^{y_i^*} \tau x \mathrm{d}x - \int_{y_i^*}^{B_i} \tau \left|x - x_i^s\right| \mathrm{d}x$$

where the border b_i is fixed. Unlike the central city government, any suburban government cares about the total commuting costs borne by its residents, which are given by

$$\int_{b_i}^{y_i^*} \tau x \mathrm{d}x + \int_{y_i^*}^{B_i} \tau \left|x - x_i^s\right| \mathrm{d}x = \tau \left[\frac{(y_i^*)^2 - b_i^2}{2} + \frac{(B_i - y_i^*)^2}{4}\right]$$

as well as by the residents' wage bill

$$\int_{b_i}^{y_i^*} w_0 \mathrm{d}x + \int_{y_i^*}^{B_i} w_i \mathrm{d}x = (y_i^* - b_i)(I - T_0) + (B_i - y_i^*)(I - K - T_i).$$

However, this government neglects the impact of its tax policy on the central city's fiscal basis and the commuting costs within the whole metropolitan area.

Local governments set non-cooperatively their business tax rates with the aim of maximizing the welfare of their residents. Specifically, the central city maximizes W_0 with respect to T_0, while the ith suburban jurisdiction maximizes W_i with respect to T_i. A marginal increase in T_i rises the share of jobs in the CBD as well as the aggregate value of commuting costs paid by the ith jurisdictions' residents. Raising T_i also reduces employment in the ith SBD and the wage paid by firms located in this SBD. Regarding the central city, an increase in T_0 has no impact on the commuting costs within its jurisdiction because they all work in the CBD. By contrast, it affects its resident's welfare through the wage paid by the CBD firms. Last, corporate taxes affect all the residents through the head tax they pay within their jurisdiction by changing the revenue the local governments collect from firms.

Differentiating W_0 (W_i) with respect to T_0 (T_i) yields:

$$\frac{\mathrm{d}W_0}{\mathrm{d}T_0} = \frac{2mK}{3\tau} - \frac{2m\left(2T_0 - \Sigma_i T_i/m\right) + 3\tau\lambda_0}{3\tau} + \frac{L}{3} \tag{21}$$

and

$$\frac{\mathrm{d}W_i}{\mathrm{d}T_i} = -\frac{2T_i}{3\tau} \tag{22}$$

where $\mathrm{d}^2W_0/\mathrm{d}T_0^2 < 0$ and $\mathrm{d}^2W_i/\mathrm{d}T_i^2 < 0$ hold. Using (22), we obtain

$$T_i^* = 0. \tag{23}$$

In other words, *the suburban governments neither tax nor subsidize firms.* Plugging (23) into (21) and solving for T_0, we get the equilibrium business tax set in the central city:

$$T_0^* = \frac{K}{2} + \frac{\tau\left(L - 3\lambda_0\right)}{4m}. \tag{24}$$

When λ_0 increases, the central city government collects more taxes from its residents, which allows it to lower its business tax rate. As a consequence, more jobs are located in the CBD, and thus the central labor pool expands. Observe that T_i^* and T_0^* are independent of F because the public good does not affect firms' profits.

Given the equilibrium tax rates, two cases may arise in the last stage of the game. In the first one, *the metro is monocentric* ($y_i^* = B_i = B$). This amounts to assuming that

$$\lambda_0 \geq \hat{\lambda}_0 \equiv \frac{5L}{3} - \frac{2mK}{3\tau}. \tag{25}$$

Note also that (25) always holds when K exceeds $5\tau B/2$ because $\hat{\lambda}_0 < 0$. In other words, communication costs are sufficiently large to deter the decentralization of jobs. More generally, the metro is monocentric at the tax competition outcome when at least one of the following conditions holds: the population size of the central city is large, commuting costs are low, communication costs are high, and the number of suburban jurisdictions is large. Under these circumstances, even though the suburban jurisdictions do not tax firms, wages that would be paid by SBD firms are too low to attract firms and workers away from the CBD.

In a monocentric metro, the business tax is given by

$$T_0^* = \frac{K}{2} + \frac{\tau(L - 3\hat{\lambda}_0)}{4m} = K - \tau B \tag{26}$$

which is always nonnegative because $K < \tau B$ implies $\hat{\lambda}_0 > L$. As shown by (26), a deeper institutional fragmentation raises the corporate tax set by the central city government. Indeed, since there are no SBDs, the central city government has no incentive to reduce its tax rate when the degree of fragmentation increases. On the contrary, because its population rises with m, the central city government can shift the cost of the public good toward firms without affecting the attractiveness of the CBD.

In the second case, the central city population is smaller than $\hat{\lambda}_0$, so that *the metro is polycentric*. For this to happen, it must be that $\hat{\lambda}_0 > 0$, that is, $K < 5\tau B/2$. In other words, jobs are decentralized at the tax competition outcome when at least one of the following conditions is satisfied: (i) the population is large, (ii) commuting costs are high, (iii) communication costs are low, and (iv) the number of suburban jurisdictions is small. Under these circumstances, the business tax set in the CBD, which is given by (24), is higher than (26) because $\lambda_0 < \hat{\lambda}_0$. Indeed, a higher business tax rate generates a wage drop so that the welfare loss rises with the number of CBD workers.

