JOB MARKET PAPER

Information Management by a Budget constrained Principal

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Abstract

This paper contributes to the debate on transparency in the public sector by considering one of its major features: a limited budget. In a principal-agent model with moral hazard and bounds on transfers, I study an information design problem where the principal can either choose to be transparent and fully reveal information before the agent takes action, or remain opaque. On the one hand, transparency makes it possible for the principal to engage her limited budget only when the expected gain is worth the implementation cost. On the other hand, opacity does not allow for tailor-made contracts but rather ensures optimal incentives. I show that transparency is more likely to be optimal for the principal when the task is less valuable and the budget is lower. Furthermore, the optimal information structure is derived, requiring that the agent be told only what action to undertake.

JEL classification: D23, D81, D82, D86

Keywords: Moral hazard, Limited Liability, Budget Constraint, Information Design

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1 Introduction

Transparency in the public sector is subject to ongoing debate. Economic intuition calls for an increase in the available information in agency relationships, and several countries have engaged on this path through reforms such as the No Child Left Behind Act (NCLB) in the United States or the legislation introduced in 2000 in the United Kingdom requiring public disclosure of local government results. Nevertheless, these transparency improvements are subject to resistance from both public employees and managers. Several recent papers, both within and outside the political economy literature, identify efficiency justifications for these oppositions in terms of incentives. In Besley and Smart (2007), for agents with uncertain ability, better information improves selection but increases rent seeking. In career concern models, Prat (2005) shows that information on an agent's action encourages him to disregard useful private signals, while Levy (2007a,b) shows that better outcomes can be insured by using secretive committees in which members do not reveal their votes. Other potential perverse effects of transparency are considered in Gavazza and Lizzeri (2007) regarding the congestion consequences of public disclosure of school and teacher quality, and in Dranove et al. (2003) who show that report cards push doctors to select their patients.

My paper contributes to the study of this link between incentives and transparency by focusing on a major feature of the public sector, i.e. a limited budget. Indeed, unlike in firms, no monetary benefits can be directly matched to public activity, and incentives cannot be insured through sharing profits. In the public sector, the budget constraint caps ex-post remunerations. This results in two biases for incentives. First, a limited budget implies that incentives are not affordable in every circumstances. Then, since huge rewards cannot be promised in case of very productive actions, incentives are less cost efficient.

In such constrained situations, the choice of transparency is a crucial instrument to make the best use of the limited budget. Managers can broadly choose between two strategies. They can either opt for a *transparent* approach and design case-by-case incentives or choose an *opaque* approach associated with batched incentives. In a transparent approach, managers specify what to do and when to do it, while an opaque approach comes with a general recommendation. A venerable illustration of these strategies can be found in Hegel (1818) regarding the difference between religious behaviors in Ancient Rome and Greece. In Ancient Rome, piety was administrative. A calendar specified which gods to worship, what sacrifices to make, and the associated blessings i.e. the gods' plan was transparent. However, in Ancient Greece, believers were required to be constantly honorable and moral. In particular circumstances, this behavior resulted in heroic actions that could be noticed and rewarded by the gods i.e. the gods' plan was opaque.

These strategies have both advantages and drawbacks. Managers who take a transparent approach makes it possible for managers are able to choose the employee's action that corresponds best to the circumstances. The disadvantage is that, this strategy implies contracting to each circumstance. When taking an opaque approach, the same action is implemented in every circumstance. This only requires one contract, but it lacks flexibility.

I focus here on a principal-agent model with moral hazard and risk-neutral protagonists, where the effort can have different productivities and featuring bounds on payments. In this model, incentive instruments are constrained, the agent is protected by limited liability, transfers are subject to an upper bound, i.e. the budget constraint. The principal can choose between a transparent or opaque management strategy. I study this question as an ex-ante information design problem with full commitment and contracts in which the principal can choose to reveal the productivity of the effort before (transparent approach) or after (opaque approach) the agent's action. The question here is thus to determine which disclosure policy the principal should **commit to** so as to maximize her expected profits rather than an informed principal issue.

The principal faces a trade-off when choosing between transparent and opaque management strategies. On the one hand, transparent management means that the principal can implement effort only when productivity is such that the expected gain is worth the corresponding implementation cost; she can therefore engage her limited budget in *relevant incentives*. On the other hand, opaque management allows the principal to implement the effort in every circumstance and compensate the agent on average, ensuring higher *incentive efficiency*. I show that transparency is more likely to be optimal for managers operating under tight constraints i.e. transparent management is more likely to be optimal for the principal when the task is least valuable and the budget is lowest.

Transparent and opaque management strategies are extreme information structures since the agent is either fully informed about the productivity of his effort or not informed at all. In an extension, I let the principal design the information structure and show that the optimal information structure requires opaque management conditional on the productivity being sufficiently high and the use of only two signals: one inducing effort and the other not. This shares a common intuition with the findings of Myerson (1986), which were established with very general mechanisms, and extends a result of informational design stated in Kamenica and Gentzkow (2011) to moral hazard setups. This information structure may not be optimal from a welfare maximizer perspective since it implements the effort in fewer states than is welfare optimal. As a consequence, the welfare maximizer may prefer opacity and want to establish a no communication rule that prevents the principal from persuading her agent. Finally, extending the discussion about information to technological matters, I let the principal choose between a distribution of the productivity of the effort and its mean preserving spread and show that, for low values of the available budget, the principal prefers facing more risky environments.

Literature Review Incentive theory has shown that there should be no loss when more information is available (see Hölmstrom (1979), Shavell (1979), Gjesdal (1982), Grossman and Hart (1983), Singh (1985)). In these papers, the principal learns something in addition to the outcome. She can either choose to condition the payment to this new data or not. Nevertheless, in some circumstances such public information cannot be ignored once it is revealed. This is the case in Crémer (1995), where the principal cannot commit to not using the additional knowledge, which makes it difficult to use credible threats. Hence, the principal may prefer to keep her agent at arm's length, i.e. non-informed and thus trade information for incentive power. In a similar vein, Sobel (1993) shows, in an agency framework with optimal contracts, that full disclosure is not optimal when the agent is not risk neutral. Indeed, in this setup, full disclosure imposes supplementary constraints on the principal, who must compensate the agent in each state, rather than on average. In contrast with this article, the current paper reveals a trade-off between transparency and opacity. This comes from the fact that I impose constraints on the principal's problem. The first set of constraints concerns the productivity of effort. I allow some states to be unproductive, that is to say, states where the expected benefit of the effort does not exceed its expected costs. The ability to contract for each state in a tailor-made way with full disclosure thus becomes a valuable instrument for the principal. The second set of constraints affects the available transfers. I consider a model with limited liability and a limited budget capping the payments.

The effects of introducing of a lower bound on the principal's available transfers has been extensively discussed in agency frameworks; Innes (1990), Sappington (1983) and Demski et al. (1988), to name just a few, offer characterizations of the optimal contracts in such configurations. Nevertheless, the consequences of introducing an upper bound are surprisingly understudied. Jewitt et al. (2008) explore the impact of the introduction of an upper and a lower bound on the optimal incentives structure and show that limited payments may lead to option-like contracts. In a closely related model to the one presented here, Poblete and Spulber (2012) draw a general characterization of an optimal contract in which the net benefits of the principal and agent are non-decreasing in the outcome. Their results reveal a trade-off between providing the agent with incentives for performance and compensating the principal for this investment. The model presented here departs from these approaches in two ways. First, the constraint upon the incentive means is exogenous, and is not a function of the outcome. This assumption makes it possible to consider cases where outcome is not monetary. Second, I study this question as an information design problem.

As an information design problem, the question addressed in this paper is related to the optimal information disclosure question considered in Rayo and Segal (2010) and Tamura (2014) and to the Bayesian persuasion question considered in Kamenica and Gentzkow (2011). However, these articles examine Sender-Receiver interactions in which available information directly determines the receiver's action and associated pay-offs. In a moral hazard framework, available information also impacts the incentive problem. In this sense, my question of information design is more closely related to the transparency question considered in Jehiel (2015). In this article, Jehiel shows, at a highly general level and for a class of problems including moral hazard, that full transparency can be generically improved upon using a dimensionality argument and the possibility that the action and the state can be locally varied. My results, established at a lower level of generality and with optimal contracts, echo this finding and develop the discussion on how constraints impact this non-transparency result. I show that the non-transparency result still holds when constraints are introduced on transfers, but that stronger constraints are associated with more transparency. Indeed, transparency allows the principal to implement effort only when the expected gain is worth the corresponding implementation cost. In contrast, opacity makes it possible to implement effort in every state with a cost-minimizing contract. Therefore, when the principal's problem is more constrained, the principal is more likely to choose incentive relevance over incentive efficiency. This trade-off is discussed, but not solved, in Jehiel (2015) and a similar arbitrage is identified in Lazear (2006), where the optimal rules for high-stakes testing depend on the costs of action and monitoring.

Interestingly, this trade-off makes it possible to study the issue of firm boundaries from a different perspective. Transparency allows the principal to implement the most appropriate action. This echoes the "employment contract" defined by Simon (1951) where the head of an organization can choose an employee's action from an agreed set once uncertainty is lifted. In my model, a similar distinction between the contracts associated with the two management strategies is identified. If the current state is revealed before the agent's action, the principal can order her preferred action state by state in a tailor-made way. When the agent acts before learning the state, then the principal and agent agree on a final outcome and a corresponding transfer. As developed by Coase (1937),giving the employer this authority reduces coordination problems. However, leaving the agent with no bargaining power may discourage him from undertaking profitable actions. This issue was tackled by the "modern transaction cost economics" established by Williamson (1975, 1985). Since then, the literature on firms' boundaries has moved the focus from coordination problems to efficiency considerations. In the model presented here, with complete contract, I find that management strategy fundamentally changes incentives and that opaqueness leads to stronger incentives.

The rest of the paper is organized as follows. In section 2, I present the model and the baseline case in which final outcome is the only observable information. Sections 3 and 4 characterize the optimal incentive schemes with a budget constraint on the principal's side respectively with opaque and transparent strategies. In section 5, I study the trade-off between incentive relevance and incentive efficiency. In section 6, the optimal information structure for the principal is derived. Section 7 discusses its welfare implications, while section 8 extends the discussion to technological choices and section 9 concludes. Proofs, when not in the text, can be found in the appendix.

2 The model and the no-information case

A risk-neutral principal (she) hires a risk neutral agent (him) for a job under full commitment. The agent is protected by limited liability and cannot be prohibitively punished. The outcome is either a success with a positive value R for the principal or a failure with no value, without loss of generality. The agent undertakes a non-observable action, he can exert the required effort and endure a cost c, or not. The effort affects the probability of success: p is the probability of success when effort is exerted, and p_0 is the corresponding probability when the agent shirks with $p > p_0$. The agent can face different situations affecting the productivity of his action. Therefore, the impact of the agent's effort on success depends on the current state of the world, and the productivity p is distributed according to the positive density $f(p)$ with distribution function $F(p)$ over $[\underline{p}, \overline{p}]$. I assume for sake of simplicity that the probability of success when the agent shirks, p_0, does not change with the state of the world, and let $\Delta_p \equiv p - p_0$.

