Liquidity, Government Bonds and Sovereign Debt Crises *

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Job market paper

Abstract

This paper analyses the European financial crisis through the lens of the sovereign bond liquidity. Using novel data we show that government securities are the prime collateral in the European repo market, which is becoming a primary source of funding for the banking system in the Euro area. We document that the repo haircuts on peripheral government bonds sharply increased during the crisis, reducing their liquidity and amplifying the movements in the yields of these securities. We study the impact of a rise in haircut to public bonds on the business cycle and on asset prices through a dynamic stochastic general equilibrium model with liquidity frictions. The model predicts a flight to liquidity and a fall in investments which reduces the economic activity. We show that the government may alleviate the negative impact of the liquidity shock by issuing more short-term bonds which provide an alternative liquid instrument to investors. The negative impact of a rise in haircuts on the value of bonds is confirmed by a Bayesian structural vector autoregressive model for the Irish economy.

JEL classification: E44, E58, G12

Key words: Repo, haircuts, government bonds, financial crisis, liquidity shock

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1 Introduction

The recent financial crisis was characterised by the increase in the yield spreads of sovereign bonds issued by peripheral countries of the Eurozone (Greece, Ireland, Italy, Portugal and Spain), which has weakened their fiscal position and reduced their capacity to roll over the debt. Since the creation of the monetary union has eliminated the exchange rate risk, two main factors may explain why countries in the periphery of the Eurozone have been paying higher interest rates on the public debt than countries in the core: credit risk and liquidity.\footnote{The share of outstanding government debt issued in euro is more than 90% in all of the Eurozone countries (Eurostat, 2013).}

Credit risk derives from the government’s probability of default. Weak fiscal and macroeconomic fundamentals of a country induce investors to ask higher compensations for holding government securities due to their increased expected losses. In addition, fears of default and self-validating expectations may also drive up the yields of government securities issued by countries that cannot press new currency as highlighted by the theoretical contributions of Calvo (1988), Cole and Kehoe (2000) and Corsetti and Dedola (2012).\footnote{Tristani and Höröahl (2013) and Dewachter et al. (2013) find that economic fundamentals are the primary determinant of European bond yield spreads during the crisis, but their evolution is also associated with an unobservable non-fundamental factor that they interpret as the manifestation of self-fulfilling dynamics.}

Liquidity is a broad concept that traditional theories by Keynes (1936) and Hicks (1967) refer to as the capacity of an asset to store wealth and protect its owner from a shortage of revenue. Modern corporate finance distinguishes between market liquidity and funding liquidity. Market liquidity is the ease to sell an asset with limited price impact; when it is difficult to exchange a security, its market liquidity is low and its price deviates from the fundamentals. Empirical works which disentangle credit and market liquidity effects in the bond yields find compelling evidence that market liquidity has contributed to the rise of intra-euro spreads during the crisis (Manganelli and Wolswijk, 2009; Favero et al., 2010 and Monfort and Renne, 2013).

Funding liquidity is the ease with which investors can obtain funding: as they typically borrow against an asset, in this paper we define funding liquidity as the “pledgeability” of a security, that is its capacity to serve as collateral in the secured interbank market, especially in the market of repurchase agreements (repos). Following this interpretation, we investigate the role of the sovereign bond liquidity in the European financial crisis.

The share of outstanding government debt issued in euro is more than 90% in all of the Eurozone countries (Eurostat, 2013). Tristani and Höröahl (2013) and Dewachter et al. (2013) find that economic fundamentals are the primary determinant of European bond yield spreads during the crisis, but their evolution is also associated with an unobservable non-fundamental factor that they interpret as the manifestation of self-fulfilling dynamics.
Government bonds are shown to be the privileged asset to guarantee secured funding on which European banks increasingly rely to meet their liquidity needs. This goes in the same direction of Gennaioli et al. (2014a) who find that banks, in a large panel of countries, hold a sizeable amount of government securities. The predominant motivation is that of storing liquidity and posting them as collateral in borrowing arrangements rather than for their high returns or for carry trade operations (Acharya and Steffen, 2013), or because they exempt from capital adequacy requirement (Livshits and Schoors, 2009) or in response to the governments’ pressure (Broner et al., 2014). The role of the government debt as collateral in the interbank lending market is emphasised by Bolton and Jeanne (2011) in analysing the contagion effect of a sovereign default in financially integrated economies such as the Eurozone. In line with the findings of Andritzky (2012), they also show that European banks considerably increased their exposure to foreign government bonds before the onset of the financial crisis, involving a diversification of their collateral base.

In the European money market, the collateral may be redeployed by the cash lender to secure other repos, this way stretching the intermediation chain. For example, the Italian debt could be employed as collateral by a French bank to borrow from a German bank, which in turn could re-use the same bond for a repo with a Dutch bank. Paraphrasing Kiyotaki and Moore (2003), before the crisis European banks held government securities not only for their maturity value but also for their exchange value and these papers circulated as means of savings; hence they could be considered as money or a medium of exchange.

Nevertheless, we show that the increase in the sovereign risk during the crisis led to a rise in the repo haircuts on government bonds issued by peripheral countries, reducing their pledgeability and funding liquidity. Consequently, leveraged investors were forced to sell illiquid bonds of the periphery and purchase liquid bonds of the core with lower haircuts in order to avoid a contraction of their funding, contributing to the widening of the yield spreads.

The observed pullback of foreign investors from the sovereign bonds of the periphery and the

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3The European stress test in 2010 has revealed that in most countries of the Eurozone foreign banks owned a larger share of the government’s debt than domestic banks and that banks invested more than half of their government debt portfolio in the debt of foreign governments.

4Singh (2011) provides evidence on the importance of the re-use of collateral for financial intermediation and calculates the velocity of pledged collateral, finding a reduction in the velocity after the failure of Lehman Brother. The ERSB (2014) estimates that in Europe approximately 94% of securities pledged to the banks is eligible for re-use and that on average banks re-use collateral once (that is the collateral multiplier is 2).
repatriation of the debt (Andritzky, 2012; Brutti and Sauré, 2013) could derive not only from the rise in the risk of sovereigns and the tightening of prudential rules, but also from a reduction in their liquidity as collateral in the European interbank repo market.\footnote{Angelini, Grande and Panatta (2014) highlight other factors that explain the resurgence of the home bias in the bond holding after 2008, such as the fear of a break up of the Eurozone, the higher risk-adjusted return on government bonds than on loans and the awareness that a sovereign default would have a pervasive effect on banks even without the government suspension of debt payment.}

Building on Del Negro, Eggertsson, Ferrero and Kiyotaki (2012), hereafter DEFK, we explore this liquidity channel of the Eurozone crises through a DSGE model with liquidity frictions. Similarly to European banks, entrepreneurs choose to hold sovereign bonds as a way to store liquidity for financing future investments. Indeed, even if the returns on public bonds are lower than those on private assets, entrepreneurs can ease their funding constraint by borrowing against government securities.\footnote{This echoes Holmstrom and Tirole (1998) who show that firms that cannot pledge any of their future income are willing to pay a premium on assets that are able to store liquidity and help them in state of liquidity shortage.} However, an unanticipated liquidity shock can suddenly reduce their liquidity equivalently to a rise in the repo haircut and entrepreneurs can pledge a smaller fraction of bonds to finance investments. Further, the price of government securities falls since they become less attractive. We test the impact of a policy experiment in response to the liquidity shock, consisting in issuing one-period bonds, that are completely liquid and not subject to liquidity constraints. This unconventional policy, by providing the investors with an alternative liquid means of savings, eases their funding constraint and reduces the contraction of investments.\footnote{In this set-up we abstract from the risk of sovereign bonds to focus on a pure liquid channel. Several authors introduce probability of default in DSGE models. For instance, Coimbra (2014) analyses the feedback effects of the rise in the risk premium of government bonds in a model in which banks face a Value-at-Risk constraint. Bocola (2014) develops a quantitative model to estimate the liquidity and credit risk channels in the pass-through of sovereign risk.}

The negative impact of a liquidity shock on the value of government bonds is confirmed empirically by the impulse response function of a structural vector autoregressive model (SVAR) which includes yields of Irish 10-year government bonds, the haircut on these securities applied the main European clearing house and the sovereign CDS premiums. We find that bond yields increase after a liquidity shock and a credit risk shock.

Related literature. This paper is related to four strands of research. Several works analyse the U.S. repo market and its role in the liquidity crisis of 2007-2008. Adrian and Shin (2009, 2010) shows that the procyclical leverage of US investment banks amplifies fluctuations in asset prices and that they adjust the balance sheets through variations in repos. Brunnermeier (2009) points...
out that prior to the crisis financial intermediaries were heavily exposed to maturity mismatch through their increased reliance on overnight and short-term repos and asset-backed commercial papers (ABCPs). Gorton and Metrick (2010, 2012) argue that the financial crisis was a run in the securitized-banking system characterized by a general rise in repo haircuts which shrank the banking liquidity in a similar way of massive deposit withdrawals in traditional bank runs and triggered a re-sale of assets. In contrast to that study, Copeland et al. (2010) find that in tri-party repo market the haircuts were stable between 2008 and 2010 and Krishnamurthy et al. (2013) argue that the main cause of the liquidity crisis was a credit crunch in the ABCP market. Despite the interest towards the U.S. repo market, few academic articles explore the repo market in the Euro area, and we know little about the size, the evolution and implications for the sovereign-debt crises.

This paper attempts to fill this gap and examines the structural characteristics of the European repo market, the differences with the US market and its development in time of crisis. A second strand emphasises the negative feedback between the level of haircuts and the value of the underlying collateral. Gârleanu and Pedersen (2009) incorporate margin constraints into a consumption capital asset pricing model and show that when margin constraints bind, higher margins raise the required returns on assets, which lowers asset prices. Aschcraft et al. (2010) present an OLG model in which a reduction in the haircut on a security eases the funding constraint and lowers its required return. Empirically, they show that the Federal Reserve by cutting the haircuts through the Term Asset-Backed Securities Loan Facility (TALF) increased the yields of the eligible securities. Brunnermeier and Pedersen (2009) endogenise changes in haircuts and changes in asset prices through a loss spiral and a margin spiral, which create a strong interaction between funding liquidity and market liquidity. The fall in the price of an asset reduces its value as collateral and force investors to sell some of its assets to obtain funding for maintaining other obligations, depressing its value even further (loss spiral). The higher volatility of the asset price leads to a rise in haircuts, which in turns strengthens the deleveraging (margin spiral).

