Sustainable International Monetary Policy Cooperation

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Policy Rhetoric I

Dr. John Lipsky, Acting Managing Director of the IMF, stated on 21 June 2011:

> *"A (second) success is the remarkable increase in global policy cooperation that has taken place in the wake of the 2008-09 global financial crisis. When the world last faced such grave danger—during the Great Depression—countries acted in their own, perceived self-interest with beggar-thy-neighbor policies that in fact deepened the downturn. This time, countries acted together to tackle the crisis."*

Policy Rhetoric II

Janet Yellen:

> *[I]f the United States were to go it alone with tough policies, we could see our financial institutions flee in a race to the bottom. But I'm convinced that won't happen. We are working closely with our international counterparts to develop coherent and comprehensive approaches, aided by the Basel Committee on Banking Supervision, the Financial Stability Board, the International Monetary Fund, and other institutions.*

Motivation

- Literature to date, two extreme assumptions:
  - **Cooperation** by countries, or,
  - **Non-Cooperation** (strategic: Markov perfection, i.e., Nash)
- However, it is unrealistic that:
  - each country can fully commit to Cooperation, or,
  - Non-Cooperation fully rationalizes observed policy reality:
    - Anecdotal, observed, rhetoric
    - Logically: may exist better policy equilibrium

Motivation $\mapsto$ Question

Normative inquiry

If neither typical policy paradigms are logically palatable, ...

then, *how to design* incentive-feasible international monetary policy cooperation?

- An idealized architecture for endogenous cooperation:
  - *Endogenous* sustainability of *some* cooperative regime
  - A distorted version of traditionally *assumed cooperative* regime
  - Sustained by a cross-country and history-contingent contract.
- We call it: *sustainable cooperation* regime.

Question

Inquiry in three parts:

1. When does an assumed cooperation fail to be incentive feasible?

2. How does an incentive-feasible or sustainable equilibrium look like?

3. Welfare implications: winners *or* losers?

What we do

- Linear-quadratic (LQ) framework for optimal monetary policy analysis
  - Benigno and Benigno (2006).
- Calculate and study equilibria under:
  - assumed cooperation (Ramsey),
  - assumed non-cooperation (Nash), and
  - *sustainable cooperation*.

What we do

- Under sustainable cooperation, countries maximize the global social welfare subject to
  - general equilibrium conditions, and
  - a set of *sustainability constraints* for each country:

$$V_t = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} U_s \geq W(y_{t-1}, y^*_{t-1}, \tau_t).$$

What we find

- Cooperation cannot be assumed to be incentive feasible when the countries are asymmetric: ▸ Examples
  - ▹ Counter-example #1: Asymmetric volatility of markup shocks
    - e.g., Home positive markup shock.
    - Home has an incentive to deviate from cooperation to non-cooperation.
  - ▹ Counter-example #2: Unequal country sizes (c.f., Benigno 2002, not discussed today; same story)
- Dynamics of sustainable cooperation "in between" those under cooperation and non-cooperation.

What we find

- Sustainable cooperation balances temptation to manipulate terms of trade with promise of welfare redistribution between countries:
  - Pseudo Pareto weight on each country's welfare *endogenously* shifts toward Home's welfare
  - to keep the home country incentivized within the sustainable cooperation regime.
- The social welfare under sustainable cooperation is on/near the Pareto frontier.

The rest of the talk

- LQ framework (Benigno-Benigno, JME, 2006)
- Equilibria under:
  - cooperation,
  - non-cooperation, and
  - **sustainable cooperation**.
- Welfare comparison with Pareto frontier (if have time)

Overview

- We use an (approximate) Linear-Quadratic (LQ) framework
  - Optimizing households and firms
- Two countries—Home and Foreign. Each country:
  - Differentiated goods
  - Calvo sticky prices
  - No international labor mobility; no physical capital
- Internationally complete markets trading in Arrow securities;
- Law of one price and PCP: Real exchange rate is always unity and consumption equalization.

Phillips curves

- Competitive quilibrium summarized by functional equations "NK-Phillips" (Home and Foreign*):

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + k\mu_t + k\left[(\rho+\eta)y_t + \frac{1}{2}(1-\rho)s_t\right], \quad (1)$$

$$\pi_t^* = \beta \mathbb{E}_t \pi_{t+1}^* + k\mu_t^* + k\left[(\rho+\eta)y_t^* + \frac{1}{2}(1-\rho)s_t^*\right]. \quad (2)$$

  - π_t : producer-price inflation,
  - y_t : output,
  - s_t : terms of trade (TOT),
  - μ_t : exogenous Markovian markup shock.

