Collusion in Capacity Under Irreversible Investment

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Abstract

This paper studies the possibility of collusion under irreversible investment in production capacity. In usual models of collusion, there exists a short run incentive to deviate, balanced by the future punishment endured by the deviating firm. I show that the irreversibility of investment also creates a long run incentive for deviation. Indeed, once a firm has invested, it is committed to its new capacity for the future, due to the irreversibility of investment. A large investment reduces the competitor's incentive to invest in the future. Therefore, when a firm deviates from a collusive agreement, it builds a large capacity in order to reduce the future punishment. The deviating firm may then reach a dominant position in the market. When firms become more patient, the short run incentive for deviation is reduced but the long run incentive is increased. Collusion may thus be easier to sustain for less patient firm. This result, which goes in the opposite direction of the literature on collusion, suggests that folk theorems are not the right tool to study collusion in dynamic - but not repeated - games.

Keywords: Capacity investment and disinvestment. Dynamic games. Markov-perfect equilibrium. Real options. Collusion.

JEL classification: D43 L13 L25

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1 Introduction

In many industries, the production of firms is limited by their infrastructure: the number of factories or equipments that firms have. This capacity of production may evolve in time, as firms invest in new infrastructure. However, the cost of building a new capacity is usually sunk and investments are then (at least partially) irreversible. These features of capacity limitation and irreversible investment impact the way competition works. This is in particular the case in the Lysine market.

Lysine is an essential amino acid that stimulates growth and lean muscle development in hogs, poultry, and fish. It has no substitutes. By the late 1960s, Japanese biotechnology firms had discovered a bacterial fermentation technique that transformed the production of lysine. It involves the fermentation of dextrose into lysine and requires some specific chemical infrastructure. The cost of this infrastructure is then sunk.[1] This technique is considerably cheaper than conventional extraction methods. By the end of the 1980s, there were three major players in the lysine market: Ajinomoto and Kyowa Hakko based in Japan; and Sewon based in South Korea. In 1988, ADM acquired a fermentation technique for lysine and began the production of the world's largest lysine factory in 1989. ADM's entry was effective in February 1991.

This entry of ADM in the lysine market has led to the creation of one of the most famous cartel. It occurred from June 1992 to June 1995, date at which the FBI raided the headquarters of the participating firms. It was the first global price-fixing conspiracy to be convicted by the US or EU antitrust authorities in the 40, and the monetary penalties rose up to $305 million. The cartel started slightly after the entry of Archer-Daniels-Midland Company (ADM) in the Lysine business. However, between the end of 1992 and early 1993, ADM invested massively to increase its production capacity, passing from a capacity of 60 000 tones per year to a capacity of 113 000 tones. At the same time, ADM started to cheat from the collusive agreement, and a price war began in March 1993. The price war ended in November 1993, after what the cartel lived a peaceful life until the action of the FBI.[2]

[1]There is some possibility to transform a Lysine plan to produce another amino acid, however this is costly.

[2]For more information on the Lysine cartel, see the books of Eichenwald (2000) and Connor (2000), and

This cartel case emphasizes one point: capacities may evolve during collusion times. Several papers focus on the impact of constant capacity constraint on collusion, but very few deal with the strategic evolution of capacity. This work attempts to fill that gap.

More precisely, this work shows the existence of long run incentives for deviation. Indeed, to deviate from a collusive equilibrium, a firm has to invest in new units of capacity in order to increase its production. In doing so, the deviating firm commits to its new level of capacity due to the irreversibility of investment. This reduces the incentives of its opponent to invest in order to punish the deviating firm. In some case, it even prevents the opponent to implement any punishment. A firm may thus increase its long run profit by deviating from the collusive equilibrium. This contrasts with the usual theory of collusion, where the deviation only permits to increase the short run profit of the firm, whereas the long run profit is reduced due to the punishment. This is consistent with the Lysine case, where the deviation of ADM in 1993 and the building of new capacity permits the firm to increase its market share in the American market from 44 percent in 1992 to 57 percent in following years.

In the literature on collusion and capacity constraint, Compte et al (2002) focus on the case of price competition with inelastic demand under fixed capacity constraints, and shows that the larger firm is the one with the most incentives to deviate. Under Cournot competition, linear demand and soft capacity constraint, Vasconcelos (2005) finds that it is the smallest firm which has the most incentives to deviate. Under a similar framework, Bos and Harrington (2010) show that, when the cartel is not inclusive, the deviation may be driven by medium-sized firms. In all these works, asymmetry between firms' capacity hinders collusion.

Fabra (2006) and Knittel and Lepore (2010) focus on the impact of capacity constraint on collusion when there is demand evolution, and show that collusive price can be counter-cyclical. Garrod and Olczak (2014) show that, under imperfect monitoring, capacity constraints may help to detect a deviation.

All these works assume that capacities are fixed and study the impact of capacities on collusion.[3] To my knowledge, there are only two papers focusing on the evolution of

the articles of Connor (2001), Roos (2006) and Connor (2014).

[3] Compte et al (2002) and Vasconcelos (2005) studies the impact of a change in the capacity distribution

capacity during collusion. Paha (2013) adapts the model of Besanko and Doraszelski (2004) to simulate the investment behavior of the firms when they collude in price but not in capacity. Besides his result on the effect of uncertainty on cartel formation, Paha presents counter-intuitive evidence that a low discount rate can hurt collusion. In a duopoly model with linear demand and partially irreversible investment, Feuerstein and Gersbach (2005) studies a Grim trigger strategy in which the cooperative behavior is for each firm to install the half of the monopoly capacity, and the punitive behavior is to invest until the marginal value of capacity equalizes the price of investment. They show that these strategies can form an equilibrium, but for a discount rate lower than in case of fully reversible investment.

In the present paper, I study a Cournot duopoly with irreversible investment in capacity in a discrete time setting. More precisely, at each time, the production of each firm is determined by its level of capacity. Firms may decide to increase their capacity through buying some assets. The price of investment is linear. The results depend on the size of a time period, which can be interpreted as the degree of flexibility of the firms' investment decisions. When the time period's size is large, it takes time for the firm to punish a deviation, whereas when the time period's size is closed to zero, firms react instantly to any deviation. This model can thus be viewed as a generalization of Feuerstein and Gersbach (2005).

In this framework the non-cooperative equilibrium is for the firms to invest only at the beginning of the game. If both firms start with small enough initial capacity, both firms invest to the same Cournot level of capacity. When one of the firm starts with an initial capacity higher than the Cournot level, the firm is committed to this level of capacity due to the irreversibility of investment, and thus its opponent reacts by investing to a level of capacity inferior to the Cournot level. The firm with the high initial capacity does not invest.

To study collusion, I focus on a particular kind of strategy, the Grim-trigger strategy for status quo. It consists for the firm to keep their initial capacity as long as its opponent keeps its own initial level of capacity. When one of the firms deviates, the other responds by investing as in the non-cooperative equilibrium. However, if the deviating firm has installed a capacity larger than the Cournot outcome, it has committed itself to this capacity for the

due to a merger, and Knittel and Lepore (2010) endogenize the choice of capacity at the beginning of the game. However, at the time when collusion is implemented, the capacities are fixed.

rest of the game, and the non-cooperative equilibrium is for the punishing firm to invest less than the Cournot outcome. This is the long run effect of the deviation: the deviating firm can gain a size advantage during the deviation, which persist for the rest of the game. The usual short run effect is also present: there is a period, between the implementation of the deviation and the implementation of the punishment, when the deviating firm is the only firm to have a capacity larger than the collusive capacities. Both short run and long run effects are important to determine the incentives for the firm to deviate.