We document this adverse feedback loop between levels of haircuts and value of underlying collateral.

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8Herdahl and King (2008) compare the development of the repo market in the Euro area, in US and in UK in the first stage of the global financial crisis, but it does not cover the recent period characterized by tensions in sovereign-debt markets. Mancini et al. (2014) and Boissel et al. (2014) focus on the repo transactions performed on the principal European anonymous electronic trading platform (Broker Tec, Eurex Repo and MTS).
collateral during the most acute phase of the European financial crisis: haircuts on peripheral
government securities surged following the rise in the bond yields. The effects of the fall in the
value of the collateral are described by several models with financial frictions, such as Schleifer
The shift from unsecured to secured lending in the European money market during the crisis
is akin to a rise in agency frictions which obliges borrowing banks to provide collateral whose
value exceeds the loan by a determined amount that is the haircut. The impact of a temporary
shock which reduces the borrower’s net worth and the value of the asset used as input in the
production process and as collateral translates in a persistent decline of output and price of the
asset.\footnote{In the models in which the borrowing constraints derive from costly state of verification (Bernanke and
Gertler, 1989; Carlstrom and Fuerst, 1997; Bernanke, Gertler and Gilchrist, 1999) the cost of external financing
is increasing in the borrowing since higher leverage requires more monitoring. In Bernanke, Gertler and Gilchrist
(1999) the negative shock hitting the net worth also reduces the price of capital arising an amplification mechanism
similar to Kiyotki and Moore (1997).}

The increase in the haircuts reinforces this mechanism as it reduces the leverage of investors
exacerbating the downturn. We study this channel by modelling a rise in the margins as a
liquidity shock following Kiyotaki and Moore (2012), hereafter KM, which is the seminal paper
that incorporates liquidity frictions into a real business cycle model.\footnote{While KM focus on the market liquidity of an asset and its resaleability, we study the funding liquidity and pledgeability in terms of repo haircuts. Liquidity is modelled in different ways in macroeconomic models. In Gertler and Kiyotaki (2010) liquidity shocks affect interbank funds and they model variations in haircuts as tightening of the incentive constraint of banks. In Calvo (1999) liquidity is a parameter that enters in the utility function making the land more valuable. Beningno and Nasticò (2013) consider liquidity as the property of an asset to be exchanged for goods.} DEFK combine the KM
framework with a monetary DSGE model including nominal and real rigidities and the zero lower
bound condition in line with Christiano et al. (2005) and Smets and Wouters (2007) in order
to estimate the impact of a liquidity shock on private financial assets and the consequences of
the credit facilities implemented by the Federal Reserve. We propose a model that departs from
DEFK in two directions. First, equity is illiquid and public securities are almost completely
- but not fully - liquid. Funding constraints are tighter since entrepreneurs cannot acquire
resources by disposing of private assets, emphasising the role of public bond to store liquidity.
In order to reproduce the rise in the repo haircuts on the peripheral sovereign debt, all the
dynamics of the model takes place through a liquidity shock hitting the public bonds. Second,
we compare the impact of this shock in a laissez-faire economy and within an economy in which
the government reacts with an unconventional policy intervention by issuing liquid papers in the form of one-period bonds. These differences derive from the observation that while in the U.S. the high pre-crisis liquidity of private asset (such a commercial papers and asset-backed securities) suddenly dried up, in Europe - where the market of private financial assets is far less liquid and government bonds are the liquid instrument for financial investment - the peculiar feature of the crisis was instead the reduction of sovereign bond liquidity.

The remainder of the paper is organized as follows: Section 2 defines the terms commonly employed for the market of repurchase agreements, describes the data and the European repo market; Section 3 presents the model and Section 4 the results; Section 5 examines empirically the impact of a rise in haircuts on Irish government bond yields and Section 6 concludes.

2 The European repo market

2.1 Definitions and data

We start by explaining the main characteristics of a repo contract, important in understanding the results of the analysis, and by describing the data used to investigate the European market of repurchase agreement. A repo transaction is an agreement between two parties on the sale and subsequent repurchase of securities at an agreed price. It is equivalent to a secured loan, with the difference that legal title of the securities passes from the seller to the buyer which may re-use them as collateral in other repo transactions. In order to protect the lender from the risk of a reduction in the value of collateral, repos involve overcollateralization and the difference between the value of the cash and the value of collateral is the haircut or initial margin. The haircut takes account of the unexpected loss that the lender in a repo may face due to the difficulty of selling that security in response to a default by the borrower. Accordingly, it is at the same time an indicator of funding liquidity from the perspective of the cash borrower and of market liquidity from the perspective of the cash lender. Figure 1 shows an example of bilateral repo.\footnote{According to the involvement of intermediaries between the lender and the borrower, repos can be distinguished in two types. In bilateral repos, the lender and the borrower transact directly with each other, selecting the collateral, initiating the transfer of cash and securities, and conducting collateral valuation. In tri-party repos, a third party intermediates the transaction providing operational services to the parties, in particular the selection and valuation of collateral securities, but does not participate in the risk of transaction.}

At time t, the cash borrower (securities dealer, commercial bank, hedge fund) posts €100 securities
as collateral and receives a €90 loan from the cash lender (commercial bank, investment fund, money market fund) with a haircut of 10%. At time \( t+k \), the borrower returns the cash with an interest of 1.1% (repo rate) and receives back the collateral. If repo is used to finance the purchase of a security, the haircut is equivalent to the inverse of the leverage. To hold €100 securities the investor can borrow up to €90 from the repo lender and must come up with €10 of its own capital, so the maximum leverage is 10. In this example, a rise in haircut by 10 percentage points reduces the borrower liquidity to €80 and the leverage to 5.

Figure 1: Bilateral repo contract

The determinants of haircuts vary according to the repo structure. In bilateral repos, the haircut reflects mainly the creditworthiness of the borrower. In repo transactions which involve a central clearing counterparty (CCP) interposing between the borrower and the lender and bearing the counterparty credit risk, haircuts are settled on the basis of the CCP internal rules and depend on the quality of the underlying collateral and its market risk.

Data. Because of the lack of comprehensive information on the repos in the Eurozone, different sources are used in this study to shed light on this market, in particular Bankscope and the European Repo Market Survey (ERMS). Bankscope provides data on the banks' balance sheet at yearly frequency showing the stock of repos and reverse repos held by financial institutions. The ERMS provides information on the size and composition of the European repo market, including the types of repos traded, the rates, the collateral, the cash currency and the maturity.

\[\text{The advantage of this database is that it allows to compare different sources of funding of European financial institutions. Nevertheless, it presents three main limits: first, it lacks of important breakdowns, such as counterparty, maturity and currency, preventing a more granular analysis of the European repo market and it does not separate repos as interbank transactions from conventional monetary policy operations. Second, the period covered by the database starts from 2006. Lastly, for several banks data on repos are missing, making impossible an overview of the aggregated banking system.}\]
It is a semi-annual survey conducted by the International Capital Market Association (ICMA) starting from 2001. The survey asks a sample of banks in Europe for the value of their repo contracts that were still outstanding at close of a business days excluding the value of repos transacted with central banks as part of official monetary policy operations, this allows to focus the interbank secured lending. The ERMS also reports the average of haircuts for various categories of collateral (governments bonds, public agencies, corporate bonds, covered bonds, mortgage-back securities, other asset-backed securities, convertible bonds, equity), but it does not provide information about the haircuts on government bonds at national level. We collected data on the haircuts applied by the main European clearing houses to the 10-year sovereign bonds. Variations in the margins settled by the LCH. Clearnet Ltd on Irish and Portuguese bonds are found by reading the RepoClear Margin Rate Circulars in the website of the clearing house.

2.2 Structural characteristics

Figure 2 exhibits the extraordinary expansion of the European repo market in the last decade. The ERMS has collected data on repos held by 67 banks that continuously participated to all the surveys from which we have subtracted reverse repos in order to focus on their liabilities. The repo market tripled in the run-up of the crisis and after a short contraction between 2008 and 2009 rapidly recovered, reaching around €3 trillion and approaching the about $4.4 trillion of the U.S. repo market estimated by the Federal Reserve of New York in 2009 based on the average daily amount outstanding of the primary dealers repo financing (Acharya and Öncü, 2012).

Table 1 displays the funding structure of the largest European commercial banks, for which Bankscope reports data on repos. For the biggest five institutions in the sample repos account for around 10% of their liabilities. Further, for those banks secured borrowing is larger than

\[ \text{LCH Clearnet Ltd is a clearing house operating in the euro area and providing clearing services covering Austrian, Belgian, Dutch, German, Irish, Finnish, Portuguese, Slovakian, Slovenian, Spanish and UK government debts, on cash and repos transactions (cash bonds and repo trades in Italian and French markets are cleared by LCH.Clearnet SA). In 2012 LCH.Clearnet cleared approximately 50% of the global interest rate swap market, and is the second largest clearer of bonds and repos in the world. Information of haircuts are extracted from the webpage: http://www.lchclearnet.com/risk_management/ltd/margin_rate_circulars/repoclear/default.asp} \]

\[ \text{Since Bankscope does not distinguish between repos in the private market and repos as monetary policy operations of the ECB, the table reports data on 2010 to exclude the large-scale LTROs in 2011 and 2012.} \]
unsecured borrowing. This finding is in line with the European Money Market Survey conducted
by the ECB, which reports that the largest ten banks account for 62% of the total repo turnover,
that 90% of the total are bilateral and among them 71% are cleared by CCPs (figure 4). Moreover,
it confirms the importance of secured transactions in the European money market. These results
point out that the largest European banks tend to post a collateral in order to access to the
short-term wholesale funding\footnote{the European money market survey also shows that the bulk of repos is overnight or with a maturity of one
week.}.