In addition to this canonical framework, I introduce a constraint on the principal's side. The principal faces an upper bound B on her ex-post payments. As a consequence, she may not offer bonuses as high as she would like, even if they yield positive net profits. Such an exogenous budget can generally be applied in circumstances where performance does not immediately translate into monetary benefits for the principal, which is always the case in the public sector.

As a baseline case, I consider that the only observable information is the outcome. Therefore, to design her incentive scheme, the principal relies only upon the observed outcome, which might be a success or a failure. Due to limited liability, the principal cannot punish the agent in case of failure or promise a strictly positive bonus, which would encourage shirking. Thus, in order to implement the effort in every state, it is optimal for the principal to give a strictly positive bonus b only when a success occurs. This bonus saturates the incentive compatibility constraint and $b = c/E(\Delta_p)$.[2] However, this bonus cannot exceed the available incentive budget of the Principal B. As a consequence, the principal cannot implement the effort if $c/E(\Delta_p) > B$. Moreover, the principal will choose to implement the effort only if the value of a success is worth the implementation cost i.e. $R \geq E(p)c/E(\Delta_p)^2$.

3 Opaque Management

I let the principal choose between two management strategies. The principal can reveal the productivity of the effort before (transparent approach) or after (opaque approach) the agent's action. I study this question as an ex-ante information design problem with full

[2]In this situation the incentive compatible constraint takes the standard form $\int_{\underline{p}}^{\overline{p}} pbf(p)\,\mathrm{d}p - c \geq p_0 b$ and the participation constraint is always satisfied if the incentive constraint is saturated.

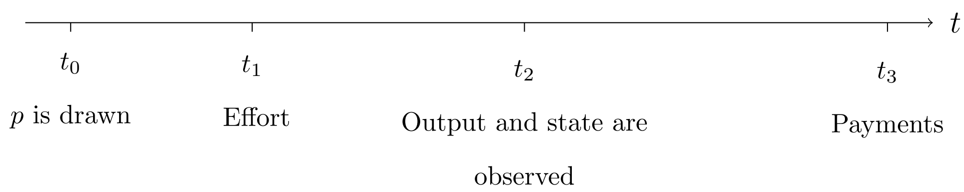


Figure 1: Contracting timing with opaque management

commitment and contracts. The issue studied here is thus not an informed principal situation. In this section, I consider the opaque management strategy whose contractual timing is displayed in Figure 1. In this framework, the principal cannot restrain the implementation of effort to a particular interval. She can either implement a global effort, or none. Since the state of the world is known ex-post, the principal can condition her payment to the realized p. Indeed, a success does not convey the same information in all states. The likelihood that the agent has exerted an effort when the principal observes a success is an increasing function of p.

As shown by Innes (1990) for limited liability models, it is optimal for the principal to pay the agent only in the most informative state, $\overline{p}$. However, this incentive strategy requires an infinite budget, which is not available to the principal in this model. To solve the incentive problem in the most efficient way, the principal has to put the incentive weight on the most informative states. The principal thus has to give the maximum bonus i.e. all of the available budget B, when a success is observed in the states with the highest p.[3] The size of this interval is derived from the incentive compatibility constraint and is a function of the available budget. The principal does not have to choose what to give, but when to give, and the bonus $b_O(p)$ is a step function of p, as illustrated by Figure 2. The principal determines a threshold value of p noted $p^*(B)$ such that

$$\int_{p^*(B)}^{\overline{p}} B\Delta_p f(p)\,\mathrm{d}p = c \qquad (IC_O)$$

[3]Since the principal is budget constrained, she can not condition her payment to a single state. She has to pay the agent in case of success in an interval of p. The principal select the interval where a success is the most informative about the agent's action. Therefore, this interval takes the form $[X, \overline{p}]$ because the likelihood ratio $\int_X^{\overline{p}} pf(p)\,\mathrm{d}p / \int_X^{\overline{p}} p_0 f(p)\,\mathrm{d}p$ is an increasing function of X.

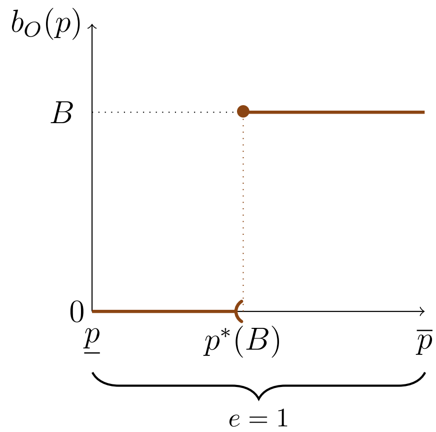


Figure 2: Optimal bonus with a budget B and opaque management

The function $p^*(B)$ increases in B. When the budget increases, the principal is able to condition her payment to more informative states. Note that the agent is paid when his effort has the highest effect on the outcome.

Proposition 1 *If $B \geq c/E(\Delta_p)$, the optimal contract with opaque management is:*

$$\begin{cases} b_O(p) = B & \textit{if} \quad p \geq p^*(B) \\ \\ b_O(p) = 0 & \textit{otherwise} \end{cases}$$

And no incentives if not.

Nevertheless, this incentive scheme is feasible only if $B \geq c/E(\Delta_p)$. If $B = c/E(\Delta_p)$ the principal has to pay as soon as a success is observed, no matter what the current state is. This is the baseline case exposed in Section 2, and the worst-case scenario when ex-post information is available for contracting. The other extreme case appears when B tends towards infinity. In this case, the principal can condition the payment to a success observed in the best state, $\overline{p}$ (see the appendix for a demonstration of this limit result).

The expected profit for the principal with ex-post information is noted $V_O(B, R)$ and can be expressed with the optimal contract:

$$V_O(B,R) = \int_{\underline{p}}^{\overline{p}} pRf(p)\,\mathrm{d}p - \int_{p^*(B)}^{\overline{p}} pBf(p)\,\mathrm{d}p$$

This function increases in B and R. A higher budget does not increase the gains but diminishes the implementation cost. A higher budget allows the principal to condition her payment to more informative states and increases the incentive efficiency, while an increase in R leads to higher expected gains.

4 Transparent Management

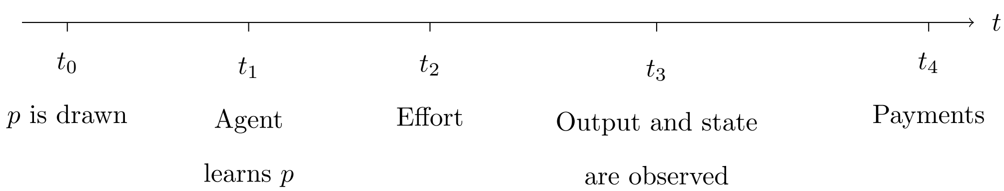


Figure 3: Contracting timing with transparent management

I consider here a case where the current state of the world is revealed before the agent's action. The contracting timing with transparent management is displayed in Figure 3. In this framework, the principal has to build a contract for each state of the world. For all $p \in [\underline{p}, \overline{p}]$, the principal receives a signal about the outcome at the end of the relationship, i.e. success or failure. To implement the effort, it is optimal for the principal to pay a strictly positive bonus if, and only if, a success is observed. This bonus, in state p, is noted $b_T(p)$ and saturates the incentive constraint. Therefore the optimal bonus is such that $b_T(p) = c/\Delta_p$ and is a decreasing function of p.[4] If the distance between p and p_0 decreases, the value of the bonus increases. However, this payment cannot be higher than the incentive budget B. As a consequence, the principal cannot implement the effort in state p if $p < c/B + p_0 \equiv \hat{p}(B)$ as illustrated in Figure 4. Moreover, the principal will implement the effort in state p if $p(R - b_T(p)) \geq p_0 R$ i.e. state p is productive. When p decreases, the expected result decreases and the implementation cost increases. The last productive state is noted $\widetilde{p}(R)$.

[4] Here the agent's incentive constraint takes the standard form $pb_T(p) - c \geq p_0 b_T$ and the participation constraint is satisfied if the incentive constraint is saturated.

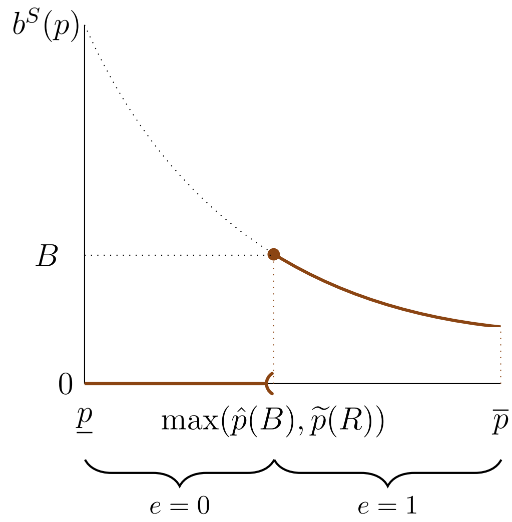


Figure 4: Optimal bonus with a budget B and transparent management

Therefore, there are states in which the principal can simply not implement an effort, either because her budget is not sufficiently high, or because the state is not sufficiently productive. Note that this limit does not apply to opaque management since the effort is implemented in every state.

Proposition 2 *The optimal contract with transparent management is:*

$$\begin{cases} b_T(p) = \frac{c}{\Delta_p} & \textit{if} \quad p \geq \max\{\hat{p}(B), \widetilde{p}(R)\} \\ \\ b_T(p) = 0 & \textit{otherwise} \end{cases}$$

The principal's expected profit in this configuration is noted $V_T(B, R)$ and can be expressed with the optimal contract:

$$V_T(B, R) = \int_{\underline{p}}^{\max\{\hat{p}(B), \widetilde{p}(R)\}} p_0 R f(p) \, \mathrm{d}p + \int_{\max\{\hat{p}(B), \widetilde{p}(R)\}}^{\overline{p}} p[R - b_T(p)] f(p) \, \mathrm{d}p$$

This function increases with B and R. A higher budget allows the principal to implement effort in more, but less productive, states. This function is constant when B is such that $\hat{p}(B) < \widetilde{p}(R)$.

5 Trade-off between opaque and transparent management

5.1 If all the states are productive

If all the states are productive, $R \geq \underline{p}c/(\underline{p}-p_0)^2$, the principal wishes to implement the effort for all $p \in [\underline{p}, \overline{p}]$. In this case, comparing $V_T(B,R)$ and $V_O(B,R)$ amounts to comparing the corresponding implementation cost. Since the principal's objective with opaque management is precisely to minimize this cost, this strategy is always preferable when feasible.

Proposition 3 *If all the states are productive and $B \geq c/E(\Delta_p)$, the principal chooses opaque management.*

Proof. On the one hand, when the agent chooses his action before the state is revealed, the implementation cost reaches its maximum value, $cE(p)/E(\Delta_p)$, when $B = c/E(\Delta_p)$. On the other hand, when the agent chooses his action after the state is revealed, the implementation of the effort in every state is possible only if $B \geq c/\Delta_{\underline{p}}$ and induces a cost $\int_{\underline{p}}^{\overline{p}} cp/\Delta_p f(p)\mathrm{d}p$. Let $g(p) = cp/\Delta_p$, these implementation costs can be respectively written as $g[E(p)]$ and $E[g(p)]$. The function g is convex in p and the Jensen's inequality insures that $g[E(p)] \leq E[g(p)]$. Since both $V_O(B,R)$ and $V_T(B,R)$ increase in B, the principal prefers the opaque management if this strategy is feasible i.e. $B \geq c/E(\Delta_p)$.

