The optimal inflation rate with discount factor heterogeneity

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Abstract

This paper considers a framework in which the relevant frictions faced by the monetary authority are price stickiness and monopolistic competition, and shows that deviations from long-run price stability are optimal in the presence of discount factor heterogeneity. I derive analytical solutions for the optimal inflation rate in two different cases. First, in a standard New Keynesian model in which the heterogeneity in discount factors is imposed exogenously. Second, in a model with a perpetual youth structure in which it arises endogenously. I find that the optimal inflation rate is positive when the social discount factor is greater than the discount factor used by firms when evaluating profit flows, zero when the two are equal, and negative when the planner is more impatient than firms. A baseline calibration of the perpetual youth model suggests values of the optimal inflation rate comprised between 0.2% and 1%.

Keywords: optimal inflation rate, sticky prices, discount factor heterogeneity

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1 Introduction

In standard New Keynesian models, the monetary authority does not face a short-run trade-off between stabilizing inflation and real activity. Stabilizing inflation also implies stabilizing the welfare-relevant output gap, a result referred to as the "divine coincidence" (Blanchard and Galí 2007) in the literature. However these models do feature a long-run relationship between the levels of inflation and output (see for example Ascari and Rossi 2012). This suggests that the central bank should take advantage of this relationship and adopt a non-zero inflation target when output is inefficiently low. Yet, in most studies, the optimal long-run inflation rate is found to be exactly zero.

This paper shows that this result rests on the equality of social and private (here firms's) discount factors. When heterogeneity in discount factors is introduced in an environment in which the relevant frictions faced by the monetary authority are price stickiness and the market power of firms, the optimal inflation rate becomes positive or negative depending on the relative degree of impatience of firms and the planner. I show this result analytically in two different frameworks. In order to build intuition, I first use a standard New Keynesian model in which the heterogeneity in discount factors is imposed exogenously. In a second step, I modify this model by incorporating an overlapping generations structure, as in Blanchard's (1985) and Yaari's (1965) perpetual youth model. In this framework, the wealth difference between young and old households drives a wedge between the steady-state value of the stochastic discount factor, which is used by firms when evaluating profit flows, and the household subjective discount factor, which is used by the planner when evaluating utility flows.

In both models, inflation has an effect on economic activity both because it influences firms' markups (or inversely their marginal costs) and because it creates costs that drive a wedge in the aggregate resource constraint. Depending on the pricing model under consideration, these costs may arise through price dispersion or because resources are expended adjusting prices. In what follows, I refer to them as the "resource costs of inflation". Given general assumptions about price setting, the Ramsey problem of the monetary authority can be shown to reduce to a simple formula. The optimal inflation rate is chosen so that a weighted average of the discounted effects of inflation on marginal costs and on resource costs in all periods is equalized to zero, where the weights depend on the level of steady-state distortions. The paper then considers two different price-setting models widely used in the literature, one in which firms face price adjustments costs à la Rotemberg (1982), and another in which prices are set on the basis of staggered Calvo (1983) contracts. Analytical expressions for the objects of interest, the discounted effects of inflation on marginal costs and on resource costs evaluated in steady state, can be obtained. Their behavior can be analyzed to understand the determinants of the optimal long-run inflation rate.

The following results can be drawn from the evaluation of the simple New Keynesian model. First, the resource costs of inflation are minimized for a zero inflation rate, irrespective of the values of the private and social discount rates. Second, higher inflation in a given period has two opposite effects on marginal costs. Because some firms cannot reset their prices, or because they have to pay price adjustment costs, it erodes current markups. Moreover, firms choosing their optimal price in previous periods anticipate this higher inflation and set a higher markup. Since these effects do not even out, inflation has a non-zero impact on average marginal cost. The problem faced by the monetary authority is thus intrinsically dynamic. Under commitment, the central bank internalizes the effects of its choices on the behavior of the private sector. When fixing the time t inflation rate, it takes into account that this choice will have consequences in other periods and weights these consequences according to their distance in the future. This discounting effect perfectly offsets the effect of inflation on average marginal cost when the private discount rate — which partly determines the slope of the long-run Phillips curve — and the social discount rate — which determines the strength of the weighting effect — are equal. In that case, the discounted effect of inflation on marginal costs is equal to zero for an inflation rate of zero; long-run price stability is optimal. This result is no longer valid when private and social discount rates differ. When the private discount rate is higher, the monetary authority has more leverage over markups and the optimal inflation rate is positive. When the social discount rate is higher, the central bank puts a higher relative weight on earlier periods when anticipated inflation leads to higher markups and lower output, and the optimal inflation rate is negative.

These results are confirmed in the model incorporating a perpetual youth structure. In this environment, although a simple expression for the discounted effects of inflation on marginal costs cannot be derived, it is still possible to solve analytically for the Ramsey steady state to which the monetary authority would like to converge. I find that long-run price stability is not optimal unless the steady-state stochastic discount factor of households is equal to the discount factor used by the planner. If that discount factor is the household subjective discount factor, as I assume, non-zero inflation rates are warranted. The wedge between the stochastic discount factor and the subjective discount factor depends on a key parameter, the death probability, for which previous studies have used values comprised between 0.03 and 0.13 at a quarterly frequency. Using this range, I find that the optimal inflation rate is comprised between 0.06% and 0.3% in a baseline calibration. When steady-state distortions are larger, these values increase to 0.2% and 1%.

Previous papers have considered New Keynesian models incorporating a perpetual youth structure to study how monetary policy should response to stock market conditions (Castelnuovo and Nisticò 2010, Nisticò 2012), or asset price bubbles (Galí 2017)[1] . Del Negro et al. (2015) show that such a framework can provide a resolution to the excess sensitivity of macroeconomic variables to announcements about the future path of the policy rate in standard New Keynesian models, the so-called "forward guidance puzzle". This paper builds on these contributions and shows that the heterogeneity in discount factors that arises endogenously in such a model provides a rationale for non-zero inflation targets. The paper is also related to a wide literature expertly surveyed by Schmitt-Grohé and Uribe (2010) that has sought to understand the determinants of the optimal rate of inflation. Other more recent contributions by Bilbiie et al. (2014), Adam and Weber (2017), Coibion et al. (2012), and Carlsson and Westermark (2016) show respectively that endogenous producer entry and exit, firm-level heterogeneity in productivity, the zero lower bound on nominal interest rates, and the combination of labor market frictions and nominal wage staggering may provide justifications for moderately positive inflation targets. Numerous papers, among which Goodfriend and King (1997) and King and Wolman (1999), have looked specifically at the consequences of the presence of sticky prices. Goodfriend and King (1997) showed that the rate of inflation that maximizes the average level of welfare is slightly positive in a model of staggered price adjustment à la Calvo (1983). King and Wolman (1999) make the distinction between such a "golden rule" rate of inflation that maximizes average utility and a "modified golden rule" rate of inflation, equal to zero, that maximizes the discounted sum of flow utilities at different dates. With respect to these studies, the contribution of this paper is to show in a clear analytical way that the optimality of zero inflation rests on the equality of social and private discount factors. In numerous macroeconomic models, this property is not verified and the combination of price stickiness and monopolistic competition may thus provide a rationale for non-zero inflation targets.

The paper is organized as follows. Section 2 considers a standard New Keynesian model and shows analytically that the optimal inflation rate depends on the gap between social and private discount

[1] Other studies focusing on the conduct of monetary policy in perpetual youth models include, among others, Piergallini (2006) and Nisticò (2016).

factors. Section 3 confirms this result in a perpetual youth model in which the heterogeneity in discount factors arises endogenously, and solves for numerical values of the optimal inflation rate in a baseline calibration. Section 4 concludes.

2 Discount factor heterogeneity and optimal long-run inflation in the standard New Keynesian model

2.1 A simple formula for the optimal inflation rate

Consider a standard New Keynesian model without capital, as outlined for example in Galí (2008), which is modified to allow firms, households, and the social planner to have different discount factors. The dynamics of the economy can be described by a Euler equation (1), an equation equating labor supply and labor demand (2), a resource constraint (3), as well as equations describing the price-setting behavior of firms, the behavior of monetary policy, and the dynamics of a technological factor Z_t.

$$\beta_h E_t \frac{1+I_t}{\Pi_{t+1}} \frac{U_c(C_{t+1})}{U_c(C_t)} = 1 \tag{1}$$

$$\frac{v_L(L_t)}{U_c(C_t)} = mc_t F_L(L_t) \tag{2}$$

$$\frac{F(L_t)}{s_t} = C_t \tag{3}$$

where β_h is the discount factor of households, I_t the nominal interest rate set by the central bank, Π_t gross inflation, C_t consumption, L_t the number of hours worked, mc_t the marginal cost of firms, and s_t a wedge arising from the presence of costly price adjustment. Household utility depends positively on consumption and negatively on the number of hours supplied in a separable way $U(C_t, L_t) = U(C_t) - v(L_t)$. Firms produce with labor according to the production technology $F(L_t)$. In what follows, I assume the following functional forms for analytical convenience; $F(L_t) = Z_t L_t$, $U(C_t) = log(C_t)$, and $v(L_t) = \chi \frac{L_t^{1+\varphi}}{1+\varphi}$, where φ is the inverse of the Frisch elasticity of labor supply and χ is a scaling factor. Depending on the model of price-setting behavior under consideration, the resource costs of inflation

s_t and marginal cost mc_t are potentially functions of all past, current, and future inflation rates

$$mc_t = \Omega(\Pi_{t-i}, ..., \Pi_t, ..., E_t\Pi_{t+i}) \tag{4}$$

$$s_t = \Upsilon(\Pi_{t-i}, ..., \Pi_t, ..., E_t\Pi_{t+i}) \tag{5}$$

where equation (4) is the Phillips Curve of the model, Ω and Υ depend on the price-setting model, and $i \in \mathbb{N}^+$.

