Modelling Choice and Valuation in Decision Experiments

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Abstract

This paper develops a parsimonious descriptive model of individual choice and valuation in the kinds of experiments that constitute a substantial part of the literature. It argues that most of the best-known 'regularities' observed in those experiments arise from a tendency for participants to perceive probabilities and payoffs in a particular way. This model organises much more of the data than any other extant model and generates a number of novel testable implications. It also helps identify the conditions under which expected utility theory and *all* of its current main rivals are liable – indeed, *bound* – to fail.   97 words

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Introduction

This paper develops a parsimonious descriptive model of risky choice which can organise much of the most influential experimental evidence of systematic departures from conventional decision theory. Focusing on the kind of tasks which constitute much of the evidence – that is, choices between pairs of lotteries involving no more than three outcomes, and/or valuations of such lotteries – it will be shown that individuals who behave according to this model will violate all but one of the key axioms of rational choice – the only exception being transparent dominance.

The paper is organised as follows. Section 1 sets up the basic framework. Section 2 models the perception of probabilities and shows that one simple proposition about the way that probabilities are handled is enough to ensure that the axioms of independence, betweenness and transitivity are all *bound* to fail in one way or another. This section identifies a number of novel predictions of regularities which, if borne out by further tests, would contradict the class of rank-dependent models that are currently regarded as offering the best alternative to standard expected utility theory. Section 3 models an analogous proposition about the way that payoffs are perceived, and this allows the model to explain a number of other regularities which cannot be accommodated by expected utility theory or any of its main rivals. Section 4 discussed the relationship between the model proposed here and a number of other axiomatic or behavioural models which have attempted to organise various subsets of regularities. Section 5 concludes.

1. The Modelling Framework

Before outlining the particular framework for this model, three remarks.

First, although there is one decision theoretic tradition which involves identifying some set of general axioms (often selected on the basis of their supposed normative appeal) and then deriving the implications for any given class of cases, this paper is not in that tradition. Rather, the model in this paper is a behavioural model, concerned with why people may be liable to act in certain systematic ways in response to particular kinds of stimuli. The model is set out formally in order to allow implications to be derived, but it is important to keep in mind that it is a model of decisions often made quite quickly[1] and on the basis of perceptions rather than after

[1] Many experiments ask participants to make numerous decisions within single sessions, and once they become familiar with the tasks, many participants spend only a few seconds on each one: for example,

long deliberation involving complex calculation. The structure of the model is therefore intended to capture *tendencies* in the ways perceptions are formed and judgments are made: it is *not* suggested that people actually make calculations strictly according to the formulae.

Second, although it is intended to explain observed patterns of behaviour, it *is* only a model, and therefore simplifies. In particular, although actual responses are susceptible to ‘noise’ and error, the exposition initially abstracts from that and presents a deterministic model[2]. The exposition also abstracts from failures of procedure invariance and framing effects (Tversky and Kahneman, 1986). Such effects undoubtedly influence behaviour, but the claim being made in this paper is that we can explain many regularities without needing to invoke those additional effects. On the other hand, the proposed model rests on just two basic propositions involving just two free parameters and it would be surprising if this were sufficient to account for *all* of the many regularities observed in the relevant class of decision experiments. But that is not the claim. This is not a theory of everything. Nevertheless, those two basic propositions combine to organise many more of the known regularities than any other single model currently available, and in the course of doing so, help us to identify where – and more importantly, perhaps, *why* – those other models are liable to fall short.

Third, in addition to organising much of the existing data, the model also allows/predicts patterns which have not (yet) been much observed, except somewhat serendipitously, as by-products of designs with other primary objectives. Where I am aware of such evidence, I shall refer to it. But this is principally a theoretical paper: it aims to provide a new model and it will identify a number of ways in which that model is differentiated from existing alternatives. It suggests directions for new experimental research, but the conduct of such research is beyond the scope of this paper.

Having made those preliminary points, let us now turn to the formulation of the model. The bulk of the experimental data used to test theories of risk come from

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Moffatt (2005) analysed a pairwise choice dataset where mean decision times mostly ranged between 3 and 8 seconds per choice. This may be a somewhat extreme case, but it would not be uncommon to find the great majority of participants taking no more than 15 or 20 seconds to process most of the kinds of decisions presented in many choice / valuation experiments.

[2] Section 4 will discuss (briefly) the question of extending the model to allow for the stochastic nature of decision behaviour. As noted above, we are dealing with data generated quite quickly and somewhat impressionistically and it would be surprising if there were not some stochastic component in such data; but the model abstracts from that and focuses on what may be regarded as ‘central tendencies’.

decisions that can be represented in terms of pairs of alternative lotteries, each involving no more than three monetary payoffs.

Figure 1: The Basic Pairwise Choice Format

	p_3	p_2	p_1
S	x_3	x_2	x_1
R	x_3	x_2	x_1
	q_3	q_2	q_1

Figure 1 shows a basic template for such cases. Payoffs[3] are $x_3 > x_2 > x_1 \geq 0$ and the probabilities of each payoff under the safer lottery S are, respectively, p_3, p_2 and p_1, while the corresponding probabilities for the riskier lottery R are q_3, q_2 and q_1, with $q_3 > p_3$, $q_2 < p_2$ and $q_1 > p_1$.

Although this template is broad enough to accommodate any pairwise choice or valuation involving up to three payoffs, the great majority of experimental tasks involve simpler formats – most commonly, with S being either a sure thing (where $p_2 = 1$) or else a two-payoff lottery (with $p_3 = 0$; $1 > p_2, p_1 > 0$) versus a two-payoff R lottery ($q_2 = 0$, $1 > q_3, q_1 > 0$). A certainty equivalent valuation can be modelled as the case where p_2 is set at 1 and the respondent is asked to identify the value of x_2 which makes her indifferent between S and R.

Within this framework, a choice can be seen as a judgment between two arguments pulling in opposite directions. The argument in favour of R is that it offers some chance – the difference between q_3 and p_3 – of getting x_3 rather than x_2. Against that, the argument in favour of S is that it offers a better chance – in this case, the difference between q_1 and p_1 – of getting x_2 rather than x_1.

Most decision theories represent the force of these arguments in some formal way and then propose, in effect, that choice depends on the relative force of the competing arguments. To illustrate, consider expected utility theory (EUT). Under EUT, the advantage that R offers over S on the payoff dimension is given by the

[3] The great majority of experiments involve non-negative payoffs. The framework can accommodate negative amounts (i.e. losses); but to avoid complicating the exposition unnecessarily, the initial focus will be upon non-negative payoffs, and the issue of losses will be addressed later.

subjective difference between x_3 and x_2 – that is, $u(x_3)$-$u(x_2)$, where u(.) is a von Neumann-Morgenstern utility function – which is weighted by the probability of that difference, q_3-p_3. Correspondingly, the advantage that S offers over R is the utility difference $u(x_2)$-$u(x_1)$, weighted by the probability of that difference, q_1-p_1. Denoting strict preference by $\succ$ and indifference by ~, EUT entails:

$$S \begin{smallmatrix}\succ \\ \sim \\ \prec\end{smallmatrix} R \Leftrightarrow (q_1-p_1).[u(x_2)-u(x_1)] \begin{smallmatrix}> \\ = \\ <\end{smallmatrix} (q_3-p_3).[u(x_3)-u(x_2)] \qquad (1)$$

Alternatively, Tversky and Kahneman's (1992) cumulative prospect theory (CPT), modifies this expression in two ways: it draws the subjective values of payoffs from a value function v(.) rather than a vNM utility function u(.); and it involves the nonlinear transformation of probabilities into decision weights, here denoted by $\pi(.)$. Thus for CPT we have:

$$S \begin{smallmatrix}\succ \\ \sim \\ \prec\end{smallmatrix} R \Leftrightarrow [\pi(q_1)-\pi(p_1)].[v(x_2)-v(x_1)] \begin{smallmatrix}> \\ = \\ <\end{smallmatrix} [\pi(q_3)-\pi(p_3).[v(x_3)-v(x_2)] \qquad (2)$$

Under both EUT and CPT, it is as if an individual maps each payoff to some subjective utility/value, weights each of these by some (function of) its probability, and thereby arrives at an overall evaluation or 'score' for each lottery which is then used to compare alternatives, with the decision rule being to choose the one with the highest evaluation. By assigning to each lottery a score that is determined entirely by the interaction between the decision maker's preferences and the characteristics of that particular lottery (that is, independent of any other lotteries in the available choice set), such models guarantee respect for transitivity. In addition, the functions and procedures which govern subjective values and decision weights may be specified in ways which guarantee respect for monotonicity and first order stochastic dominance[4].

[4] The original form of prospect theory – Kahneman and Tversky (1979) – involved a method of transforming probabilities into decision weights which allowed violations of first order stochastic dominance – an implication to which some commentators were averse. Quiggin (1982) proposed a more complex 'rank-dependent' method of transforming probabilities into decision weights which seemed to broadly preserve the spirit of the original while ensuring respect for first order stochastic dominance. CPT uses a version of this method.

However, for any theory to perform well descriptively, its structure needs to correspond with the way participants perceive stimuli and act on those perceptions. If the way they actually make judgments runs counter to some feature(s) of a theory, the observed data are liable to diverge systematically from the implications of that theory. It is a central proposition of this paper that participants' perceptions and judgments *are* liable to operate in ways which run counter to the assumptions underpinning most decision theories, including EUT and CPT. In particular, there is much psychological evidence suggesting that many people do not evaluate alternatives separately and purely on the basis of the 'absolute' levels of their attributes, but that their judgments and choices are also liable to be influenced by 'relative' considerations – see, for example, Stewart *et al.* (2003). In the context of pairwise choices between lotteries, this may entail individuals making between-lottery comparisons of both probabilities and payoffs, and having their perceptions and judgments systematically affected by such comparisons.

To help bridge from a conventional theory such as EUT to a model which allows for between-lottery considerations, rearrange Expression (1) as follows:

$$S \begin{matrix}\succ \\ \sim \\ \prec\end{matrix} R \iff (q_1-p_1)/(q_3-p_3) \begin{matrix}> \\ = \\ <\end{matrix} [u(x_3)-u(x_2)]/[u(x_2)-u(x_1)] \qquad (3)$$

A verbal interpretation of this is: "S is judged preferable to / indifferent to / less preferable than R according to whether the *relative force of the argument for S compared with R on the probability dimension* – that is, $(q_1-p_1)/(q_3-p_3)$ – is greater than / equal to / less than the *relative force of the argument for R compared with S on the payoff dimension* – namely, $[u(x_3)-u(x_2)]/[u(x_2)-u(x_1)]$.

Under EUT, the ratios on each side are simply the ratios of two values determined independently and solely on the basis of *within*-lottery features[5]. But if participants in experiments also make comparisons *between* the two alternatives, their

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[5] This is also true for CPT: the CPT counterpart to Expression (3) takes the form:

$$S \begin{matrix}\succ \\ \sim \\ \prec\end{matrix} R \iff [\pi(q_1)-\pi(p_1)]/[\pi(q_3)-\pi(p_3)] \begin{matrix}> \\ = \\ <\end{matrix} [v(x_3)-v(x_2)]/[v(x_2)-v(x_1)]$$

and the $\pi(.)$'s are all derived via an algorithm that operates entirely *within* their respective lotteries.

perceptions of these ratios may be influenced by considerations that are absent from theories such as EUT and CPT, with the result that observed patterns of response may deviate systematically from the patterns entailed by those models.