As for the equilibrium wages, (4) implies that $w_0^* = I - T_0^*$ and $w^* = I - K$. The wage paid in the CBD exceeds that paid in a SBD if and only if

$$\lambda_0 > \frac{L}{3} - \frac{2mK}{3\tau} \equiv \bar{\lambda}_0.$$

Observe that $\bar{\lambda}_0 < \hat{\lambda}_0$ always holds. Therefore, when $\bar{\lambda}_0 < \lambda_0 < \hat{\lambda}_0$, each suburban jurisdiction hosts a SBD where the wage is lower than in the CBD. By contrast, when $\lambda_0 < \bar{\lambda}_0$, wages are higher in the SBDs than in the CBD ($w_0^* < w^*$). Indeed, the central city being very small, its government must set a high corporate tax to finance the public good, which holds back the wage earned by the CBD workers.

The next two propositions summarize our results.

Proposition 4 *Regardless of the metro structure, the central city government always sets a positive business tax.*

Proposition 5 *When the metro is polycentric, the suburban governments do not tax or subsidy firms.*

The positive tax differential reflects the asymmetry between the central and edge cities discussed in Section 2. The intuition behind this result was spelled out by Baldwin and Krugman (2004): a locale endowed with a comparative advantage can set a higher tax rate because more firms are located there.

Finally, observe that, at the tax rates (23) and (24), all suburban jurisdictions share the same economic border with the central city:

$$y_i^* = \frac{B}{6} + \frac{K}{3\tau} + \frac{\lambda_0}{2m} \equiv y^*. \tag{27}$$

In other words, when the population size of the central city is given, *the equilibrium economic structure of the metro is symmetric even though its institutional structure need not be so.* When the administrative borders of the central city are fixed, a higher metro population leads to larger employment centers. However, the CBD grows at a lower rate than the SBDs. Moreover, the suburban employment level decreases with the population size of the central city. Moreover, lowering commuting costs fosters the centralization of jobs in the CBD.

4.2.2 Head tax

It remains to determine the equilibrium head taxes. As long as suburban jurisdictions attract people, the corresponding governments must tax their residents to finance the local public good. Since $T_i^* = 0$, the head tax in the ith suburban jurisdiction is equal to the public good cost per capita:

$$t_i^* = \frac{F}{\lambda_i}. \tag{28}$$

The head tax increases (decreases) with b_i (B_i) because the fiscal basis of the ith suburban jurisdiction shrinks (expands). For the same reason, a larger number of suburban jurisdictions leads to a higher head tax because B decreases with m.

As for the central city, its budget constraint implies that the equilibrium head tax depends on the size of the CBD, namely

$$t_0^* = \frac{F - T_0^* m y^*}{\lambda_0}.$$

Since a tax is levied on the CBD firms and $\lambda_0 > \lambda_i$, the head tax in the central city is smaller than the public good cost per capita, whence smaller than in the edge cities. Thus, everything else being equal tax competition is beneficial to the central city residents. Note also that lowering communication costs does not impact on the head tax paid in suburban jurisdictions but entices the central city government to substitute a higher head tax to a lower business tax. Furthermore, the head tax decreases with the total population L since a higher population is associated with a wider labor pool, thus a larger number of firms located in the CBD. This in turn generates a higher business tax revenue.

Proposition 6 summarizes our results.

Proposition 6 *The equilibrium head tax in the central city is lower than the equilibrium rate prevailing in each suburban jurisdiction.*

Note that this proposition critically depends on the assumption that the cost of the local public good is the same across jurisdictions. If this cost is higher in the central city, the business tax paid by the CBD firms may not be sufficient for the head tax in the central city to be lower than in the suburban jurisdictions. The same holds if the central city must supply and finance metropolitan public goods. In this case, the fiscal burden may become sufficiently high for some workers, whence some firms, to quit the CBD toward SBDs, thus worsening the central city's fiscal problem.

We are now equipped to determine the equilibrium utility level that emerges at the end of the game. To this end, observe first that if the land rent in the central city is shifted upward by, say, a fixed amount a, the aggregate land rent in this city is increased by $a\lambda_0$. In this case, each workers receives an additional transfer equal to a. As a result, we have a degree of freedom that allows choosing r_0. We select the value of r_0 is such that the land rent is continuous at the border b_i: $R_0(b_i) = R_i^0(b_i)$. Thus, the land rent defined over the segment $[0, y^*]$ is continuous and given by $R_0(x) = r_0 - \tau x$.

Since the consumers located at the various borders b_i have the same well-being, the equilibrium utility level is given by the utility level of the consumer located at city 1's outer limit:

$$V^* = I - K - \tau\frac{B_1 - y^*}{2} - \alpha\left(B_1 - b_1\right) - \frac{F}{B - b_1} + \frac{\tau}{B - b_1}\left[\frac{(y^* - b_1)^2}{2} + \frac{(B_1 - y^*)^2}{4}\right] \quad (29)$$

when $y^* < B_1$ and by

$$V^* = I - K - \alpha\left(B_1 - b_1\right) - \frac{F}{B_1 - b_1} + \frac{\tau(B_1 - b_1)}{2} \quad (30)$$

when all jobs are located in the CBD.

Let us make a pause. So far, we have determined necessary conditions for a tax equilibrium. Though often neglected in the literature, proving the existence of a (pure-strategy) Nash equilibrium in tax games is a fairly problematic issue (Laussel and Le Breton, 1998). Hence, the solution to the above necessary conditions need not be an equilibrium. In particular, the fact that the economic border of the central city must be the same for all edge cities may not be consistent with the assumption of different institutional borders. In other words, we do not know yet whether a tax equilibrium exists. In the next section, we will impose a condition on the b_i and B_i for such an equilibrium to exist.

