

Monetary Policy, Leverage, and Bank Risk-Taking*

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Abstract

We provide a theoretical foundation for the claim that prolonged periods of easy monetary conditions increase bank risk taking. The net effect of a monetary policy change on bank monitoring (an inverse measure of risk taking) depends on the balance of three forces: interest rate pass-through, risk shifting, and leverage. When banks can adjust their capital structures, a monetary easing leads to greater leverage and lower monitoring. However, if a bank's capital structure is fixed, the balance depends on the degree of bank capitalization: when facing a policy rate cut, well capitalized banks decrease monitoring, while highly levered banks increase it. Further, the balance of these effects depends on the structure and contestability of the banking industry, and is therefore likely to vary across countries and over time.

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1 Introduction

The recent global financial crisis has brought the relationship between monetary policy and bank risk taking to the forefront of the economic policy debate. Many observers have blamed loose monetary policy for the credit boom and the ensuing crisis in the late 2000s, arguing that, in the run up to the crisis, low interest rates and abundant liquidity led financial intermediaries to take excessive risks by fueling asset prices and promoting leverage. The argument is that had monetary authorities raised interest rates earlier and more aggressively, the consequences of the bust would have been much less severe. More recently, a related debate has been raging on whether continued exceptionally low interest rates are setting the stage for the next financial crisis.¹

Fair or not, these claims have become increasingly popular both in academia and in the business press. Surprisingly, however, the theoretical foundations for these claims have not been much studied and hence are not well understood. Macroeconomic models have typically focused on the quantity rather than the quality of credit (e.g. the literature on the bank lending channel) and have mostly abstracted from the notion of risk. Papers that consider risk (e.g., financial accelerator models in the spirit of Bernanke and Gertler, 1989) explore primarily how changes in the stance of monetary policy affects the riskiness of borrowers rather than the risk attitude of the banking system. In contrast, excessive risk-taking by financial intermediaries operating under limited liability and asymmetric information has been the focus of a large banking literature which, however, has largely ignored monetary policy.² This paper is an attempt to fill this gap.

We develop a model of financial intermediation where banks can engage in costly monitoring to reduce the credit risk in their loan portfolios. Monitoring effort and the pricing (i.e., interest rates) of bank assets and liabilities are endogenously determined and, in equilibrium, depend on a benchmark monetary policy rate. We obtain three main findings. First, for the case where a bank's capital structure is fixed exogenously, we find that the effects of monetary policy changes on bank monitoring and, hence, portfolio risk critically depend on a bank's leverage: a monetary easing will lead highly capitalized banks to monitor less, while the opposite is true for poorly capitalized banks.

¹See, for example, Rajan (2010), Taylor (2009), or Borio and Zhu (2008).

²Diamond and Rajan (2009) and Farhi and Tirole (2009) are recent exceptions, although these deal with the effects of expectations of a "macro" bailout rather than the implications of the monetary stance. Reviews of the older literature are in Boot and Greenbaum (1993), Bhattacharya, Boot, and Thakor (1998), and Carletti (2008).

We then endogenize banks' capital structures by allowing them to adjust their capital holdings in response to monetary policy changes. For this case we find that a cut in the policy rate will lead banks to increase their leverage. Reflecting this increase in leverage, our third main finding is that once leverage is allowed to be optimally chosen, a policy rate cut will unambiguously lower bank monitoring and increase risk taking.

These results are consistent with the evidence collected by a growing empirical literature on the effects of monetary policy on risk-taking (see, for example, Maddaloni and Peydró, 2010 and Ioannidou et al., 2009; Section 2 gives a brief survey). A negative relationship between bank risk and the real policy rate is also evident in data from the U.S. Terms of Business Lending Survey, as graphically illustrated in Figure ???. In this figure, bank risk is measured using the weighted average internal risk rating assigned to loans by banks from the U.S. Terms of Business Lending Survey³ and the real policy rate is measured using the nominal federal funds rate adjusted for consumer price inflation.⁴ Both variables are detrended by deducting their linear time trend and we use quarterly data from the second quarter of 1997 until the fourth quarter of 2008.

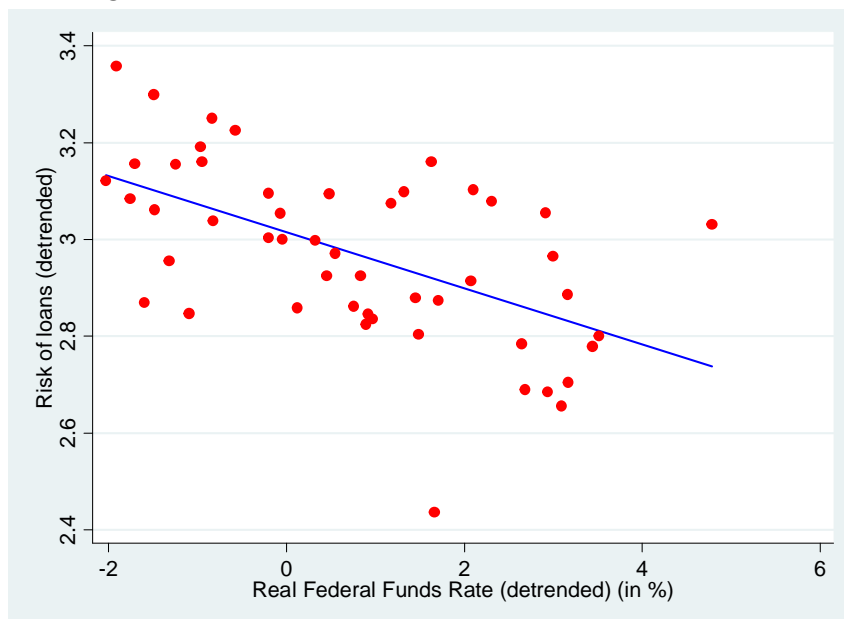
Our model is based on two standard assumptions. First, banks are protected by limited liability and choose the degree to which to monitor their borrowers or, equivalently, choose the riskiness of their portfolios. Since monitoring effort is not observable, a bank's capital structure has a bearing on its risk-taking behavior. Second, monetary policy affects the cost of a bank's liabilities through changes in the risk-free rate. Under these two assumptions, we show that the balance of three coexisting forces - interest-rate pass-through, risk shifting, and leverage - determines how monetary policy changes affect a bank's risk taking.

The first is a pass-through effect that acts through the asset side of a bank's balance sheet. In our model, monetary easing reduces the policy rate, which is then reflected in a reduction of the interest rate on bank loans. This, in turn, reduces the bank's gross return conditional on

³The U.S. Terms of Business Lending Survey, which is a quarterly survey on the terms of business lending of a stratified sample of about 400 banks conducted by the U.S. Federal Reserve Bank. The survey asks participating banks about the terms of all commercial and industrial loans issued during the first full business week of the middle month in every quarter. The publicly available version of this survey encompasses an aggregate version of the terms of business lending, disaggregated by type of banks. Loan risk ratings vary from 1 to 5 and are increasing in risk. We use the weighted average risk rating score aggregate across all participating banks as measure of bank risk.

⁴The effective federal funds rate is a volume-weighted average of rates on trades arranged by major brokers and calculated daily by the Federal Reserve Bank of New York using data provided by the brokers. As inflation rate we use the three-month average change in the U.S. consumer price index.

Figure 1: U.S. bank risk and the real federal funds rate



its portfolio repaying, reducing the incentive for the bank to monitor. This effect is akin to the portfolio reallocation effect present in portfolio choice models. In these models, when monetary easing reduces the real yield on safe assets, banks will typically increase their demand for risky assets.⁵

Second, there is a standard risk-shifting effect that operates through the liability side of a bank's balance sheet. Monetary easing lower the costs of a bank's liabilities. Everything else equal, this increases a bank's profit when it succeeds and thus creates an incentive to limit risk taking in order to reap those gains. The extent of this effect, however, depends critically on the degree of limited liability protection afforded to the bank.⁶ To see why, consider a fully leveraged bank that is financed entirely through deposits/debt. Under limited liability, this bank will suffer no losses in case of failure. A policy rate cut will increase the bank's expected net return on all assets by lowering the rate it has to pay on deposits. The bank can maximize this effect by reducing the risk of its portfolio, choosing a safer portfolios for which there is a higher probability the bank will have to repay depositors. In contrast, for a bank fully funded by capital, the effect of a decrease in the

⁵The exception would be banks with decreasing absolute risk aversion who, instead, would decrease their holdings of risky assets (Fishburn and Porter, 1976).

⁶This is similar to what happens in models that study the effects of competition for deposits on bank stability (Hellmann, Murdoch, and Stiglitz, 2000, Matutes and Vives, 2000, Cordella and Levy-Yeyati, 2003).

cost of its liabilities will, all other things equal, increase the expected net return uniformly across portfolios and have little or no effect on the bank's risk choices.

When banks' capital structures are exogenously determined, the net effect of a monetary policy change on bank monitoring depends on the balance of these two effects. This, in turn, depends on a bank's capital structure as well as the structure of the market in which it operates. The risk-shifting effect is stronger the more beneficial is the limited liability protection to the bank. This effect is therefore greatest for fully leveraged banks, and is lowest for banks with zero leverage who as a result have no limited liability protection. In contrast, the magnitude of the pass-through effect depends on how policy rate changes are reflected in changes to lending rates. Thus, the magnitude of this effect depends on the market structure of the banking industry: it is minimal in the case of a monopolist facing an inelastic demand function, when the pass-through onto the lending rate is zero; and it is maximal in the case of perfect competition, when lending rates fully reflect policy rate changes. It follows that the net effect of a monetary policy change may not be uniform across times, banking systems or individual banks. Following a policy rate cut, monitoring will decrease when leverage is low and increase when leverage is high. The position of this threshold level of leverage will, in turn, depend on the market structure of the banking industry.

By contrast, a third force comes into play once we allow banks to optimally modify their capital structure in response to a monetary policy change. On the one hand, banks have an incentive to be levered since holding capital is costly. On the other hand, capital serves as a commitment device to limit risk taking and helps reduce the cost of debt and deposits. Banks with limited liability tend to take excessive risk since they do not internalize the losses they impose on depositors and bondholders. Bank capital reduces this agency problem: the more the bank has to lose in case of failure, the more it will monitor its portfolio and invest more prudently. When investors cannot observe a bank's monitoring but can only infer its equilibrium behavior, higher capital (i.e., lower leverage) will lower their expectations of a bank's risk-taking and, thus, reduce the bank's cost of deposits and debt. Given that a policy rate cut reduces the agency problem associated with limited liability, it follows that the benefit from holding capital will also be reduced. In equilibrium, therefore, lower policy rates will be associated with greater leverage. This result provides a simple micro-foundation for the empirical regularities documented in recent papers, such as in Adrian and

Shin (2009). The addition of this leverage effect tilts the balance of the other two effects: all else equal, more leverage means more risk taking. Our model's unambiguous prediction when banks' capital structures are endogenous is consistent with the claim that monetary easing leads to greater risk taking.