The next section discusses how we might allow for between-lottery comparisons *on the probability dimension*, and identifies the possible implications for a variety of decision scenarios involving just three payoffs. Section 3 will do something similar for the payoff dimension. But first, the probability dimension.

2. Modelling Probability Judgments

We start with one of the most widely replicated of all experimental regularities: the form of 'Allais paradox' that has come to be known as the 'common ratio effect' (CRE) – see Allais (1953) and Kahneman and Tversky (1979).

Consider the two pairwise choices shown in Figure 2.

Figure 2: An Example of a Pair of 'Common Ratio Effect' Choices

Choice #1

	1	
S_1	30	
R_1	40	0
	0.8	0.2

Choice #2

	0.25		0.75
S_2	30		0
R_2	40	0	
	0.2	0.8	

In terms of the template in Figure 1, $x_3 = 40$, $x_2 = 30$ and $x_1 = 0$. In Choice #1, $p_2 = 1$ (so that $p_3 = p_1 = 0$) while $q_3 = 0.8$, $q_2 = 0$ and $q_1 = 0.2$. Substituting these values into Expression (3), the implication of EUT is that

$$S_1 \overset{\succ}{\underset{\prec}{\sim}} R_1 \Leftrightarrow 0.2/0.8 \overset{>}{\underset{<}{=}} [u(40)-u(30)]/[u(30)-u(0)] \qquad (4)$$

Choice #2 can be derived from Choice #1 by scaling down the probabilities of x_3 and x_2 by the same factor – in this example, by a quarter – and increasing the probabilities of x_1 accordingly. Applying EUT as above gives

$$S_2 \begin{matrix} \succ \\ \sim \\ \prec \end{matrix} R_2 \iff 0.05/0.2 \begin{matrix} > \\ = \\ < \end{matrix} [u(40)\text{-}u(30)]/[u(30)\text{-}u(0)] \qquad (5)$$

The expression for the relative weight of argument for R versus S on the payoff dimension is the same for both (4) and (5) – i.e. [u(40)-u(30)]/[u(30)-u(0)]. Meanwhile, the expression for the relative weight of argument for S versus R on the probability dimension changes from 0.2/0.8 in (4) to 0.05/0.2 in (5). Since these two ratios are equal, the implication of EUT is that the balance of relative arguments is exactly the same for both choices: an EU maximiser should either pick S in both choices, or else pick R on both occasions

However, countless experiments using CRE pairs like those in Figure 2 find otherwise: many individuals violate EUT by choosing S_1 in Choice #1 and R_2 in Choice #2, while the opposite departure – choosing R_1 and S_2 – is relatively rarely observed. CPT can accommodate this asymmetry. To see how, consider the CPT versions of (4) and (5):

$$S_1 \begin{matrix} \succ \\ \sim \\ \prec \end{matrix} R_1 \iff [1\text{-}\pi(0.8)]/\pi(0.8) \begin{matrix} > \\ = \\ < \end{matrix} [v(40)\text{-}v(30)]/[v(30)\text{-}v(0)] \qquad (6)$$

$$S_2 \begin{matrix} \succ \\ \sim \\ \prec \end{matrix} R_2 \iff [\pi(0.25)\text{-}\pi(0.2)]/\pi(0.2)] \begin{matrix} > \\ = \\ < \end{matrix} [v(40)\text{-}v(30)]/[v(30)\text{-}v(0)] \qquad (7)$$

As with EUT, the relative argument on the payoff dimension is the same for both expressions. But the nonlinear transformation of the probabilities means that the relative strength of the argument for S versus R on the probability dimension decreases as we move from Choice #1 to Choice #2. Using the parameters estimated in Tversky and Kahneman (1992), $[1\text{-}\pi(0.8)]/\pi(0.8) \approx 0.65$ in (6) and

$[\pi(0.25)-\pi(0.2)]/\pi(0.2) \approx 0.12$ in (7). So any individual for whom $[v(40)-v(30)]/[v(30)-v(0)]$ is less than 0.65 but greater than 0.12 will choose S_1 in Choice #1 and R_2 in Choice #2, thereby exhibiting the 'usual' form of CRE violation of EUT. Thus this pattern of response is entirely compatible with CPT.

However, there may be other ways of explaining that pattern. This paper proposes an alternative account which gives much the same result *in this scenario* but which has quite different implications from CPT for some other cases.

To help set up the intuition behind this model, we start with Rubinstein's (1988) idea that some notion of *similarity* might explain the CRE, as follows[6]. In Choice #1, the two lotteries differ substantially on both the probability and the payoff dimensions; and although the expected value of 32 offered by R_1 is higher than the certainty of 30 offered by S_1, the majority of respondents choose S_1, a result which Rubinstein ascribed to risk aversion operating in such cases. However, the effect of scaling down the probabilities of the positive payoffs in Choice #2 may be to cause many respondents to consider those scaled-down probabilities to be *similar* (or "approximately the same", or "inconsequential")[7], and therefore respondents might pay less attention to them and give decisive weight instead to the dimension which remains *dis*similar – namely, the payoff dimension, which favours R_2 over S_2.

Similarity theory can be deployed to explain a number of other regularities besides the CRE (see, for example, Leland (1994), (1998)). However, one possible limitation might be the dichotomous nature of the similarity judgment: that is, above some (not very clearly specified) threshold, two stimuli are considered dissimilar and processed as under EUT; but below that threshold, they become similar and the difference between them is then regarded as inconsequential.

Nevertheless, the similarity notion reflects two thoughts: first, that the individual is liable to make between-lottery comparisons of probabilities; and second, that although the *objective ratio* of the relevant probabilities remains the same as both are scaled down by the same factor, the smaller *difference* between them in Choice #2 affects the *perception* of that ratio. The model in this paper incorporates those two ideas in a way that enables us to derive a number of new implications.

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[6] Tversky (1969) used a notion of similarity to account for violations of transitivity: these will be discussed in Section 3.

[7] Rubinstein (1988, p.148) acknowledged and appeared to agree with a suggestion by Margalit that the phrase 'is approximately the same as' might be better than 'is similar to'; and Leland (1998) has been inclined to refer to the 'inconsequentiality' of the difference.

When setting the model up, it makes the notation more compact if we replace q_1-p_1 by b_S (to signify that this is the pro**b**ability that S will give a better outcome) and correspondingly replace q_3-p_3 – the probability difference favouring R – by b_R. We also replace the pa**y**off advantage offered by R over S – that is, x_3-x_2 – by y_R, and replace the payoff advantage of S over R – namely, x_2-x_1 – by y_S. Finally, we model the '*perceived* relative argument for S compared with R on the probability dimension' as some function of b_S and b_R denoted by $\phi(b_S, b_R)$, and we correspondingly represent the '*perceived* relative argument for R compared with S on the payoff dimension' by some function of y_R and y_S denoted by $\xi(y_R, y_S)$. The general decision rule is that participants in experiments choose according to the balance of *perceived* arguments, so that

$$S \; \begin{matrix} \succ \\ \sim \\ \prec \end{matrix} \; R \quad \Leftrightarrow \quad \phi(b_S, b_R) \; \begin{matrix} > \\ = \\ < \end{matrix} \; \xi(y_R, y_S) \qquad (8)$$

Because this choice rule is at the heart of it, this model will be called the *perceived relative argument model* (PRAM).

Developing the between-lottery similarity insight, PRAM models $\phi(b_S, b_R)$ so that in common ratio scenarios where $b_S/b_R < 1$ – as is the case in the great majority of experiments to date – $\phi(b_S, b_R)$ becomes less and less consequential as b_S becomes smaller and smaller, even though the objective ratio remains constant. There may be a variety of functional forms that could correspond with this basic intuition about perception, but one which is fairly simple involves formulating $\phi(b_S, b_R)$ as follows:

$$\phi(b_S, b_R) = (b_S/b_R)^{(b_S + b_R)^{\alpha}} \quad \text{where } \alpha \leq 0 \qquad (9)$$

To repeat a point made at the beginning of Section 1, it is *not* being claimed that individuals consciously calculate the modified ratio according to (9), any more than the proponents of CPT claim that individuals actually set about calculating decision weights according to the somewhat complex rank-dependent algorithm in that model. What the CPT algorithm is intended to capture is the idea of some probabilities being underweighted and others being overweighted when individual lotteries are being evaluated, with this underweighting and overweighting tending to be systematically associated with payoffs according to their rank within the lottery. Likewise, what the formulation in (9) aims to capture is the idea that differences interact with ratios in a way which is consistent with *perceptions* of the relative weight of a ratio being influenced by between-lottery considerations.

To illustrate this, consider how (9) is applied to the CRE scenario in Figure 2. Notice first that when $\alpha = 0$, $(b_S+b_R)^{\alpha} = 1$, so that $\phi(b_S, b_R)$ is then simply b_S/b_R: that is, the perceived relative argument coincides with the objective ratio. On this reading, α might be thought of as a person-specific behavioural characteristic: someone for whom $\alpha = 0$ is someone who takes probabilities and their ratios just as they are, as EUT supposes; whereas someone for whom α is less than 0 is liable to have their judgment of ratios influenced by differences.

Notice also that when $b_S+b_R = 1$ (which means that probabilities are scaled up to their maximum extent), all individuals (whatever their α) perceive the ratio as it objectively is. This should not be taken too literally. The intention is *not* to insist that there is no divergence between perceived and objective ratios when the decision problem is as scaled-up as it can be. At this point, for at least some people, there *might* be some divergence in the opposite direction[8]. However, all the model needs to assume for present purposes is that when $b_S+b_R = 1$, the perceived relative argument for S versus R takes the objective ratio as its baseline value.

As the probabilities are scaled down, b_S+b_R falls. For a scenario like the one in Figure 2, where $b_S/b_R < 1$, this entails the value of $\phi(b_S, b_R)$ being lower in Choice #2 than in Choice #1. At the same time, since the set of payoffs is the same, $\xi(y_R, y_S)$ remains constant across these choices. So the effect of reducing $\phi(b_S, b_R)$ – that is, the effect of the probability advantage of S being perceived as smaller and more inconsequential – is that $\phi(b_S, b_R)$ may be greater than $\xi(y_R, y_S)$ in #1 but $\phi(b_S, b_R)$ may be less than $\xi(y_R, y_S)$ in #2. Thus PRAM also allows the non-EU combination of S_1 and R_2 so often observed, while disallowing the opposite combination of R_1 and S_2.

However, although PRAM and CPT have much the same implications for pairs of choices like those in Figure 2, there are other common ratio scenarios for

[8] In the original version of prospect theory, Kahneman and Tversky (1979) proposed that $p_2 = 1$ might involve an extra element – the 'certainty effect' – reflecting the possibility that certainty might be especially attractive; but CPT does not require any special extra weight to be attached to certainty and weights it as 1.

which they make opposing predictions. To see this, consider a different pair of choices. A 'scaled-up' Choice #3 involves S_3 offering 25 for sure – written (25, 1) – and R_3 offering a 0.2 chance of 100 and a 0.8 chance of 0, written (100, 0.2; 0, 0.8). Scaling down q3 and p2 to one-quarter of those values produces a Choice #4 between S_4 = (25, 0.25; 0, 0.75) and R_4 = (100, 0.05; 0, 0.95).

