The consequences of a one-sided externality in a dynamic, two-agent framework*

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February 11, 2015

Abstract

We discuss a dynamic model where all agents contribute to a global externality, but only those in a specific region suffer from it. We model this in a dynamic setting via a two agent, non-cooperative overlapping generations model and analyze the consequences for economic growth and intertemporal choices. We find that multiple steady states may result from this asymmetry. In particular, if the agent who is affected by the externality has to spent a large share of his income to offset it, then he may be stuck in an environmental poverty trap. We provide conditions for the existence of, and local convergence to, the equilibria, as well as a condition for the global convergence to the poverty trap.

While, in addition to maintenance expenditures, externalities tend to be addressed via studying taxes, investment in R&D or alike, we focus on capital market integration. Specifically, agents in the affected region can open up their capital market to enable capital inflows. We investigate whether an open capital market improves or worsens their welfare. While we do find that capital market integration eliminates the environmental poverty trap, we show that capital market integration is not always in both agents' interest. In particular, we provide conditions under which the agents prefer autarkic or integrated capital markets.

JEL classification: Q27; Q56; F2.
Keywords: overlapping generations; poverty trap; capital market integration; environmental quality.

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*The authors are grateful to Gianluigi Vernasca and participants at the EAERE conference in Rome, 2011. This article previously circulated under the title "Is Capital Market Integration a Remedy for the Environmental Poverty Trap?"

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1 Introduction

We often observe situations in which an agent, be it an individual, a region or a country, is unconcerned with a negative externality that it imposes upon other agents. This is the starting point of almost all investigations into environmental economics. The new perspective we bring into this issue is to consider situations beyond the classical inefficiencies where neither the standard policy interventions nor a Coasian bargaining process work.

The situation we have in mind is inspired by global climate change, but easily extends to most situations with a one-sided externality. Evidence from the IPCC (Change 2007) suggests that some agents are heavily impacted by the adverse effects of a temperature rise, while others are almost unaffected. The fact that some countries are expected to be harmed little by climate change may, at least partly, explain their lack of interest to join international climate change agreements or to undertake climate action (Finus 2003). Moreover, the heavily affected agents are typically nations with low per capita income. Hence, they face difficulties in allocating funds to mitigate emissions or to adapt to environmental change without compromising economic growth.

A Coasian perspective suggests that nations with high impacts should pay other nations to release less emissions. In the present context this could imply transfer payments from the poor to the rich. To defuse this fairness issue, we assume that the marginal abatement costs are constant and do not differ among agents. Hence there are no efficiency gains from international coordination. However, poor countries can stipulate economic growth by opening up their capital market to foreigners. The resulting growth in GDP frees up resources to be spent on mitigation and adaptation activities. This is the key point we discuss in our paper.

There are several questions that naturally arise from the setting described above. How do agents act when faced with a one-sided externality? What, if any, are the dynamic consequences of this for consumption, economic growth, and environmental quality? How could potential problems be avoided if policy makers cannot follow the standard toolbox, comprising taxes, subsidies, command-control? In this regard, a first objective of this article is to present and discuss a dynamic framework in which one agent imposes a one-sided externality upon another one and study implications for economic growth and the environment.

Our modeling approach borrows heavily from John and Pecchenino's (1994) model and extends it to a two-agent version. In particular, we assume that, while both agents affect environmental quality, only one of them suffers from its deterioration or gains from its improvement since it is local to him alone. Hence, this approach is consistent with empirically-relevant issues such as upstream polluter and downstream pollutee, or climate change where only one agent is expected to be affected. It is precisely this asymmetry which drives our results, which be believe to be novel to the literature.

We find that multiple steady states can result from this asymmetry. In particular, the agent who is affected by the externality may get stuck in a situation where fighting the externality absorbs most of his savings such that no funds are left for capital accumulation. We call this the environmental poverty trap. We show that, even though both agents hold the same technologies and primary factor endowments, if the agent who is affected by the externality is sufficiently poor in terms of initial capital endowments, then he may be stuck in this trap. We provide conditions for the existence of, and local convergence to, the equilibria, as well as a condition for the global convergence to the poverty trap.

Economists tend to assume the existence of a social planner or international agency which may intervene with the standard toolbox of taxes, subsidies, quotas or command-and-control. The first-best solution to treating an externality like this is then easily calculated and generally well-known. However, if one places this model more deeply into an international setting of regions or countries, it is unlikely that agents would be willing to be controlled by a planner unless it is in their mutual benefit. In other words, we do not believe in such an agency for the situation described above. The standard forms of economic interventions named above are unlikely to be beneficial to the non-affected agent in our setting.[1]

Thus, our second contribution is to advocate a novel approach to at least partly address this externality if policy interventions in form of taxes or subsidies as well as command-and-control methods are impossible. We investigate whether capital market integration alleviates the environmental poverty trap. It turns out that capital market integration will eliminate the poverty trap. We show that this has a positive effect on the environment while the effect on welfare is ambiguous. Specifically, we find that a negative welfare effect occurs for a large and reasonable set of parameters. In particular, we show that poor and small agents fare better with integrated capital markets while rich agents, or those agents able to sufficiently impact their own environmental quality, should not integrate capital markets for environmental reasons alone.

Our article relates to the established literature as follows.[2] In Copeland and Taylor (1994), trade increases welfare if the environment is a local public good, although scale, composition and technique effects add up to higher pollution. Their extension to a pure public good (Copeland and Taylor 1995) yields less stringent conclusions. We assume the environment to be a local public good as in Copeland and Taylor (1994), but take a

[1] Nevertheless, it may be sometimes worthwhile for the affected agent to subsidize the other agent to reduce his impact on the externality, see e.g. (Hoel and Schneider 1997).

[2] Our modeling structure partially relates to the environmental economics literature dealing with closed-loop differential games. Mäler and de Zeeuws (1998) study a differential game of the acid rain problem. They consider N regions that minimize the cost of emission reduction, where emission reductions help to reduce the acid rain problem and thereby the damage from acid rain. Fernandez (2002) builds upon Mäler and de Zeeuws (1998) and studies a two-region differential game of managing waster quality in a border waterway under trade liberalization and no trade. The advantage of our approach is that, by setting our model within the standard OLG framework, we can analyze consumption, savings and abatement decisions as well as allow for a study of income and changes in interest rates. The changes in the interest rates prove pivotal for the results of this article. This approach then allows us to place our results in close comparison to single-agent models like John and Pecchenino (1994).

dynamic approach.[3] Our results cannot be fully compared to those in Copeland and Taylor (1994), as we study capital market integration and not trade in goods. However, our result is that the environment always improves from capital market integration, while welfare in the polluted region nevertheless may decrease. The difference in results arises since in our case capital market integration links the returns to capital, while in Copeland and Taylor trade induces the South to produce with dirtier technologies.

There are other contributions to the environmental economics, overlapping generations literature that derive multiple steady states.[4] Prieur (2009) gives conditions under which a zero-maintenance equilibrium may arise which may lead to multiple steady states. In a recent contribution, Bella (2013) has shown that a poverty trap may occur in an endogenous growth model with environmental quality. The multiple steady states in these articles are caused by specific assumptions, like non-linearities or conditions on the utility function. In our case, the environmental poverty trap is a result of an international externality. In this respect, our model is close to John and Pecchenino (2002). They analyze the use of transfers for a cooperative and non-cooperative, short-run and long-run solution to a two-country overlapping generation model. The main differences between their and our model is that they treat environmental quality as a flow, and that they assume the same utility function for both countries. In contrast, in our model environmental quality is a stock, and we have an asymmetry in the utility function - while one country is concerned with environmental quality, the other is not. This leads to distinct conclusions, namely to our environmental poverty trap and the focus on capital market integration. Furthermore, our modeling approach allows us to obtain explicit results.

This article is structured as follows. Section 2 introduces the theoretical model. Section

[3] The predominant approach in the literature is static, see e.g. Rauscher (1991), Chichilnisky (1994), Copeland and Taylor (1994), (1995)), as examples. For overviews we refer the reader to Esty (2001), Copeland and Taylor (2004) as well as Jayadevappa and Chatre (2000).

[4] In a recent survey, Azariadis (2011) describes what further mechanisms - apart from environmental ones - may give rise to poverty traps.

3 studies the model without international capital mobility. In section 4 we analyze the model by allowing for free trade through integrated capital markets. We derive the changes implied by the move from autarky to international capital markets in section 5. Finally, section 6 concludes.

2 The Model

The model extends John and Pecchenino's overlapping generations model to a two-region perspective. We assume one region, called *Home*, to benefit from a public good. This public good is subject to a negative externality arising from pollution at *Home* and from the other region, called *Abroad*. Pollution is a by-product of consumption in both regions. Contrary to *Home*, region *Abroad* does not benefit from the public good. We, thus, deal with a directed cross-border externality imposed upon one region only. The dynamics in this model arise through capital accumulation and the effects of consumption on environmental quality. In this sense, we elaborate on the model by John and Pecchenino (1994), which will allow a direct comparison. We explicitly restrict the analysis to equal levels of total factor productivity as well as full employment, perfect information and perfect capital markets. Our intention is to show that, even though we assume everything else equal, there are already novel results from assuming a one-sided externality.

Environmental quality

Environmental quality Q_t deteriorates from emissions that come from consumption, c_t, and improves through abatement, A_t, with a one–period time lag. Time is taken discrete, with $t = 0$ as initial period. Variables applying to Abroad are denoted with tilde. Hence

$$Q_{t+1} = Q_t - \beta(c_t + \tilde{c}_t) + \gamma(A_t + \tilde{A}_t), \tag{1}$$

where $\beta > 0$ denotes emissions per unit consumption and $\gamma > 0$ the effectiveness of abatement. We assume $Q_0 > 0$, $c_t \geq 0$, $A_t \geq 0$, and $Q_t \geq 0$ for all t. Unlike John and Pecchenino (1994), environmental quality does not return to its natural level in case there is no human interference. This assumption implies that every time that pollution or abatement efforts change they induce a new steady state in environmental quality. For example, once the forest on a slope is cut the earth gets washed off and this leads to changes that nature itself cannot reverse. Similarly, a climate-induced sea level rise shrinks the habitable surface with nature not being able to restore itself. Only very costly abatement efforts would be able to restore the original state.[5]

The representative firm

In each region there is a representative firm that maximizes profits Π_t subject to capital K_t and labor L_t under perfect competition. We normalize the output price to unity and assume a constant returns to scale production function $Y_t = F(K_t, L_t)$. In intensive form we write $k_t = K_t/L_t$, $y_t = Y_t/L_t$ and $y_t = f(k_t)$. The representative firm maximizes profits $\Pi_t = f(k_t)L_t - R_tK_t - w_tL_t$, taking as given the payments to wages, $w_t = f(k_t) - f'(k_t)k_t$, and the interest payments on capital, $R_t = f'(k_t)$. We assume $f(k_t) = k_t^\alpha$, with $\alpha \in (0, 1)$, hence $w_t = (1-\alpha)k_t^\alpha$ and $R_t = \alpha k_t^{\alpha-1}$. We follow De La Croix and Michel (2002) by assuming that capital depreciates fully during the course of one generation.