Focusing on the limit case in which the time period's size tends to zero permits to isolate the long run effect. In that case, there is a parallel between the decision of the Stackelberg leader and the optimal deviation. Indeed, the punishing firm decides its capacity taking into account the capacity already installed by the deviating firm. Collusion is then possible only if the Stackelberg profit is inferior to half of the monopoly profit. A low discount rate is then harmful for collusion, as more patient firms are more likely to invest, and then to prefer becoming the Stackelberg leader than staying at the half of the monopoly profit. When the time period's size is positive, the short run effect may compensate the long run and the impact of the discount rate on collusion depends on which effect dominates.

These findings provide intuition on the results of Paha (2013) and Feuerstein and Gersbach (2005). In Paha (2013), the counter-intuitive effect of low discount rate on the possibility of collusion comes from the domination of the long run effect on the short run effect. Feuerstein and Gersbach (2005) finds the opposite result (on discount rate) because the profit of the Stackelberg leader with linear demand is higher than half of the monopoly profit. Therefore, the long term effect does not permit collusion to be implemented (with linear demand), collusion is more difficult with irreversible than with reversible investment as the short run effect must compensate the negative long run effect. This depends on the form of the profit assumed, and the introduction of a simple quadratic cost of production leads to a different results.

Section 2 presents the model and the non-cooperative equilibrium. Section 3 focus on the collusive equilibrium for status quo, with the short run and long run effect of deviation. Section 4 concludes.

2 A Framework with Irreversible investment

2.1 Model

I consider a market with two firms (A and B) competing *à la Cournot* with homogenous product, in a dynamic time setting similar to the one of Sannikov and Skrzypacz (2007). Time is continuous and the horizon is infinite, but firms only take decision at discrete times $t = \{0, \tau, 2\tau, ...\}$. The game is therefore discrete.[4] The continuous time framework permits to vary τ, the duration of time periods between which investment decisions are made.

More precisely, at each time $t = n\tau$ ($n \in \mathbb{N}$) firms simultaneously decide to extend (or not) their capacity through buying some assets. Firms starts with an initial capacity k_0^i (for $i \in \{A, B\}$) and the capacity of firm i at time t is thus $k_t^i = k_{t-\tau}^i + I_t^i$, where I_t^i denotes the investment of firm i at time t. During the time interval $[t, t+\tau[$ the quantity produced by firm i is determined by its capacity leading to a production constraint:

$$\text{for all } s \in [t, t+\tau[,\ q_s^i = k_t^i. \tag{1}$$

The price of the good is a function of the total quantity produced, $P\left(q_t^A + q_t^B\right)$ and the production cost of firm i is $c(q_t^i)$. The price of investment, p^+, is linear, and firms face the same discount factor δ. The profit of firm i made during the time interval $[t, t+\tau[$ is then fully determined by the firms' capacity at time t:

$$\int_0^\tau \delta^t \left[P\left(q_s^i + q_s^{-i}\right) k^i - c(q^i)\ \right] dt = \frac{(1-\delta^\tau)}{\ln(1/\delta)} \left[P\left(k_t^i + k_t^{-i}\right) k_t^i - c(k_t^i)\ \right] \tag{2}$$

where k_t^{-i} is the capacity of the opponent of firm i. For a vector of capacity k_t, and a discount rate $\delta \in]0,1[$, the inter-temporal profit of firm i is then given by:

$$\Pi_i = \sum_{t=0,\tau,..} \delta^t \left[(1-\delta^\tau)\, \pi_i(k_t) - p^+ I_t^i\right], \tag{3}$$

where $\pi_i(k_t)$ is defined by:

$$\pi_i(k_t) = \frac{P\left(k_t^i + k_t^{-i}\right) k_t^i - c(k_t^i)}{\ln(1/\delta)}. \tag{4}$$

[4] Even if firms' decisions are taken at discrete date, this game is not a repeated game in stricto sensus: the stage games are not independent as the capacity installed at time t is present at tim $t+\tau$.

π_i is the inter-temporal profit of firm i when there is no investment during the game.

I assume that the strategies of the firms are markovian, meaning that the investment at time t is a function of the capacities of the industry at time $t-\tau$, $I_t^i = I^i(k_{t-\tau})$. In that case, according to the dynamic principle, the profit of firm i can be rewritten:

$$\Pi_i(k) = (1-\delta^\tau)\,\pi_i\,(k + I(k)\,) - p^+ I^i(k) + \delta^\tau \Pi_i\,(k + I(k)\,)\,. \tag{5}$$

$I^*(k)$ is then a Markovian equilibrium if and only if, for each $i \in \{A, B\}$,

$$I^{*i}(k) = \arg\max_{I^i} \left\{ (1-\delta^\tau)\,\pi_i\left(k^i + I^i,\ k^j + I^{*j}(k)\right) - p^+ I^i(k) + \delta^\tau \Pi_i\left(k^i + I^i,\ k^j + I^{*j}(k)\right) \right\}, \tag{6}$$

where $I^{*j}(k)$ is the level of investment of the opponent of firm i, and the equilibrium profits, Π^*, are defined by:

$$\Pi_i^*(k) = (1-\delta^\tau)\,\pi_i(k + I^*(k)\,) - p^+ I^i(k) + \delta^\tau \Pi_i^*\,(k + I^*(k)\,)\,. \tag{7}$$

In order to ensure the existence of the equilibria I make the following assumption:

A: $c(.)$, is a twice-differentiable positive function such that $c' \geq 0$, $c'' \geq 0$. $P(.)$ is also a twice-differentiable positive function, with $P' < 0$, $P'' < 0$ when P is strictly positive.

2.2 Non-cooperative Equilibrium

As usual in dynamic competition, there exist a multiplicity of equilibria. The issue is to determine which equilibrium is the non-cooperative equilibrium. In a classic model of repeated cournot competition, the non-cooperative equilibrium is given by the repetition of stage game equilibrium. However, in this model, the periods are linked by the variable of capacity. I claim that the non-cooperative equilibrium is the regular Markovian equilibrium, defined as the the Markovian equilibrium whose strategy are partially differentiable in regard to the state variable. There are three reasons to do so. First, partially differentiability implies continuity, which seems natural, as it implies that a little change in the state of the industry will lead to similar reaction. Furthermore, as it is shown in theorem 1, the regular markovian equilibrium is unique. The second point is that this equilibrium is similar to the Nash equilibrium of the one-shot game, i.e. the game in which investments can only be done in the beginning of the game. Finally, when firms start with the same level of capacity, this equilibrium correspond to the classic equilibrium of repeated cournot competition.

In order to state proposition 1, let k^i_{Irr} be the best response capacity of firm i, i.e. the level of capacity the firm i will install if firm j install a level k^j, and firm i has no initial capacity:

$$\frac{\partial \pi_i}{\partial k^i}(k^i_{Irr}, k^j) = p^+. \tag{8}$$

As the profits are symmetric for both firm, $k^A_{Irr}(.) = k^B_{Irr}(.)$ and the best response capacity will be written $k_{Irr}(.)$ in the following. The Cournot level, k_C, is then given by:

$$k_{Irr}(k_C) = k_C. \tag{9}$$

The Cournot level is the equilibrium of the game if both firm have no initial capacities. When firms have initial capacities, the equilibrium is given by proposition 1.