Our contribution to the existing literature is twofold. First, we provide a model that isolates the effect of monetary policy changes on bank risk taking independently of other macroeconomic considerations related to asset values, liquidity provision, etc. The model provides a theoretical foundation for some of the regularities recently documented in the empirical literature, including the inverse relationship between monetary conditions and leverage, and the tendency for banks to load up on risk during extended periods of loose monetary policy. While our treatment of monetary policy is obviously minimal (we take monetary policy as exogenous and abstract from other effects linked to the macroeconomic cycle), our paper can help bridge the gap between macroeconomic and banking models. Second, our framework can help reconcile the somewhat dichotomous predictions of two important strands of research: the literature on the flight to quality and that on risk shifting linked to limited liability. The paper also contributes to the ongoing policy debate on whether macroprudential tools should complement monetary policy to safeguard macrofinancial stability. We discuss this issue further in the concluding section.

The paper proceeds as follows: Section 2 presents a brief survey of related theoretical and empirical work. Section 3 introduces the model and examines the equilibrium when the bank capital structure is exogenous. Section 4 solves the endogenous capital structure case. Section 5 examines the role of market structure, while Sections 6 and ?? present some numerical examples and stylized facts. Section 7 concludes. Proofs are mostly relegated to the appendix.

2 Related Literature

Our paper is related to a well established literature studying the effects of monetary policy changes on credit markets. The literature on financial accelerators posits that monetary policy tightening leads to more severe agency problems by depressing borrowers' net worth (see models in the spirit of Bernanke and Gertler, 1989, and Bernanke et al., 1996). The result is a flight to quality: firms more affected by agency problems will find it harder to obtain external financing. However, this

says little about the riskiness of the marginal borrower that obtains financing because monetary tightening increases agency problems across the board, not just for firms that are intrinsically more affected by agency problems. Thakor (1996) focuses on the quantity rather than the quality of credit. Yet, his model has implications for bank risk taking. In Thakor (1996), banks can invest in government securities or extend loans to risky entrepreneurs. The impact of monetary policy on the quantity of bank credit and thus on the riskiness of the bank portfolio depends on its relative effect on the bank intermediation margin on loans and securities. While the impact on portfolio risk is not explicitly studied, if monetary easing were to reduce the rate on securities more than that on deposits, the opportunity cost of extending loans would fall and the portion of a bank's portfolio invested in loans would increase; otherwise, the opposite would happen.

Rajan (2005) identifies in the “search for yield” a related, but different, mechanism through which monetary policy changes may affect risk taking. He argues that financial institutions may be induced to switch to riskier assets when a monetary policy easing lowers the yield on their short-term assets relative to that on their long-term liabilities. This is a result of limited liability. If yields on safe assets remain low for a prolonged period, continued investment in safe assets will mean that a financial institution will need to default on its long-term commitments. A switch to riskier assets (and higher yields) may increase the probability that it will be able to match its obligations. Dell’Ariccia and Marquez (2006a) find that when banks face an adverse selection problem in selecting borrowers, monetary policy easing may lead to a credit boom and lower lending standards. This is because banks’ incentives to screen out bad borrowers are reduced when their cost of funds is lowered.

More recently, Farhi and Tirole (2009) and Diamond and Rajan (2009) have examined the role of “macro bailouts” and collective moral hazard on banks’ liquidity decisions. When banks expect a strong policy response by the monetary authorities should a large negative shock occur (a mechanism often referred to as the “Greenspan put”), they will tend to take on excessive liquidity risk. This behavior, in turn, will increase the likelihood that the central bank will indeed respond to a shock by providing the necessary liquidity to the banking system. Unlike in this paper, their focus is on the reaction function of the central bank (the policy regime) rather than on the policy stance. Agur and Demertzis (2010) present a reduced form model of bank risk taking to focus on

how monetary policymakers should balance the objectives of price stability and financial stability. Drees et al. (2010) find that the relationship between the policy rate and risk taking depends on whether the primary source of risk is the opaqueness of a security or the idiosyncratic risk of the underlying investment.

Our paper also relates to a large theoretical literature examining the effects of limited liability, leverage, and deposit rates on bank risk taking behavior. Several papers (for example, Matutes and Vives, 2000, Hellmann, Murdoch, and Stiglitz, 2000, Cordella and Levy-Yeyati, 2000, Repullo, 2004, and Boyd and De Nicolò, 2005) have focused on how competition for deposits (i.e., higher deposit rates) exacerbates the agency problem associated with limited liability and may inefficiently increase bank risk taking.⁷ This effect is similar to the risk-shifting effect identified in this paper: more competition for deposits increases the equilibrium deposit rate, compressing intermediation margins and thus reducing a bank’s incentives to invest in safe assets.

The framework we use is based on Dell’Ariccia and Marquez (2006b) and Allen, Carletti, and Marquez (2010). In particular, the latter shows how banks may choose to hold costly capital to reduce the premium demanded by depositors. They, however, ignore the effects of monetary policy and do not examine how leverage moves in response to policy rate changes. Our result that leverage is decreasing in the policy rate is also related to that in Adrian and Shin (2008). In their paper, leverage is limited by the moral hazard induced by the underlying risks in the environment. In our model, an increase in the policy rate exacerbates the agency problem associated with limited liability, which in turn leads to a reduction in leverage.

Finally, there is a small, but growing, empirical literature that links monetary policy and bank risk taking. For example, Lown and Morgan (2006) show that credit standards in the U.S. tend to tighten following a monetary contraction. Similarly, Maddaloni and Peydró (2010) find that credit standards tend to loosen when overnight rates are lowered. Moreover, using Taylor rule residuals, they find that holding rates low for prolonged periods of time softens lending standards even further. Similarly, Altunbas et al. (2010) find evidence that “unusually” low interest rates over an extended period of time contributed to an increase in banks’ risk-taking. Jimenez et al. (2008) and Ioannidou, Ongena, and Peydró (2009) use detailed information on borrower quality

⁷Boyd and De Nicolò (2005) also show that when moral hazard on the borrowers side is taken into account, the result may be reversed.

from credit registry databases for Europe and Bolivia. They find a positive association between low interest rates at loan origination and the probability of extending loans to borrowers with bad or no credit histories (i.e., risky borrowers).

3 A Simple Model of Bank Risk Taking

Banks face a negatively sloped demand function for loans, $L(r_L)$, where r_L is the gross interest rate the bank charges on loans. We assume for tractability that the demand function is linear, $L = A - br_L$. In section 5, we examine the impact of alternative market structures.⁸

Loans are risky and a bank's portfolio needs to be monitored to increase the probability of repayment. The bank is endowed with a monitoring technology, allowing the bank to exert monitoring effort q which also represents the probability of loan repayment. This monitoring effort entails a cost equal to $\frac{1}{2}cq^2$ per dollar lent.⁹ An alternative interpretation of this assumption is that banks have access to a continuum of portfolios characterized by a parameter $q \in [0, 1]$, with returns $r_L - \frac{1}{2}cq$ and probability of success q .¹⁰

Banks fund themselves with two different types of liabilities. A portion k of a bank's liabilities represents a cost irrespective of the bank's profit, while a portion $1 - k$ is repaid only when the bank succeeds. Consistent with other existing models, k can represent the portion of bank assets financed with bank equity or capital. In this case, $1 - k$ would be interpreted as the fraction of the bank's portfolio financed by deposits. However, k can be also interpreted more generally as an inverse measure of the degree of limited liability protection accorded to banks. For now, we treat k as exogenous. In Section 4, we examine the case where banks can adjust k in response to a change in monetary policy.

For simplicity, we assume that the deposit rate is fixed and equal to the policy rate, $r_D = r^*$. (We will relax this assumption later.) This is consistent with the existence of deposit insurance,

⁸The assumption of a downward sloping demand curve for loans is supported by broad empirical evidence (e.g., Den Haan, Sumner, and Yamashiro, 2007). More generally, the pass-through will depend on the cost structure of bank liabilities, including the proportion of retail versus wholesale deposits (Flannery, 1982). Berlin and Mester (1999) show that markups on loans decrease as market rates increase, implying that as market rates increase, there is less than a one for one increase in loan rates.

⁹For a model in the same spirit but where banks choose among portfolios with different risk/return characteristics, see Cordella and Levy-Yeyati (2003).

¹⁰This latter interpretation correspond to the classic risk shifting problem between bondholders and shareholders, in that shareholder can choose between investments that have a lower probability of success, but that payoff more conditional on success.

for instance. Equity, however, is more costly, with a yield $r_E = r^* + \xi$, with $\xi \geq 0$, which is consistent with an equity premium as a spread over the risk-free rate. Alternatively, the cost r_E can be interpreted as the opportunity cost for shareholders of investing in the bank.¹¹

We structure the model in two stages. For a fixed policy rate r^* , in stage 1 banks choose the interest rate to charge on loans, r_L . In the second stage, banks then choose how much to monitor their portfolio, q .

3.1 Equilibrium when Leverage is Exogenous

We solve the model by backward induction, starting from the last stage. The bank's expected profit can be written as:

$$\Pi = \left(q(r_L - r_D(1 - k)) - r_E k - \frac{1}{2} c q^2 \right) L(r_L), \quad (1)$$

which reflects the fact that the bank's portfolio repays with probability q . When the bank succeeds, it receives a per-loan payment of r_L and earns a return $r_L - r_D(1 - k)$ after repaying depositors. When it fails, it receives no revenue, but, because of limited liability, does not need to repay depositors. The term $r_E k$ represents the cost of equity to the bank or, equivalently, the opportunity cost of bank shareholders, which is borne irrespective of the bank's revenue.

Taking the loan rate r_L as given, the first order condition for bank monitoring can be written as

$$\frac{\partial \left(q(r_L - r_D(1 - k)) - r_E k - \frac{1}{2} c q^2 \right)}{\partial q} L(r_L) = 0,$$

which implies

$$\hat{q} = \min \left\{ \frac{r_L - r_D(1 - k)}{c}, 1 \right\}. \quad (2)$$

Since $r_D = r^*$, we obtain immediately from (2) that the direct (i.e., for a given lending rate) effect of a policy rate hike on bank monitoring is non-positive, $\frac{\partial \hat{q}}{\partial r^*} \leq 0$. This is consistent with most of the literature on the effects of deposit competition on risk taking (see for example Hellmann et al., 2000). One way to interpret this result is that the short-term incentives banks with severe maturity mismatches have to monitor will be reduced by an unexpected increase in the policy rate.

¹¹We assume that the premium on equity, ξ , is independent of the policy rate r^* . This is consistent with our goal to isolate the effect of an exogenous change in the stance of monetary policy. However, from an asset pricing perspective these are likely to be correlated through underlying common factors which may drive the risk premium as well as the risk free rate. Our results continue to hold as long as the within period correlation between ξ and r^* is sufficiently different from (positive) one.

We can now solve the first stage of the model where banks choose the loan interest rate. Assuming that an interior solution exists, we substitute \hat{q} into the expected profit function and obtain:¹²

$$\Pi(\hat{q}) = \left(\frac{(r_L - r_D(1-k))^2}{2c} - r_E k \right) L(r_L). \quad (3)$$

Maximizing (3) with respect to the loan rate yields the following first order condition:

$$\frac{\partial \Pi(\hat{q})}{\partial r_L} = L(r_L) \frac{r_L - r_D(1-k)}{c} + \frac{\partial L(r_L)}{\partial r_L} \frac{(r_L - r_D(1-k))^2}{2c} - r_E k \frac{\partial L(r_L)}{\partial r_L} = 0. \quad (4)$$

From (4) we obtain our first result.