We are now in a position to derive a simple formula that relates the optimal inflation rate to the discounted effects of inflation on marginal costs and on resource costs. I consider the problem of a monetary authority acting under commitment and look at the steady state to which it would like to converge in the long run. In this scenario, the central bank chooses a path $\{I_t\}_{t=0}^{\infty}$ in order to maximize intertemporal utility $E_t \sum_{i=0}^{\infty} (\beta_s)^i U(C_{t+i}, L_{t+i})$ subject to the constraints of the competitive economy, where β_s is the social discount factor. These constraints are equations (1), (2), (3), (4), and (5). The problem can be simplified in several ways. First, note that it is possible to use equations (2) and (3) to solve for consumption C_t and hours L_t as a function of mc_t and s_t

$$L_t = \left[\frac{mc_t s_t}{\chi}\right]^{\frac{1}{1+\varphi}} \tag{6}$$

$$C_t = Z_t \left[\frac{mc_t}{\chi}\right]^{\frac{1}{1+\varphi}} s_t^{-\frac{\varphi}{1+\varphi}} \tag{7}$$

Second, as emphasized before, mc_t and s_t are potentially functions of all past, present, and future inflation rates. This implies, through equation (1), that choosing a sequence $\{I_t\}_{t=0}^{\infty}$ is equivalent to choosing a sequence for inflation $\{\Pi_t\}_{t=0}^{\infty}$. Thus, the Ramsey problem amounts to choosing $\{\Pi_t\}_{t=0}^{\infty}$ to maximize the following objective function

$$U = E_t \sum_{t=0}^{\infty} \beta_s^t \left\{ log \left[Z_t \left[\frac{mc_t}{\chi}\right]^{\frac{1}{1+\varphi}} s_t^{-\frac{\varphi}{1+\varphi}} \right] - \frac{mc_t s_t}{1+\varphi} \right\} \tag{8}$$

where the link between s_t, mc_t and inflation rates in different periods is given by equations (4) and (5). In steady state, the first-order condition of this problem can be expressed as

$$\left[mc(\Pi)^{-1} - s(\Pi)\right]\left(\sum_{j=-\infty}^{\infty}(\beta_s)^j \frac{\partial mc_{-j}}{\partial \Pi}\right) - \left[\varphi s(\Pi)^{-1} + mc(\Pi)\right]\left(\sum_{j=-\infty}^{\infty}(\beta_s)^j \frac{\partial s_{-j}}{\partial \Pi}\right) = 0 \tag{9}$$

where the absence of time subscript denotes steady-state variables. This equation makes clear that the optimal inflation rate depends on: 1) the discounted effect of current inflation on marginal costs in all periods; 2) the discounted effect of current inflation on resource costs in all periods; 3) weights that are related to the degree of steady-state inefficiency. In particular, note that mc is equal to 1 in an efficient steady state with zero inflation, and lower than 1 when the steady state is inefficient. Furthermore, s is exactly equal to 1 when inflation is zero, and larger than 1 when inflation is different from zero. Thus, the first term in equation (9) is equal to zero in an efficient steady state with zero inflation. Indeed, in such a situation, output is at its optimal level and the monetary authority has no incentives to influence it through marginal costs. In the following section, I leave aside this particular case and consider that the steady state is inefficient.

2.2 A specific case: Rotemberg pricing

This section takes a closer look at the elements in equation (9) when firms face deadweight costs of adjusting prices à la Rotemberg (1982). In appendix 5.1, I also derive analytical formulas for the objects presented in equation (9) when firms have an exogenous probability of being able to reset their price (Calvo 1983).

The deadweight costs of adjusting prices have the same composition as the consumption basket and are proportional to aggregate output $\Psi_t = \frac{\phi^p}{2}(\Pi_t - 1)^2 Y_t$. In such a setting, the optimal pricing condition of firms leads to the following relationship between marginal cost and inflation

$$1 - \theta + \theta mc_t - \phi^p \Pi_t \left(\Pi_t - 1\right) + \beta_f \phi^p E_t \Pi_{t+1}\left(\Pi_{t+1} - 1\right) \frac{1 - \frac{\phi^p}{2}(\Pi_t - 1)^2}{1 - \frac{\phi^p}{2}(\Pi_{t+1} - 1)^2} = 0 \tag{10}$$

where θ is the elasticity of substitution between goods, and β_f is the discount factor used by firms when discounting future profit flows. Moreover s_t is given by

$$s_t = \frac{1}{1 - \frac{\phi^p}{2}(\Pi_t - 1)^2} \tag{11}$$

Thus with Rotemberg pricing, s_t is a function of Π_t and mc_t is a function of Π_t and $E_t\Pi_{t+1}$. (9) simplifies to

$$\Lambda - \Gamma = 0$$

where $\Lambda = \left[mc(\Pi)^{-1} - s(\Pi)\right]\left(\beta_s^{-1}\frac{\partial mc_{-1}}{\partial \Pi} + \frac{\partial mc}{\partial \Pi}\right)$ is the effect of inflation on utility operating through marginal cost and $\Gamma = \left[\varphi s(\Pi)^{-1} + mc(\Pi)\right]\frac{\partial s}{\partial \Pi}$ is the effect of inflation on utility operating through the resource costs of price adjustment. They are equal to

$$\Lambda = \left[mc(\Pi)^{-1} - s(\Pi)\right]\frac{\phi^p}{\theta}\left[\left(\frac{\beta_f}{\beta_s} - 1\right)(1 - 2\Pi) + \left(\beta_f - \frac{\beta_f}{\beta_s}\right)\frac{\phi^p \Pi\left(\Pi - 1\right)^2}{1 - \frac{\phi^p}{2}(\Pi - 1)^2}\right] \tag{12}$$

$$\Gamma = \left[\varphi s(\Pi)^{-1} + mc(\Pi)\right]\frac{\phi^p(\Pi - 1)}{\left[1 - \frac{\phi^p}{2}(\Pi - 1)^2\right]^2} \tag{13}$$

Not surprisingly, since resource costs depend only on current inflation, they are minimized when inflation is equal to zero ($\Pi = 1$). More interestingly, the discounted sum of the effect of current inflation on past and present marginal costs depends on the ratio of the social and private discount factors, β_s and β_f. When the two are equal, $\Lambda = 0$ when $\Pi = 1$. This result can be understood by considering the effect of current inflation on the sum of past and present marginal costs

$$\frac{\partial mc_{-1}}{\partial \Pi} + \frac{\partial mc}{\partial \Pi} = (1 - \beta_f)\frac{\phi^p}{\theta}(2\Pi - 1) \tag{14}$$

Since adjusting prices is costly, firms do not pass on the entirety of movements in marginal costs to prices and current inflation is associated with a reduction in markups. According to the same logic, expected future inflation leads firms to set higher markups in order to minimize future price adjustments costs. However, these effects are asymmetric. Since firms discount the future, higher inflation in t has a larger positive impact on marginal cost at time t than a negative impact at time $t-1$. In other words, the model features a positive long-run relationship between inflation and marginal cost. These outcomes, which happen in successive periods, are weighted differently by the Ramsey planner. Events occuring at time $t-1$, which are closer in the future, receive a higher weight than events occuring at time t. When social and private discount rates are equal, this weighting effect fully offsets the effect of inflation on average markups and the effect of inflation on the *discounted* sum of markups is equal to zero. The inflation rate is then set equal to zero in order to minimize resource costs. However, when social and private discount rates differ, one effect dominates the other and the optimal inflation rate is different from zero. This can be seen in Figure 1, which plots Λ and Γ for different values of the ratio β_f/β_s.

When $\beta_f = \beta_s$, Λ is equal to zero for $\Pi = 1$ (first panel). For an unchanged β_s, a decrease in β_f leads to a strenghtening of the long-run relationship between inflation and marginal cost. The effect of steady-state inflation on the average markup is now larger and the average markup effect dominates the weighting effect; Λ is positive for a large range of values of Π and the optimal inflation rate is positive (panel 2). In the third case, the long-run relationship between inflation and marginal cost is identical as in the first case. However, the monetary authority now puts a larger relative weight on the adverse effects of inflation happening before time t than on its beneficial effects happening at time t; Λ is negative for a large range of values of Π and the optimal inflation rate is negative (third panel).

3 Optimal long-run inflation in a New Keynesian model with a perpetual youth structure

In order to build intuition, the preceding analysis focused on a simple model and imposed exogenously the heterogeneity in discount factors. I now develop a model in which this heterogeneity arises endogenously. I incorporate an overlapping generations structure, as in Blanchard's (1985) and Yaari's (1965) perpetual youth model, in the simple New Keynesian model of section 2.

