The Market for Standard Essential Patents

Marc Bourreau[*], Yann Ménière[†], Tim Pohlmann[‡]

January 4, 2015

Abstract

In this paper, we develop a theoretical framework to analyze the incentives of Standard Essential Patents (SEPs) owners to trade SEPs. We first show that the size of a SEP portfolio determines an owner's ability to charge royalties to standard implementors. We then study the causes and consequences of SEP trading.

Keywords: Standard Essential Patents; Patent Portfolio; IP Rights Trading; Licensing.

JEL Codes: L1, L24, O3.

[*]Telecom ParisTech, Department of Economics and Social Sciences, and CREST-LEI, Paris. Email: marc.bourreau@telecom-paristech.fr.

[†]Centre d'Economie Industrielle MINES ParisTech, Paris, France. Email: yann.meniere@mines-paristech.fr.

[‡]Centre d'Economie Industrielle MINES ParisTech, Paris, France. Email: tim.pohlmann@mines-paristech.fr.

1 Introduction

A patent is said to be essential to a standard when it covers part of this standard's specifications. Standard essential patents (SEPs) differ from ordinary patents, because their value primarily depends on the market success of the related standard. Since standards are used across entire industries, SEPs can generate licensing revenues on a very large scale. Moreover, the technological specifications of standards are by definition public information. Therefore, licensing a SEP does not require further transfer of technical information (e.g., related know-how) to enable the licensee to use the technology; it simply consists of having users pay for implementing the standard.[1]

Another important aspect of SEPs is that they usually come in packages. Current IT standards like GSM, UMTS, LTE, MPEG or Blu-Ray are highly sophisticated technology platforms, embodying numerous innovative components. As a result, each of these standards may incorporate dozens or even hundreds of different SEPs, and the technology sponsors of a standard frequently hold and license several SEPs as a bundle. Because standards are collectively developed by industry sponsors, it is also frequent that several firms own SEP portfolios on the same standard.

In recent years, an increasing activity of SEP trading has been observed between firms in standard-related industries, and especially in telecoms. This evolution reflects a more general trend towards the trade of IP rights, including non-essential patents. However, SEPs are explicitly held as priority targets, including in the most prominent operations – such as the acquisition of Nortel's portfolio (ca 5,000 patents) by the Rockstar consortium (led by Apple and Microsoft) or the acquisition of Motorola Mobility (17,000 patents) by Google in 2011. The effect of these transfers is to change the size and distribution of SEPs portfolio among firms around standards.

Our purpose in this paper is to develop a theoretical framework accounting for the role of the portfolio size in the market for licenses, the incentives for SEPs owners to sell or buy these assets,

[1] According to antitrust authorities, the market for one SEP licenses is a relevant market per se.

and the consequences of SEP trading for the entire industry. We proceed in two steps. We first show how size of a SEP portfolio determines an owner's ability to charge royalties to standard implementors. We then focus on the causes and consequences of SEP trading.

We first show that the royalty charged by a SEP holder depends on its portfolio size. Our argument proceeds from the premise that the market power conferred by a SEP is contingent on its strength as a patent right. Indeed, the actual scope of patents is often ambiguous and disputable, and the validity of a patent may be challenged in courts. As a consequence, the value of a SEP portfolio as a means to collect royalties ultimately depends on its owner's ability to successfully enforce it in courts. We show that a SEP holder may be unable to set up a licensing program if its portfolio is not large enough to sustain a credible threat of enforcement. Beyond this critical portfolio size, the *enforcement margin* is binding, so that the licensor can charge higher royalty the stronger (larger) its portfolio. There is yet another threshold of portfolio size beyond which the licensor is bound by demand, thereby charging a monopoly royalty whatever the number of SEPs in its portfolio.

This model revisits the well-known tragedy of anticommons, according to which the fragmentation of IP owners increases the cumulative royalty price for licensees. More precisely, we characterize two distinct effects, namely *double-marginalization* and *royalty stacking,* that are due to enforcement-bound and demand-bound licensors, respectively. We show in particular that royalty stacking has a stronger impact on cumulative royalties, which can be partially internalized by strong *demand-bound* licensors.

We finally highlight two different types of motives for trading SEPs. On the one hand, buying patents can be a means for an enforcement-bound licensor to charge higher royalties, thereby raising the cumulative royalty cost paid by manufacturers. On the other hand, buying patents or patent portfolios can be a means for demand-bound licensors to mitigate the *double-marginalization* and *royalty stacking* problems. We show that the former effect dominates unless the set of SEP owners is dominated by a single *demand-bound* licensor that can correctly internalize the external effect of the transaction. This result moreover holds for single SEP transfers as well as the sale of entire portfolios. Therefore, the existence of a market for SEPs tends to increase the cumulative royalty charged to manufacturers when the standard is not sponsored by one dominant technology sponsor.

The paper is related to a first stream of literature on the licensing of essential patents. The problem of double marginalization when essential patents are licensed by different owners has been studied in a number of papers, under the strong assumption that these patents are perfect complements (Shapiro, 2001; Aoki & Nagaoka, 2004; Kim, 2004; Ménière & Parlane, 2010; Lévêque & Ménière, 2011). Important contributions by Lerner and Tirole (2004, 2014) have relaxed this assumption in different ways, thereby enabling a richer analysis of the price setting mechanisms for SEP licenses. Lerner and Tirole (2004) focus on the licensing of SEPs - either bilaterally or through a patent pool - after the standard specifications have been defined. They show that in this context, prices may be bound by the demand margin (then resulting in double marginalization) but also, in some cases, by the competition margin – that is, the circumvention of the patent by the potential licensee. Lerner and Tirole (2014) in turn consider the problem of ex ante competition between patented functionalities that are candidate for being included in the standard.

The rest of the paper is organized as follows. In Section 2, we set up the model and study how the size of SEP portfolio affects the level of royalties an owner can charge. In Section 3, we analyze the gain from SEP trading for owners, and in Section 4 we study two specific cases, namely, a merger of SEP portfolios, and a divestiture. Finally, we conclude.

2 Model

We consider a product market where the technological standard embodies k Standard Essential Patents (SEPs) owned by $n \leq k$ patent holders. Patent rights are probabilistic; each SEP has the same probability $\theta \in (0, 1)$ of being held valid by a court when challenged.[2] Patent holder $i \in N = \{1, ..., n\}$ has a portfolio of k_i SEPs, with $\sum_{i=1}^{n} k_i = k$. It can set up a FRAND licensing program, whereby it charges uniform per-unit royalties r_i to downstream producers of standard-compliant goods for using its k_i SEPs. The patent holders are not involved in the downstream product market.

There is a large number of downstream producers, which are identical and offer each a fixed quantity $\overline{q}$ of a homogeneous good. The demand function in the downstream market is given by $Q = D(p)$, and it is decreasing and differentiable. There is free entry, and the producers that enter the market compete in prices. For the sake of simplicity, we assume that manufacturers incur no production costs except for the per-unit royalties they have to pay for using the SEPs.