Under CPT, the counterpart of $\phi(b_S, b_R)$ is $[1-\pi(0.2)]/\pi(0.2)$ in Choice #3, while in Choice #4 it is $[\pi(0.25)-\pi(0.05)]/\pi(0.05)$. Using the transformation function from Tversky and Kahneman (1992), the value of $[1-\pi(0.2)]/\pi(0.2)$ in Choice #3 is approximately 2.85 while the value of $[\pi(0.25)-\pi(0.05)]/\pi(0.05)$ in Choice #4 is roughly 1.23. So individuals for whom $[v(x_3)-v(x_2)]/[v(x_2)-v(x_1)]$ lies between those two figures are liable to choose S_3 in Choice #3 and R_4 in Choice #4, thereby entailing much the same form of departure from EUT as in Choices #1 and #2.

However, in this case PRAM has the opposite implication. In the maximally scaled-up Choice #3, $\phi(b_S, b_R) = b_S/b_R = 0.8/0.2 = 4$. In Choice #4, the same b_S/b_R ratio is raised to the power of $(b_S+b_R)^{\alpha}$ where $b_S+b_R = 0.25$, and where $\alpha < 0$. The precise value of $\phi(b_S, b_R)$ depends on α, but for all $\alpha < 0$, reducing b_S+b_R increases the exponent on b_S/b_R above 1. So in scenarios like this, where $b_S/b_R > 1$, the effect of scaling down the probabilities is to give relatively *more* weight to b_S and relatively *less* to b_R, thereby *increasing* $\phi(b_S, b_R)$. This allows the possibility of someone choosing R_3 and S_4, but works against choosing S_3 and R_4. In other words, for scenarios where $b_S/b_R > 1$, PRAM has the *opposite* implication to CPT. The reason is that under these circumstances PRAM entails b_R becoming more and more inconsequential as it tends towards zero, in contrast with the assumption made by CPT, where the probability transformation function entails that low probabilities associated with high payoffs will generally be substantially *over*weighted.

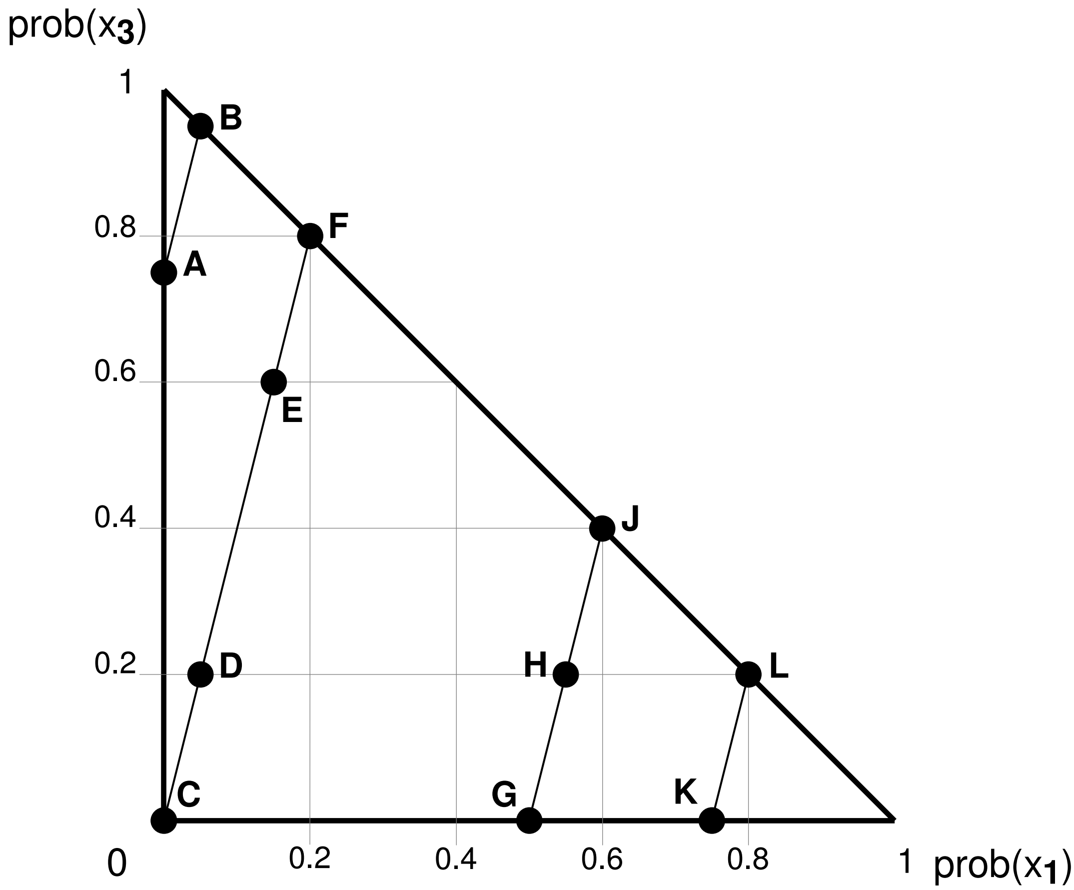
This suggests a straightforward test to discriminate between CPT and PRAM: namely, presenting experimental participants with scenarios involving choices like #3 and #4 which have $b_S/b_R > 1$ alongside choices like #1 and #2 where $b_S/b_R < 1$. Indeed, one might have supposed that such tests have already been conducted. But in fact, evidence from common ratio scenarios where $b_S/b_R > 1$ are thin on the ground and I have been unable to find any cases where this ratio is as high as 4, as in choices #3 and #4. Such evidence as I have found gives tentative encouragement to the PRAM prediction: for example, Battalio *et al.* (1990) report a study where their Set 2 (in their

Table 7) involved choices where $(x_3–x_2)$ = \$14 and $(x_2–x_1)$ = \$6 with b_S/b_R = 2.33. Scaling down by one-fifth resulted in 16 departures from EUT (out of a sample of 33), with 10 of those switching from R in the scaled-up pair to S in the scaled-down pair (in keeping with PRAM) while only 6 exhibited the 'usual' common ratio pattern. Another instance can be found in Bateman *et al.* (2006). In their Experiment 3, 100 participants were presented with two series of choices involving different sets of payoffs. In each set there were CRE questions where b_S/b_R was 0.25, and in both sets a clear pattern of the usual kind was observed: the ratio of S_1&R_2 : S_2&R_1 was 37:16 in Set 1 and 29:5 in Set 2. In each set there were also CRE questions where b_S/b_R was greater than 1 – although only 1.5 – and in these cases the same participants generated S_1&R_2 : S_2&R_1 ratios of 13:21 in Set 1 and 10:16 in Set 2. One can only speculate about whether the 'counter' asymmetries in the latter cases would have been stronger if the b_S/b_R ratio in these latter cases had been nearer to the inverse of the 0.25 ratio in the former cases.

So although the existing evidence in this respect is suggestive, it is too sparse to be conclusive; and as will become clear, there is also often a dearth of extant evidence in other areas where PRAM has different implications from CPT. Nevertheless, there is *some* evidence, and the remainder of this section will identify a number of other ways in which the implications of PRAM differ from those generated by existing theories.

When examining and comparing the implications of various decision theories for the kinds of choices that fit the Figure 1 template, many authors have found it helpful to represent such choices visually by using a Marschak-Machina (M-M) triangle – see Machina (1982) – as shown in Figure 3. The vertical edge of the triangle shows the probability of x_3 and the horizontal edge shows the probability of x_1. Any residual probability is the probability of x_2.

Figure 3: A Marschak-Machina Triangle



The eleven lotteries labelled A through L (letter I omitted) represent different combinations of the same set of $\{x_3, x_2, x_1\}$. So, for example, if those payoffs were, respectively, 40, 30 and 0, then C would offer the certainty of 30 while F would be (40, 0.8; 0, 0.2): that is, C and F would be S_1 and R_1 from Choice #1. Likewise, K = (30, 0.25; 0, 0.75) is S_2 in Choice #2, while L = (40, 0.2; 0, 0.8) is R_2 in that choice.

The gradient of the lines connecting the various lotteries is 4 in every case (i.e. the inverse of the ratio b_S/b_R, which is 0.25 for any pair of lotteries on the same line). An EU maximiser's indifference curves in any triangle are all straight lines with gradient $[u(x_2)-u(x_1)]/[u(x_3)-u(x_2)]$ – i.e. the inverse of y_R/y_S in the notation used above. So she will either always prefer the more south-westerly of any pair on the same line (if $b_S/b_R > y_R/y_S$) or else always prefer the more north-easterly of any such pair, with this applying to any pair of lotteries in the triangle connected by a line with gradient 4.

CPT also entails each individual having a well-behaved indifference map (i.e. all indifference curves with a positive slope at every point, no curves intersecting) but CPT allows these curves to be nonlinear. Although the details of any particular map will vary with the degree of curvature of the value function and the weighting function[9], the usual configuration can be summarised broadly as follows: indifference curves fan out as if from the south-west of the right-angle of the triangle, tending to be convex on the more south-easterly side of the triangle but more concave to the north-west, and particularly flat close to the right hand end of the bottom edge of the triangle while being much steeper near to the top of the vertical edge.

PRAM generates *some* implications which appear broadly compatible with that CPT configuration; but there are other implications which are quite different. To show this, Table 1 takes a number of pairs from Figure 3 and lists them according to the value of $\phi(b_S, b_R)$ that applies to each pair. The particular value of each $\phi(b_S, b_R)$ will depend on the value of α for the individual in question; but so long as $\alpha < 0$, we can be sure that the pairs will be ordered from lowest to highest $\phi(b_S, b_R)$ as in Table 1. This allows us to say how any such individual will choose, depending on where his $\xi(y_R, y_S)$ stands in comparison with $\phi(b_S, b_R)$. We do not yet need to know more precisely how $\xi(y_R, y_S)$ is specified by PRAM, except to know that it is a function of the three payoffs and is the same for all choices involving just those three payoffs[10].

Table 1: Values of $\phi(b_S/b_R)$ for Different Pairs of Lotteries from Figure 3

Value of $\phi(b_S/b_R)$	**Pair**
$0.25^{(0.25)^{\alpha}}$	**A** vs **B**, **C** vs **D**, **E** vs **F**, **G** vs **H**, **H** vs **J**, **K** vs **L**,
$0.25^{(0.50)^{\alpha}}$	**G** vs **J**,
$0.25^{(0.75)^{\alpha}}$	**C** vs **E**, **D** vs **F**,
$0.25^{(1)^{\alpha}}$	**C** vs **F**,

[9] In Tversky and Kahneman (1992), their Figure 3.4(a) shows an indifference map for the payoff set $\{x_3 = 200, x_2 = 100, x_1 = 0\}$ on the assumption that $v(x_i) = x_i^{0.88}$ and supposing that the weighting function estimated in that paper is applied.

[10] This requirement is met by EUT, where $\xi(y_R, y_S) = [u(x_3)-u(x_2)]/[u(x_2)-u(x_1)]$, and by CPT, where $\xi(y_R, y_S) = [v(x_3)-v(x_2)]/[v(x_2)-v(x_1)]$. Although the functional form for $\xi(y_R, y_S)$ proposed by PRAM is different from these, it will be seen in the next Section that the PRAM version of $\xi(y_R, y_S)$ is also constant for any particular set $\{x_3, x_2, x_1\}$.