5.2 If some states are not productive

If some states are not productive, the principal faces a trade-off between incentive relevance and incentive efficiency. Transparent management makes it possible to implement the effort in a particular state only when the expected gain is worth the expected cost, while opaque management allows the effort to be implemented in every state with a cost-minimizing contract. The principal's choice between these two strategies depends on the values of the budget B and the success R. The principal prefers opaque management if, and only if:

$$\int_{\underline{p}}^{\max\{\hat{p}(B),\widetilde{p}(R)\}} R\Delta_p f(p)\,\mathrm{d}p \geq \int_{p^*(B)}^{\overline{p}} pBf(p)\,\mathrm{d}p - \int_{\max\{\hat{p}(B),\widetilde{p}(R)\}}^{\overline{p}} pb_T(p)f(p)\,\mathrm{d}p \qquad \text{(C)}$$

The effects of a rise in available budget and value of success on this trade-off are ambiguous. With opaque management, if R increases, the expected gain increases, and if B increases, the principal is able to promise a higher bonus in more informative states and the implementation cost decreases. With transparent management, if R increases, the expected gain increases, more states become productive, and the principal wants to implement effort for lower values of p. With this management strategy, an increase in the budget allows effort to be implemented in more but less productive states. However, the principal may face two constraints with transparent management. Indeed, she may want to implement effort in more states regarding the value of R but be unable to afford the corresponding implementation costs due to her limited budget, or she may be able to implement effort in more states regarding B but not want to due to the value of a success.

For some values of R and B the principal's choice can be identified. If the value of a success is sufficiently low, $R < \overline{p}c/(\overline{p} - p_0)^2$, or if the budget is too small, $B < c/(\Delta_{\overline{p}})$, it is never optimal for the principal to implement effort, no matter what the considered management strategy. Moreover, if $B < c/(E[p] - p_0)$ opaque management is not feasible and, as stated in **Proposition 3**, if $B \geq c/(E[p] - p_0)$ and all the states are productive, then opaque management is always preferred. For all the other pairs (R, B), the two possible constraints in the transparent management described above have to be taken into account. Let $B(R)$ be the relation between the available budget and the value of success such that $\hat{p}(B) = \widetilde{p}(R)$. This relation is distribution free and defined as:

$$B(R) = \frac{2cR}{\sqrt{4cp_0R + c^2} + c}$$

This function is increasing and concave in R. Along this relation, the last productive state is the last state in which the effort is implementable. Above this relation, there are more states in which effort is implementable than there are productive states. In such circumstances, if B increases, with R remaining constant, the implementation cost of opaque management decreases and nothing changes for the transparent approach. Therefore, such a variation always favors an opaque strategy. A similar observation can be made below the relation $B(R)$, where the binding constraint in transparent management relies on the available budget. In this circumstance, an increase in R, with B remaining constant, is more beneficial to opaque management than to transparent management. The study of the impact of the other

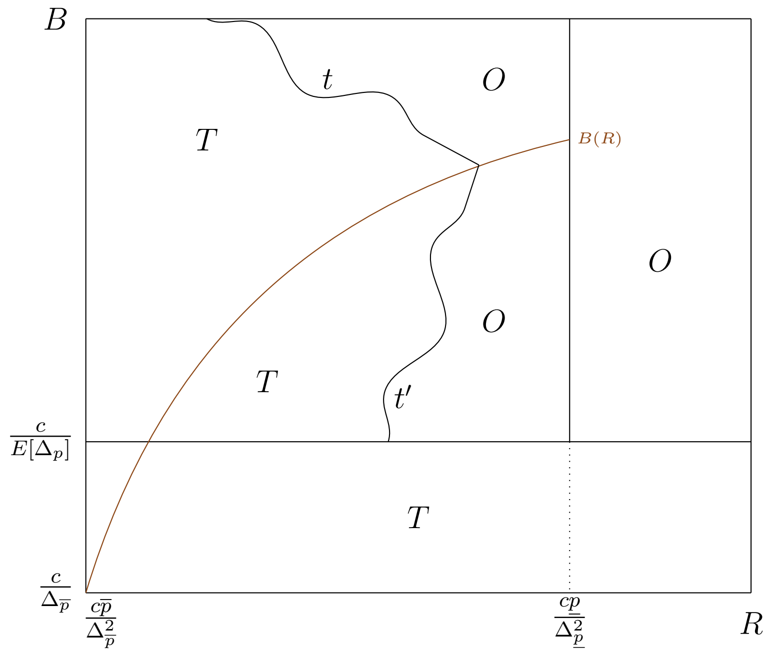


Figure 5: Trade-off between opaque and transparent management

variations on the trade-off, namely an increase in R above $B(R)$ and in B below, is far more complex and depends on the distribution of productivity. In particular, the trade-off may reverse more than once, as depicted in Figure 5. An example featuring uniform distribution is developed below to illustrate this issue. Nevertheless, the comparative statics of **Proposition 4** state that the less valuable the task and the lower the budget, the more likely it is that transparency will be optimal for a principal.

Proposition 4 *When all of the states are not productive and $B \geq c/E[\Delta_p]$. Let $B(R)$ be such that $\hat{p}(B) = \widetilde{p}(R)$ and $B^{-1}(R)$ the inverse of this bijection.*

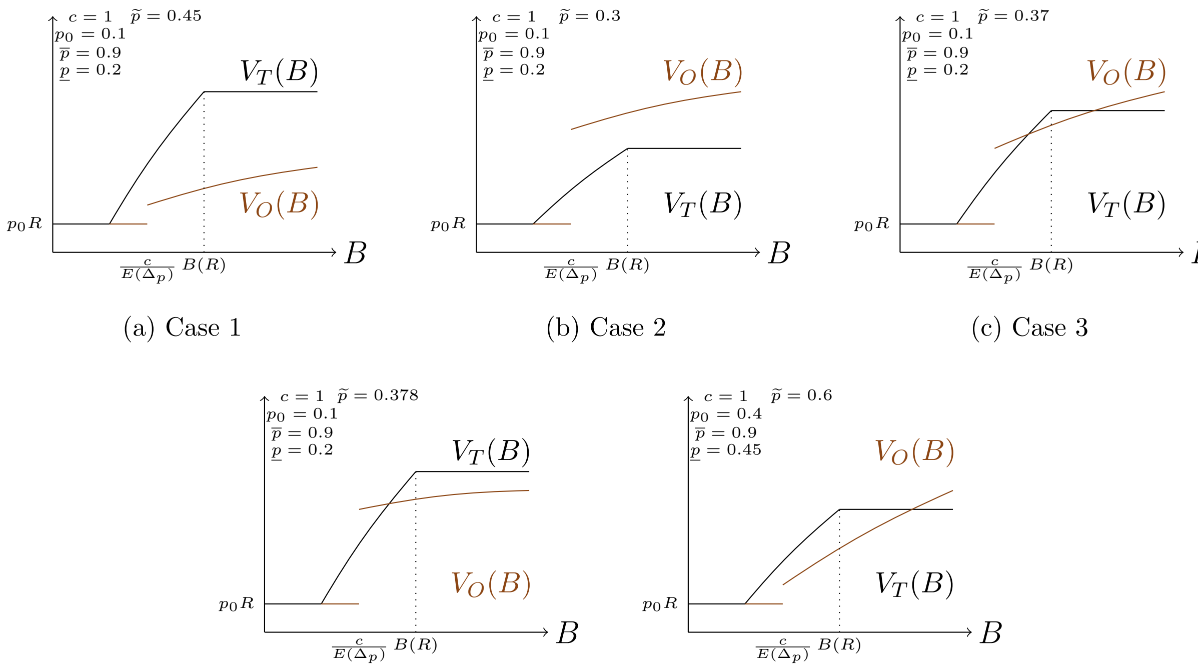
(i) The function $t : E \rightarrow F$ where $E = \{R \mid \frac{c\overline{p}}{\Delta^2_{\overline{p}}} \leq R \leq \frac{c\underline{p}}{\Delta^2_{\underline{p}}}\}$ and $F = \{B \mid B \geq B(R)\}$ such that $\forall B \geq t(R)$, $V_O(B,R) \geq V_T(B,R)$ is surjective.

(ii) The function $t' : E' \rightarrow F'$ where $E' = \{B \mid \frac{c}{E[\Delta_p]} \leq B\}$ and $F' = \{R \mid R \geq B^{-1}(R)\}$ such that $\forall R \geq t'(B)$, $V_O(B,R) \geq V_T(B,R)$ is surjective.

(iii) On the relation $B(R)$, there exists a unique R_O such that opaque management is preferred for all $R \geq R_O$.

Proof. In circumstances considered in (i), an increase in B, with R remaining constant, decreases the implementation cost of the opaque strategy and does not impact $V_T(R,B)$. Therefore, an increase in B is always beneficial to the opaque approach. In circumstances considered in (ii), $dV_O(R,B)/dR = E[\Delta_p]$ and $dV_T(R,B)/dR = E[\Delta_p] - \int_{\underline{p}}^{\hat{p}(B)} \Delta_p f(p)\,\mathrm{d}p$ and an increase in R is always beneficial to the opaque approach. The proof of (iii) comes directly from (i) and (ii).

Example featuring uniform distribution



(a) Case 1 (b) Case 2 (c) Case 3

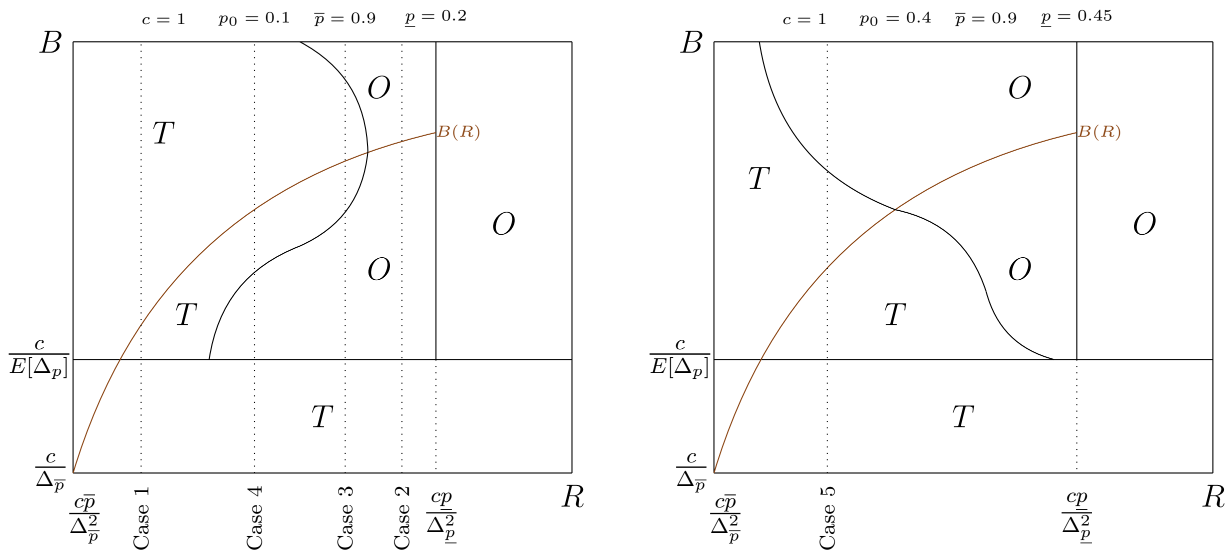
(d) Case 4 (e) Case 5

Figure 6: Simulations of $V_T(B)$ and $V_O(B)$ with uniform distribution

With a uniform distribution, it is possible to fully solve the trade-off above the relation $B(R)$. Consider the derivatives of the expected profits with respect to R. Simple calculations lead to:[5]

$$\frac{dV_O(R,B)}{dR} > \frac{dV_T(R,B)}{dR}$$

[5] $E[\Delta_p] > \frac{1}{\overline{p}-\underline{p}}[p_0(\widetilde{p}(R)-\underline{p}) + \frac{\overline{p}^2-\widetilde{p}(R)^2}{2} + \widetilde{p}(R)'(p_0R - R\widetilde{p}(R) + \frac{\widetilde{p}(R)c}{\widetilde{p}(R)-p_0})] \Leftrightarrow \frac{c\widetilde{p}(R)}{(\underline{p}-p_0)^2} > R$