Proposition 1 *There exist a degree of capitalization, \tilde{k} , such that, for $k < \tilde{k}$, bank monitoring decreases with the monetary policy rate, $\frac{d\hat{q}}{dr^*} < 0$, while for $k > \tilde{k}$ it increases with the policy rate, $\frac{d\hat{q}}{dr^*} > 0$.*

The intuition behind this result is that a tightening of monetary policy leads to an increase in both the interest rate a bank charges on its loans and that it pays on its liabilities. The first effect, which reflects the pass-through of the policy rate on loan rates, increases the incentives to monitor. The second effect, the risk-shifting effect, decreases monitoring incentives to the extent that it applies to liabilities that are repaid only in case of success. Indeed, from 2 is evident that an increase in the cost of capital affects the bank's monitoring effort only through its effect on the lending rate. Thus, for a bank funded entirely through capital, the risk-shifting effect disappears. In contrast, an increase in the interest rate on deposits will also have a direct negative impact on \hat{q} . In addition, a tightening of monetary policy leads to a compression of the intermediation margins, $r_L - r_D$. Thus, for a bank entirely funded with deposits, the risk-shifting effect will dominate. In between the two extremes of full leverage or zero leverage, the bank's capital structure determines the net effect of a monetary policy change on risk taking. Banks with a higher leverage ratio will react to a monetary policy tightening by taking on more risk, while those with a lower leverage ratio will do the opposite.

¹²It is straightforward to see that there always exist values of c that guarantee an interior solution for q . Later, we demonstrate numerically that an interior solution to the full model, where also bank leverage (k) is endogenous, exists. In other words, there is a wide range of parameter values for which the first order conditions characterize the equilibrium.

It is worth noting that the results so far are obtained under the assumption that the pricing of deposits is insensitive to risk (i.e., q), but does reflect the underlying policy rate r^* . This would be consistent with the existence of deposit insurance, so that depositors are not concerned about being repaid by the bank, but nevertheless want to receive a return that compensates them for their opportunity cost, which would be incorporated in the policy rate r^* .¹³ In what follows, we show that the result in Proposition 1 is not driven by depositors' insensitivity to risk, but rather by the bank's optimizing behavior given its desire to maximize its expected return, which incorporates not only the return conditional on success but also the probability of success.

Assume now that depositors must be compensated for the bank's expected risk taking. Depositors cannot directly observe q . However, from observing the capital ratio k they can infer the bank's equilibrium monitoring behavior, \hat{q} . Given an opportunity cost of r^* , depositors will demand a promised repayment r_D such that $r_D E[q|k] = r^*$, or in other words $r_D = \frac{r^*}{E[q|k]}$. The timing is as before, with the additional constraint that depositors' expectations about bank monitoring, $E[q|k]$, must in equilibrium be correct, so that $E[q|k] = \hat{q}(r_D|k)$. It is worth noting that this introduces an incentive for the bank to hold some capital. Equity is more expensive than deposits (or debt), but it allows the bank to commit to a higher q and thus reduces the yield investors demand on instruments subject to limited liability (i.e. debt or deposits). We exploit this aspect further in the next section.

We can now state the following result, which parallels that in Proposition 1.

Proposition 2 *Suppose that depositors require compensation for risk, so that $r_D = \frac{r^*}{E[q|k]}$. Then there exist a degree of capitalization, \tilde{k} , such that, for $k < \tilde{k}$, bank monitoring decreases with the monetary policy rate, $\frac{d\hat{q}}{dr^*} < 0$, while for $k > \tilde{k}$ it increases with the policy rate, $\frac{d\hat{q}}{dr^*} > 0$.*

4 Endogenous Capital Structure

So far, we have assumed that the bank's degree of leverage or capitalization is exogenous. This setting could apply, for instance, to the case of individual banks that would optimally like to choose a level of capital below some regulatory minimum. For such banks, changes in the policy

¹³Keeley (1990) formally shows that when deposits are fully protected by deposit insurance, the supply of deposits will not depend on bank risk.

rate would not be reflected in their capitalization decisions since the regulatory constraint would be binding. In this section, we extend the model to allow for an endogenous capital structure and contrast our results with those above for the case of exogenous leverage. As capital structure will be endogenous, we adopt the framework introduced at the end of the previous section and allow unsecured investors to demand compensation for the risk they expect to face (in other words, we eliminate deposit insurance).¹⁴

Specifically, consider the following extension to the model. At stage 1, banks choose their desired capitalization ratio k . At stage 2, unsecured investors observe the bank's choice of k and set the interest rate they charge on the bank's liabilities. The last two stages are as before in that banks choose the lending interest rate and then the extent of monitoring.

4.1 Equilibrium

As before, we solve the model by backward induction. The solutions for the last two stages are analogous to those in the previous section. At stage three, unsecured investors will demand a promised return of $r_D = \frac{r^*}{E[q|k]}$. As we show below, this provides the bank with an incentive to hold some capital to reduce the cost of borrowing. Formally, the objective function is to maximize bank profits with respect to the capital ratio k :

$$\max_k \Pi = \left(\hat{q}(\hat{r}_L - r_D(1 - k)) - r_E k - \frac{1}{2} c \hat{q}^2 \right) L(\hat{r}_L),$$

subject to

$$r_D = \frac{r^*}{E[q|k]},$$

where $\hat{q} = \hat{q}(r_L; k)$ is the equilibrium choice of monitoring induced by the bank's choice of the loan rate r_L and capitalization ratio k , and $\hat{r}_L = \hat{r}_L(k)$ is the optimal loan rate given k . In other words, the bank takes into account the influence of its choice of k on its subsequent loan pricing and monitoring decisions.

The first order condition for k can be expressed as

$$\frac{d\Pi}{dk} = \frac{\partial \Pi}{\partial k} + \frac{\partial \Pi}{\partial r_L} \frac{dr_L}{dk} + \frac{\partial \Pi}{\partial q} \frac{dq}{dk} = \frac{\partial \Pi}{\partial k} = 0$$

¹⁴In practice, it may be more realistic to assume that some fraction of bank liabilities are insured or insensitive to risk, while the remaining fraction are uninsured so that their pricing must reflect the expected amount of risk, such as for subordinated debt. Allowing for these two kinds of liabilities in no way affects our results, as we illustrate in Section 6, where we incorporate both insured and uninsured deposits into our numerical examples.

since the last two terms are zero from the envelope theorem. Substituting, this becomes

$$\frac{d\Pi}{dk} = \left((r_L - q) \frac{\partial q}{\partial k} - (r_E - r^*) \right) L(r_L) = 0,$$

which characterizes the bank's optimal choice of \hat{k} . As we show below in Proposition 3, \hat{k} is strictly positive for a broad range of parameter values.

We can now use this to establish the following result.

Proposition 3 *Equilibrium bank leverage decreases with the monetary policy rate: $\frac{d\hat{k}}{dr^*} > 0$.*

The proposition establishes that, when an internal solution \hat{k} for the capitalization ratio exists, then \hat{k} will be increasing in r^* . Put differently, a low monetary policy rate will induce banks to be more leveraged (i.e., to hold less capital).

A policy rate hike increases the rate the bank has to pay on its debt liabilities and exacerbates the bank's agency problem - note that at $r^* = 0$, a limit case where the principal is not repaid at all, there is no moral hazard and $\hat{q} = q^*$. This effect is essentially the same as in the flight-to-quality literature (see for example Bernanke et al., 1989). It follows that as the policy rate increases so does the benefit from holding capital, the only commitment device available to the bank to reduce moral hazard. Put differently, investors will allow banks to be more levered when the policy rate is low relative to when it is high. A similar result is in Adrian and Shin (2008), where leverage is a decreasing function of the moral hazard induced by the underlying risks in the environment, and evidence of this behavior is documented in Adrian and Shin (2009).

The following result characterizes banks' loan pricing decisions as a function of the monetary policy rate, and will be useful in establishing the next main result.

Lemma 1 *When bank leverage, the loan rate, and the level of monitoring are all optimally chosen with respect to the monetary policy rate r^* , the optimal loan rate \hat{r}_L is increasing in r^* : $\frac{d\hat{r}_L}{dr^*}$.*

The intuition for the lemma is straightforward: when the monetary policy rate increases, this raises the opportunity cost on all forms of financing. Consequently, in equilibrium the rate that the bank charges on any loans also increases. In other words, there is at least some pass through of the changes in the bank's costs of funds onto the price of (bank) credit, which is reflected in a higher loan rate.

We can now state our next main result:

Proposition 4 *When bank leverage is optimally chosen to maximize profits, monitoring will always increase with the monetary policy rate: $\frac{d\hat{q}}{dr^*} > 0$.*

In contrast to the result in Proposition 1, when bank leverage is endogenous we have that bank monitoring always increases when the policy rate r^* increases. Relative to the case where leverage is exogenous, here monetary policy tightening affects bank monitoring through the additional channel of a decrease in leverage, as per Proposition 3. Proposition 4 complements this result along the dimension of bank monitoring, so that the aggregate effect of an increase in the monetary policy rate is for banks to be less levered and to take less risk (i.e., monitor more). Conversely, reductions in r^* that would accompany monetary easing should lead to more highly levered banks and reduced monitoring effort.

It bears emphasizing that the clear cut effect of a change in the monetary policy rate arises only when banks are able to adjust their capital structures (i.e., k) in response to changes in r^* . Changes in bank leverage are, therefore, an important additional channel through which changes in monetary policy affect bank behavior. Moreover, Proposition 4 shows that the leverage effect can be sufficiently strong to overturn the direct effect on bank risk taking identified in Proposition 2 for the case where leverage is exogenously given. At the same time, to the extent that some banks may be constrained by regulation from adjusting their capital structures (for instance, if their optimal capital holdings are below the minimum mandated by capital adequacy regulation), we may in practice observe cross sectional differences in banks' reactions to monetary policy shocks.

5 Extension: The role of market structure

This section examines the effect of alternative assumptions on the structure of the loan market. We look at two diametrically opposed cases: First, a perfectly competitive credit market, where banks take the lending rate as given, which is determined by market clearing and a zero profit condition for the banks; and second, a monopolist facing a loan demand function that is perfectly inelastic up to some fixed loan rate R . This upper limit can be interpreted as either the maximum return on projects, or as the highest rate consistent with borrowers satisfying their reservation utilities.

Under these two extreme structures, we show that our results when leverage is endogenous continue to hold qualitatively. Specifically, when the capital ratio k is endogenously determined, the leverage effect dominates and monetary easing will increase bank risk taking. If banks are unable to adjust their capital structures, however, the loan market structure does matter for how monetary policy affects risk taking. Intuitively, the pass-through of the monetary policy rate on lending rates is higher the more competitive is the market. It follows that intermediation margins are less sensitive to monetary policy changes in more competitive markets. And this, in turn, results in a diminished risk shifting effect and consequently a smaller region of leverage for which monetary easing causes risk taking to decrease.

5.1 The Perfect Competition Case

Consider the following modification of our model to incorporate perfect competition. At stage 1, given a fixed policy rate, the lending rate is set competitively so that banks make zero expected profits in equilibrium. At stage 2, banks choose their desired leverage (or capitalization) ratio k . At stage 3, unsecured investors observe the bank's choice of k and set the interest rate they charge on the bank's liabilities, r_D . And in the last stages, as before, banks choose the extent of monitoring.