The timing of the game is the following. In the first stage, the SEP owners can trade some (or all) of the SEPs in their portfolio to other SEP owners or to outsiders. In the second stage, the SEP owners set simultaneously FRAND licensing terms for producers, in the shadow of patent litigation. We look for the subgame-perfect Nash equilibrium of this game.

With this timing, we wish to investigate how the distribution of SEP ownership among patent holders affect their licensing strategies. As usual, we move backwards, and start with the last stage of the game where SEP owners set the FRAND licensing terms.

[2] We exclude the extreme values (0 and 1), otherwise there would be no uncertainty on the patents' strength.

2.1 Licensing SEPs

At the second stage, the patent holders set their per-unit royalties simultaneously. We can thus analyze the licensing decision of patent holder $i \in N$, taking as given the cumulative royalties $R_{-i} = \sum_{j \neq i} r_j$ set by the other licensors. The timing of the licensing subgame is as follows:[3]

2a. Each SEP owner i sets and announces the royalty rate r_i that it will charge to all manufacturers for using its patent portfolio.

2b. Manufacturers enter the market. Upon entry, each manufacturer decides whether to take a license from SEP owner i or not. If it does, it pays the royalty rate r_i. Otherwise, it uses the technology without a valid license.

2c. Manufacturers compete in prices.

2d. SEP owner i can decide to enforce its patent rights in courts against the manufacturers that did not take a license. In case of litigation, both parties incur a litigation cost L. If it is held infringing by the court, a manufacturer has to pay damages d per unit of output, where d is exogenous. Otherwise, it can use the SEPs from owner i at no cost. As an alternative, both parties can agree on a settlement royalty to save litigation costs. All litigations between SEP owners and manufacturers occur simultaneously.

Before solving this licensing subgame, consider the case where patent holder i sues a manufacturer for infringement. The manufacturer wins the litigation if and only if all of firm i's patents are invalidated and/or declared non infringed by the court. Given that the patent holder owns k_i

[3] Note that with this timing, if manufacturers had market power and were producing variable quantities, they could influence the SEP owners' litigation decisions at the last stage of the game.

An alternative timing would be to assume that competition takes places after the litigation stage. In this case, if manufacturers produce a variable output, the SEP owners' litigation decisions are interdependent. For example, a SEP owner has more incentives to sue a given infringing manufacturer if the other SEP owner do not sue it (in which case its costs are very low, and hence, its quantity is high) than if all SEP owners sue it (in which case it has to pay high damages, leading to a low quantity).

SEPs, this event happens with probability $(1-\theta)^{k_i}$. The probability that patent holder i wins the litigation is thus

$$\omega(k_i) = 1 - (1-\theta)^{k_i}. \tag{1}$$

Equation (1) shows that the probability that the SEP owners wins the litigation increases with its portfolio size k_i, at a decreasing rate (i.e., $\omega'(k_i) > 0$ and $\omega''(k_i) < 0$). Therefore, a large SEP portfolio gives market power to a SEP owner for the negotiation of licensing terms, as we will show below.

2.2 Critical portfolio size

At the last stage of the licensing subgame (i.e., Stage 2d), each SEP owner i has to decide whether to sue a manufacturer j that did not take a license and is active in the downstream market. From (1), since the manufacturer sells a fixed quantity of output $\overline{q}$, firm i obtains the expected net profit $\omega(k_i)\,d\overline{q} - L$ from suing the infringer. Therefore, suing is profitable and credible if and only if $k_i \geq \underline{k}$, where $\underline{k}$ is the lowest integer such that

$$\omega(k_i) \geq \frac{L}{d\overline{q}}. \tag{2}$$

Using (1), we find that $\underline{k} = \lfloor Log(1 - L/(d\overline{q}))/Log(1-\theta) \rfloor$.

It is clear from condition (2) that the critical size of the SEP portfolio, $\underline{k}$, is larger if total output is spread across a large number of (small) manufacturers, i.e., $\overline{q}$ is low. Conversely, setting up a FRAND licensing program with a credible threat of litigation is easier in front of a small number of (large) manufacturers.

If $k_i < \underline{k}$, patent holder i will not go to court against an active and infringing manufacturer. If $k_i \geq \underline{k}$, the patent holder will go to court, and it is profit increasing for SEP owner i and manufacturer j to agree on a settlement royalty $\omega(k_i)\,dq$, because they can both save the litigation cost L.[4]

2.3 Downstream competition

Moving backwards, at Stage 2c, there is perfect competition between downstream manufacturers. Let ρ_j^i denote manufacturer j's payment to SEP owner i, per unit of output. We have

$$\rho_j^i = \begin{cases} r_i & \text{if the manufacturer has accepted the license from firm } i \\ d & \text{if it has refused the license and } k_i \geq \underline{k} \\ 0 & \text{if it has refused the license and } k_i < \underline{k} \end{cases}.$$

Manufacturer j's marginal cost at this stage, c_j, is then given by $c_j = \sum_{i \in N} \rho_j^i$. Finally, let $\underline{c} = \min_j c_j$. Since there is free entry in the downstream market, we can assume that there are at least two downstream firms with marginal cost $\underline{c}$. The equilibrium price in the downstream market is then $p = \underline{c}$.

2.4 Entry and licensing decisions

We now consider the downstream firms' decisions to take SEP licenses at Stage 2b, given the royalties $r = (r_i)_{i \in N}$ proposed by patent holders and their respective portfolios $k = (k_i)_{i \in N}$. Each manufacturer j has to make a choice $\gamma_i^j \in \{0, 1\}$ on whether to take a license from SEP holder i ($\gamma_i^j = 1$) or instead to refuse the license and possibly incur infringement damages ($\gamma_i^j = 0$).

If $k_i < \underline{k}$, patent holder i will not sue the manufacturer. The manufacturer therefore decides not to take a license, i.e., $\gamma_i^j = 0$. Otherwise, if $k_i \geq \underline{k}$, the patent holder will go to court against the manufacturer in case of infringement. The manufacturer, in this case, has to trade off between

[4] In our setting, therefore, there will never be any litigation in equilibrium, only settlements.

paying the royalties r_i and paying the damages $\omega(k_i)\,d$ (after settlement). It therefore decides to take a license from firm i if and only if $r_i \leq \omega(k_i)\,d$.

In sum, we have

$$\gamma_i^j = \begin{cases} 0 & \text{if} \quad k_i < \underline{k} \\ 0 & \text{if} \quad k_i \geq \underline{k} \text{ and } r_i > \omega(k_i)\,d \\ 1 & \text{if} \quad k_i \geq \underline{k} \text{ and } r_i \leq \omega(k_i)\,d \end{cases} \cdot$$

It follows that the equilibrium price in the downstream market minimizes the unit cost of the downstream firms:

$$p^* = \min_j \sum_{i \in N} \left[\gamma_i^j r_i + \left(1 - \gamma_i^j\right) \omega(k_i)\, d \times Id_{k_i \geq \underline{k}}\right] \equiv \underline{c}.$$

Finally, there is entry of m downstream manufacturers with unit cost $\underline{c}$ until $m\overline{q} = D(\underline{c})$.