Proposition 1 *There exists an unique regular markov perfect equilibrium given by:*

$$I^{*i}(k) = \max\left\{k^{*i}(k) - k^i_0,\ 0\right\}, \tag{10}$$

where $k^{*i}(k)$ is defined by:

$$k^{*i}(k) = \begin{cases} k_{Irr}(k^j) \ \textit{if}\ k^j > k_C \\ k_C \ \textit{if}\ k^j \leq k_C \end{cases}. \tag{11}$$

All investments are done in the beginning of game, and the equilibrium path is $\{I^*(k_0), 0, 0, ..\}$.

Proposition 1 presents the three possible scenarios depending on the initial capacities. If initial capacities are small enough, both firms invests, and the equilibrium is the Cournot equilibrium.[5] If one of the firms has a large initial capacity and the other firm a small one, the biggest firm wishes to reduce its capacity but is constrained by the irreversibility of capacity. The smaller firm adjust then its capacity to the initial capacity of its opponent, investing until its best response level. Finally, when both firms have too large initial capacity, no one invests and firms keep their initial capacities forever.

[5]With a marginal cost of investment p^+.

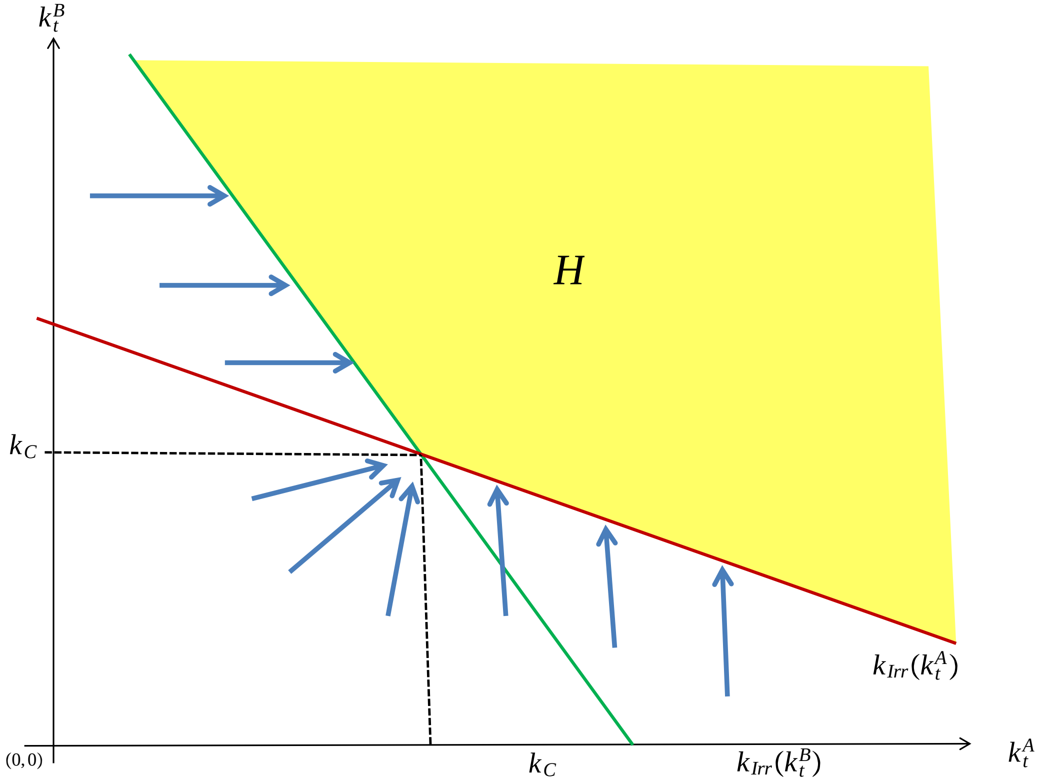

Figure 1: non-cooperative equilibrium

3 Collusive agreement for status quo

3.1 Agreement for status quo

In this section I consider a particular collusive strategy, the collusive agreement for status quo. It consists for each firm to keep its initial capacity as long as the other firm also keeps its initial capacity, and to invest as in the non-cooperative equilibrium if the other firm has invested. There are several reasons to use this particular Grim-trigger strategy. In real cartel agreements, firms usually agree on quantities proportional to the capacity owned at the time of the agreement. If the Grim-trigger strategy is the most considered strategy in the literature, some recent work focus on other equilibria with renegotiation. However, in this case, the irreversibility of investment provides any renegotiation after the punishment, as the firms cannot reduce their investment (nor their production).

More precisely, I focus on the Grim trigger strategy for status quo:

$$I_{Coll}(k) = \begin{cases} 0 \text{ if } k = k_0 \\ I^*(k) \text{ elsewhere} \end{cases} . \tag{12}$$

The objective of this section is to characterize the set of initial capacity such that (12) is a markovian equilibrium:

$$\Psi = \left\{ k_0 \in \mathbb{R}^2_+ \mid I_{Coll} \text{ verifies (6) and (7)} \right\}. \tag{13}$$

I assume in the following that the initial capacities are sufficiently low for both firms to invest in the non-cooperative equilibrium (as defined in the previous section). If both capacities are high and no firms invest at the non-cooperative equilibrium, then the agreement for status quo is obviously an equilibrium, as it coincides with the non-cooperative equilibrium. In that case, there is no real collusion, as firms behave exactly in the same way in both equilibria. If one of the firms has a capacity higher than the Cournot capacity, given by (9), and the other firm has a capacity small enough for the firm to invest in the non-cooperative equilibrium, there is no possibility for a status quo agreement. Indeed, when the largest firm keeps its capacity constant, the best response of the smallest firm is to invest until the capacity level (8). This corresponds to the non-cooperative equilibrium behavior, and the best response to the status quo agreement is, for the smallest firm, to deviate from the collusive agreement.

Assume that firm i deviates from the agreement for status quo by investing in a level of capacity I_d. At the time of the investment decision, its opponent does not know that firm i deviates and therefore keeps its initial capacity. When the investment is realized, the opponent observes the deviation and the non-cooperative equilibrium is played. However, at this time the capacity of firm i is no more its initial capacity, but its capacity of deviation, $k_d^i = k_0^i + I_d$. The non-cooperative equilibrium played after the deviation may be impacted by this evolution of capacity.

The form of the non-cooperative equilibrium gives two features on the optimal deviation.

If the capacity of deviation is inferior to the Cournot level ($k_d^i < k_C$), then the non-cooperative equilibrium is for both firms to invest until the Cournot level. In that case, by investing directly to the Cournot level, firm i does not change the non-cooperative equilibrium established after the realization of the deviation, but it increases its profit during the time when the capacity of deviation is installed, but not the punishment capacity. The optimal deviation should then be superior to the Cournot level. This also implies that the deviating firm i does not invest after the deviation.

Furthermore, there exists a level of capacity, $k_{Irr}^{-1}(k_0^j)$, such that, if the deviating capacity is larger than this level of capacity, the punishing firm does not invest after the deviation. Indeed, due to the irreversibility of investment, the deviating firm is committed to a capacity too large for the other firm to invest at the non-cooperative equilibrium competition. If the deviating capacity is larger than this level, then reducing the deviating capacity does not change the reaction of the punishing firm, but permits to increase the profit of the deviating firm.[6] The optimal deviation should then be superior to this level of deviation.