Again, we solve the model by backward induction. As for the case where banks have market power analyzed in Sections 3 and 4, solving for the equilibrium monitoring and imposing $r_D = \frac{r^*}{E[q|k]}$ implies, as before

$$\hat{q} = \frac{r_L + \sqrt{r_L^2 - 4cr^*(1-k)}}{2c}. \quad (5)$$

We first consider the case where k is exogenous. For this case, we impose a zero profit condition,

$$\hat{\Pi} = L \left(\hat{q}r_L - r^* - k\xi - \frac{c}{2}\hat{q}^2 \right) = 0,$$

to obtain r_L as a function of r^* and k . We can now state the following result.

Proposition 5 *In a perfectly competitive market, for a fixed capitalization ratio k , bank monitoring increases with the monetary policy rate, $\frac{d\hat{q}}{dr^*} > 0$, for $k \in (0, 1]$, with $\frac{d\hat{q}}{dr^*} = 0$ for $k = 0$.*

This result contrasts with that obtained in Propositions 1 and 2 for the case where banks have market power. There, the effect of a change in monetary policy on risk taking depended on the

degree of bank capitalization, k , with decreased risk taking as the monetary policy rate increases for a sufficiently low level of k . Here $\frac{d\hat{q}}{dr^*}$, the bank's response to changes in monetary policy in terms of monitoring, remains non-negative over its entire range, although it is still increasing in k . This result stems from the fact that the pass-through of the policy rate onto the loan rate is maximum in the case of perfect competition, and must perfectly reflect the increase in the policy rate. It follows that the pass-through effect dominates the risk-shifting effect, so that the region where $\frac{d\hat{q}}{dr^*} < 0$ disappears.

We next endogenize the capital ratio k , as in Section 4. Banks maximize

$$\max_k \Pi = L \left(\hat{q} r_L - r^* - k\xi - \frac{c}{2} \hat{q}^2 \right),$$

which for the case of perfect competition gives

$$\hat{k} = 1 - r_L^2 \frac{\xi (r^* + \xi)}{cr (r^* + 2\xi)^2}.$$

To obtain the lending rate, we impose the zero profit condition for banks:

$$\hat{\Pi} = L \left(\hat{q}(\hat{k}) r_L - r^* - \hat{k}\xi - \frac{c}{2} \hat{q}^2(\hat{k}) \right) = 0. \quad (6)$$

From (6) we can solve for the equilibrium lending rate, capital, and monitoring as: $\hat{r}_L = \sqrt{\frac{2cr^*(r^*+2\xi)^2}{3r^*\xi+(r^*)^2+2\xi^2}}$,

$\hat{k} = \frac{r^*\xi+(r^*)^2}{3r^*\xi+(r^*)^2+2\xi^2}$, and $\hat{q} = \sqrt{\frac{r^*4(r^*+\xi)^2}{2c(3r^*\xi+(r^*)^2+2\xi^2)}}$. From these we immediately obtain the following result.

Proposition 6 *In a perfectly competitive market, equilibrium bank leverage decreases with the monetary policy rate: $\frac{d\hat{k}}{dr^*} > 0$. And, when bank leverage is optimally chosen to maximize profits, monitoring will always increase with the monetary policy rate: $\frac{d\hat{q}}{dr^*} > 0$.*

This result extends Propositions 3 and 4 to the case of perfect competition and establishes that even when credit markets are perfectly competitive, monetary easing in equilibrium lead banks to both hold less capital and take on more risk once one incorporates banks' ability to adjust their optimal leverage ratios.

5.2 A Monopolist Facing Inelastic Demand

Here, we assume as in the main part of the paper that banks can choose the interest rate to charge, but also that there is a fixed demand for loans, L , as long as the lending rate does not exceed a

fixed value of R . This setting can be interpreted as one where each borrower has a unit demand for loans and R is the borrower's reservation loan rate. Demand becomes zero for $r_L > R$. This eliminates any pricing effects on loan quantity and allows us to focus on a case where the loan rate is not responsive to changes in the cost of funding since, given the fixed, inelastic demand, it will always be set at the maximum value of $\hat{r}_L = R$.

We can solve for \hat{q} , imposing the condition that $r_D = \frac{r^*}{E[q|k]}$, and obtain

$$\hat{q} = \frac{R + \sqrt{R^2 - 4cr^*(1-k)}}{2c}, \quad (7)$$

from which we can state the following claim.

Claim 1 For $k \in [0, 1)$ fixed, a monopolist bank facing a demand function that is perfectly inelastic for $r_L \leq R$ will always decrease monitoring when the policy rate is raised: $\left. \frac{d\hat{q}}{dr} \right|_k < 0$. For $k = 1$, $\left. \frac{d\hat{q}}{dr} \right|_k = 0$.

Proof: From 7 we can immediately write:

$$\frac{d\hat{q}}{dr} = -\frac{1-k}{\sqrt{R^2 - 4cr^*(1-k)}} < 0.$$

■

This result stands in stark contrast with what we obtained in Proposition 5 for the case of perfect competition when leverage is exogenous. There, irrespective of the level of leverage, risk taking was always decreasing in the policy rate. Here, risk taking is always increasing in the policy rate. The difference stems precisely from the extent to which the bank passes onto the lending interest rate changes in its costs stemming from changes in the policy rate. If demand is inelastic, the pass-through is zero as the lending rate is always held at its maximum, R , and thus cannot adjust further when the monetary policy rate changes. Therefore, the impact of a change in the policy rate on monitoring, \hat{q} , operates solely through the liability side of the bank balance sheet, reducing the bank's return in case of success and leading it to monitor less. Put differently, there is only a risk-shifting effect. By contrast, in the perfect competition case the pass-through is at its maximum and the impact of a change in r^* on the lending rate dominates the risk shifting effect.

This result holds in a more general setting. For example, in our main model it can be shown that the leverage threshold below which a monetary policy tightening leads to an increase in risk-taking is lower the flatter is the loan demand function. Again, as demand becomes more elastic - which can be interpreted as the market becoming more competitive - the interest rate pass-through increases, making the net effect of a change in the policy rate on monitoring more positive.¹⁵

To study the effect of a change in monetary policy when the monopolist bank can choose the capitalization ratio k , we maximize bank profits with respect to k :

$$\max_k \Pi = L \left(\widehat{q}R - r - k\xi - \frac{c}{2}\widehat{q}^2 \right).$$

This gives the first order condition

$$-\frac{r^*}{2} - \xi + \frac{r^*R}{2\sqrt{R^2 - 4cr^*(1-k)}} = 0,$$

that has solution

$$\widehat{k} = 1 - R^2 \frac{\xi(r^* + \xi)}{cr^*(r^* + 2\xi)^2}. \quad (8)$$

We can substitute the solution \widehat{k} back into the formula for \widehat{q} to obtain

$$\widehat{q} = R \frac{(r + \xi)}{c(r + 2\xi)}. \quad (9)$$

It is now immediate that Proposition 6 extends to this case: $\frac{d\widehat{k}}{dr^*} > 0$ and $\frac{d\widehat{q}}{dr^*} > 0$ when the bank can adjust its target capital ratio in response to a change in the monetary policy rate.

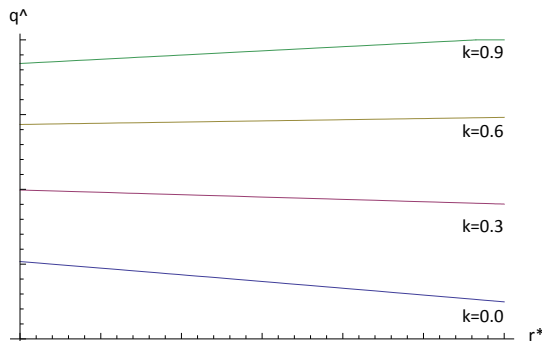
6 A Numerical Example

In this section, we present some simple numerical simulations of the model. The purpose is twofold. First, we want to provide an intuitive graphical illustration of the effects identified in this paper. Second, since most of our analysis relies on internal solutions for several of the choice variables in the model, the example serves to demonstrate that there is a broad set of parameter values for which such solutions indeed exist.

For the linear demand function $L = A - br_L$ as above, we assume that $A = 100$ and $b = 8$. We also assume that 35 percent of the bank's liabilities consist of insured deposits and the rest is

¹⁵ A formal proof for this result can be obtained on request from the authors.

Figure 2: Bank monitoring, \hat{q} , as a function of the monetary policy rate r^* for different values of bank capitalization, k .



uninsured and therefore must be priced to reflect its risk. This is to provide some realism to the numbers and also to cover both cases considered in our analysis. Finally, we set the monitoring cost parameter $c = 9$ and the equity premium, ξ , to 6 percent.¹⁶

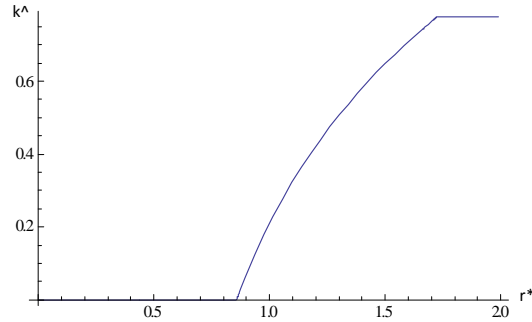
Figure 2 illustrates Proposition 1. The equilibrium probability of loan repayment for different levels of k is plotted as a function of the policy rate. The chart covers a broad range of real interest rate values (from negative 10 percent to positive 20 percent) encompassing the vast majority of realistic cases. From this picture it is easy to see how the response of a bank's risk taking to a change in the monetary policy rate depends on its capitalization. For low levels of k , bank monitoring \hat{q} decreases with the policy rate r^* , and the opposite happens at high levels.¹⁷

When we allow the bank to change its target leverage ratio, an additional effect emerges and the short-term ambiguity in the relationship between risk-taking and the policy rate is resolved. As the policy rate increases, so does the agency problem associated with limited liability. The bank's response is to decrease its leverage ratio to limit the increase in the interest rate it has to pay on its uninsured liabilities. Figure 3 describes this relationship. The equilibrium leverage ratio is plotted against the real policy interest rate. Note that, for illustrative purposes, the chart covers an extremely wide range of interest rates from minus 100 percent to plus 100 percent, which are well beyond what typically occurs in practice. At extremely low values of the policy rate (below minus 15 percent), the agency problem is sufficiently small that the bank finds it optimal to be fully

¹⁶ An equity premium of 6 percent is consistent with the historical average spread between U.S. stock returns and risk-free interest rate as reported in Mehra and Prescott (1985).

¹⁷ In our numerical example, the threshold value for k at which the relationship between the policy rate and bank risk taking reverses is about 0.55, which is a fairly high capitalization ratio in practice.

Figure 3: Optimal bank capitalization, \hat{k} , as a function of the real policy interest rate r^* .



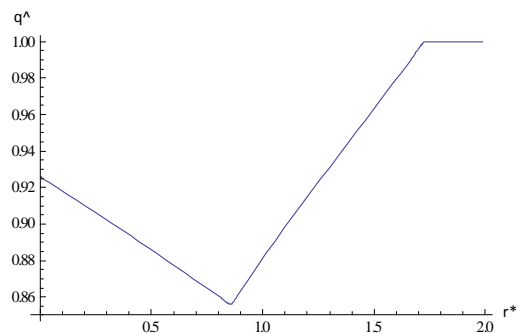
levered (more technically, k hits the zero-lower-bound corner solution). For more realistic ranges of the interest rate, the model admits an internal solution and bank capital k increases with the policy interest rate. However, the slope of this relationship is decreasing in the policy rate. Eventually, the relationship becomes flat once it hits its upper bound (this corresponds to $\hat{q}(k) = 1$, see below).