(a) Case 1, 2, 3 and 4

(b) Case 5

Figure 7: Trade-off with uniform distribution

Therefore, an increase in R, with B remaining constant, is more beneficial to the opaque strategy than to the transparent strategy and tips the scales in favor of opaque management. Below the relation $B(R)$, ambiguity cannot be resolved, and even with uniformly distributed p, it is not possible to analytically derive conditions for the principal's choice. The relative positions of the expected increasing and concave profits, $V_O(B,R)$ and $V_T(B,R)$, vary for the uniform distribution with the values of the parameters as illustrated with simulations in Figure 6 and in the corresponding diagrams in Figure 7. However, all of the possible expected profits share the same profile. The opaque strategy is not feasible if $B < c/E(\Delta_p)$ and the transparent strategy is always preferred since the principal is able to implement the effort in the most productive states with this strategy. When B increases, the implementation cost decreases with the opaque strategy because the principal can promise a higher bonus in case of success in more informative states. When B tends towards infinity, $V_O(B,R)$ converges to $E(p)R - \overline{p}c/\Delta_{\overline{p}}$. In the transparent scenario, an increase in B makes it possible for the principal to implement the effort in more but less productive states. Hence, $V_T(B,R)$ is increasing and concave in B. When $B > B(R)$, $V_T(B,R)$ is constant since the principal does not implement the effort in unproductive states.

Several configurations should be considered. On the one hand, opaque strategy may yield higher expected profits when it becomes feasible. This configuration is the one of panels

(b), (c) and (d) on Figure 6. An increase in B has an ambiguous effect on the comparison of $V_O(B,R)$ and $V_T(B,R)$. Indeed, an increase in B may be more beneficial to the opaque strategy, this is Case 2 of panel (b) in Figure 6 drawn separately in comparative diagram of panel (a) of Figure 7. Or it might be more beneficial to the transparent strategy: this is Case 4 illustrated in panel (d) in Figure 6 and panel (a) in Figure 7. This configuration appears when there are a large number of unproductive states. In the last possible configuration, Case 3, similar to Case 4, an increase in B is more beneficial to the transparent strategy when this increase is used to implement the effort in productive states. When $V_T(B,R)$ reaches its constant value, an increase in B is more beneficial to the opaque strategy since the principal can promise a higher bonus in more informative states. This case, illustrated in panel (a) in Figure 7, appears when there are a large number of unproductive states and the best states are sufficiently informative. On the other hand, opaque strategy may not yield a higher expected profit when it becomes feasible. This configuration is shown on panels (a) and (e) in Figure 6. Only two possible cases exist, i.e. either the limit value of $V_O(B,R)$ is higher than $V_T[B(R),R]$ and an increase in B is more beneficial to the opaque strategy as illustrated in panel (b) in Figure 7, or it is not, and an increase in B is more beneficial to the transparent strategy as illustrated by Case 1 in panel (a) of Figure 7.

6 Optimal information structure

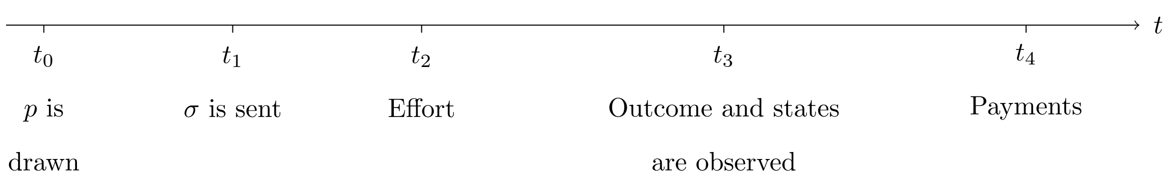


Figure 8: Contracting timing with an optimal information structure

Only two extreme information structures have been considered so far. The purpose of this section is to identify the optimal information structure for the principal. I maintain the two main assumptions: the current state of the world is public information at the end of the contracting relationship, and there is full commitment. As a consequence, the principal cannot lie about the value of p and can commit to any information structure. Full commitment thus rules out the informed principal problem and there is no incentive compatibility constraint on the principal's side.

Moreover, since the current state is a public information at the end of the contractual relationship, it is always optimal for the principal to condition her payment on the observed value of p. Indeed, if this information is available before the effort, it is also available afterwards and, following Holmström (1979) and the *informativeness principle*, the principal should use it. Therefore, the relevance of the information structure depends on the information delivered to the agent before his effort. The informational timing is displayed in Figure 8. This information is conveyed by a signal σ, sent by the principal. Principal and agent share a prior about the distribution of p. They believe $F(p)$ to be the cumulative function of this random variable. The probability that message σ will occur is λ_σ and the corresponding posterior of the agent is $F_\sigma(p)$. Bayesian rationality requires that the agent's posteriors must be Bayes-plausible i.e. on average the principal will not lie. This condition is natural in this model since the current state is revealed ex-post.

I consider here a class of information structures where signals can take a finite number of values, $\sigma \in \{1, 2, ..., n\}$.[6] This assumption insures that for any Bayes-plausible distribution of posteriors there is a set of signals that induces it, and Bayesian rationality requires that:

$$\sum_{\sigma=1}^{n} \lambda_\sigma F_\sigma(p) = F(p)$$

Hence, Bayes's rule makes it possible to study the incentive problems separately given a signal σ and a corresponding posterior $F_\sigma(p)$. When receiving signal σ, the agent learns that p belongs to a Borel set S_σ such that $S_\sigma \subset [\underline{p}, \overline{p}]$. I assume that $P = \{S_\sigma | 1 \leq \sigma \leq n\}$ forms a partition of $[\underline{p}, \overline{p}]$. This assumption insures that each state p belongs to one and only one Borel set S_σ.[7]

For each set, the agent chooses between two actions: he can either work or shirk. The principal associates a recommendation with each signal and faces an incentive compatibility constraint for each signal inducing the effort. Therefore, the principal is always better off by employing only one signal to induce the effort, since it allows her to aggregate the agent's various incentive constraints into one constraint that is easier to satisfy. This result stated in **Proposition 5** shares a common intuition with the finding established with very general

[6] This restriction is used in Kamenica and Gentzkow (2011) but is more substantive here since the set of possible productivity is infinite.

[7] Similar assumptions are made in Board (2009) for defining *groups*.

mechanisms in Myerson (1986), i.e. providing more information to the agent makes the incentive constraints harder to satisfy.

Proposition 5 *In the optimal information structure, the principal uses only two signals: one induces the effort, the other does not.*

A similar intuition can be found in Kamenica and Gentzkow (2011) where, in the optimal information structure, there are no more signals than available actions. However, the result of **Proposition 5** is established in a principal agent model with moral hazard in which information impacts both the agent's action and the implementation cost. This thus demonstrates that limiting the number of signals to the number of possible actions is also optimal when dealing with an agent, and not only with a receiver, and that a transparent strategy is never optimal. Let us label the signal inducing effort as "go" and the other as "no go". When receiving the signal "go" (respectively "no go"), the agent learns that p belongs to a Borel set S_{go} ($S_{no\,go}$) which is the union of all of the sets inducing effort (respectively shirking).

The last step in the characterization of the optimal information structure concerns the arrangement of the sets S_{go} and $S_{no\,go}$. To address this question it is sufficient to ask if it is optimal for the principal to change her recommendation more than once along p. To introduce the formal argument the following definition is useful.

Definition 1 *Two sets, S_i and S_j where $i \neq j$, overlap if there exists $p_H > p_M > p_L$, such that $p_H, p_L \in S_i$ and $p_M \in S_j$.*

Using this definition, the question becomes: is it optimal for the principal that S_{go} and $S_{no\,go}$ should overlap? On the one hand, since the principal uses ex-post information, the optimal bonus in the optimal information structure is that of the opaque management strategy characterized in section 3. That is to say, the principal promises a bonus B when a success occurs in the most informative states, and promises nothing for the lowest p in the set S_{go}. As a consequence, it is in the principal's best interest to include the highest values of p in S_{go} since it insures higher incentive efficiency. On the other hand, including the highest value of p in S_{go} increases the expected gain. Hence, as stated in **Lemma 1** it is not optimal for the principal that S_{go} and $S_{no\,go}$ should overlap.

Lemma 1 *On the optimal information structure, S_{go} and $S_{no\,go}$ cannot overlap.*

Therefore, it is possible to conclude from **Proposition 5** and **Lemma 1** that in the optimal information structure the principal uses only two signals $\sigma = go$ and $\sigma = no\ go$ and that the associated sets S_{go} and $S_{no\,go}$ are such that for all $p \in S_{go}$ and $p' \in S_{no\,go}$, $p \geq p'$ as stated in **Proposition 6**.

Proposition 6 *In the optimal information structure, there exists a unique $x \in [\underline{p}, \overline{p}]$ such that the principal sends out signal "go" if $p \in [x, \overline{p}]$ and signal "no go" otherwise.*

Unsurprisingly, the optimal information structure combines those features of the transparent and opaque management strategies that are beneficial to the principal i.e. the principal benefits from the incentive efficiency of the opaque strategy and the ability to not implement the effort in the less productive states. The optimal informational structure is the opaque management strategy conditional to a unique threshold value of p labeled x.[8] Due to this threshold-based structure, it is possible to derive the two posteriors.

$$F_{no\,go}(p) = \min\left\{\frac{F(p)}{F(x)}, 1\right\} \qquad\qquad F_{go}(p) = \max\left\{\frac{F(p) - F(x)}{1 - F(x)}, 0\right\}$$

The expected profit with the optimal information structure is $V^*(x, B, R)$ where $p^*(x, B)$ is such that the agent receives the bonus B in case of success for all $p \in [p^*(x, B), \overline{p}]$.

$$V^*(x, B, R) = \int_{\underline{p}}^{x} p_0 R f(p)\,\mathrm{d}p + \int_{x}^{\overline{p}} pRf(p)\,\mathrm{d}p - \int_{p^*(x,B)}^{\overline{p}} pBf(p)\,\mathrm{d}p$$

[8] In this model, the set of possible productivity is infinite and there should be no *a priori* restriction on the possible structure of the signals. In particular, any state p could be stochastically associated with several signals. In the class of information structures considered here, characterized by a finite and countable set of signals and the fact that P forms a partition of $[\underline{p}, \overline{p}]$, this possibility is not considered. To the best of my knowledge, the online appendix of Kamenica and Gentzkow (2011) and a recent working paper by Yamashita (2016) constitute the state-of-the-art on this issue, employing a more general class of possible information structure, but they are not fully conclusive. In the model presented here, I conjecture that, employing a more general class of information structure, the equilibrium is essentially unique up to mixing at the threshold.

