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Understanding financial derivatives during the South Sea Bubble: the case of the South Sea subscription shares

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South Sea Company subscription shares were compound call options on the firm's own fully-paid shares. From the description of shares found in *6 Geo.1, c.4*, a theory of their pricing is developed. A method for computing subscription share values is also developed. Calculated theoretical values for subscription shares are compared to the shares' historical values and a close correspondence between the two is demonstrated. The prices of the subscriptions relative to fully-paid share prices thus appear to be explainable using simple financial economic theory and to have been formed quite rationally. There is no obvious evidence of barriers to arbitrage or inefficiencies in the markets for fully-paid shares and subscription shares during the financial crisis known as the South Sea Bubble.

JEL classifications: G13, K22, N23, N83.

1. Introduction

In this paper we demonstrate the existence of something whose existence has never been noted before. During the South Sea Bubble of 1720 there was not only a market for South Sea shares—a market that famously rose to dizzying heights before it came crashing down—there were also markets for financial derivatives related to South Sea shares. We do not refer here to the occasional option contract that was privately negotiated and drawn up between two individuals.[1] We refer instead to financial derivative markets that attracted a trade amongst large numbers of people. These individuals traded contracts that were fixed in form and were issued by the South Sea Company itself. The trading values for these contracts were reported everyday in what passed for the financial

[1] Such contracts were common enough in 1720 for we have examples of them (*Add.MS* 22,639, fff. 193, 195, and 203) and even some abstracts of ledgers for what appears to be a specialist options broker (*HLRO* Box 158).

press of that time. We refer here to the markets for the South Sea subscription shares, for surely they were financial derivatives. In fact, they were call options. In point of fact, we shall show that they were compound call options on South Sea fully-paid shares.

The South Sea Bubble was a prominent landmark in the financial economic history of the western world. One of the major themes in Dickson's (1967) history of the Financial Revolution, for example, was that the South Sea Bubble shook (literally) British financial institutions into the forms that they kept largely for the rest of the 18th century. This paper of course makes a contribution to that historical literature, but to whom else outside of a circle of interested historians would this paper be interesting? Our hopeful answer is that this history will be interesting, as well, to the specialized financial economist. Economic analyses of modern market crashes and bubbles regularly depend upon information that can be found only in markets for financial derivatives. We know that modern investment professionals are used to thinking of derivatives as components of schemes for insuring against loss of value in portfolios. Even when such schemes do not succeed, we might see in the prices of financial derivatives the traces of what investment professionals had hoped to achieve. There might be evidence of a change in the market price of risk or the first evidence of the arrival of essential information that has not yet reached other financial markets. Any future analysis of the South Sea Bubble will be made easier if the existence of derivatives markets and their rational operation can be demonstrated.

The historical literature generally concludes, however, that market participants were widely irrational and inept in 1720. This conclusion is now so integral to South Sea historiography that it has moved nearly beyond question and study and has taken on 'mythic' status, according to Hoppit (2002). In this paper we cannot explain the South Sea Bubble in any wide sense, nor can we overturn the more narrow conclusion that the fundamental value of South Sea (fully-paid) shares was somehow irrationally determined in 1720. We can, however, place one significant qualification upon the usual 'mythic' conclusions; regardless of what determined the market values for South Sea shares in 1720, the relative values of South Sea subscription shares are readily understandable using standard pricing theory. Empirical support for such theory will allow us to conclude that between the markets for South Sea subscription and fully-paid shares there were no significant barriers to arbitrage or inefficiencies that could arise from irrational behaviour. We shall thus be able to point to this one part of the financial markets of 1720 and shall be able to conclude that irrationality did prevail there.

This paper is divided into several sections and is supported by appendices provided as supplementary data and programs accessible via links to the on-line version of the paper. In the next section the problems in understanding South Sea subscription share values are presented; the relationship with modern day partly-paid shares is discussed. In the section after that a theory is developed that says the South Sea subscription shares were compound call options. In the third section a simple method is proposed for computing the values of subscription shares based upon the compound call option theory. In the fourth section the data to be analysed are described and their limitations are discussed. Essential facts about returns' distributions of South Sea fully-paid shares in 1720 are developed. These facts are used to calibrate the computational model of subscription share values employed in section six. In that section we use the computational model to test the performance of our compound call option theory. Discussion of our results and of possible further research is found in the concluding section of the paper.