3.1 Model

3.1.1 Households

In every period j a new cohort is born with mass γ, and each cohort has a constant probability of dying γ which does not depend on j. The budget constraint for members of cohort j is given by

$$C_{j,t} + \int_0^1 Q_t(i) S_{j,t}(i) di + \frac{B_{j,t}}{P_t(1+R_t)} \leq \frac{1}{1-\gamma}\left[\frac{B_{j,t-1}}{P_t} + \int_0^1 \left(Q_t(i) + D_t(i)\right) S_{j,t-1}(i) di\right] + \frac{W_t}{P_t} L_{j,t} - T_t \tag{15}$$

where $S_{j,t}(i)$ is the number of shares issued by firm i and bought by individuals of cohort j. These shares offer (real) dividends $D_t(i)$ and their real price is $Q_t(i)$. $B_{j,t}$ are bonds that promise a unit of currency tomorrow and cost $(1+R_t)^{-1}$ today. As in Blanchard (1985), households enter an annuity contract in which the fraction γ of members dying in each period leaves its wealth to those who remain alive. $C_{j,t}$ and $L_{j,t}$ are consumption and labor supply for cohort j; W_t, T_t and P_t are nominal wages, lump sum taxes, and the price level in period t. Household utility is

$$E_t \sum_{s=0}^{\infty} (\beta(1-\gamma))^s \left[log(C_{j,t+s}) + \chi log(1 - L_{j,t+s})\right] \tag{16}$$

where β is the subjective discount factor of households. The future is discounted at the rate $\beta(1-\gamma)$ to take into account the probability of dying. We obtain two intertemporal conditions for shares and bonds and the household's intratemporal condition for the optimal choice of consumption and hours worked

$$Q_t(i) = \beta E_t \frac{C_{j,t}}{C_{j,t+1}} \left(Q_{t+1}(i) + D_{t+1}(i)\right) \tag{17}$$

$$\beta E_t \frac{C_{j,t}}{C_{j,t+1}} \frac{1+R_t}{\Pi_{t+1}} = 1 \tag{18}$$

$$\chi \frac{C_{j,t}}{1-L_{j,t}} = \frac{W_t}{P_t} = w_t \tag{19}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate between periods $t-1$ and t. The stochastic discount factor between t and $t+1$, conditional on surviving, is

$$\beta_{t,t+1} = \beta E_t \frac{C_{j,t}}{C_{j,t+1}} \tag{20}$$

This discount factor is common to all cohorts as can be seen from equation (18). Following Piergallini (2006), Castelnuovo and Nisticò (2010), and Nisticò (2012), one can derive from these equations a policy function for consumption

$$C_{j,t} = \frac{1-\beta(1-\gamma)}{1+\chi} \left[\Omega_{j,t} + E_t \sum_{s=0}^{\infty} \beta_{t,t+s} (1-\gamma)^s \left(w_{t+s} - T_{t+s}\right)\right] \tag{21}$$

where $\Omega_{j,t} = \frac{1}{1-\gamma}\left(\frac{B_{j,t-1}}{P_t} + \int_0^1 \left(Q_t(i) + D_t(i)\right) S_{j,t-1}(i)di\right)$ is financial wealth held by cohort j at the beginning of period t. Households of each cohort consume out of financial and human wealth with propensity $\frac{1-\beta(1-\gamma)}{1+\chi}$, where human wealth is equal to the discounted sum of expected future labor income adjusted for taxes.

Aggregation

Consider any variable $X_{j,t}$ for a cohort born at time j and define the aggregate variable

$$X_t \equiv \sum_{j=-\infty}^{t} \gamma(1-\gamma)^{t-j} X_{j,t} \tag{22}$$

where $\gamma(1-\gamma)^{t-j}$ is the mass of cohort j. The aggregation of the labor supply condition and of the policy function for consumption gives

$$\chi C_t = w_t \left(1 - L_t\right) \tag{23}$$

$$C_t = \frac{1-\beta(1-\gamma)}{1+\chi}\left[\Omega_t + E_t \sum_{s=0}^{\infty} \beta_{t,t+s}(1-\gamma)^s \left(w_{t+s} - T_{t+s}\right)\right] \tag{24}$$

Aggregating equation (20) gives

$$E_t\beta_{t,t+1}\left(\frac{C_{t+1}}{1-\gamma} - \frac{\gamma}{1-\gamma}C_{t+1,t+1}\right) = \beta C_t \tag{25}$$

Since new cohorts are born with zero financial wealth, we have that

$$C_{t,t} = \frac{1-\beta(1-\gamma)}{1+\chi}\left[E_t \sum_{s=0}^{\infty} \beta_{t,t+s}(1-\gamma)^s \left(w_{t+s} - T_{t+s}\right)\right] \tag{26}$$

Using equation (24), this implies

$$C_{t+1,t+1} = C_{t+1} - \frac{1-\beta(1-\gamma)}{1+\chi}\Omega_{t+1} \tag{27}$$

We finally obtain

$$C_t = \frac{1}{\beta}E_t\beta_{t,t+1}C_{t+1} + \frac{\gamma}{1-\gamma}\frac{1-\beta(1-\gamma)}{\beta\left(1+\chi\right)}E_t\beta_{t,t+1}\Omega_{t+1} \tag{28}$$

When $\gamma = 0$, we have the familiar Euler equation of the representative agent case. When $\gamma \neq 0$, a wedge related to financial wealth Ω_{t+1} appears. Indeed, while standard Euler equations hold at the cohort level (see equation (18)), this is no longer true in the aggregate. Expected consumption is not only related to current consumption, interest rates, and expected inflation since 1) a proportion $(1-\gamma)$ of agents in each cohort will die between t and $t+1$, 2) new agents will appear in $t+1$ for which past Euler equations do not hold. Since these new households are born without any financial wealth, their $t+1$ consumption is lower than the consumption of surviving households by a factor depending on the amount of financial wealth held by the latter. This explains the presence of the Ω_{t+1} term in equation (28).

3.1.2 Firms

They produce according to the technology $Y_{it} = Z_t L_{it}$ and face quadratic price adjustment costs $\Phi_{it} = \frac{\phi^p}{2}(\frac{P_{it}}{P_{it-1}} - 1)^2 Y_t$. These costs have the same composition as the aggregate consumption basket and are proportional to aggregate output. Since firms belong to households, they use the household stochastic discount factor. They choose P_{it+s} to maximize

$$E_t \sum_{s=0}^{\infty} \beta_{t,t+s} \left[\frac{P_{it+s}}{P_{t+s}} Y_{it+s} - w_{t+s} L_{it+s} - \frac{\phi^p}{2} (\frac{P_{it+s}}{P_{it+s-1}} - 1)^2 Y_{t+s} \right]$$

subject to $Y_{it+s} = Z_{t+s} L_{it+s}$ and $Y_{it+s} = \left(\frac{P_{it+s}}{P_{t+s}}\right)^{-\theta} Y^D_{t+s}$ where Y^D is aggregate demand. We obtain a nonlinear Phillips Curve relating marginal cost to inflation

$$(1-\theta) + \theta \frac{w_t}{Z_t} - \phi^p \Pi_t (\Pi_t - 1) + E_t \beta_{t,t+1} \phi^p \Pi_{t+1} (\Pi_{t+1} - 1) \frac{Z_{t+1} L_{t+1}}{Z_t L_t} = 0 \tag{29}$$

3.1.3 Government and resource constraint

The government budget constraint is of the form

$$P_t G_t = P_t T_t \tag{30}$$

where T_t are lump-sum taxes that appear in the household budget constraint, and $G_t = \omega Y_t$ is a constant fraction of total output. The profits of firms are given by

$$D_t S_{t-1} = \int_0^1 D_t(i) S_{t-1}(i) di = Y_t - w_t L_t - \frac{\phi^p}{2} (\Pi_t - 1)^2 Y_t \tag{31}$$

The number of shares is normalized to one. Moreover, there is zero net supply of bonds in equilibrium ($B_t = 0$). After substituting the expression for profits and the government budget constraint in the aggregate household budget constraint, we obtain

$$C_t = Z_t L_t \left[1 - \omega - \frac{\phi^p}{2}(\Pi_t - 1)^2\right] \tag{32}$$

3.1.4 Equilibrium conditions

A competitive equilibrium is a set of plans $\{C_t, L_t, \Pi_t, \beta_{t,t+1}, Q_t, \Omega_t\}$ satisfying the following equations

$$C_t = \frac{1}{\beta} E_t \beta_{t,t+1} C_{t+1} + \frac{\gamma}{1-\gamma} \frac{1-\beta(1-\gamma)}{\beta\,(1+\chi)} E_t \beta_{t,t+1} \Omega_{t+1} \tag{33}$$

$$(1-\theta) + \theta \frac{\chi C_t}{Z_t\,(1-L_t)} - \phi^p \Pi_t (\Pi_t - 1) + E_t \beta_{t,t+1} \phi^p \Pi_{t+1} (\Pi_{t+1} - 1) \frac{Z_{t+1} L_{t+1}}{Z_t L_t} = 0 \tag{34}$$

$$C_t = Z_t L_t \left[1 - \omega - \frac{\phi^p}{2}(\Pi_t - 1)^2\right] \tag{35}$$

$$Q_t = E_t \beta_{t,t+1} \left(Q_{t+1} + Z_{t+1} L_{t+1} \left(1 - \frac{\phi^p}{2}(\Pi_{t+1} - 1)^2\right) - \frac{\chi C_{t+1}}{1 - L_{t+1}} L_{t+1} \right) \tag{36}$$

$$\Omega_t = Q_t + Z_t L_t \left(1 - \frac{\phi^p}{2}(\Pi_t - 1)^2\right) - \frac{\chi C_t}{1 - L_t} L_t \tag{37}$$

$$E_t \frac{R_t}{\Pi_{t+1}} \beta_{t,t+1} = 1 \tag{38}$$

given specifications for the exogenous process $\{Z_t\}$ and monetary policy $\{R_t\}$.

3.1.5 The stochastic discount factor in steady state

In steady state, the Euler Equation can be expressed as

$$\tilde{\beta} = \beta \left[1 + \frac{\gamma}{(1-\gamma)} \frac{(1-\beta(1-\gamma))}{(1+\chi)} \frac{\Omega}{C}\right]^{-1} \tag{39}$$

where $\tilde{\beta}$ is the steady-state value of the stochastic discount factor. $\tilde{\beta}$ differs from β by a factor $\left[1+\frac{\gamma}{(1-\gamma)}\frac{(1-\beta(1-\gamma))}{(1+\chi)}\frac{\Omega}{C}\right]^{-1}$ which disappears when $\gamma = 0$. Financial wealth is equal to the expected discounted value of future dividends and is positive in steady state. We have $\frac{\Omega}{C} > 0$ and $\tilde{\beta} < \beta$. The stochastic discount factor, which is used by firms when evaluating future profits flows and appears in the Phillips Curve, is thus different from the subjective discount factor of households in steady state. Indeed, in order for wealth to be accumulated, consumption growth must be positive at the cohort level. This happens only if the real return on savings, the inverse of the stochastic discount factor, is larger than the subjective rate of time preference.