Figure 3 shows that sovereign bonds are the predominant assets in the collateral pool of
the European repo market, accounting for around 80% of the total, and this share was stable
during the crisis. Furthermore, the ERMS shows that haircuts on government bonds in the
tri-party segment are lower than the ones on other assets, such as corporate bonds, covered
bonds, mortgage-back securities and equity. Looking at the composition of government securities,
German bonds account for the largest share, although their supply is lower than French and in
particular Italian bonds (Bolton and Jeanne, 2011). This reflects their lower riskiness and suggest
that are the most liquid bonds in the Euro area.

To summarize, government bonds are the most liquid assets and the preferred collateral in the
European repo market. This is in contrast with the U.S. secured market, in which private assets
are more largely employed as collateral\footnote{Krishnamurthy et al. (2013), Fitch Ratings (2012) and European Parliament (2013).}.

Another difference between the two markets concerns
their structure, since tri-party repos are the predominant arrangements in the U.S. while bilateral CCP-cleared repos constitute the biggest share in Europe. Consequently, increases of haircuts in the US market can be explained by mechanisms of adverse selection (Gorton and Metrick, 2008) and information acquisition sensitivity (Dang et al., 2011 a and Dang et al. 2011 b), because the cash borrower may have more information about the quality of the asset used as collateral, e.g. mortgage-backed securities. By contrast, in the European market the larger use of CCPs, which bear the credit risk, and of government securities as collateral reduces problems of asymmetric information. However, variations in haircuts may have a more destabilizing effect on the value of the underlying collateral since the European repo market is highly concentrated in few clearing houses (LCH Clearnet Ltd, LCH Clearnet SA, Eurex and CC&G) and large banks, which may react in a synchronic way to a change in haircuts.

### 2.3 Developments during the crisis

We next examine the evolution of the European repo market during the financial crisis. Figure 4 compares the dynamics of secured and unsecured borrowing in a sample of 101 European banks included by the European Money Market Survey. After the onset of the crisis we observe a shift of funding from unsecured to secured following the rise in counterparty risk. Furthermore,
bilateral CCP-cleared repos steadily increased within the secured segment.

The expansion of the European repo market in time of crisis is contrast with the U.S. market which evaporated during the period 2007-2009. More than a consequence of the recent crisis, this evolution seems to derive from a structural transformation in the business model of the largest European financial institutions in the last decade, which rely more massively on short-term collateralised debt as a source of funding for their activities, similarly to the US investment banks (see Adrian and Shin (2009) and Brunnermeier (2009)).

The recent financial crisis also affected the composition of sovereign bonds within the pool of collateral (see figure 3). In particular, the share of government securities issued by peripheral countries reduced. Italian bonds fell from 10% of the total in June 2011 to 7% in December 2011, when the tensions on the sovereign debt was at the peak. This could reflect not only of their higher riskiness - since in the CCP-cleared transaction it is the clearing house which bears the risk - but also of their lower funding liquidity determined by the increases in repo haircuts. For instance, LCH Clearnet SA on 8th November 2011 published the variation in repo margins on Italian government securities by between 3 and 5.5 percentage points and accordingly on 9th November the haircut on the 10-year Italian bonds increased from 6.65 to 11.65 percentage points. The yields of the same asset increased from 6.39 to a high record level of 7.25 percentage points.
Figure 4: Interbank transactions in the European money market

Repos and unsecured borrowing (total turnover)  Shares of bilateral and tri-party repos (in percent of the total)

Source: European Repo Survey

between November 7th and November 9th (figure 5). During this period, several economic and political events occurred that likely affected the levels of the Italian bond yields, such as downgrades in the credit rating and the political instability. However, it is evident the short run impact of the rise in haircut to Italian bonds whose yields skyrocketed.

Figure 5: Yields on 10-year Italian (LHS) and German (RHS) sovereign bonds

Black line: change in the haircut on Italian bonds by LCH Clearnet SA

17The Financial Times reported the following: “Italian bonds are in the perfect storm at the moment. Real money investors are running away and those investors using Italian bonds to finance will also be clearing the desk now.”, “LCH Clearnet SA raises margin on Italian bonds”, Financial Times 9th November 2011
Larger increases in haircuts were experienced by Irish and Portuguese bonds in 2011. Figure 6 compares the evolution of the yields on 10-year bonds and the haircuts to these securities settled by LCH Clearnet Ltd, the most important European CCP.

Following the rise in the yields, the haircuts on both the securities increased from a low level of 15% before the crisis to 80%. Following the initial example, now the investors can obtain only €20 posting Irish or Portuguese bonds which are worth €100 in a repo transaction. Figure 6 shows the strong procyclicality of the haircut due to the fact that LCH Clearnet Ltd systemically increases the haircut when the spread with the German government bonds exceeds 450 basis points.

Raises in haircuts to peripheral bonds in turn may have increased the pressure on the sovereign debt increasing the yields of these assets. Higher initial margins diminish the ability of leveraged investors to borrow and tighten their funding constraint, increasing the shadow cost of capital. As a consequence, they reduce their position on the bonds with higher haircuts and shift their portfolio towards securities with lower margins to relax their funding constraint. At that point, bond prices are more driven by liquidity considerations rather than by movements in fundamentals. The next session presents a model to study this mechanism.

Figure 6: Yields (LHS) and haircuts (RHS) on 10-year government bonds

Ireland

Portugal

Sources: Global finance data and LCH Clearnet website

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18 In the appendix are reported the haircuts settled by LCH Clearnet SA and CC&G on 10-year European government bonds.
3 The model

The model is an infinite horizon closed economy populated by a continuum of households of measure one. The members of each representative household are either entrepreneurs or workers. The other agents are intermediate and final good firms, labour agencies, capital producers and the government. The intermediaries in the production process and the labour market introduce nominal rigidities in the model that also features capital adjustment costs. Household can invest in two risk-free assets characterized by different liquidity: other households’ equity and government bonds. The government conducts conventional fiscal and monetary policy by setting the nominal interest rate and collecting taxes. There is no risk of sovereign default in the model, but long-term bonds are subject to a liquidity shock that is the only shock perturbing the economy. In response to this shock the government may implement an unconventional policy which consists in increasing the supply of one-period bonds which are more liquid than long-term bonds.

3.1 Households

Structure. Within the representative household there is a mass 1 continuum of family members $j \in [0,1]$ that are workers and entrepreneurs. Each entrepreneur $j \in [0,\gamma)$ invests and each worker $j \in (\gamma, 1]$ supplies labor; both types return their earnings to the household. Their profession is determined at the beginning of each period by an idiosyncratic shock; with probability $\gamma$, they become entrepreneurs and with probability $(1-\gamma)$ they become workers. By the law of large number, $\gamma$ and $1-\gamma$ also represent the fraction of entrepreneurs and workers, respectively. At the end of each period, all members share consumption goods and assets, but resources cannot be reallocated among members within the period.

Preferences. The household’s objective is to maximize the utility function

$$
\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_s H_s(j)) = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{C_s^{1-\sigma}}{1-\sigma} - \frac{\xi}{1+\eta} \int_{\gamma}^{1} H_s(j)^{1+\eta} dj \right]
$$

(1)

where $\mathbb{E}_t$ denotes the conditional expectation, $\beta$ is the subjective discount factor, $\sigma$ is the coefficient of relative risk aversion, $\xi$ is a scaling parameter that can be chosen to match a target value for the steady state level of hours, and $\eta$ is the inverse Frisch elasticity of labour supply.
Utility depends positively upon the sum of all consumption goods bought by different household members \( (C_t = \int_0^1 C_t(j) dj) \) and negatively upon the workers' labour supply.

**Portfolio.** Households buy physical capital \( K_t \) at price \( q_t \) and they lend it to intermediate good producers earning a constant dividend stream \( r_t \). They also own government bonds \( B^L_t \) with price \( Q^L_t \) defined as perpetuities with coupons which decay exponentially, as in Woodford (2001). A bond issued at date \( t \) pays \( \lambda^{k-1} \) at date \( t+k \) and \( \lambda \in [0, \beta] \) is the coupon decay factor that parametrises the duration of bonds of the bonds which corresponds to \( (1-\lambda \beta)^{-1} \). We define this bond as long-term to differentiate from the short-term bond \( B^S_t \) which is a one period bond with zero coupon, i.e \( \lambda = 0 \), with price \( Q^S_t \). We assume that the supply of short-term bond is very limited and that they account for a little share of the households' portfolio. In addition, they hold \( N^O_t \) claims on other households' capital and sell claims on own capital to other household \( N^I_t \). The financing structure implies the households' balance sheet at the beginning of period \( t \) in table [2].

**Table 2: Household’s balance sheet (financial assets)**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital stock: ( q_t K_t )</td>
<td>Equity issued: ( q_t N^I_t )</td>
</tr>
<tr>
<td>Others’ equity: ( q_t N^O_t )</td>
<td></td>
</tr>
<tr>
<td>Long-term bonds: ( Q^L_t \frac{B^L_t}{P_t} )</td>
<td>Net worth: ( q_t N_t + Q^L_t \frac{B^L_t}{P_t} + Q^S_t \frac{B^S_t}{P_t} )</td>
</tr>
<tr>
<td>Short-term bonds: ( Q^S_t \frac{B^S_t}{P_t} )</td>
<td></td>
</tr>
</tbody>
</table>

Following the simplification of KM, we assume that equity issued by the other households and the unmortgaged capital stock \( (K_t - N^I_t) \) yield the same returns, have the same value and depreciate at the same rate, so they are perfect substitute and can be summed together and defined as equity.

\[19\] This assumption avoids that household’s members hold government bonds only with short maturity because are more liquid and is consistent with the evidence supported by Eurostat (2013) that in Europe one-year bonds account for just 5% of the total public debt and that the average maturity is between 6 and 7 years, which is matched in the calibration of the model. In Leeper and Zhou (2013) government may find optimal to issue bonds with long maturity because they facilitate the intertemporal smoothing of inflation and output gap. In the steady state the prices of long-term and short-term bonds are linked by a non-arbitrage condition.
\[ N_t = N_t^O + K_t - N_t^I \]  

(2)

At the end of each period households also receive profits \( D_t \) and \( D_t^I \) respectively from a non-tradable portfolio of intermediate goods producing firms and capital producing firms from which they receive per-period profits of. The budget constraint of the typical household member \( j \) can be written as the following

\[
C_t(j) + p_I^t I_t(j) + q_t [N_{t+1}(j) - I_t(j) - \lambda N_t] + Q_t^L \left[ \frac{B_{t+1}^L(j)}{P_t} - \lambda \frac{B_t^L}{P_t} \right] + Q_t^S B_{t+1}^S(j) \\
= r^L_t N_t + \frac{B_t^L}{P_t} + \frac{B_t^S}{P_t} - H_t(j) + D_t + D_t^I - T_t
\]

(3)

where \( P_t \) denotes the price level, \( p_I^t \) is the cost of a unit of new capital in terms of the consumption goods, differing from 1 because of capital adjustment cost, \( H_t(j) \) and \( W_t(j) \) are the working hours and nominal wage for workers \( j \). Households members allocate their resources (LHS) between purchase of non-storable consumption good, investment in new capital - if they are entrepreneurs - and savings in different assets (equity, long-term bonds and short-term bonds). They finance their activities (RHS) with returns on equity, on long-term bonds, short-term bonds, wages - if they are workers - and the dividends of final and intermediate firms net to taxes.