Figure 4 illustrates the relationship between the bank's monitoring effort/probability of repayment and the real policy rate for the case with endogenous leverage. For extremely low values of the real policy rate (exactly the values for which $\hat{k} = 0$), bank monitoring \hat{q} is decreasing in the policy rate. The intuition is straightforward. At these levels \hat{k} is in a corner (at zero) and does not move when the policy rate changes. It follows that the result related to a fixed capital structure applies. And since $\hat{k} = 0$, we obtain that $\frac{d\hat{q}}{dr^*} = \frac{d\hat{q}}{dr^*} \Big|_{k=0} < 0$. For the most realistic real policy rate range between minus 10 percent and plus 20 percent, \hat{q} admits an internal solution and is increasing in r^* . Eventually, at a very high real interest rate (about 80 percent), \hat{q} hits its upper bound, which is exactly when the relationship between \hat{k} and r^* becomes flat.

7 Discussion and Conclusions

This paper provides a theoretical foundation for the claim that prolonged periods of easy monetary conditions increase bank risk taking. In our model, the net effect of a monetary policy change on bank monitoring (an inverse measure of risk taking) depends on the balance of three forces: interest rate pass-through, risk shifting, and leverage. When banks can adjust their capital structures, a monetary easing leads to greater leverage and lower monitoring. However, if a bank's capital

Figure 4: Equilibrium bank monitoring, $\hat{q}(k)$, as a function of the real policy interest rate.



structure is instead fixed, the balance will depend on the degree of bank capitalization: when facing a policy rate cut, well capitalized banks will decrease monitoring, while highly levered banks will increase it. Further, the balance of these effects will depend on the structure and contestability of the banking industry, and is therefore likely to vary across countries and over time.

There are several potential extensions to our analysis that are useful to discuss. First, we model monetary policy decisions as exogenous changes in the real yield on safe assets. Of course, this is an approximation. In particular, we abstract from how central banks respond to the economic cycle and inflation pressures when choosing their policy stance. The next step should be to take into account the role of the interaction of the monetary policy stance with the real cycle in determining bank risk-taking. A promising avenue in this direction may be to augment the model to examine how borrowers' incentives change over the cycle.

Another important simplifying assumption is that the cost of equity is independent from the bank's leverage. Yet, our results would continue to hold in a more complex setting where the required return to equity is increasing in the degree of bank leverage. In this case, it is straightforward to see that, everything else equal, equilibrium leverage would be lower than in our base model since an increase in capitalization would have the additional benefit of reducing equity costs. Also, leverage would continue to be decreasing in the policy rate, although the exact shape of this relationship would depend on the functional form assumed for the cost of equity as a function of leverage.

A third simplification in the paper is that we focus on credit risk and abstract from other

important aspects of the relationship between monetary policy and risk taking, such as liquidity risk.¹⁸ While other frameworks may be better suited to study this issue (see, for example, Farhi and Tirole, 2009, and Stein, 2010), our model could be adapted to capture risks on the liability side of the bank's balance sheet. For instance, banks might choose to finance themselves through expensive long-term debt instruments or cheaper short-term deposits, which, however, carry a greater liquidity risk. In that context, the trade-off for a bank would be between a wider intermediation margin and a greater risk of failure should a liquidity run ensue. Hence, dynamics similar to those in this paper could be obtained. We leave all these extensions to future research.

The model has clear testable implications. First, in situations where banks are relatively unconstrained in raising capital and can adjust their capital structures, the model predicts a negative relationship between the policy rate (in real terms) and measures of bank risk. Second, in situations where banks face constraints, such as when their desired capital ratios are already below regulatory minimums for capital regulation, this negative relationship between the policy rate and bank risk is less pronounced for poorly capitalized banks and in less competitive banking markets. Third, the model predicts a negative relationship between the policy rate and bank leverage. While we provide some simple empirical evidence in support of a negative relationship between policy rate and bank risk, and policy rate and leverage, we leave more rigorous empirical analysis of these relationships to future research.

The findings in this paper bear on the debate about how to integrate macro-prudential regulation into the monetary policy framework to meet the twin objectives of price and financial stability (see, for example, Blanchard et al., 2009). Whether a trade off between the two objectives emerges will depend on the type of shocks the economy is facing. For instance, no trade-off between price and financial stability may exist when an economy nears the peak of a cycle, when banks tend to take the most risks and prices are under pressure. Under these conditions, monetary tightening will decrease leverage and risk taking and, at the same time contain price pressures. In contrast, a trade-off between the two objectives would emerge in an environment, such as that in the runup to the current crisis, with low inflation but excessive risk taking. Under these conditions, the policy

¹⁸ A growing literature focuses on funding liquidity risk of banks and the adverse liquidity spirals that such risk could generate in the event of negative shocks (see Diamond and Rajan, 2008; Brunnermeier and Pedersen, 2009; and Acharya and Viswanathan, 2010) and on the role of monetary policy in altering bank fragility in the presence of liquidity risk (Acharya and Naqvi, 2010; and Freixas et al., 2010).

rate cannot deal with both objectives at the same time: Tightening may reduce risk-taking, but will lead to an undesired contraction in aggregate activity and/or to deflation. Other (macroprudential) tools are then needed.

In this context, the potential interaction between banking market conditions, monetary policy decisions, and bank risk-taking implied by our analysis can be seen as an argument in favor of the centralization of macro-prudential responsibilities within the monetary authority. And the complexity of this interaction points in the same direction. How these benefits balance with the potential for lower credibility and accountability associated with a more complex mandate and the consequent increased risk of political interference is a question for future research.

8 Appendix

Proof of Proposition 1: Since $\hat{q} = \frac{r_L - r^*(1-k)}{c}$, $\frac{d\hat{q}}{dr^*} = \frac{1}{c} \left(\frac{d\hat{r}_L}{dr^*} - (1-k) \right)$. To find $\frac{d\hat{r}_L}{dr^*}$, start by substituting $\hat{q} = \frac{r_L - r^*(1-k)}{c}$ into the expected profit function, and given $r_E = r^* + \xi$, we obtain

$$\Pi = \left(q(r_L - r^*(1-k)) - r_E k - \frac{1}{2} c q^2 \right) L(r_L) \quad (10)$$

$$= \left(\frac{r_L - r^*(1-k)}{c} (r_L - r^*(1-k)) - (r^* + \xi) k - \frac{1}{2} c \left(\frac{r_L - r^*(1-k)}{c} \right)^2 \right) L(r_L) \quad (11)$$

$$= \left(\frac{1}{2c} (r_L - r^*(1-k))^2 - k(\xi + r^*) \right) L(r_L) \quad (12)$$

The first order condition with respect to r_L is

$$\frac{\partial \Pi}{\partial r_L} = \frac{1}{c} (r_L - r^*(1-k)) L(r_L) + \frac{\partial L(r_L)}{\partial r_L} \left(\frac{1}{2c} (r_L - r^*(1-k))^2 - k(\xi + r^*) \right) = 0.$$

Define the identity $G \equiv \frac{\partial \Pi}{\partial r_L} = 0$. We can now use the Implicit Function Theorem, that $\frac{d\hat{r}_L}{dr^*} = -\frac{\frac{\partial G}{\partial r^*}}{\frac{\partial G}{\partial r_L}}$.

For the denominator, differentiate G with respect to r_L to get the following second order condition:

$$\begin{aligned} \frac{\partial G}{\partial r_L} &= \frac{1}{c} L(r_L) + \frac{1}{c} (r_L - r^*(1-k)) \frac{\partial L(r_L)}{\partial r_L} + \frac{\partial^2 L(r_L)}{\partial r_L^2} \left(\frac{1}{2c} (r_L - r^*(1-k))^2 - k(\xi + r^*) \right) \\ &\quad + \frac{\partial L(r_L)}{\partial r_L} \frac{1}{2c} (r_L - r^*(1-k)) \end{aligned}$$

Since $\frac{\partial^2 L(r_L)}{\partial r_L^2} = 0$, this becomes

$$\frac{\partial G}{\partial r_L} = \frac{1}{c} L(r_L) + \frac{\partial L(r_L)}{\partial r_L} \frac{3}{2c} (r_L - r^*(1-k)).$$

We can rewrite the FOC with respect to r_L as

$$L(r_L) = -\frac{\partial L(r_L)}{\partial r_L} \left(\frac{1}{2} (r_L - r^*(1-k)) - \frac{k(\xi + r^*)}{\frac{1}{c} (r_L - r^*(1-k))} \right), \quad (13)$$

and substitute into $\frac{\partial G}{\partial r_L}$ to obtain

$$\frac{\partial G}{\partial r_L} = \frac{1}{c} \frac{\partial L(r_L)}{\partial r_L} \left(r_L - r^*(1-k) + c \frac{k}{r_L - r^*(1-k)} (\xi + r^*) \right) < 0,$$

which establishes the second order condition as negative.

We can now differentiate G with respect to r^* .

$$\frac{\partial G}{\partial r^*} = -\frac{1}{c} (1-k) L(r_L) - \frac{1}{c} \frac{\partial L(r_L)}{\partial r_L} ((r_L - r^*(1-k))(1-k) + ck).$$

Using again the first order condition expressed as in (13), we can substitute this into the above to get

$$\frac{\partial G}{\partial r^*} = -\frac{\partial L(r_L)}{\partial r_L} \left((1-k) \left(\frac{\frac{1}{2c} (r_L - r^*(1-k))^2 + k(\xi + r^*)}{(r_L - r^*(1-k))} \right) + k \right) > 0,$$

which, combined with the fact that $\frac{\partial G}{\partial r_L} < 0$, establishes that $\frac{d\hat{r}_L}{dr^*} = -\frac{\frac{\partial G}{\partial r^*}}{\frac{\partial G}{\partial r_L}} > 0$. Clearly, as $k \rightarrow 0$, the expression for $\frac{\partial G}{\partial r^*}$ converges to

$$\frac{\partial G}{\partial r^*} = -\frac{\partial L(r_L)}{\partial r_L} \frac{1}{2c} (r_L - r^*) > 0.$$

To sign $\frac{d\hat{q}}{dr^*}$, however, we need to compare $\frac{d\hat{r}_L}{dr^*}$ to 1:

$$\begin{aligned} \left. \frac{d\hat{r}_L}{dr^*} \right|_{k=0} &= -\frac{\frac{\partial G}{\partial r^*}}{\frac{\partial G}{\partial r_L}} = -\frac{-\frac{1}{c} \frac{\partial L(r_L)}{\partial r_L} \frac{1}{2} (r_L - r^*)}{\frac{1}{c} \frac{\partial L(r_L)}{\partial r_L} (r_L - r^*)} \\ &= \frac{1}{2} < 1, \end{aligned}$$

so that $\frac{d\hat{q}}{dr^*} = \frac{1}{c} \left(\frac{d\hat{r}_L}{dr^*} - (1-k) \right) = \frac{1}{c} \left(\frac{1}{2} - 1 \right) < 0$ for $k = 0$.