2. The subscription shares and the problem of their pricing

The subscriptions shares were but one part of what was a complex and changing South Sea Company capital structure. This capital structure and the changes in it proposed to be made in 1720 were central to the larger speculation about the future of the South Sea Company.[2] The subscription shares were planned to sit alongside already existing and other projected equity and debt liabilities of the Company in a legal framework described in a very long and unstructured Parliamentary Act, *6 Geo.1, c.4*.

Near the end of *6 Geo.1, c.4* is a passage devoted to defining the terms under which subscription finance could be raised and any special abilities or disabilities that would attach to subscription shares.[3] The Act also defined the uses to which subscription finance could be applied. *6 Geo.1, c.4* thus fulfilled in part the role that would be played today by a share prospectus, as well as being an expression of corporate law. The Company would leave some of the other provisions of subscription shares, such as the instalment schedules, to later definition. The important features of subscription shares defined by this Act were as follows:

(i) If a subscriber failed to pay an instalment, the Company could deprive him of any 'Share, Dividend, Annuity, or Profits' that he might otherwise be entitled to (Appendix 2, lines 27–9);

(ii) If a subscriber failed to pay an instalment, the Company could also 'stop the Transfers or Assignments' of the defaulter's interests in the firm (Appendix 2, lines 33–4);

(iii) The defaulter's liability for a missed payment, plus 5 per cent per annum interest, would be met from the 'Shares and Stocks of such Defaulter'; this meant that if the subscriber persisted in default for a space of three months, the Company, or anyone whom the Company designated, could sell the defaulter's 'Stocks' (Appendix 2, lines 35–44);

(iv) If in the event of such a forfeiture to the Company, the forfeited shares would be sold by the Company for whatever they were worth. If the value of such a

[2] The finer details and history of this capital structure are provided in the first of our supplementary data appendices, Appendix 1.

[3] We now refer the interested reader to supplementary Appendix 2, which is a line-numbered transcription of this passage from the Act. This passage can be found in the uniform *Stautes at Large*, Volume 5, in Section 98 of the Act (http://www.parl18c.soton.ac.uk/soton/digbib/view?did=c1:6821223&p=301).

sale was in excess of the defaulter's liability to the Company, the defaulter would receive the excess value—the 'Overplus', as it was called (Appendix 2, line 45).

These four provisions in *6 Geo.1, c.4*, when taken together, allow us to build a theory of how the prices of subscription shares should have behaved. Items (iii) and (iv) are the important ones. Any subscriber considering default would be liable to the Company only to the extent of his holding of shares in the Company. In other words, default would result in a simple forfeiture of the shares to the Company. Forfeiture, however, could not be finalized until three months had passed, after which the defaulter could restore himself to ownership of the shares by paying the missed instalment plus three-months' interest. This could be thought of as an additional option to 'wait-and-see', but the cost of exercising this option was so low (only the interest cost of waiting), we may treat it as simply an extension of the instalment schedule itself. Indeed, it is no wonder that we have so much evidence that instalments for each of the four subscription issues were heavily in arrears.[4] It cost subscribers practically very little to delay the payments of their instalments for at least three months.

We find modern examples of partly-paid shares that impose the penalty of forfeiture if instalments are not paid and in which the issuer continues to incur an obligation to the owner of the partly-paid share as a result of forfeiture. This is a feature of the South Sea subscription shares that plays actually a very important role in the theory that will be presented in the next section. The restriction of the South Sea subscriber's liability to his share holdings, however, is also very important. This is a feature of partly-paid shares not to be generally found today, except perhaps only in the partly-paid shares of so-called No Liability (NL) companies (Morris, 1997).

The payment schedules of instalments were left to the Company to declare. In the cases of each of the four subscription issues the Company declared a fixed schedule of payments.[5] The first two issues' schedules were defined in April 1720 and the Company stayed with these schedules until December. One thing the Company did not do was to reserve to itself the option of suddenly and surprisingly making a call for an instalment. When it made any surprise announcement about the instalment schedules, it was always for a delay in a scheduled payment. Having a fixed instalment schedule is another feature of the South Sea subscription shares that makes the construction of a pricing theory for them relatively easy to achieve.