3.2 The optimal long-run inflation rate

This section derives the implications for optimal inflation of this heterogeneity in discount factors. For simplicity, I abstract from government spending and assume $\omega = 0$.

3.2.1 Sources of inefficiency in the economy

I consider the problem of a social planner who is not concerned with the distribution of consumption and labor between cohorts, but only with aggregates. This planner discounts the future at the rate β_s. The intertemporal social welfare function is given by

$$W_0^s = E_0 \sum_{t=0}^{\infty} \beta_s^t \left[log(C_t) + \chi log(1 - L_t)\right] \tag{40}$$

The planner maximizes welfare by choosing C_t and L_t subject to the aggregate resource constraint $C_t = Z_t L_t$. The optimality condition dictating the choice between consumption and labor is

$$\frac{\chi C_t}{1 - L_t} = Z_t \tag{41}$$

Comparing equations (34) and (41) shows that the decentralized equilibrium is efficient when $\theta \to \infty$ and $\phi^p = 0$. Thus, just as in the standard New Keynesian model, distortions in the decentralized equilibrium arise solely because of monopolistic competition and costly price adjustment. The introduction of a demographic structure does not in itself create more inefficiencies (given the welfare function defined above)[2].

[2] If, instead, the planner assigned an even weight to the lifetime utility of each individual, it would be optimal to equalize consumption across cohorts. See Nisticò (2016) for more details.

3.2.2 Analytical derivation

We can derive a simple formula for the optimal inflation rate analogous to equation (9). I solve for C_t and L_t as functions of s_t and mc_t, where $s_t = \frac{1}{1-\frac{\phi^p}{2}(\Pi_t-1)^2}$ and $mc_t = \frac{w_t}{Z_t}$, using equations (23) and (32). While s_t only depends on time t inflation, mc_t can be shown to depend on all current and future inflation rates Π_{t+i} $\forall i = 0, ..., +\infty$. After substituting the expressions of C_t and L_t in the social welfare function, deriving W_0^s with respect to Π_t, and evaluating the resulting expression in steady state, we obtain

$$\left(\frac{1}{mc} - \frac{1+\chi}{\chi s + mc}\right)\left(\sum_{j=0}^{\infty} \beta_s^{-j} \frac{\partial mc_{-j}}{\partial \Pi}\right) + (1+\chi)\left(\frac{1}{s} - \frac{\chi}{\chi s + mc}\right)\frac{\partial s}{\partial \Pi} = 0 \tag{42}$$

In a zero inflation efficient steady state, $s = mc = 1$ and the term involving marginal cost disappears. The equation becomes $\frac{\partial s}{\partial \Pi} = 0$, which is verified for $\Pi = 1$. The result of section 2 is confirmed in this setup: zero inflation is optimal even in the presence of discount factor heterogeneity if there are no distortions arising from monopolistic competition. In that case, the decentralized allocation is efficient. Turning to a situation in which $mc < 1$, the optimal inflation rate depends on the objects $\sum_{j=0}^{\infty} \beta^{-j} \frac{\partial mc_{-j}}{\partial \Pi}$ and $\frac{\partial s}{\partial \Pi}$. However, an analytical expression for the discounted effects of inflation on marginal costs cannot be obtained in this framework. As a consequence, equation (42) cannot be used to obtain an analytical characterization of the optimal long-run inflation rate. Instead, I consider an alternative way of formulating the Ramsey problem in which the planner chooses directly a sequence $\{C_t, \Pi_t, L_t, Q_t, \beta_{t,t+1}, D_{t+1}\}$ to maximize (40) subject to the constraints of the competitive economy.

$$Max_{C_t,\Pi_t,L_t,Q_t,\beta_{t,t+1},D_{t+1}} \; E_0 \sum_{s=0}^{\infty} \beta_s^t \left[log(C_t) + \chi log(1-L_t)\right]$$

subject to

$$Z_t L_t \left[1 - \frac{\phi^p}{2}(\Pi_t - 1)^2\right] - C_t = 0$$

$$(1-\theta) + \theta \frac{\chi C_t}{Z_t (1-L_t)} - \phi^p \Pi_t (\Pi_t - 1) + E_t \beta_{t,t+1} \phi^p \Pi_{t+1} (\Pi_{t+1} - 1) \frac{Z_{t+1} L_{t+1}}{Z_t L_t} = 0$$

$$E_t \beta_{t,t+1} (Q_{t+1} + D_{t+1}) - Q_t = 0$$

$$\frac{1}{\beta}E_t\beta_{t,t+1}C_{t+1}+\frac{\gamma}{1-\gamma}\frac{1-\beta(1-\gamma)}{\beta\left(1+\chi\right)}E_t\beta_{t,t+1}\left(Q_{t+1}+D_{t+1}\right)-C_t=0$$

$$Z_tL_t\left(1-\frac{\phi^p}{2}(\Pi_t-1)^2\right)-\frac{\chi C_t}{1-L_t}L_t-D_t=0$$

The first-order conditions of this problem are presented in the appendix, where I also prove the following result analytically.

Proposition: In a model with monopolistic competitition, quadratic price adjustment costs, and a perpetual youth structure, optimal inflation in the Ramsey equilibrium is different from zero in steady state unless the social discount factor coincides with the steady-state value of the discount factor used by firms when evaluating future profit flows.

The following condition obtains in the zero-inflation Ramsey steady state

$$\frac{1}{\theta\chi}\left(\frac{\chi C}{1-L}-1\right)\phi^p\left(1-\frac{\tilde{\beta}}{\beta_s}\right)=0 \tag{43}$$

Given positive values of steady-state consumption and hours worked, this condition is verified in two distinct cases. Either the steady state is efficient and $\frac{\chi C}{1-L}=1$, or the social discount factor β_s is equal to the steady-state stochastic discount factor $\tilde{\beta}$. If none of these conditions are met, the optimal long-run inflation rate is non zero. A planner concerned with the welfare of all generations, born or unborn, would discount the future at the rate $\beta_s=\beta$. In that case, long-run price stability is not optimal.

3.2.3 Numerical analysis

I now solve for the optimal inflation rate in a baseline calibrated version of the model. The discount factors β_s and β are assumed to be equal and fixed at 0.995. The elasticity of substitution between goods θ is set equal to 6, which implies a steady-state markup of 20%. The price adjustment cost parameter ϕ^p is chosen according to the following logic. The linearized Phillips Curve of the model is observationally equivalent to the one derived under Calvo pricing, and structural estimates of New Keynesian models find an elasticity of inflation with respect to marginal cost ω of 0.5 (Lubik and Schorfheide 2004). In my model $\omega=\frac{\theta-1}{\phi^p}$, which implies that $\phi^p=12$. Alternatively, assuming an average contract duration of 4 quarters, the coefficient ω under Calvo pricing would be equal to 0.0846. This implies $\phi^p = 59$. I choose an intermediate value $\phi^p = 40$. The key parameter driving a wedge between the steady-state stochastic discount factor and the household subjective discount factor is the turnover rate γ. If one interprets the model literally, γ represents an average probability of dying for individuals, which is likely to be very low at a quarterly frequency. Alternatively, one can see the perpetual youth model as capturing transitions in and out of hand-to-mouth status and other forms of wealth re-setting such as household default. Following this interpretation, Del Negro et al. (2015) use values of γ ranging from 0.03 to 0.06. In the model of Castelnuovo and Nisticò (2010), this turnover rate is estimated to be much higher at about 0.13. I thus experiment with values ranging from 0.03 to 0.13. Finally, the Frisch elasticity of labor supply depends on level of hours worked L_t. I calibrate χ so that it is equal to 2 in a zero-inflation steady state. This implies that steady-state hours are equal to 1/3.

The results are presented in figure 3, where the annualized optimal inflation rate is plotted for different values of γ. The deviations from price stability are moderate. The optimal inflation rate ranges from about 0.06% when $\gamma = 0.03$ to 0.32% when $\gamma = 0.13$. As shown in equation (42), the weights attached to the discounted effect of inflation on marginal costs and resource costs depend on the degree of monopolistic competition. As steady-state distortions increase, it becomes more interesting to influence the level of output through marginal costs. This can be seen in figure 4, which repeats the same exercise for $\theta = 4$. In that case, the optimal inflation rate ranges from 0.22% for $\gamma = 0.03$ to 1.11% when $\gamma = 0.13$. I also consider an economy with both price and wage markups, in which nominal wages are flexible. Both markups are fixed at 20%. In this scenario, the optimal inflation rate ranges from 0.2% for $\gamma = 0.03$ to 0.98% when $\gamma = 0.13$ (see also figure 4).

4 Conclusion

This paper analyzes the determinants of the optimal long-run inflation rate in a framework in which the relevant frictions faced by the central bank are monopolistic competition and price stickiness. Previous studies have concluded that the optimal inflation rate to which a monetary authority acting under commitment would want to converge in the long run in such an environment is equal to zero. Instead, this paper shows that its value depends on the gap between social and private discount factors.

This result is derived analytically in two different frameworks. First, in a standard New Keynesian model in which the heterogeneity in discount factors is imposed exogenously. Second, in a model with a perpetual youth structure in which it arises endogenously. In both models, the underlying intuition is similar. By engineering a positive rate of inflation, the central bank can lower the long-run level of markups, but incurs resource costs or costs related to price dispersion. This suggests that the inflation rate should be set to a slighly positive value in order to strike a balance between those costs and benefits. However, the problem faced by the monetary authority is intrinsically dynamic. Inflation in a given period t has consequences on markups both in period t and in periods preceding t through anticipated inflation. These real consequences of inflation are weighted differently by the planner, depending on their distance in the future. When social and private discount factors are equal, the weighting effect perfectly offsets the effect of inflation on average markups, and the optimal long-run inflation rate is equal to zero. This is no longer true when social and private discount factors differ. In the perpetual youth model, since Euler equations do not hold for all households, the steady-state stochastic discount factor, which is used by firms to evaluate profit flows, is lower than the household subjective discount factor, which is used by the planner to evaluate utility flows. As a result, the average markup effect dominates the weighting effect and the optimal inflation rate is positive. A credible calibration of the model suggests an optimal inflation rate comprised between 0.2% and 1%.