- $\beta \in (0,1)$ discount factor, $\rho > 0$ CRRA, $\eta > 0$ Frisch elasticity, and $k = (1-\alpha)(1-\alpha\beta)/\left[\alpha(1+\sigma\eta)\right] > 0$.

Terms of Trade

- Complete markets, law of one price ...
- Equilibrium TOT is given by

$$s_t = p_F - p_H = y_t - y_t^*, \tag{3}$$

  where p_H and p_F are Home and Foreign producer prices.
- An increase (decrease) in s_t means deterioration (improvement) in the TOT.

First-order TOT feedback on inflation

- Note that $s_t^* = -s_t$.
- When countries are more risk averse, $\rho > 1$,
  - An increase in s_t lowers Home marginal cost and hence inflation via NKPC: $(1-\rho)s_t < 0$.
  - It also raises foreign inflation since $(1-\rho)s_t^* > 0$: resembles positive markup shock to Foreign inflation.
- When $\rho < 1$, the opposite is true.
- When $\rho = 1$, two countries are "insular" in equilibrium response.
  - One country's shock does not affect the other.

Welfare functions

The Home policymaker has the welfare function:

$$
\begin{aligned}
V_0 &= -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t \qquad (4) \\
&= -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} (\eta+\rho)\left(y_t - \frac{1}{\eta+\rho}\mu_t\right)^2 + \frac{\sigma}{k}\pi_t^2 + \frac{1}{2}(1-\rho)s_t^2 \\ +(\eta+\rho)\left(y_t^* + \frac{1}{\eta+\rho}\mu_t^*\right)^2 + \frac{\sigma}{k}\left(\pi_t^*\right)^2 \end{array} \right].
\end{aligned}
$$

(Equivalent: household utility approximated up to second order.)

Welfare functions, cont'd

Foreign's welfare function:

$$V_0^* = -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t^* \tag{5}$$

$$= -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} (\eta+\rho)\left(y_t^* - \frac{1}{\eta+\rho}\mu_t^*\right)^2 + \frac{\sigma}{k}\left(\pi_t^*\right)^2 \\ +\frac{1}{2}(1-\rho)\left(s_t^*\right)^2 \\ +(\eta+\rho)\left(y_t + \frac{1}{\eta+\rho}\mu_t\right)^2 + \frac{\sigma}{k}\pi_t^2 \end{array} \right].$$

Note: The targets of Home and Foreign output are

- non-zero and asymmetric across countries
- function of markup shocks, μ and μ^*
- The policymakers wish that the *welfare-relevant* output gaps in Eqs. (4) and (5) are close to zero.

Terms of trade externality

- But that creates a policy driven TOT externality on other's welfare:
- For example, Foreign welfare worsens if Home sets $y_t = \frac{1}{\eta+\rho}\mu_t$.
- The externality is present even if countries are "insular"
  (when $\rho = 1$).
- This is further strengthened or weakened by a feedback effect via NKPC
  (when $\rho \neq 1$).

Global welfare function

- The global welfare function is

$$\begin{aligned} V_0^W &= \frac{1}{2}V_0 + \frac{1}{2}V_0^*, \\ &= -\frac{1}{2}\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Big[(\eta+\rho)\, y_t^2 + (\eta+\rho)\, (y_t^*)^2 \\ &\quad + \frac{1-\rho}{2} s_t^2 + \frac{\sigma}{k}\pi_t^2 + \frac{\sigma}{k}\,(\pi_t^*)^2 \Big] \end{aligned} \tag{6}$$

  Here, the targets of Home and Foreign output are zero for both countries.

  - Non-zero output targets in (4) and (5) stem from the TOT externality under non-cooperation.
  - Here, the cooperation objective (6) internalizes this externality.
  - Details

Cooperation

- Policymakers in cooperation jointly maximize the global welfare function (6) subject to equilibrium conditions (1)-(2) (NKPCs) and (3) (TOT).
- The FONCs in the cooperation regime are (1)-(3), along with

$$-\sigma\pi_t = y_t - y_{t-1}, \tag{7}$$
$$-\sigma\pi_t^* = y_t^* - y_{t-1}^*. \tag{8}$$

- Commitment (in terms of domestic interest):
  - History-dependent policies.
- Internalized TOT externality:
  - The optimal rules are inward-looking, i.e., the optimal trade-off only involves own country variables.