5 The Case of a Symmetric Metro

Since both the socially optimal metro and the equilibrium economic borders of the central city are symmetric (see (7) and (27)), it is reasonable to focus on the case where the borders of the central city and of the metro are symmetric: $b_i = b$ and $B_i = B$. In addition, in a symmetric metro a tax equilibrium always exists and is symmetric, which vastly simplifies the comparison between the market outcome and the social optimum. Last, the SBDs have the same size given by

$$B - y^* = \frac{5B}{6} - \frac{K}{3\tau} - \frac{\lambda_0}{2m}$$

which decreases with the number of suburban jurisdictions since L exceeds λ_0. Plugging $\lambda_0 = mb$ into this expression yields the common economic border of the central city

$$y^* = \frac{B}{6} + \frac{K}{3\tau} + \frac{b}{2} \tag{31}$$

which rises with both the central city and metro populations. The symmetry assumption also allows for a simple determination of the internal structure of the metro. The condition $y^* < B$ holds if and only if

$$b < \hat{b} \equiv \frac{5B}{3} - \frac{2K}{3\tau}. \tag{32}$$

Thus, for the metro to be polycentric, $5B\tau$ must exceed $2K$, that is, the population size of the metro must be sufficiently large for the structure of the metro to be polycentric. When (32) does not hold, all jobs are concentrated in the CBD and the urban labor market is integrated (see Appendix A). In this case, we fall back on the standard monocentric city model of urban economics (Mills, 1967; Zenou, 2009).

Furthermore, $y^* > b$ if and only if

$$b < \tilde{b} \equiv \frac{B}{3} + \frac{2K}{3\tau}$$

which means that the central city population cannot be too large for the CBD to attract suburbanite workers.

5.1 Is the CBD too large or too small?

Comparing (8) and (14) reveals that the CBD reaches its first-best size if and only if $T_0 = T_i$, whereas fiscal competition yields a positive tax differential equal to T_0^*.

Proposition 7 *Assume a symmetric and polycentric metro. Then, corporate tax competition yields insufficient concentration of jobs and firms in the CBD. Furthermore, under corporate tax harmonization, the distribution of firms between the CBD and the SBDs is socially optimal.*

In accordance with the literature, we find that fiscal competition delivers an inefficient outcome, which takes here the concrete form of too small a CBD. In other words, a fragmented metro in which tax competition prevails is inefficient. Note that this argument disregards the fact a higher number of firms in the CBD is likely to generate agglomeration economies that rise firms' productivity. The downsizing of the CBD triggered by tax competition may, therefore, generate additional efficiency losses. This suggests that "the central city government is too small." By contrast, corporate tax harmonization within the metro leads to the optimal level of concentration of firms and jobs within the CBD.

Though Proposition 3 suggests an ambiguous relationship between fragmentation and the CBD size, tax competition among jurisdictions makes this relationship clear-cut: *institutional fragmentation fosters the centralization of jobs*. This result seems to contradict Glaeser and Kahn (2001) who observe that jobs and firms are located farther away from the CBD when institutional fragmentation is deeper. However, one should keep in mind that the decentralization of jobs goes hand in hand with lower communication costs. When this additional effect is taken into account, (31) shows that combining a higher value for m and a lower one for K may well result in bigger edge cities and a smaller CBD.

Furthermore, the business tax

$$T_0^* = \frac{K}{2} + \frac{\tau}{4}(B - 3b)$$

decreases with b. Indeed, a larger central city accommodates more jobs and firms. The fiscal basis being broader, the government can set a lower business tax. Consider now that an increase in the metro population. If this new population is absorbed by a spatial extension of the suburban jurisdictions, then the business tax rate T_0^* raises. Indeed, since the edge cities also get larger, corporate tax competition gets softer. This allows the central city government to raise its business tax. However, if the inflow is accompanied by the creation of new jurisdictions (m rises), then T_0^* may increase or decrease.

5.2 Do public good spillovers matter?

So far, we have neglected the possibility for the suburbanites working in the CBD to consume the public services provided in the central city. Instead, we now assume that in-commuters cannot be excluded from the consumption of these services. In this case, suburbanites working in the CBD benefit from both the public good provided in their own jurisdiction and in the central city. In what follows, we assume that the suburbanites' utility gain associated with the consumption of the central city public good, $g^{sp} \leq g$, exceeds the congestion cost generated by the larger number of users, αmy, for otherwise, cross-commuters would not consume the central public services. Note that g^{sp} may be interpreted as the extent of the spillover.

The indirect utility of a consumer living in the central city becomes

$$V_0(x) = w_0 - R_0(x) - \tau x + g - \alpha m y - t_0 + \frac{ALR_0}{mb}$$

while the indirect utility of a suburbanite working in the central city is given by

$$V_i^0(x) = w_0 - R_i(x) - \tau x + g + g^{sp} - \alpha\lambda - t + \frac{ALR}{B-b}.$$

The non-arbitrage condition for the marginal worker is now such that

$$w_0 - w_i = \tau\frac{3y - B}{2} - g^{sp} + \alpha m y. \tag{33}$$

It follows from (33) that the equilibrium economic border of the central city is given by

$$y^{sp} = \frac{\tau B}{3\tau + 2\alpha m} + \frac{2\left(K + g^{sp} - T_0 + T_i\right)}{3\tau + 2\alpha m}$$

which increases with g^{sp} and decreases with α.