So if an individual's $\xi(y_R, y_S)$ is lower than even the lowest value of $\phi(b_S, b_R)$ in the table – that is, lower than $0.25^{(0.25)^\alpha}$ – the implication is that $\phi(b_S, b_R) > \xi(y_R, y_S)$ for all pairs in that table, meaning that in every case the safer alternative – the one listed first in each pair – will be chosen. In such a case, the observed pattern of choice will be indistinguishable from that of a risk averse EU maximiser.

However, consider an individual for whom $\xi(y_R, y_S)$ is higher than the lowest value of $\phi(b_S, b_R)$ but lower than the next value on the list: i.e. the individual's evaluation of the payoffs is such that $\xi(y_R, y_S)$ is greater than $0.25^{(0.25)^\alpha}$ but less than $0.25^{(0.50)^\alpha}$. Such an individual will choose the safer (first-named) alternative in the choice between G and J and in all of the other choices lower down the table; but he will choose the riskier (second-named) alternative in every one of the pairs in the top row. This results in a number of patterns of choice which violate EUT; and although some of these are compatible with CPT, others are not.

First, $\xi(y_R, y_S)$ now lies in the range which produces the usual form of CRE – choosing C = (30, 1) over F = (40, 0.8; 0, 0.2), but choosing L = (40, 0.2; 0, 0.8) over K = (30, 0.75; 0, 0.25) . In fact, a $\xi(y_R, y_S)$ which lies anywhere between $0.25^{(0.25)^\alpha}$ and $0.25^{(1)^\alpha}$ will produce this pattern. This pattern is compatible with CPT.

Second, this individual is now liable to violate betweenness. Betweenness is a corollary of linear indifference curves which means that any lottery which is some linear combination of two other lotteries will be ordered between them. For example, consider C, D and F in Figure 3. D = (40, 0.2; 30, 0.75; 0, 0.05) is a linear combination of C and F – it is the reduced form of a two-stage lottery offering a 0.75 chance of C and a 0.25 chance of F. With linear indifference curves, as entailed by EUT, D cannot be preferred to *both* C and F, and nor can it be less preferred than *both* of them: under EUT, if C $\succ$ F, then C $\succ$ D and D $\succ$ F; or else if F $\succ$ C, then F $\succ$ D and D $\succ$ F. The same goes for any other linear combination of C and F, such as E = (40, 0.6; 30, 0.25; 0, 0.15). But PRAM entails violations of betweenness. In this case, the individual whose $\xi(y_R, y_S)$ lies anywhere above $0.25^{(0.25)^\alpha}$ and below $0.25^{(0.75)^\alpha}$ will a) choose D over *both* C and F, and b) will choose both C and F over E.

All these choices between those various pairings of C, D, E and F *might* be accommodated by CPT, although it would require the interaction of v(.) and the weighting function to generate an S-shaped indifference curve in the relevant region of the triangle. However, to date CPT has not been under much pressure to consider how to produce such curves: as with common ratio scenarios where $b_S/b_R > 1$, there is a paucity of experimental data looking for violations of betweenness in the vicinity of the hypotenuse – although one notable exception is a study by Bernasconi (1994) who looked at lotteries along something akin to the C-F line and found precisely the pattern entailed by the PRAM analysis.

A third implication of PRAM relates to 'fanning out' and 'fanning in'. As noted earlier, CPT indifference maps are usually characterised as generally fanning out across the whole triangle, tending to be particularly flat close to the right hand end of the bottom edge while being much steeper near to the top of the vertical edge. However, steep indifference curves near to the top of the vertical edge would entail choosing A over B, whereas PRAM suggests that any value of $\xi(y_R, y_S)$ greater than $0.25^{(0.25)^{\alpha}}$ will cause B to be chosen over A. In conjunction with the choice of C over F, this would be more in keeping with fanning out in more south-easterly part of the triangle but fanning *in* in the more north-westerly area. Again, there is rather less evidence about choices in the north-west of the triangle than in the south-east, but Camerer (1995) refers to some evidence consistent with fanning *in* towards that top corner, and in response to this kind of evidence, some other non-EU models – for example, Gul (1991) – were developed to have this 'mixed fanning' property[11].

Thus far, however, it might seem that the implications of PRAM are not *radically* different from what might be implied by CPT and other non-EU variants which, between them, *could* offer accounts of each of the regularities discussed above – although, as Bernasconi (1994, p.69) noted, it is difficult for any particular variant to accommodate *all* of these patterns via the *same* nonlinear transformation of probabilities into decision weights.

However, there is a further implication of PRAM which *does* represent a much more radical departure. Although the particular configurations may vary, what CPT and most of the other non-EU variants have in common is that preferences over the lottery space can be represented by indifference maps of *some* kind. Thus transitivity

[11] However, a limitation of Gul's (1991) 'disappointment aversion' model is that it entails linear indifference curves and therefore cannot accommodate the failures of betweenness that are now well-documented.

is intrinsic to all of these models. But what Table 1 allows us to see is that PRAM entails violations of transitivity.

When $\xi(y_R, y_S)$ lies above $0.25^{(0.25)^\alpha}$ and below $0.25^{(0.50)^\alpha}$, notice the implication for the pairwise choices involving G, H and J: the riskier alternatives are picked in the choices between G and H, and between H and J; but in the choice between G and J, the safer alternative is picked. Thus we have J $\succ$ H and H $\succ$ G which, in conjunction with transitivity, would imply J $\succ$ G; but so long as $\xi(y_R, y_S) <$ $0.25^{(0.50)^\alpha}$, the opposite choice will be made between G and J: that is, PRAM entails a violation of transitivity in the form of a cycle where J $\succ$ H and H $\succ$ G but G $\succ$ J.

Nor are such cycles confined to that case and that range of values for $\xi(y_R, y_S)$. When $\xi(y_R, y_S)$ lies above $0.25^{(0.50)^\alpha}$, the choice between G and J will switch and that particular cycle will no longer occur. Yet when $\xi(y_R, y_S)$ takes a higher value still – in this example, between $0.25^{(0.75)^\alpha}$ and $0.25^{(1)^\alpha}$ – PRAM entails more cycles, this time involving C, D, E and F. Table 1 indicates that when $\xi(y_R, y_S)$ lies in this range, the riskier alternative is chosen in every case except C vs F. So this means F $\succ$ E, E $\succ$ C, but C $\succ$ F. It also means F $\succ$ D, D $\succ$ C, but C $\succ$ F.

Indeed, if PRAM is modelling perceptions appropriately, it is easy to show that, for any triple of pairwise choices derived from three lotteries on the same straight line, there will always be *some* range of $\xi(y_R, y_S)$ that will produce a violation of transitivity in the form of a 'betweenness cycle'.

To see this, set x_3, x_2, x_1 and q_3 such that an individual acting according to PRAM is indifferent between L = (x_2, 1) and N = (x_3, q_3; x_1, 1-q_3). Denoting b_L/b_N by b, indifference entails $\phi(b_L, b_N) = \xi(y_N, y_L) = b^{(1)^\alpha} = b$. Since the value of $\xi(.,.)$ is determined by the set of the three payoffs, $\xi(.,.) = b$ for all pairs of lotteries defined over this set {x_3, x_2, x_1}. Now construct any linear combination M = (L, λ; N, 1-λ) where $0 < \lambda < 1$, and consider the pairwise choices {L, M} and {M, N}. Since M is on the straight line connecting L and N, $b_L/b_M = b_M/b_N = b$. Hence $\phi(b_L, b_M) = b^{(1-\lambda)^\alpha}$ and $\phi(b_M, b_N) = b^{(\lambda)^\alpha}$. With $0 < \lambda < 1$, this entails $\phi(b_L, b_M)$, $\phi(b_M, b_N) < b$ for all $b < 1$ and $\phi(b_L, b_M)$, $\phi(b_M, b_N) > b$ for all $b > 1$. Since $\xi(.,.) = b$ for all these pairings, the implication is either the triple N $\succ$ M, M $\succ$ L, but L ~ N when $b < 1$, or else L $\succ$ M, M $\succ$ N, but N ~ L when $b > 1$. As they stand, with L ~ N, these are weak violations of transitivity; but it is easy to see that by decreasing q_3 very slightly when $b < 1$ (and thereby producing L $\succ$ N), or by increasing q_3 enough when $b > 1$ to produce N $\succ$ L, strict violations of transitivity result for (nearly) all $0 < \lambda < 1$ and $\alpha < 0$.

The implication of betweenness cycles is one which sets PRAM apart from EUT and all non-EU models that entail transitivity. But is there any evidence of such cycles? Such evidence as there is comes largely as a by-product of experiments with other objectives, but there is at least *some* evidence. For example, Buschena and Zilberman (1999) examined choices between mixtures on two chords within the M-M triangle and found a significant asymmetric pattern of cycles along one chord, although not along the other chord. Bateman *et al.* (2006) also reported such asymmetries: these were statistically significant in one area of the triangle and were in the predicted direction, although not significantly so, in another area. Finally, re-analysis of an earlier dataset turns out to yield some additional evidence that supports this distinctive implication of PRAM. Loomes and Sugden (1998) asked 92 respondents to make a large number of pairwise choices, in the course of which they faced six 'betweenness triples' where $b < 1$ – specifically, {18, 19, 20}, {21, 22, 23}, {26, 27, 28}, {29,30,31}, {34, 35, 36} and {37, 38, 39} in the triangles labelled III-VI, where b ranged from 0.67 to 0.25. Individuals can be classified according to whether they a) exhibited no betweenness cycles, b) exhibited one or more cycles only in the direction consistent with PRAM, c) exhibited one or more cycles only in the opposite direction to that implied by PRAM, or d) exhibited cycles in both directions. 35 respondents never exhibited a cycle, and 11 recorded at least one cycle in both directions. However, of those who cycled only in one direction or the other, 38 cycled in the PRAM direction as opposed to just 8 who cycled only in the opposite direction. If both propensities to cycle were equally likely to occur by chance, the probability of the ratio 38:8 is less than 0.00001; and even if all 11 'mixed cyclers' were counted against the PRAM implication, the probability of the ratio 38:19 would still be less than 0.01.

So PRAM has a striking and distinctive implication concerning transitivity over lotteries within any given triangle. As will be shown in the next section, where the functional form of $\xi(y_R, y_S)$ is discussed in more detail, these implications are not limited to lotteries within particular triangles, and a broader class of systematic intransitivities can be accommodated. However, before expanding the discussion in that way, a broader implication of the modelling of $\phi(b_S, b_R)$ merits some attention.

It was noted earlier that when $b_S/b_R > 1$, PRAM entails the opposite of the CRE pattern associated with scenarios where $b_S/b_R < 1$. It was also noted above that when $b_S/b_R < 1$, PRAM entails betweenness cycles in one direction; but when $b_S/b_R > 1$, the expected direction of cycling is reversed. The two implications are particular manifestations of the more general point that moving from $b_S/b_R < 1$ to $b_S/b_R > 1$ has the effect of turning the whole of Table 1 upside down, thereby also reversing the direction of the violations of betweenness – i.e. D is now *less* preferred than both C and F rather than being preferred to both of them, while E is now liable to be picked over both C and F rather than being less preferred than both of them.