A key assumption of the model is the presence of following funding constraints which limit the financing of new investments by entrepreneurs and determine the liquidity of the asset.

\[
N_{t+1}(j) \geq (1 - \theta) I_t(j) + \lambda N_t
\]

(4)

\[
B_{t+1}^L(j) \geq (1 - \phi_t) B_t^L
\]

(5)

\[
B_{t+1}^S(j) \geq 0
\]

(6)

Inequality 4 means that the entrepreneur can issue claims on the future output of investment but only for a fraction \( \theta \in [0, 1] \). This borrowing constraint implies that investment is partially funded internally and entrepreneurs have to retain \( 1 - \theta \) as own equity. In addition, equity
is assumed to be completely illiquid and entrepreneurs cannot sell it to obtain more resources to invest. Hence, the entrepreneurs’ equity holding at the start of the period $t+1$ must be at least the sum of the downpayment $(1 - \theta)I_t$ and depreciated equity $\lambda N_t$, where $\lambda$ is the inverse depreciation rate.

The entrepreneur can acquire additional resources by disposing of a fraction $\phi_t \in [0, 1]$ of long-term bonds, so a resaleability constraint imposes to keep the residual $(1 - \phi_t)$ of bonds in its portfolio (inequality 5). $(1 - \phi_t)$ is equivalent to the haircut in the repo market since it determines the quantity of liquidity that the borrower can obtain by pledging a government bond in secured borrowing. The assumptions on the different resaleability of equity and bonds reflect the characteristics of the European money market, in which private assets are scarcely used as collateral, while bonds are typically posted for repo transactions.

Inequality 6 implies that short-term bonds are not subject to resaleability constraint and are fully liquid, but entrepreneurs cannot borrow from the government. $\phi_t$ is the key parameter of the model characterising the liquidity of financial assets. We can think that it takes value 1 for equity, value 0 for short-term bonds and an intermediate value for long-term bonds. The dynamic of the model follows a reduction in $\phi_t$, which is paramount of a rise in the repo haircut on sovereign bonds.

At the end of the period, the assets of the households are given by

\begin{align*}
N_{t+1} &= \int N_{t+1}(j) dj, \quad (7) \\
B_{t+1}^{L} &= \int B_{t+1}^{L}(j) dj, \quad (8) \\
B_{t+1}^{S} &= \int B_{t+1}^{S}(j) dj, \quad (9)
\end{align*}

---

20 This follows from the assumption that there is not insurance market against having an investment opportunity, so that the market is incomplete. If $\theta = 1$ any investment opportunities can be shared across entrepreneurs and the market is complete.

21 Keeping the original interpretation of KM, a fall in $\phi_t$ also describes a contraction in the market liquidity of European sovereign bonds, as shown by the rise in the bid-ask spread of public securities during the crisis.
\[ K_{t+1} = K_t + \int I_t(j) \, dj, \] (10)

Next, we can take into account the specific functions of entrepreneurs workers.

### 3.1.1 Entrepreneurs

Entrepreneur \( j \in [0, \gamma] \) does not supply labour, so we set \( H_t(j) = 0 \) in equation (3) to get its budget constraint. In order to acquire new equity they can either produce it at price \( p_t^L \) or buy it in the market at price \( q_t \). For the rest of the model we assume that \( q_t > p_t^L \). In this case, entrepreneurs will use all the available liquidity for new investment projects to maximize the households’ utility. Accordingly, they issue the maximum amount of new equity and they sell as much bonds as possible, because their expected return is lower than the expected return on new investments. In equilibrium, the funding constraints all bind and the entrepreneur does not buy consume goods within the period:

\[ N_{t+1}(j) = (1 - \theta)I_t(j) + \lambda N_t \] (11)

\[ B^L_{t+1}(j) = (1 - \phi_t)B^L_t \] (12)

\[ B^S_{t+1}(j) = 0 \] (13)

\[ C(j) = 0 \] (14)

Since we have the solution for entrepreneurs, \( N_{t+1}(j)B^L_{t+1}(j), B^S_{t+1}(j), C_t(j) \) for \( j \in [0, \gamma] \), we can plug equations (11), (12), (13) and (14) into equation (??) to derive the function of investment for entrepreneurs

\[ I_t(j) = \frac{r_t^K N_t + [1 + \lambda \phi_t Q^L_t] \frac{B^L_t}{p^L_t} + Q^S_t \frac{B^S_t}{p^S_t} + D_t + D^L_t - T_t}{p^L_t - \theta q_t} \] (15)

The nominator represents the maximum liquidity available for the entrepreneurs deriving from the return on papers (equity and long-term bonds), sales of the resaleable fraction of long-term bonds after depreciation, sales of short-term bonds and the dividends net taxes. The denominator
is the difference between the price of one unit of investment goods and the value of equity issued
by the entrepreneur which indicates the amount of own resources necessary to finance for one unit
of investment. The inverse of this gap is equivalent to the leverage. Aggregating by entrepreneurs

\[ I_t = \int_0^\gamma I_t(j) dj = \gamma \frac{r_t^k N_t + [1 + \lambda \phi_t Q_t^L] B_t^L + Q_t S B_t^S + D_t + D_t^I - T_t}{p_t^L - \theta q_t} \]  

(16)

3.1.2 Workers

Workers \( j \in [\gamma, 1] \) do not invest, so \( I_t(j) = 0 \). They supply labour as demanded by firms at a
fixed wages, as the union who represents each type of worker sets wages on a staggered basis.
To determine the asset and consumption choices of workers, we first derive the household’s
decision for \( N_{t+1}, B_{t+1}^L, B_{t+1}^S \) and \( C_t \), taking \( W_t \) and \( H_t \) as given. Knowing the solution for
employers, we can then determine \( N_{t+1}(j), B_{t+1}^L(j), B_{t+1}^S(j) \) and \( C_t(j) \) for workers, given
the aggregate consumption and asset holding.

3.1.3 Households’ problem

To solve the model for the household, we aggregate the workers’ and entrepreneurs’ budget
constraint

\[ C_t + p_t^L I_t + q_t [N_{t+1} - I_t - \lambda N_t] + Q_t^L \left[ \frac{B_{t+1}^L}{P_t} - \lambda \frac{B_t^L}{P_t} \right] + Q_t S B_{t+1}^S \]

\[ = r_t^k N_t + \frac{B_t^L}{P_t} + \frac{B_t^S}{P_t} + \int_\gamma^1 W_t(j) H_t(j) + D_t + D_t^I - T_t \]  

(17)

Households maximize the utility function \((2)\) by choosing \( C_t, N_{t+1}, B_{t+1}^L \) and \( B_{t+1}^S \) subject
to the aggregate budget constraint and the investment constraint. The first order conditions for
equity, long-term bonds and short-term bonds are respectively:

\[ U'_{c,t} = \beta E_t \left( U'_{c,t+1} \left[ \frac{r_{t+1}^L + \lambda q_{t+1}}{q_t} + \frac{\gamma (q_{t+1} - p_{t+1}^L)}{P_{t+1}^L - \theta q_{t+1} q_t} \right] \right) \]  

(18)
\[ U'_{c,t} = \beta E_t \left\{ \frac{1}{\pi_{t+1}} U'_{c,t+1} \left[ \frac{1 + \gamma q_{t+1}}{Q^S_t} + \frac{(q_{t+1} - p^I_{t+1})}{p^I_{t+1} - q_{t+1}} \frac{1 + \lambda Q^L_{t+1}}{Q^L_t} \right] \right\} \] (19)

\[ U'_{c,t} = \beta E_t \left\{ \frac{1}{\pi_{t+1}} U'_{c,t+1} \left[ \frac{1}{Q^S_t} + \frac{(q_{t+1} - p^I_{t+1})}{p^I_{t+1} - q_{t+1}} \frac{1}{Q^S_t} \right] \right\} \] (20)

where \( \pi_t \) is the inflation rate defined as \( \pi_t = \frac{P_{t+1}}{P_t} \). The choice of sacrificing one unit of consumption today to purchase a paper gives a payoff which is composed by two parts. The first is the returns on the asset: \( \frac{r_{t+1} + \lambda q_{t+1}}{q_t} \) for equity, \( \frac{1 + \lambda Q^L_{t+1}}{Q^L_t} \) for long-term bonds and 1 for short-term bonds. The second part is a "liquidity premium", deriving from the fact that the papers in the entrepreneurs' portfolio relax their investment constraint. This premium is a function of the leverage \( \gamma q_{t+1} - p^I_{t+1} \), the distance between the price of equity and the price of capital goods, and the liquid returns of each asset. Thus, the bond holding eases the financing constraints more than the equity holding which makes bonds more valuable for entrepreneurs.