2.5 Enforcement and demand margins

We now turn to Stage 2a, where the SEP owners set their royalties. The analysis above shows that each SEP holder i is bound by a maximum royalty, $\overline{r}(k_i) \equiv \omega(k_i)\,d$, which represents its expected settlement royalty in case of patent litigation. This maximum royalty increases with the size of the licensor's portfolio, and it determines the set of royalties $r_i \in [0, \overline{r}(k_i)]$ that firm i can effectively charge to downstream manufacturers.

Since each SEP owner i sets a royalty $r_i \leq \overline{r}(k_i)$ for its portfolio, the equilibrium price in the downstream market is $p^* = \sum_{i \in N'} r_i \equiv R$, where $N' \subset N$ is the set of SEP owners such that $k_i \geq \underline{k}$. It follows that if $k_i \geq \underline{k}$, the program of licensor i is

$$\max_{r_i} r_i D(R)\,, \text{ s.t. } r_i \in [0, \widetilde{r}(k_i)]\,. \tag{3}$$

The unconstrained solution of this program, $\widehat{r}(R)$, is implicitly defined by $\widehat{r}(R) = -D(R)/D'(R)$.[5] This unconstrained royalty rate, which corresponds to the standard monopoly price for an iron-clad patent, balances the per-unit revenues with the negative effect on the volumes of downstream goods. However, this unconstrained solution may not be feasible for a licensor with a weak patent portfolio. Let us thus define by $\overline{k}(R)$ the minimum portfolio size required to charge the royalty $\widehat{r}_i(R)$, that is, the smallest integer $\overline{k}$ such that

$$\overline{r}\left(\overline{k}\right) \geq \widehat{r}(R).$$

We can then characterize the licensor's pricing strategy as follows:

Proposition 1 *Given R_{-i}, the cumulative per unit royalty charged by the other SEP owners, the licensing strategy of SEP owner i depends on the size of its SEP portfolio:*

(i) if $k_i < \underline{k}$, SEP owner i does not have a sufficiently large portfolio to implement a licensing program, and therefore it cannot charge royalties: $r_i(R_{-i}) = 0$;

(ii) if $k_i \geq \underline{k}$ and $k_i < \overline{k}$, the enforcement margin binds and SEP owner i charges $r_i(R_{-i}) = \overline{r}(k_i) = \omega(k_i)\, d$;

(iii) if $k_i \geq \overline{k}$, the demand margin binds and SEP owner i charges $r_i(R_{-i}) = \widehat{r}(R)$.

Figure 1 below illustrates this result. It represents the licensor's profit as a function of its portfolio size. For $k_i < \underline{k}$, the SEP owner is unable to license and makes zero profit. Beyond the critical portfolio size, it can start charging positive royalties and making profits. These profits increase with the portfolio size, as a larger portfolio raises the enforcement margin, but with decreasing returns. These decreasing returns are due to two effects: first, the strength of the SEP portfolio is a concave function of the number of SEPs; second, licensing profits are a concave

[5] The second-order condition is satisfied if $2D'(R) + \widehat{r}D''(R) \leq 0$.

function of the level of royalties, because higher royalties decrease the volume of output. Due to this second effect, licensing profits reach a maximum at $\overline{k}$, corresponding to a royalty of $r_i(R_{-i}) = \widehat{r}(R)$. Adding SEPs in the portfolio beyond this threshold does not increase the licensor's profit, since it has no benefit in raising further its per unit royalty.

(FIGURE 1)

In other words, the incremental benefit of adding one SEP to a portfolio (e.g., by filing a new essential patent) is positive only when the enforcement margin binds, and it is then higher the smaller the size of the portfolio (conditional on its size exceeding $\underline{k}$).

2.6 Royalty stacking and double marginalization

Assume the existence of a licensing equilibrium based on the initial distribution of SEPs between the n licensors. This equilibrium is defined by three subsets of SEP holders, namely, (i) a subset S of n_s strong licensors with a large portfolio $k_i \geq \overline{k}$ that are bound by the demand margin, (ii) a subset E of n_e licensors with an intermediate size of portfolio k_i, with $\underline{k} \leq k_i < \overline{k}$, which are bound by the enforcement margin, and (iii) a subset W of $n_w = n - n_s - n_e$ weak SEP owners with a small portfolio $k_i < \underline{k}$, which are unable to implement a licensing program.

The profits of SEP holders in each group are the following:

$$\begin{cases} \pi_s = \widehat{r}D(R) \\ \pi_e(k_i) = d\omega(k_i)D(R) \\ \pi_w = 0 \end{cases},$$

where $R = \overline{R} + \widehat{R}$, with $\overline{R} = d\sum_{i \in E} \omega(k_i)$ and $\widehat{R} = n_s \widehat{r}\left(\overline{R}, n_s\right)$.

To clarify the exposition, we distinguish two categories of cumulative royalties. We first call *royalty stacking* an increase of $\overline{R}$ due to an increase of n_e (i.e., an increase in the number of licensors with a portfolio of an intermediate size) or to the strengthening of the SEP portfolio of one licensor from group E. We call *double marginalization* an increase in $\widehat{R}$ due to an increase in n_s (i.e., an increase in the number of licensors with a large portfolio).

Note that there is a close relationship between royalty stacking and double marginalization. We can indeed establish that

$$\frac{\partial \widehat{R}}{\partial \overline{R}} = \frac{n_s}{\widehat{R}} \frac{\partial \widehat{R}}{\partial n_s} - 1 \equiv \varepsilon_{\widehat{R}} - 1, \tag{4}$$

where $\varepsilon_{\widehat{R}}$, the elasticity of $\widehat{R}$ to n_s, provides a measure of *double marginalization* (the full proof is available in Appendix 1). Formally, a double marginalization problem (i.e., $\varepsilon_{\widehat{R}} > 0$) occurs for any positive integer n_s whenever the licensors' royalties are strategic substitutes.[6] We will focus on this case in the following.

Proposition 2 *Strategic substitutability between licensors' royalties creates a double marginalization problem, measured by* $\varepsilon_{\widehat{R}} \in (0, 1)$*, implying a substitution between royalty stacking and double marginalization:* $\partial \widehat{R}/\partial \overline{R} = \varepsilon_{\widehat{R}} - 1 \in (-1, 0)$.

Proof. See Appendix 2. ■

Since $\varepsilon_{\widehat{R}} > 0$, strong licensors are subject to the usual double marginalization problem, whereby an increase in the number of strong licensors induces an increase in their cumulative royalties $\widehat{R}$. However, since $\partial \widehat{R}/\partial \overline{R} \in (-1, 0)$, strong licensors react to an increase in the royalty stack $\overline{R}$ by charging lower cumulative royalties $\widehat{R}$. In other words, their perception of the demand margin leads them to compensate a fall of demand due to an increased royalty stack.

[6] Formally, this is the case if $D' + \widehat{r}D'' < 0$, implying that $\varepsilon_{\widehat{R}} > 0$. Note that this condition is satisfied for a linear demand, and that it is slightly more restrictive than the profit concavity condition for a licensor $i \in S$.