Our assumption of technology being the same across regions is only for purposes of simplifying the subsequent analysis. We can readily derive that the results are not qualitatively altered by this assumption. Allowing for different technologies across regions would, however, increase the mathematical complexities substantially.

[5]Other functional forms allow for a natural regeneration rate, as in Jouvet et al. (2005). In addition, the linearity in abatement and pollution implies that unbounded growth in environmental quality is theoretically possible. We discuss the differences in section 3.4.

The agents

We assume two representative agents, one agent associated to region *Home* and the other to region *Abroad*. Agents live for two periods. They are called young in their first period of life and old in their second period. Hence each region is populated by a young and an old generation. We abstract from population growth and assume that both regions are inhabited by the same number of agents,[6] thus $L_t = \tilde{L}_t = 1$. Young agents earn labor income ($w_t \geq 0$) only. This income is spent either on capital formation ($s_t \geq 0$) or abatement ($A_t \geq 0$); there is no consumption while young.[7] Old agents spend their returns on saving completely on consumption.

The main assumption in this article, which also drives the subsequent results, is that agents Abroad do not value environmental quality, while those at Home do. The utility function of the representative agent at Home is $V = \theta \log(c_{t+1}) + \log(Q_{t+1})$, while the one from the agent Abroad is given by $\tilde{V} = \theta \log(\tilde{c}_{t+1})$. Clearly, there are many examples for this type of asymmetry. For example, assume a river runs through two countries, where the downstream country is affected by the pollution from both countries because it takes its drinking water from the river. Another example would be smaller island states and sea level rise. Yet another example would be simply cultural differences in personal attitudes. While one can make up a multitude of examples for an international externality where only one country is affected or perceives an issue to be important for utility, our main incentive here is to drive the asymmetry between the two countries to an extreme and study the implications.

[6]This assumption of equal size helps us in avoiding the need to track the effect of the population when capital markets are integrated. Apart from simplifying the analytics it also constrains the focus on technology and per capita effects.

[7]This modeling approach is abstracting from a potential coordination between young and old generations. This is precisely the reason for using an overlapping generation structure. The idea is that each generation imposes an externality on the next (in our model also on another region) and the question is how this externality affects the intergenerational decision-taking process in the absence of a planner. Another way in which this could be interpreted is that abatement is a lump-sum tax on wages of the young only and the political decision-taking process is being driven by the young generation.

Consumption of the old at Home receives a multiplicative factor $\theta > 0$, which indicates the relative weight of consumption versus the environment in the utility function. Young agents allocate resources to maximize welfare. They are not altruistic with respect to future generations or people living abroad. Both players face income constraints, given by $w_t = s_t + A_t$ and $R_{t+1}s_t = c_{t+1}$ for agents at Home, and $\tilde{w}_t = \tilde{s}_t + \tilde{A}_t$ and $\tilde{R}_{t+1}\tilde{s}_t = \tilde{c}_{t+1}$ for agents Abroad.

We assume that both agents play a Nash game, which is defined as follows. In this 2-person game the agent at Home chooses $s_t \in [0, w_t]$ which implicitly determines A_t, and agent Abroad chooses $\tilde{s}_t \in [0, \tilde{w}_t]$ which implicitly determines $\tilde{A}_t$. They have their pure strategy payoff functions V and $\tilde{V}$ that give utility $V = V(s_t, \tilde{s}_t, Q(s_t, \tilde{s}_t))$ and $\tilde{V} = V(s_t, \tilde{s}_t)$. We write the game in normal form as $\Gamma = [2, s_t, \tilde{s}_t, V(s_t, \tilde{s}_t, Q_t), V(s_t, \tilde{s}_t, Q_t)]$.

Definition 1 *A strategy profile* $\boldsymbol{s} = (s_t, \tilde{s}_t)$ *is a Nash-equilibrium of game* $\Gamma = [2, s_t, \tilde{s}_t, V(s_t, \tilde{s}_t, Q_t), V(s_t, \tilde{s}_t$ *if* $V(s_t, \tilde{s}_t, Q(s_t)) \geq V(s_t', \tilde{s}_t, Q(s_t))$ *and* $\tilde{V} = V(s_t, \tilde{s}_t') \geq \tilde{V} = V(s_t, \tilde{s}_t)$*, for all* $s_t' \in [0, w]$ *and* $\tilde{s}_t' \in [0, \tilde{w}]$.

As agents Abroad are not affected by environmental degradation, it is straight-forward to derive that their optimal abatement expenditure is zero. All their income is spent on investment while young and consequently on consumption when old. This suggests that we are dealing with a strict externality[8] where each agent is following his own rational interests. As a consequence, their consumption is defined by

$$\tilde{c}_{t+1} = \tilde{R}_{t+1}\tilde{w}_t \tag{2}$$

Agents Abroad simply save all their income and consume their savings as well as the return on those savings when old. As a consequence, they impose a scale effect on the

[8] This is a rather useful assumption since we wish to study how the effect of this pure externality might be changed if we study international capital markets as an additional link between the two regions. In addition, it is rather often a realistic assumption.

environment.

The representative agent at Home solves thus an open-loop maximization problem where he takes the consumption and abatement efforts of Abroad as given. He solves

$$\max_{\{A_t\}} \; \theta \log(c_{t+1}) + \log(Q_{t+1}) \qquad \text{subject to} \tag{3}$$

$$w_t - A_t = s_t, \tag{4}$$

$$R_{t+1} s_t = c_{t+1}, \tag{5}$$

$$Q_{t+1} = Q_t - \beta(c_t + \tilde{c}_t) + \gamma(A_t + \tilde{A}_t). \tag{6}$$

The maximization problem for Home can be written as

$$\max_{\{s_t\}} \; \theta \log\big(R_{t+1} s_t\big) + \log\big(Q_t - \beta(c_t + \tilde{c}_t) + \gamma(w_t - s_t + \tilde{A}_t)\big). \tag{7}$$

We now show that the game has a unique Nash equilibrium. The first-order condition gives

$$c_{t+1} = \frac{\theta}{\gamma} R_{t+1} Q_{t+1}, \tag{8}$$

$$s_t = \frac{\theta}{\gamma} Q_{t+1}, \tag{9}$$

$$A_t = w_t - \frac{\theta}{\gamma} Q_{t+1}. \tag{10}$$

Therefore, savings of the representative young agent at Home are proportional to expected environmental quality when old. The higher the expected future environmental quality the lower will be the abatement effort. If agents strongly value consumption over environmental quality (high θ), then this increases their savings and diminishes their abatement efforts. The larger is the effectiveness of abatement the higher will be optimal abatement since it is relatively cheaper to abate than to consume and thereby to increase utility.

Increasing income of agents at Home has two effects on the environment. On the one hand, higher income allows agents to save more, thereby increase future consumption, and consequently also pollution. This is a type of scale effect. On the other hand, agents may now direct more money into abatement, they 'green' consumption by improving the pollution-to-income ratio.

We call the equilibrium resulting from the assumptions and conditions above the temporal equilibrium. It is defined by the following definition.

Definition 2 *The* temporal equilibrium *consists of the allocations* $\{w_t, \tilde{w}_t, R_t, \tilde{R}_t, k_t, \tilde{k}_t, c_t, \tilde{c}_t, s_t, A_t, Q_t\}$, *where at every* $t = 0, 1, 2, ...$, *firms maximize profits; labor and good markets clear; agents at Home maximize* (3) *subject to* (4), (5) *and* (6); *agents Abroad have an optimal consumption defined by* (2); *net profits get distributed to the capital owners such that* $R_t = f'(k_t)$ *and* $\tilde{R}_t = f'(\tilde{k}_t)$.

3 Autarkic capital markets

We start by assuming no interactions between both regions other than the environmental externality. We call this autarkic capital markets. Then regional savings from the current generation give the regional capital stock of the next generation, $k_{t+1} = s_t$, and $\tilde{k}_{t+1} = \tilde{s}_t$. Based on the optimal allocations at the temporal equilibrium, we can derive the dynamic system that characterizes the autarkic intertemporal equilibrium. This system is given by the equations

$$\tilde{k}_{t+1} = (1-\alpha)\tilde{k}_t^{\alpha}, \tag{11}$$

$$k_{t+1} = \frac{\theta}{\gamma(1+\theta)}\left[Q_t - \beta\frac{\theta}{\gamma}\alpha k_t^{\alpha-1}Q_t - \alpha\beta\tilde{k}_t^{\alpha} + \gamma(1-\alpha)k_t^{\alpha}\right], \tag{12}$$

$$Q_{t+1} = \frac{1}{1+\theta}\left[Q_t - \beta\frac{\theta}{\gamma}\alpha k_t^{\alpha-1}Q_t - \alpha\beta\tilde{k}_t^{\alpha} + \gamma(1-\alpha)k_t^{\alpha}\right]. \tag{13}$$

We can now define the intertemporal equilibrium in the autarky case.

Definition 3 *Given the capital stocks k_0 and $\tilde{k}_0$ and the environmental quality Q_0, an* autarkic intertemporal equilibrium *is a temporal equilibrium that in addition satisfies, for all $t \geq 0$, the autarkic capital accumulation conditions $k_{t+1} = s_t$ and $\tilde{k}_{t+1} = \tilde{s}_t$.*

For completeness we assume that the dynamic system applies only for $k_t > 0$ and $Q_t > 0$. For solutions that lead to $k_{t+1} \leq 0$ and $Q_{t+1} \leq 0$ we replace the equations (12) and (13) by $k_{t+\tau} = 0$ and $Q_{t+\tau} = 0$, $\forall \tau \geq 1$.

3.1 Existence of multiple steady states

Equation (11), (12) and (13) become at steady state

$$\Gamma(Q) \equiv [(1-\alpha)\gamma - \alpha\beta]\left(\frac{\theta}{\gamma}Q\right)^{\alpha} - \theta Q - \alpha\beta(1-\alpha)^{\frac{\alpha}{1-\alpha}} = 0. \tag{14}$$

We study (14) to analyze the existence of a steady state. We shall introduce the following assumption.