Non-cooperation

- A policymaker in non-cooperation maximizes the social welfare function given the other country's response.
- The Home policymaker maximizes (4) s.t. (1)-(3), given π_t^*.
- The Foreign policymaker maximizes (5) s.t. (1)-(3), given π_t.

Non-cooperation

- The FONCs in the non-cooperation regime are (1)-(3), along with

$$-\sigma\pi_t = y_t - \xi_t - (y_{t-1} - \xi_{t-1}), \tag{9}$$

$$-\sigma\pi_t^* = y_t^* - \xi_t^* - (y_{t-1}^* - \xi_{t-1}^*). \tag{10}$$

- Note: The TOT externality is no longer internalized:

$$\xi_t = \frac{(1+\rho+2\eta)\mu_t - (1-\rho)\mu_t^*}{2(1+\eta)(\eta+\rho)}, \text{ and } \xi_t^* = \frac{(1+\rho+2\eta)\mu_t^* - (1-\rho)\mu_t}{2(1+\eta)(\eta+\rho)}$$

  stem from the home and foreign target outputs.
  - Each country would like to zero out their output gaps at the expense of other.

Parameter values

	Parameters	Values
β	subjective discount factor	0.99
η	Frisch elasticity	0.47
ρ	coefficient of relative risk aversion	(varied)
α	Calvo parameter	0.75
σ	Goods elasticity of substitution	10.0

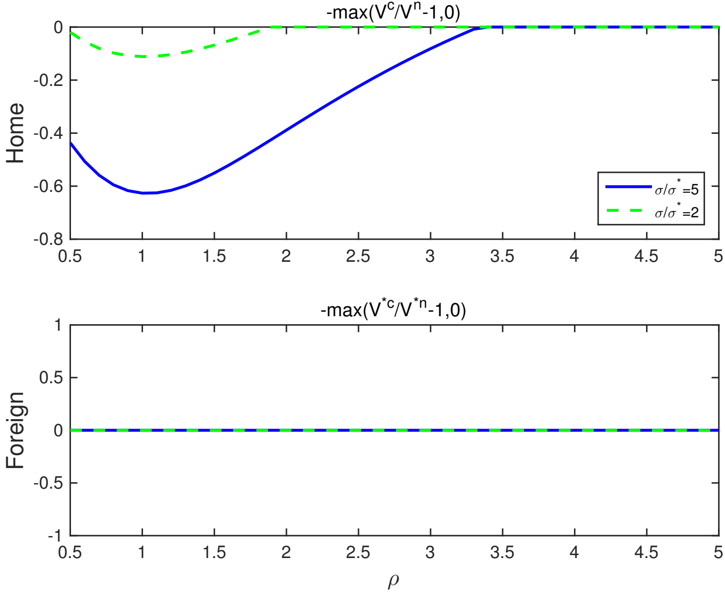
- Finite markup-shock state space: $\mu \in M = \{-\sigma_\varepsilon, 0, \sigma_\varepsilon\}$ and $\mu^* \in M^* = \{-\sigma_\varepsilon^*, 0, \sigma_\varepsilon^*\}$ for each.
- Shocks are assumed to be i.i.d. with the transition matrix $p(\mu_j|\mu_i)_{(1\times 3)} = [(1-\rho_\mu)/2, \rho_\mu, (1-\rho_\mu)/2]$ for any μ_i.
- We set $\rho_\mu = \rho_\mu^* = 0.5$, $\sigma_\varepsilon = 1.0$ and $\sigma_\varepsilon^* = \{0.2, 0.5\}$, i.e., asymmetric volatility between the Home and Foreign countries.

Question 1:

When does an assumed cooperation fail to be incentive feasible?

Incentive Infeasibility of Cooperation

$R_0 = -\max\{V_0^c / V_0^n - 1, 0\} < 0 \Leftrightarrow V_0^c < V_0^n$ (Note that $V < 0$)



Notes on TOT externality

- Suppose a positive markup shock, $\mu_t > 0$ at Home.
- This creates a temptation for Home to close the output gap, by raising y_t (with an inflation trade-off).
- But this induces a negative TOT externality on the output gap term in Foreign's welfare function, via the term $(y_t + \mu_t/(\eta + \rho))^2 > 0$.