Evidently, the government of the central city internalizes the congestion effects generated by the cross-commuters. As for the suburban governments, two cases may arise: (i) they focus only upon the welfare effects of their own supply of public services; (ii) they adopt an opportunistic behavior by internalizing the welfare effects of the central city public services consumed by their out-commuters. Consider the first case. The central city government now maximizes the social welfare function given by

$$W_0^{sp} = \lambda_0(I - T_0) + T_0\theta_0 L - F - \tau\frac{mb^2}{2} - \alpha m y^{sp}$$

while the suburban governments maximize

$$W_i^{sp} = (y^{sp} - b)(I - T_0) + (B - y^{sp})(I - K - T_i) - \tau\left[\frac{(y^{sp})^2 - b^2}{2} + \frac{(B - y^{sp})^2}{4}\right] + T_i\theta_i L - F - \alpha\lambda^2.$$

It is readily verified that the equilibrium business tax rates are given by

$$\begin{aligned} T_0^{sp} &= \frac{3\tau + 2\alpha m}{2\,(3\tau + \alpha m)}K + \frac{\tau\left[(3\tau + 2\alpha m)\,B - 9\tau b\right]}{4\,(3\tau + \alpha m)} \\ T^{sp} &= \frac{\alpha m}{3\tau + \alpha m}K + \frac{\alpha m \tau (B + 3b)}{2\,(3\tau + \alpha m)} - g^{sp}. \end{aligned}$$

It follows immediately from (24) that *the central city business tax is higher in the presence than in the absence of spillovers*. The reason is easy to grasp. The government of the central city increases its tax rate to shrink its labor pool, whence to lower the resulting congestion effects borne by its residents. Moreover, $T^{sp} < 0$ since $g^{sp} > \alpha m y^{sp}$. Indeed, everything else being equal, the consumption of the central city's public services makes this jurisdiction more attractive to suburbanites. This entices the suburban governments to subsidy firms. Therefore, $T_0^{sp} - T^{sp}$ exceeds $T_0^* - T^*$. As a consequence, *public good spillovers exacerbate the dispersion of firms within the metro* (see Proposition 7).

At the above tax rates, the economic border of the central city is such that

$$y^{sp} = \frac{K}{3\tau + \alpha m} + \frac{\tau B + 3\tau b}{2\,(3\tau + \alpha m)}$$

so that $T^{sp} = \alpha m y^{sp} - g^{sp} < 0$. Note that $y^{sp} = y^*$ when $\alpha = 0$. By contrast, when α is positive, we have $y^{sp} < y^*$. In addition, the stronger the congestion effect (a higher α), the smaller the CBD. All in all, the economic size of the central city shrinks in the presence of spillovers, thus making the cooperation between the central city and the edge cities even more compelling for the metro to be efficient.

Assume now that the suburban governments internalize the spillovers and congestion costs associated with the consumption of the central city services by their out-commuters. The objective function of the central city's government is unchanged, but suburban governments now maximize

$$W_i^{\mathbf{sp}} = W_i^{sp} + g^{sp} m y^{\mathbf{sp}} - \alpha \left(y^{\mathbf{sp}} - b\right) m y^{\mathbf{sp}}.$$

We show in Appendix B that the resulting tax rates also intensify the dispersion of firms and jobs away from the CBD. The central city's economic border is now given by

$$y^{\mathbf{sp}} = \frac{K}{3\,(\tau + \alpha m)} + \frac{\tau B + 3\tau b + 2\alpha b m}{6\,(\tau + \alpha m)} + \frac{g^{sp}}{3\,(\tau + \alpha m)}.$$

Moreover, the economic border of the central city increases (decreases) with g^{sp} (α) because the existence of spillovers make the central city more attractive as a workplace.

The existence of spillovers has both expected and unexpected redistributional implications for consumers living in the central and peripheral jurisdictions. First, the out-commuting suburbanites benefit from more public services whereas the central city's residents bear a higher congestion cost. This is not the end of the story, however. Since the business tax paid by the CBD firms is higher, the central-city's workers get a lower pay. By contrast, the SBD workers earn a higher wage as the suburban firms are subsidized. As a consequence, *central city's residents are hurt twice through an externality effect and an income effect*. The free-ridding problem between the central city and the suburban jurisdictions thus has implications that go beyond the standard consumption effects generated by spillovers.

Proposition 8 comprises a summary.

Proposition 8 *Assume that the suburbanites working in the CBD consume the central city public services. Then, the central city government taxes more the CBD firms whereas the suburban governments subsidize the SBD firms. Furthermore, the presence of spillovers leads to a smaller CBD.*

5.3 How to choose the central city limit?

The benefits of a symmetric metro are reap when we come to the choice of the border between the central city and the suburban jurisdictions. In what follows, we consider two very different institutional mechanisms. In the first one, for any given value of m, the planner whose aim is to maximize the common individual utility level within the metro chooses the border b prior to the game described in Section 4. In other words, we conduct a typical second-best analysis in which the planner first chooses the optimal administrative structure of the metro, and then lets consumers, firms and local governments to pursue their own interest. In the second scenario, we tackle the problem from a different perspective and ask whether there exists a *secession-free border*, that is, a value of b such that no coalition of consumers wants to be reallocated to the adjacent jurisdiction.

5.3.1 The optimal administrative border of the central city

Recall that for any given b workers share the same equilibrium utility level $V^*(b)$ regardless of the jurisdiction in which they live. Thus, a consumer located at $x = B$, who pays a land rent equal to zero, works in the SBD located at $x_i^s = y^*/2 + B/2$, and lives in a suburban jurisdiction where the head tax given by (28), also reach the utility level $V^*(b)$. Evidently, by choosing the administrative border of the central city, the planner also chooses its economic border (see (31)) and, therefore, the overall commuting pattern that prevails in the metro. To be precise, raising b leads to an expansion of the CBD labor pool, which in turn generates more commuting.

Insert Figure 1 about here

Two cases must be distinguished depending on the spatial pattern of jobs and firms anticipated by the planner. The proposition in Appendix C provides a detailed description of the optimal solution.