These features are summarized in the following Lemma.

Lemma 1 *The optimal deviation verifies* $k_d^{*i} \in \left[k_C, k_{Irr}^{-1}(k_0^j)\right]$.

In the following, I focus on deviations which verifies the condition given in Lemma 1, in order to find the optimal deviation. In that case, the profit of firm i is:

$$\Pi^i = (1-\delta^\tau)\,\pi_i(k_d^i, k_0^j) - p^+\left(k_d^i - k_0^i\right) + \delta^\tau \pi_i\left(k_d^i, k_{Irr}\left(k_d^i\right)\right). \tag{14}$$

The first term of the profit is made when the deviating firm has installed its capacity, but before the punishment is implemented. The second term is simply the investment cost of the deviation. The last term is made during the rest of game, when firms have reached the non-cooperative equilibrium. It depends on the deviation, as the level of punishment depends on the capacity installed by the deviation. The deviation has therefore two impacts: it increases the short run profit and it also reduces the capacity installed by the competitors on the long run, during the punishment.[7]

[6]Indeed, when the capacity of firm i is $\left(k_{Irr}^j\right)^{-1}(k_0^j)$ and the capacity of firm j is k_0^j the equilibrium of regular competition implies that no firm invests. Firm i has then no interest to invest more than $\left(k_{Irr}^j\right)^{-1}(k_0^j)$.

[7]The capacity installed in punishment, determined in (8), is decreasing due to assumption A.

3.2 Limit case: no time-to-build

This sub-section focuses on the case where the time period's size tends to zero ($\tau \to 0$). As time is continuous, the deviation is instantly detected and both the deviation capacity and the punishment capacity are instantly installed. The deviating firm does not make any profit before the implementation of its capacity neither during the time between the deviation and the punishment. Indeed, when $\tau \to 0$, the profit of the deviating firm becomes:

$$\Pi_i = \pi_i(k_d^i, k_{Irr}\left(k_d^i\right)) - p^+\left(k_d^i - k_0^i\right). \tag{15}$$

Therefore, deviating from the collusive equilibrium has no short run impact. It only influences the long run distribution of capacity.

In this limit case, there is a clear parallel between the choice of the optimal deviation and the Stackelberg game. Due to the irreversibility of investment, when a firm deviates, it commits to its new level of capacity. When the opponent observes the deviation, it reacts to this new choice of capacity. After that, firms keep their capacity forever. As the time period's size tends to zero, firms only make profit in the long run. The deviating firms is then in the position of a Stackelberg leader, whereas its opponent, which reacts to the deviation, behaves as a Stackelberg follower.

Let k_S^i be the Stackelberg capacity, as defined by the first order condition of (15):

$$\frac{\partial \pi_i}{\partial k^i}(k_S^i, k_{Irr}\left(k_S^i\right)) + \frac{\partial k_{Irr}}{\partial k^i}\left(k_S^i\right)\frac{\partial \pi_i}{\partial k^j}(k_S^i, k_{Irr}\left(k_S^i\right)) = p^+. \tag{16}$$

Lemma 2 describes the optimal deviation.

Lemma 2 *When $\tau \to 0$, the optimal deviation is to install a capacity:*

$$k_d^{*i} = \begin{cases} k_S^i \text{ if } k_0^j \leq k_{Irr}\left(k_S^i\right) \\ k_{Irr}^{-1}(k_0^j) \text{ if } k_0^j > k_{Irr}\left(k_S^i\right) \end{cases}. \tag{17}$$

If the punishing firm starts with a small initial capacity, the punishing firm will invest after the deviation. Knowing that its opponent will invest after the deviation, the optimal strategy of the deviating firm is to install the Stackelberg capacity. In that case, the condition for firm i to accept the collusive agreement is to make more profit with its initial capacity than if it installs the Stackelberg capacity and its opponent invests to the best response level of the Stackelberg capacity:

If $k_0^j \leq k_{Irr}^j(k_S^i)$,

$$\pi_i(k_0) \geq \pi_i(k_S, k_{Irr}(k_S)) - p^+\left(k_S - k_0^i\right). \tag{18}$$

When the initial capacity of the punishing firm is sufficiently large, the deviating firm invests less than the Stackelberg capacity in order to take into account the fact that the punishing firm is committed by its initial level of capacity. In that case, the punishing firm does not invest after the deviation. The condition for firm i to accept the collusive agreement is then to make more profit with its initial capacity than if it increases its capacity to the best response level:

If $k_0^j > k_{Irr}^j(k_S^i)$,

$$\pi_i(k_0) \geq \pi_i\left(k_{Irr}^{-1}(k_0^j), k_0^j\right) - p^+\left((k_{Irr})^{-1}(k_0^j) - k_0^i\right). \tag{19}$$

Figure B presents the optimal deviation and the punishment which follows in function of the initial capacities.

The conditions (18) and (19) permit to determine which firm has the more incentives to deviate from the collusive agreement.

Lemma 3 *When $\tau \to 0$, the firm with the more incentives to deviate from the collusive agreement is the firm with the smallest initial capacity:*

If (18) and (19) holds for i and $k_0^i < k_0^j$, then (18) and (19) holds for firm j.

Assume that both firms are small so that the deviation is to install the Stackelberg capacity, even when the larger firm deviates ($k_0^i \leq k_{Irr}(k_S)$ for both $i = A, B$). In that case, the total capacity after the deviation is the sum of the leader and the follower capacity of Stackelberg ($k_S + k_{Irr}(k_S)$). The price after the deviation is then the same whatever the deviating firm is. In that case, the difference between the initial capacity and the