3 Marginal gains from trading SEPs

We now study the potential gains from trading individual SEPs. For this purpose, we assume for the moment that the trade of one SEP does not make the seller or the buyer from one group (S, E or W) to another. Obviously, under this assumption, SEP trading has no effect when it involves members of the S and/or W groups, and therefore there are no marginal gains or losses from trade in these cases. Hence, we focus on transfers involving at least one member of subset E. Since the licensors from group E always have positive marginal gains of adding one SEP to their portfolio, a first obvious result is that there is a gain of transferring a SEP from a weak SEP holder $i \in W$ to a member of group E. Indeed, there is no opportunity cost for weak licensors of further reducing their portfolio size.

3.1 Trade between enforcement-bounded licensors

We consider now the case of a transfer of one SEP between two enforcement-bounded licensors, $i, j \in E$. The transfer from i to j generates the following gain from trade:

$$\Delta = \left[\pi_i\left(k_i - 1, k_j + 1\right) + \pi_j\left(k_i - 1, k_j + 1\right)\right] - \left[\pi_i\left(k_i, k_j\right) + \pi_j\left(k_i, k_j\right)\right]. \tag{5}$$

The transfer has both direct effects on the profits of the buyer and the seller. The direct effect is positive for the buyer and negative for the seller, since (i) a stronger (weaker) portfolio induces the buyer (seller) to charge higher (lower) royalties, and (ii) both of them belonging to E, their profits are (weakly) increasing in their individual royalty level. The change in the other firm's royalty level has also an indirect effect on each licensor's profit. Since higher cumulative royalties induce a lower demand, the effect of lower (higher) royalties charged by firm i (j) after the transfer is positive for buyer i (negative for seller i).

We have $\pi_i(k_i, k_j) = dw(k_i)D(R)$, with $R = \widetilde{R} + dw(k_i) + dw(k_j)$ and $\widetilde{R} = \sum_{l \neq i,j} r_l$. Denoting R' the cumulative royalties after the trade, we can write the gains from trade as

$$\Delta = d\left(w(k_i - 1) + w(k_j + 1)\right) D(R') - d\left(w(k_i) + w(k_j)\right) D(R').$$

Note that due to the concavity of $w(\cdot)$, $w(k_i - 1) + w(k_j + 1) \geq w(k_i) + w(k_j)$ if and only if $k_i > k_j$. Studying the gains from SEP trade therefore reduces to studying the variation of the function $\pi(w) = dwD(R(w))$ with respect to w.

We have

$$\frac{d\pi(w)}{dw} = d\left[D(R(w)) + wR'_w D'(R(w))\right].$$

Since $R(w) = dw + n_s\widehat{r}(w) + d\sum_{l \neq i,j} w(k_l)$, then $R'_w(w) = d + (\partial\widehat{R}/\partial\overline{R})(\partial\overline{R}/\partial w) = d\left[1 + n_s\partial\widehat{r}/\partial\overline{R}\right]$.

The gains from SEP trade are therefore given by

$$\frac{d\pi(w)}{dw} = d\left(D(R) + D'(R)wd\left[1 + n_s\frac{\partial\widehat{r}}{\partial\overline{R}}\right]\right).$$

Assuming, without loss of generality, that $k_i > k_j$, the cumulative royalties of firms i and j increase and therefore $dw > 0$. The transfer is then profitable if:

$$D(R) > -D'(R)\,\omega d\left[1 + n_s\frac{\partial\widehat{r}}{\partial\overline{R}}\right]. \tag{6}$$

The term on the left-hand side reflects the direct effect of a change in i and j's cumulative royalties. This effect is proportional to demand, that is, to the level of output $D(R)$. Since cumulative royalties increase, it is positive. The term on the right-hand side captures the variation of demand due to the increase of cumulative royalties. It is therefore proportional to the per-unit royalties initially charged by the parties: wd. Since higher aggregate royalties leads to a lower demand, this effect impacts profits negatively. If $n_s \geq 1$, it is mitigated by the reaction of strong licensors: they indeed reduce their optimal royalty to accommodate the increase in $\overline{R}$. Condition (6) thus tells us that a transfer from i to j, with $k_i > k_j$, generates positive gains from trade if the benefit of higher aggregate royalties for the two parties offsets the resulting loss in demand.

Observing that in equilibrium $\widehat{r} = -D(R)/D'(R)$, we can rewrite condition (6) as follows:

$$\widehat{r} > \omega d \left[1 + n_s \frac{\partial \widehat{r}}{\partial \overline{R}}\right]. \tag{7}$$

It follows directly that when $n_s = 0$, a transfer will take place from the larger to the smaller portfolio if together the parties' royalties do not exceed the optimal royalty $\widehat{r}$. Conversely, if the parties joint royalties generate double-marginalization, the smaller portfolio will sell to the larger one, so as to reduce their aggregate royalties.

Let us assume now that there is at least one strong licensor which is bound by the demand margin (i.e., $n_s > 0$). Condition (7) can then be expressed as follows:

$$d\omega < \frac{\widehat{r}}{\varepsilon_{\widehat{R}}}, \tag{8}$$

where $\varepsilon_{\widehat{R}} \in (0, 1)$, which is defined in (4), represents the strength of the double marginalization effect. Since $\varepsilon_{\widehat{R}} < 1$, strong licensors reduce their royalties following an increase in w, which relaxes condition (7) for $n_s > 0$ and thus further encourages a transfer towards the smaller SEP owner. Using the linear inverse demand function $p(Q) = a - bQ$, we can furthermore show that the SEP will always flow from the larger to the weaker of two enforcement-bounded licensors, thereby amplifying royalty stacking and double marginalization at the aggregate level. We summarize these results in Proposition 3:

Proposition 3 *There is always a gain from transferring a SEP between asymmetric enforcement-*

bounded licensors.

(i) If $n_s = 0$, the SEP is transferred from the weaker to the stronger licensor if the sum of their per-unit royalties is bound by the demand margin, and from the stronger to the weaker one otherwise.

(ii) (Linear demand) If $n_s \geq 1$, the SEP is always transferred from the stronger to the weaker licensor.

Proof. With the linear demand function, we obtain that:

$$\frac{\partial \widehat{r}}{\partial \overline{R}} = -\frac{1}{n_s + 1}.$$

Using this expression in (7), we find that

$$\Delta > 0 \Leftrightarrow d\left(\omega\left(k_i\right) + \omega\left(k_j\right)\right) < \left(n_s + 1\right)\widehat{r}.$$

This condition is satisfied for any $n_s \geq 1$, since by definition of $i, j \in E$ we have $d\omega\left(k_i\right) < d\omega\left(k_j\right) < \widehat{r}$. ■