The polycentric metro Let b^p be the optimal border of the central city when the metro is polycentric. The common utility level maximized by the planner is then given by:

$$V^*(b) = I - K - \tau\left(\frac{B - y^*}{2}\right) - \alpha\,(B - b) - \frac{F}{B - b} + \frac{\tau}{B - b}\left[\frac{(y^* - b)^2}{2} + \frac{(B - y^*)^2}{4}\right]$$

where

$$y^* = \frac{B}{6} + \frac{K}{3\tau} + \frac{b}{2}.$$

The direct effect of b on welfare is a priori ambiguous because a rise in b affects both the costs and benefits of sharing the financing of public services. We must also stress the presence of another direct effect, i.e. the administrative border affects the aggregate land rent per capita.

Furthermore, when the metro is polycentric, the administrative border has an indirect effect through its influence on the central city's economic border. Indeed, standard calculations reveal that

$$\frac{\mathrm{d}V^*}{\mathrm{d}y^*} = \frac{3\tau(y^* - b)}{2(B - b)} \geq 0.$$

By implication, since $\mathrm{d}y^*/\mathrm{d}b > 0$, the common utility increases with b through its positive impact on the economic size of the central city. To be precise, increasing y^* generates a hike in the share of the aggregate land rent redistributed to individuals deflated by commuting costs.

As shown in Appendix C, the location of the utility-maximizing border varies with respect to the size-fragmentation ratio ($B = L/m$) characterizing the metro. Indeed, when the metro has a large population and/or a low degree of fragmentation such that $B > B^{\max} \equiv K/\tau + 3\sqrt{F/4\alpha + \tau}$ holds, we get

$$b^p = \tilde{b} \equiv \frac{B}{3} + \frac{2K}{3\tau} < B.$$

In other words, when the size-fragmentation ratio takes on a high value, the planner chooses a border such that the labor pool of each jurisdiction coincides with the jurisdiction itself. In this case, the planner aims to promote a metro characterized by the absence of cross-border commuting, which reduces individual welfare. For this reason, it is not surprising that $B^{\max}$ decreases when the unit commuting cost rises, thus making the condition $B > B^{\max}$ less stringent. Besides, it is worth stressing that $B^{\max}$ decreases when the cost of the public good is lower (see also Appendix C). Indeed, by choosing a large central city that matches its economic borders, the planner also reduces the size of the suburban jurisdictions. Nevertheless, this is not too detrimental to the suburbanites because the cost of the public good is low.

One important question is to know whether the utility-maximizing border $b^p = \tilde{b}$ coincides with the first-best optimum (see section 3). Recall that the benevolent planner always chooses for the central city a population size smaller than the size of its labor pool. As $b^p = \tilde{b} = y^*$, this utility-maximizing border is suboptimal. To be precise, it is straightforward to verify that $\tilde{b} > \bar{b} = B/(m+1)$. As a result, when B is large the utility- maximizing central city has a population size exceeding its socially optimal one.

When the size-fragmentation ratio takes intermediate values such that $B^{\max} > B > B^{\min} = K/\tau - 3\sqrt{F/4\alpha + \tau}$, the utility-maximizing border is given by

$$b^p = B - \sqrt{\frac{12F - \tau\left(\frac{K}{\tau} - B\right)^2}{3\,(16\alpha + \tau)}} < B.$$

As above, the metro is formed by $m + 1$ independent edge cities.[7] Moreover, it can be checked that the population size of the central city still exceeds the optimal size $\bar{b}$. Thus,

[7] We show in Appendix C that when the size-fragmentation ratio reach low values such that $B^{\min} > B$, the utility-maximizing borders do not sustain a polycentric metro.

$b^p$and $\bar{b}$ implying each the existence of several jurisdictions, whether fragmentation is good or bad in practice depends on where the central city border is established. Last, we want to stress that whatever its value, b^p always decreases with m because a larger number of suburban jurisdictions requires a larger number of suburbanites, whence a smaller central city, to finance the local public good.

The monocentric metro When the planner anticipates a monocentric metro at the subgame equilibrium, the individual utility level is given by

$$V^*(b) = I - K - \alpha (B - b) - \frac{F}{B - b} + \frac{\tau}{2} (B - b)$$

and the utility-maximizing border (b^m) by

$$b^m = B - \sqrt{\frac{2F}{2\alpha - \tau}} < B$$

provided that 2α exceeds τ and $B < K/\tau - 3\sqrt{F/2(2\alpha - \tau)}$. Moreover, $b^m > 0$ if and only if $\check{F} = (2\alpha - \tau)B^2/2 > F$. This condition is more likely to hold when communication costs are high, commuting costs and the fixed cost of public good are low. Once again, the metro is fragmented in $m + 1$ competing jurisdictions.

Two comments are in order. Firstly, it is readily verified that $b^p > b^m$. In other words, when the planner finds it optimal to reduce the economic size of the central city through the emergence of SBDs, he also finds it socially desirable to increase its population size. This shows that *the administrative and economic borders of the central city need not move in the same direction*. The reason for this rather unexpected result is that the population size in the central city can shrink when the metro becomes monocentric without reducing the tax basis while allowing the suburban jurisdictions to host more people and to reduce their head tax. Secondly, in the polycentric case we have just seen that the planner aims to foster a larger central city whose population size becomes too large relative to the first-best outcome.