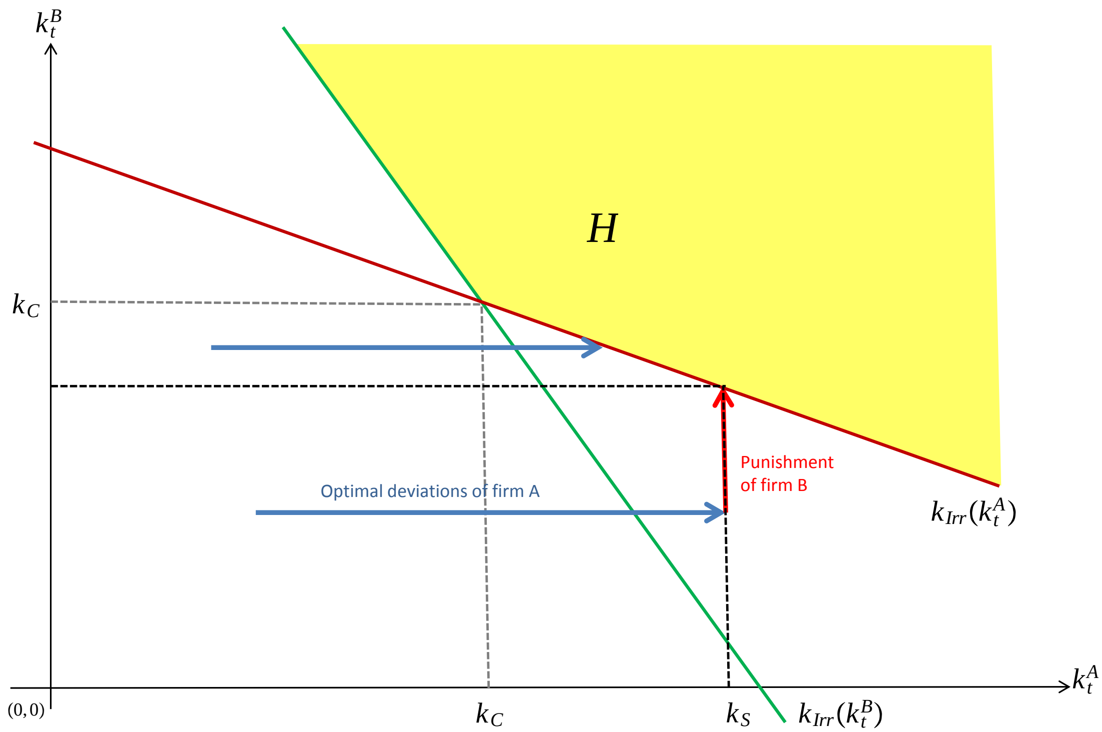

Figure 2: optimal deviation

Stackelberg capacity is more important for the smaller firm than for the larger firm. The smaller firm has then more incentives to deviate, as it gains more capacity. If both firms are large ($k_0^i \leq k_{Irr}(k_S)$ for one i) so that the deviation is to provide the other firm to invest, as described by (19), the price after the deviation is more important when the small firm deviates than when the larger firm deviates. Indeed, as the marginal profit of investment is decreasing with the firm's capacity, the smaller firm has more incentives to invest for the same total capacity in the non-cooperative equilibrium. Therefore, to provide the other firm to invest after the deviation, the larger firm has to install a capacity more important than the smaller one. This price reduction makes the deviation less profitable for the larger firm than for the smaller one. The same reasoning applies when one firm is small and behaves as in (18) and the other one is large and behaves as in (19).

Finally, Lemma 2 and 3 permits to determine the initial capacities for which there is a possibility of collusion.

Proposition 2 *When $\tau \to 0$, the set of initial capacities such that the agreement for status quo is a markov perfect equilibrium is:*

$$\Psi = \left\{ k \in \mathbb{R}_+^2 \mid (18) \text{ and } (19) \text{ holds for } i = \arg \inf_{\{A,B\}} \{k_0^i\} \right\}. \tag{20}$$

Furthermore, if k_M is the firm's capacity which maximizes the joint profit,

$$k_M = \arg\max \left\{ \pi_A(k_M, k_M) + \pi_B(k_M, k_M) - 2p^+ k_M \right\} \tag{21}$$

then, if $k_M < k_{Irr}(k_S)$, there is a possibility of collusion if and only if the stackelberg profit is inferior to half of the joint profit:

$$\pi_i(k_S, k_{Irr}(k_S)) - p^+ k_S \leq \frac{\pi_A(k_M, k_M) + \pi_B(k_M, k_M) - 2p^+ k_M}{2}. \tag{22}$$

The parallel between the optimal deviation and the choice of the leader of Stackelberg permits to have a clear condition for the possibility of collusion. When half of the monopoly profit is larger than the profit of the Stackelberg leader, there is a possibility of collusion. Half of the monopoly profit is the best collusive profit the firms can make and the profit of the Stackelberg leader is the deviating profit coming from the monopoly capacity.

In the usual theory of collusion, when firms are more patient (when their discount rate, δ, increases), the importance of the profit at the time of deviation relative to the future profit of punishment decreases, and the collusion is easier to sustain. When investment is irreversible, it is possible to differentiate the condition (18) and (19) with respect to the discount rate to determine its impact on collusion.

Lemma 4 *When $\tau \to 0$, if the discount rate increases, it becomes harder to sustain collusion:*

$$\text{If } \delta' > \delta,\ \Psi_{\delta'} \subset \Psi_\delta \tag{23}$$

When firms become more patient, their willingness to invest increases, as firms are ready to pay more in the short run to gain profit in the long run. In the case with infinitesimal time periods, deviating from the agreement only impacts the long run profit of firms. Therefore, when firms are more willing to invest, their incentives to deviate increase. This result differs from the usual theory. The reason is that the present case, with no time-to-build, focuses on the long run profitability of deviation, whereas the usual theory focuses on the short run profitability of deviation.

3.3 General case

This sub-section generalizes the results of the previous section when the time periods are positive ($\tau > 0$). In that case the incentive for deviation is a combination of short term and long term effects. Indeed, the deviating firm makes a positive profit during the time between the installation of the deviation capacity and the installation of the punishing capacity. After the installation of the punishing capacity, the deviating firm makes its long run profit as in the previous sub-section.

In order to state the optimal deviation, let $k_D^i(k_0^j)$ be the capacity that the deviating firm whishes to install if its competitor invests after the deviation, as defined by:

$$\frac{\partial \pi_i}{\partial k^i}(k_D^i, k_0^j) + \delta^\tau \left[\frac{\partial \pi_i}{\partial k^i}(k_D^i, k_{Irr}^j\left(k_D^i\right)) + \frac{\partial k_{Irr}}{\partial k^i}\left(k_D^i\right) \frac{\partial \pi_i}{\partial k^j}(k_D^i, k_{Irr}\left(k_D^i\right)) \right] = p^+. \tag{24}$$

This optimal capacity of deviation is inferior to the Stackelberg capacity ($k_D < k_S$).[8] Indeed, the fact that the deviating firm makes a short term profit reduces its incentives to invest, as the capacity which maximizes the short term profit of deviation is inferior to the one maximizing the long run profit. Lemma 5 describes the optimal deviation.

Lemma 5 *The optimal deviation is to install a capacity:*

$$k_d^{*i} = \begin{cases} k_D^i(k_0^j) \text{ } if \text{ } k_0^j \leq k_{Irr}\left(k_D^i\right) \\ k_{Irr}^{-1}(k_0^j) \text{ } if \text{ } k_0^j > k_{Irr}\left(k_D^i\right) \end{cases}. \tag{25}$$

As in the case with infinitesimal time periods, there are two possible reaction for the deviating firm. If the initial capacity of the punishing firm is small, the punishing firm will

[8]The proof of this fact is in appendix.

invest after the deviation. In that case, the optimal deviation is the one maximizing (24), and firm i accepts the collusive agreement if it makes more profit with its initial capacity than by deviating:

If $k_0^j \leq k_{Irr}^j\,(k_D^i)$,

$$\pi_i(k_0) \geq (1-\delta^\tau)\,\pi_i(k_D^i, k_0^j) + \delta^\tau \pi_i(k_D^i, k_{Irr}\left(k_D^i\right)) - p^+\left(k_D^i - k_0^i\right). \tag{26}$$

When the initial capacity of the punishing firm is sufficiently large, the deviating firm invests less than the capacity defined by (24) in order to take into account the fact that the punishing firm is committed by its initial level of capacity and will not invest after the deviation. Firm i accepts the collusive agreement if its profit is superior when it keeps its initial capacity than when it increases its capacity to the best response level:

If $k_0^j > k_{Irr}^j\,(k_D^i)$,

$$\pi_i(k_0) \geq \pi_i\left((k_{Irr})^{-1}\,(k_0^j), k_0^j\right) - p^+\left((k_{Irr})^{-1}\,(k_0^j) - k_0^i\right). \tag{27}$$

As previously, the firm with the smaller initial capacity is the one which has the more incentives to deviate from the status quo agreement.

Lemma 6 *The firm with the more incentives to deviate from the collusive agreement is the one with the smallest initial capacity:*

If (26) and (27) holds for i and $k_0^i < k_0^j$, then (26) and (27) holds for firm j.