### 3.2 Firms

Competitive final-goods producers combine intermediate goods \( Y_i \) where \( i \in [0, 1] \) indexes intermediate-goods-producing firms, to sell a homogeneous final goods \( Y_t \) according to the technology.

\[ Y_t = \left[ \int_0^1 Y_{it}^{1+\lambda_f} \, di \right]^{1+\lambda_f} \] (21)

where \( \lambda_f > 0 \). Their demand for the generic \( i^{th} \) intermediate good is

\[ Y_{it} = \left[ \frac{P_{it}}{P_t} \right]^{-\frac{1+\lambda_f}{\lambda_f}} Y_t \] (22)

where \( P_{it} \) is the nominal price of good \( i \). The zero profit condition for competitive final goods producers implies that the aggregate price level is

\[ P_t = \left[ \int_0^1 P_{it}^{-\frac{1}{\lambda_f}} \, di \right]^{-\lambda_f} \] (23)
The intermediate-goods firm $i$ uses $K_{it}$ units of capital and $H_{it}$ units of composite labor to produce output $Y_{it}$ according to the production technology

$$Y_{it} = K_{it}^\alpha H_{it}^{1-\alpha} \quad (24)$$

where $\gamma \in (0, 1)$. Intermediate-goods firms operate in a monopolistic competition and set prices on staggered basis à la Calvo (1983), taking the real wage $W_t^r$ and the rental rate of capital $r_t^K$ as given. With probability $1 - \zeta_p$, the firms can reset its price and with probability $\zeta_p$ they cannot. By the law of large number, the probability of changing the price corresponds to the fraction of firms that reset the price and they choose the price $\tilde{P}_t$ to maximize the present discounted value of profits.

### 3.2.1 Labour agencies

Competitive labour agencies combine $j$-specific labour inputs into a homogeneous composite $H_t$ according to

$$H_t = \left[ \left( \frac{1}{1 - \chi} \right)^{\frac{\lambda\omega}{1+\lambda\omega}} \int_1^1 H_t(j)^{\frac{1+\lambda\omega}{1+\lambda\omega}} dj \right]^{1+\lambda\omega} \quad (25)$$

where $\lambda\omega > 0$. Labour agencies provide the labour input to firms at the wage $W_t$ and remunerate the households for the labour supplied. Labour agencies do not have profit for their intermediation:

$$W_t H_t = \int_1^1 W_t(j) H_t(j) dj \quad (26)$$

where $W_t$ is the aggregate wage index. Labour agencies maximize the profit function $\frac{26}{26}$ subject to $\frac{25}{25}$, taking wages $W_t(j)$ as given. The first order condition determines the demand for the $j^{th}$ labour input

$$H_t(j) = \frac{1}{1 - \chi} \left[ \frac{W_t(j)}{W_t} \right]^{\frac{1+\lambda\omega}{1+\lambda\omega}} H_t \quad (27)$$

where $W_t(j)$ is the wage specific to the labour input $j$. From the zero profit condition for labor
agencies the aggregate wage index is

\[ W_t = \left[ \frac{1}{1 - \chi} \int_1^\chi W_t(j)^{\frac{\chi - 1}{\chi}} dj \right]^{-\lambda} \] (28)

Labour agencies set wages on a staggered basis, taking as given the demand for their specific labour input. Each period, labour agencies are able to reset the wage \( W_t(j) \) with probability \( 1 - \zeta \omega \) and with probability \( \zeta \omega \) they cannot and the wage remains fixed. By the law of large number, the probability of changing the wage corresponds to the fraction of workers whose wages change. Households supply whatever labour is demanded at that wage. If labour agencies can modify the wage, they will chose the wage \( \tilde{W}_t \) to maximize their utility function.

### 3.2.2 Capital-good producers

The creation of new capital is delegated to competitive capital-good producers who transforms consumption goods into investment goods. They choose the amount of investment goods to maximize the profits taking the price of investment goods \( p^I_t \) as given

\[ D^I_t = \left\{ p^I_t - \left[ 1 + f\left( \frac{I_t}{T} \right) \right] \right\} I_t \] (29)

The price of investment goods differ from the price of consumption goods because of the adjustment costs, which depends on the deviations of actual investment from its steady-state value. \( f(.)I_t \) reflect the adjustment cost and \( f(.) \) is a measure of technology illiquidity, capturing the difficulty to undo investment. We assume that \( f(1) = f'(1) = 0 \) and \( f''(1) > 0 \). The first order condition for this problem is

\[ p^I_t = 1 + f\left( \frac{I_t}{T} \right) + f\left( \frac{I_t}{T} \right) \frac{I_t}{T} \] (30)

The law of motion of capital is

\[ K_{t+1} = \lambda K_t + I_t \] (31)

We consider the following identity equation between capital and net equity
\[ K_{t+1} = N_{t+1} \]  

The resource constraint can be expressed as

\[ Y_t = C_t + \left[ 1 + f\left(\frac{I_t}{T}\right) \right] I_t \]  

(33)

Finally, considering the aggregate expression for \( D_t \) for the whole economy and \( D_t^I \), the investment function can be rewritten as:

\[ I_t = \chi r^K N_t + \left[ 1 + \lambda \phi_t Q^L_t \right] \frac{B^L_t}{P_t} + Q^S_t \frac{B^S_t}{P_t} + Y_t - w_t H_t - r^K_t + p^I_t I_t - I_t [1 + f(I_t)] - \tau_t \]  

(34)

3.3 The Government

The government conducts fiscal policy and monetary policy separately following exogenous policy rules\(^\text{22}\) The government faces the following intertemporal budget constraint:

\[ Q^L_t \left( \frac{B^L_{t+1}}{P_t} - \lambda \frac{B^L_t}{P_t} \right) + Q^S_t \frac{B^S_t}{P_t} + T_t = \frac{B^S_t}{P_t} + \frac{B^L_t}{P_t} \]  

(35)

The debt repayment is financed by the issue of new debt and a net taxes \( T_t \). A solvency condition links taxes with the outstanding beginning-of-period government debt in term of deviation from the steady state

\[ T_t - T = \psi_T \left( \frac{B^L_t}{P_t} - \frac{B^L_t}{P} \right) \]  

(36)

where \( \psi_T > 0 \) measure the elasticity of fiscal policy to variations in the size of the debt. The government sets the nominal interest rate following the feedback rule constrained by the zero lower bound condition

\[ R_t = \max \left( \pi_t^{\psi}, 0 \right) \]  

(37)

\(^{22}\)See Cui and Guillen (2013) for a study of optimal policy in a model with liquidity frictions.
where \( \psi > 1 \). Unconventional policy consists in choosing the supply of one-period bonds (in term of equity) as a function of the proportional deviations of liquidity from the steady state.

\[
\frac{B_{t+1}^S}{N_t} = \psi_B \left( \frac{\phi_t}{\bar{\phi}} - 1 \right)
\]

(38)

The price of the nominal short-term bond is the inverse of the nominal rate, so the government by setting the nominal interest rate, also sets the price of short-term bonds.

\[
Q_t^S = \frac{1}{R_t}
\]

(39)

Recent papers propose theoretical models to analyse the effects of non conventional monetary policy operations. In DEFK and KM, the government conducts open market operations through which supplies liquid assets (money or T-bills) against illiquid asset, such as mortgage-back securities. Curdia and Woodford (2009) study the impact of quantitative easing in a model with financial intermediation in which the central bank increases the supply of reserves. Gertler and Karadi (2009) and Gertler and Kiyotaki (2010) introduce agency problems between depositors and banks and study the effect of the direct lending and the discount windows policy implemented by the Federal Reserve. In this model, at the zero lower bound short-term bonds become equivalent to money and the unconventional policy studied in this model is similar to the Long Term Refinancing Operations (LTROs). When the interbank market frozen, the ECB provided large-scale funds to European banks easing their liquidity constraints. Drechsel et al. (2012) find that the banks who borrowed more from the ECB were the ones with riskier collateral benefiting the lower haircuts applied by the ECB than private repo markets. In this model, when the haircuts to public papers increase the government can respond by injecting in the economy more liquid papers.
4 Numerical simulation

4.1 Calibration

The model is calibrated at quarterly frequency to match the economy of Ireland because of the rich dynamics of the haircut on the Irish bonds that is used to calibrate the process of the liquidity parameter. Table 3 reports the calibrated values. The steady state of the liquidity is 0.75 which is equivalent to one minus the level of the haircuts on Irish bonds before the crisis (25%). We assume that follows an AR(1) process and we estimate its parameters from the dynamics of Irish haircut during the period of crisis. The autoregressive coefficient $\rho = 0.99$ and we take the standard deviation of the residuals $\sigma^2$ to calibrate the size on the liquidity shock. The other parameter characterizing the financial frictions $\theta$ is 0.5. The liquidity share is defined as

$$ l_s(t) = \frac{\phi Q_t B_t^{L+1}}{\phi Q_t B_t^{L+1} + \gamma P_t K_{t+1}} \quad (40) $$

The nominator is the liquid part of the public debt (the total of Irish government gross liabilities times the liquidity parameter) and the denominator is equal to the value of the total productive capital (OECD Economic Outlook, 2013). The average of this ratio between 2000 and 2011 is 0.43. The share of non depreciated capital (one minus depreciation rate) $\lambda$ is 0.973, giving the duration of long-term bond $(1 - \lambda \beta)^{-1}$ of 6.9 years, which is the average of maturity of the Irish debt (Eurostat, 2013).