A 1

$$(1-\alpha)^{\frac{\alpha}{\alpha-1}}(\alpha^{\frac{2\alpha-1}{1-\alpha}} - \alpha^{\frac{\alpha}{1-\alpha}})\big((1-\alpha) - \alpha\beta/\gamma\big)^{\frac{1}{1-\alpha}} > \beta/\gamma. \tag{15}$$

This assumption will prove useful for the existence of a steady state. It implies, for the reasonable level of $\alpha = 0.3$, that $\beta/\gamma < 0.625$.

Proposition 1 *Under assumption A1 there exist two steady states to the dynamic system* (11), (12) *and* (13), *given by $\{k_l, \tilde{k}, Q_l\}$, $\{k_h, \tilde{k}, Q_h\}$, where $Q_l < Q_h$ and $k_l < k_h$, and one corner state $\{0, \tilde{k}, 0\}$. If A1 holds with equality, then there exists one steady state, $\{\hat{k}, \tilde{k}, \hat{Q}\}$ and one corner state $\{0, \tilde{k}, 0\}$. Otherwise there exists only the corner state $\{0, \tilde{k}, 0\}$.*

All proofs are available in the Appendix.

Note that $(1-\alpha)\gamma > \alpha\beta$ is a straightforward necessary condition for the existence of a steady state. It requires that, if young agents at Home spend all of their income on abatement then they must be able to offset more than the emissions from Home. The necessary and sufficient condition in equation (15) is homogenous of degree zero in β and γ, hence only β/γ matters. This ratio gives the costs of abating the emissions out of one unit consumption in terms of GDP.[9]

We will also focus some attention on the trivial steady state at $\{0, \tilde{k}, 0\}$. For the reasonable level of $\alpha = 0.3$ condition (15) requires that $\beta/\gamma < 0.625$. Thus, if γ is not sufficiently effective compared to β, then no interior steady state for the region Home will exist in the autarkic case. In this case, region Abroad will still grow to its interior steady state given by $\tilde{k}$, while region Home will continue to spend on environmental quality despite decreasing capital stocks.

The question now is whether this trivial steady state is a possibility despite the fact that assumption A1 holds, meaning that all parameter conditions allow for an interior steady state. This we analyze by studying the local dynamics around the steady states, and we also provide some conditions on the global dynamics.

3.2 Dynamics and comparative statics

In the previous section we provided the necessary and sufficient condition (15) for the existence of a steady state. Furthermore, we know that, for a non-degenerate set of parameters, two interior steady states exist and one corner state. We now provide conditions for the stability of the steady states which give an analytical understanding of the *environmental poverty trap*.

Proposition 2 *Considering the dynamic equations* (11), (12) *and* (13) *that describe the*

[9] A change in β/γ could be understood as either cleaner consumption (e.g. less energy consuming) or better abatement technology.

autarkic intertemporal equilibrium, then the low steady state $\{\tilde{k}, k_l, Q_l\}$ *is unstable and the high steady state* $\{\tilde{k}, k_h, Q_h\}$ *is stable. Thus, for all* $k_t < k_l$ *the system converges to the corner state, while for all* $k_t > k_l$ *the system converges to the high steady state.*

To add some precision, there exists a Saddle-node bifurcation when assumption A1 holds with equality. Excluding the corner state, then for a sufficiently small β/γ ratio we have two steady states, one stable one and one unstable one. For an increasing β/γ ratio these steady states approach each other and collide when assumption A1 is satisfied with equality. If A1 does not hold, which would be the case for a too large β/γ ratio, then no steady state (apart from the corner state) exist.

The proposition above supports our argument for the existence of the *environmental poverty trap* for $k_0 \in (0, k_l)$. If Home is initially endowed with a per capita capital stock less than k_l, then fighting pollution will freeze resources necessary to maintain its current per capita capital stock. Home will experience negative growth rates ending up in an economic collapse.

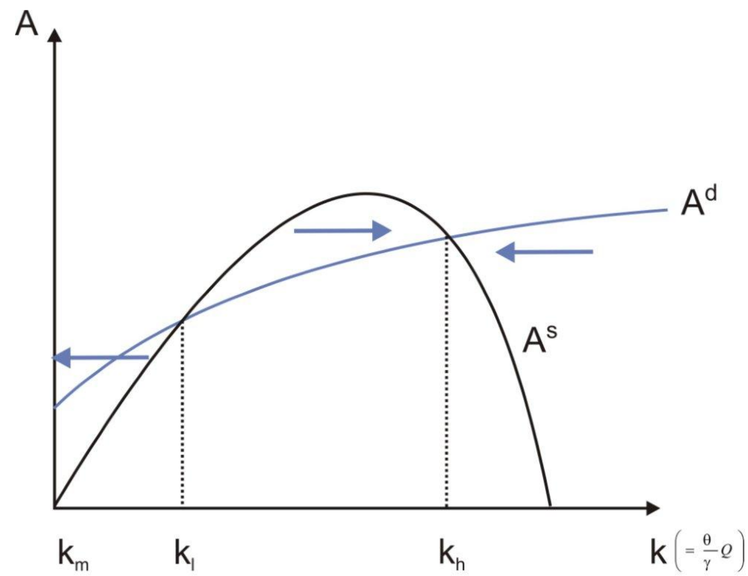
The fact that overlapping generation models with environmental quality can generate multiple equilibria is not new to the literature. For example, John and Pecchenino (1994), focusing on one region, suggest that multiple equilibria may occur if the production function takes a different form than Cobb-Douglas, or if one allows for increasing returns. In our case multiple equilibria are due to the international externality, and occur despite focusing on the Cobb-Douglas case.[10]

Figure 1 gives the intuition for the dynamics in an k–A diagram.[11] Since $w = (1-\alpha)k^\alpha$ as well as $w - A = k$, we define $A^s \equiv (1-\alpha)k^\alpha - k$ as steady state abatement supply. Abatement is used to offset emissions from Abroad and Home, hence equation (1) must

[10]Some articles in the dynamic games literature are related to our framework here. Mäler and De Zeeuw (1998) or Fernandez (2002) miss the link through capital accumulation and, by not being able to explicitly study multi-agent control problems with concave production functions, cannot obtain our result of the environmental poverty trap.

[11]A Q–A diagram would have the same shape because Q is linear in k (see (9)).

Figure 1: Capital - Abatement dynamics in the autarky case



hold with $\gamma A = \beta\,(c + \tilde{c})$. This yields $A^d \equiv \alpha(\beta/\gamma)k^\alpha + (\beta/\gamma)\tilde{c}$, where $\tilde{c} \equiv \alpha(1-\alpha)^{\frac{\alpha}{1-\alpha}}$. We dub this term A^d as it gives the steady state demand for abatement given the capital stock at home. Figure 1 shows equilibrium k as the intersection of A^s and A^d. The graph also illustrates that k_l is unstable. For a small perturbation of k to the left of k_l, abatement does not suffice to keep up with emissions, hence the economy at Home will collapse. For a small perturbation to the right of k_l, abatement exceeds emissions, hence the economy at Home will improve its environmental quality and increase its capital stock. Beyond k_h, abatement falls short of emissions again, pulling the economy back to k_h.

The results of the comparative statics are also easily recognized in Figure 1 and will be proved below. Note that Home's steady state capital stock depends only on the β–to–γ ratio while environmental quality is sensitive to the level of γ. Furthermore, the extent of the poverty trap depends on consumption Abroad. The risk to fall into the poverty trap increases with Abroad approaching its steady state consumption. The stable steady state $\{k_h, \tilde{k}, Q_h\}$ is sensitive with respect to key parameters in an expected way. Assume, for example, a greening of consumption - i.e. emissions per unit of consumption,

β, decrease.[12] As a consequence, both environmental quality and capital at Home increases. Moreover, the risk to fall into the poverty trap decreases. An improvement in abatement technology, γ, works exactly in the same direction. An increase in the willingness to pay for environmental quality, θ, however, leaves the capital stocks unaffected, while potential steady state environmental quality improves. Summing up, we get[13]

$$\frac{dQ_h}{d\beta} < 0, \quad \frac{dQ_h}{d\gamma} > 0, \frac{dQ_h}{d\theta} < 0.$$

A policy maker aiming for technological innovations through lower β or higher γ therefore not only reduces the risk of falling into the poverty trap, it increases environmental quality and capital. Whereas fostering environmental consciousness (higher θ) has no effect on the poverty trap and capital, it improves environmental quality.

[12] To illustrate the greening of consumption, assume that houses are endowed with solar cells instead of obtaining electricity from a coal fired plant.

[13] The proofs are as follows. Using the implicit function theorem at $\Gamma(Q) = 0$ gives

$$\frac{dQ}{d\beta} = \alpha \frac{(\theta/\gamma Q)^{\alpha} + (1-\alpha)^{\frac{\alpha}{1-\alpha}}}{\alpha\big((1-\alpha)\gamma - \alpha\beta\big)(\theta/\gamma)^{\alpha} Q^{\alpha-1} - \theta}.$$

We require $\alpha\big((1-\alpha)\gamma - \alpha\beta\big)(\theta/\gamma)^{\alpha} Q^{\alpha-1} < \theta$ for $\frac{dQ}{d\beta} < 0$. Plugging the explicit solution for $\hat{Q}$ into this condition and rearranging gives $\left(\frac{\hat{Q}}{Q}\right)^{1-\alpha} < 1$. We know that $Q_l < \hat{Q} < Q_h$, hence the sign of the comparative statics prevails.

Using the implicit function theorem at $\Gamma(Q) = 0$ gives

$$\frac{dQ}{d\gamma} = -\frac{(1-\alpha)2 + \alpha 2\beta/\gamma}{\alpha\big((1-\alpha)\gamma - \alpha\beta\big)(\theta/\gamma)^{\alpha} Q^{\alpha-1} - \theta}(\theta/\gamma Q)^{\alpha}.$$

We require $\alpha\big((1-\alpha)\gamma - \alpha\beta\big)(\theta/\gamma)^{\alpha} Q^{\alpha-1} > \theta$ for $\frac{dQ}{d\gamma} < 0$. The same argument as above applies.

Using the implicit function theorem at $\Gamma(Q) = 0$ gives

$$\Big(Q - \alpha\big((1-\alpha)\gamma - \alpha\beta\big)(\theta/\gamma)^{\alpha}\theta^{-1}Q^{\alpha}\Big)d\theta - \Big(\alpha\big((1-\alpha)\gamma - \alpha\beta\big)(\theta/\gamma)^{\alpha}Q^{\alpha-1} - \theta\Big)dQ = 0.$$

Simplifying gives $\frac{dQ}{d\theta} = -Q/\theta < 0$.