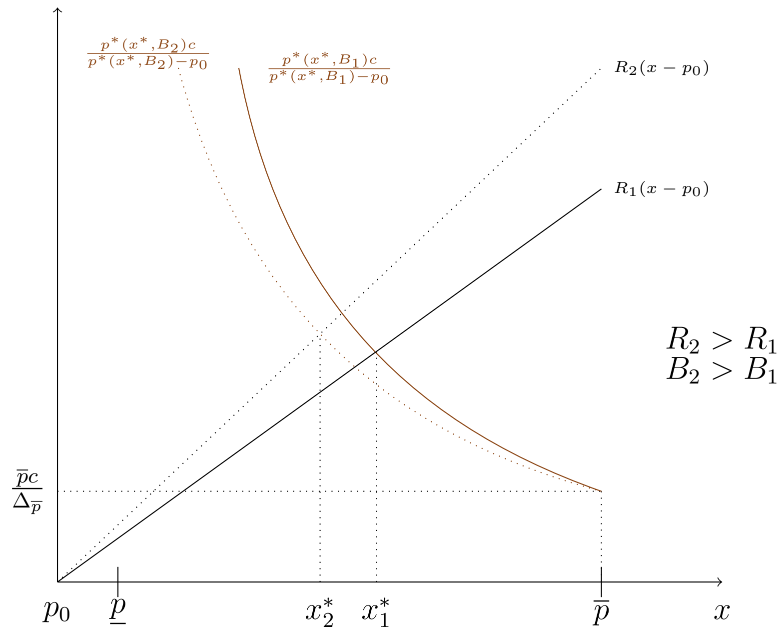


Figure 9: Determination of x^* with the uniform distribution

The optimal value of x labeled x^* can be obtained through the following optimality condition:

$$\frac{\partial V^*(x^*, B, R)}{\partial x^*} = 0 \Leftrightarrow x^* = p_0 + \frac{c}{R}\frac{p^*(x^*, B)}{\Delta_{p^*(x^*,B)}}$$

Note that x^* is the solution of the following equation depicted with the uniform distribution in Figure 9:

$$R(x^* - p_0) = \frac{p^*(x^*, B)c}{\Delta_{p^*(x^*,B)}}$$

The left-hand side of this equation is independent from B and increasing in R, while the right-hand side is independent of R and decreasing in B. Indeed, the right-hand side is decreasing in the likelihood that the agent exerted the effort when a success is observed in state $p^*(x^*, B)$, which is increasing in B since a higher budget makes it possible for the principal to condition her payment on more informative states. As a consequence, x^* is decreasing in R and in B for all possible distributions of the productivity of effort as stated in **Proposition 7**.

Proposition 7 *For any distribution of states, the optimal threshold x^* is decreasing in R and B.*

The intuition behind this result is similar to that in Section 5. If both the value of the task performed by the agent and the budget increase, then the principal wants to implement the effort in more states and so extends the support of the signal "*go*". As a consequence, this signal becomes more opaque for the agent.

7 Welfare implications

Let us leave the principal's objective aside for a moment and consider the problem from the perspective of a welfare maximizer. The traditional candidate for this entity is the government and is a natural choice for the model presented here, whose major application is the public sector. The definition used for welfare is simply the sum of the principal's expected profit and the agent's expected payment minus the cost, and can be computed for all of the information structures. Let $W_O(B,R)$, $W_T(B,R)$ and $W^*(B,R)$ respectively be welfare with the opaque, transparent and optimal information structures.

$$\begin{cases} W_O(B,R) = & \int_{\underline{p}}^{\overline{p}} pRf(p)\,\mathrm{d}p - c \\ \\ W_T(B,R) = & \int_{\underline{p}}^{\max\{\hat{p}(B),\widetilde{p}(R)\}} p_0Rf(p)\,\mathrm{d}p + \int_{\max\{\hat{p}(B),\widetilde{p}(R)\}}^{\overline{p}} [pR-c]f(p)\,\mathrm{d}p \\ \\ W^*(B,R) = & \int_{\underline{p}}^{x^*} p_0Rf(p)\,\mathrm{d}p + \int_{x^*}^{\overline{p}} [pR-c]f(p)\,\mathrm{d}p \end{cases}$$

In this framework, the government and the principal attach the same value to the public service. If the welfare maximizer is the government, then the principal is the head of the institution in charge of providing a public service whose social value is R. The principal may be, for instance, a hospital director, a prison governor, or a police chief, and there is no reason to think that a hospital director would attach a lower to a patient's recovery than the government.

However, the principal has to handle the difficulties associated with project management, embodied in this model by the moral hazard issue. The consequence is a misalignment between the government's objective and the principal's objectives. The incentive problem is

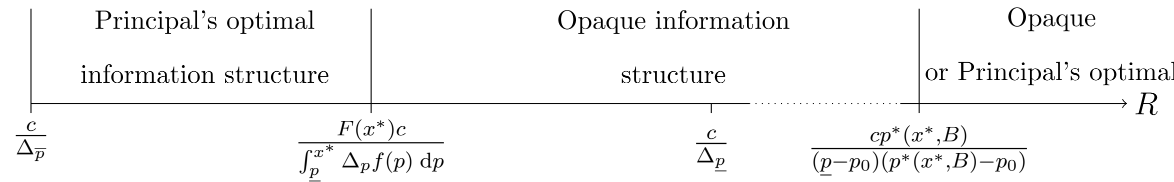


Figure 10: Best information structure for a welfare maximizer

neutral for the government, which seeks to maximize the probability of success as long as the expected gain is worth the “true” cost of the effort i.e. the government wants the effort to be implemented in state p if $R \geq c/\Delta_p$, while the principal has to take into account the implementation cost due to her managerial responsibilities.

Therefore, the optimal information structure for the principal may not be optimal for the government. Let p^W be the last productive state from a welfare-maximizer perspective. It is possible to show that the principal implements the effort in fewer states than it is welfare optimal.

Lemma 2 *For any distribution of states,* $p^W \leq x^* \ \forall \ R$ *and* B*.*

Proof. The misalignment between the government's objective and the principal's objective comes from the moral hazard issue. The incentive constraint faced by the principal takes its most relaxed form when there is no budget constraint i.e. when B tends towards infinity. In this case, $\lim_{B \to +\infty} x^* = p_0 + c/R \cdot \overline{p}/\Delta_{\overline{p}}$ which is always higher than $p^W = p_0 + c/R$. *A fortiori*, $p^W \leq x^* \ \forall \ R$ and B since x^* is decreasing in B.

To better align the principal's objective, the governement could use two instruments, i.e. the budget, and rules on information management. On the one hand, since x^* is decreasing in B, a higher budget makes it optimal for the principal to implement the effort in more welfare-productive states. On the other hand, if $R \geq c/\Delta_{\underline{p}}$ then all of the states are welfare productive and welfare maximizer prefers the opaque management strategy in which the principal implements the effort for all p. When R decreases, the government faces a trade-off between losing welfare productive states and paying for unproductive activities. The government prefers the opaque management strategy as long as:

$$W_O(B,R) \geq W^*(B,R) \Leftrightarrow R \geq \frac{F(x^*)c}{\int_{\underline{p}}^{x^*} \Delta_p f(p)\ \mathrm{d}p}$$

Hence, if the value of the task is sufficiently high, the government will want to establish a no-communication rule that will prevent the principal from persuading her agent and denying the implementation of effort in welfare productive states as depicted in Figure 10. The government becomes indifferent between the opaque and optimal management strategies when $R \geq cp^*(x^*,B)/[(\underline{p}-p_0)(p^*(x^*,B)-p_0)]$ because when R is great enough, the principal implements the effort in all of the states with the optimal information structure.[9]

8 Technological choices

The distribution of the states plays a major role in both management strategies. With a transparent strategy, this distribution determines the probability of facing productive and affordable efforts, while with the opaque strategy, the distribution of productivity affects the implementation cost by giving the probabilities of the most informative states. Intuitively, in the public sector, the distribution of states could be interpreted as the essential characteristic of the population targeted by the public service. I present this issue as a technological choice for the principal. Comparing two distributions, I focus on two general orders, i.e. the first and the second order stochastic dominance. These orders require few assumptions and are easy to interpret.

When a cumulative function G first order stochastically dominates the cumulative F, the probability of obtaining at least p is higher with G for every p.

Definition 2 *The distribution G is first-order stochastic dominant over F, G FOSD F, if and only if $G(p) \leq F(p)$ for all $p \in [\underline{p}, \overline{p}]$.*

[9]Note that the transparent management strategy has not been considered. This is because if there is no rule forbidding it, the principal will always choose the optimal information strategy over the transparent one. Suppose that the government has the ability to force the principal to choose a transparent strategy. Then, the last state in which effort is implemented is $\max\{\hat{p}(B), \widetilde{p}(R)\}$. Consider the particular case where $\hat{p} = \widetilde{p}$. In this case, $x^* \leq \hat{p} = \widetilde{p}$ is equivalent to $p^*(x^*,B) \geq \hat{p} = \widetilde{p}$. Since the incentive compatibility constraint requires that $\int_{p^*(x^*,B)}^{\overline{p}} \Delta_p f(p)\ \mathrm{d}p = (1-F(x^*))(\hat{p}-p_0)$ it must be the case that $p^*(x^*,B) \geq \hat{p} = \widetilde{p}$. Furthermore, since x^* is increasing in R and in B, $x^* \leq \max\{\hat{p}(B), \widetilde{p}(R)\}$.

Therefore the technology characterized by G is "better" than the one characterized by F, the best states are more likely to occur. The notion of second order stochastic dominance allows us to order the distributions in terms of risk. We use mean-preserving spread of the simple type (Rothschild and Stiglitz (1970)).

Definition 3 *The distribution G is second-order stochastic dominant over F, G SOSD F, if and only if $S_G(p) \leq S_F(p)$ for all $p \in [\underline{p}, \overline{p}]$ and $S_G(\overline{p}) = S_F(\overline{p})$, where*

$$S_F(p) = \int_{\underline{p}}^{p} F(x)\mathrm{d}x$$

The end-point equality of the S-functions imposes equal means, and F is going to be riskier than G, F is a mean-preserving spread of G.

If F is a mean-preserving spread of G, then some of the weight associated with the modal value of p is redistributed to the tails, and the probability of obtaining extreme values of p is higher with the cumulative F. This distribution is then more risky than G.

8.1 The choice of technology with opaque management

When the principal and the agent observe the current state ex-post, along with the outcome, a "better" technology is always preferred. This results from the fact that the best states associated with the higher probability of obtaining the value R are more likely to occur, increasing the incentive efficiency. Let $V_{NI}^{F}(B)$ be the expected profit of the principal with a cumulative function F for the p.