This property may be relevant in a wide range of macroeconomic environments. In models incorporating financial frictions, whether on the side of firms or households, and in models considering the flows of creation and destruction in the labor and goods markets, private and social discount factors are generally different. In such settings, the combination of price stickiness and monopolistic competition may thus provide a rationale for non-zero inflation targets.

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5 Appendix

5.1 Standard NK model - Analytical formulas for optimal inflation with Calvo pricing

In the model of price setting based on Calvo (1983), each firms may reset its price only with probability $1-\alpha$ in any given period, independent of the time elapsed since the last adjustment. The optimal pricing condition of firms is

$$E_t \sum_{s=t}^{\infty} (\alpha \beta_f)^{s-t} \left(\prod_{i=1}^{s} \Pi_{t+i}^{-1} \right)^{-\theta} \left\{ p_t^o \left(\prod_{i=1}^{s} \Pi_{t+i}^{-1} \right) - \frac{\theta}{\theta - 1} mc_s \right\} = 0 \tag{44}$$

where $p_t^o = \frac{P_t^o}{P_t}$ is the relative price chosen by firms which can reset prices, and P_t is the aggregate price in the economy at time t. The dynamics of inflation are described by the following equation

$$1 = (1-\alpha)(p_t^o)^{1-\theta} + \alpha \Pi_t^{\theta - 1} \tag{45}$$

Moreover, price dispersion leads to a dispersion in the quantity of varieties produced by firms. Since aggregate output is a concave function of the quantity of each variety, this dispersion leads to a costly decrease in output which is captured by

$$s_t = (1-\alpha)\left(p_t^o\right)^{-\theta} + \alpha \Pi_t^{\theta} s_{t-1} \tag{46}$$

In this environment s_t not only depends on the time t inflation rate but also on all past inflation rates Π_{t-i} $\forall i = \mathbb{N}^+$. Symmetrically, mc_t depends on the time t inflation rate and on all future expected inflation rates $E_t \Pi_{t+i}$ $\forall i \in \mathbb{N}^+$. As in the case of Rotemberg pricing, we can rewrite (9) as a function of the effect of inflation on utility operating through marginal costs $\mathit{\Lambda}$, and as the effect of inflation on utility operating through price dispersion Γ

$$\Lambda - \Gamma = 0$$

where

$$\Lambda = \left[mc(\Pi)^{-1} - s(\Pi)\right] \left(... + \beta_s^{-n} \frac{\partial mc_{-n}}{\partial \Pi} + ... + \beta_s^{-1} \frac{\partial mc_{-1}}{\partial \Pi} + \frac{\partial mc}{\partial \Pi} \right)$$

$$\Gamma = \left[\varphi s(\Pi)^{-1} + mc(\Pi)\right] \left(\frac{\partial s}{\partial \Pi} + \beta_s \frac{\partial s_{+1}}{\partial \Pi} + ... + \beta_s^n \frac{\partial s_{+n}}{\partial \Pi} + ...\right)$$

After using equations (44) to (46), and under the assumption that $\alpha \Pi^{\theta} < 1$[3], we can find an analytical solution for both expressions (see below)

$$\Lambda = \left[mc(\Pi)^{-1} - s(\Pi)\right] \frac{(\theta-1)}{\theta} p^o(\Pi) \frac{\alpha \Pi^{\theta-2}}{1\text{-}\alpha\beta_f \Pi^{\theta-1}} \frac{\left[(1-\frac{\beta_f}{\beta_s}\Pi)(1-\alpha\frac{\beta_f}{\beta_s}\Pi^{\theta-1}) + (\theta-1)(1-\Pi)\frac{\beta_f}{\beta_s}(1-\alpha\Pi^{\theta-1})\right]}{(1-\alpha\Pi^{\theta-1})(1-\alpha\frac{\beta_f}{\beta_s}\Pi^{\theta-1})}$$

$$\Gamma = \left[\varphi s(\Pi)^{-1} + mc(\Pi)\right] \frac{\theta(1-\alpha)(p^o(\Pi))^{-\theta}\alpha\Pi^{\theta-2}}{(1-\alpha\beta_s\Pi^{\theta})(1-\alpha\Pi^{\theta})(1-\alpha\Pi^{\theta-1})}(\Pi-1)$$

Figure 2 plots Λ and Γ for different values of the ratio β_f/β_s. Several things can be learned from this figure. First, the discounted effect of inflation on present and future price dispersion terms is equal to zero when $\Pi = 1$. When $\Pi < 1$, an increase in inflation reduces price dispersion today and in the future (Γ is negative). Symmetrically, when $\Pi > 1$, a decrease in inflation reduces price dispersion today and in the future (Γ is positive). Second, the discounted effect of current inflation on past and present marginal costs $\mathit{\Lambda}$ is equal to zero when the social and private discount rates β_s and β_f are equal, and when steady-state inflation is equal to zero. The rationale for this result is similar to the one outlined above with Rotemberg pricing. The average markup is decreasing in current inflation, and increasing in future expected inflation. The effect operating through current inflation tends to dominate the one operating through anticipated inflation for low positive rates of inflation (Goodfriend and King 1997). The average markup is indeed minimized for such a rate. However, events happening before time t loom larger for the Ramsey planner than events happening at time t. That is, the detrimental effects of current inflation on past markups are given a higher weight than its beneficial effects on current markups. When social and private discount rates coincide, these average markup and weighting effects offset each other for an inflation rate of zero.

[3]Even for a high value of price stickiness $\alpha = 0.8$ and a high elasticity of substitution between goods $\theta = 11$, this is valid as long as quarterly steady-state inflation is lower than 2.05%. When $\alpha = 0.9$ and $\theta = 11$, quarterly steady-state inflation has to be lower than 0.96%.

Derivation of Λ

We start with the pricing condition

$$E_t \sum_{s=t}^{\infty} (\alpha\beta_f)^{s-t} \frac{U_c(C_s)}{U_c(C_t)} \left(\frac{P_t}{P_s}\right)^{-\theta} Y_s \left\{ p_t^o \frac{P_t}{P_s} - \frac{\theta}{\theta-1} mc_s \right\} = 0$$

This equation states that firms set their optimal price so that on average their relative price $\frac{P_t^o}{P_s}$ is set as a constant markup over marginal cost mc_s over the lifetime of the price. The weights used when averaging are $(\alpha\beta_f)^{s-t} \frac{U_c(C_s)}{U_c(C_t)} \left(\frac{P_t}{P_s}\right)^{-\theta} Y_s$. They depend on the stochastic discount factor of firms $\beta_f^{s-t} \frac{U_c(C_s)}{U_c(C_t)}$, on the probability that the price will still be effective α^{s-t}, and on the state of demand in period s $\left(\frac{P_t}{P_s}\right)^{-\theta} Y_s$. Imposing market clearing $C_s = Y_s$, and assuming log utility, we obtain

$$E_t \sum_{s=t}^{\infty} (\alpha\beta_f)^{s-t} \left(\frac{P_t}{P_s}\right)^{-\theta} p_t^o \frac{P_t}{P_s} = E_t \sum_{s=t}^{\infty} (\alpha\beta_f)^{s-t} \left(\frac{P_t}{P_s}\right)^{-\theta} \frac{\theta}{\theta-1} mc_s$$

We can express marginal cost as a function of the current period optimal price p_t^o, future inflation rates $\Pi_{t+1}, \Pi_{t+2}, ...$, and future marginal costs $mc_{t+1}, mc_{t+2}, ...$

$$mc_t = \frac{\theta-1}{\theta} p_t^o E_t \left[1 + \alpha\beta_f \Pi_{t+1}^{\theta-1} + (\alpha\beta_f)^2 (\Pi_{t+1}\Pi_{t+2})^{\theta-1} + ...\right]$$

$$-\alpha\beta_f E_t \Pi_{t+1}^{\theta} mc_{t+1} - (\alpha\beta_f)^2 E_t \left(\Pi_{t+1}\Pi_{t+2}\right)^{\theta} mc_{t+2} + ...$$

This equation is also valid one period ahead

$$mc_{t+1} = \frac{\theta-1}{\theta} p_{t+1}^o E_{t+1} \left[1 + \alpha\beta_f \Pi_{t+2}^{\theta-1} + (\alpha\beta_f)^2 (\Pi_{t+2}\Pi_{t+3})^{\theta-1} + ...\right]$$

$$-\alpha\beta_f E_{t+1} \Pi_{t+2}^{\theta} mc_{t+2} - (\alpha\beta_f)^2 E_{t+1} \left(\Pi_{t+2}\Pi_{t+3}\right)^{\theta} mc_{t+3} + ...$$

Substituting $\alpha\beta_f E_t \Pi_{t+1}^{\theta} mc_{t+1}$ in the equation for mc_t, we obtain

$$mc_t = \frac{\theta-1}{\theta} p_t^o E_t \left[1 + \alpha\beta_f \Pi_{t+1}^{\theta-1} + (\alpha\beta_f)^2 (\Pi_{t+1}\Pi_{t+2})^{\theta-1} + ...\right]$$

$$-\alpha\beta_f E_t \Pi_{t+1}^{\theta} \frac{\theta-1}{\theta} p_{t+1}^o \left[1+\alpha\beta_f \Pi_{t+2}^{\theta-1} + (\alpha\beta_f)^2 (\Pi_{t+2}\Pi_{t+3})^{\theta-1} + ...\right]$$