At the other extreme, as $k \rightarrow 1$, we have

$$\frac{\partial G}{\partial r^*} = -\frac{\partial L(r_L)}{\partial r_L} > 0,$$

which again establishes that $\frac{d\hat{r}_L}{dr^*} > 0$ for $k = 1$. Given $\left. \frac{d\hat{q}}{dr^*} \right|_{k=1} = \frac{1}{c} \frac{d\hat{r}_L}{dr^*}$, we can conclude that $\frac{d\hat{q}}{dr^*} > 0$ for $k = 1$.

By continuity, there must exist a value of k , \tilde{k} , such that $\frac{d\hat{q}}{dr^*} < 0$ for $k < \tilde{k}$, and $\frac{d\hat{q}}{dr^*} > 0$ for $k > \tilde{k}$. The final step is to show that such a value is unique. Given our assumption of a linear demand function, we can without loss of generality write this as $L(r_L) = A - br_L$. We can now substitute for \hat{q} into the bank's profits to obtain

$$\Pi = \left(\frac{c}{2} \left(\frac{r_L - r^*(1-k)}{c} \right)^2 - k(r^* + \xi) \right) (A - br_L).$$

From this we obtain the *FOC* with respect to r_L ,

$$\frac{\partial \Pi}{\partial r_L} = (A - br_L) \left(\frac{r_L - r^*(1-k)}{c} \right) - b \left(\frac{c}{2} \left(\frac{r_L - r^*(1-k)}{c} \right)^2 - k(r^* + \xi) \right) = 0.$$

Solving yields

$$\hat{r}_L = \frac{1}{3b} \left(A + 2br^*(1-k) + \sqrt{(A - br^*(1-k))^2 + 6kb^2c(r^* + \xi)} \right), \quad (14)$$

and substituting into \widehat{q} we obtain

$$\widehat{q} = \frac{\left(A - br^*(1 - k) + \sqrt{(A - br^*(1 - k))^2 + 6kb^2c(r^* + \xi)} \right)}{3bc}.$$

This expression for \widehat{q} is clearly increasing in k , and is decreasing in r^* for values of k near 0, and increasing in r^* for values of k near 1. Tedious calculations show that, for value of c such that $\widehat{q} < 1$ (i.e., for which we have an interior solution), in addition we have $\frac{\partial^2 \widehat{q}}{\partial r^* \partial k} > 0$ for all $k \in (0, 1)$. Therefore, there is a unique point \widetilde{k} for which $\frac{d\widehat{q}}{dr^*} = 0$, as desired. ■

Proof of Proposition 2: In the absence of deposit insurance, rational depositors will demand an interest rate commensurate to the expected probability of repayment, $r_D = \frac{r^*}{E[\widehat{q}]}$. Recall that, assuming an interior solution, we have $\widehat{q} = \frac{r_L - r_D(1 - k)}{c}$. Since in equilibrium depositors' expectations must be correct, we can substitute for r_D as $r_D = \frac{r^*}{E[\widehat{q}]}$ and rearrange to get

$$q^2 - r_L q + r^*(1 - k) = 0.$$

Following Allen et al. (2010), we solve for q and take the larger root:

$$\widehat{q}(k) = \frac{1}{2c} \left(r_L + \sqrt{r_L^2 - 4cr^*(1 - k)} \right). \quad (15)$$

This implies

$$\left. \frac{d\widehat{q}(k)}{dr} \right|_k = \frac{1}{2c} \left(\left. \frac{dr_L}{dr} \right|_k + \frac{-2c(1 - k) + r_L \left. \frac{dr_L}{dr} \right|_k}{\sqrt{r_L^2 - 4cr^*(1 - k)}} \right). \quad (16)$$

The deposit rate is obtained from the maximization of the bank's profit, and is determined by the following FOC (after substituting $L(r_L) = A - br_L$):

$$\frac{\partial \Pi}{\partial r_L} = (A - br_L) \left(\frac{r_L - r_D(1 - k)}{c} \right) - b \left(\frac{c}{2} \left(\frac{r_L - r_D(1 - k)}{c} \right)^2 - k(r^* + \xi) \right) = 0.$$

Solving gives

$$\widehat{r}_L = \frac{1}{3b} \left(A + 2br_D(1 - k) + \sqrt{(A - br_D(1 - k))^2 + 6kb^2c(r^* + \xi)} \right). \quad (17)$$

Differentiating r_L with respect to k we obtain

$$\frac{dr_L}{dr^*} = \frac{2}{3} \frac{dr_D}{dr^*} (1 - k) + \frac{bck + \frac{1}{3} \frac{dr_D}{dr} (1 - k) (br_D(1 - k) - A)}{\sqrt{(A - br_D(1 - k))^2 + 6kb^2c(r^* + \xi)}}.$$

Evaluated at $k = 1$, this expression becomes $\frac{dr_L}{dr^*} = \frac{bc}{\sqrt{A^2 + 6b^2c(r^* + \xi)}} > 0$. This immediately implies that at $k = 1$, $\frac{d\hat{q}(k)}{dr^*} = \frac{dr_L}{dr^*} > 0$.

Now consider the case $k = 0$. At $k = 0$, $\frac{dr_L}{dr^*}$ becomes $\frac{dr_L}{dr} = \frac{1}{3} \frac{dr_D}{dr}$. Thus we have $\frac{dr_L}{dr^*} = \frac{dr_D}{3}$. And since $r_D = \frac{r}{\hat{q}}$,

$$\frac{dr_D}{dr^*} = \frac{1}{3} \left(\frac{\hat{q} - r^* \frac{d\hat{q}}{dr}}{\hat{q}^2} \right).$$

Plugging this into (16), we get

$$\frac{d\hat{q}(k)}{dr^*} = \frac{1}{2c} \left(\frac{1}{3} \left(\frac{\hat{q} - r^* \frac{d\hat{q}}{dr}}{\hat{q}^2} \right) + \frac{-2c + r_L \frac{1}{3} \left(\frac{\hat{q} - r^* \frac{d\hat{q}}{dr}}{\hat{q}^2} \right)}{\sqrt{r_L^2 - 4cr^*}} \right),$$

which solving for $\frac{d\hat{q}(k)}{dr^*}$ yields:

$$\frac{d\hat{q}(k)}{dr^*} = \frac{\hat{q} \left(r_L + \sqrt{r_L^2 - 4cr^*} - 6c\hat{q} \right)}{r^* \left(r_L + \sqrt{r_L^2 - 4cr^*} \right) + 6\hat{q}^2 c \sqrt{r_L^2 - 4cr^*}}. \quad (18)$$

The denominator of (18) is positive, and remembering that at $k = 0$,

$$\hat{q}(k) = \frac{1}{2c} \left(r_L + \sqrt{r_L^2 - 4cr^*} \right),$$

we can write the numerator of (18) as

$$\hat{q} (2c\hat{q} - 6c\hat{q}) = -4\hat{q}^2 < 0.$$

This tells us that $\frac{d\hat{q}(k)}{dr^*} < 0$ at $k = 0$, as desired. ■

Proof of Proposition 3: As in Proposition 2, in the absence of deposit insurance, rational depositors will demand an interest rate commensurate to the expected probability of repayment, $r_D = \frac{r^*}{E[\hat{q}]}$. As before, this yields an equilibrium expression for bank monitoring as

$$\hat{q}(k) = \frac{1}{2c} \left(r_L + \sqrt{r_L^2 - 4cr^* (1 - k)} \right). \quad (19)$$

Also, again using the fact that in equilibrium we must have $r_D = \frac{r^*}{\hat{q}}$, we can rewrite the profit function as:

$$\Pi = \left(\hat{q}\hat{r}_L - r^*(1 - k) - r_E k - \frac{1}{2} c\hat{q}^2 \right) L(\hat{r}_L).$$

The first order condition with respect to k is

$$\frac{\partial \Pi}{\partial k} = \left(r^* - r_E + \frac{\partial \hat{q}}{\partial k} (\hat{r}_L - c\hat{q}) \right) L(\hat{r}_L) + \frac{\partial \Pi}{\partial \hat{r}_L} \frac{\partial \hat{r}_L}{\partial k} = 0.$$

The second term, $\frac{\partial \Pi}{\partial \hat{r}_L} \frac{\partial \hat{r}_L}{\partial k}$, is zero by the envelope theorem, which implies a first order condition of

$$r^* - r_E + \frac{\partial \hat{q}}{\partial k} (\hat{r}_L - c\hat{q}) = 0. \quad (20)$$

The second order condition can now be written as

$$\frac{\partial^2 \Pi}{\partial k^2} = \frac{\partial L}{\partial \hat{r}_L} \frac{\partial \hat{r}_L}{\partial k} \left(r^* - r_E + \frac{\partial \hat{q}}{\partial k} (\hat{r}_L - c\hat{q}) \right) + L(\hat{r}_L) \left(\frac{\partial \hat{q}}{\partial k} \left(\frac{\partial \hat{r}_L}{\partial k} - c \frac{\partial \hat{q}}{\partial k} \right) + \frac{\partial^2 \hat{q}}{\partial k^2} (\hat{r}_L - c\hat{q}) \right).$$

The first term is zero from (20), leaving only

$$\frac{\partial^2 \Pi}{\partial k^2} = \frac{\partial \hat{q}}{\partial k} \left(\frac{\partial \hat{r}_L}{\partial k} - c \frac{\partial \hat{q}}{\partial k} \right) + \frac{\partial^2 \hat{q}}{\partial k^2} (\hat{r}_L - c\hat{q}). \quad (21)$$

To sign this expression, we use the following auxiliary result.

Lemma 2 *Around the optimal leverage ratio \hat{k} , the optimal loan rate \hat{r}_L is increasing in k : $\left. \frac{\partial \hat{r}_L}{\partial k} \right|_{\hat{k}} > 0$.*

Proof of Lemma 2: From the first order conditions with respect to r_L we have

$$\frac{\partial \Pi}{\partial r_L} = qL(r_L) + \frac{\partial L(r_L)}{\partial r_L} \left(\hat{q}(r_L - r_D(1-k)) - r_E k - \frac{1}{2} c \hat{q}^2 \right) + \frac{\partial \Pi}{\partial q} \frac{\partial q}{\partial r_L} = 0.$$

Since the last term is zero by the envelope theorem, we can write:

$$\hat{q}L(r_L) + \frac{\partial L(r_L)}{\partial r_L} \left(\hat{q}(r_L - r_D(1-k)) - r_E k - \frac{1}{2} c \hat{q}^2 \right) = 0. \quad (22)$$

Define $Z \equiv \frac{\partial \Pi}{\partial r_L} = 0$. Then, using the Implicit Function Theorem we have $\left. \frac{\partial \hat{r}_L}{\partial k} \right|_{\hat{k}} = -\frac{\frac{\partial Z}{\partial k}}{\frac{\partial Z}{\partial r_L}}$:

$$\begin{aligned} \frac{\partial Z}{\partial r_L} &= \hat{q} \frac{\partial L(r_L)}{\partial r_L} + L(r_L) \frac{\partial \hat{q}}{\partial r_L} + \hat{q} \frac{\partial L(r_L)}{\partial r_L} + \frac{\partial^2 L(r_L)}{\partial r_L^2} \left(\hat{q}(r_L - r_D(1-k)) - r_E k - \frac{1}{2} c \hat{q}^2 \right) + \\ &\quad \frac{\partial L(r_L)}{\partial r_L} \frac{(\hat{q}(r_L - r_D(1-k)) - r_E k - \frac{1}{2} c \hat{q}^2)}{\partial q} \frac{\partial q}{\partial r_L}, \end{aligned}$$

where the last two terms are zero: the first because of the linearity of the loan demand function, and the second because of the envelope theorem. This means:

$$\frac{\partial Z}{\partial r_L} = 2\hat{q} \frac{\partial L(r_L)}{\partial r_L} + L(r_L) \frac{\partial \hat{q}}{\partial r_L}.$$