3.3 A more global result

One issue with proposition 2 is that the analysis of the stability of the steady states holds only locally. Thus, the results only apply in sufficiently close proximity to the steady states. Indeed, they carry forward as long as capital Abroad, $\tilde{k}_t$, is sufficiently close to its steady state. However, proposition 2 may not hold any longer for $\tilde{k}$ sufficiently far away from its steady state. Indeed, proposition 2 may be weakened in that case. For this we reduce the dynamic system (11), (12) and (13) to one equation. It will be non-autonomous and, thus, according to the Hartman-Grobman theorem, linearization will not be possible.

We first solve the difference equation $\tilde{k}_{t+1} = (1-\alpha)\tilde{k}^{\alpha}$. The explicit solution for this difference equation can be written as

$$\tilde{k}_t = (1-\alpha)^{\frac{1}{\alpha}\left(\sum_{\tau=0}^{t}\frac{(1)_\tau \alpha^\tau}{\tau!}-1\right)}\tilde{k}_0^{\alpha^t} \equiv g(\alpha, \tilde{k}_0, t), \tag{16}$$

where $(1)_\tau$ is the Pochhammer symbol (Abramowitz and Stegun 1972) and denotes the falling factorial $(x)_y = x(x+1)...(x+y-1)$. Using equation (9) and (16) in (12) gives us the equation that characterizes the dynamics of the autarkic intertemporal equilibrium

$$k_{t+1} = \frac{\theta}{\gamma(1+\theta)}\left[\frac{\gamma}{\theta}k_t + \big((1-\alpha)\gamma - \alpha\beta\big)k_t^{\alpha} - \alpha\beta g(\alpha, \tilde{k}_0, t)^{\alpha}\right] \equiv \Psi(k_t, t). \tag{17}$$

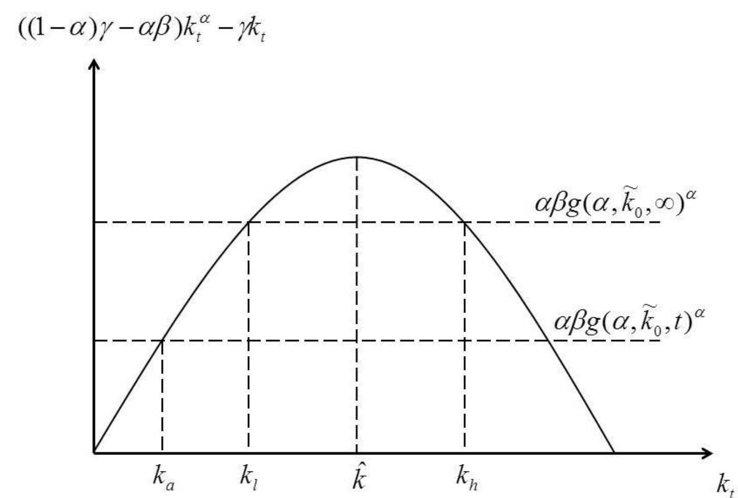
We are now in the position to summarize our results on the global dynamics in the following proposition.

Proposition 3 *Assume $\tilde{k}_t \leq \tilde{k}$, $k_t < k_l$ and A1 holds. Further assuming there $\exists \psi > 0$, such that ψ solves, for $\tau \geq 0$, $\tilde{k}_\tau = \psi k_\tau$. Then $k_{\tau+t} \leq k_\tau$, $\forall t > \tau \geq 0$, if*

$$\left[\frac{((1-\alpha)\gamma - \alpha\beta) - \gamma k_0^{1-\alpha}}{\alpha\beta}\right]^{\frac{1}{\alpha}} < \psi \leq \frac{(1-\alpha)^{\frac{1}{1-\alpha}}}{k_0}.$$

This proposition draws attention to the fact that smaller values of $k_0 < k_l$ may still allow for convergence to the high steady state. This happens if $\tilde{k}_t$ is sufficiently smaller than k_t. Figure 2 provides a graphical illustration of our result above. If $\alpha\beta g(\alpha, \tilde{k}_0, t)^\alpha$ is smaller than $((1-\alpha)\gamma - \alpha\beta)k_t^\alpha - \gamma k_t$, then the capital stock and environmental quality at Home grow. Our study of the local dynamics in the previous section only allowed us to conclude that, for $k_t > k_l$, region Home would be out of the environmental poverty trap. However, if region Abroad starts sufficiently far away from its own steady state, such that $k_t > k_a$ as depicted in Figure 2, then capital stock and environmental quality at Home may grow. Whether they continues to grow until $\{k_h, Q_h\}$, or even temporarily further, depends on how fast capital Abroad grows. If it is the case that it grows sufficiently slowly such that $k_t > k_l$, then region Home may leave the poverty trap despite the fact that its initial capital stock was below k_l.

Figure 2: A graphical illustration of the global dynamics



To summarize, the autarky case has shown that if one region obtains utility from a public good that can be both improved and harmed by all regions (in our case two), then the region subject to the local public good may find itself in a poverty trap. The trap arises since this region must spend increasing amounts of its capital returns on improving the public good.

3.4 Some remarks and extensions

Clearly, the results above are based on a reduced-form model. We assumed an absence of technical progress and assumed both production functions are the same, we have a strictly one-sided externality, we assumed that there is no self-regeneration mechanism in environmental quality, and we assumed that agents can only look ahead for one period. We now discuss how the results of the model may be augmented if one were to take these extensions into account.

3.4.1 The role of technical change

In the previous analysis we assumed an absence of technical change, and furthermore assumed that both production technologies are the same. We now relax these assumptions and take a look at how our previous results are augmented. In particular, let us assume that the production function at Home takes the form $f(k_t) = A_t k_t^{\alpha}$, while the production function Abroad is of form $\tilde{f}(\tilde{k}_t) = \tilde{A}_t \tilde{k}_t^{\alpha}$. This then augments the dynamic system of the autarkic intertemporal equilibrium defined by equations (11) to (13) and we now obtain

$$\begin{aligned} \tilde{k}_{t+1} &= (1-\alpha)\tilde{A}_t \tilde{k}_t^{\alpha}, && (18) \\ k_{t+1} &= \frac{\theta}{\gamma} Q_{t+1}, && (19) \\ Q_{t+1} &= \frac{1}{1+\theta}\Big[Q_t + \big(\gamma(1-\alpha) - \alpha\beta\big) A_t k_t^{\alpha} - \alpha\beta \tilde{A}_t \tilde{k}_t^{\alpha}\Big]. && (20) \end{aligned}$$

Then $Q_{t+1} \geq Q_t$ if

$$Q_t \leq \frac{1}{\theta}\Big[\big(\gamma(1-\alpha) - \alpha\beta\big) A_t k_t^{\alpha} - \alpha\beta \tilde{A}_t \tilde{k}_t^{\alpha}\Big]. \quad (21)$$

It is now clear that our previous results carry forward if the technology level Abroad is not too low relative to the one at Home. More specifically, assume that $A_t = A$ and $\tilde{A}_t = \tilde{A}$, then Proposition 3 can easily be re-written to take this productivity difference into account. In this case we find that if Abroad has a sufficiently high total factor productivity (which is currently constrained to 1), then the consumption externality of Abroad would be too large and abatement from Home could not compensate for this externality.

Instead, let us assume that productivity changes over time, then again our previous results carry forward if technical change Abroad is not to slow relative to the one at Home. Assume, for example, that there are perfect spillovers in technical change, such that $A_t = \tilde{A}_t$. In this case technical change provides two scale effects. One, it will scale up the right-hand side of the inequality constraint (21), so that $Q_{t+1} \geq Q_t$ is more likely to hold. Two, it will scale up $\tilde{k}_t$ faster, so that the difference $\big(\gamma(1-\alpha) - \alpha\beta\big)A_t k_t^\alpha - \alpha\beta\tilde{A}_t\tilde{k}_t^\alpha$ is more likely to become negative, which will more easily induce the environmental poverty trap.

What we view as important is that, despite both agents having access to the same technology, we obtain a poverty trap. This places a stronger emphasis on the international spillover, which in turn is inducing the poverty trap.

3.4.2 Both agents care about the environment

The result that k_t converges to zero if $k_0 \in (0, k_l)$ partly hinges on our assumption on the preferences of Abroad. Since Abroad is not affected by this externality, then if the externality they impose upon Home is sufficiently large, this will lead to economic collapse. If Abroad would also be impacted by the externality, then this would induce Abroad to have positive levels of abatement. Vernasca (2005) has a modeling approach along these lines. While that article utilizes a more general utility function and allows for environmental quality for both agents, it does not allow for region-dependent capital stocks. As a result, there is no environmental poverty trap in the paper and no policy discussion of the effect of capital market integration. Furthermore, there are no results on global stability in Vernasca (2005). With the externality in the utility function of Abroad there will always be an interior solution to environmental quality in the first-order condition of Abroad. Hence, Abroad and Home would invest in abatement, and consequently environmental quality will not be driven to zero. Though this may still lead to multiple equilibria, we would not observe the economic collapse. Instead, depending on the weight that the environment had in Abroad's preferences, we would see k_t to converge to low levels instead of zero. Our results would, thus, essentially be unchanged but less drastic.

3.4.3 Adding a self-regeneration in environmental quality

One may wish to know how the result above is augmented if we allow for a natural regeneration rate $m \in (0,1)$. For example, a possible functional form is $Q_{t+1} = m(X - Q_t) + Q_t - \beta(c_t + \tilde{c}_t) + \gamma(A_t + \tilde{A}_t)$, taken from Jouvet, Michel and Rotillon (2005), with X denoting the natural state and m the speed at which nature returns to the natural state. In this case, equation (15) takes the form

$$\Gamma(Q) \equiv [(1-\alpha)\gamma - \alpha\beta]\left(\frac{\theta}{\gamma}Q\right)^{\alpha} - \theta Q + m(X - Q) - \alpha\beta(1-\alpha)^{\frac{\alpha}{1-\alpha}} = 0.$$

Clearly, if the natural regeneration is sufficiently slow, or for a low natural level of the environment, we recover our result in Proposition 1. In contrast, for $mX > \alpha\beta(1-\alpha)^{\frac{\alpha}{1-\alpha}}$, the steady state can be unique. Hence, in terms of generality we lose very little by assuming that environmental quality cannot regenerate itself, but instead we gain much in terms of analytical results.