Thus marginal cost is equal to the expected discounted sum of real revenues when prices are set optimally today minus the appropriately discounted expected discounted sum of real revenues when prices are set optimally tomorrow. This last expression can be rewritten as

$$mc_t = \frac{\theta-1}{\theta} p_t^o + \frac{\theta-1}{\theta} E_t \left(p_t^o - p_{t+1}^o \Pi_{t+1}\right) \left[\alpha\beta_f \Pi_{t+1}^{\theta-1} + (\alpha\beta_f)^2 \left(\Pi_{t+1}\Pi_{t+2}\right)^{\theta-1} + ...\right]$$

We can now derive the effect of future expected inflation on marginal cost

- $\frac{\partial mc_t}{\partial \Pi_t} = \frac{\theta-1}{\theta} \frac{\partial p_t^o}{\partial \Pi_t} \left[1+\alpha\beta_f \Pi_{t+1}^{\theta-1} + (\alpha\beta_f)^2 (\Pi_{t+1}\Pi_{t+2})^{\theta-1} + ...\right]$
- $\frac{\partial mc_t}{\partial E_t \Pi_{t+1}} = \frac{(\theta-1)^2}{\theta} E_t \left(p_t^o - p_{t+1}^o \Pi_{t+1}\right) \Pi_{t+1}^{\theta-2} \left[\alpha\beta_f + (\alpha\beta_f)^2 (\Pi_{t+2})^{\theta-1} + ...\right]$
- $-\frac{\theta-1}{\theta} \left(\frac{\partial p_{t+1}^o}{\partial \Pi_{t+1}} \Pi_{t+1} + p_{t+1}^o\right) \left[\alpha\beta_f \Pi_{t+1}^{\theta-1} + (\alpha\beta_f)^2 \left(\Pi_{t+1}\Pi_{t+2}\right)^{\theta-1} + ...\right]$
- $\frac{\partial mc_t}{\partial E_t \Pi_{t+2}} = \frac{(\theta-1)^2}{\theta} E_t \left(p_t^o - p_{t+1}^o \Pi_{t+1}\right) \Pi_{t+2}^{\theta-2} \left[(\alpha\beta_f)^2 \Pi_{t+1}^{\theta-1} + (\alpha\beta_f)^3 (\Pi_{t+1}\Pi_{t+3})^{\theta-1} + ...\right]$

Under the assumption that $\alpha\beta_f\Pi^{\theta}$is lower than 1, we obtain in steady state

- $\frac{\partial mc}{\partial \Pi} = \frac{\theta-1}{\theta} \frac{\partial p^o}{\partial \Pi} \frac{1}{1-\alpha\beta_f \Pi^{\theta-1}}$
- $\frac{\partial mc}{\partial \Pi_{+1}} = \frac{(\theta-1)^2}{\theta} p^o (1-\Pi) \frac{\Pi^{-1}\alpha\beta_f \Pi^{\theta-1}}{1-\alpha\beta_f \Pi^{\theta-1}} - \frac{\theta-1}{\theta} \alpha\beta_f \Pi^{\theta-1} \left(\frac{\partial p^o}{\partial \Pi} \Pi + p^o\right) \frac{1}{1-\alpha\beta_f \Pi^{\theta-1}}$
- $\frac{\partial mc}{\partial \Pi_{+2}} = \frac{(\theta-1)^2}{\theta} p^o (1-\Pi) \frac{\Pi^{-1}\left(\alpha\beta_f \Pi^{\theta-1}\right)^2}{1-\alpha\beta_f \Pi^{\theta-1}}$

For $n \geq 2$, all derivatives share a common form

- $\frac{\partial mc}{\partial \Pi_{+n}} = \frac{(\theta-1)^2}{\theta} p^o (1-\Pi) \frac{\Pi^{-1}\left(\alpha\beta_f \Pi^{\theta-1}\right)^n}{1-\alpha\beta_f \Pi^{\theta-1}}$

We can now compute what the planner cares about, the discounted effect of inflation on marginal cost

$$\Xi = \frac{\Lambda}{mc(\Pi)^{-1} - s(\Pi)} = \sum_{i=0}^{\infty} (\beta_s^{-1})^i \frac{\partial mc}{\partial \Pi_{+i}}$$

After some algebra, we obtain

$$\Xi = \frac{(\theta-1)}{\theta} p^o \frac{\alpha \Pi^{\theta-2}}{1\text{-}\alpha\beta_f \Pi^{\theta-1}} \frac{\left[(1-\frac{\beta_f}{\beta_s}\Pi)(1-\alpha\frac{\beta_f}{\beta_s}\Pi^{\theta-1}) + (\theta-1)(1-\Pi)\frac{\beta_f}{\beta_s}(1-\alpha\Pi^{\theta-1})\right]}{(1-\alpha\Pi^{\theta-1})(1-\alpha\frac{\beta_f}{\beta_s}\Pi^{\theta-1})}$$

where $p^o = \left[\frac{1-\alpha\Pi^{\theta-1}}{1-\alpha}\right]^{\frac{1}{1-\theta}}$.

Derivation of Γ

We have

$$s_t = (1-\alpha)\left(p_t^o\right)^{-\theta} + \alpha \Pi_t^\theta s_{t-1}$$

Iterating backward, we obtain

$$s_t = (1-\alpha)\left[\left(p_t^o\right)^{-\theta} + \alpha \Pi_t^\theta \left(p_{t-1}^o\right)^{-\theta} + \alpha^2 \left(\Pi_t \Pi_{t-1}\right)^\theta \left(p_{t-2}^o\right)^{-\theta} + ...\right]$$

We can derive the effect of current and past inflation on s_t

- $\frac{\partial s_t}{\partial \Pi_t} = -\theta(1-\alpha)\left(p_t^o\right)^{-\theta-1}\frac{\partial p_t^o}{\partial \Pi_t} + (1-\alpha)\theta\Pi_t^{\theta-1}\left[\alpha\left(p_{t-1}^o\right)^{-\theta} + \alpha^2\Pi_{t-1}^\theta\left(p_{t-2}^o\right)^{-\theta} + ...\right]$
- $\frac{\partial s_t}{\partial \Pi_{t-1}} = -\theta(1-\alpha)\alpha\Pi_t^\theta\left(p_{t-1}^o\right)^{-\theta-1}\frac{\partial p_{t-1}^o}{\partial \Pi_{t-1}} + (1-\alpha)\theta\Pi_{t-1}^{\theta-1}\left[\alpha^2\Pi_t^\theta\left(p_{t-2}^o\right)^{-\theta} + \alpha^3\left(\Pi_t\Pi_{t-2}\right)^\theta\left(p_{t-3}^o\right)^{-\theta} + ...\right]$
- $\frac{\partial s_t}{\partial \Pi_{t-2}} = -\theta(1-\alpha)\alpha^2\left(\Pi_t\Pi_{t-1}\right)^\theta\left(p_{t-2}^o\right)^{-\theta-1}\frac{\partial p_{t-2}^o}{\partial \Pi_{t-2}} + (1-\alpha)\theta\Pi_{t-2}^{\theta-1}\left[\alpha^3\left(\Pi_t\Pi_{t-1}\right)^\theta\left(p_{t-3}^o\right)^{-\theta} + ...\right]$

Under the assumption that $\alpha\Pi^\theta$ is lower than 1, we obtain in steady state

- $\frac{\partial s}{\partial \Pi} = \theta(1-\alpha)(p^o)^{-\theta}\frac{\alpha\Pi^{\theta-2}}{(1-\alpha\Pi^{\theta-1})(1-\alpha\Pi^\theta)}(\Pi-1)$
- $\frac{\partial s}{\partial \Pi_{-1}} = \alpha\Pi^\theta\frac{\partial s}{\partial \Pi}$
- $\frac{\partial s}{\partial \Pi_{-2}} = (\alpha\Pi^\theta)^2\frac{\partial s}{\partial \Pi}$

For $n \geq 2$, all derivatives share a common form

- $\frac{\partial s}{\partial \Pi_{-n}} = (\alpha\Pi^\theta)^n\frac{\partial s}{\partial \Pi}$

We can now compute what the planner cares about, the discounted effect of inflation on price dispersion

$$\Phi = \frac{\Gamma}{\varphi s(\Pi)^{-1} + mc(\Pi)} = \sum_{i=0}^{\infty}(\beta_s)^i\frac{\partial s}{\partial \Pi_{-i}}$$

After some algebra, we find

$$\Phi = \frac{\theta(1-\alpha)(p^o)^{-\theta}\alpha\Pi^{\theta-2}}{(1-\alpha\beta_s\Pi^\theta)(1-\alpha\Pi^\theta)(1-\alpha\Pi^{\theta-1})}(\Pi-1)$$