We can rewrite $Z = 0$ as

$$L(r_L) = -\frac{\frac{\partial L(r_L)}{\partial r_L} (\widehat{q}(r_L - r_D(1-k)) - r_E k - \frac{1}{2}c\widehat{q}^2)}{\widehat{q}}.$$

Thus

$$\begin{aligned} \frac{\partial Z}{\partial r_L} &= 2\widehat{q} \frac{\partial L(r_L)}{\partial r_L} - \frac{\frac{\partial L(r_L)}{\partial r_L} (\widehat{q}(r_L - r_D(1-k)) - r_E k - \frac{1}{2}c\widehat{q}^2)}{\widehat{q}} \frac{\partial \widehat{q}}{\partial r_L} \\ &= \frac{1}{\widehat{q}} \left(2\widehat{q}^2 \frac{\partial L(r_L)}{\partial r_L} - \frac{\partial L(r_L)}{\partial r_L} \left(\widehat{q}(r_L - r_D(1-k)) - r_E k - \frac{1}{2}c\widehat{q}^2 \right) \frac{\partial \widehat{q}}{\partial r_L} \right), \end{aligned}$$

and, since r_D is already determined at this stage, we can substitute for \widehat{q} in the above as $\widehat{q} = \frac{r_L - r_D(1-k)}{c}$ and write the second order condition as

$$\begin{aligned} \frac{\partial Z}{\partial r_L} &= \frac{1}{\widehat{q}} \frac{\partial L(r_L)}{\partial r_L} \left(\frac{3}{2}\widehat{q}^2 + \frac{r_E k}{c} \right) \\ &= \frac{\partial^2 \Pi}{\partial r_L^2} = \frac{1}{\widehat{q}} \frac{\partial L(r_L)}{\partial r_L} \left(\frac{3}{2} \left(\frac{r_L - r_D(1-k)}{c} \right)^2 + \frac{r_E k}{c} \right) < 0, \end{aligned}$$

which verifies the second order condition.

Now, to compute $\frac{\partial Z}{\partial k}$, we first write Z in a way that reflects the equilibrium condition that $r_D = \frac{r^*}{\widehat{q}}$, since r_D is determined after k and r^* are chosen:

$$Z = \widehat{q}L(r_L) + \frac{\partial L(r_L)}{\partial r_L} \left(\widehat{q}r_L - r^*(1-k) - r_E k - \frac{1}{2}c\widehat{q}^2 \right) = 0.$$

We can now differentiate this to obtain

$$\begin{aligned} \frac{\partial Z}{\partial k} &= \frac{\partial \widehat{q}}{\partial k} L(r_L) + \frac{\partial L(r_L)}{\partial r_L} (r^* - r_E) + \frac{\partial L(r_L)}{\partial r_L} (r_L - c\widehat{q}) \frac{\partial \widehat{q}}{\partial k} \\ &= \frac{\partial \widehat{q}}{\partial k} L(r_L) + \frac{\partial L(r_L)}{\partial r_L} \left(-\xi + (r_L - c\widehat{q}) \frac{\partial \widehat{q}}{\partial k} \right). \end{aligned}$$

However, from (20), the FOC with respect to k , we know that the term in brackets is zero. This means that, for $\widehat{q}(k) = \frac{1}{2c} \left(r_L + \sqrt{r_L^2 - 4cr^*(1-k)} \right)$,

$$\frac{\partial Z}{\partial k} = \frac{\partial \widehat{q}}{\partial k} L(r_L) = \frac{L(r_L) r^*}{\sqrt{r_L^2 - 4cr^*(1-k)}} > 0.$$

Thus, $\left. \frac{\partial \widehat{r}_L}{\partial k} \right|_{\widehat{k}} = -\frac{\frac{\partial Z}{\partial k}}{\frac{\partial Z}{\partial r_L}} > 0$, as desired. ■

We can now use Lemma 2 to establish that, around the equilibrium value of capital \widehat{k} , $\frac{\partial \widehat{r}_L}{\partial k} > 0$. From this, it also follows that $\frac{\partial \widehat{q}}{\partial k} > 0$. We therefore need to sign $\left(\frac{\partial \widehat{r}_L}{\partial k} - c \frac{\partial \widehat{q}}{\partial k} \right)$. From (19), we can

write

$$c \frac{\partial \hat{q}}{\partial k} = \frac{1}{2} \frac{\partial \hat{r}_L}{\partial k} + \frac{cr^* + \frac{1}{2} \frac{\partial \hat{r}_L}{\partial k} \hat{r}_L}{\sqrt{r_L^2 - 4cr^*(1-k)}}.$$

Thus

$$\frac{\partial \hat{r}_L}{\partial k} - c \frac{\partial \hat{q}}{\partial k} = \frac{1}{2} \frac{\partial \hat{r}_L}{\partial k} - \frac{cr^* + \frac{1}{2} \frac{\partial \hat{r}_L}{\partial k} \hat{r}_L}{\sqrt{r_L^2 - 4cr^*(1-k)}} = \frac{1}{2} \frac{\partial \hat{r}_L}{\partial k} \left(1 - \frac{\hat{r}_L}{\sqrt{r_L^2 - 4cr^*(1-k)}} \right) - \frac{cr^*}{\sqrt{r_L^2 - 4cr^*(1-k)}},$$

which is negative because $\hat{r}_L \geq \sqrt{r_L^2 - 4cr^*(1-k)}$ for any $k \leq 1$. Note as well that

$$\frac{\partial^2 \hat{q}}{\partial k^2} = \frac{\partial^2}{\partial k^2} \left(\frac{1}{2c} \left(r_L + \sqrt{r_L^2 - 4cr^*(1-k)} \right) \right) = -2c \frac{(r^*)^2}{\left(\sqrt{r_L^2 - 4cr^* + 4ckr^*} \right)^3} < 0.$$

It follows that profits are concave in k .

Define now $G \equiv \frac{\partial \Pi}{\partial k} = 0$ and $H = \frac{\partial^2 \Pi}{\partial k^2} < 0$. Using the implicit function theorem, we then have

$$\frac{d\hat{k}}{dr^*} = -\frac{\frac{\partial G}{\partial r^*}}{H}.$$

Since the denominator is negative, the sign of $\frac{d\hat{k}}{dr^*}$ will be the same as that of $\frac{\partial G}{\partial r^*}$. Note that

$r^* - r_E = r^* - (r^* + \xi) = -\xi$. Then, the numerator is

$$\frac{\partial G}{\partial r^*} = \frac{\partial \left(-\xi + \frac{\partial \hat{q}}{\partial k} (\hat{r}_L - c\hat{q}) \right)}{\partial r^*} = \frac{\partial \hat{q}}{\partial k} \left(\frac{\partial \hat{r}_L}{\partial r^*} - c \frac{\partial \hat{q}}{\partial r^*} \right) + (\hat{r}_L - c\hat{q}) \frac{\partial^2 \hat{q}}{\partial k \partial r^*}. \quad (23)$$

The first term is positive since $\frac{\partial \hat{q}}{\partial k} > 0$, $\frac{\partial \hat{r}_L}{\partial r^*} > 0$, and $\frac{\partial \hat{q}}{\partial r^*} < 0$. The second term depends on the sign of $\frac{\partial^2 \hat{q}}{\partial k \partial r^*}$, which is given by

$$\frac{\partial^2}{\partial k \partial r^*} \left(\frac{1}{2c} \left(r_L + \sqrt{r_L^2 - 4cr^*(1-k)} \right) \right) = \frac{r_L^2 - 2cr^*(1-k)}{\left(r_L^2 - 4cr^*(1-k) \right)^{\frac{3}{2}}} > 0.$$

It follows that $\frac{d\hat{k}}{dr^*} > 0$, as desired. ■

Proof of Lemma 1: We can write $\frac{d\hat{r}_L}{dr^*} = \frac{\partial \hat{r}_L}{\partial k} \Big|_{\hat{k}} \frac{d\hat{k}}{dr^*} + \frac{d\hat{r}_L}{dr^*} \Big|_{\hat{k}}$, where the notation $\frac{d\hat{r}_L}{dr^*} \Big|_{\hat{k}}$ refers to the derivative of the equilibrium loan rate with respect to the monetary policy rate, for a given fixed capital ratio k . As above, $\frac{\partial \hat{r}_L}{\partial k} \Big|_{\hat{k}}$ is the derivative of the loan rate around the equilibrium level of capital, \hat{k} . Therefore, we have that the first term, $\frac{\partial \hat{r}_L}{\partial k} \Big|_{\hat{k}} \frac{d\hat{k}}{dr^*}$, is positive from Lemma 2 and Proposition 3. Therefore, the only remaining term to sign is $\frac{d\hat{r}_L}{dr^*} \Big|_{\hat{k}}$. For this, recall again the first order condition for profit maximization with respect to r_L obtained in (22):

$$\frac{\partial \Pi}{\partial r_L} = \hat{q}L(r_L) + \frac{\partial L(r_L)}{\partial r_L} \left(\hat{q}(r_L - r_D(1-k)) - r_E k - \frac{1}{2} c \hat{q}^2 \right) = 0.$$

We again define $Z \equiv \frac{\partial \Pi}{\partial r_L} = 0$. Then, using the Implicit Function Theorem we have $\frac{dr_L}{dr^*} = -\frac{\frac{\partial Z}{\partial r^*}}{\frac{\partial Z}{\partial r_L}}$.