Lemma 6 permits to determine the initial capacities for which there is a possibility of collusion.

Proposition 3 *The set of initial capacities such that the agreement for status quo is a markov perfect equilibrium is:*

$$\Psi = \left\{ k \in \mathbb{R}_+^2 \mid (26) \text{ and } (27) \text{ holds for } i = \arg \inf_{\{A,B\}} \{k_0^i\} \right\}. \tag{28}$$

There is no clear condition ensuring that there is a possibility of collusion (meaning that Ψ is not restrict to the Cournot equilibrium). However, when condition (22) is valid, there is a possibility of collusion for τ small enough. When firms are more patient, the short term profit of the deviation is reduced, but the long run profit increases. The impact of the discount rate on the possibility of collusion is then ambiguous.

4 Conclusion

This work shows that the irreversibility of investment creates a long run incentive for the deviation. Indeed, when a firm wishes to deviate from the collusive agreement, it has to increases its capacity of production. By doing so, it benefits from a short run increases of profits before its opponent starts to increase its production in punishment, but it also commit to this capacity for the rest of the game, due to the irreversibility of investment. This commitment reduces the incentive of its opponent to invest in the punishment regime, and permits the deviating firm to gain a long run advantage in term of size of capacity.

This is consistent with the Lysine example, in which the entry firm, ADM, invests during the first period of collusion (1992-1993), leading to the break of the cartel. During the following price war, the increase of capacity of ADM permits the firm to increase its market share at the expense of its competitors. ADM keeps its new market share for the second collusive period which starts in the end of 1993.

This work rests on two hypotheses, the fact that firms produce always at full capacity and the fact that the industry starts under capacity (which can be due to an entry, or positive demand chock). These leads to two different directions for future works. First, firms can have unused capacity than can be observed by the competitors, which can be used to enforce cartel agreements. Knowing how this unused capacity can facilitate collusion and how it effects the firms' investment in capacity seems a promising topic for future research. The second direction is the impact of market evolution expectation. Firms may face growing, or uncertain market, which will impact their possibility of collusion. What is the impact of market evolution and does these evolutions can create a market under capacity and facilitate cartel creation look interesting research issues.

5 Annex

Proof of Proposition 1:

For a given strategy of firm j, $I^j(k)$, the profit of firm Π_i verifies:

$$\Pi_i(k) = \max_{I^i} \left\{ (1-\delta^\tau)\, \pi_i \left(k^i + I^i, k^j + I^j(k) \right) + \delta^\tau \Pi_i \left(k^i + I^i, k^j + I^j(k) \right) - p^+ I^i \right\} \quad (29)$$

Let $V_i(k)$ be defined by:

$$V_i(k) = (1-\delta^\tau)\, \pi_i(k) + \delta^\tau \Pi_i(k).$$

Equation (29) can then be rewritten as:

$$\Pi_i(k) = \max_{I^i} \left\{ V_i \left(k^i + I^i, k^j + I^j(k) \right) - p^+ I^i \right\}$$

If V_i is differentiable, the optimal investment of firm i is thus determined by the implicit equation:

$$\frac{\partial V_i}{\partial k_i} \left(k^i + I^i, k^j + I^j(k) \right) = p^+. \quad (30)$$

Unfortunately, there is no reason a priori for V_i to be differentiable as the cost of investment is not differentiable in 0 (in fact, Π_i is not differentiable at the equilibrium and neither V_i). However, if $I^j(.)$ is left-hand differentiable, by (3) Π_i is also left hand differentiable, and thus V_i is left hand differentiable. The optimal level of investment is determined by:

$$\frac{\partial_i^- V}{\partial k_i} \left(k^i + I^i, k^j + I^j(k) \right) = p^+, \quad (31)$$

where $\frac{\partial^- \cdot}{\partial k_i}$ denotes the left-hand differential of V_i.

Therefore, if $\frac{\partial^- V_i}{\partial k_i}(k^i, k^j) \geq p^+$ and $\frac{\partial^- V_j}{\partial k_i}(k^i, k^j) \geq p^+$, then no firms invest, and (k^i, k^j) is a steady point. By (30), firms reach a steady point after their first investment is realized. As investment is irreversible and there is no depreciation of capacities, a steady point is defined as a point where no firms invests at the equilibrium, and, by (29), the intertemporal profit function at a steady state is given by:

$$\Pi_i \left(k^i, k^j \right) = \pi_i \left(k^i, k^j \right). \quad (32)$$

This permits to rewrite (29), the program of firm i for a given $I^j(k)$:

$$\Pi_i = (1-\delta^\tau)\, \pi_i \left(k^i + I^i, k^j + I^j(k) \right) + \delta^\tau \pi_i \left(k^i + I^i, k^j + I^j(k) \right) - p^+ I^i. \quad (33)$$

Therefore, the best response of the firm is to not invests if

$$\frac{\partial \pi_i}{\partial k_i}\left(k^i, k^j + I^j(k)\right) \geq p^+, \tag{34}$$

and to invest a level I defined by the implicit equation:

$$\frac{\partial \pi_i}{\partial k_i}\left(k^i + I, k^j + I^j(k)\right) = p^+. \tag{35}$$

If the initial capacities are such that

$$\forall i \in \{1,2\},\ \frac{\partial \pi_i}{\partial k_i}\left(k_0^i, k_0^j\right) \geq p^+, \tag{36}$$

then there is no investment. If the initial capacities are both inferior to k_C, the equilibrium is k_C, and if firm j has an initial capacity superior to k_C, but such that $k_0^i < k_{Irr}(k_0^j)$, then firm i invests until the level $k_{Irr}(k_0^j)$. ■

Lemma 1 is shown in the text. The results of section 3.2, when there is no time-to-build, are the limit case of the results of section 3.3, when there is a time-to-build, and therefore are shown after the results of section 3.3.

Proof of footnote numero 8:

By assumption A, π is concave, and $\frac{\partial \pi_i}{\partial k^i} < 0$. In particular:

$$\frac{\partial \pi_i}{\partial k^i}(k_D, k_0) < p^+. \tag{37}$$

Therefore:

$$\delta^\tau p^+ < p^+ - (1-\delta^\tau)\frac{\partial \pi_i}{\partial k^i}(k_D, k_0). \tag{38}$$

Equation (24) implies that

$$p^+ - (1-\delta^\tau)\frac{\partial \pi_i}{\partial k^i}(k_D, k_0) = \delta^\tau\left[\frac{\partial \pi_i}{\partial k^i}(k_D, k_{Irr}(k_D)) + \frac{\partial k_{Irr}}{\partial k^i}(k_D)\frac{\partial \pi_i}{\partial k^j}(k_D, k_{Irr}(k_D))\right], \tag{39}$$

and equation (16) implies that

$$\delta^\tau p^+ = \delta^\tau\left[\frac{\partial \pi_i}{\partial k^i}(k_S, k_{Irr}(k_S)) + \frac{\partial k_{Irr}}{\partial k^i}(k_S)\frac{\partial \pi_i}{\partial k^j}(k_S, k_{Irr}(k_S))\right]. \tag{40}$$

Then,

$$\frac{\partial \pi_i}{\partial k^i}(k_S, k_{Irr}(k_S)) + \frac{\partial k_{Irr}}{\partial k^i}(k_S)\frac{\partial \pi_i}{\partial k^j}(k_S, k_{Irr}(k_S)) < \frac{\partial \pi_i}{\partial k^i}(k_D, k_{Irr}(k_D)) + \frac{\partial k_{Irr}}{\partial k^i}(k_D)\frac{\partial \pi_i}{\partial k^j}(k_D, k_{Irr}(k_D)), \tag{41}$$

and, as $\frac{\partial \pi_i}{\partial k^i}\left(., k_{Irr}\left(.\right)\right)+\frac{\partial k_{Irr}}{\partial k^i}\left(.\right)\frac{\partial \pi_i}{\partial k^j}\left(., k_{Irr}\left(.\right)\right)$ is decreasing by assumption H and by (8),

$$k_S > k_D.$$

■

Lemma 5 is the result of the maximization of (14) under the constraint $k_d^i \leq \left(k_{Irr}\right)^{-1}\left(k_0^j\right)$, coming from Lemma 1.