Notes on TOT externality and offsetting force, $\rho \neq 1$

- When $\rho \neq 1$, there exists a second force:
  - By attempting to close Home output gap through raising y_t, s_t will increase.
  - Foreign also have incentive to close its output gap, by reducing (or raising) y_t^*, but this also directly hurts Home's welfare.
  - This force acts as a self-disciplining device on Home to exploit the TOT externality. In fear of the retaliation from Foreign, Home refrains from exploiting the TOT externality.

Question 2:

Features of sustainable cooperation plan?

Sustainable cooperation

- A state-contingent contract at period 0 between countries.
  - Note the assumption of the timeless perspective.
- Encodes no incentive to deviate from cooperation by respecting sustainability constraints

$$V_t = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} U_s \geq W(y_{t-1}, y^*_{t-1}, \tau_t), \qquad (11)$$

$$V^*_t = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} U^*_s \geq W^*(y_{t-1}, y^*_{t-1}, \tau_t). \qquad (12)$$

  - W and W^* are the welfare functions for Home and Foreign under the non-cooperation regime.
  - $\tau_t = [\mu_t, \mu_{t-1}, \mu^*_t, \mu^*_{t-1}]'$ is a vector of exogenous shocks.

FONCs: Memory and Incentives

Policymakers maximize the global welfare function (6) subject to (1)-(3) and sustainability constraints (11) and (12).

Constrained efficient policy trade-offs:

$$-\sigma\pi_t = y_t - \zeta_t - z_t(y_{t-1} - \zeta_{t-1}), \quad (13)$$

$$-\sigma\pi_t^* = y_t^* - \zeta_t^* - z_t(y_{t-1}^* - \zeta_{t-1}^*), \quad (14)$$

where (like peeling onions ...)

- $z_t = \frac{\Psi_{t-1}+\Psi_{t-1}^*}{\Psi_t+\Psi_t^*} \in (0,1]$ alleviates incentive to deviate:
  - $\Psi_t = \Psi_{t-1} + \psi_t$ and $\Psi_t^* = \Psi_{t-1}^* + \psi_t^*$, given $\Psi_{-1} = \Psi_{-1}^* = 1/2$
  - ψ_t and ψ_t^* are Lagrange multipliers on the sustainability constraints.

  Details

- $\zeta_t = \frac{(1+\rho+2\eta)\vartheta_t-(1-\rho)\vartheta_t^*}{2(1+\eta)(\eta+\rho)}$ and $\zeta_t^* = -\frac{(1+\rho+2\eta)\vartheta_t^*-(1-\rho)\vartheta_t}{2(1+\eta)(\eta+\rho)}$ are endogenous shifters to trade-off ...

FONCs, cont'd

- Also, ϑ_t and ϑ_t^* are endogenously determined as

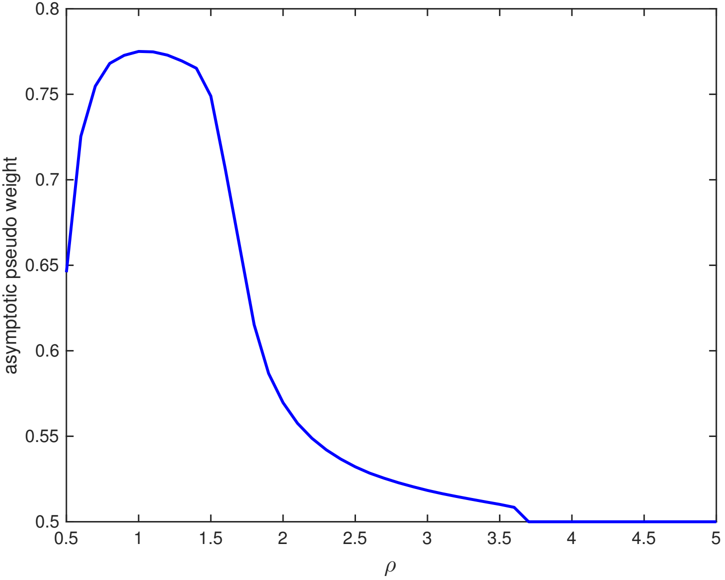
$$\begin{aligned}
\vartheta_t &= (2\nu_t - 1)\mu_t \\
&\quad -\beta\mathbb{E}_t \underbrace{(z_{t+1}^{-1} - 1)\left[I_{t+1} D_1 W(y_t, y_t^*; \tau_{t+1}) + I_{t+1}^* D_1 W^*(y_t, y_t^*; \tau_{t+1})\right]}_{=\Xi_{t+1}}, \\
\vartheta_t^* &= (2\nu_t - 1)\mu_t^* \\
&\quad +\beta\mathbb{E}_t \underbrace{(z_{t+1}^{-1} - 1)\left[I_{t+1} D_2 W(y_t, y_t^*; \tau_{t+1}) + I_{t+1}^* D_2 W^*(y_t, y_t^*; \tau_{t+1})\right]}_{=\Xi_{t+1}^*}.
\end{aligned}$$