This need not be the case when jobs and firms are concentrated in the CBD. Indeed, we have

$$b^m \gtreqqless \bar{b} \Leftrightarrow F \lesseqqgtr \dot{F} \equiv \frac{L^2(2\alpha - \tau)}{2\,(m+1)^2}$$

with $\dot{F} < \check{F}$. Thus, when the fixed cost of public good is low enough ($F < \dot{F}$), the population of the central city exceeds its first-best city size, while the opposite holds for $\dot{F} < F < \check{F}$.

The next propositions summarize our results.

Proposition 9 *Assume a metro in which the planner chooses the limit of the central city.*

(i) The utility-maximizing border is such that the metro is always formed by several jurisdictions supplying the public good.

(ii) The central city reaches its smallest size when the metro is monocentric.

(iii) The utility-maximizing border generates a central city whose size exceeds the socially optimal size when the metro is polycentric. In the monocentric case, the central city can host a population larger or smaller than the first-best city size.

As shown in Appendix C, a third case arises when commuting costs are high relative to congestion costs, the cost of the public good is high and the metropolitan population low. Under these circumstances, the metro is formed by m identical and independent cities which are each endowed with their own CBD. This extreme case is not interesting for our purpose as it involves a vanishing central city.

5.3.2 Does a border-stable city limit exist?

Consider the ith radial axis. As seen above, regardless of the value of b_i, the consumers located along the ith axis reach the same level of utility $V^*(b_i)$ given by (29) or (30). At the equilibrium, any border between the central city and a suburban jurisdiction is robust against any individual deviation. However, if consumers can coordinate their move, this need not be true. Consider a coalition $\Delta > 0$ of consumers such that $0 < \Delta < b_i$ and $0 < \Delta < B - b_i$. Since they are closer to b_i, the consumers located in the interval $[b_i - \Delta, b_i]$ with $\Delta > 0$ are those who have the highest incentive to join the ith jurisdiction by shifting the border from b_i to $b_i - \Delta$. Similarly, when the consumers are located in $[b_i, b_i + \Delta]$, they may want to join the central city and shift the border from b_i to $b_i + \Delta$.

Our purpose being to find out whether a secession-free city limit exists, we focus on arbitrarily small coalitions. Let Ω (Ω_0) be the utility gains obtained by those consumers when they join the suburban jurisdiction i (the central city). The border b_i^* is stable if $\mathrm{d}\Omega_0/\mathrm{d}\Delta \leq 0$ at $\Delta = 0$ and $\mathrm{d}\Omega/\mathrm{d}\Delta \leq 0$ at $\Delta = 0$. Because $|\mathrm{d}\Omega_0/\mathrm{d}\Delta| = |\mathrm{d}\Omega/\mathrm{d}\Delta|$, b^* is *border-stable* if $\mathrm{d}\Omega_0/\mathrm{d}\Delta = \mathrm{d}\Omega/\mathrm{d}\Delta = 0$ when $\Delta = 0$.

Consider first the case in which all jobs are located in the central city ($y^* = B$). Remember that the differences between jurisdictions in taxes, congestion costs and aggregate land rent are capitalized in the land rent, the consumers located along the ith axis reach the same level of utility V^* regardless of the value of b_i. As in 4.1.2, let $i = 1$ be the suburban jurisdiction such that the cost C_i takes on its lowest value. Under these circumstances, the equilibrium utility level is given by (30) and depends only upon b_1. As a result, we may focus on the edge city 1.

It is readily verified that

$$\Omega_0 = -\Delta\left[\frac{F}{(B - b_1 - \Delta)(B - b_1)} - \alpha + \frac{\tau}{2}\right] \qquad \Omega_1 = \Delta\left[\frac{F}{(B - b_1 + \Delta)(B - b_1)} - \alpha + \frac{\tau}{2}\right]$$

where $\Omega_0 = \Omega_1 = 0$ when $\Delta = 0$. Furthermore, we have

$$\left.\frac{\mathrm{d}\Omega_0}{\mathrm{d}\Delta}\right|_{\Delta=0} = \alpha - \frac{F}{(B-b_1)^2} - \frac{\tau}{2} \qquad \left.\frac{\mathrm{d}\Omega_1}{\mathrm{d}\Delta}\right|_{\Delta=0} = -\alpha + \frac{F}{(B-b_1)^2} + \frac{\tau}{2}$$

the solution of which is unique and given by

$$\check{b}_1 = b^m$$

provided that $2\alpha - \tau > 0$. As a result, when the metro is monocentric, empowering consumers the right to change jurisdictions' border yields an outcome identical to the second-best border. Indeed, consumers' decisions to join another jurisdiction generate no externality across jurisdictions because their decisions are capitalized into the price of land.

Assume now that jobs are dispersed within the metro. In this event, a border change along one radial axis generates an inter-jurisdictional externality which is not capitalized into land values. Recall that the economic border of the metro is given by

$$y^* = \frac{B}{6} + \frac{K}{3\tau} + \frac{b_1 + b_i + \Sigma_{k\neq 1,i} b_k}{2m}.$$

As a result, when a coalition of consumers changes the administrative border of the ith radial axis (with $i \neq 1$), the economic border of all jurisdictions are affected, something that cannot happen in a monocentric metro since $y^* = B$. Specifically, since $(\mathrm{d}V^*/\mathrm{d}y^*)(\mathrm{d}y^*/\mathrm{d}b_i) > 0$ for $i \neq 1$ when $y^* > b_1$, consumers along the ith radial axis have an incentive to join the central city. Indeed,

$$\Omega_0 = 3\tau\Delta\frac{\Delta + 4m(y^* - b_1)}{16m^2\lambda_1} \geq 0 \qquad \Omega_i = -3\tau\Delta\frac{4m(y^* - b_1) - \Delta}{16m^2\lambda_1}$$

which implies that $\mathrm{d}\Omega_0/\mathrm{d}\Delta > 0$ and $\mathrm{d}\Omega_i/\mathrm{d}\Delta < 0$ at $\Delta = 0$. Therefore, regardless of the value of b_1, the stable border takes on its highest value, that is, $\check{b}_i = y^*$ where y^* depends on the central city's population size. As a consequence, at the symmetric outcome, the stable border is such that $\check{b} = y^*(\check{b})$, which implies $\check{b} = \tilde{b}$. Note that $\check{b} > b^p$ except when $B > B^{\max}$ where $\check{b} = b^p = \tilde{b}$ (see Section 5.3.1). Indeed, when B is very large the externality generated by the border effect becomes sufficiently strong to jeopardize the identity between the two solutions.