And there is one more implication. Consider what happens when the payoffs are changed from gains to losses (e.g., by putting a minus sign in front of each x_i). Applying this to Figure 2, the S lottery now involves a sure loss of 30 – that is, S = (-30, 1) – while R = (-40, 0.8; 0, 0.2). In this case, $b_S/b_R = 4$, so that the 'reverse CRE' is entailed by PRAM. Although there is a dearth of evidence about scenarios where $b_S/b_R > 1$ in the domain of gains, there is a good deal more evidence from the domain of losses, ever since Kahneman and Tversky (1979) reported the reverse CRE in their Problems 3' & 4', and 7' and 8', and dubbed this 'mirror image' result the 'reflection effect'. It is clear that PRAM also entails the reflection effect, not only in relation to CRE, but more generally, as a consequence of inverting the value of b_S/b_R when positive payoffs are replaced by their 'mirror images' in the domain of losses.

Finally, by way of drawing this section to a close, are there any well-known regularities within the M-M triangle that PRAM does *not* explain? It would be remarkable if a single formula on the probability dimension involving just one 'perception parameter' α were able to capture absolutely every well-known regularity as well as predicting several others. It would not be surprising if human perceptions were susceptible to more than just one effect, and there may be other factors entering into similarity judgments besides the one proposed here. For example, Buschena and Zilberman (1999) suggested that when all pairs of lotteries are transformations of some base pair such as {C, F} in Figure 3, the distances between alternatives in the M-M triangle would be primary indicators of similarity – which is essentially what the current formulation of $\phi(b_S, b_R)$ proposes (to see this, compare the distances in Figure

3 with the values of $\phi(b_S, b_R)$ in Table 1). However, Buschena and Zilberman modified this suggestion with the conjecture that if one alternative but not the other involved certainty or quasi-certainty, this might cause the pair to be perceived as less similar, and if two alternatives had different support, they would be regarded as more dissimilar.

An earlier version of this model (Loomes, 2006) proposed incorporating a second parameter (called β) into $\phi(b_S, b_R)$ with a view to capturing something of this sort, and thereby distinguishing between two pairs such as {C, D} and {K, L} which are equal distances apart on parallel lines. The effect of β was to allow C and D to be judged more dissimilar from each other than K and L, since C and D involved a certainty being compared with a lottery involving all three payoffs, whereas K and L involved two payoffs each. On this basis, with $b_S/b_R < 1$, the model allowed the combination of C $\succ$ D with K $\prec$ L, but not the opposite. And this particular regularity has been reported in the literature: it is the form of Allais paradox that has come to be known since Kahneman and Tversky (1979) as the 'common consequence effect': this effect is compatible with CPT, but if PRAM is restricted to the 'α-only' form of $\phi(b_S, b_R)$, there is no such distinction between {C, D} and {K, L} and this form of PRAM does not account for the common consequence effect.

So why is β not included in the present version? Its exclusion should not be interpreted as a denial of the possible role of other influences upon perceptions: on the contrary, as stated above, it would be remarkable if every aspect of perception on the probability dimension could be reduced to a single expression with just one free parameter. But in order to focus on the explanatory power provided by that single formulation, and to leave open the question of how best to modify $\phi(b_S, b_R)$ in order to allow for other effects on perception, there is an argument for putting the issue of a β into abeyance until we have more information about patterns of response in scenarios which have to date been sparsely investigated. If the α-only model performs well but (as seems likely) is insufficiently flexible to provide a full description of behaviour, the data collected in the process of testing may well give clues about the kinds of additional modifications that may be appropriate.

However, the more immediate concern is to extend the model beyond sets of decisions consisting of no more than three payoffs between them. To that end, the next section considers how perceptions might operate on the payoff dimension.

3. Modelling Payoff Judgments

As indicated in Expression (3) and footnote 3 above, the EUT and CPT ways of modelling 'the relative argument for R compared with S on the payoff dimension' are, respectively, $[u(x_3)-u(x_2)]/[u(x_2)-u(x_1)]$ and $[v(x_3)-v(x_2)]/[v(x_2)-v(x_1)]$. That is, these models, like many others, map from the objective money amount to an individual's subjective value of that amount via a utility or value function, and then suppose that the relative argument for one alternative against another can be encapsulated in terms of the ratio of the differences between these subjective values. So modelling payoff judgments may be broken down into two components: the subjective difference between any two payoffs; and how pairs of such differences are compared and perceived.

Consider first the conversion of payoffs into subjective values/utilities. It is widely accepted that – in the domain of gains at least – v(.) or u(.) are concave functions of payoffs, reflecting diminishing marginal utility and/or diminishing sensitivity. Certainly, if we take the most neutral base case – S offering some sure x_2, while R offers a 50-50 chance of x_3 or x_1 – it is widely believed that most people will choose S whenever x_2 is equal to the expected (money) value of R; and indeed, that many will choose S even when x_2 is somewhat less than that expected value – this often being interpreted as signifying risk aversion in the domain of gains. In line with this, PRAM also supposes that payoffs map to subjective values via a function c(.), which is (weakly) concave in the domain of gains[12]. To simplify notation, $c(x_i)$ will be denoted by c_i.

On that supposition, the basic building block of $\xi(y_R, y_S)$ is $(c_3-c_2)/(c_2-c_1)$, which is henceforth denoted by c_R/c_S. This is the counterpart to b_S/b_R in the specification of $\phi(b_S, b_R)$. So to put the second component of the model in place, we apply the same intuition about similarity to the payoff dimension as was applied to probabilities, and posit that the *perceived* ratio is liable to diverge more and more

[12] Actually, the strict concavity of this function, although it probably corresponds with the way most people would behave when presented with 50-50 gambles, is not necessary in order to produce many of the results later in this section, where a linear c(.) is sufficient. And since there are at least some commentators who think that the degree of risk aversion seemingly exhibited in experiments is surprisingly high – see, for example, Rabin (2000) – it may sometimes be useful (and simpler) to work on the basis of a linear c(.) and abstract from any concavity as a source of what may be interpreted as attitude to risk. The reason for using c(.) rather than u(.) or v(.) is to keep open the possibilities of interpretations that may differ from those normally associated with u(.) or v(.).

from the 'basic' ratio c_R/c_S the more different c_R and c_S become. Because the c_i's refer to payoffs rather than probabilities, there is no counterpart to b_S+b_R having an upper limit of 1. So, as a first and very simple way of modelling perceptions in an analogous way, let us specify $\xi(y_R, y_S)$ as:

$$\xi(y_R, y_S) = (c_R/c_S)^{\delta} \text{ where } \delta \geq 1 \tag{10}$$

Under both EUT and CPT, $\delta = 1$ (i.e. $c(.) = u(.)$ under EUT and $c(.) = v(.)$ under CPT). However, when $\delta > 1$, the bigger of c_R and c_S receives 'disproportionate' attention, and this disproportionality increases as c_R and c_S become more and more different. So in cases where $c_R/c_S > 1$, doubling c_R while holding c_S constant has the effect of more than doubling the perceived force of the relative argument favouring R. Equally, when $c_R/c_S < 1$, halving c_R while holding c_S constant weakens the perceived force of the argument for R to something less than half of what it was.

With $\xi(y_R, y_S)$ specified in this way, a number of results can be derived. In so doing, the strategy will be to abstract initially from any effect due to any nonlinearity of $c(.)$ by examining first the implications of setting $c_i = x_i$.

First, we can derive the so-called *fourfold pattern of risk attitudes* (Tversky and Kahneman, 1992) whereby individuals are said to be risk-seeking over low-probability high-win gambles, risk-averse over high-probability low-win gambles, risk-seeking over high-probability low-loss gambles and risk-averse over low-probability high-loss gambles.

This pattern is entailed by PRAM, even when $c(.)$ is assumed to be linear within and across gains and losses. To see this, start in the domain of gains and consider an R lottery of the form $(x_3, q_3; 0, 1\text{-}q_3)$ with the expected value x_2 (= $q_3.x_3$). Fix S = $(x_2, 1)$ and consider a series of choices with a range of R lotteries, varying the values of q_3 and making the adjustments to x_3 necessary to hold the expected value constant at x_2. Since all of these choices involve $b_S+b_R = 1$, $\phi(b_S, b_R) = (1–q_3)/q_3$. With $c_i = x_i$, we have $\xi(y_R, y_S) = [(x_3\text{-}x_2)/x_2]^{\delta}$. With $x_2 = q_3.x_3$, this gives $\xi(y_R, y_S) = [(1–q_3)/q_3]^{\delta}$, which can be written $\xi(y_R, y_S) = [\phi(b_S, b_R)]^{\delta}$. When $q_3 > 0.5$, $\phi(b_S, b_R)$ is less than 1 and so with $\delta > 1$, $\xi(y_R, y_S)$ is even smaller: hence S is chosen in preference to R, an observation that is conventionally taken to signify risk aversion.

However, whenever $q_3 < 0.5$, $\phi(b_S, b_R)$ is greater than 1 and $\xi(y_R, y_S)$ is bigger than $\phi(b_S, b_R)$, so that now R is chosen over S, which is conventionally taken to signify risk seeking. Thus we have the first two elements of the 'fourfold attitude to risk' – risk-aversion over high-probability low-win gambles and risk-seeking over low-probability high-win gambles in the domain of gains. And it is easy to see that if we locate R in the domain of losses, with q_3 now the probability of 0 and with the expected value of R held constant at $q_1.x_1 = x_2$, the other two elements of the fourfold pattern – risk-aversion over low-probability high-loss gambles and risk-seeking over high-probability low-loss gambles – are also entailed by PRAM.

The fact that these patterns can be obtained even when c(.) is linear breaks the usual association between risk attitude and the curvature of the utility/value function and suggests that at least part of what is conventionally described as risk attitude might instead be attributable to the way that the perceived relativities on the probability and payoff dimensions vary as the skewness of R is altered. If c(.) were nonlinear – and in particular, if it were everywhere concave, as u(.) is often supposed to be, the above results would be modified somewhat: when $q_3 = 0.5$ and $x_2/x_3 = 0.5$, $(c_3\text{-}c_2)/c_2 < 0.5$, so that S would be chosen over R for $q_3 = 0.5$, and might continue to be chosen for some range of q_3 below 0.5, depending on the curvature of c(.) and the value of δ. Nevertheless, it could still easily happen that below some point, there is a range of q_3 where R is chosen. Likewise, continuing concavity into the domain of losses is liable to move all of the relative arguments somewhat in favour of S, but there may still be a range of high-probability low-loss R which are chosen over S. In short, and in contrast with CPT, PRAM does use convexity in the domain of losses to explain the fourfold pattern.

Still, even if they reach the result by different routes, PRAM and CPT share the fourfold pattern implication. However, there is a related regularity where they part company: namely, the preference reversal phenomenon and the cycle that is its counterpart in pairwise choice. In the language of the preference reversal phenomenon (see Lichtenstein and Slovic, 1971, and Seidl, 2000) a low-probability high-win gamble is a \$-bet while a high-probability low-win gamble is a P-bet. The widely-replicated form of preference reversal occurs when an individual places a higher certainty equivalent value on the \$-bet than on the P-bet but picks the P-bet in a straight choice between the two. Denoting the bets by \$ and P, and their certainty equivalents as sure sums of money $CE_\$$ and CE_P such that $CE_\$ \sim \$$ and $CE_P \sim P$, the 'classic' and frequently-observed reversal occurs when $CE_\$ > CE_P$ but $P \succ \$$. The opposite reversal – placing a higher certainty equivalent on the P-bet but picking the \$-bet in a straight choice – is relatively rarely observed.