  where $\nu_t = \frac{\Psi_t}{\Psi_t + \Psi_t^*} \in (0,1)$ is called a *pseudo weight*, given $\nu_{-1} = 1/2$.
- For example, if the Home sustainability constraint is binding, ν_t gets close to one.
- $I_t = 1$ when the Home sustainability constraint is binding in period t; $I_t = 0$ otherwise.

Computation

- The solution for the sustainable cooperation is obtained only numerically.
- Policy function iteration method with occasionally binding constraints (Kehoe and Perri, 2002; Sunakawa, 2015). ▸ Details
- Approximation scheme: linear projections onto family of spline basis functions.
- We check that Euler approximation errors are negligible.
- Also check for alternative initial policy function guesses.

Pseudo Pareto weight



Features of Sustainable Cooperation

- Pseudo weight shifts in favor of Home country.
- When $\rho = 1$, Home policymaker is tempted to manipulate the TOT in favor of the own welfare.
- The state-contingent contract needs to take this into account by redistributing welfare weights from Foreign to Home.
- When ρ goes away from unity, the sustainability constraints tend to bind less: temptation is offset by a threat of Foreign retaliation (via feedback of s_t onto Foreign inflation).

Pareto weights

- Pseudo weight, ν_t is related to how countries manipulate the TOT as in the non-cooperation regime.
- We can show the following propositions:

Proposition

$\nu_t = 1 - z_t(1 - \nu_{t-1}) > \nu_{t-1}$ when $\psi_t > 0$ and $\nu_t = z_t \nu_{t-1} < \nu_{t-1}$ when $\psi_t^* > 0$.

Proposition

(i) When $\nu_t = 1/2$ for all $t \geq 0$, i.e., the sustainability constraint never binds, $\zeta_t = \zeta_t^* = 0$.
(ii) When $\nu_t \to 1$, $\zeta_t = \xi_t$ and $\zeta_t^* = -\xi_t^*$ approximately hold.
(iii) When $\nu_t \to 0$, $\zeta_t = -\xi_t$ and $\zeta_t^* = \xi_t^*$ approximately hold;

Pareto weights, cont'd

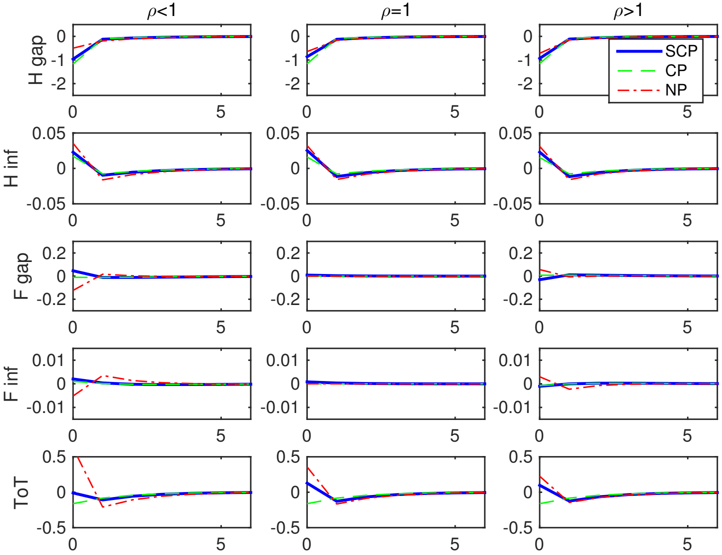
- When either of the sustainability constraints is binding ($\psi_t > 0$ or $\varphi_t^* > 0$),

$$z_t = \frac{\Psi_{t-1} + \Psi_{t-1}^*}{\Psi_{t-1} + \Psi_{t-1}^* + \psi_t + \psi_t^*} < 1$$

  holds.