Proof of Lemma 6:

Let $\alpha(x)$ be defined by

$$\alpha(x)=\begin{cases}\left(1-\delta^{\tau}\right)\pi_i(k_D^i,k_0^j)-p^{+}\left(k_D^i-k_0^i\right)+\delta^{\tau}\pi_i\left(k_D^i,k_{Irr}\left(k_D^i\right)\right) \text{ if } x\leq k_{Irr}\left(k_D^i\left(x\right)\right)\\ \pi_i\left(\left(k_{Irr}\right)^{-1}(x),x\right)-p^{+}\left(k_{Irr}\right)^{-1}(x) \text{ if } x>k_{Irr}\left(k_D^i\left(x\right)\right)\end{cases}. \tag{42}$$

As the deviation is the optimal one, the derivative of α is (see equation (24) and (8)):

$$\alpha'(x)=\begin{cases}\dfrac{\left(1-\delta^{\tau}\right)k_D^i(x)P'(k_D^i(x)+x)}{\ln(1/\delta)} \text{ if } x\leq k_{Irr}\left(k_D^i\left(x\right)\right)\\ \dfrac{P'\left(\left(k_{Irr}^j\right)^{-1}(x)+x\right)\left(k_{Irr}^j\right)^{-1}(x)}{\ln(1/\delta)} \text{ if } x>k_{Irr}\left(k_D^i\left(x\right)\right)\end{cases}. \tag{43}$$

With this notation, firm i agrees to collude if and only if:

$$\pi_i(k_0)-p^{+}k_0^i-\alpha(k_0^j)\geq 0 \tag{44}$$

Let h be a function defined by:

$$h(x)=\frac{P(T)x-c(x)}{\ln(1/\delta)}-p^{+}x+\alpha(x), \tag{45}$$

where T is a constant which verifies $T<k_D^i(x)+x$ if $x\leq k_{Irr}\left(k_{D^*}^i\left(x\right)\right)$ and $T<\left(k_{Irr}^j\right)^{-1}(x)+x$ if $x>k_{Irr}\left(k_D^i\left(x\right)\right)$. Then, the derivative of h is:

$$h'(x)=\begin{cases}\dfrac{P(T)+\left(1-\delta^{\tau}\right)k_{D^*}^i(x)P'(k_D^i(x)+x)-c'(x)}{\ln(1/\delta)}-p^{+} \text{ if } x\leq k_{Irr}\left(k_D^i\left(x\right)\right)\\ \dfrac{P(T)+P'\left(\left(k_{Irr}\right)^{-1}(x)+x\right)\left(k_{Irr}\right)^{-1}(x)-c'(x)}{\ln(1/\delta)}-p^{+} \text{ if } x>k_{Irr}\left(k_D^i\left(x\right)\right)\end{cases}. \tag{46}$$

As $P(T) > P(k_D^i(x)+x)$ if $x \leq k_{Irr}\left(k_D^i\left(x\right)\right)$ and $P(T) > \left(k_{Irr}\right)^{-1}(x)+x$ if $x > k_{Irr}\left(k_D^i\left(x\right)\right)$, the derivative of h si positive. Then, if $k_0^j > k_0^i$,

$$\frac{P(T)k_0^j - c(k_0^j)}{\ln(1/\delta)} - p^+k_0^j + \alpha(k_0^j) \geq \frac{P(T)k_0^i - c(k_0^i)}{\ln(1/\delta)} - p^+k_0^i + \alpha(k_0^i) \tag{47}$$

By assuming that $T = k_0^i + k_0^j$, (47) can be rewrite:

$$\pi_j(k_0) - p^+k_0^j - \alpha(k_0^i) \geq \pi_i(k_0) - p^+k_0^i - \alpha(k_0^j). \tag{48}$$

Therefore, if $k_0^j > k_0^i$, if firm i agrees for collusion, then firm j also agrees, as:

$$\pi_i(k_0) - p^+k_0^i - \alpha(k_0^j) \geq 0 \Rightarrow \pi_j(k_0) - p^+k_0^j - \alpha(k_0^i). \tag{49}$$

■

Proposition 3 comes from the combination of Lemma 5 and 6. In section 3.2, Lemma 2 is a corollary of Lemma 5 and Lemma 3 is corollary of Lemma 6.

Proof of Proposition 2:

The first part of proposition 2 comes from the combination of Lemma 2 and 3.

To state the second part of proposition 2, remark that the incentive for collusion of firm i (IC_{coll}) can be written:

$$IC_{\text{coll}} = \begin{cases} \pi_i(k_0) - p^+k_0^i - \left(\pi_i(k_S, k_{Irr}\left(k_S\right)) - p^+k_S^i\right) \text{ if } k_0^j \leq k_{Irr}\left(k_S\right) \\ \pi_i(k_0) - p^+k_0^i - \left(\pi_i\left(k_{Irr}^{-1}(k_0^j), k_0^j\right) - p^+k_{Irr}^{-1}(k_0^j)\right) \text{ if } k_0^j > k_{Irr}\left(k_S\right) \end{cases}.$$

For a given initial state, the collusive agreement for status quo is a markovian equilibrium if and only if this incentive for collusion is positive. To show if there is or not a possibility for collusion, I determine the initial state maximizing the incentive for collusion and study the sign of IC_{coll} in this particular state.

As the smallest firm has more incentive to deviate than its opponent, the initial state maximizing IC_{coll} is symmetric. In the following, k_0 design alternatively the capacity of one of the firms or the vector of capacity, (k_0, k_0).

First, assume that $k_0 > k_{Irr}\left(k_S\right)$. Then, the incentive to collude is:

$$IC_{\text{coll}} = \pi_i(k_0, k_0) - p^+k_0 - \left(\pi_i\left(k_{Irr}^{-1}(k_0)\,, k_0\right) - p^+k_{Irr}^{-1}(k_0)\right). \tag{50}$$

The second term, $\pi_i\left((k_{Irr})^{-1}(k_0), k_0\right) - p^+ (k_{Irr})^{-1}(k_0)$ is the value of the maximization of $\pi_i(k_S^i, k_{Irr}(k_S^i)) - p^+ k_S^i$ in the boundary $k_{Irr}^{-1}(k_0)$. Therefore, its differential is positive (as $k_{Irr}^{-1}(k_0) < k_S$). The first term is maximized in $k_M < k_{Irr}(k_S^i)$, as the profit is symmetric and:

$$k_M = \arg\max \left\{\pi_A(k_M, k_M) + \pi_B(k_M, k_M) - 2p^+ k_M\right\} = \arg\max \left\{\pi_i(k_M, k_M) - p^+ k_M\right\}. \tag{51}$$

Therefore, the differential of $\pi_i(k_0, k_0) - p^+ k_0$ is negative, as for the differential of IC_{coll}, and the initial state maximizing IC_{coll} should be inferior to $k_{Irr}(k_S)$.