Let X be some sure amount of money such that $CE_\$ > X > CE_P$. Then the 'classic' preference reversal translates into the choice cycle $\$ \succ X$, $X \succ P$, $P \succ \$$. However, this cycle and the preference reversal phenomenon are both incompatible with CPT and other models which have transitivity built into their structure: if $\$ \succ X$ and $X \succ P$ – which is what the fourfold pattern entails when X is the expected value of the two bets – then transitivity requires $\$ \succ P$ in any choice between those two, and also requires that this ordering be reflected in their respective certainty equivalents. Any strong asymmetric pattern of cycles and/or any asymmetric disparity between choice and valuation cannot be explained by CPT or any other transitive model[13].

By contrast, PRAM entails both the common form of preference reversal and the corresponding choice cycle. To see this, consider Figure 4.

Figure 4: A {\$, P} Pair with Expected Value = X

	λq	$(1-\lambda)q$	$1-q$
\$	$X/\lambda q$	0	0
P	X/q	X/q	0

In line with the parameters of most preference reversal experiments, let the probabilities be set such that $1 > q > 0.5 > \lambda q > 0$. The case is simplest when both bets have $x_1 = 0$ and the same expected value, X. To simplify the exposition still further and show that the result does not require any nonlinearity of c(.), let $c_i = x_i$. We have already seen from the discussion of the fourfold pattern that under these conditions, when X is a sure sum equal to the expected value of both bets, $\$ \succ X$ and $X \succ P$. For a

[13] Kahneman and Tversky (1979) are very clear in stating that prospect theory is strictly a theory of pairwise choice, and they did not apply it to valuation (or other 'matching') tasks. In their 1992 exposition of CPT they repeat this statement about the domain of the theory, and to the extent that they use certainty equivalent data to estimate the parameters of their value and weighting functions, they do so by inferring these data from an iterative choice procedure. Strictly speaking, therefore, CPT can only be said to have an implication for choices – and in this case, choice cycles (which it does not allow). Other rank-dependent models make no such clear distinction between choice and valuation and therefore also entail that valuations should be ordered in the same way as choices.

cycle to occur, PRAM must also allow P $\succ$ \$. To see what PRAM entails for this pair, we need to derive $\phi(b_P, b_\$)$ and $\xi(y_\$, y_P)$.

Since $b_P = (1-\lambda)q$ and $b_\$ = \lambda q$, $\phi(b_P, b_\$) = \left((1-\lambda)/\lambda\right)^{(q)^{\alpha}}$ (11)

And since $y_\$ = [(1-\lambda)X/\lambda q]$ and $y_P = X/q$, $\xi(y_\$, y_P) = \left((1-\lambda)/\lambda\right)^{\delta}$ (12)

Thus the choice between P and \$ depends on whether λ is greater than or less than 0.5 in conjunction with whether q^{α} is greater than or less than δ. Since α and δ are person-specific parameters, consider first an individual whose perceptions are such that $q^{\alpha} > \delta \geq 1$. In cases where $\lambda > 0.5$ and therefore $(1-\lambda)/\lambda < 1$, such an individual will judge $\phi(b_P, b_\$) < \xi(y_\$, y_P)$ and will pick the \$-bet, so that no cycle occurs. But where $\lambda < 0.5$, that same individual will judge $\phi(b_P, b_\$) > \xi(y_\$, y_P)$ and will pick the P-bet, thereby exhibiting the cycle \$ $\succ$ X, X $\succ$ P, P $\succ$ \$. Since PRAM supposes that valuations are generated within the same framework and on the basis of the same person-specific parameters as choices, \$ $\succ$ X entails $CE_\$ > X$ and X $\succ$ P entails $X > CE_P$, that such an individual will also exhibit the classic form of preference reversal, $CE_\$ > CE_P$ in conjunction with P $\succ$ \$.

Next consider an individual whose perceptions are such that $\delta > q^{\alpha} \geq 1$. For such an individual, $\lambda < 0.5$ entails $\phi(b_P, b_\$) < \xi(y_\$, y_P)$ so that she will pick the \$-bet and no cycle will be observed. But in cases where $\lambda > 0.5$, she will judge $\phi(b_P, b_\$) >$ $\xi(y_\$, y_P)$ and will pick the P-bet, thereby exhibiting the cycle \$ $\succ$ X, X $\succ$ P, P $\succ$ \$. So although this individual will exhibit a cycle under different values of λ than the first individual, the implication is that any cycle she does exhibit will be in same direction – namely, the direction consistent with the classic form of preference reversal.

Thus under the conditions in the domain of gains exemplified in Figure 4, PRAM entails cycles in the expected direction but not in the opposite direction. On the other hand, if we 'reflect' the lotteries into the domain of losses by reversing the sign on each non-zero payoff, the effect is to reverse all of the above implications: now the model entails cycles in the opposite direction.

Besides the large body of preference reversal data (again, see Seidl, 2000) there is also empirical evidence of this asymmetric patterns of cycles – see, for example, Tversky, Slovic and Kahneman (1990) and Loomes, Starmer and Sugden (1991). In addition, the opposite asymmetry in the domain of losses was reported in Loomes & Taylor (1992).

Those last two papers were motivated by a desire to test regret theory (Bell, 1982; Fishburn, 1982; Loomes and Sugden, 1982 and 1987), which has the same implications as PRAM for these parameters. But the implications of regret theory and PRAM diverge under different parameters. To see this, scale all the probabilities of positive payoffs (including X, previously offered with probability 1) down by a factor p (and in the case of X, add a 1-p probability of zero) to produce the three lotteries shown in Figure 5.

Figure 5: A {\$', P', X'} Triple, all with Expected Value = p.X

	λpq	$(1-\lambda)pq$	$p(1-q)$	$1-p$
\$'	$X/\lambda q$	0	0	0
P'	X/q	X/q	0	0
X'	X	X	X	0

Since the payoffs have not changed, the values of $\xi(.,.)$ for each pairwise choice are the same as for the scaled-up lotteries. However, scaling the probabilities down changes the $\phi(.,.)$ values. In the choice between \$ and X when p = 1, $\phi(b_X, b_\$)$ is $\left((1-\lambda q)/\lambda q\right)^{(1)^\alpha}$ which reduces to $[(1-\lambda q)/\lambda q]$; and with $\lambda q < 0.5$, this is smaller than $\xi(y_\$, y_X) = [(1-\lambda q)/\lambda q]^\delta$ when $\delta > 1$. However, as p is reduced, p^α increases, and at the point where it becomes larger than δ, $\phi(b_{X'}, b_{\$'})$ becomes greater than $\xi(y_{\$'}, y_{X'})$ so that the individual now chooses X' over \$'. Likewise, when p = 1, the scaled-up X was chosen over the scaled-up P; but as p is reduced, $\phi(b_{X'}, b_{P'})$ falls and becomes smaller than $\xi(y_{P'}, y_{X'})$ at the point where p^α becomes larger than δ. So once this point is reached, instead of $\$ \succ X$ and $X \succ P$ we have $P' \succ X'$ and $X' \succ \$'$.

Whether a 'reverse' cycle is observed then depends on the choice between \$' and P'. Modifying and combining Expressions (11) and (12) we have

$$P' \begin{matrix}\succ\\ \sim\\ \prec\end{matrix} \$' \quad \Leftrightarrow \quad \phi(b_{P'}, b_{\$'}) = \left((1-\lambda)/\lambda\right)^{(pq)^{\alpha}} \begin{matrix}>\\ =\\ <\end{matrix} \left((1-\lambda)/\lambda\right)^{\delta} = \xi(y_{\$'}, y_{P'}) \qquad (13)$$

so that S' will be chosen in cases where $(1-\lambda)/\lambda > 1$ and $(pq)^{\alpha} > \delta$. In such cases – and $(1-\lambda)/\lambda > 1$ is typical of many preference reversal experiments – the result will be the cycle P' $\succ$ X', X' $\succ$ \$', \$' $\succ$ P'. The opposite cycle will not occur once p has fallen sufficiently to produce $p^{\alpha} > \delta$ (although, of course, the value of p at which this occurs may vary greatly from one individual to another).

Such 'similarity cycles' were reported by Tversky (1969) and were replicated by Lindman and Lyons (1978) and Budescu and Weiss (1987). More recently, Bateman *et al.* (2006) reported such cycles in two separate experiments with rather different payoff parameters than those used by Tversky. Those experiments had been designed primarily to explore the CRE, and the data concerning cycles were an unintended by-product. Even so, there were four triples that fitted the Figure 5 format and in all four of these, similarity cycles outnumbered cycles in the opposite direction to a highly significant extent.

Such an asymmetry is contrary to the implications of regret theory[14]. However, as shown earlier, PRAM not only entails similarity cycles in scaled-down choices but also entails the opposite asymmetry in scaled-up choices – a predominance of what might be called 'regret cycles'. Moreover, this novel and rather striking implication of the model turns out to have some empirical support. Following the first two experiments reported in Bateman *et al.* (2006), a third experiment was conducted in which every pairwise combination of four scaled-up lotteries, together with every pairwise combination of four scaled-down lotteries, were presented in conjunction with two different λ sets of payoffs. All these choices were put to the same individuals in the same sessions under the same experimental conditions. The results are reported in Day and Loomes (2006): there was a clear tendency for regret cycles to

[14] More will be said about this in the discussion in Section 4.

predominate when the lotteries were scaled up, while there was a strong asymmetry favouring similarity cycles among the scaled-down lotteries.

There is a variant upon this last result for which some evidence has recently appeared. Look again at X' in Figure 5: it is, in effect, a P-bet. Likewise, P' from Figure 5 could be regarded as a $-bet. Finally, let us relabel $' in Figure 5 as Y, a 'yardstick' lottery offering a higher payoff – call it x* – than either of the other two. Instead of asking respondents to state certainty equivalents for P and $, we could ask them to state probability equivalents for each lottery – respectively, PE_P and $PE_\$$ – by setting the probabilities of x* that would make them indifferent between that lottery and the yardstick[15]. If, for some predetermined probability (such as λpq in Figure 5), the individual exhibits a 'similarity cycle' Y $\succ$ $, $ $\succ$ P, P $\succ$ Y, then the probability equivalence task requires setting the probability of x* at something less than λpq in order to establish $PE_\$ \sim S$, while it involves setting the probability of x* at something greater than λpq in order to generate $PE_P \sim P$. Thus for valuations elicited in the form of probability equivalents, PRAM entails the *opposite* of the classic preference reversal i.e. it allows the possibility of $PE_P > PE_\$$ in conjunction with $ $\succ$ P. A recent study by Butler and Loomes (2007) reported exactly this pattern: a substantial asymmetry in the direction of 'classic' preference reversals when a sample of respondents gave certainty equivalents for a particular {$, P} pair; and the opposite asymmetry when that same sample were asked to provide probability equivalents for the very same {$, P} pair.

There are other implications of PRAM omitted for lack of space[16], but the discussion thus far is sufficient to show that PRAM is not only fundamentally different from CPT and other non-EU models that entail transitivity but also that it diverges from one of the best-known nontransitive models in the form of regret theory. This may therefore the moment to focus attention on the essential respects in which PRAM differs from these other models, and to consider in more detail the

[15] Such tasks are widely used in health care settings where index numbers (under EUT, these are the equivalent of utilities) for health states lying somewhere between full health and death are elicited by probability equivalent tasks, often referred to as 'standard gambles'.