Other parameters are standard in the literature and are taken from DEFK: the discount factor $\beta = 0.99$, the inverse Frisch elasticity of labor supply $\eta = 1$, the capital share $\alpha = 0.4$, the arrival rate of investment opportunity in each quarter $\gamma = 0.05$. The degree of monopolistic competition in labor and product markets are calibrated symmetrically assuming a steady state markup of 10% ($\delta_p = \delta_w = 0.75$). The average duration of price and wage contracts is 4 quarters. Concerning the policy rules, feedback coefficient on inflation in the monetary policy rule is 1.5 to guarantee a uniquely determined equilibrium. Transfers slowly adjust to the government debt ($\psi_T = 0.1$) and the coefficient of the intensity of government intervention $\psi_B$ is -0.127.
Table 3: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>( \beta )</td>
<td>Discount factor</td>
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<td>( \sigma )</td>
<td>Relative risk aversion</td>
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<td>( \gamma )</td>
<td>Probability of investment opportunity</td>
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<tr>
<td>( \lambda )</td>
<td>Inverse depreciation rate</td>
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</tr>
<tr>
<td>( \alpha )</td>
<td>Capital share</td>
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<td>( f'(1) )</td>
<td>Adjustment cost parameter</td>
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<tr>
<td>( \eta )</td>
<td>Inverse Frish elasticity</td>
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<td>( \zeta_p = \zeta_w )</td>
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<tr>
<td>( \delta_p = \delta_w )</td>
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<td>( \theta )</td>
<td>Borrowing constraint</td>
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</tr>
<tr>
<td>( \phi )</td>
<td>Resaleability constraint</td>
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</tr>
<tr>
<td>( L )</td>
<td>Steady-state liquidity/GDP</td>
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</tr>
<tr>
<td>( \psi_e )</td>
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<td>( \psi_T )</td>
<td>Transfer rule coefficient</td>
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<td>( \psi_B )</td>
<td>Government intervention coefficient</td>
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<td>( \rho_{\phi} )</td>
<td>Autoregressive coefficient in the productivity and liquidity</td>
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<tr>
<td>( \sigma_{\phi} )</td>
<td>Size of the liquidity shock</td>
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4.2 Results

4.2.1 The impact of a liquidity shock

We first analyse the economy in which the government does not respond to the negative liquidity shock. Figure 7 shows the response of output, investment, consumption and prices of financial assets to a drop in \( \phi_t \). The model suggests a contraction of 0.14% on impact, mostly due to the fall in investment by 0.25%. The reduction of liquidity of long-term bonds reduces investments because it tightens the entrepreneurs’ funding constraint and shrinks the funds they can obtain from the sale of the bonds. The presence of nominal rigidities is at the root of the fall in consumption. With flexible price the contraction in output would generate deflationary expectations leading to a reduction in the real interest rate inducing the household to spend more. The fall in the prices of equity and long-term bonds have a negative wealth effect on consumption. In addition, the decrease in the value of equity amplifies the fall in investment since the size of the required downpayment per unit of investment increases reducing the leverage of entrepreneurs.
The persistence of the liquidity shock induces entrepreneurs to anticipate lower future resaleability leading to a flight to liquidity towards the more liquid short-term bonds whose price increases. It results in a rise in the spread between the value of the more liquid and the less liquid securities. This mechanism is consistent with the widening of the yield spread between liquid bonds of the core and illiquid bonds of periphery in the midst of the European financial crisis.

Figure 7: Impulse Response Function to a negative liquidity shock

4.2.2 The effects of the policy intervention

We consider now the scenario in which the government responds to the drop in \( \phi_t \) by issuing more short-term bonds. Figure 8 compares the impulse responses to a negative liquidity shock in case the government does not intervenes (blue line) and in case in which does react (red line). The model predicts that this unconventional policy succeeds in alleviating the contractionary effect of the shock. Output decrease by 0.04% and investment by 0.1%, respectively 0.1% and 0.15% less than in the laissez-faire economy. The fall in consumption drastically lessens, mainly because the reduction in the price of equity is less pronounced, which also reduces the deleveraging of entrepreneurs. The increased supply of liquid short term bond relaxes the funding constraint of entrepreneurs which shifts their portfolio towards the most liquid bonds. Consequently, the price of long-term bonds contracts even more and the price of short-term bonds increases less
than in a laissez-faire economy because of the higher supply.

Figure 8: The effects of policy intervention

4.2.3 The zero lower bound

Finally we assess the consequences of the liquidity shock in the two scenarios considering the zero lower bound condition. Figure 9 displays that when the monetary policy is constrained by the ZLB the impact of a liquidity shock is amplified in the context of both a laissez-faire economy and the economy in which the government conducts unconventional policy (dashed lines). This is more evident for consumption because the nominal interest rate cannot be reduced in the negative region and monetary policy cannot boost consumption expenditure through a reduction in the real interest rate. Concerning financial assets, the unconventional policy has a smaller impact on the price of long-term and short-term bonds, while reduces the drop in the price of equity. The two LTROs implemented by the ECB in December 2011 and February 2012 have reduced the yields of sovereign bonds through two channels. First, the injection of 1 trillion Euro of cheap funding to European banks relaxed their funding constraint facilitating their investment in domestic bonds. Second, it alleviated the liquidity strain on peripheral bonds following the increased haircuts in the private repo market, since banks could post the illiquid bonds as
collateral in the secured loans with the ECB. The first channel is captured by the model, since the ECB provided money with very low interest rates and low haircuts, with a similar impact of the increased supply of liquid bonds in the banks’ funding. The unconventional policy analysed in this paper leads to a substitution between long-term and short-term bonds. In this set-up, the LTROs would be equivalent to an increase in $\phi_t$ which increases the resealability of long-term bonds because entrepreneurs can obtain additional funds from the ECB with haircuts lower than in the private market.

Figure 9: The effects of policy intervention

![Graphs showing the effects of policy intervention](image)

Note: dashed lines are the IRFs under the zero lower bound

5 Empirical analysis

We now study the dynamic relationship between funding liquidity, credit risk and yields of government bonds during the period of crisis and we examine whether the data is consistent with the view that increases in haircuts have a negative impact on the value of bonds. We include the credit risk as European clearing houses set the haircuts as a function of the market risk of the assets. First, we test if credit risk leads funding liquidity and if funding liquidity leads the yields. Second, we estimate the impulse response functions (IRF) of a credit risk shock and a liquidity shock with a structural vector autoregressive (SVAR) model. We focus the analysis
on the Irish sovereign debt market because the evolution of the 10-year bond yields and the repo haircut to these securities provide us with a laboratory to study the interaction between those variables. From November 2010 to July 2011 the yields rose from around 8% to 14% and then decreased until 7% in February 2012. During the same period, the haircut applied by LCH Clearnet Ltd grew from 15% to 80% before reducing to 25% following the fall in the yields. We censored the sample from 01-11-2010 to 01-12-2011, because during this time the Security Market Program (SMP) was already active, but we exclude the large-scale LTROs in December 2011 and in February 2012 which could have altered the relationship between the variables. Even though purchases of Irish bonds in the secondary market through the SMP is likely to have affected the bond yields and credit risk of Irish bonds, data on the composition of the acquired assets by nationality of issuance and maturity are not publicly available.\footnote{Ghyseler, Idier, Manganelli and Vergote (2014) and Easer and Shwaab (2014) find that the SMP was successful in reducing bond yields of Irish bonds as well as Italian, Portuguese, Greek and Spanish bonds.} However, we can control for the shift in market sentiment that could be derived from the announcement of the adoption of this policy in May 2010.

Credit risk is measured by changes in CDS spread on Irish sovereign debt and funding liquidity by the levels of the haircut applied by the main European clearing house, LCH Clearnet Ltd, as shown by figure [7]. Empirical literature generally consider the cost of funding in the unsecured market, such as the Euribor-Eonia spread, as proxy for funding liquidity. Consistent with our initial interpretation of funding liquidity and because of the shrink in the unsecured interbank market during the crisis, we propose an alternative indicator which is specific to an asset and not to a market rate. Irish Government-specific credit risk is measured by the spread of a senior five-year dollar-denominated CDS contract obtained from Datastream and the yields on Irish bonds are taken from the Global Finance database. All the data are at daily frequency.

As preliminary analysis, we start by studying the copula function between haircuts and yields. The marginal cumulative distribution function and the joint cumulative distribution functions are estimated non parametrically by kernel methods. Figure [10] displays the scatter. There is a strong and positive link between the two variables, in particular for the upper-tail probabilities as we can see at the top-right corner of the graph. This is confirmed by the coefficient of linear correlation parameter of copula which is 0.59.
We next analyse the dynamic interaction between credit risk, funding liquidity and bond yields. In order to investigate whether there exists a lead-lag relationship between the variables, we use a Granger causality test to assess if past value of the variable $i$ helps predict the variable $j$, above and beyond the information contained in past values of $j$ alone. To do so, we estimate the following reduced-form VAR including in the following order haircut, CDS spread and 10-year sovereign bond yields. This exercise is close to Pellizzon et al. (2014) who examine the dynamic interaction between credit risk and market liquidity in the Italian sovereign debt market; they find that credit risk Granger causes market liquidity, which could have an indirect impact on the bond yields. However, they do not find the opposite relationship significant.

$$Y_t = \alpha_0 + \sum_{j=1}^{p} A_j Y_{t-j} + \epsilon_t$$

where $y_t$ is $(3 \times 1)$ vector, $\alpha_0$ is a $(3 \times 1)$ vector of intercepts, $A_j$ is a $(3 \times 3)$ matrix of coefficients, , $\epsilon_t$ is a $(3 \times 1)$ vector of residuals and $\epsilon_t$ is i.i.d. $N(0, \Sigma)$.

Because of the limited size of the sample the model is estimated with Bayesian techniques. Diffuse priors based on OLS estimation on the overall sample are used. In particular, we take the Normal and Inverse Gamma distributions for the estimation of the coefficients and the volatilities. The appendix present the details of the estimation strategy.
Table 4: Granger causality tests

<table>
<thead>
<tr>
<th>Test</th>
<th>F-statistic</th>
<th>critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>∆ yields → ∆ haircut</td>
<td>9.582</td>
<td>0.908</td>
</tr>
<tr>
<td>∆ haircut → ∆ yields</td>
<td>3.305</td>
<td>0.456</td>
</tr>
<tr>
<td>∆ yields → ∆ CDS spread</td>
<td>6.222</td>
<td>0.456</td>
</tr>
<tr>
<td>∆ CDS spread → ∆ yields</td>
<td>9.205</td>
<td>0.695</td>
</tr>
</tbody>
</table>

Table 4 show the F statistics for the test. Both variations in CDS spreads and haircut are significant to predict variations in the bond yields, suggesting that credit risk and funding liquidity lead the value of the government securities. In turn, bond yields Granger causes credit risk and funding liquidity. The latter results may be explain by the fact that LCH Clearnet Ltd changes the haircut when the spread with the German bond yields exceed 450 basis points and considers other indicators for credit risk, which is consistent with the significance of the test of the variations in CDS spreads on the variations in haircuts. All in all, this exercise shows the rich dynamics between funding liquidity, credit risk and bond yields and suggests that credit risk may have an indirect impact on the yields not only via market liquidity, as found by Pellizzon et al. (2014), but also via funding liquidity.