5.2 NKPY model - Ramsey steady state

The first-order conditions of the Ramsey problem are

- Π_t: $-(\lambda_{1t}+\lambda_{5t})Z_tL_t\phi^p(\Pi_t-1)-\lambda_{2t}\phi^p\left(2\Pi_t-1\right)+\frac{\beta_{t-1,t}}{\beta_s}\lambda_{2t-1}\phi^p\left(2\Pi_t-1\right)\frac{Z_tL_t}{Z_{t-1}L_{t-1}}=0$
- D_t: $-\lambda_{5t}+\frac{\beta_{t-1,t}}{\beta_s}\left[\lambda_{3t-1}+\frac{\gamma}{1-\gamma}\frac{1-\beta(1-\gamma)}{\beta(1+\chi)}\lambda_{4t-1}\right]=0$
- Q_t: $-\lambda_{3t}+\frac{\beta_{t-1,t}}{\beta_s}\left[\lambda_{3t-1}+\frac{\gamma}{1-\gamma}\frac{1-\beta(1-\gamma)}{\beta(1+\chi)}\lambda_{4t-1}\right]=0$
- $\beta_{t,t+1}$: $\lambda_{2t}E_t\left(\phi^p\Pi_{t+1}(\Pi_{t+1}-1)\frac{Z_{t+1}L_{t+1}}{Z_tL_t}\right)+\lambda_{3t}E_t\left(Q_{t+1}+D_{t+1}\right)+\lambda_{4t}E_t\left(\frac{1}{\beta}C_{t+1}+\frac{\gamma}{1-\gamma}\frac{1-\beta(1-\gamma)}{\beta(1+\chi)}\left(Q_{t+1}+D_{t+1}\right)\right)=0$
- L_t: $-\frac{\chi}{1-L_t}+(\lambda_{1t}+\lambda_{5t})Z_t\left(1-\frac{\phi^p}{2}(\Pi_t-1)^2\right)+\lambda_{2t}\left(\theta\chi\frac{C_t}{Z_t(1-L_t)^2}-E_t\beta_{t,t+1}\phi^p\Pi_{t+1}(\Pi_{t+1}-1)\frac{Z_{t+1}L_{t+1}}{Z_tL_t^2}\right)-\lambda_{5t}\chi C_t\frac{1}{(1-L_t)^2}+\frac{\beta_{t-1,t}}{\beta_s}\phi^p\Pi_t(\Pi_t-1)\frac{Z_t}{Z_{t-1}L_{t-1}}\lambda_{2t-1}=0$
- C_t: $\frac{1}{C_t}-\lambda_{1t}+\lambda_{2t}\frac{\theta\chi}{Z_t(1-L_t)}-\lambda_{4t}-\lambda_{5t}\frac{\chi}{1-L_t}L_t+\frac{\beta_{t-1,t}}{\beta_s\beta}\lambda_{4t-1}=0$

In steady state

- Π_t: $-(\lambda_1+\lambda_5)L\phi^p(\Pi-1)-\lambda_2\phi^p\left(2\Pi-1\right)\left(1-\frac{\tilde{\beta}}{\beta_s}\right)=0$
- D_t: $-\lambda_5+\frac{\tilde{\beta}}{\beta_s}\left[\lambda_3+\frac{\gamma}{1-\gamma}\frac{1-\beta(1-\gamma)}{\beta(1+\chi)}\lambda_4\right]=0$
- Q_t: $-\lambda_3(1-\frac{\tilde{\beta}}{\beta_s})+\frac{\tilde{\beta}}{\beta_s}\frac{\gamma}{1-\gamma}\frac{1-\beta(1-\gamma)}{\beta(1+\chi)}\lambda_4=0$
- $\beta_{t,t+1}$: $\lambda_2\phi^p\Pi(\Pi-1)+\lambda_3\left(Q+D\right)+\lambda_4\left(\frac{1}{\beta}C+\frac{\gamma}{1-\gamma}\frac{1-\beta(1-\gamma)}{\beta(1+\chi)}\left(Q+D\right)\right)=0$
- L_t: $\frac{-\chi}{1-L}+(\lambda_1+\lambda_5)\left(1-\frac{\phi^p}{2}(\Pi-1)^2\right)+\lambda_2\left(\theta\chi\frac{C}{(1-L)^2}+\tilde{\beta}\phi^p\Pi(\Pi-1)\frac{1}{L}(\frac{1}{\beta_s}-1)\right)-\lambda_5\chi C\frac{1}{(1-L)^2}=0$
- C_t: $\frac{1}{C}-\lambda_1+\lambda_2\frac{\theta\chi}{1-L}-\lambda_4(1-\frac{\tilde{\beta}}{\beta_s\beta})-\lambda_5\frac{\chi}{1-L}L=0$

The third equation implies

$$\lambda_3=\frac{\frac{\tilde{\beta}}{\beta_s}\frac{\gamma}{1-\gamma}\frac{1-\beta(1-\gamma)}{\beta(1+\chi)}\lambda_4}{1-\frac{\tilde{\beta}}{\beta_s}}$$

Substituting in the second

$$\lambda_5=\lambda_3=\frac{\frac{\tilde{\beta}}{\beta_s}\frac{\gamma}{1-\gamma}\frac{1-\beta(1-\gamma)}{\beta(1+\chi)}\lambda_4}{1-\frac{\tilde{\beta}}{\beta_s}}=\frac{\frac{\tilde{\beta}}{\beta_s}\alpha\lambda_4}{1-\frac{\tilde{\beta}}{\beta_s}}$$

where $\alpha = \frac{\gamma}{1-\gamma}\frac{1-\beta(1-\gamma)}{\beta(1+\chi)}$. The previous system of equations simplifies to

- $-(\lambda_1 + \frac{\frac{\tilde{\beta}}{\beta_s}\alpha\lambda_4}{1-\frac{\tilde{\beta}}{\beta_s}})L\phi^p(\Pi-1) - \lambda_2\phi^p\,(2\Pi-1)\left(1-\frac{\tilde{\beta}}{\beta_s}\right) = 0$
- $\lambda_2\phi^p\Pi(\Pi-1) + \lambda_4\left(\frac{1}{\beta}C + \frac{\alpha}{1-\frac{\tilde{\beta}}{\beta_s}}\,(Q+D)\right) = 0$
- $\frac{-\chi}{1-L} + (\lambda_1 + \frac{\frac{\tilde{\beta}}{\beta_s}\alpha\lambda_4}{1-\frac{\tilde{\beta}}{\beta_s}})\left(1-\frac{\phi^p}{2}(\Pi-1)^2\right) + \lambda_2\left(\theta\chi\frac{C}{(1-L)^2} + \tilde{\beta}\phi^p\Pi(\Pi-1)\frac{1}{L}(\frac{1}{\beta_s}-1)\right) - \frac{\frac{\tilde{\beta}}{\beta_s}\alpha\lambda_4}{1-\frac{\tilde{\beta}}{\beta_s}}\chi C\frac{1}{(1-L)^2} = 0$
- C_t: $\frac{1}{C} - \lambda_1 + \lambda_2\frac{\theta\chi}{1-L} - \lambda_4\left[(1-\frac{\tilde{\beta}}{\beta_s\beta}) + \frac{\frac{\tilde{\beta}}{\beta_s}\alpha}{1-\frac{\tilde{\beta}}{\beta_s}}\frac{\chi}{1-L}L\right] = 0$

We can solve for λ_4 as a function of λ_2 using the second equation of this new system

$$\lambda_4 = -\lambda_2\frac{\phi^p\Pi(\Pi-1)}{\frac{1}{\beta}C + \frac{\alpha}{1-\frac{\tilde{\beta}}{\beta_s}}\,(Q+D)}$$

We now express λ_1 as a function of λ_2 using the last equation

$$\frac{1}{C} + \lambda_2\left(\frac{\theta\chi}{1-L} + \frac{\phi^p\Pi(\Pi-1)}{\frac{1}{\beta}C + \frac{\alpha}{1-\frac{\tilde{\beta}}{\beta_s}}\,(Q+D)}\left[1-\frac{\tilde{\beta}}{\beta_s\beta} + \frac{\frac{\tilde{\beta}}{\beta_s}\alpha}{1-\frac{\tilde{\beta}}{\beta_s}}\frac{\chi}{1-L}L\right]\right) = \lambda_1$$

Substituting in the third equation

$$\lambda_2 = \frac{\frac{\chi}{1-L} - \frac{1}{C}\left(1-\frac{\phi^p}{2}(\Pi-1)^2\right)}{\left(\frac{\theta\chi}{1-L} + \frac{\phi^p\Pi(\Pi-1)}{\frac{1}{\beta}C + \frac{\alpha}{1-\frac{\tilde{\beta}}{\beta_s}}(Q+D)}\left[(1-\frac{\tilde{\beta}}{\beta_s\beta}) + \frac{\frac{\tilde{\beta}}{\beta_s}\alpha}{1-\frac{\tilde{\beta}}{\beta_s}}\frac{\chi}{1-L}L\right]\right)\left(1-\frac{\phi^p}{2}(\Pi-1)^2\right) + \theta\chi\frac{C}{(1-L)^2} + \phi^p\Pi(\Pi-1)\frac{1}{L}(\frac{\tilde{\beta}}{\beta_s}-\tilde{\beta}) + \frac{\frac{\tilde{\beta}}{\beta_s}\alpha\phi^p\Pi(\Pi-1)\left(\chi\frac{C}{(1-L)^2} - 1 + \frac{\phi^p}{2}(\Pi-1)^2\right)}{\left(\frac{1}{\beta}-\frac{\tilde{\beta}}{\beta_s\beta}\right)C + \alpha(Q+D)}}$$

In a zero-inflation steady state, this equation becomes

$$\lambda_2 = \frac{\frac{\chi}{1-L} - \frac{1}{C}}{\frac{\theta\chi}{1-L} + \theta\chi\frac{C}{(1-L)^2}}$$

Evaluating in the first equation

$$\frac{1}{\theta\chi}\left(\frac{\chi}{1-L} - \frac{1}{C}\right)\phi^p\left(1-\frac{\tilde{\beta}}{\beta_s}\right) = 0$$

When $\beta_s = \beta$, this happens only when $\theta \to \infty$ (first term tends towards zero) or when $\gamma \to \infty$ (second term tends towards zero). Zero inflation is not optimal unless the steady state is efficient or agents are infinitely lived.