The following proposition summarizes our main result.

Proposition 10 *Assume that jurisdictions compete in tax. The border-stable city limit and the utility-maximizing border are the same except when the metro is polycentric and the inequality* $B > B^{\max}$ *holds.*

6 Conclusion

UNFINISHED

In this paper, we have studied the economic and institutional organization of large metropolitan areas. We model jurisdiction formation as a result of a trade-off between the benefits of a large central city and the benefits of edge cities. A larger number of edge cities reduces commuting and congestion costs but raises communication costs and public expenditures. In addition, institutional fragmentation triggers fiscal competition between the central and edge cities, which results to an excessive decentralization of jobs.[8] By developing the residential areas, the creation of suburb cities may also contribute to excessive commuting and inefficiency patterns of economic and demographic structure. In addition, a large central city allows for a higher average wage and a lower tax burden on consumers.

[8] The competition among cities within the metropolitan areas to attract mobile tax base can be fierce, even in the high centralized countries. For example, competition among cities of the Beijing-Tianjin metropolitan region is becoming fierce (Tang and Xu, 2008). Even if China is a unitary state, Beijing competes fiercely with nearby municipalities.

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Appendix A

Assume that $b \geq \tilde{b}$, which means $y^* \leq b$. In this event, (20) no longer describes the total welfare in the central city, which is now given by the following expression:

$$\begin{aligned} W_0 &= (I-T_0)my^* + (b-y^*)\textstyle\sum_{i=1}^m (I-T_i-K) - m\tau\frac{(y^*)^2}{2} - m\tau\frac{b^2-(y^*)^2}{2} \\ &\quad -\alpha\lambda_0^2 - (F - T_0 m y^*) \end{aligned}$$

where y^* is still given by (14). It is then readily verified that

$$\frac{\mathrm{d}W_0}{\mathrm{d}T_0} = \textstyle\sum_{i=1}^m (K+T_i)\frac{\mathrm{d}y^*}{\mathrm{d}T_0} < 0. \tag{A.1}$$

Since y^* decreases with T_0, the above inequality implies that, for any T_i, the best reply $T_0^*(T_i)$ must be such that

$$y[T_0^*(T_i), T_i] = b.$$

As for the total welfare in the ith suburban jurisdiction, it is becomes

$$\begin{aligned} W_i &= (B-b)(I-T_i-K) - \frac{\tau}{2}\left(\frac{B+y}{2}-b\right)^2 - \frac{\tau}{2}\left(B-\frac{B-y}{2}\right)^2 \\ &\quad -\alpha\lambda_i^2 - [F-(B-y)T_i]. \end{aligned}$$

Differentiating W_i with respect to T_i yields the first-order condition:

$$\frac{\mathrm{d}W_i}{\mathrm{d}T_i} = b - y^* - \left[\frac{\tau}{2}(y^*-b) + T_i\right]\frac{\mathrm{d}y^*}{\mathrm{d}T_i} = 0$$

with $\mathrm{d}^2W_i/\mathrm{d}T_i^2 < 0$. Because $b - y^* = 0$ must hold in equilibrium, the above equality implies that $T_i^* = 0$ in (A.1). Plugging this value into (14) and solving for T_0 yields

$$T_0^* = K + \frac{3\tau}{2}\left(\frac{B}{3} - b\right).$$

In sum, the marginal worker is located within the metro ($y^* < b$) or at the city border ($y^* = b$). As a consequence, the assumption $y > b$ made above is a weak restriction.

Appendix B

When the suburban governments internalize the costs and benefits generated by the consumption of the central city's public services, the equilibrium tax rates are given by

$$\begin{aligned} T_0^{\mathbf{sp}} &= \frac{(3\tau+2\alpha m)(\tau L + 2Km + 2g^{sp}m)}{12m(\tau+\alpha m)} + \frac{(3\tau+2\alpha m)\, g^{sp}}{6(\tau+\alpha m)} + \frac{2\alpha\lambda_0(2\alpha m - 3\tau) - 9b\tau^2}{12(\tau+\alpha m)} \\ T_i^{\mathbf{sp}} &= -\frac{\alpha(\tau L + 2Km)}{6(\tau+\alpha m)} - \frac{\alpha\left[2mg^{sp} - \lambda_0(3\tau + 4m\alpha)\right]}{6(\tau+\alpha m)}. \end{aligned}$$

The business tax set by the central city government thus increases with the extent of the spillover whereas the subsidy paid the suburban governments increases.

Appendix C

Observe that $\tilde{b} = \hat{b} = B$ if and only if $\tau B = K$. Thus, for $\underline{b} = B$ to hold, it must be that $K = \tau B$, a zero-measure case. >From now on, we assume that $K \neq \tau B$.