3.2 Trade between demand-bounded and enforcement-bounded licensors

We consider now a SEP trade between a strong (demand-bounded) and a medium-strength (enforcement-bounded) licensor, that is, a trade from $i \in S$ to $j \in E$. We will assume in the following that $k_s > \overline{k}$, so that a transfer to firm j does not lead firm i to charge lower royalties (the case where $k_s = \overline{k}$ is accounted for in the previous subsection). The potential gains of a transfer from $i \in S$ to $j \in E$ are then given by:

$$\Delta = \left[\pi_e\left(k_i - 1, k_j + 1\right) - \pi_e\left(k_i, k_j\right)\right] + \left[\pi_s\left(k_i - 1, k_j + 1\right) - \pi_s\left(k_i, k_j\right)\right], \tag{9}$$

where the first term into brackets on the right-hand side represents the buyer' gain of being able to charge higher royalties due to a larger portfolio, while the second term captures the seller's loss from lower demand due to higher cumulative royalties paid by manufacturers. Since the diminution of k_i has no effect on royalties, we have $\Delta > 0$ if and only if $\partial\pi_e(k_i, k_j)/\partial k_j + \partial\pi_s(k_i, k_j)/\partial k_j > 0$. We find that

$$\frac{\partial\pi_e(k_i, k_j)}{\partial k_j} = dw'(k_j)\left[D(R) + D'(R)dw(k_j)\left(1 + \frac{\partial\widehat{R}}{\partial\overline{R}}\right)\right],$$

and that

$$\frac{\partial\pi_s(k_i, k_j)}{\partial k_j} = dw'(k_j)\left[\frac{\partial\widehat{r}}{\partial\overline{R}}D(R) + D'(R)\widehat{r}\left(1 + \frac{\partial\widehat{R}}{\partial\overline{R}}\right)\right].$$

Therefore, the transfer from i to j is profitable if and only if

$$D(R)\left[1 + \frac{\partial\widehat{r}}{\partial\overline{R}}\right] + \left[\widehat{r} + d\omega(k_i)\right]D'(R)\left[1 + n_s\frac{\partial\widehat{r}}{\partial\overline{R}}\right] > 0. \tag{10}$$

This condition again balances a margin effect and a demand effect of a change in the royalty structure. The margin effect is positive, but it includes the reaction of the seller, who reduces its royalty to adapt to an increase of $\overline{R}$. The demand effect is negative, but it is also moderated by the reaction of all strong licensors. Note that since $\partial\widehat{r}/\partial\overline{R} < 0$, all strong licensors experience a fall in their per-unit margins, but this externality is not internalized by the parties in the transaction.

Using the fact that $\widehat{r} = -D/D'$, condition (10) becomes

$$\widehat{r} + d\omega(k_i) < \frac{1}{\varepsilon}\left[\widehat{r} + \frac{\partial\widehat{r}}{\partial n_s}\right], \tag{11}$$

where $\partial\widehat{r}/\partial n_s \in (-\widehat{r} + n_s, 0)$. This condition is comparable to condition (8) with one important difference, namely that the seller integrates as an opportunity cost the need for her to reduce its royalty after the sale.

Proposition 4 *There is always a gain from transferring a SEP between a strong and a medium-strength licensor:*

(i) If $n_s = 1$, the SEP is transferred from the medium-strength to the strong licensor.

(ii) (Linear demand) If $n_s \geq 2$, the SEP is transferred from the strong to the medium-strength licensor.

Proof. See Appendix 4. ■

This contrasted result is due to the external effect of double marginalization when there is more than one strong licensor. Indeed, all licensors benefit from enhanced demand due to double margin mitigation. This benefit being spread across all licensors, this however implies that the firm that buys SEPs from smaller licensors in order to mitigate double margins can only reap a fraction of this benefit. The problem is worse in presence of other strong licensors, because the latter will strategically react to an effort to reduce cumulative royalties by raising their own royalty level. By contrast, medium-strength licensors are not bound by the demand margin and therefore benefit from enhanced demand without raising their royalties. As a result, double margins mitigation by a strong licensor is privately profitable only when there is no other strong licensor to free ride on this move.

In presence of several strong licensors, it is more profitable for each of them to sell away their useless patents to medium strength licensors who are better positioned to monetize them. While this generates a direct gain from trade for the two parties in the transfer, this also results in higher cumulative royalties paid by manufacturers, and thus a negative double margins externality which is detrimental to the industry as a whole.

3.3 Competitive demand for SEPs

We study now SEP transfers through a second price sealed bid auction (SPSBA). In the following, we focus first on a SEP trade initiated by a weak SEP owner $w \in W$. We posit that $n_s + n_e \geq 2$, which enables competition between SEP buyers. Using the linear demand system, we can derive the following proposition:

Proposition 5 *(Linear demand) A SEP auctioned by a weak licensor is purchased by the strongest licensor if:*

(i) $n_s = 1$*, or*

(ii) $n_s = 0$ *and the weakest (i) and strongest (j) SEP owners are such that* $d\omega(k_i) + d\omega(k_j) > \widehat{r}$*.*

Otherwise, the SEP is purchased by the weakest licensor within E*.*

Proof. See Appendix 5. ■

This proposition states that a dominant licensor is able to preempt SEPs that are for sale only if it is the only dominant owner. Note that by dominant we mean here not only a unique strong licensor. In case there is no such strong licensor, it may also be the strongest medium size licensor if, when added to the weakest licensor's royalty, its royalty exceeds the strong portfolio royalty $\widehat{r}$.

By contrast, the weakest licensor preempts SEPs for sale if there is no dominant licensor, because it has then the strongest benefit of strengthening its portfolio. As a result, the royalty stack paid by manufacturers increases. Interestingly, this is also true when there are more than one strong licensor. This again increases the royalty stack, which forces strong licensors to cut their own royalty to compensate for the reduced demand. Although a single strong licensor would then be able to preempt the SEP, this is not possible anymore when there are at least two strong licensors. This is due to the externality between strong licensors. The benefit of preempting the SEP would be to prevent increased royalty stacking. However, this would equally benefit all strong licensors, not only the buyer. As a result, none of them is ready to incur the private cost generating this public benefit.

Note that this result can be easily extended to SEP trading between active licensors. Indeed there are always gains from trade between firms with different portfolio sizes, but the (potential) seller does not wholly capture them if the price he gets is the bid of the second best bidder. Since there is an opportunity cost of letting the SEP go to the highest bidder (for instance, the one with the smallest portfolio) in exchange for the second highest bidder's willingness to pay, there may be cases where the SEP owner does not want to sell at this second price. However, we know that even in this case the highest bidder's willingness to pay exceeds the reservation value. Hence, trade can always take place between SEP holders with asymmetric portfolio sizes.

4 Trading SEPs portfolios

4.1 Merging portfolios

We analyze here the bilateral gains of merging portfolios. Since such a merger results in removing one licensor, it necessarily results in a decrease of total cumulative royalties. This is also true when two medium-strength merge their portfolios, since the royalty is an increasing but concave function of the portfolio size. Hence, the only possible motive for merging portfolios is to mitigate royalty stacking and/or double-marginalization.

Against this background, our results (reported in the proposition below) show again that SEP concentration is possible only through the creation or reinforcement of one single dominant licensor.