Proposition 8 *If G FOSD F then $V_{NI}^{G}(B) \geq V_{NI}^{F}(B)$ for all B.*

If the principal can choose between a safe technology and a risky technology, she will always select the risky one.

Proposition 9 *If F is a mean-preserving spread of G then $V_{NI}^{F}(B) \geq V_{NI}^{G}(B)$, for all B.*

This results from the fact that the value of ex-post information increases with the dispersion of the distribution. The incentive efficiency gains on the expected implementation costs increase with the risk faced by the principal and the agent.[10]

[10] Juan-José Ganuza and José S. Penalva, "Signal Orderings Based on Dispersion and the Supply of Private Information in Auctions", *Econometrica*, 2010.

8.2 The choice of technology with transparent management

When the principal and the agent know the current states before contracting, then the principal wants the state in which she can implement effort to occurs. That is to say, she wants $(1 - F(\hat{p}(B)))$ to be as high as possible. Since $(1 - G(p)) > (1 - F(p))$ when G *FOSD* F the principal always prefers the "best" technology.

Proposition 10 *If G FOSD F then $V_I^G(B) \geq V_I^F(B)$ for all B.*

The attitude of the principal towards risk with ex-ante information is not unequivocal. Two effects with opposite directions intervene: the probability of being in an implementable state (1) and the associated implementation costs (2).

$$V_I^G(B) - V_I^F(B) = \underbrace{(\Delta_{\hat{p}} R - \frac{\hat{p}c}{\Delta_{\hat{p}}})(F(\hat{p}(B)) - G(\hat{p}(B)))}_{(1)} - \underbrace{\int_{\hat{p}(B)}^{\overline{p}} (R + \frac{p_0 c}{\Delta_p^2})(G(p) - F(p))\,\mathrm{d}p}_{(2)}$$

For low values of B, the principal will prefer the risky distribution which increases the probability of encountering a state in which effort is implementable. When the budget rises, the principal favors a safer environment in which the worst states are less likely to happen.

Proposition 11 *If F is a Mean Preserving Spread of G, and the densities only cross twice, there exists a value of B noted B^{MPS} such that*

$$\begin{cases} V_I^G(B) < V_I^F(B) & \text{if} \quad B < B^{MPS} \\ V_I^G(B) = V_I^F(B) & \text{if} \quad B = B^{MPS} \\ V_I^G(B) > V_I^F(B) & \text{if} \quad B > B^{MPS} \end{cases}$$

9 Conclusive remarks

In a principal agent model with moral hazard, limited liability, and a continuum of states, I introduce a budget constraint on the principal's side. This simple restriction leads to a significant modification of the canonical results. In this constrained world, the management of public information is the key adjustment variable by which the principal can make best use of her limited incentive power. Choosing between opaque and transparent management, the principal faces a trade-off between incentive efficiency and incentive relevance; where the latter is more likely to be optimal when the task is least valuable and the budget at its lowest. The optimal information structure is derived under full commitment and requires that the agent simply be told what action to undertake. The optimal information structure combines the incentive optimality of opaqueness and the ability to not engage the principal's limited budget in unworthy tasks. This is done through a threshold-based instrument: the agent is told to exert effort only if the task is productive enough, but not the exact productivity. I hence show that introducing a budget constraint on the principal's side mitigates a broad insight obtained from the recent literature, i.e. that opaqueness is optimal for incentive provision and that one should proceed cautiously on this issue, especially in the public sector.

This work opens several paths for future research, especially regarding the issue of information leaks. The results presented here are conditional on the assumption that the principal has full control over information. Obviously, this assumption does not hold if the agent can access information, and the principal may want to engage in costly actions to prevent this access and benefit from the incentive efficiency associated with opacity. Furthermore, this work provides a convenient theoretical framework to study incentives in circumstances where performance does not immediately translate into monetary benefits, a defining characteristic of the public sector.

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A Appendix

A.1 Proof of the limit

Let F and G be two cumulative functions of p. Without information $p^*(B)$ is the solution of $\displaystyle\int_{p^*(B)}^{\overline{p}} B\Delta_p f(p)\,\mathrm{d}p = c$ and $q^*(B)$ is the solution of $\displaystyle\int_{q^*(B)}^{\overline{p}} B\Delta_p g(p)\,\mathrm{d}p = c$.

Note that

$$\begin{aligned} E_F(C) = & \int_{p^*(B)}^{\overline{p}} pBf(p)\,\mathrm{d}p \\ E_G(C) = & \int_{q^*(B)}^{\overline{p}} pBg(p)\,\mathrm{d}p \end{aligned}$$

It is possible to rewrite these expected cost using the incentive constraints above.

$$\begin{aligned} E_F(C) = & \tfrac{c}{B} + \textstyle\int_{p^*(B)}^{\overline{p}} p_0 f(p)\,\mathrm{d}p = \tfrac{c}{B} + p_0(1 - F(p^*(B))) \\ E_G(C) = & \tfrac{c}{B} + \textstyle\int_{q^*(B)}^{\overline{p}} p_0 g(p)\,\mathrm{d}p = \tfrac{c}{B} + p_0(1 - G(q^*(B))) \end{aligned}$$

The following step limits can be computed

$$\begin{aligned} \lim_{B\to+\infty} p^*(B) = \overline{p} \quad ; \quad \lim_{p\to\overline{p}} F(p) = 1 \\ \lim_{B\to+\infty} q^*(B) = \overline{p} \quad ; \quad \lim_{p\to\overline{p}} G(p) = 1 \end{aligned}$$

We thus find that

$$\lim_{B\to+\infty} E_F(C) - E_G(C) = \lim_{B\to+\infty} p_0(G(q^*(B)) - F(p^*(B))) = 0$$

And F and G are equivalent when B tends towards infinity. In particular if F is uniform

$$\lim_{B\to+\infty} E_F(C) = \frac{\overline{p}c}{\Delta_{\overline{p}}}$$

A.2 Proof of proposition 5

We need to prove that it is optimal for the principal to use only one signal inducing shirking (1) and only one inducing the effort (2).

(1) Due to limited liability, it is always optimal for the principal to set a null bonus when she does not implement the effort. Therefore, the contract faced by the agent is identical for all the σ inducing shirking and the principal can use only one signal without loss of generality.

(2) Consider two signals, $\sigma = i$ and $\sigma = j$, where $i \neq j$, inducing the effort. Let $b(p)$ be the optimal bonus promised by the principal. Note that this bonus is a function of p, since the principal can condition her payment to the current state publicly known ex-post, and that this bonus as the same structure as the one described in Section III. The incentive compatibility constraint when signal i occurs is:

$$\int_{\underline{p}}^{\overline{p}} \Delta_p b(p) \,\mathrm{d}F_i(p) - c \geq 0 \tag{IC_i}$$

The incentive compatibility constraint when signal j occurs is:

$$\int_{\underline{p}}^{\overline{p}} \Delta_p b(p) \,\mathrm{d}F_j(p) - c \geq 0 \tag{IC_j}$$

Consider now the signal $\sigma = k$ merging the two signal above. Note that

$$F_k(p) = \frac{\lambda_i F_i(p) + \lambda_j F_j(p)}{\lambda_i + \lambda_j}$$

And,

$$\begin{aligned} \int_{\underline{p}}^{\overline{p}} \Delta_p b(p) \,\mathrm{d}F_k(p) - c = & \frac{\lambda_i}{\lambda_i + \lambda_j} \overbrace{\Big[\int_{\underline{p}}^{\overline{p}} \Delta_p b(p) \,\mathrm{d}F_i(p) - c \Big]}^{IC_i} \\ & + \frac{\lambda_j}{\lambda_i + \lambda_j} \underbrace{\Big[\int_{\underline{p}}^{\overline{p}} \Delta_p b(p) \,\mathrm{d}F_i(p) - c \Big]}_{IC_j} \geq 0 \end{aligned} \tag{IC_k}$$

Hence, if both (IC_i) and (IC_j) are satisfied, the incentive compatibility constraint with the signal k is satisfied as well and the principal is better off with one signal. This can be replicated with any number of signals inducing the effort. It is therefore optimal to use only one signal inducing the effort.

A.3 Proof of Lemma 1

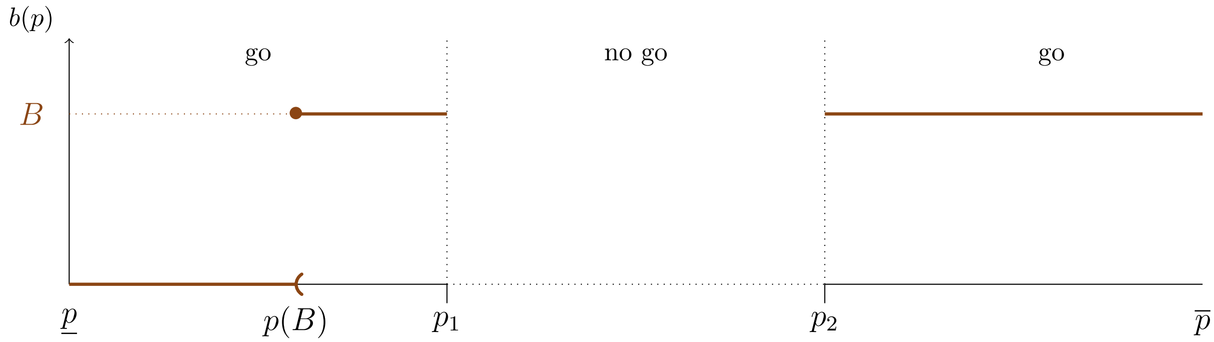

Figure 11: When S_{go} and $S_{no\,go}$ overlap

Consider an information structure where $S_{go} = S_i \cup S_j$, $[\underline{p}, \overline{p}] = S_{go} \cup S_{no\,go}$ and the borel sets S_i, S_j and $S_{no\,go}$ are such that:

$$S_i = \{p \in [\underline{p}, \overline{p}],\ \underline{p} \leq p \leq p_1\}$$
$$S_j = \{p \in [\underline{p}, \overline{p}],\ p_2 \leq p \leq \overline{p}\}$$
$$S_{no\,go} = \{p \in [\underline{p}, \overline{p}],\ p_1 < p < p_2\}$$

Where $\underline{p} < p_1 < p_2 < \overline{p}$. In this information structure, for all p_L, p_M and p_H belonging respectively to S_1, $S_{no\,go}$ and S_2, $p_L < p_M < p_H$. Hence, S_{go} and $S_{no\,go}$ overlap. This information structure is displayed in Figure 9. Let $b(p)$ be the bonus paid by the principal in state p. In this structure, the agent exerts the effort if he receives the signal $\sigma = go$ and the corresponding incentive constraint is:

$$\int_{\underline{p}}^{\overline{p}} \Delta_p b(p)\ \mathrm{d}F_{go}(p) \geq c$$

It is optimal for the principal to use ex-post information to design her incentive scheme and the optimal bonus is the one of the opaque management strategy. Due to limited liability, the principal promises a strictly positive bonus only when a success is observed in the most informative states. The principal promises a budget B when a success occurs in an interval. Let $p(B)$ be the lower bound of this interval. I assume that $p(B) \in [\underline{p}, p_1]$. The implications of this assumption are discussed below. Using bayes' rule, the incentive constraint can be written as:

$$\frac{\lambda_i}{\lambda_{go}} \int_{p(B)}^{p_1} \Delta_p B \, \mathrm{d}F_i(p) + \frac{\lambda_j}{\lambda_{go}} \int_{p_2}^{\overline{p}} \Delta_p B \, \mathrm{d}F_j(p) \geq c$$

It is possible to split the set S_i in two sub-sets labelled $S_{i,1}$ and $S_{i,2}$. In the first one the agent receives a bonus null in case of success while he receives B in $S_{i,2}$. Using the bayes's rule once more the incentive constraint can be finally rewritten as:

$$\frac{\lambda_{i,2}}{\lambda_{go}} \, E[\Delta_p \, B \mid p(B) \geq p \geq p_1] + \frac{\lambda_j}{\lambda_{go}} \, E[\Delta_p \, B \mid p_2 \geq p \geq \overline{p}] \geq c \qquad (IC^b)$$

I let the principal rearrange this information structure by shifting some weight from S_i to $S_{no\,go}$ under the constraint that λ_{go} is not affected by this operation. This restriction eases the construction of the main argument but is not necessary as it is discussed below. The principal can face two cases. She can either shift weights associated with a null bonus (1) or with a bonus B in case of success (2).