3.4.4 A remark on the planning horizon

One might wonder whether the previous result still holds if one takes into account a longer planning horizon for the Home region. What we shall show is that while a longer planning horizon may reduce the possibility of being in an environmental trap, it may not necessarily avoid it. In the next section we conclude that a fool-proof way to avoid the trap is capital market integration.

Let us assume that the Home planner at time t is also altruistic and takes the impact of the current choices on the next generation's welfare into account. Mathematically, this then leads to the utility function

$$\theta \log\left(c_{t+1}\right) + \log\left(Q_{t+1}\right) + \delta\Big(\theta \log\left(c_{t+2}\right) + \log\left(Q_{t+2}\right)\Big), \tag{22}$$

where $\delta > 0$ is the degree of altruism. For simplicity, we assume that the Home planner still takes capital accumulation as exogenous and thus does not view his impact on the future capital stock. Substituting the constraints and maximizing leads to the optimality condition

$$\frac{\theta}{s_t} = \frac{\gamma}{Q_{t+1}} + \delta\left(\frac{\gamma}{Q_{t+2}} + R_{t+1}\frac{\beta}{Q_{t+2}}\right). \tag{23}$$

Without altruism (i.e. $\delta = 0$), this optimality condition reduces to equation (9). Allowing for altruism (or an extended planning horizon), the Home planner takes into account that his current consumption and abatement decision affect environmental quality at time $t+2$. This is represented by the second term on the right-hand side of equation (23). This term is always positive, and thus leads to lower optimal savings for the planner and consequently higher abatement levels.

While this makes clear that incorporating further time periods into Home's planning horizon reduces the chance that, for a given capital level abroad (i.e. consumption level), Home will end up in the environmental trap, it does not inform us whether Home may be able to avoid the environmental trap altogether by extending the planning horizon. For this we take a backward approach and look at the law of motion for environmental quality, given by

$$Q_{t+1} = Q_t - \beta(c_t + \tilde{c}_t) + \gamma(A_t + \tilde{A}_t). \qquad (24)$$

We know that region Abroad fully consumes its wages, while the maximum that region Home may spend on abatement is also given by its wages. However, in this case there would be no capital accumulation in region Home. Thus, region Home has to choose the highest level of abatement such that capital is still constant. Based on the constraint $w_t = s_t + A_t$, with $s_t = k_{t+1}$, $w_t = (1-\alpha)k_t^\alpha$, this is given by $A = (1-\alpha)^2\big(\alpha(1-\alpha)\big)^{\frac{\alpha}{1-\alpha}}$.[14] Assuming thus that abatement is at its potential maximum, then in this case environmental quality accumulates according to $Q_{t+1} = Q_t - \beta(1-\alpha)\tilde{k}_t^\alpha + \gamma(1-\alpha)^2\big(\alpha(1-\alpha)\big)^{\frac{\alpha}{1-\alpha}}$. Consequently, a trap in environmental quality can only be avoided if the maximum of abatement can exceed the maximum of pollution from region Abroad, which is given by $\beta(1-\alpha)^{\frac{1}{1-\alpha}}$. This leads to the condition

$$Q_{t+1} = Q_t - \beta(1-\alpha)^{\frac{1}{1-\alpha}} + \gamma(1-\alpha)^2\big(\alpha(1-\alpha)\big)^{\frac{\alpha}{1-\alpha}}. \qquad (25)$$

As a result, region Home can only avoid the environmental trap if

$$(1-\alpha)^2\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)^{\frac{2\alpha-1}{1-\alpha}} \geq \beta/\gamma.$$

The left-hand side is a concave, monotonically decreasing function which goes from 1 to zero. At the standard value of $\alpha = 0.3$ the left-hand side is equal to 0.359 implying that the abatement effectiveness must be roughly three times larger than the emissions per unit of

[14] This comes from assuming that capital is kept constant and abatement is at the maximum. Thus we have $(1-\alpha)k^\alpha - k = A$, and the maximum of $(1-\alpha)k^\alpha - k$ arises when $k = \big(\alpha(1-\alpha)\big)^{\frac{1}{1-\alpha}}$.

consumption in order for the Home region's abatement to able to compensate for Abroad's externality.

To conclude, even when disregarding any optimality conditions and allowing for maximum sustainable abatement, it turns out that there are limits to region's Home ability to avoid the environmental trap. In what follows we suggest one way that does not depend on long-term planning (or other standard economic tools like taxation) to help region Home avoid the trap, namely capital market integration.

4 Integrated capital markets

We now consider the globalization case. Assume integrated capital markets, where free capital mobility between the regions prevails. This implies

$$k_{t+1} + \tilde{k}_{t+1} = s_t + \tilde{s}_t, \text{ and} \tag{26}$$

$$R_{t+1} = \tilde{R}_{t+1}. \tag{27}$$

As a consequence, in the integrated capital markets case, global savings make the global capital stock. Free capital mobility implies that the returns to capital in each region must be equal. The equality of returns condition is given by equation (27). We can now define the integrated intertemporal equilibrium.

Definition 4 *Given the capital stocks k_0 and $\tilde{k}_0$ and the environmental quality Q_0, an* integrated intertemporal equilibrium *is a temporal equilibrium that in addition satisfies, for all $t \geq 0$, the integrated capital accumulation condition $k_{t+1} + \tilde{k}_{t+1} = s_t + \tilde{s}_t$ and the equality of returns condition $R_{t+1} = \tilde{R}_{t+1}$, as well as the conditions* (30) *and* (31).

The equality of returns condition implies that $k_{t+1} = \tilde{k}_{t+1}$. Substituting the optimality conditions (9) and (11) we obtain $2k_{t+1} = \frac{\theta}{\gamma} Q_{t+1} + (1-\alpha)k_t^\alpha$, or, equivalently,

$$Q_{t+1} = \frac{\gamma}{\theta}\big(2k_{t+1} - (1-\alpha)k_t^\alpha\big). \tag{28}$$

Using equation (1), solving equation (28) for k_t^α and substituting, we derive

$$Q_{t+1} = \frac{1}{1+\theta}\Big[Q_t + \big((1-\alpha)\gamma - 2\alpha\beta\big)k_t^\alpha\Big]. \tag{29}$$

This implies the dynamic system

$$k_{t+1} = 1/2\Bigg[\frac{\theta}{(1+\theta)\gamma}\Big[Q_t + \big((1-\alpha)\gamma - 2\alpha\beta\big)k_t^\alpha\Big] + (1-\alpha)k_t^\alpha\Bigg], \tag{30}$$

$$Q_{t+1} = \frac{1}{1+\theta}\Big[Q_t + \big((1-\alpha)\gamma - 2\alpha\beta\big)k_t^\alpha\Big]. \tag{31}$$

We label the steady states in this case with subscript i. The explicit solution for Q_i and k_i are

$$Q_i = \frac{((1-\alpha)\gamma - 2\alpha\beta)}{\theta}\left(\frac{(1-\alpha)\gamma - \alpha\beta}{\gamma}\right)^{\frac{\alpha}{1-\alpha}}, \tag{32}$$

$$k_i = \left(\frac{(1-\alpha)\gamma - \alpha\beta}{\gamma}\right)^{\frac{1}{1-\alpha}}. \tag{33}$$

We introduce the following assumption now.

A 2 $(1-\alpha)\gamma > 2\alpha\beta$.

This assumption is important for the existence of a steady state in the integrated intertemporal equilibrium.

Proposition 4 *Under assumption A2 there exists a unique steady state to the dynamic system* (30) *and* (31) *that characterizes the integrated intertemporal equilibrium.*

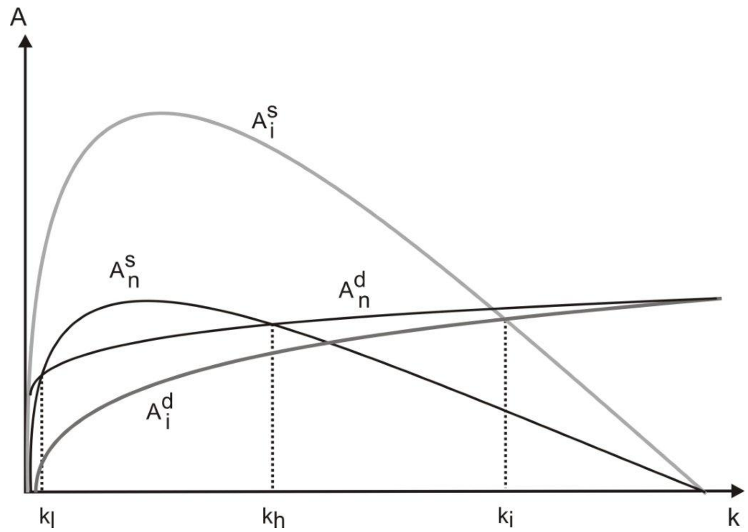
For the proof it is enough to show that a necessary and sufficient condition for the positivity of Q_i and k_i is $(1-\alpha)\gamma > 2\alpha\beta$.

We compare the stringency of the condition A1 for the existence of a steady state in the autarky versus the condition A2 in the integrated case. We re-write condition A2 as $\frac{(1-\alpha)}{2\alpha} > \beta/\gamma$. In this case we know a steady state with integrated capital markets exists. Let us now take the maximum possible level of β/γ that, in the limit, would still guarantee us the existence of a steady state in the integrated capital market case. Define this as $(\beta/\gamma)^{\max} \equiv \frac{1-\alpha}{2\alpha}$. Studying condition A1 gives us that an increasing β/γ ratio makes the existence of a steady state in the autarkic case less likely, since the left-hand side is decreasing in β/γ while the right-hand side is increasing in β/γ. This implies that if condition A1 is satisfied for $(\beta/\gamma)^{\max}$, then it will also be satisfied for all $\beta/\gamma \leq (\beta/\gamma)^{\max}$. If condition A1 is not satisfied for $(\beta/\gamma)^{\max}$, then it will be satisfied for a $\beta/\gamma < (\beta/\gamma)^{\max}$ and we can conclude that a steady state in the autarkic case is less likely than in the integrated case (meaning that less β/γ ratios allow for the existence of an autarkic steady state). Thus, substituting $(\beta/\gamma)^{\max} = \frac{1-\alpha}{2\alpha}$ into condition A1 gives an inequality that is only a function of α. We define this function as $\Phi(\alpha) \equiv (\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}) - 2^{\frac{\alpha}{1-\alpha}}$. Studying $\Phi(\alpha)$ gives us that for any $\alpha \in (0,1)$ this function is negative. As a consequence, the existence of a steady state in the autarkic case is less likely than in the integrated capital market case for it imposes a stronger constraint on the parameters.