Case 1. Suppose that the planner anticipates a monocentric metro. The common utility level is then given by

$$V^*(b) = I - K - \alpha\,(B-b) - \frac{F}{B-b} + \frac{\tau}{2}\,(B-b)$$

which is strictly concave. Differentiating this expression with respect to b yields

$$b^m = B - \sqrt{\frac{2F}{2\alpha - \tau}} \text{ if } 2\alpha > \tau$$

where $b^m > 0$ if and only if $(2\alpha - \tau)B^2 > 2F$.

If $2\alpha > \tau$, we have

$$b^m = B - \sqrt{2F/2\alpha - \tau}.$$

Furthermore, $b^m \geq \hat{b}$ if and only if

$$B < \frac{K}{\tau} - 3\sqrt{\frac{F}{2(2\alpha - \tau)}}\ .$$

If this inequality does not hold, b^m is not consistent with the maximization program of the planner who anticipates a monocentric configuration.

If $(2\alpha - \tau)B^2 < 2F$, then $V^*(b)$ is strictly decreasing over $[0, B]$. In this case, the planner wants to make the central city as small as possible. In the limit, $b = 0$ which means that the metro is polycentric and formed by m independent cities.

Case 2. Assume now that the planner anticipates that jobs and firms will be dispersed and denote by b^p the optimal border of the central city. In this case, the planner maximizes the following utility function:

$$V^*(b) = I - K - \tau\left(\frac{B-y^*}{2}\right) - \alpha\,(B-b) - \frac{F}{B-b} + \frac{\tau}{B-b}\left[\frac{(y^*-b)^2}{2} + \frac{(B-y^*)^2}{4}\right]$$

where

$$y^* = \frac{B}{6} + \frac{K}{3\tau} + \frac{b}{2}.$$

It is readily verified that $V^*(b)$ is strictly concave if and only if

$$F > F_{\min} \equiv \frac{\tau}{12}\left(\frac{K}{\tau} - B\right)^2.$$

2.a. Assume $F > F_{\min}$. Differentiating $V^*(b)$ with respect to b yields

$$b^p = B - 2\sqrt{\frac{12F - \tau\left(\frac{K}{\tau} - B\right)^2}{3\,(16\alpha + \tau)}} < B. \tag{C.1}$$

It must be that $y^* < B$ or, equivalently, $b^p < \hat{b}$. Moreover, the central city attracts suburbanites if and only if y^*exceeds b^p or equivalently $b^p \leq \tilde{b}$. We must check when these two conditions hold. First, $b^p < \hat{b}$ if and only if

$$B > B^{\min} = \frac{K}{\tau} - 3\sqrt{\frac{F}{4\alpha + \tau}}$$

while $b^p \leq \tilde{b}$ if and only if

$$B < B^{\max} = \frac{K}{\tau} + 3\sqrt{\frac{F}{4\alpha + \tau}}.$$

Therefore, (C.1) is the border selected by the planner if and if $B^{\min} < B < B^{\max}$.

Two more cases are be investigated. In the first one, $B > B^{\max}$, which means that the metro has a large population or a low degree of fragmentation. Since $B > B^{\max}$ implies $\hat{b} > \tilde{b}$, we have:

$$b^p = \tilde{b} \equiv \frac{B}{3} + \frac{2K}{3\tau}.$$

In the second case, we have $B < B^{\min}$, whence $\hat{b} < \tilde{b}$, and thus $b^p = \hat{b}$. However, b^p must be lower than $\hat{b}$ for the metro to be polycentric. As a consequence, $b^p = \hat{b}$ is not consistent with the planner's anticipation of a polycentric metro.

2.b. When $F \leq F_{\min}$, the function $V^*(b)$ is convex. Recall that $\hat{b} < \tilde{b}$ if and only if $B < K/\tau$. As a result, $V^*(b)$ has two local maximizers at $b = 0$ and $b = \hat{b}$ if $K/\tau > B > 2K/5$, or at $b = 0$ and $b = \tilde{b}$ if $B > K/\tau$.

First, it is readily verified that $V(\hat{b}) > V(0)$ for all $F \leq F_{\min}$. However, $b^p = \hat{b}$ implies the concentration of firms and jobs in the CBD, which contradicts the planner's anticipation of a polycentric metro. Second, $V(\tilde{b}) > V(0)$ if and only if

$$F < \tilde{F} \equiv (\tau B - K)\, \frac{3B - 2\frac{K}{\tau} + 16\frac{\alpha}{\tau}B}{24}$$

where $\tilde{F} > F_{\min}$ since $B > K/\tau$. Thus, $F < \tilde{F}$ holds when $\hat{b} > \tilde{b}$. As a result,

$$b^p = \tilde{b} < B$$

if $F < F_{\min}$ and $B > K/\tau$.

To sum up, we have:

Proposition 11 *When $(2\alpha - \tau)B^2 > 2F$ and $B < K/\tau - 3\sqrt{F/2(2\alpha - \tau)}$, the utility-maximizing border is given by $b^m = B - \sqrt{2F/2\alpha - \tau} > 0$ and the metro is monocentric.*

Proposition 12 *The utility-maximizing border sustains a polycentric metro and is given by*

- $b^p = B/3 + 2K/3\tau$ *when (i) $F > F^{\min}$ and $B > B^{\max}$ or (ii) $F < F^{\min}$ and $B > K/\tau$;*
- $b^p = B - 2\sqrt{12F - \tau\left(\frac{K}{\tau} - B\right)^2/3\,(16\alpha + \tau)}$ *when $F > F^{\min}$ and $B^{\max} > B > B^{\min}$;*
- *when $2\alpha - \tau < 2F/B^2$ and $B < 2K/5\tau$, the metro is split into m independent and identical cities .*