The denominator we know is negative from the proof of Lemma 2. For the numerator, we have

$$\begin{aligned}\frac{\partial Z}{\partial r^*} &= \frac{\partial \hat{q}}{\partial r^*} L(r_L) - \frac{\partial L(r_L)}{\partial r_L} + \frac{\partial L(r_L)}{\partial r_L} (r_L - c\hat{q}) \frac{\partial q}{\partial r^*} \\ &= \frac{\partial \hat{q}}{\partial r^*} L(r_L) - \frac{\partial L(r_L)}{\partial r_L} \left(1 - (r_L - c\hat{q}) \frac{\partial q}{\partial r^*}\right).\end{aligned}$$

Now, using the fact that $\hat{q} = \frac{1}{2c} \left(r_L + \sqrt{r_L^2 - 4cr^*(1-k)}\right)$, we know that

$$\frac{\partial \hat{q}}{\partial r^*} = -\frac{1-k}{\sqrt{r_L^2 - 4cr^*(1-k)}}.$$

For ease of exposition, let us define $W = \sqrt{r_L^2 - 4cr^*(1-k)}$. We can substitute this into $\frac{\partial Z}{\partial r^*}$ to obtain

$$\frac{\partial Z}{\partial r^*} = -\frac{1-k}{W} L(r_L) - \frac{\partial L(r_L)}{\partial r_L} \left(1 - \left(r_L - c\frac{1}{2c}(r_L + W)\right) \left(-\frac{1-k}{W}\right)\right).$$

We can rewrite $Z = 0$ as

$$L(r_L) = -\frac{\frac{\partial L(r_L)}{\partial r_L} (\hat{q}r_L - r^*(1-k) - r_E k - \frac{1}{2}c\hat{q}^2)}{\hat{q}} = -\frac{\frac{\partial L(r_L)}{\partial r_L} (\hat{q}r_L - r^* - k\xi - \frac{1}{2}c\hat{q}^2)}{\hat{q}},$$

and we can substitute into the above

$$\frac{\partial Z}{\partial r^*} = \frac{\partial L(r_L)}{\partial r_L} \left(\frac{1-k}{W} \frac{(\hat{q}r_L - r^* - k\xi - \frac{1}{2}c\hat{q}^2)}{\hat{q}} - \left(1 - \left(r_L - c\frac{1}{2c}(r_L + W)\right) \left(-\frac{1-k}{W}\right)\right) \right).$$

Substituting now for \hat{q} and simplifying yields

$$\frac{\partial Z}{\partial r^*} = \frac{\partial L(r_L)}{\partial r_L} \left(-\frac{1}{4r^*H} (r^*(r_L + W) + 2k\xi(r_L - W) + kr^*(r_L + W)) \right).$$

From the equilibrium solution for \hat{q} , we know that

$$2c\hat{q} = r_L + \sqrt{r_L^2 - 4cr^*(1-k)} = r_L + W.$$

This allows us to write

$$\frac{\partial Z}{\partial r^*} = \frac{\partial L(r_L)}{\partial r_L} \left(-\frac{1}{4r^*\sqrt{r_L^2 - 4cr^*(1-k)}} \left(r^*2c\hat{q} + 2k\xi \left(r_L - \sqrt{r_L^2 - 4cr^*(1-k)} \right) + kr^*2c\hat{q} \right) \right).$$

It must also be that

$$2(r_L - c\hat{q}) = 2r_L - \left(r_L + \sqrt{r_L^2 - 4cr^*(1-k)} \right) = r_L - \sqrt{r_L^2 - 4cr^*(1-k)}.$$

This term shows up in the expression above for $\frac{\partial Z}{\partial r^*}$. We can therefore substitute this back into $\frac{\partial Z}{\partial r^*}$ to obtain

$$\begin{aligned}\frac{\partial Z}{\partial r^*} &= -\frac{\partial L(r_L)}{\partial r_L} \frac{1}{4r^* \sqrt{r_L^2 - 4cr^*(1-k)}} (r^*2c\hat{q} + 2k\xi(2(r_L - c\hat{q})) + kr^*2c\hat{q}) \\ &= -\frac{\partial L(r_L)}{\partial r_L} \frac{1}{4r^* \sqrt{r_L^2 - 4cr^*(1-k)}} (2r^*c\hat{q}(1+k) + 4k\xi(r_L - c\hat{q})) > 0,\end{aligned}$$

since $\frac{\partial L(r_L)}{\partial r_L} < 0$. Therefore, $\left. \frac{d\hat{r}_L}{dr^*} \right|_{\hat{k}} = -\frac{\frac{\partial Z}{\partial r^*}}{\frac{\partial Z}{\partial r_L}} > 0$, as desired. ■

Proof of Proposition 4: From the proof of Proposition 3, we have that since $r_D = \frac{r^*}{q}$, we can rewrite the profit function as

$$\Pi = \left(\hat{q}\hat{r}_L - r^*(1-k) - r_E k - \frac{1}{2}c\hat{q}^2 \right) L(\hat{r}_L).$$

The first order condition with respect to k is

$$\frac{\partial \Pi}{\partial k} = r^* - r_E + \frac{\partial \hat{q}}{\partial k} (\hat{r}_L - c\hat{q}) = -\xi + \frac{\partial \hat{q}}{\partial k} (\hat{r}_L - c\hat{q}) = 0. \quad (24)$$

This has to be satisfied as an identity in equilibrium: $\frac{\partial \Pi}{\partial k} \equiv 0$ for any value of r^* at the equilibrium choice of k .

Now consider the following derivative:

$$\begin{aligned}\frac{d}{dr^*} \left(\frac{\partial \Pi}{\partial k} \right) &= \frac{\partial}{\partial r^*} \left(-\xi + \frac{\partial \hat{q}}{\partial k} (\hat{r}_L - c\hat{q}) \right) \\ &= \frac{\partial \hat{q}}{\partial k} \left(\frac{d\hat{r}_L}{dr^*} - c \frac{d\hat{q}}{dr^*} \right) + \frac{\partial q^2}{\partial k \partial r^*} (\hat{r}_L - c\hat{q}).\end{aligned}$$

Given that $\frac{\partial \Pi}{\partial k}$ is identically equal to zero, this expression must also equal zero: $\frac{d}{dr^*} \left(\frac{\partial \Pi}{\partial k} \right) = 0 \Leftrightarrow$

$$\frac{\partial \hat{q}}{\partial k} \left(\frac{d\hat{r}_L}{dr^*} - c \frac{d\hat{q}}{dr^*} \right) + \frac{\partial q^2}{\partial k \partial r^*} (\hat{r}_L - c\hat{q}) = 0. \quad (25)$$

We can compute

$$\frac{\partial q^2}{\partial k \partial r^*} = \frac{\partial^2}{\partial k \partial r^*} \left(\frac{1}{2c} \left(r_L + \sqrt{r_L^2 - 4cr^*(1-k)} \right) \right) = \frac{r_L^2 - 2cr^*(1-k)}{(r_L^2 - 4cr^*(1-k))^{\frac{3}{2}}} > 0. \quad (26)$$

We know already that $\frac{d\hat{q}}{dk} > 0$, and that $\hat{r}_L - c\hat{q} \geq 0$. Therefore, the only way for the equilibrium condition $\frac{d}{dr^*} \left(\frac{\partial \Pi}{\partial k} \right) = 0$ to be satisfied is if $\frac{d\hat{r}_L}{dr^*} - c \frac{d\hat{q}}{dr^*} < 0$. However, since (25) only holds around

the equilibrium value of capital, \widehat{k} , we can apply Lemma 1 to sign $\frac{d\widehat{r}_L}{dr^*}$ as positive. It then follows that $\frac{d\widehat{q}}{dr^*} > 0$. ■

Proof of Proposition 5: We start from the zero profit condition for a given k :

$$Z \equiv \widehat{\Pi} = L \left(\widehat{q}r_L - r^* - k\xi - \frac{c}{2}\widehat{q}^2 \right) = 0.$$

This condition can be used to determine the equilibrium loan rate r_L .

From (5) we can write

$$\frac{d\widehat{q}}{dr^*} = \frac{1}{c} \left(\frac{1}{2} \frac{dr_L}{dr^*} + \frac{\frac{1}{2}r_L \frac{dr_L}{dr^*} - c(1-k)}{\sqrt{r_L^2 - 4cr^*(1-k)}} \right). \quad (27)$$

Applying the Implicit Function Theorem, we obtain $\frac{dr_L}{dr} = -\frac{\frac{\partial Z}{\partial r}}{\frac{\partial Z}{\partial r_L}}$. It is easy to show that

$$\frac{\partial Z}{\partial r} = -\frac{(1-k) \left(r_L - \sqrt{r_L^2 - 4cr^*(1-k)} \right)}{2\sqrt{r_L^2 - 4cr^*(1-k)}} - 1 < 0,$$

and

$$\frac{\partial Z}{\partial r_L} = \frac{\left(r_L + \sqrt{r_L^2 - 4cr^*(1-k)} \right)^2}{4c\sqrt{r_L^2 - 4cr^*(1-k)}} > 0.$$

This gives us that

$$\frac{dr_L}{dr} = -\frac{\frac{\partial Z}{\partial r}}{\frac{\partial Z}{\partial r_L}} = \frac{2c(1-k) \left(r_L - \sqrt{r_L^2 - 4cr^*(1-k)} \right)}{\left(r_L + \sqrt{r_L^2 - 4cr^*(1-k)} \right)^2} + \frac{4c\sqrt{r_L^2 - 4cr^*(1-k)}}{\left(r_L + \sqrt{r_L^2 - 4cr^*(1-k)} \right)^2} > 0.$$

We can now substitute into (27) and note that at $k = 0$, $\frac{d\widehat{q}}{dr} = 0$. And, at $k = 1$, $\frac{d\widehat{q}}{dr} = \frac{4r_L}{(r_L + \sqrt{r_L^2})^2} > 0$. ■

Proof of Proposition 6: After substituting in $\widehat{q} = \frac{r_L + \sqrt{r_L^2 - 4cr^*(1-k)}}{2c}$, maximizing profits

$$\max_k \Pi = L \left(\widehat{q}r_L - r^*(1-k) - r_E k - \frac{c}{2}\widehat{q}^2 \right)$$

gives the first order condition

$$\frac{\partial \Pi}{\partial k} = -\frac{r^*}{2} - \xi + \frac{r^*r_L}{2\sqrt{r_L^2 - 4cr^*(1-k)}} = 0.$$

We can solve this to obtain

$$\widehat{k} = 1 - r_L^2 \frac{\xi (r^* + \xi)}{cr^* (r^* + 2\xi)^2}.$$

We now impose zero profits to obtain the lending rate

$$\widehat{r}_L = \sqrt{\frac{2cr^* (r^* + 2\xi)^2}{3r^*\xi + r^{*2} + 2\xi^2}}.$$

Plugging back into \widehat{k} yields

$$\widehat{k} = \frac{r^*\xi + r^{*2}}{3r^*\xi + r^{*2} + 2\xi^2}. \quad (28)$$

From (28) we immediately obtain

$$\frac{d\widehat{k}}{dr^*} = \frac{\xi (4\xi^3 + 10r^*\xi^2 + 2r^{*3} + 8r^{*2}\xi)}{(r^{*3} + 4\xi^3 + 8r^*\xi^2 + 5r^{*2}\xi) (3r^*\xi + r^{*2} + 2\xi^2)} > 0.$$

This means that leverage is decreasing in the policy rate. We can also write

$$\widehat{q} = \sqrt{\frac{r^{*4} (r^* + \xi)^2}{2c (3r^*\xi + r^{*2} + 2\xi^2)}},$$

from which is immediate that there always exists a c large enough that $\widehat{q} < 0$. More precisely,

$$\frac{r^{*4} (r^* + \xi)^2}{2c (3r^*\xi + r^{*2} + 2\xi^2)} < 1 \iff r^{*4} (r^* + \xi)^2 < 2c (3r^*\xi + r^{*2} + 2\xi^2) \iff \frac{2r^* (r^* + \xi)^2}{3r^*\xi + r^{*2} + 2\xi^2} < c.$$

Now note that

$$\frac{d\widehat{q}}{dr^*} = \frac{(4r^*\xi + r^{*2} + 2\xi^2)}{\sqrt{\frac{2r^*(\xi+r^*)}{c(r^*+2\xi)} c (r^* + 2\xi)^2}} = \frac{(4r^*\xi + r^{*2} + 2\xi^2)}{\sqrt{2cr^* (\xi + r^*) (r^* + 2\xi)^3}} > 0,$$

as desired. ■

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