Proposition 5 *Considering the dynamic equations* (30) *and* (31) *that describe the integrated intertemporal equilibrium, then the unique steady state* $\{k_i, k_i, Q_i,\}$ *is asymptotically stable if assumption A2 holds.*

Figure 3 provides a numerical illustration similar to Figure 1. It shows both cases, integrated and autarkic capital markets. As suggested in Proposition 4, in order to ensure the existence of a steady state we can impose a simpler and weaker constraint on the key parameters than in the autarky case. Young agents at Home now must be able to abate emissions from world consumption, i.e. $(1-\alpha)\gamma > 2\alpha\beta$. Assuming $\alpha = 0.3$ as in most

Figure 3: Capital - Abatement dynamics. Autarkic and integrated capital markets



numerical studies, we require $\beta/\gamma < 1.16$, such that abatement costs due to one unit of consumption must not exceed 1.16. Recall that this ratio must be less than 0.625 in the autarky case. The graph shows first A^s with $A^s = 2\left((1-\alpha)k^\alpha - k\right)$, and A^d which is given by $\gamma A = \beta(c + \tilde{c})$. Since world consumption is determined by world capital income, we get $A^d = 2\frac{\alpha\beta}{\gamma}k^\alpha$. The intersection of A^d and A^s gives k_i. As in the autarky case, steady state capital and abatement do depend on the β–to–γ ratio but not on the level of these parameters. Given that $(1-\alpha)\gamma > 2\alpha\beta$ holds, we see that abatement supply may exceed abatement demand for any $k < k_i$, while abatement supply will be lower than abatement demand for any $k > k_i$. We thus observe converging dynamics to k_i.

Our general conclusion is that integrated capital markets, by linking the capital stock of the two regions, prevents the environmental poverty trap that exists in the autarkic case. The economic intuition is as follows. In autarky, the currently young generation at Home may heavily invest into abatement at the cost of savings, trading off future GDP for environmental quality. Their decision thus has a negative effect on the following generation, whose labor income depends on their ancestors' savings. Under integrated capital markets, investments from Abroad can substitute for domestic savings, making sure that labor income does not dry up. This result is also robust to virtually any modification in the production function or labor amount, as long as the inputs are complements and integrated capital markets imply that international capital stocks are linked proportionately.

The steady state Q_i varies in the relevant parameters as intuition suggests. We obtain[15]

$$\frac{dQ_i}{d\beta} < 0, \quad \frac{dQ_i}{d\gamma} > 0, \quad \frac{dQ_i}{d\theta} < 0.$$

We conclude that the steady state environmental quality responds to changes in the key parameters in the same way that steady state environmental quality in the autarkic case does.

5 Incentives to integrate capital markets

In this section we compare integrated capital markets and autarkic capital markets. We are especially interested in the effect on welfare from capital market integration, since this represents the long-run incentives for each region to move from autarky to globalized capital markets.

Proposition 6 *Given a steady state in autarky exists, then*

1. $Q_i > Q_h$ *and* $k_i > k_h$,

2. $\tilde{k}_n > \tilde{k}_i$,

[15]The comparative statics with respect to β and θ are trivial. For $\frac{dQ_i}{d\gamma}$ we get after some manipulations

$$\frac{dQ_i}{d\gamma} = \frac{(1-\alpha)2\gamma\left((1-\alpha)\gamma - \alpha\beta\right) + \alpha 2\beta\left((1-\alpha)\gamma - 2\alpha\beta\right)}{(1-\alpha)\gamma\left((1-\alpha)\gamma - \alpha\beta\right)\theta}\left(\frac{(1-\alpha)\gamma - \alpha\beta}{\gamma}\right)^{\frac{\alpha}{1-\alpha}}.$$

This is positive for $(1-\alpha)\gamma > 2\alpha\beta$.

3. $2k_i > \hat{k}_n + k_n$.

In words, capital market integration both increases the environmental quality and the capital stock at Home compared to the autarky case. In contrast, we find that capital Abroad is higher without capital market integration. Finally, the steady state world capital stock in the autarky case is lower than in the integrated capital case.

It is immediate from Proposition 6 that Abroad's savings decline with capital market integration, since $\tilde{k}_i < \tilde{k}$ and $\tilde{s} = (1-\alpha)\tilde{k}^{\alpha}$. Home's savings increase, because $s = \frac{\theta}{\gamma}Q$, and $Q_i > Q_h$. Proposition 6 shows that environmental quality benefits from capital market integration despite the scale effect through increased global capital. This result contrasts with Copeland and Taylor (1994) who find that free trade increases income but increases world pollution if income levels differ between regions. We find that income levels do not play the same role once capital markets are integrated. Instead, integrating capital markets allows to free capital for abatement which then helps to prevent further environmental deterioration. We, furthermore, notice that capital market integration will alleviate the environmental poverty trap that persists in the autarkic case for sufficiently poor regions. The model predicts that the steady state capital stock at Home in autarky is smaller than Abroad, see equation (33). Once capital markets integrate, capital will flow from Abroad to Home. Since the young generations draw income from labor only, the young from Abroad suffer from a capital drain while those at Home gain from the capital inflow. From Home's perspective, this is a positive income effect for the young. When old, however, generations live on capital income only. Now we get the reverse income effect. Agents abroad benefit from higher rates of returns while those at home suffer from lower interest rates relative to the returns in autarky. Agents at home may face an additional adverse effect from opening up capital markets. Supposing that agents abroad gain from capital market integration, they increase consumption hence pollution.

We have shown that the world gets richer in terms of GDP and environmental quality. However, this does not necessarily imply that both regions can benefit from capital market integration because of distributional effects on regional welfare. The following result sheds some light on the distributional effects.

Result 1 *We find that indirect steady state utility Abroad always increases following capital market integration, while indirect utility at Home may increase or decrease.*

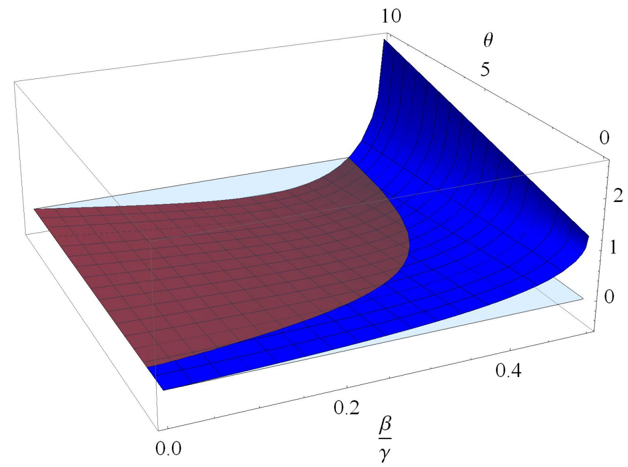
For Abroad it is possible to show this result analytically, while for Home we resort to a numerical illustration that nevertheless covers all reasonable parameter ranges.

Abroad's indirectly utility is a monotonic transformation of its consumption. Hence, it sufficies to compare Abroad's consumption in autarky, $\tilde{c}_n$, and integrated markets, $\tilde{c}_i$. We derive $\tilde{c}_n = \alpha(1-\alpha)^{\frac{\alpha}{1-\alpha}}$, while $\tilde{c}_i = \alpha(1-\alpha)(1-\alpha-\alpha\beta/\gamma)^{\frac{2\alpha-1}{1-\alpha}}$. Then, substituting the explicit solutions for $\tilde{c}_n$ and $\tilde{c}_i$ into $\tilde{c}_i > \tilde{c}_n$ and simplifying leads to $(1-\alpha)^{1-2\alpha} > (1-\alpha-\alpha\beta/\gamma)^{1-2\alpha}$. This is true for those parameter combinations that allow for an interior steady state.

To show the indirect utility difference at Home, we graphically illustrate it for all feasible parameter combinations and a given $\alpha = 0.3$. This leaves the emission coefficient β and abatement effectiveness γ as free parameters where we have to obey $\beta/\gamma < 0.625$ in order to ensure the existence of a steady state under autarky. Figure 4 shows the welfare difference at Home under integration minus autarky. Our result is that Home can benefit from capital market integration for high β/γ ratios or for low values of θ.

These results show that for a sufficiently high preference towards the environment (low θ) Home is always able to improve its welfare through capital market integration. However, if Home's preferences are mostly directed towards consumption and it can easily offset the international emissions (low β/γ ratio), then capital market integration will not be beneficial for Home unless it is in the environmental poverty trap. As a consequence we conclude that, if the initial capital stock at Home (and Abroad) is such that Home is in

Figure 4: Indirect Home utility in integrated vs. autarky markets for $\alpha = 1/3$.



Explanation: The red region is where indirect utility in autarky is higher and the blue where utility in integrated markets is larger.

the environmental poverty trap, then it is always beneficial for Home to integrate capital markets. On the converse, if Home's initial level of capital is already sufficiently high, then whether Home should integrate capital markets or not depends on the relative weight of the environment in its preferences, on the shape of the production function, and on the effectiveness of abatement versus the dirtiness of consumption, as shown above.

6 Conclusion

In this article we present a two-region, dynamic general equilibrium model essentially based on the original model of John and Pecchenino (1994). Though we carry most of their assumptions forward, we find that extending their original model to a two-region model where one region is affected by an international externality while the other is not, leads to important differences. Our main finding is the existence of an *environmental poverty trap* in the case of autarkic capital markets.

An environmental poverty trap occurs if the region that is affected by the international externality is unable to increase abatement sufficiently to overcome the detrimental effects of consumption on this externality. In this case the affected region will reduce savings and spend an increasing amount of money to protect itself from the effect of this externality. This leads the region to be stuck in a deteriorating spiral that we dub the environmental poverty trap. However, with a sufficiently high level of capital, the affected region may protect itself from the damages of the externality while still holding enough capital back for economic growth.

Our study also suggests important results for the effect of capital market integration on environmental quality and the environmental poverty trap. We, firstly, show that that long-run environmental quality always benefits from capital market integration. This occurs since the steady state stock of capital in the region affected by the externality increases compared to the autarky case, which allows to divert a larger share of income towards abatement efforts. Secondly, we find that regions starting with low initial capital stocks can escape the environmental poverty trap by opening up their borders to foreign direct investments. This, again, is due to an income effect since higher capital stocks support higher labor income, hence more resources will be available that may be spent on abatement. That result will also be robust to virtually any modification in the production function or labor amount, as long as the inputs are complements and international capital markets imply that international capital stocks are linked proportionately. For regions already beyond the danger of a poverty trap, capital market integration still improves environmental quality but does not automatically imply welfare gains. On the contrary, the inflow of capital lowers the marginal return on capital which in turn has a negative impact on the returns to savings. For reasonable parameter combinations (e.g. a high preference towards consumption, or relatively costly abatement) we show that capital market integration may decrease welfare for regions subject to an environmental externality.