Proposition 6 *(Linear demand) Merging two SEP portfolios is a profitable operation in three cases only:*

(*i*) *If $n_s = 0$, two medium-strength licensors find it profitable to merge if the sum of their* *pre-merger royalties is bound by the demand margin;*

(*ii*) *If* $n_s = 1$, *the strong licensor finds it profitable to absorb any medium-strength licensor;*

(*iii*) *If* $n_s = 2$, *the strong licensors find it profitable to merge into a single one.*

Proof. See Appendix 6. ■

4.2 Portfolio divestiture

Finally, we consider the possibility that a strong licensor splits its portfolio so as to enable the creation of a new licensor. This requires that the SEP seller $i \in S$ has a strong enough portfolio, that is, $k_i \geq \overline{k}+\underline{k}$. Consider that such a strong licensor sells a package of $k_i - \overline{k} \equiv k_e$ SEPs to a new entrant. Given n_s and $\overline{R}$ before the transfer, this sale is profitable if

$$\pi_s\left(\overline{R}'\right) + \pi_e\left(k_e\right) > \pi_s\left(\overline{R}\right), \tag{12}$$

where $\overline{R}' = \overline{R} + d\omega\left(k_e\right)$. Considering the linear demand system, we show that this condition is satisfied for any $n_s \geq 2$, but not for $n_s = 1$.

Proposition 7 *(Linear demand) A strong licensor that has a large enough portfolio to set up a second medium-strength licensing program finds it profitable to do so if and only if there is at least a second licensor* ($n_s \geq 2$).

Proof. See Appendix 7. ■

Note that a similar outcome can be obtained if a new entrant manages to purchase useless SEPs from several strong licensors. Accordingly, the condition for entry to occur is relaxed and becomes $\sum\limits_{i\in S}\left(k_i - \overline{k}\right) > \underline{k}$.

5 Conclusion

(To be completed)

References

Aoki, R., Nagaoka, S. (2004). "The Consortium Standard and Patent Pools," *The Economic Review* 55(4), 345-357.

Choi, J.-P. (2010). "Patent Pools and Cross-Licensing in the Shadow of Patent Litigation," *International Economic Review* 51(2), 441-460.

Choi, J.-P., Gerlach, H. (2013). "A Theory of Patent Portfolios," CESifo Working Paper No. 4405.

Kim, S.-H. (2004). "Vertical structure and patent pools," *Review of Industrial Organization* 25, 231-250.

Lemley, M., Shapiro, C. (2005). "Probabilistic Patents," *Journal of Economic Perspectives* 19(2), 75-98.

Lemley, M., Shapiro, C. (2007). "Patent Hold-Up and Royalty Stacking," *Texas Law Review* 85(7), 1991-2049.

Lerner, J., Tirole, J. (2004). "Efficient Patent Pools," *American Economic Review* 94(3), 691-711.

Lerner, J., Tirole, J. (2014). "Standard-Essential Patents," *Journal of Political Economy*, forthcoming.

Lévêque, F., Ménière, Y. (2011). "Patent Pool Formation: Timing Matters," *Information Economics and Policy* 23(3-4), 243-251.

Ménière Y., Parlane, S. (2010), "Licensing of complementary patents: comparing the royalty, fixed-fee and two-part tariff regimes," *Information Economics and Policy* 22(2), 178-191.

Scott-Morton, F., Shapiro, C. (2014). "Strategic Patent Acquisitions," *Antitrust Law Journal* 79(2), 463-499.

Shapiro, C. (2001). "Navigating the Patent Thicket: Cross Licenses, Patent Pools, and Standard Setting," NBER Chapters, in: *Innovation Policy and the Economy* 1, 119-150, National Bureau of Economic Research, Inc.

Appendix

1. Proof of Equation (4)

The equilibrium royalty rate for strong (demand-bounded) licensors is given by $f(\widehat{r}, R) = D(R) + \widehat{r}D'(R) = 0$. Applying the envelop theorem to this condition, we obtain that:

$$\frac{\partial \widehat{r}}{\partial n_s} = -\frac{\partial f/\partial n_s}{\partial f/\partial \widehat{r}} = -\frac{\widehat{r}\left[D' + \widehat{r}D''\right]}{n_s\left[D' + \widehat{r}D''\right] + D'}, \tag{13}$$

since, using the fact that $R = \overline{R} + n_s\widehat{r}$, we have $\partial f/\partial \widehat{r} = n_s\left[D' + \widehat{r}D''\right] + D'$ and $\partial f/\partial n_s = \widehat{r}\left[D' + \widehat{r}D''\right]$. Similarly, we obtain that:

$$\frac{\partial \widehat{r}}{\partial \overline{R}} = -\frac{D' + \widehat{r}D''}{n_s\left[D' + \widehat{r}D''\right] + D'} = \frac{1}{\widehat{r}}\frac{\partial \widehat{r}}{\partial n_s}. \tag{14}$$

Hence, using (14),

$$\frac{\partial \widehat{R}}{\partial n_s} = \widehat{r} + n_s\frac{\partial \widehat{r}}{\partial n_s} = \widehat{r}\left[1 + n_s\frac{\partial \widehat{r}}{\partial \overline{R}}\right].$$

Using the fact that $\widehat{R} = n_s\widehat{r}$, it follows that:

$$\frac{1}{\widehat{r}}\frac{\partial \widehat{R}}{\partial n_s} = \frac{n_s}{\widehat{R}}\frac{\partial \widehat{R}}{\partial n_s} = 1 + n_s\frac{\partial \widehat{r}}{\partial \overline{R}} = 1 + \frac{\partial \widehat{R}}{\partial \overline{R}}.$$

Therefore,

$$\frac{\partial \widehat{R}}{\partial \overline{R}} = \frac{n_s}{\widehat{R}}\frac{\partial \widehat{R}}{\partial n_s} - 1.$$

2. Proof of Proposition 2

As $\widehat{R} = n_s \widehat{r}$, we can express double marginalization as

$$\frac{\partial \widehat{R}}{\partial n_s} = \widehat{r} + n_s \frac{\partial \widehat{r}}{\partial n_s}.$$

Using (13), this expression becomes

$$\frac{\partial \widehat{R}}{\partial n_s} = \frac{\widehat{r} D'}{n_s \left[D' + \widehat{r} D'' \right] + D'}.$$

Since $D' \leq 0$, we have $\partial \widehat{R} / \partial n_s \geq 0$ if and only if $n_s \left[D' + \widehat{r} D'' \right] + D' < 0$. A sufficient condition is that $D' + \widehat{r} D'' < 0$. Note that this condition holds when r_i and r_j $(i \in S, i \neq j)$ are strategic substitutes, since this is the case if and only if $D'(R) + r_i D''(R)$ for all r_i and r_j. Note that $D' + \widehat{r} D'' < 0$ also implies that

$$\frac{\partial \widehat{r}}{\partial \overline{R}} = \frac{1}{\widehat{r}} \frac{\partial \widehat{r}}{\partial n_s} = -\frac{Q' + \widehat{r} Q''}{n_s \left[Q' + \widehat{r} Q'' \right] + Q'} < 0, \ \frac{\partial \widehat{R}}{\partial \overline{R}} = n_s \frac{\partial \widehat{r}}{\partial \overline{R}} < 0,$$

and that

$$\varepsilon_{\widehat{R}} = \frac{n_s}{\widehat{R}} \frac{\partial \widehat{R}}{\partial n_s} > 0.$$

Finally, we have

$$\varepsilon - 1 = \frac{\partial \widehat{R}}{\partial \overline{R}} < 0,$$

which implies that $\varepsilon_{\widehat{R}} < 1$ and that $\partial \widehat{R} / \partial \overline{R} \in (-1, 0)$.