In order to assess the impact of a rise in haircuts on government bond yields avoiding endogeneity problems and reverse causality issues, we estimate the IRF of a structural liquidity shock and credit risk shock. Structural shocks are recovered using the triangular Cholesky decomposition. The main assumption of this identification scheme is that haircuts respond to yields with one lag. This is due to the one-day delay between the communication of the clearing house of variations in haircuts and the implementation of this decision. Variations in haircut are assumed to affect simultaneously CDS spread and yields. The second assumption is that credit risk responds instantaneously to variations in the haircuts but not to variations in the yields. Figure 11 compares the impulse response function of a shock of haircut and a shock of CDS spreads on the yields of Irish government bonds to disentangle the liquidity and credit risk channels.

Yields rise significantly following a credit risk shock and a after a negative liquidity shock, suggesting that the two channels are important for the dynamics of the yields. In particular, this seems to confirm the mechanism of margin spiral for Irish government securities during the most acute phase of tensions in the sovereign-debt market in 2011 and a negative feedback loop
between the price of government bonds and the haircuts.

Figure 11: Impulse response function of a liquidity shock and a credit risk shock

6 Conclusions

This paper has explored the liquidity channel of the European sovereign debt crises. It has shown that government securities play a key role as collateral in the European repo market easing the interbank lending in the Eurozone. Nevertheless, during the crisis repo haircuts of peripheral government bonds drastically augmented in the wake of the raise in sovereign risk. The escalation of sovereign yields and haircuts suggest an alternative channel to the European “twin crises” - the combination of sovereign weakness and banking fragility - in addition to the balance sheet channel (Gennaioli et al., 2014 b) and bail-out channel (Acharya et al., 2011). Banks accumulated government bonds to store liquidity and use them as collateral to lever up, increasing their exposure on foreign bonds due to the European financial integration. When the sovereign risk of peripheral countries rose, repo haircuts of public bonds surged drying up interbank liquidity and weakening the banking system. Banks had to reduce simultaneously their position on peripheral bonds with higher haircuts causing a fire sale which depressed the value of these securities even more resulting in further increases in haircuts. We studied the
consequences of a reduction in the liquidity of public bonds with a model incorporating liquidity frictions to simulate the impact of a raise in haircuts on government securities. The model exhibits a fall in output, investment, consumption and in the price of long-term government bonds. The contractionary effects of this shock can be alleviated by a policy response consisting in issuing new short-term bonds to provide the investor with an alternative liquid asset that relaxes the funding constraint of investors. We have assessed empirically the main results of the model by estimating the impulse response function of a liquidity shock in a SVAR model including yields, haircut and CDS spreads for Ireland to disentangle liquidity and credit risk effects. The theoretical model can be extended in several dimensions. One is to introduce risk of sovereign default to endogenise the liquidity parameter as a function of the debt riskiness and a Value-at-Risk constraint for entrepreneurs. When the probability of default rises, the haircut also increases creating a feedback between the liquidity and riskiness of the government bonds. A second extension would be that of consider an open economy with two countries conducting independent fiscal policy and sharing the monetary policy in a typical framework of a monetary union in order to study the impact of a liquidity shock on the bond issued by one country and the possible policy responses of the central bank.
References


Appendix A: Figures

Figure 12: Yields and haircuts on 10-year government bonds (LCH.Clearnet SA)
- Italy
- Spain

Figure 13: Haircuts on 10-year government bonds (CC&G)

Appendix B: Solving the model

B1. Optimality conditions in the good market and labour market

At each period $1 - \zeta_p$ intermediate-goods firms set the price $\hat{P}_{it}$ to maximize the present discounted value of profits.
subject to the demand for its own goods (22) and conditional on not changing its price. The problem of intermediate goods producers can be solved in two steps. First, they choose the optimal amount of inputs (capital and labor) and they minimize the costs, \( w_t H_t - r_t K_t \), subject to the production of intermediate goods (24). The first order condition is

\[
\frac{K_t}{H_t} = \frac{\alpha}{1 - \alpha} \frac{w_t}{r_t} = \frac{K_t}{H_t}
\]

(43)

Since the capital-labour ratio at the firm-level is independent of firm-specific variables, then the marginal cost \( mc_{it} \), i.e. the Lagrange multiplier on the constraint, is also independent of firm-specific variables

\[
m_{ct} = \left( \frac{r_t}{\alpha} \right)^{\alpha} \left( \frac{w_t}{1 - \alpha} \right)^{1 - \alpha} = mc_t
\]

(44)

In the second step the \((1 - \zeta_p)\) firms that can change the price, will choose \( \tilde{P}_{it} \) to maximize

\[
E_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{t-s} \left[ \frac{\tilde{P}_{it}}{P_s} - (1 + \lambda_f)mc_s \right] Y_s(i) = 0
\]

(45)

We focus on a symmetric equilibrium in which all firms choose the same price \( \tilde{P}_{it} = \bar{P}_t \). Let \( \bar{p}_t = \tilde{P}_t / P_t \) the optimal relative price. The first order condition for optimal price settings becomes

\[
E_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{t-s} \left[ \frac{\bar{p}_t}{\pi_{t,s}} - (1 + \lambda_f)mc_s \right] \left( \frac{\bar{p}_t}{\pi_{t,s}} \right)^{-\frac{\lambda_f}{s\pi_t}} Y_s = 0
\]

(46)

By the law of large number, the probability of changing the price coincides with the fraction of firms who change the price in equilibrium. From the zero profit condition (23), inflation depends on the optimal reset price according to

\[
1 = (1 - \zeta_p)\bar{p}_t^{-\frac{1}{\lambda_f}} + \zeta_p \left( \frac{1}{\pi_t} \right)^{-\frac{1}{\lambda_f}}
\]

(47)

Finally, since the ratio of capital-output is independent of firm-specific factors, the aggregate
production function is

$$K_t^\gamma H_t^{1-\gamma} = \int_0^1 Y_{it} di = \sum_{s=0}^\infty \zeta_p (1-\zeta_p)^{1-s} \left( \frac{\tilde{p}_{t-s}}{\pi_{t-s,t}} \right)^{1+\lambda_{it}} Y_i$$

where $K_t = \int_0^1 K_{it} di$ and $H_t = \int_0^1 H_{it} di$

Each period, $1-\zeta_\omega$ labour agencies are able to reset the wage $W_t(j)$ to minimize the present discount value of the disutility of work

$$E_t \sum_{s=t}^\infty (\beta \zeta_\omega)^{s-t} \left[ C_s^{1-\sigma} \frac{1}{1-\sigma} - \frac{\xi}{1+\eta} \int_{\chi}^1 H_s(j)^{1+\nu} dj \right]$$

subject to (17) and (27). The first order condition for this problem is

$$E_t \sum_{s=t}^\infty (\beta \zeta_\omega)^{s-t} C_s^{1-\sigma} \left[ \frac{W_{t+s}(j)}{P_s} - (1+\lambda_\omega) \frac{\omega H_s(j)^{\nu}}{C_s^{1-\sigma}} \right] H_s(j) = 0$$

We focus on a symmetric equilibrium in which all agencies choose the same wage. Let $w_t = W_t/P_t$. From equation (28) the law of motion of real wage is

$$w_t = \frac{\chi}{\zeta_\omega} w_{t+s} + \zeta_\omega \left( \frac{w_{t-1}}{\pi_t} \right)^{1-\zeta_\omega}$$

**B2. Equilibrium conditions**

To solve the model we define $L_{t+1} = B^L_{t+1}/P_t$, as real long-term bonds. The total factor productivity and resaleability constraint $(A_t, \phi_t)$ follow an exogenous AR(1) process and there are 4 endogenous state variables: the aggregate capital stock, the nominal short-term bond, the real long-term bond and the real wage rate from the previous period $(K_t, B^S_t, L_t, w_{t-1})$. The recursive competitive equilibrium is defined as 9 endogenous quantities $(I_t, C_t, Y_t, H_t, K_{t+1}, N_{t+1}, B^S_{t+1}, L_{t+1}, T_t)$ and 11 prices $(q_t, Q^L_t, Q^S_t, p^I_t, \tilde{w}_t, \tilde{w}_t, \tilde{p}_t, \pi_t, r^K_t, mc_t, R_t)$ as a function of state variables $(K_t, B^S_t, C_t, Y_t, L_t, w_{t-1}, A_t, \phi_t)$, which satisfies the 19 equilibrium conditions (18, 19, 20, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 49, 50). Once all the market clearing condition and the government budget constraints are satisfied, the household budget constraint is satisfied by Walras’ Law.
B3. Steady state

In the steady-state economy there is no change in the total factor productivity, resaleability, nominal price level, prices and endogenous quantities. The steady-state versions of the Euler conditions are respectively

\[
\beta^{-1} = \frac{r^k + \lambda q}{q} + \frac{\gamma(q - 1)r^k}{q(1 - \theta q)} \quad (A.1)
\]

\[
\beta^{-1} = \frac{1 + \lambda Q^L}{Q^L} + \frac{\gamma(q - 1)}{1 - \theta q} \frac{1 + \lambda Q^L}{Q^L} \quad (A.2)
\]

\[
\beta^{-1} = \frac{1}{Q^S} + \frac{\gamma(q - 1)}{1 - \theta q} \quad (A.3)
\]

where in steady state \( p^I = 1 \) because \( f(1) = f(1) = 0 \) from equation (30). No arbitrage condition in steady state is

\[
\frac{1}{Q^S} = \frac{1 + \lambda Q^L}{Q^L} \quad (A.4)
\]

The capital-labour ratio is given by condition (43)

\[
\frac{K}{H} = \frac{\alpha}{1 - \alpha} \frac{w}{r^k} \quad (A.5)
\]