This suggests the following policy implications. Regions that are subject to the environmental poverty trap should open their capital markets since linking capital markets internationally will allow for income increases that may be spent on supporting abatement efforts. On the other hand, regions that are rich enough for sufficiently high abatement efforts should not resort to international capital mobility for environmental reasons alone, since this might reduce long-run welfare.

Though our model is somewhat stylized, by focusing on two regions that possess the same technologies and differ only by the fact that one is affected by the environmental quality whereas the other is not, we still find the existence of an environmental poverty trap in the autarkic case. Our intuition is that this result carries forward for regions of different sizes, different production technologies as well as to $N > 2$ regions. Several extensions are nevertheless interesting to study in this setting. Firstly, one could study the role of international cooperation. This could provide useful results for international treaties and policies. Secondly, one could assume different technologies or population levels. This will induce different levels of the capital stock in the integrated case and thereby lead to more complicated results. Nevertheless, it is well-known that poorer regions tend to have lower levels of total factor productivity and, therefore, we expect some kinds of conditional convergence even in the integrated market case. Furthermore, introducing catching up in technology would imply the same result as ours.

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7 Appendix

Proof of Proposition 1

Since the optimality condition $k_t = \theta/\gamma Q_t$ implies that k_t is proportional to Q_t, it suffices to prove the existence for one equation. A necessary condition for the existence of a steady state is $(1-\alpha)\gamma > \alpha\beta$, since $\Gamma(Q) < 0$ for all $Q \geq 0$ otherwise. A necessary and sufficient condition for the existence of a steady state is $\exists \hat{Q}$, such that $\Gamma'(\hat{Q}) = 0$ and $\Gamma(\hat{Q}) > 0$. $\hat{Q}$ is thus the maximum of $\Gamma(Q)$ since $\Gamma''(Q) < 0$ for $(1-\alpha)\gamma > \alpha\beta$. Graphically speaking, we know $\Gamma(Q) < 0$ at $\Gamma(0)$ and at $\Gamma(\infty)$. Then we require $\Gamma(Q) > 0$ for some positive Q. In that case, two steady states will exist (and one corner state), or one for a degenerate set of parameters (and one corner state). $\hat{Q}$ is given by

$$\hat{Q} = \frac{1}{\theta}\left[\frac{\alpha\big((1-\alpha)\gamma - \alpha\beta\big)}{\gamma^{\alpha}}\right]^{\frac{1}{1-\alpha}}.$$

Substituting this solution into $\Gamma(Q) > 0$ gives the condition for the existence of a steady state

$$(1-\alpha)^{\frac{\alpha}{\alpha-1}}(\alpha^{\frac{2\alpha-1}{1-\alpha}} - \alpha^{\frac{\alpha}{1-\alpha}})\big((1-\alpha) - \alpha\beta/\gamma\big)^{\frac{1}{1-\alpha}} > \beta/\gamma.$$

We show that there exists a β/γ ratio that satisfies this equality for a given $\alpha \in (0,1)$. The necessary condition for β/γ constrains its range to $\beta/\gamma \in (0, \frac{1-\alpha}{\alpha})$. Within this range, the right-hand side of the inequality above is increasing from zero to $\frac{1-\alpha}{\alpha}$, while the left-hand side decreases from $\phi(\alpha) \equiv (1-\alpha)(\alpha^{\frac{2\alpha-1}{1-\alpha}} - \alpha^{\frac{\alpha}{1-\alpha}}) > 0$ to zero. Therefore, there exists a ratio of $\beta/\gamma \in (0, \frac{1-\alpha}{\alpha})$ such that the inequality above is satisfied. In this case we obtain $Q_l < \hat{Q} < Q_h$, where Q_l is the low steady state and Q_h the high one. The rest of the proposition follows trivially. ■

Proof of Proposition 2

We analyze the system (11) to (13) around the two steady states. The Jacobian is

$$\begin{bmatrix} \alpha & 0 & 0 \\ -\frac{\alpha^2\beta\theta}{(1-\alpha)\gamma(\theta+1)} & \frac{\alpha\theta}{\gamma(\theta+1)}\left(\frac{Q(1-\alpha)\beta\theta k^{\alpha-2}}{\gamma}+(1-\alpha)\gamma k^{\alpha-1}\right) & \frac{\theta}{\gamma(\theta+1)}\left(1-\frac{k^{\alpha-1}\alpha\beta\theta}{\gamma}\right) \\ -\frac{\alpha^2\beta}{(1-\alpha)(\theta+1)} & \frac{\alpha}{\theta+1}\left(\frac{Q(1-\alpha)\beta\theta k^{\alpha-2}}{\gamma}+(1-\alpha)\gamma k^{\alpha-1}\right) & \frac{1}{\theta+1}\left(1-\frac{k^{\alpha-1}\alpha\beta\theta}{\gamma}\right) \end{bmatrix}.$$

The eigenvalues of this Jacobian are $\lambda_1 = 0$, $\lambda_2 = \alpha$, and

$$\lambda_3 = \frac{1}{1+\theta}\left(1 - k^{\alpha-2}\gamma^{-2}\alpha\theta(k\gamma(\beta+(-1+\alpha)\gamma) + Q(-1+\alpha)\beta\theta)\right).$$

The eigenvalues λ_1 and λ_2 are evident, but for λ_3 we only have an implicit solution in terms of k and Q. We can substitute $\frac{\gamma}{\theta}k = Q$, which holds from the first-order conditions and get

$$\lambda_3 = \frac{1}{1+\theta}\left(1 + k^{\alpha-1}\gamma^{-1}\alpha\theta((1-\alpha)\gamma - \alpha\beta)\right).$$

Substituting the explicit solution of $\hat{Q}$ into the equation above gives $\lambda_3|_{Q=\hat{Q}} = 1$. We now only need to know the slope of λ_3 when k changes. Thus,

$$\frac{\partial\lambda_3}{\partial k} = -\frac{1-\alpha}{1+\theta}k^{\alpha-1}\gamma^{-1}\alpha\theta((1-\alpha)\gamma - \alpha\beta) < 0,$$

since $(1-\alpha)\gamma - \alpha\beta > 0$ by assumption. Conclusively, for $k < \hat{k}$ we know that $\lambda_3 > 1$. Therefore, the low steady state $\{\tilde{k}, k_l, Q_l\}$ is unstable in $\{k_l, Q_l\}$. On the other hand, for $k > \hat{k}$ we know that $\lambda_3 < 1$, and therefore Q_h is asymptotically stable. ■

Proof of Proposition 3

$k_{t+1} < k_t$ if $k_t > \Psi(k_t, t)$. Since we assume that $\tilde{k}_t \leq \tilde{k}$ and $k_t < k_l$, then $\tilde{k}_t$ is monotonically increasing to its steady state $\tilde{k}$. By continuity and monotonicity of $\gamma k_t\big((1-\alpha)\gamma - \alpha\beta\big)k_t^\alpha$, this implies that if $k_{t+1} < k_t$ for one $t \geq 0$, this also applies for all t. Thus, if we show this holds for $t = 0$, then our result applies for any t. Substituting $\tilde{k}_0 = \psi k_0$ into equation (17) at $t = 0$ and assuming that $k_0 > \Psi(k_0, 0)$ leads to the inequality

$$k_0 > \frac{\theta}{\gamma(1+\theta)}\left[\frac{\gamma}{\theta}k_0 + \big((1-\alpha)\gamma - \alpha\beta\big)k_0^\alpha - \alpha\beta(\psi k_0)^\alpha\right].$$

Solving this for ψ gives the first result in the proposition. The second inequality in the proposition above comes from the assumption $\tilde{k}_t \leq \tilde{k}$. Since we assume that $\psi k_0 \leq \tilde{k}$ and $\tilde{k} = (1-\alpha)^{\frac{1}{1-\alpha}}$, the result follows immediately. ■

Proof of Proposition 5

The Jacobian at the steady state $\{\tilde{k}_i, k_i, Q_i\}$ is given by

$$\begin{bmatrix} \frac{\alpha(2\alpha\beta\theta+(\alpha-1)\gamma(2\theta+1))}{2(\alpha(\beta+\gamma)-\gamma)(\theta+1)} & \frac{\theta}{2(\theta+1)\gamma} \\ \frac{\alpha\gamma(2\alpha\beta+(\alpha-1)\gamma)}{(\alpha(\beta+\gamma)-\gamma)(\theta+1)} & \frac{1}{\theta+1} \end{bmatrix}.$$

The two eigenvalues are

$$\begin{aligned} \lambda_a \quad = \quad & \frac{1}{4(\theta+1)((1-\alpha)\gamma-\alpha\beta)}\Big(2((1-\alpha)\gamma-\alpha\beta)(1+\alpha\theta)+(1-\alpha)\alpha\gamma \\ & +\sqrt{((2-\alpha)(1-\alpha)\gamma-2\alpha\beta)2+4\alpha^2(2\beta+(1-\alpha)\gamma)((1-\alpha)\gamma-\alpha\beta)\theta+4\alpha^2((1-\alpha)\gamma-\alpha\beta)2\theta^2)}\Big), \end{aligned} \tag{34}$$

and

$$\begin{aligned} \lambda_b \quad = \quad & \frac{1}{4(\theta+1)((1-\alpha)\gamma-\alpha\beta)}\Big(2((1-\alpha)\gamma-\alpha\beta)(1+\alpha\theta)+(1-\alpha)\alpha\gamma \\ & -\sqrt{((2-\alpha)(1-\alpha)\gamma-2\alpha\beta)2+4\alpha^2(2\beta+(1-\alpha)\gamma)((1-\alpha)\gamma-\alpha\beta)\theta+4\alpha^2((1-\alpha)\gamma-\alpha\beta)2\theta^2}\Big). \end{aligned} \tag{35}$$