[4] The earliest discussion in the Company as to how forfeitures of subscription shares would be managed was 2 June (*Minute Book of the Committee of the Treasury*, HLRO B64, p.17). There was discussion in July about the large numbers of subscribers who had failed to pay their deposits (Boyer, 1720, vol. 20, p.132).

[5] Supplementary Appendix 3 contains a full history of the Company's management of the instalment schedules.

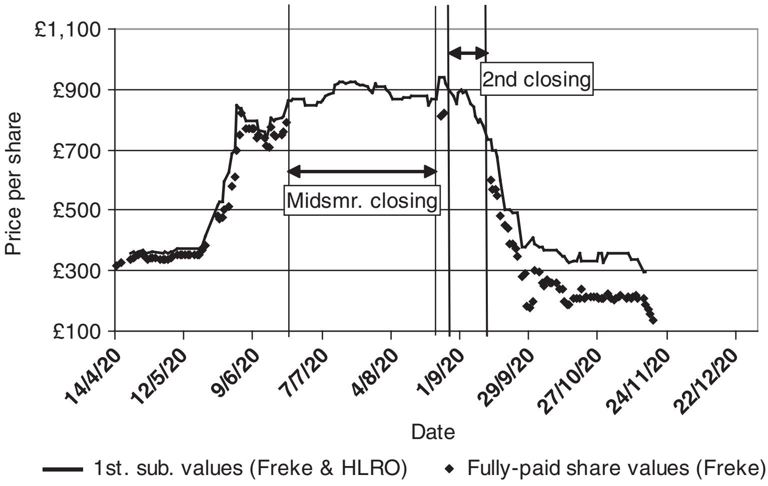

Fig. 1. South Sea fully-paid and 1st. subscription share values, 1720
Source: Freke (1720) and *HLRO* Box 158 (see text)

With a fixed instalment schedule it is possible to calculate the present value of instalments yet to be paid. Since nothing would have prevented an owner of a subscription share from paying all the instalments and obtaining a fully-paid share, it straightforwardly follows that the value of a fully-paid share would have been a lower bound on the total value of a subscription contract. That is, the trading value of a subscription share plus the present value of the cost of converting it into a fully-paid share (the present value of the instalments yet to be paid) would have been no lower than the value of a fully-paid share. This lower bound—a very minimal arbitrage bound—allows us to illustrate quite straightforwardly the challenge in understanding the values of subscription shares. In Figs 1 and 2 are respectively illustrated the value of subscription shares (plus the present values of their instalments) alongside the values of fully-paid shares.

Although we can remark that fully-paid share values do obviously provide a lower bound on the subscription share values, more remarkable still is how far above fully-paid share values subscription values are. Can we understand why subscription shares had such value? The important clue is found in the levels of the fully-paid share values themselves. When fully-paid values were high, such as on 20 June 1720, the following array of facts is found. A fully-paid share was worth about £760 (denote it $P_{fully\text{-}paid}$) and a subscription share (from the 1st series) was worth about £590 (denote it $P_{subscription}$). The difference between the two values was £170. The present value of the remaining seven instalments (calls) of £30 each would have to have been smaller than £210, but a realistic value (at about a 4% or 5% per annum discount rate) would also have been greater than £200. Thus we see that the value of a subscription contract was