Since in the steady state all firms charge the same price, \( \tilde{p} = 1 \) and the real marginal cost is equal to the inverse of markup

\[
mc = \left( \frac{r^k}{\alpha} \right)^\alpha \left( \frac{w}{1 - \alpha} \right)^{(1-\alpha)} \frac{1}{1 + \delta_f} \quad (A.6)
\]

Plugging these two equations into the production function (29) at the steady state we obtain the capital-output ratio which is a function of the rental rate of capital.

\[
\frac{Y}{K} = \frac{(1 + \delta_f)r^k}{\alpha} \quad (A.7)
\]
Equation (A.6) can be rewritten as a function of the rental rate

\[ w = (1 - \alpha) \left( \frac{1}{1 + \delta_f} \right)^{\frac{1 \alpha}{\gamma}} \left( \frac{\alpha}{\gamma} \right)^{\frac{w}{\gamma}} \] (A.8)

In steady state, the real wage is equal to a markup over the marginal rate of substitution between labor and consumption

\[ w = (1 + \delta_w) \left( \frac{H}{(1 - \gamma)} \right)^\eta \] (A.9)

Assuming that \( B^S = 0 \) and considering \( K = N \), the investment function in steady state is

\[ I = \gamma r^k K + (1 - \lambda p^B \phi) B^L + \frac{\delta_f}{1 + \lambda f} Y - T \] (A.10)

Steady-state investment are also the depreciated steady-state capital

\[ \frac{I}{K} = (1 - \lambda) \] (A.11)

The resource constraint is

\[ Y = C + I \] (A.12)

Finally, from the Government budget constraint the steady-state tax is

\[ T = B(p^B - \lambda p^B - 1) \] (A.13)

**B4. Log-linear approximation**

Define \( \bar{x}_t = log\left( \frac{x_t}{\bar{x}} \right) \) where \( x \) is the steady-state value of \( x_t \). The log-linearized equilibrium conditions are the following:

**Investments**:

\[ (1 - \gamma) \lambda \bar{r}^t + y_t - \theta \lambda \phi \bar{q} = \gamma \lambda \phi \bar{q} - \gamma \lambda \phi \bar{q} Q^L_t - \gamma (1 + \lambda \phi Q^L) \frac{\bar{r}}{K} \bar{L}_t + \gamma (1 + \lambda \phi Q^L) \frac{\bar{r}}{K} \bar{K}_t - \gamma Q^S \bar{B}^S_t - \gamma r^k \bar{N}_t + \gamma \frac{\bar{r}}{K} \bar{T}_t - \gamma \frac{Y}{K} \bar{Y}_t + \gamma \frac{(1 - \alpha)r^k}{\alpha} (\bar{w} + \bar{H}_t) + \gamma r^k \bar{K}_t = 0 \] (B.1)
Euler equation for equity:

\[-\sigma \hat{C}_t = -\sigma E_t[\hat{C}_{t+1}] - \hat{q}_t + \beta \frac{r^K}{q} (1 + \gamma \frac{2}{1+\theta q}) E_t[\hat{r}_{t+1}]
\]

\[+ \beta \gamma r^K \frac{1 - \theta}{(1 - \theta q)^2} E_t[\hat{q}_{t+1}] - \beta \gamma r^K \frac{1 - \theta}{(1 - \theta q)^2} E_t[\hat{p}_{t+1}]
\] (B.2)

Euler equation for long-term bonds:

\[-\sigma \hat{C}_t = -\sigma E_t[\hat{C}_{t+1}] - \hat{Q}_t - E_t[\hat{\pi}_{t+1}] + \beta \lambda \gamma \frac{q-1}{1 - \theta q} \phi E_t[\hat{\omega}_{t+1}] + \beta \gamma \lambda \phi \frac{q-1}{1 - \theta q} E_t[\hat{Q}_t]
\]

\[+ \beta \gamma [Q_L + \lambda \phi] \frac{1 - \theta}{(1 - \theta q)^2} E_t[\hat{q}_{t+1}] - \beta \gamma [Q_L + \lambda \phi] \frac{(1 - \theta) q}{(1 - \theta q)^2} E_t[\hat{p}_{t+1}]
\] (B.3)

Euler equation for short-term bonds:

\[-\sigma \hat{C}_t = -\sigma E_t[\hat{C}_{t+1}] - \hat{Q}_t - E_t[\hat{\pi}_{t+1}] + \beta \gamma \frac{q-1}{1 - \theta q} E_t[\hat{p}_{t+1}]
\]

\[+ \beta \gamma (1 - \theta) q \frac{1 - \theta}{(1 - \theta q)^2} E_t[\hat{q}_{t+1}] - \beta \gamma (1 - \theta) q \frac{1 - \theta}{(1 - \theta q)^2} E_t[\hat{p}_{t+1}]
\] (B.4)

Resource constraints:

\[\hat{Y}_t = \frac{I}{Y} \hat{I}_t + \frac{C}{Y} \hat{C}_t
\] (B.5)

The resource constraint:

\[\hat{m}c_t = (1 - \alpha) \hat{w}_t + \alpha \hat{r}_t - \hat{A}_t
\] (B.6)

The Phillips curve:

\[\pi_t = \frac{(1 - \zeta \beta)}{(1 - \zeta)} \hat{m}c_t + \beta E_t[\hat{\pi}_{t+1}]
\] (B.7)

The capital-labor ratio:

\[\hat{K}_t = \hat{w}_t - \hat{r}_t + \hat{H}_t
\] (B.8)

The law of motion for aggregate wages:
\[ \hat{w}_t = (1 - \zeta_w) \hat{w} + \zeta_w (\hat{w}_{t-1} - \hat{\pi}_t) \]  \hspace{1cm} (B.9)

Wage-setting decision

\[
\left( 1 + \eta \frac{1 + \delta_w}{\delta_w} \right) \hat{w}_t - (1 - \zeta_w \beta) \eta \frac{1 + \delta_w}{\delta_w} \hat{w}_t
\]

\[
= (1 - \zeta_\beta) \left( \eta \hat{H}_t + \sigma C_t \right) + \zeta_w \beta \left( 1 + \eta \frac{1 + \delta_w}{\delta_w} \right) \hat{E}_t \left( \hat{w}_{t+1} + \hat{\pi}_{t+1} \right) \]  \hspace{1cm} (B.10)

Aggregate production function

\[ \hat{Y}_t = \alpha \hat{K}_t + (1 - \alpha) \hat{H}_t \]  \hspace{1cm} (B.11)

The first order condition for capital producers

\[ \hat{p}_t = f''(1) \hat{H}_t \]  \hspace{1cm} (B.12)

Identity condition equity and capital

\[ \hat{K}_{t+1} = \hat{N}_{t+1} \]  \hspace{1cm} (B.13)

Law of motion of capital

\[ \hat{K}_{t+1} = (1 - \lambda) \hat{I}_t + \lambda \hat{K}_t \]  \hspace{1cm} (B.14)

Government budget constraint:

\[ \frac{T}{K} \hat{T}_t = \frac{L}{K} (1 + \lambda Q^L) \hat{L}_t - \frac{L}{K} (1 + \lambda Q^L) \hat{\pi}_t + \hat{B}_t^S + (1 - \lambda) (Q^L \frac{L}{K}) \hat{Q}_t^L + Q^L \frac{L}{K} \hat{L}_{t+1} + Q^S \hat{B}_{t+1}^S \]  \hspace{1cm} (B.15)

Tax rule:

\[ \frac{T}{K} \hat{T}_t = \psi_t [\frac{L}{K} (\hat{L}_t - \hat{\pi}_t)] \]  \hspace{1cm} (B.16)
The interest rate rule

\[ \hat{R}_t = \psi \hat{\pi}_t \]  

(B.17)

Government rule for issuing short-term bonds

\[ \hat{B}_t^S = \psi k \hat{\phi}_t \]  

(B.18)

Price of short-term bonds

\[ \hat{R}_t = -\log(Q^S) \]  

(B.19)

**Appendix C: Bayesian VAR**

Consider a VAR(p) model

\[ Y_t = \alpha_0 + \sum_{j=1}^{p} A_j Y_{t-j} + \epsilon_t \]  

(52)

where \( y_t \) is a (M x 1) vector, \( \alpha_0 \) is a (M x 1) vector of intercepts, \( A_j \) is a M x M matrix of coefficients, \( \epsilon_t \) is a (M x 1) vector of residuals and \( \epsilon_t \) is i.i.d. \( N(0, \Sigma) \).

An alternative way to write the VAR is the following. Let \( y \) be MT x 1 vector \( (y_1', ..., y_T') \) and \( \epsilon \) stacked conformably. Let \( x_t = (1, y_1', ..., y_{t-p}') \) and \( X' = [x_1, x_2, ..., x_T] \). K = 1 + Mp is the number of coefficients in each equation of VAR and X is a T x K matrix.

The VAR can be rewritten as:

\[ y = (I_M \otimes X)\alpha + \epsilon \quad ; \quad \epsilon \sim N(0, \Sigma \otimes I_M) \]  

(53)

Conjugate priors with Normal and Inverse Gamma distributions are used for the estimation of \( \alpha \) and \( \epsilon \).

\[ \alpha | \Sigma \sim N(\alpha^*, \Sigma \otimes V^*) \]  

(54)
\[ \Sigma^{-1} \sim W(S^{-1*}, \nu^*) \]  

where \( \alpha^*, V^*, S^{-1*}, \nu^* \) are the hyperparameters set

The posterior distributions have the form

\[ \alpha|\Sigma \sim N(\bar{\alpha}, \Sigma \otimes \bar{V}) \]  

\[ \Sigma^{-1}|y \sim W(\bar{S}^{-1}, \bar{\nu}) \]  

where

\[ \bar{V} = (V^{-1*} + X'X)^{-1} \]
\[ \bar{A} = \bar{V}[V^{-1*}A^* + X'X\bar{A}] \]
\[ \bar{S} = S + S^* + \bar{A}'X'X\bar{A} + A^*V^{-1}\bar{A} - \bar{A}'(V^{-1*} + X'X)\bar{A}' \]
\[ \bar{\nu} = T + \nu^* \]