- When the sustainability constraints never bind, $\zeta_t = \zeta_t^* = 0$ and $\nu_t = 1/2$ hold; the solution becomes the same as in the cooperation regime.
- When the Home sustainability constraint binds, $z_t < 1$ and the pseudo weight on each country's social welfare shifts to keep the Home country within the sustainable cooperation regime.

Sustainable Cooperation: Halfway-house Dynamics



Question 3: Welfare Redistribution

Winners or Losers?

Pareto planner's problem

- Consider a supranational institution that maximizes

$$\max\left[\lambda V_0 + (1-\lambda)V_0^*\right]$$

where $\lambda \in [0,1]$ a Pareto weight, subject to NKPCs (1) and (2).

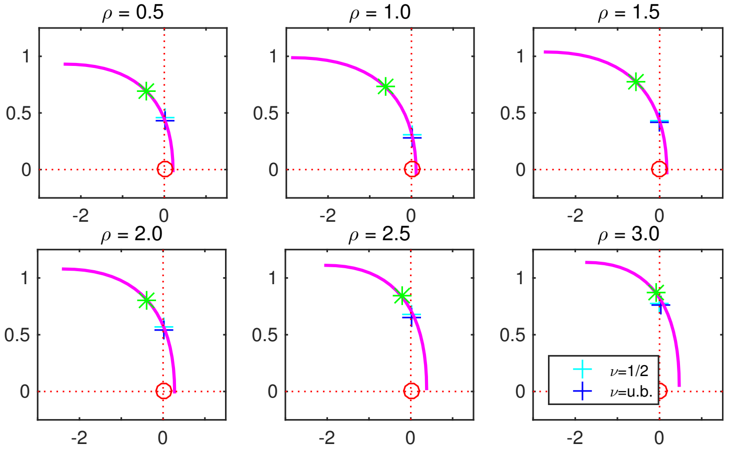
- This alternative λ-Pareto planner has optimal trade-off:

$$-\sigma\pi_t = y_t - \tilde{\zeta}_t - (y_{t-1} - \tilde{\zeta}_{t-1}),$$
$$-\sigma\pi_t^* = y_t^* - \tilde{\zeta}_t^* - (y_{t-1}^* - \tilde{\zeta}_{t-1}^*),$$

where $\tilde{\zeta}_t = (2\lambda - 1)\xi_t$ and $\tilde{\zeta}_t^* = -(2\lambda - 1)\xi_t^*$

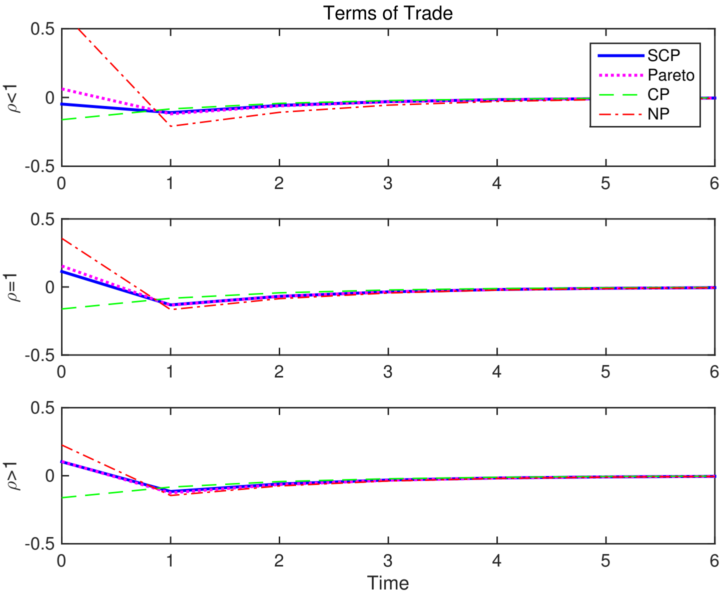
- ... Each point on Pareto frontier is an ex-ante welfare pair $(V_0, V_0^*)(\lambda)$ indexed by a λ.
- Welfare comparison is done with Time-0 metrics (i.e., stochastic steady state) with ν at $1/2$ or the upper bound.