[16] In the earlier formulation of the model (Loomes, 2006) some indication was given of the way in which the model could accommodate other phenomena, such as Fishburn's (1988) 'strong' preference reversals. Reference was also made to possible explanations of violations of the reduction of compound lotteries axiom and of varying attitudes to ambiguity. Details are available from the author on request.

possible lessons not only for those other models but for the broader enterprise of developing decision theories and using experiments to try to test them.

4. Relationship With, And Implications For, Other Models

The discussion so far has focused principally on the way that PRAM compares with and diverges from EUT and from CPT (taken to be the 'flagship' of non-EU models), with some more limited reference to other variants in the broad tradition of 'rational' theories of choice. In the paragraphs immediately below, more will be said about the relationship between PRAM and these models. However, as noted in the introduction, PRAM is more in the tradition of psychological/behavioural models, and in the latter part of this section there will be a discussion of the ways in which PRAM may be seen as building upon, but differentiated from, those models.

First, the most widely used decision model, EUT, is a special case of PRAM where $\alpha = 0$ and $\delta = 1$. This means that individuals are assumed to act as if all differences and ratios on both the probability and utility dimensions are perceived and processed exactly as they are, save only for random errors. PRAM shows that once we allow interactions which affect the judgments and perceptions of these ratios, many implications of EUT fail descriptively.

However, the ability of alternative models to accommodate such failures may also be limited by the extent to which they rule out such interactions. So CPT fails for two main reasons. First, although it replaces u(.) by v(.), it makes essentially the same assumption in terms of a consequence carrying its assigned value into every scenario, with differences and ratios between those values being processed independently and exactly as they are – that is, as if $\delta = 1$. So the kinds of choice cycles described above as 'regret' and 'similarity' cycles cannot be accounted for. Second, although CPT and other rank-dependent models allow probabilities to be transformed nonlinearly, and can even assign the same probability a different weight depending on its 'rank' within a lottery and the magnitudes of the other probabilities in that same lottery, CPT disallows any *between*-lottery influences on this transformation[17].

[17] It is interesting to consider why CPT, a model that was initially inspired by insights about psychology and psychophysics, should permit effects from comparisons *within* a lottery but – even though it is explicitly a theory of *pairwise* choices – should disallow such effects *between* lotteries. The answer *may* be found in the evolution of the model. The original (1979) form of prospect theory made no such within-lottery comparisons: probabilities were simply converted via a nonlinear transformation function. But this had the result that the weights generally did not add up to 1, which allowed effects that were regarded as normatively undesirable or behaviourally implausible and that had to be

Other models can achieve some CPT-like results by a different within-lottery route: for example, disappointment theory (Bell, 1985; Loomes and Sugden, 1986) keeps probabilities as they are, but allows within-lottery interactions between payoffs in ways which can accommodate certain violations of independence. However, what rank-dependent models and disappointment theory have in common is that they effectively assign 'scores' to each lottery as a whole which that lottery carries with it into every choice and valuation task. In short, by restricting such interactions to within-lottery comparisons and ruling out any between-lottery effects, these models cannot account for betweenness cycles which undermine the existence of any well-behaved indifference map within the Marschak-Machina triangle.

By contrast, regret theory allows between-lottery comparisons – but only on the payoff dimension. Essentially, it modifies the utility of any one payoff on the basis of the other payoff(s) offered by other lotteries under the same state of the world. In the 1987 formulation of regret theory, the *net advantage* of one payoff over another is represented by the $\psi(.\,,.)$ function, which is assumed to be strictly convex, so that for all $x_3 > x_2 > x_1$, $\psi(x_3, x_1) > \psi(x_3, x_2) + \psi(x_2, x_1)$[18]. This enables the model to accommodate regret cycles, classic preference reversals and some violations of independence (although these latter require the additional assumption of statistical independence between lotteries). However, regret theory does not allow for any between-lottery interactions on the probability dimension – in fact, it takes probabilities exactly as they are – and therefore cannot account for violations of the sure-thing principle, nor similarity cycles, nor betweenness cycles under assumptions of statistical independence[19].

Many non-EU models of the kind referred to above – and especially those designed to appeal to an audience of economists – have been influenced by the desire to meet criteria of rationality and/or generality and have therefore tried to minimise

controlled by other means such as an 'editing' phase to spot them and eliminate them. The rank-dependent procedure was a later development, proposed as a way of closing such 'gaps' and guaranteeing respect for dominance and transitivity. But the latter goal is driven more by normative precepts than by psychological insight; and this 'arranged marriage' between the various insights and goals may be seen as the reason why CPT ends up in a 'halfway house' position when viewed from the PRAM perspective.

[18] Notice the resemblance to the PRAM formulation. If one were to take the differences between the pairs of payoffs and put them over any common denominator Z to get a measure of the relative force of each difference, PRAM would imply the same inequality i.e. $[(x_3-x_1)/Z]^\delta > [(x_3-x_2)/Z]^\delta + [(x_2-x_1)/Z]^\delta$ for all $\delta > 1$.

[19] Regret theory *can* produce cycles over triples involving a set of just three payoffs by manipulating the juxtaposition of those payoffs. Such 'juxtaposition effects' – see, for example, Loomes (1988) – can also be shown to be implied by PRAM. Details can be obtained from the author on request.

departures from the baseline of EUT and to invoke alternative axioms or principles driven by normative considerations. However, if there are between-lottery interactions operating on perceptions in the way modelled by PRAM, those axioms are bound to be transgressed. Thus any such model will fail in one way or another to accommodate the evidence and/or will need to invoke certain supplementary assumptions or forms of special pleading to try to cope with those data.

Models from a more psychological/behavioural may be less encumbered by such rigidities. Nevertheless, as discussed below, when such models are viewed from a PRAM perspective, it turns out that they too have imposed certain assumptions which limit their capacity to account for the evidence – except by invoking special additional assumptions of their own.

For example, Shafir *et al.* (1993) proposed an ‘advantage’ model (AM) which accommodates *some* departures from EUT and shares *some* insights with PRAM. However, that model was concerned exclusively with choices between binary lotteries and money or probability equivalences for such lotteries. Thus it does not address tasks where one or both lotteries have more than two payoffs, which necessarily limits its scope relative to PRAM: by its nature, it does not deal with any lotteries in the interior of the M-M triangle, and therefore cannot deal with violations of betweenness or betweenness cycles. In addition, AM invokes different parameters for gains and losses, and calls on an additional principle, denoted by (*) – see their p.336 – to allow each of those parameters to vary further according to the nature of the task. This is in contrast with PRAM, which applies the same person-specific parameters across the board to all choice and equivalence tasks.

To see why AM needs to invoke different parameters and principles for different situations, consider how that model handles the most basic choice problem. Adapting AM to the notation used in the current paper, the simplest choice involves a pair $S = (x_2, p_2)$ and $R = (x_3, q_3)$ where $x_3 > x_2$ and $p_2 > q_3$ and where the expected money values are, respectively, $EMV_S = x_2 \times p_2$ and $EMV_R = x_3 \times q_3$. The AM choice rule is then:

$$S \overset{\succ}{\underset{\prec}{\sim}} R \;\Leftrightarrow\; EMV_S(p_2-q_3) \overset{>}{\underset{<}{=}} EMV_R[(x_3-x_2)/x_3]k_G \qquad (14)$$

where k_G is a weight representing the relative importance placed upon the payoff and probability advantages. In simple choices, the expectation is that most people will place more weight on probabilities than payoffs, so including k_G in the payoff part of the expression suggests $k_G < 1$. When simple choices involve losses rather than gains, a different weight k_L is used instead. Supplementary principle (*) invokes 'compatibility' in equivalence tasks, so that the same person's *k*'s may be different for money equivalences than for straight choices, and different again for probability equivalences. And while, as stated earlier, I do not deny that such additional considerations may come into play, PRAM does not require them in order to accommodate the evidence, whereas without them the explanatory power of AM is greatly reduced.

The reason why AM is relatively limited and why it therefore needs supplementary assumptions may be found by examining the restrictions on PRAM implicit in Expression (14). EMV_S is weighted by the *simple difference* between probabilities and the interaction between difference and ratio, which is crucial to the PRAM modelling of the *perceived* relative argument favouring S, is absent from (14). So although AM can accommodate the 'usual' common ratio effect when $q_3/p_2 \geq 0.5$, applying it to cases where q_3/p_2 is considerably less than 0.5 would entail an even stronger 'fanning out' pattern, whereas the data (and PRAM) suggest that the usual effect is moderated or even reversed in such cases. And while AM can (just about) accommodate Tversky's (1969) similarity cycles, it can only do so by invoking a value of k_G "somewhat outside the common range, which is compatible with the fact that it [Tversky's evidence] characterizes a pre-selected and therefore somewhat atypical group of subjects" (Shafir et al., 1993, p.351). However, examples of similarity cycles reported in Bateman et al. (2006) cannot be accounted for[20]. Meanwhile, explaining the typical form of preference reversal requires (*) to be invoked to allow a rather different k_G to be used for valuation than for choice because AM is not generally compatible with the kinds of *choice cycles* that mimic the preference reversal phenomenon. This limitation relative to PRAM appears to stem

[20] For example, in Experiment 2 described there, participants chose between pairs of lotteries {P, R}, {R, S} and {P, S} where the three lotteries were: P = (£25, 0.15); R = (£15, 0.20); S = (£12, 0.25). Out of 21 participants (from a total of 149 in the sample) who exhibited choice cycles, 20 were in the 'Tversky' direction i.e. $P \succ R$, $R \succ S$, but $S \succ P$. However, there is no value of k_G compatible with this cycle. In particular, $R \succ S$ requires $k_G > 0.25$, while $S \succ P$ requires $k_G < 0.154$. So although there may be some cycles compatible with AM, there are also some very strong asymmetric patterns which the model does not readily accommodate.

from the fact that the $[(x_3-x_2)/x_3]$ term on the right hand side of (14) does not actually use the ratio of relative advantages (which would require the denominator to be x_2) and does not allow for the perceptual effects represented in PRAM by raising the ratio to the power δ. In the absence of modelling that effect, AM tries to compensate with a combination of (*) and k_G.

The 'contrast-weighting' model proposed by Mellers and Biagini (1994), with its emphasis on the role of similarity, is closer in both spirit and structure to PRAM. The key idea is that similarity between alternatives along one dimension/attribute tends to magnify the weight given to differences along the other dimension(s). The model is framed in terms of *strength of preference* for one option over another. Applied to a pair of lotteries where $S = (x_2, p_2; 0, 1\text{-}p_2)$ and $R = (x_3, q_3; 0, 1\text{-}q_3)$ and where $x_3 > x_2$ and $p_2 > q_3$, the judged strength of preference for S over R is given by $u(x_2)^{\alpha(p)}.\pi(p_2)^{\beta(x)} - u(x_3)^{\alpha(p)}.\pi(q_3)^{\beta(x)}$ where u(.) gives the utilities of the payoffs and π(.) represents the subjective probabilities of receiving those payoffs, while α(p) and β(x) are, respectively, the contrast weights applied as exponents to those indices.