Define $Z = (((2-\alpha)(1-\alpha)\gamma - 2\alpha\beta)2 + 4\alpha^2(2\beta + (1-\alpha)\gamma)((1-\alpha)\gamma - \alpha\beta)\theta + 4\alpha^2((1-\alpha)\gamma - \alpha\beta)2\theta^2$, then $Z > 0$ if $(1-\alpha)\gamma - \alpha\beta > 0$, which holds by assumption. Making use of the characteristic equation, we find that

$$\begin{aligned} f(\lambda) &= \lambda^2 + \frac{(2\gamma + \alpha\gamma - \alpha^2\gamma - 2\alpha(\beta+\gamma) - 2\alpha(-\gamma + \alpha(\beta+\gamma))\theta)}{(-2\gamma + 2\alpha(\beta+\gamma) + 2(-\gamma + \alpha(\beta+\gamma))\theta)}\lambda \\ &\quad + \frac{-\alpha\gamma + \alpha^2\gamma}{(-2\gamma + 2\alpha(\beta+\gamma) + 2(-\gamma + \alpha(\beta+\gamma))\theta)}, \\ &= \lambda^2 + M\lambda + N. \end{aligned}$$

Then, asymptotic stability prevails if $1 + M + N > 0$, $1 - M + N > 0$ and $N < 1$. These conditions are equivalent to

$$\frac{(1-\alpha)\theta}{1+\theta} > 0,$$

$$\frac{(1-\alpha)\alpha\gamma}{(1+\theta)((1-\alpha)\gamma - \alpha\beta)} < 2,$$

and

$$1 + \alpha > \frac{(1-\alpha)(\alpha\beta - \gamma)}{((1-\alpha)\gamma - \alpha\beta)(1+\theta)}.$$

A necessary and sufficient condition for the second inequality to be satisfied is $(1-\alpha)\gamma - 2\alpha\beta > 0$. The first and third condition are satisfied if $(1-\alpha)\gamma - \alpha\beta > 0$. ■

Proof of Proposition 6

Part 1) We prove $Q_i > Q_h$ by showing that $Q_i > Q_l$ and $\Gamma(Q_i) < 0$. Substituting Q_i into function $\Gamma(Q)$ and assuming $\Gamma(Q_i) < 0$ gives

$$\begin{aligned} &((1-\alpha)\gamma - \alpha\beta)(\theta/\gamma)^\alpha((1-\alpha)\gamma - 2\alpha\beta)^\alpha\theta^{-\alpha}\left(\frac{(1-\alpha)\gamma - \alpha\beta}{\gamma}\right)^{\frac{\alpha\alpha}{1-\alpha}} \\ &-((1-\alpha)\gamma - 2\alpha\beta)\left(\frac{(1-\alpha)\gamma - \alpha\beta}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} - \alpha\beta(1-\alpha)^{\frac{\alpha}{1-\alpha}} < 0. \end{aligned} \qquad (36)$$

We transform equation (36) in two steps. First, rewriting gives

$$-\left(\frac{(1-\alpha)\gamma-\alpha\beta}{\gamma}\right)^{\frac{\alpha}{1-\alpha}}((1-\alpha)\gamma-2\alpha\beta)\left[1-\left(\frac{(1-\alpha)\gamma-\alpha\beta}{(1-\alpha)\gamma-2\alpha\beta}\right)^{1-\alpha}\right]<\alpha\beta(1-\alpha)^{\frac{\alpha}{1-\alpha}},$$

and re-writing the right-hand side gives

$$\alpha\beta(1-\alpha)^{\frac{\alpha}{1-\alpha}}=(1-\alpha)^{\frac{\alpha}{1-\alpha}}\Big((1-\alpha)\gamma-\alpha\beta-(1-\alpha)\gamma+2\alpha\beta\Big).$$

Second, dividing by $(1-\alpha)\gamma-2\alpha\beta$ and multiplying by $-\gamma^{\frac{\alpha}{1-\alpha}}$ gives

$$\Big((1-\alpha)\gamma-\alpha\beta\Big)^{\frac{\alpha}{1-\alpha}}\left[1-\left(\frac{(1-\alpha)\gamma-\alpha\beta}{(1-\alpha)\gamma-2\alpha\beta}\right)^{1-\alpha}\right]>((1-\alpha)\gamma)^{\frac{\alpha}{1-\alpha}}\left[1-\frac{(1-\alpha)\gamma-\alpha\beta}{(1-\alpha)\gamma-2\alpha\beta}\right].$$

The terms in the square brackets are always negative. Also, it is easy to see that $\Big((1-\alpha)\gamma-\alpha\beta\Big)^{\frac{\alpha}{1-\alpha}}<((1-\alpha)\gamma)^{\frac{\alpha}{1-\alpha}}$. Define $\psi\equiv\frac{(1-\alpha)\gamma-\alpha\beta}{(1-\alpha)\gamma-2\alpha\beta}$. Then $1-\psi^{1-\alpha}>1-\psi$ if $\psi>1$. Since $\psi>1$ is necessary for the existence of a steady state, we know $\Gamma(Q_i)<0$. This shows that either $Q_i<Q_l$ or $Q_i>Q_h$.

To show that $Q_i>Q_l$ we show that $Q_i>\hat{Q}$ for the β/γ ratio that leads to a steady state in the autarky case. We proof by contradition and assuming $Q_i<\hat{Q}$ gives, after some manipulation, the condition

$$\frac{\beta}{\gamma}<\frac{1-\alpha}{\alpha}\frac{1-\alpha^{\frac{1}{1-\alpha}}}{2-\alpha^{\frac{1}{1-\alpha}}}.$$

The domain of β/γ that leads to an interior steady state is given in equation (15). We now need to show that this domain is smaller than the domain of β/γ that implies $Q_i>\hat{Q}$. The maximum of the left-hand side of equation (15) is when $\beta/\gamma\to 0$. Simplifying leads to $\frac{1-\alpha}{\alpha}\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}$. Since this is smaller than $\frac{1-\alpha}{\alpha}\frac{1-\alpha^{\frac{1}{1-\alpha}}}{2-\alpha^{\frac{1}{1-\alpha}}}$, for any $\alpha\in(0,1)$, we conclude that $Q_i>\hat{Q}$ if a steady state in the autarky case exists. Hence, together with the result $\Gamma(Q_i)<0$ this implies that $Q_i>Q_h>Q_h$.

The second part of point 1) states that $k_i > k_h$. Since $Q_i > Q_l$, $k_h = \theta/\gamma Q_h$ and $k_i = \left(\frac{(1-\alpha)\gamma-\alpha\beta}{\gamma}\right)^{\frac{1}{1-\alpha}}$, if we solve for $\Gamma(k_i)$ and find $\Gamma(k_i) < 0$, then $k_i < k_l$ or $k_i > k_h$. Thus, substituting k_i into $\Gamma(k)$ gives

$$\Gamma(k_i) = \big((1-\alpha)\gamma - \alpha\beta\big)\left(\frac{(1-\alpha)\gamma-\alpha\beta}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} - \gamma\left(\frac{(1-\alpha)\gamma-\alpha\beta}{\gamma}\right)^{\frac{1}{1-\alpha}} - \alpha\beta(1-\alpha)^{\frac{\alpha}{1-\alpha}}.$$

Simplifying leads to

$$\Gamma(k_i) = -\alpha\beta(1-\alpha)^{\frac{\alpha}{1-\alpha}} < 0.$$

We thus know that either $k_i < k_l$ or $k_i > k_h$. We compare k_i and $\hat{k}$. Since $\hat{k} = \alpha^{\frac{1}{1-\alpha}} k_i$, and $\alpha^{\frac{1}{1-\alpha}} < 1$, then we know that $k_i > \hat{k}$. This implies that $k_i > k_h$.

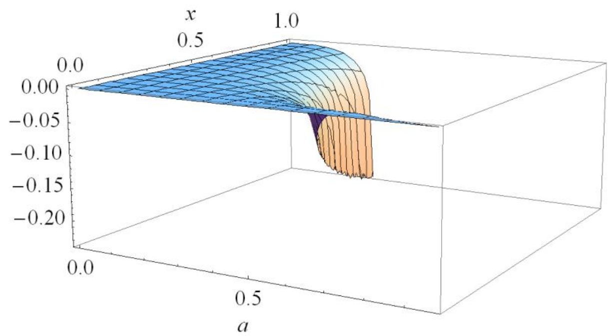
In part 2) we note that $\tilde{k}_n > \tilde{k}_i$. The proof is as follows. Since $k_i = ((1-\alpha)-\alpha\beta/\gamma)^{\frac{1}{1-\alpha}}$, and $\hat{k}_n = (1-\alpha)^{\frac{1}{1-\alpha}}$, then $\tilde{k}_n > \tilde{k}_i$ follows directly.

Proof of part 3) Comparing the total capital stock in the autarky case and in the integrated case is done as follows. Total capital in the autarky case is $k_h + \tilde{k}_n$, while total capital in the integrated case is $2k_i$. The first step is to show that $2k_i > \tilde{k}_n$. Substituting the explicit solutions and simplifying we obtain $\beta/\gamma < \frac{1-\alpha}{\alpha}(2^{1-\alpha} - 1)$. Comparing again to the maximum of the left-hand side of equation (15), we find that $\frac{1-\alpha}{\alpha}(2^{1-\alpha} - 1)$ is larger. Thus, we conclude that $2k_i > \tilde{k}_n$. In the second step we substitute $2k_i - \tilde{k}_n$ into $\Gamma(k)$ and if we obtain a negative sign, then we know that total steady state capital in the autarky case is lower than in the integrated case. After substituting and assuming $\Gamma(2k_i - \tilde{k}_n) < 0$, we obtain

$$\Theta \equiv (1-\alpha-\alpha\beta/\gamma)(2(1-\alpha-\alpha\beta/\gamma)^{\frac{1}{1-\alpha}} - (1-\alpha)^{\frac{1}{1-\alpha}})^{\alpha} - 2(1-\alpha-\alpha\beta/\gamma)^{\frac{1}{1-\alpha}} + (1-\alpha)^{\frac{1}{1-\alpha}} - \alpha(1-\alpha)^{\frac{\alpha}{1-\alpha}}\beta/\gamma < 0.$$

We cannot prove analytically that the above condition holds. However, using Mathematica, we can show graphically that the above condition holds. This is shown in Figure 5 below, where $x = \beta/\gamma$ and the values of Θ are depicted on the vertical axis. Thus, $\Gamma(2k_i - \tilde{k}_n) < 0$

Figure 5: Value of Θ for $\alpha \in (0, 1)$ and $\beta/\gamma \in (0, 1)$.



Note that this result holds for any $\beta/\gamma > 0$. We restrict the β/γ ratio for presentation purposes only.

for the feasible range of $\alpha \in (0, 1)$ and β/γ such that the sufficient condition (15) is satisfied. Combined with the previous results we know that total capital stock in the autarky case is lower than in the integrated case. ■