(1) Consider the first scenario and let the principal shift some weight from the worst states of S_i i.e. the worst states of $S_{i,1}$, to the worst states of $S_{no\,go}$. Let label this weight $\lambda_S^{(1)}$. Note that $\lambda_S^{(1)}$ is the probability that the current state belongs to the shifted interval. This operation modifies sets S_i and $S_{no\,go}$. Let label $S_i^{(1)}$ and $S_{no\,go}^{(1)}$ the corresponding modified sets where $\underline{p} < \underline{p}^{(1)} < p_1 < p_1^{(1)} < p_2 < \overline{p}$ are:

$$S_i^{(1)} = [\underline{p}^{(1)}, p_1^{(1)}]$$
$$S_{no\,go}^{(1)} = [\underline{p}, \underline{p}^{(1)}[\bigcup]p_1^{(1)}, p_2]$$

λ_{go} is not affected by this operation if and only if:

$$F(\underline{p}^{(1)}) = F(p_1^{(1)}) - F(p_1) \equiv \lambda_S^{(1)}$$

Let $b^{(1)}(p)$ be the bonus after the shift. Hence, the incentive constraint after the shift is:

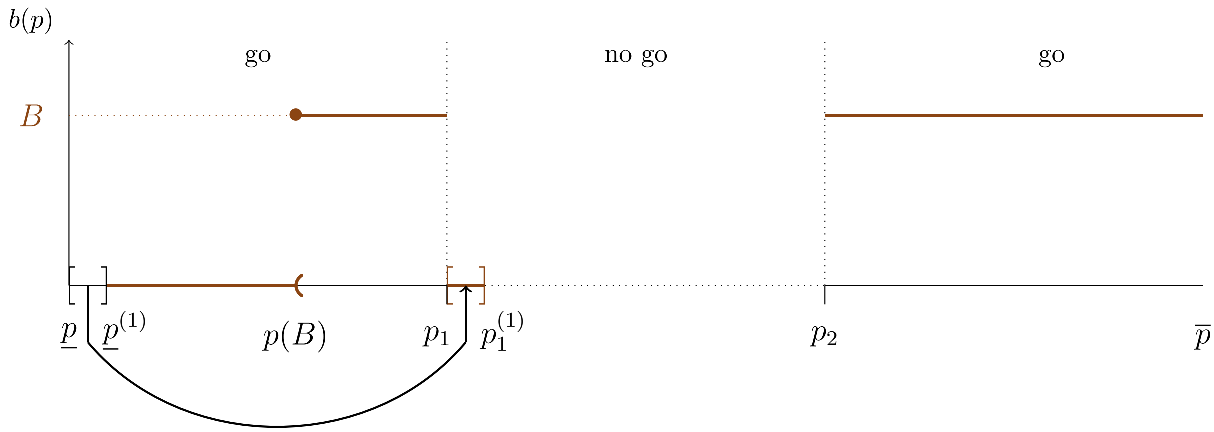

Figure 12: Weight shifting in Case (1)

$$\frac{\lambda_{i,2}^{(1)}}{\lambda_{go}} E\left[\Delta_p\, b^{(1)}(p) \mid \underline{p}^{(1)} \leq p \leq p_1^{(1)}\right] + \frac{\lambda_j}{\lambda_{go}} E\left[\Delta_p\, b^{(1)}(p) \mid p_2 \leq p \leq \overline{p}\right] \geq c \qquad (IC^{(1)})$$

A possible, but not optimal, candidate for $b^{(1)}(p)$ is:

$$\begin{cases} b^{(1)}(p) = B & \text{if} \quad p \in S_{i,2} \cup S_j \\ b^{(1)}(p) = 0 & \text{otherwise} \end{cases}$$

This bonus is clearly not optimal since the principal does not place the incentive weight on the most informative states. Note that the constraint upon λ_{go} is detrimental to the shifted problem since the principal cannot use the incentive efficiency of more informative states to include more states in $S_{go}^{(1)}$. With this candidate for $b^{(1)}(p)$, $IC^{(1)}$ is identical to IC^b and the effect of the shift can be studied through the expected gains only. Let V^b and $V^{(1)}$ be the expected profits for the principal respectively before and after the shift and study the comparison:

$$V^{(1)} - V(b) = R\lambda_S^{(1)}\left[E[\Delta_p \mid p_1 \leq p \leq p_1^{(1)}] - E[\Delta_p \mid \underline{p} \leq p \leq \underline{p}^{(1)}]\right]$$

This comparison is strictly positive and the principal is strictly better off after the shift even if the selected incentive scheme is not optimal. Let me come back here on the assumption about the position of $p(B)$. If $p(B) \in S_j$ the argument expose of case (1) applies to the totality of the interval S_i and the principal is always better off by shifting some weight from S_i to $S_{no\,go}$.

(2) Consider now the case where the principal shifts some weight labelled $\lambda_S^{(2)}$ from the worst states associated with the bonus B of set S_i i.e. the worst states of set $S_{i,2}$, to the worst states of set $S_{no\,go}$. This operation modifies sets S_i and $S_{no\,go}$. Let label $S_i^{(2)}$ and $S_{no\,go}^{(2)}$ the corresponding modified sets where $\underline{p} < p(B) < p^{(2)}(B) < p_1 < p_1^{(2)} < p_2 < \overline{p}$ are such that:

$$S_i^{(2)} = [\underline{p}, p(B)] \cup [p^{(2)}(B), p_1^{(2)}]$$
$$S_{no\,go}^{(2)} =]p(B), p^{(2)}(B)[\cup]p_1^{(2)}, p_2[$$

The constraint on λ_{go} commands that:

$$F(p_1) - F(p(B)) = F(p_1^{(2)}) - F(p_2(B))$$

Let $b^{(2)}(p)$ be the bonus after this shift. The incentive constraint after this shift becomes:

$$\frac{\lambda_{i,2}^{(2)}}{\lambda_{go}} E[\Delta_p b^{(2)}(p) | p^{(2)}(B) \leq p \leq p_1^{(2)}] + \frac{\lambda_j}{\lambda_{go}} E[\Delta_p b^{(2)}(p) | p_2 \leq p \leq \overline{p}] \geq c \qquad (IC^{(2)})$$

Due to the constraint on λ_{go}, the principal can not use the totality of the available budget and reduce the interval of remuneration. Note that, in a similar way to case (1), the constraint on λ_{go} is detrimental to the shifted problem. The amount of budget used by the principal in the incentive scheme is labelled $B^{(2)}$ and the best candidate for the $b^{(2)}(p)$ is:

$$\begin{cases} b^{(2)}(p) = B^{(2)} & \text{if} \quad p \in S_{i,2}^{(2)} \cup S_j \\ \\ b^{(2)}(p) = 0 & \text{otherwise} \end{cases}$$

The amount of budget used in this incentive scheme is strictly lower than the one used in the non-shifted problem, $B^{(2)} < B$, since:

$$B^{(2)} = \frac{c}{\frac{\lambda_{i,2}^{(2)}}{\lambda_{go}} E[\Delta_p | p^{(2)}(B) \leq p \leq p_1^{(2)}] + \frac{\lambda_j}{\lambda_{go}} E[\Delta_p | p_2 \leq p \leq \overline{p}]}$$

And,

$$B = \frac{c}{\frac{\lambda_{i,2}}{\lambda_{go}} E[\Delta_p | p(B) \leq p \leq p_1] + \frac{\lambda_j}{\lambda_{go}} E[\Delta_p | p_2 \leq p \leq \overline{p}]}$$

Let $V^{(2)}$ be the expected profits for the principal after the shift and study the comparison between V^b and $V^{(2)}$. In this comparison, both expected gains and implementation cost have to be considered. Regarding the expected gains, the argument exposed in case (1) holds in case (2) since the probability of success increases in expectation. Regarding the implementation cost it is easy to see that:

$$\begin{gathered} \left[\lambda_{i,2}^{(2)} E[\Delta_p \mid p^{(2)}(B) \leq p \leq p_1^{(2)}] + \lambda_j E[\Delta_p \mid p_2 \leq p \leq \overline{p}]\right] B^{(2)} \\ < \\ \left[\lambda_{i,2} E[\Delta_p \mid p(B) \leq p \leq p_1] + \lambda_j E[\Delta_p \mid p_2 \leq p \leq \overline{p}]\right] B \end{gathered}$$

Therefore, the principal is strictly better off after the shift even if the selected incentive scheme is not optimal. The arguments exposed in case (1) and (2) can be replicated in all information structures where S_{go} and $S_{no\,go}$ overlap and the principal can always improve the expected profit by shifting some weight from the worst states of S_{go} to the worst states of $S_{no\,go}$. As a consequence, S_{go} and $S_{no\,go}$ cannot overlap in the optimal information structure.

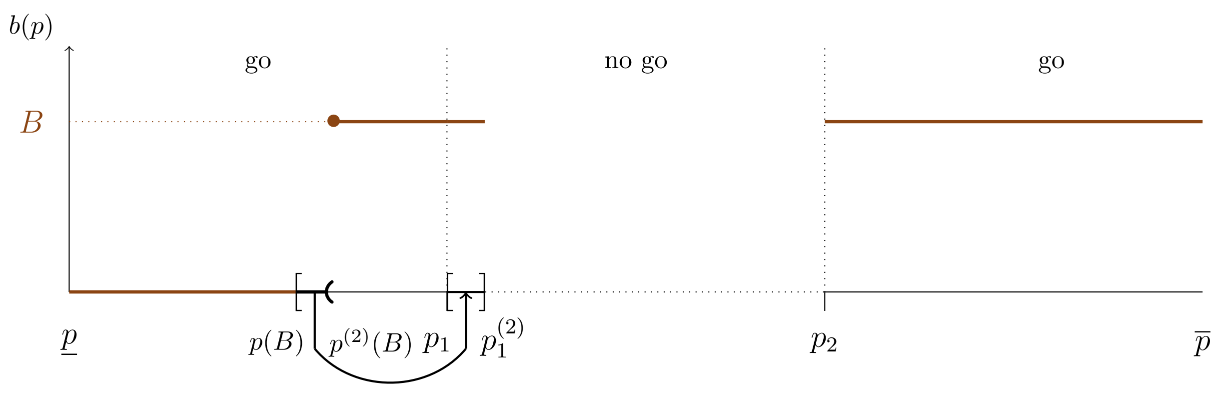

Figure 13: Weight shifting in Case (2)

A.4 Proof of proposition 6

Following **Proposition 5** and **Lemma 1**, in the optimal information structure, all the states where effort is exerted by the agent have to be consecutive and the principal use only one signal inducing this action. Therefore, if the current state is above a certain threshold noted x, the principal optimally sends the signal “go”.