To make the comparison with PRAM easier to see, let us suppose that choice between S and R maps to strength of preference in an intuitive way, so that S is chosen when strength of preference for S over R is positive and R is chosen when that strength of preference is negative. On that basis, and with some straightforward rearrangement, we have:

$$S \begin{matrix} \succ \\ \sim \\ \prec \end{matrix} R \iff \left(\frac{\pi(p_2)}{\pi(q_3)}\right)^{\beta(x)} \begin{matrix} > \\ = \\ < \end{matrix} \left(\frac{u(x_3)}{u(x_2)}\right)^{\alpha(p)} \qquad (15)$$

which might be read as saying that the choice between S and R depends on whether the strength of preference favouring S on the probability dimension is greater than, equal to, or less than the strength of preference favouring R on the payoff dimension. Put into this form, it is easier to identify the difference between PRAM and this contrast weighting (CW) model. PRAM expresses the basic ratio of arguments within each dimension in a form which can take values less than or greater than 1 (depending on the relative magnitudes of advantage within a dimension) and then expresses the perception of each ratio as a continuous nonlinear function which reflects interactions between ratios and differences. The CW model proposed in Mellers and Biagini (1994) takes its exponents $\alpha(p)$ and $\beta(x)$ as depending just on the *absolute differences* between p_2 and q_3 and between x_3 and x_2, and as taking one of just two values – one when differences are 'small' and the two indices in question are judged 'similar' and another when differences are 'large' and the two indices are judged 'dissimilar'. In this respect, the CW model has much in common with the similarity analysis suggested by Rubinstein (1988) and Leland (1994, 1998), using a dichotomous similar/dissimilar judgment. However, it was precisely in order to overcome the limitations of such a formulation and to allow many more diverse applications that PRAM was developed. Mellers and Biagini note on p.507 that "a more general representation would allow weights that are a continuous function of the absolute difference along a dimension", but they do not provide such a representation. PRAM might be seen as developing the CW/similarity insights broadly in the direction which Mellers and Biagini considered would be useful.

A somewhat different line of development was pursued by Gonzalez-Vallejo (2002). The primary focus of that paper was to embed a deterministic similarity 'core' in a stochastic framework. Using the terminology from that paper, the deterministic difference between two alternatives is denoted by d, and the decision maker chooses the option with the deterministic advantage if and only if $d \geq \delta + \varepsilon$, where δ is a 'personal decision threshold' and ε is a value representing noise/random disturbance, drawn from a distribution with zero mean and variance σ^2.

For the pair of basic lotteries $S = (x_2, p_2; 0, 1\text{-}p_2)$ and $R = (x_3, q_3; 0, 1\text{-}q_3)$ where $x_3 > x_2$ and $p_2 > q_3$, Gonzalez-Vallejo's Equation (3) gives the deterministic term as $d = [(p_2\text{-}q_3)/p_2] – [(x_3\text{-}x_2)/x_3]$. In this formulation, S is preferred to / indifferent to / less preferred than R according to whether the *proportional* advantage of S over R on the probability dimension is greater than, equal to or less than the *proportional* advantage of R over S on the payoff dimension – with, in both cases, this proportion being the difference expressed as a fraction of the higher value. Because of the centrality of the difference between these proportions, Gonzalez-Vallejo calls this the proportional difference (PD) model.

Notice that, as expressed here, PD effectively says that the deterministic component amounts to a preference for the alternative with the higher expected money value. If the money values of the payoffs were replaced by their von

Neumann-Morgenstern utilities, the deterministic component would amount to a 'core' preference for the alternative with the higher EU; and if payoffs were mapped via v(.) to a value function and probabilities were converted to decision weights in the manner proposed by rank-dependent models, the core preference would correspond with CPT or some other rank-dependent variant. So departures from expected value / expected utility / subjective expected value maximisation models are accounted for by PD in terms of the way that an individual's decision threshold δ departs from 0.

In this respect, δ plays a role not unlike that played by k_G in Shafir *et al.* (1993). And as with AM, the only way the PD model can accommodate a wide variety of different regularities is by allowing δ to vary from one regularity to another. A particular problem caused by the proportionality at the core of this model is that scaling down p_2 and q_3 by the same factor leaves d unchanged, so that the usual CRE would require δ to change systematically according to the scaling of the probabilities. That would also be required in order to allow both similarity cycles and regret cycles to be accommodated[21]. Likewise, the 'fourfold attitude to risk' patterns would require not only the size but also the sign of δ to change from one choice to the next: choosing a small-probability high-payoff lottery over a sure sum with the same EMV (i.e. where d = 0) requires a δ that favours the payoff proportion, whereas choosing the same sure sum over a large-probability moderate-payoff lottery with the same EMV (so that d is still 0) requires a δ of the opposite sign. In short, to accommodate a wide variety of different departures from EV/EU maximisation, we need PD to specify how δ varies from one set of tasks and parameters to another. Gonzalez-Vallejo does not provide such a theory. Arguably, PRAM makes such a theory unnecessary, since it accounts for the diverse effects within the same 'core' specification.

The other issue addressed by Gonzalez-Vallejo (2002) and in a different way by Mellers and Biagini (1994) is the stochastic nature of actual choice behaviour. Although Gonzalez-Vallejo's approach to this was to use a standard Fechnerian error term, that is not the only way of incorporating a stochastic element into choice behaviour: as discussed by Loomes and Sugden (1995), a 'random preference'

[21] In addition, explaining the choice cycles analogous to classic preference reversals (if it could be done at all) would typically require a positive sign on δ (because the riskier lotteries generally have higher EVs) whereas the δ needed to explain Tversky's similarity cycle (Gonzalez-Vallejo, 2002, p.143) was negative (with EVs falling as the lotteries became riskier).

specification, in the spirit of Becker, DeGroot and Marschak (1963) may be an alternative route to take. However, a comprehensive discussion of the strengths and weaknesses of different variants of 'error' specification, as well as the issues raised for fitting models and testing hypotheses, could constitute a whole new paper, and is beyond the scope of the present enterprise[22]. Suffice it to say that PRAM *could* be adapted to either approach, but the incorporation of a stochastic element by allowing any individual's behaviour, as well as any sample's behaviour, to be modelled in terms of some distribution over both α and δ would appear to be a route that could be profitably investigated in future research. Meanwhile, taking a deterministic form of PRAM as reflecting some 'central tendency' values of α and δ is sufficient for the purposes of the present paper.

More recently still, Brandstatter et al. (2006) have proposed a 'priority heuristic' (PH) model to explain a number of regularities. Whereas most of the models discussed above say little or nothing about the order in which people process a choice or valuation task, PH suggests a sequence of comparisons of features of a problem with stopping and decision rules at each stage. On this basis, the PH model can accommodate a number of the well-known regularities in choice. But it turns out to be poor at dealing with some patterns that seem easy to predict just by looking at them and offers no guidance about equivalence judgments.

The essence of the problem here is encapsulated in the second part of the title of the paper: "making choices without trade-offs". A rule is either satisfied or it is not, and this dichotomous structure of the model causes it to neglect more holistic considerations, which can then only be dealt with by invoking another heuristic. As the authors acknowledge (p.425-6), PH's predictive power is poor in cases involving large discrepancies between expected values working in the opposite direction to the PH sequence of rules[23]. This reflects the model's lack of a trade-off mechanism that would allow such expected value differentials to play a suitably weighted role. In the absence of such a mechanism, PH also offers no obvious way of handling equivalence tasks, despite the fact that participants seem perfectly able to make such judgments. Although this issue is not addressed by Brandstatter et al., one supposes that

[22] Loomes (2005) discusses the differences between various kinds of 'error' model and shows how the appropriate null and alternative hypotheses may be quite different, depending on the error model used.
[23] An example given on p.425 involves a choice between A = (88, 0.74) and B = (19, 0.86) where PH predicts choosing B but where the majority of the sample actually picked A, whose expected value is four times that offered by B. Similar failures were apparent in a number of other pairs.

equivalences would require a further set of rules. It would be interesting to see what such a set would entail, how it would relate to the choice rules – and how well it would be able to accommodate the conjunctions between certainty equivalents, probability equivalents and choices discussed above. PRAM requires no such additional set(s) of rules/principles: the appropriate trade-offs are intrinsic to the model, and the same two free parameters can be applied equally well to the various equivalences as to pairwise choices.

5. Concluding Remarks

The past thirty years have seen the development of an array of 'alternative' theories which try in different ways to account for the many well-established regularities observed in individual decision experiments: see Starmer (2000) for a review of "the hunt for a descriptive theory of choice under risk"; and Rieskamp *et al.* (2006) for a review from a more psychological perspective.

However, no single theory has so far been able to organise more than a (fairly limited) subset of the evidence. This has been something of a puzzle, because all of the regularities in question are generated by the same kinds of people. In fact, in some experiments, the very same group of individuals exhibit many of them one after the other in the same session. So it would seem that there really ought to be a single model of individual decision making under risk that is able to account for *most if not all* of them.

It has been argued above that PRAM (or something very much like it) offers a solution to that puzzle by representing the way that many participants make pairwise choices and judge equivalences in cases where there are no more than three payoffs – this being the nature of the great majority of experimental designs. Using some simple propositions about perception and judgment, PRAM shows how a typical sample of participants may, between them, be liable to exhibit *all* of the following regularities: the common ratio effect; violations of betweenness; betweenness cycles; the reflection effect and 'fourfold' attitudes to risk; 'similarity' cycles; 'regret' cycles; and preference reversals involving both certainty and probability equivalences. Moreover, all of these results were generated without requiring any curvature of c(.), nor any special assumptions about framing effects, reference points, failures of procedural invariance, and so on.

However, the development of alternative decision theories during the past thirty years has often been influenced by the desire to incorporate/defend particular assumptions or axioms for normative reasons. But if the experimental data are actually generated by PRAM-like perceptions influenced by between-lottery comparisons, any model which disallows such between-lottery influences on either or both dimensions is liable to fail descriptively. The data simply will not fit such theories, and the price to be paid for trying to force them into the wrong mould is that various supplementary assumptions or forms of special pleading have to be invoked and/or that the estimates arising from fitting such mis-specified models could be seriously misleading.

On the other hand, it has to be acknowledged that although pairwise comparisons involving no more than three payoffs have been the staple diet of individual decision experiments, they are only a small subset of the kinds of risky decisions which are of interest to psychologists, economists and decision theorists. What if the kinds of between-lottery effects modelled by PRAM are specific to – or at least, particularly pronounced in – these two-alternative three-payoff cases? If this is the case, how far can we extrapolate from these data to other scenarios?

For example, suppose we want a model which organises behaviour when decision makers are choosing between a larger number of more complex risky prospects. Perhaps the types of pairwise comparisons modelled by PRAM are less important in such cases: indeed, perhaps they are superseded altogether by other judgmental considerations. It *might* be that a model which fails on almost every front in the special class of experimental pairwise choices could do much better in other scenarios which bring additional and/or different judgmental processes into play[24]. This raises the possibility that the usefulness of any particular theory as a descriptive model of decision behaviour may depend on the characteristics of the class of problems to which it is being applied; and different models may be more or less successful in different kinds of scenarios. At the very least, this points to a need for experimental research to pay more attention not only to other areas of the M-M triangle and to choices connected by lines with different gradients within that triangle, but also to choices involving more complex lotteries and/or larger choice sets.

[24] There is some tentative support for this suggestion in Bateman et al. (2006) which shows that when participants were asked to rank larger sets of prospects, the usual CRE pattern, which has been so widely and strongly found in pairwise choice designs, was greatly attenuated.

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