Now, assume that $k_0 < k_{Irr}(k_S)$. Then, the incentive to collude is:

$$IC_{\text{coll}} = \pi_i(k_0) - p^+ k_0^i - \left(\pi_i(k_S, k_{Irr}(k_S)) - p^+ k_S^i\right),$$

which is maximized in k_M.

■

Proof of lemma 4:

The scheme of proof is to fixe a vector of initial capacity such that firm i has an incentive to deviate, and to show that firm i has an incentive to deviate for a higher δ. The proof is decomposed in two part, depending if $k_0^j \leq k_{Irr}(k_S^i)$ or not.

Before that, remark that the best response of capacity, $k_{Irr}(x)$, is a increasing function function of the discount rate, δ (for all $x \in \mathbb{R}_+$). Indeed, the implicit equation defining k_{Irr}, (8), can be rewritten:

$$P'(x + k_{Irr}(x))\, k_{Irr}(x) + P(x + k_{Irr}(x)) - c'(k_{Irr}(x)) = \ln\left(\frac{1}{\delta}\right) p^+. \tag{52}$$

This gives, by differentiation:

$$\frac{\partial k_{Irr}(x)}{\partial \delta} = -\frac{1}{\delta} \frac{p^+}{\left[P''(x + k_{Irr}(x))\, k_{Irr}(x) + 2P'(x + k_{Irr}(x)) - c''(k_{Irr}(x))\right]}. \tag{53}$$

By assumption A, the second derivative of the firm's payoff is negative, and therefore $\frac{\partial k_{Irr}(x)}{\partial \delta} > 0$, meaning that a more patient firm invest a higher quantity.

Assume that $k_0^j \leq k_{Irr}(k_S^i)$. Then, firm i has incentive to deviate if and only if:

$$\pi_i(k_0) - p^+ k_0^i < \pi_i(k_S, k_{Irr}(k_S)) - p^+ k_S^i, \tag{54}$$

which is equivalent to

$$P\left(k_0^i + k_0^{-i}\right) k_0^i - c(k_0^i) < P\left(k_S + k_{Irr}\left(k_S\right)\right) k_0^i - c(k_S^i) - \ln\left(\frac{1}{\delta}\right) p^+ \left(k_S - k_0^i\right). \tag{55}$$

By definition of k_S this can be rewritten,

$$P\left(k_0^i + k_0^{-i}\right) k_0^i - c(k_0^i) < \max_x \left\{ P\left(x + k_{Irr}\left(x\right)\right) x - c(x) - \ln\left(\frac{1}{\delta}\right) p^+ \left(x - k_0^i\right) \right\}. \tag{56}$$

The first term of the inequality does not depend of δ. To show that an increase of δ increase the incentive to deviate, it is enough to show that the derivative of the second term is positive. Let $V(\delta) = \max_{x\in\mathbb{R}_+} \left\{ P\left(x + k_{Irr}\left(x\right)\right) x - c(x) - \ln\left(\frac{1}{\delta}\right) p^+ \left(x - k_0^i\right) \right\}$. Then, the envelope theorem implies that:

$$V'(\delta) = \frac{\partial k_{Irr}\left(k_S\right)}{\partial \delta} P'\left(k_S + k_{Irr}\left(k_S\right)\right) k_S + \frac{1}{\delta} p^+ \left(k_S - k_0^i\right), \tag{57}$$

by using (56),

$$V'(\delta) = \frac{p^+}{\delta}\left(\left(k_S - k_0^i\right) - \frac{P'\left(k_S + k_{Irr}\left(k_S\right)\right) k_S}{P''\left(k_S + k_{Irr}\left(k_S\right)\right) k_{Irr}\left(k_S\right) + 2P'\left(k_S + k_{Irr}\left(k_S\right)\right) - c''\left(k_{Irr}\left(k_S\right)\right)}\right), \tag{58}$$

thus

$$V'(\delta) = \frac{p^+}{\delta} k_S \left(\frac{P''\left(k_S + k_{Irr}\left(k_S\right)\right) k_{Irr}\left(k_S\right) + P'\left(k_S + k_{Irr}\left(k_S\right)\right) - c''\left(k_{Irr}\left(k_S\right)\right)}{P''\left(k_S + k_{Irr}\left(k_S\right)\right) k_{Irr}\left(k_S\right) + 2P'\left(k_S + k_{Irr}\left(k_S\right)\right) - c''\left(k_{Irr}\left(k_S\right)\right)}\right) - \frac{p^+}{\delta} k_0^i. \tag{59}$$

As $P' < 0$,

$$\frac{P''\left(k_S + k_{Irr}\left(k_S\right)\right) k_{Irr}\left(k_S\right) + P'\left(k_S + k_{Irr}\left(k_S\right)\right) - c''\left(k_{Irr}\left(k_S\right)\right)}{P''\left(k_S + k_{Irr}\left(k_S\right)\right) k_{Irr}\left(k_S\right) + 2P'\left(k_S + k_{Irr}\left(k_S\right)\right) - c''\left(k_{Irr}\left(k_S\right)\right)} > 1, \tag{60}$$

and $V'(\delta) > 0$ as $k_S > k_0^i$ by assumption ($k_0^i < k_C$).

Assume now that $k_0^j > k_{Irr}\left(k_S^i\right)$. To simplify the notation, let $k_{Irr}^{-1} = \left(k_{Irr}\right)^{-1}\left(k_0^j\right)$. Then, firm i has incentive to deviate if and only if:

$$P\left(k_0^i + k_0^{-i}\right) k_0^i - c(k_0^i) < P\left(k_{Irr}^{-1} + k_0^j\right)\right) k_{Irr}^{-1} - c\left(k_{Irr}^{-1}\right) - \ln\left(\frac{1}{\delta}\right) p^+ \left(k_{Irr}^{-1} - k_0^i\right). \tag{61}$$

Let $V(\delta) = P\left(k_{Irr}^{-1} + k_0^j\right) k_{Irr}^{-1} - c\left(k_{Irr}^{-1}\right) - \ln\left(\frac{1}{\delta}\right) p^+ \left(k_{Irr}^{-1} - k_0^i\right)$. The derivative of the second term is given by:

$$V'(\delta) = \frac{\partial k_{Irr}^{-1}}{\partial \delta}\left(P'\left(k_{Irr}^{-1} + k_0^j\right) k_{Irr}^{-1} + P\left(k_{Irr}^{-1} + k_0^j\right) - c'\left(k_{Irr}^{-1}\right) - \ln\left(\frac{1}{\delta}\right) p^+\right) + \frac{1}{\delta} p^+ \left(k_{Irr}^{-1} - k_0^i\right). \tag{62}$$

As $k_{Irr}^{-1} < k_S$, $P'\left(k_{Irr}^{-1} + k_0^j\right) k_{Irr}^{-1} + P\left(k_{Irr}^{-1} + k_0^j\right) - c'\left(k_{Irr}^{-1}\right) - \ln\left(\frac{1}{\delta}\right) p^+$ is positive. Furthermore, as k_{Irr} increases with δ, k_{Irr}^{-1} also increases with δ and thus $V'(\delta) > 0$.

$\blacksquare$

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