A.5 Proof of proposition 7

From the optimality condition on x^*, simple derivation allows to conclude that x^* is decreasing in R. Partial derivation of the expression of x^* with respect to B leads to

$$\frac{\partial x^*}{\partial B} = -\frac{c}{R}\frac{p_0}{\Delta^2_{p^*(x^*,B)}}\frac{\partial p^*(x^*,B)}{\partial B}$$

Where

$$\frac{\partial p^*(x^*,B)}{\partial B} = \frac{\int_{p^*(x^*,B)}^{\overline{p}} \Delta_p \,\mathrm{d}F_{go}(p)}{B\Delta_{p^*(x^*,B)} f_{go}[p^*(x^*,B)]} > 0$$

Note that this partial derivative is positive which makes it possible to conclude that x^* is decreasing in B.

A.6 Proof of proposition 8

We first want to know the evolution of $p^*(B)$ when the environment gets better, when the weights associated with the good states increase. We note $q^*(B)$ the corresponding threshold of $p^*(B)$ with the cumulative G. To study the distance between $p^*(B)$ and $q^*(B)$ we focus on the incentive compatibility constraints. This comparison for any x taken from the support of p can be written as follow :

$$\int_{x}^{\overline{p}} \Delta_p g(p)\,\mathrm{d}p - \int_{x}^{\overline{p}} \Delta_p f(p)\,\mathrm{d}p = (x - p_0)(F(x) - G(x)) + \int_{x}^{\overline{p}} (F(p) - G(p))\,\mathrm{d}p$$

As long as $G\ FOSD\ F$ this difference is always positive and $q^*(B) \geq p^*(B)$.

Then, we have to compare the expected profits to determine the principal's choice. We know that $q^*(B) \geq p^*(B)$ and $\int_{q^*(B)}^{\overline{p}} \Delta_p g(p)\,\mathrm{d}p = \int_{p^*(B)}^{\overline{p}} \Delta_p f(p)\,\mathrm{d}p$. We show that these facts are not compatible with $F(p^*(B)) \geq G(q^*(B))$.

$$\int_{q^*(B)}^{\overline{p}} \Delta_p g(p)\,\mathrm{d}p = \int_{p^*(B)}^{\overline{p}} \Delta_p f(p)\,\mathrm{d}p$$
$$\Leftrightarrow (q^*(B) - p_0)G(q^*(B)) - (p^*(B) - p_0)F(p^*(B)) = \int_{p^*(B)}^{\overline{p}} F(p)\,\mathrm{d}p - \int_{q^*(B)}^{\overline{p}} G(p)\,\mathrm{d}p$$

Suppose that $G(q^*(B)) = F(p^*(B))$ we can rewrite this as

$$\int_{p^*(B)}^{\overline{p}} F(p)\,\mathrm{d}p - \int_{q^*(B)}^{\overline{p}} G(p)\,\mathrm{d}p = F(p^*(B))(q^*(B) - p^*(B)) + \underbrace{\int_{p^*(B)}^{q^*(B)} (F(p) - F(p^*(B)))\,\mathrm{d}p + \int_{q^*(B)}^{\overline{p}} (F(p) - G(p))\,\mathrm{d}p}_{+}$$

Note that

$$\int_{p^*(B)}^{\overline{p}} F(p)\,\mathrm{d}p - \int_{q^*(B)}^{\overline{p}} G(p)\,\mathrm{d}p = F(p^*(B))(q^*(B) - p^*(B))$$

Therefore $G(q^*(B))$ can not be equal to $F(p^*(B))$ and, a fortiori, $G(q^*(B))$ can not be smaller than $F(p^*(B))$. This means that the probability of being in the interval of remuneration is smaller with distribution G and so are the implementation costs. Since $\int_{\underline{p}}^{\overline{p}} pRg(p)\,\mathrm{d}p \geq \int_{\underline{p}}^{\overline{p}} pRf(p)\,\mathrm{d}p$ the better technology is always preferred by the Principal.

A.7 Proof of proposition 9

We want here to compare expected profits associated to a more risky distribution with ex-post information. We assume that G $SOSD$ F. Therefore, $S_G(p) < S_F(p)$ $\forall p \in [\underline{p}, \overline{p}]$. Since we consider mean preserving spreads the only argument of the principal's choice is the expected implementation costs. In fact, $V_{NI}^F \geq V_{NI}^G \Leftrightarrow F(p^*(B)) \geq G(q^*(B))$.
The first thing to do is to study the impact of the new distribution on the optimal threshold $p^*(B)$. To do that we focus on the difference of the incentive compatibility constraints. We set:

$$\begin{aligned}\gamma(x) &= \int_x^{\overline{p}} \Delta_p f(p)\,\mathrm{d}p - \int_x^{\overline{p}} \Delta_p g(p)\,\mathrm{d}p \\ &= (x - p_0)(G(x) - F(x)) + \int_x^{\overline{p}} (G(p) - F(p))\,\mathrm{d}p\end{aligned}$$

Using the fact that F is a Mean Preserving Spread of G, the function $\gamma(x)$ can be rewritten as follow:

$$\gamma(x) = \int_{\underline{p}}^{x} (G(p) - F(p))\,\mathrm{d}p - \Delta_x(F(x) - G(x))$$

Deriving this with respect to x, we get:

$$\gamma'(x) = \Delta_x[g(x) - f(x)]$$

The variations of γ solely depend on the difference of the density. In order to simplify the proof, we assume that the densities only cross twice: for $x = \alpha$ and in $x = \beta$ with $\alpha < \beta$. Since the cumulative functions cross once, the first crossing of the densities happens before this point and the second after. Consider the expression of $\gamma(\alpha)$:

$$\gamma(\alpha) = -\Delta_\alpha[F(\alpha) - G(\alpha)] + S_F(\alpha) - S_G(\alpha)$$

This can be seen as a sum, minus the last term weighted by a difference of probabilities, and has therefore a positive value since $S_F(x) - S_G(x) > 0$, $\forall x \in [\underline{p}, \overline{p}]$. Regarding, $\gamma(\beta)$, since G $SOSD$ F and $F(\beta) < G(\beta)$, is easy to see that:

$$\gamma(\beta) = -\Delta_\beta[F(\beta) - G(\beta)] + S_F(\beta) - S_G(\beta) > 0$$

Therefore, $\gamma(x)$ is positive for all $x \in [\underline{p}, \overline{p}]$ and $p^*(B)$ has to be higher than $q^*(B)$ to insure that:

$$\int_{p^*(B)}^{\overline{p}} \Delta_p f(p)\,\mathrm{d}p = \int_{q^*(B)}^{\overline{p}} \Delta_p g(p)\,\mathrm{d}p$$

This equality is equivalent to:

$$\frac{\overbrace{\int_{\underline{p}}^{q^*(B)} F(p)\,\mathrm{d}p - \int_{\underline{p}}^{q^*(B)} G(p)\,\mathrm{d}p}^{+} + \overbrace{\int_{q^*(B)}^{p^*(B)} F(p)\,\mathrm{d}p - [p^*(B) - q^*(B)]F(p^*(B))}^{+}}{\Delta_{q^*(B)}} = F(p^*(B)) - G(q^*(B))$$

We can conclude that G $SOSD$ F implies $F(p^*(B)) > G(q^*(B))$ and $V_{NI}^F > V_{NI}^G$. This proof holds if densities cross more than twice.

A.8 Proof of proposition 10

With ex-ante information, a change of the distribution does not imply a modification of the interval of remuneration. Therefore if G $FOSD$ F the probability of being in better states is higher with G. The probability of getting at least X is higher with G. In particular, the Principal has a higher probability of getting at least $\hat{p}(B)$, she will then choose the best technology.

$$V_I^G - V_I^F = \underbrace{(\Delta_{\hat{p}} R - \frac{\hat{p}c}{\Delta_{\hat{p}}})(F(\hat{p}) - G(\hat{p})) - \int_{\hat{p}}^{\overline{p}} (p(R - \frac{c}{p - p_0}))'(G(p) - F(p))\,\mathrm{d}p}_{+}$$

A.9 Proof of proposition 11

We want here to compare expected profits associated to a more risky distribution and ex-ante information. We assume that G $SOSD$ F, these cumulative functions cross only once and the associated densities cross twice. To study the distance with the corresponding expected profit $V_I^G(B)$ and $V_I^F(B)$, we set the function $\Gamma(\hat{p}(B))$ such that:

$$\Gamma(\hat{p}(B)) = \int_{\hat{p}(B)}^{\overline{p}} [\Delta_p R - pb(p)][f(p) - g(p)]\,\mathrm{d}p$$

We study the variation of this function with respect to $\hat{p}(B)$, the relation $\hat{p}(B) = c/B + p_0$ allows us to derive the variations with respect to B. The derivative of Γ with respect to $\hat{p}(B)$ is:

$$\Gamma'(\hat{p}(B)) = -[\Delta_{\hat{p}(B)} - \hat{p}(B)b(\hat{p}(B))][f(\hat{p}(B)) - g(\hat{p}(B))]$$

Hence, the variations of Γ solely depend on the difference of the densities. Since F is a Mean Preserving Spread of G, $\Gamma(\underline{p}) = \Gamma(\overline{p}) = 0$, the function reaches its lowest value when the densities cross the first time ($\hat{p}(B) = \alpha$) and its highest value when they cross the second time($\hat{p}(B) = \beta$). Since, $f(p) > g(p)$ for all $p \in [B, \overline{p}]$, it easy to see that:

$$\Gamma(\beta) = \int_{\beta}^{\overline{p}} [\Delta_p R - pb(p)][f(p) - g(p)]\,\mathrm{d}p > 0$$

In the same way, since $f(p) > g(p)$ for all $p \in [\underline{p}, \alpha]$ it is easy to see that:

$$\begin{aligned}\Gamma(\alpha) &= \int_{\alpha}^{\overline{p}} [\Delta_p R - pb(p)][f(p) - g(p)]\,\mathrm{d}p \\ &= -\int_{\underline{p}}^{\alpha} [\Delta_p R - pb(p)][f(p) - g(p)]\,\mathrm{d}p < 0\end{aligned}$$

Therefore, $\Gamma(\hat{p}(B)) = 0$ has a unique solution, noted B^{MPS}. Hence, for small values of the budget, $B < B^{MPS}$, the principal prefers the more risky distribution and, when $B > B^{MPS}$, the principal prefers the less risky distribution.