Pareto Frontier



- $*$ $\frac{1}{2}$-Pareto CP allocation: not incentive feasible!
- $\circ$ NP allocation: worst incentive feasible
- $+$ SCP allocation: $\approx \lambda$-Pareto allocation

Operational "Pareto" Rule ≈ SCP



Key Takeaways I

1. *Assumed cooperation outcome* not always and everywhere incentive feasible:
   - Deviation incentive larger when
     - CRRA becomes closer to unity, and
     - asymmetric volatility (markup shocks) exists among the countries.

   Yet ... assuming non-cooperative monetary policy may not be realistic nor logically welfare consistent:
   - Countries can do better if there were some contract/understanding about give-and-take policy risk sharing.

Key Takeaways II

2. A better-than-non-cooperative regime under a *sustainable cooperation* contract:
   - "halfway house" between non-cooperation and cooperation outcomes.
   - consequence of endogenous redistribution via shifting pseudo Pareto weights.

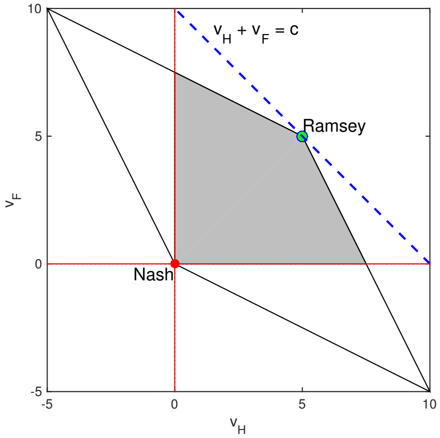
Key Takeaways III

3. There is a corresponding Pareto allocation that is payoff equivalent to the Sustainable Cooperation solution.
   - In the case where countries have asymmetric shock volatilities: equivalent Pareto planner assigns a higher Pareto weight to country with more volatile shock
   - Nevertheless, by construction, under such a Sustainable Cooperation plan, each country is still better off than acting under Non-cooperation.

More in paper: Results robust to asymmetric country size.

APPENDICES

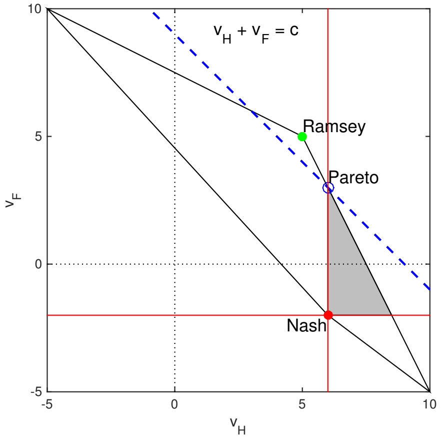
Insights from Prisoners' Dilemma Supergames



Home/Foreign	C	NC
C	(5,5)	(-5,10)
NC	(10,-5)	(0,0)

Ramsey is incentive-feasible and attainable as an outcome of the SPE.

Insights from Prisoners' Dilemma Supergames



Home/Foreign	C	NC
C	(5,5)	(-5,10)
NC	(10,-5)	(6,-2)

Ramsey is not incentive-feasible. One of the attainable equilibria is chosen so as to maximize the sum of Home and Foreign welfare. Back

Terms of trade externality

- This potential externality is shown in an intermediate step in approximating the social welfare:

$$U_t = \frac{1}{2}s_t - \frac{1-\rho}{2}(\underbrace{y_t - \frac{1}{2}s_t}_{=c_t})^2 + \frac{1+\eta}{2}y_t^2 + \frac{\sigma}{k}\pi_t^2,$$

$$U_t^* = -\frac{1}{2}s_t - \frac{1-\rho}{2}(\underbrace{y_t^* + \frac{1}{2}s_t}_{=c_t^*})^2 + \frac{1+\eta}{2}\left(y_t^*\right)^2 + \frac{\sigma}{k}\left(\pi_t^*\right)^2,$$

- $U_t + U_t^*$ yields the global social welfare; s_ts are canceled out.
- If U_t and U_t^* are considered separately, s_t is replaced by quadratic approximation of the home and foreign Phillips curves, which results in non-zero output targets. Back