3. Linear demand

We consider a linear inverse demand function $p = a - bQ$. The equilibrium price in the downstream market is given by $p^* = \overline{R} + \sum_{i \in S} r_i$. The demand for the final good is then

$$D(R) = \frac{a - \left(\overline{R} + \sum_{i \in S} r_i\right)}{b},$$

and the program of firm $i \in S$ is $\max_{r_i} r_i D(R)$. The (monopoly) royalty rate that solves this program is

$$\widehat{r} = \frac{a - \overline{R}}{n_s + 1}. \tag{15}$$

The demand for the final good at the equilibrium of the downstream market is then

$$D(R) = \frac{a - \overline{R}}{b\,(n_s + 1)}.$$

Finally, the licensing profits of a strong and medium-strength licensors are:

$$\pi_s = \frac{1}{b}\left(\frac{a - \overline{R}}{n_s + 1}\right)^2,$$

and

$$\pi_e\,(k_i) = d\omega\,(k_i)\,\frac{a - \overline{R}}{b\,(n_s + 1)}, \tag{16}$$

respectively.

4. Proof of Proposition 4

After multiplying both sides of (11) by ε, substracting $\widehat{r}$ on both sides, and factorizing by $\widehat{r}$ on the right hand side, we obtain

$$\widehat{r} + d\omega\left(k_i\right) < \frac{1}{\varepsilon}\left[\widehat{r} + \frac{\partial \widehat{r}}{\partial n_s}\right]$$

$$\Leftrightarrow$$

$$\varepsilon d\omega\left(k_i\right) < \widehat{r}\left(1 - \varepsilon + \frac{1}{\widehat{r}}\frac{\partial \widehat{r}}{\partial n_s}\right)$$

Using the fact that $1 - \varepsilon = -\partial\widehat{R}/\partial\overline{R}$ from (4) and that $\partial\widehat{R}/\partial\overline{R} = n_s\partial\widehat{r}/\partial\overline{R} = (n_s/\widehat{r})(\partial\widehat{r}/\partial n_s)$, the previous condition becomes

$$\varepsilon d\omega\left(k_i\right) < \left(1 - n_s\right)\frac{\partial \widehat{r}}{\partial n_s}. \tag{17}$$

or, by using that $\varepsilon = (\partial\widehat{R}/\partial n_s)/\widehat{r}$ and that $\partial\widehat{r}/\partial n_s = \widehat{r}\partial\widehat{r}/\partial\overline{R}$,

$$\frac{1}{\widehat{r}}\frac{\partial \widehat{R}}{\partial n_s}d\omega\left(k_i\right) < \widehat{r}\left(1 - n_s\right)\frac{\partial \widehat{r}}{\partial \overline{R}}$$

Note first that for $n_s = 1$ this condition becomes

$$\frac{1}{\widehat{r}}\frac{\partial \widehat{R}}{\partial n_s}d\omega\left(k_i\right) < 0,$$

which is impossible since $\partial\widehat{R}/\partial n_s > 0$. If $n_s \geq 2$, we rewrite condition (17) for the linear demand system. We find that $\varepsilon = 1/(n_s + 1)$ and we obtain that condition (17) is equivalent to

$$d\omega\left(k_i\right) < \widehat{r}\left(n_s - 1\right),$$

which is always true for $n_s > 1$.

5. Proof of Proposition 5

Auctioning the SEP makes it possible to derive its price based on the potential buyers' bids. We first consider in Step 1 two potential buyers $i, j \in E$ such that $k_i < k_j$. Then, in Step 2, we study the case where $i \in S$ and $j \in E$.

Step 1: $i, j \in E$. Licensor i 's bid for the SEP is equal to its benefit for having the SEP, minus its opportunity cost of letting j acquire the SEP:

$$b_i = \left[\pi_i\left(k_i+1, k_j\right) - \pi_i\left(k_i, k_j\right)\right] - \left[\pi_i\left(k_i, k_j+1\right) - \pi_i\left(k_i, k_j\right)\right],$$

and similarly for licensor j. Note that $b_i, b_j > 0$ since $\pi_i\left(k_i+1, k_j\right) - \pi_i\left(k_i, k_j\right) > 0$ and $\pi_i\left(k_i, k_j+1\right) - \pi_i\left(k_i, k_j\right) < 0$ for any $i, j \in E$. We $b_i > b_j$ if

$$d\left[w\left(k_i+1\right) + w\left(k_j\right)\right] D\left(R'\right) > d\left[w\left(k_i\right) + w\left(k_j+1\right)\right] D\left(R\right).$$

Therefore, if $k_i < k_j$, which implies that $w\left(k_i+1\right) + w\left(k_j\right) > w\left(k_i\right) + w\left(k_j+1\right)$, then $b_i > b_j$ if $\pi\left(w\right) = dwD(R(w))$ is increasing in w. From the analysis in Section 3.1, this is the case if and only if

$$\widehat{r} > \left[1 + n_s \frac{\partial \widehat{r}}{\partial \overline{R}}\right] d\left[\omega\left(k_i\right) + \omega\left(k_j\right)\right]. \tag{18}$$

It follows that the firm with the smaller (larger) portfolio buys the SEP is (18) is (not) satisfied. We can generalize this result to all potential buyers in E as follows.

Let $E = \{i, 2, ., , j\}$, with $k_i \leq k_2 \leq .. \leq k_j$ and $k_i \neq k_j$. Let $k_{\min}$ be the lowest k such that (18) is not satisfied for $d\left[\omega\left(k\right) + \omega\left(k_j\right)\right]$. It follows that j will outbid any licensor l with $k_l \geq k_{\min}$.

Let now $k_{\max}$ be the highest k such that (18) is satisfied for $d\left[\omega\left(k\right)+\omega\left(k_i\right)\right]$. It follows that i will outbid any licensor l with $k_l \leq k_{\max}$. However, if any firm with $k > k_{\max}$ exists, it will in turn outbid i. And since by construction $k_j \geq k_{\max}$, it will then outbid any $l \in E$. Hence, the SEP will go to j unless $k_j \leq k_{\max}$, that is unless (18) is satisfied for $d\left[\omega\left(k_i\right)+\omega\left(k_j\right)\right]$ so that the best bidder is i.

The following lemma summarizes this result:

Lemma 1 *Let i and j have respectively the weakest and strongest portfolio within E, with $k_i \neq k_j$. Then i is the highest bidder within E if*

$$\widehat{r} > \left[1 + n_s \frac{\partial \widehat{r}}{\partial \overline{R}}\right] d\left[\omega\left(k_i\right)+\omega\left(k_j\right)\right],$$

and j is the highest bidder otherwise.

Step 2. $i \in S$ and $j \in E$ We can now extend the analysis to SEP acquisition by strong owners. Following the same reasoning as above for Step 1, we obtain that a strong SEP owner $i \in S$ can overbid a medium strength one $j \in E$ if there exists a net gain of transferring a SEP from the former to the latter. This corresponds to the condition below:

$$\varepsilon\left[\widehat{r} + d\omega\left(k_i\right)\right] < \widehat{r} + \frac{\partial \widehat{r}}{\partial n_s}$$

Using Propositions 2 to 4, we can now sum up our results in the linear demand case as follows.