5.3 Figures

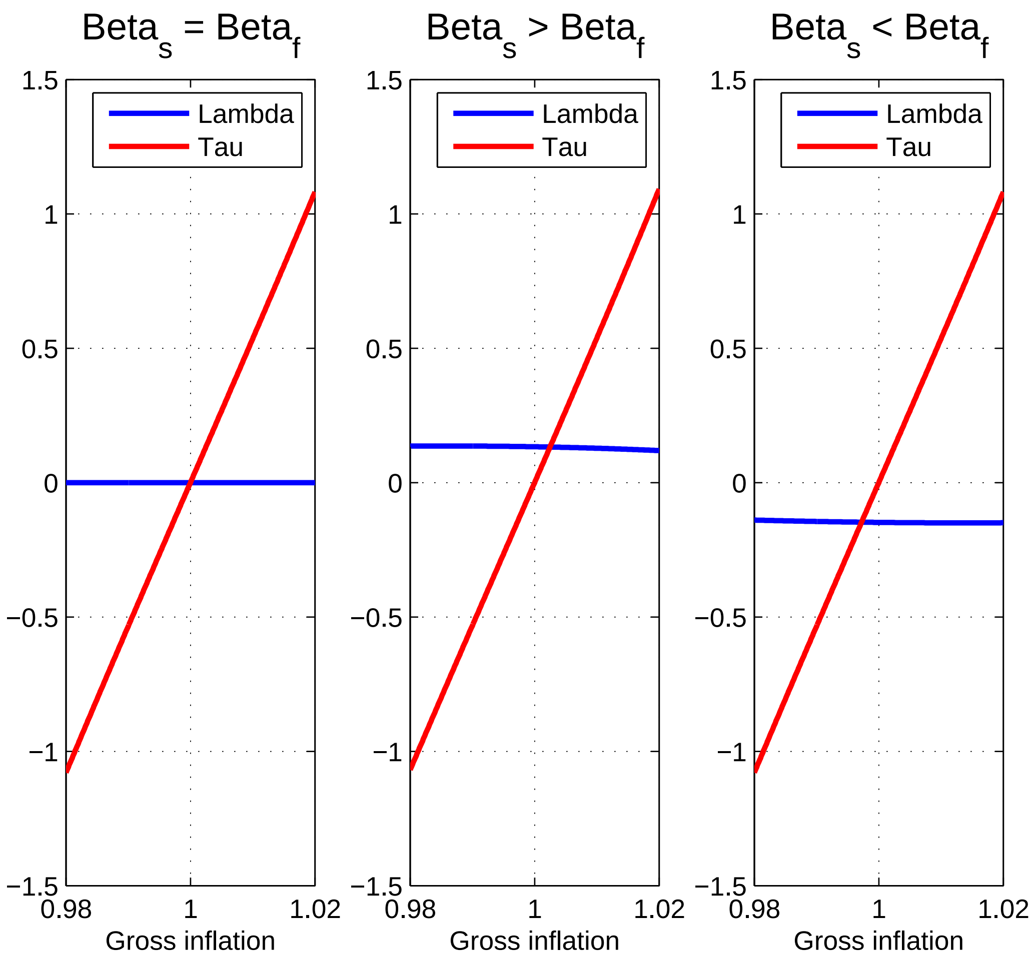

Figure 1: Λ and Γ according to the value of $\frac{\beta_f}{\beta_s}$ with Rotemberg pricing. In the first case, $\beta_f = \beta_s = 0.995$. In the second case, $\beta_s = 0.995$ and $\beta_f = 0.9\beta_s$. In the third case, $\beta_f = 0.995$ and $\beta_s = 0.9\beta_f$. The other parameters are calibrated as follows: $\phi^p = 40$, $\theta = 6$, $\varphi = 0.5$.

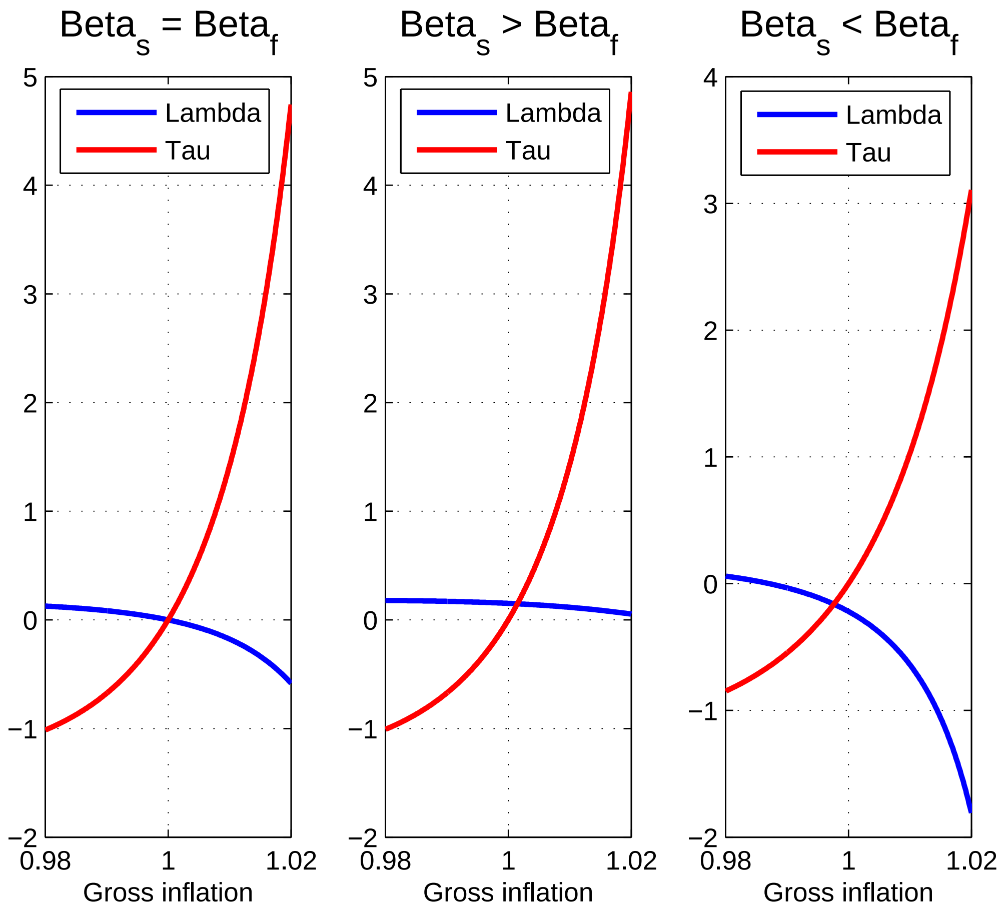

Figure 2: Λ and Γ according to the value of $\frac{\beta_f}{\beta_s}$ with Calvo pricing. In the first case, $\beta_f = \beta_s = 0.995$. In the second case, $\beta_s = 0.995$ and $\beta_f = 0.9\beta_s$. In the third case, $\beta_f = 0.995$ and $\beta_s = 0.9\beta_f$. The other parameters are calibrated as follows: $\alpha = 0.75$, $\theta = 6$, $\varphi = 0.5$.

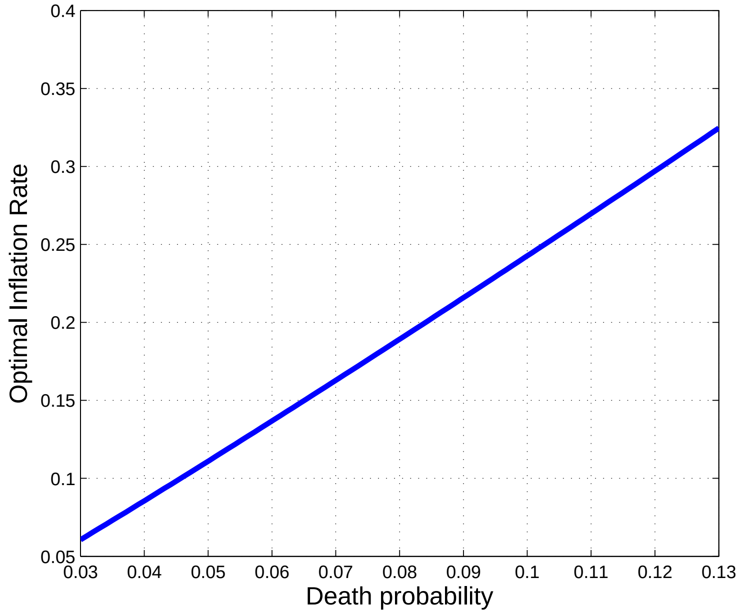

Figure 3: Optimal long-run inflation rate according to the death probability γ in the New Keynesian perpetual youth model - Baseline calibration.

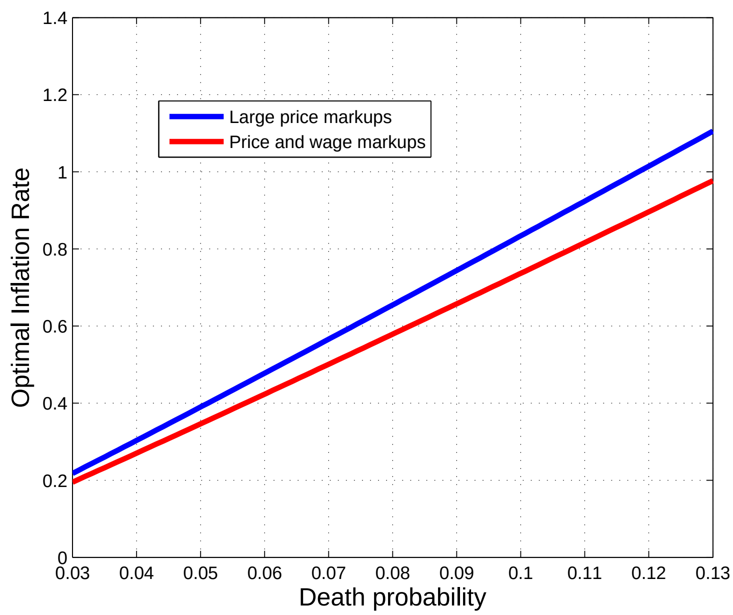

Figure 4: Optimal long-run inflation rate according to the death probability γ in the New Keynesian perpetual youth model - Models with large steady-state distortions.