Lagrangean

Lagrangean in Period 0 is

$$
\begin{aligned}
\mathcal{L}_0 \quad = \quad & -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} U_t + \frac{1}{2} U_t^* \right. \\
& + \phi_t \left(-\pi_t + \beta \mathbb{E}_t \pi_{t+1} + \frac{k}{2} \left[(1 + \rho + 2\eta) \, y_t - (1 - \rho) \, y_t^* \right] + k \mu_t \right) \\
& + \phi_t^* \left(-\pi_t^* + \beta \mathbb{E}_t \pi_{t+1}^* + \frac{k}{2} \left[(1 + \rho + 2\eta) \, y_t^* - (1 - \rho) \, y_t \right] + k \mu_t^* \right) \\
& - \psi_t \left[\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u_s + W(y_{t-1}, y_{t-1}^*, \tau_t) \right] \\
& \left. - \psi_t^* \left[\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u_s^* + W^*(y_{t-1}, y_{t-1}^*, \tau_t) \right] \right\} .
\end{aligned}
$$

Lagrangean, cont'd

Shuffling the terms,

$$\begin{aligned}\mathcal{L}_0 &= -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Psi_t U_t + \Psi_t^* U_t^* \right. \\ &+\phi_t \left(-\pi_t + \frac{k}{2} \left[(1+\rho+2\eta)\, y_t - (1-\rho)\, y_t^* \right] + k\mu_t \right) - \phi_{t-1}\pi_t \\ &+\phi_t^* \left(-\pi_t^* + \frac{k}{2} \left[(1+\rho+2\eta)\, y_t^* - (1-\rho)\, y_t \right] + k\mu_t^* \right) - \phi_{t-1}^*\pi_t^* \\ &\left. -\psi_t W(y_{t-1}, y_{t-1}^*, \tau_t) - \psi_t^* W^*(y_{t-1}, y_{t-1}^*, \tau_t) \right\},\end{aligned}$$

where $\Psi_t = \Psi_{t-1} + \psi_t$ and $\Psi_t^* = \Psi_{t-1}^* + \psi_t^*$ given $\Psi_{-1} = \Psi_{-1}^* = 1/2$.

Back

Computation

- Policy functions satisfy the following equilibrium conditions:

$$\begin{aligned}
h_\pi(s_{-1},\tau) &= \beta \sum_{u'} P(u'|u) h_\pi(s,u',u) + k\mu \\
&\quad +(k/2)\left[(1+\rho+2\eta)\, h_y(s_{-1},\tau) - (1-\rho)\, h_{y^*}(s_{-1},\tau)\right], \\
h_{\pi^*}(s_{-1},\tau) &= \beta \sum_{u'} P(u'|u) \pi^*(s,u',u) + k\mu^* \\
&\quad +(k/2)\left[(1+\rho+2\eta)\, h_{y^*}(s_{-1},\tau) - (1-\rho)\, h_y(s_{-1},\tau)\right], \\
h_\pi(s_{-1},\tau) &= -\sigma^{-1}(\eta+\rho)[h_y(s_{-1},\tau) - \zeta(s_{-1},\tau) \\
&\quad - h_z(s_{-1},\tau)(y_{-1} - \zeta_{-1}(s_{-1},\tau))], \\
h_{\pi^*}(s_{-1},\tau) &= -\sigma^{-1}(\eta+\rho)[h_{y^*}(s_{-1},\tau) - \zeta^*(s_{-1},\tau) \\
&\quad - h_z(s_{-1},\tau)(y^*_{-1} - \zeta^*_{-1}(s_{-1},\tau))], \\
V(s_{-1},\tau) &= -U(s_{-1},\tau) + \beta \sum_{u'} P(u'|u) V(s,u',u) \geq W(y_{-1},y^*_{-1},\tau) \\
V^*(s_{-1},\tau) &= -U^*(s_{-1},\tau) + \beta \sum_{u'} P(u'|u) V^*(s,u',u) \geq W^*(y_{-1},y^*_{-1},\tau)
\end{aligned}$$

- This system has a recursive structure with regard to $h_\pi(s_{-1},\tau)$, $h_{\pi^*}(s_{-1},\tau)$, $\Xi(s_{-1},\tau)$, $\Xi^*(s_{-1},\tau)$, $V(s_{-1},\tau)$, and $V(s_{-1},\tau)$.

Back