If $n_s = 0$ and $n_e \geq 2$, Proposition 2 and Lemma 1 together imply that the SEP will be bought by the strongest SEP owner in E if $d\omega\left(k_i\right) + d\omega\left(k_j\right) < \widehat{r}$, and by the weakest one otherwise.

When $n_s \geq 1$, Proposition 3 and Lemma 1 together imply that the weakest SEP owner in E always makes the best bid within E. If $n_s = 1$, Proposition 4 in turn implies that the single strong licensor is ready to pay more than any medium size licensor. If $n_s > 1$, Proposition 4 finally implies that the bid of a strong licensor is higher than the bid of any $i \in E$, of which the strongest bidder is the one with the weakest portfolio.

6. Proof of Proposition 6

Case 1: Portfolio merger between strong licensors. In this case, the merger removes one strong licensor, letting the royalty stack $\overline{R}$ unchanged. The merger is profitable if

$$\pi_s \left(n_s - 1, \overline{R}\right) > 2\pi_s \left(n_s, \overline{R}\right),$$

which can also be expressed for the linear demand system as

$$A\left(\frac{a-\overline{R}}{n_s}\right)^2 > 2A\left(\frac{a-\overline{R}}{n_s+1}\right)^2$$

$$\Leftrightarrow$$

$$n_s\left(n_s - 2\right) < 1.$$

It follows that the merger is profitable if and only if $n_s = 2$.

Case 2: Portfolio merger between medium-strength licensors. The merger is profitable if

$$\pi_e\left(k_i + k_j\right) > \pi_e\left(k_i\right) + \pi_e\left(k_j\right). \tag{19}$$

Replacing profits by their expressions in (19) and rearranging gives

$$\left[\omega\left(k_i\right) + \omega\left(k_i\right) - \omega\left(k_i + k_i\right)\right]\left[d\omega\left(k_i + k_i\right) - \left(a - \overline{R}\right)\right] > 0.$$

Obviously, the first term into brackets is positive, so this condition simplifies into

$$d\omega\left(k_i + k_i\right) > a - \overline{R}. \tag{20}$$

Observe now that with a portfolio size $k_i + k_j$ the merged licensor may now be bound by the demand margin (formally, $k_i+k_j \geq \overline{k}$). Denoting by $\widehat{r}\left(\overline{R}', n_s + 1\right)$, with $\overline{R}' = \overline{R} - d\left[\omega\left(k_i\right) + \omega\left(k_j\right)\right]$, the optimal (demand-bound) royalty in case the new entity becomes a strong licensor (so that now $|S| = n_s + 1$), we have thus:

$$d\omega\left(k_i + k_i\right) \leq \widehat{r}\left(\overline{R}', n_s + 1\right).$$

Therefore, condition (20) cannot be satisfied if

$$\widehat{r}\left(\overline{R}', n_s + 1\right) < a - \overline{R}.$$

Noting that

$$\begin{aligned} \widehat{r}\left(\overline{R}', n_s + 1\right) &= \frac{a - \overline{R} + d\left[\omega\left(k_i\right) + \omega\left(k_i\right)\right]}{n_s + 2} \\ \widehat{r}\left(\overline{R}, n_s\right) &= \frac{a - \overline{R}}{n_s + 1}, \end{aligned}$$

this condition becomes

$$d\left[\omega\left(k_i\right) + \omega\left(k_i\right)\right] < \left(n_s + 1\right)^2 \widehat{r}\left(\overline{R}, n_s\right).$$

Obviously, condition (20) cannot be satisfied for any $n_s \geq 1$, and thus the merger is not profitable in this case. Assuming now that $n_s = 0$, condition (20) may be satisfied if $d\left[\omega\left(k_i\right) + \omega\left(k_i\right)\right] >$

$\widehat{r}\left(\overline{R}, n_s\right)$. Using the fact that $a - \overline{R} = \widehat{r}\left(\overline{R}, 0\right)$, it is the case if

$$d\omega\left(k_i + k_i\right) > \widehat{r}\left(\overline{R}, 0\right).$$

Case 3: Portfolio merger between a strong and a medium-strength licensor. The condition for a profitable merger is

$$\pi_s\left(n_s, \overline{R} - d\omega\left(k_i\right)\right) > \pi_s\left(n_s, \overline{R}\right) + \pi_e\left(k_i\right). \tag{21}$$

Observe first that given n_s and $\overline{R}$ we have

$$\pi_s\left(n_s, \overline{R}\right) = A\left(\frac{a - \overline{R}}{n_s + 1}\right)^2 = A\widehat{r}\left(n_s, \overline{R}\right)\frac{a - \overline{R} - \widehat{r}\left(n_s, \overline{R}\right)}{n_s},$$

and thus, with $\overline{R}' = \overline{R} - d\omega\left(k_i\right)$:

$$\pi_s\left(n_s, \overline{R}'\right) = A\left(\frac{a - \overline{R}'}{n_s + 1}\right)^2 = A\widehat{r}\left(n_s, \overline{R}'\right)\frac{a - \overline{R} - \widehat{r}\left(n_s, \overline{R}'\right)}{n_s}.$$

Inequality (21) can then be expressed as follows:

$$\widehat{r}\left(n_s, \overline{R}'\right)\frac{a - \overline{R} - \widehat{r}\left(n_s, \overline{R}'\right)}{n_s} > \widehat{r}\left(n_s, \overline{R}\right)\frac{a - \overline{R} - \widehat{r}\left(n_s, \overline{R}\right)}{n_s} + d\omega\left(k_i\right)\frac{a - \overline{R}}{n_s + 1}.$$

After multiplying by n_s and $(n_s + 1)$, replacing $\widehat{r}\left(n_s, \overline{R}\right)$ and $\widehat{r}\left(n_s, \overline{R}'\right)$ by their expressions, and simplifying we obtain

$$\left(a - \overline{R}'\right)^2 - \left(a - \overline{R}\right)^2 > \left(n_s + 1\right) d\omega\left(k_i\right)\left(a - \overline{R}\right).$$

Developing the brackets in square on the left-hand side, dividing by $d\omega(k_i)$, and substracting $2\left(a-\overline{R}\right)$ on each side finally gives

$$d\omega(k_i) > (n_s - 1)\left(a - \overline{R}\right)$$

$$\Leftrightarrow$$

$$d\omega(k_i) > (n_s - 1)^2\, \widehat{r}\left(n_s, \overline{R}\right).$$

Obviously, this condition is satisfied only for $n_s = 1$.

7. Proof of Proposition 7

Using the linear demand system, condition (12) becomes

$$\left(\frac{a-\overline{R}'}{n_s+1}\right)^2 + d\omega(k_e)\,\frac{a-\overline{R}'}{n_s+1} > \left(\frac{a-\overline{R}}{n_s+1}\right)^2,$$

which, after simplifying and rearranging, can in turn be expressed as

$$(n_s - 1)(n_s + 1)\,\widehat{r} > n_s d\omega(k_e).$$

Obviously, this condition is satisfied for any $n_s \geq 2$, but not for $n_s = 1$.