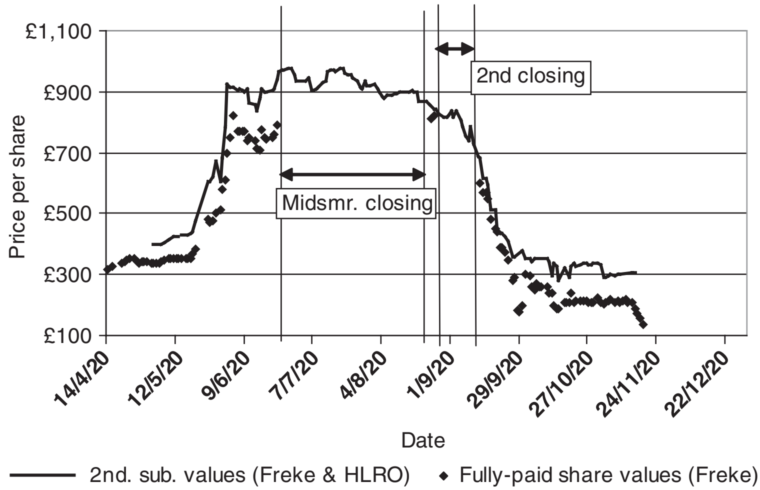

Fig. 2. South Sea fully-paid and 2nd. subscription share values, 1720
Source: Freke (1720) and *HLRO* Box 158 (see text)

a bit more than £790 on 20 June. Contrast this with 25 October 1720 when fully-paid shares were worth about £210. 1st series subscription shares were worth about £130 and at the same time the present value of their remaining seven instalments was still a bit more than £200. Thus when South Sea share values had reached low levels in the autumn of 1720, the value of subscription contracts relative to fully-paid shares actually rose. The figures strongly suggest that subscription share values, relative to fully-paid share values, followed a nonlinear function of the level of fully-paid shares. This is the hallmark of an option value.

But what kind of option could be embodied in the subscription shares that would give them such high values when fully-paid share values were low? Like any call option on shares, the subscription shares certainly gave the right to their owners to own a fully-paid share at a low price in the contingency that share prices rose and continued to stay above the issue-price of the subscription shares. But the subscription shares also required that their owners make a sequence of decisions to pay instalments and each instalment that was paid only secured to the payer the right to make another instalment payment later. How would we value such rights? That question is answered in the next section.

This is not the first scholarly examination of the South Sea subscription shares and their prices. Dale *et al.* (2005) assumed that the subscription issues were packages of fractional shares and the subscriber was required to make instalment payments with no possible recourse to default. The natural conclusion they drew from such a counterfactual assumption was that subscription share values and fully-paid share values should have been locked together into linear pricing relationships. Of course, when no sensible linear relationships were found by these authors in data, such as those in Figs 1 and 2, they concluded that irrational pricing relationships were the only possible explanation. There is, however, strong evidence that these authors did not examine the relevant law (*6 Geo.1, c.4*) and ignored as well much evidence in pamphlet literature and in Company minutes about how the subscription shares worked (Shea, 2007).

3. Theory of South Sea subscription share pricing

Presenting a theory of South Sea subscription share pricing is quite simple when subscription contracts have the very simple structure that the South Sea subscription contracts had. With a failure to pay an instalment there was to be an eventual forfeiture of the subscription share to the Company. The Company would take possession of the subscription share for the purpose only of extracting the value of the instalment that was due to it. When the Company disposed of the forfeited share, the defaulting subscriber would receive any excess value (the overplus). These were the happy circumstances of the South Sea Company's subscription contracts, as defined in *6 Geo.1, c.4*, that were discussed in the previous section. Even though a subscriber would have no reason to default if the present value of the subscription share ($P_{subscription}$) was greater than a due instalment (call it K), it is nevertheless important to remember that if he were to default for any reason, he would still be due to be paid whatever the Company was able to sell the subscription share for, minus his liability of K to the Company. Under these conditions, we shall see that a subscription share will amount to what can be called an n-fold compound call option on a fully-paid share. By a compound call option we mean a series of call options on call options.

Imagine a subscription share for which the number of instalments due to be paid is n. Let us now consider the value of that share after n-1 instalments have been paid and only one instalment remains to be paid.

Proposition 1 After n-1 instalments of K each have been paid, an n-instalment subscription share becomes nothing more than a call option upon one fully-paid share.

Proof The exercise price of this call option is the last instalment itself (K) and the exercise date is the date on which the last instalment is to be paid. If the last instalment is paid, the subscriber has (in net terms) the value of one fully-paid share less the instalment (K). If the subscriber instead defaults upon the last instalment, the Company receives the subscription share in forfeit. When the Company sells the subscription share for whatever it is worth, the defaulting subscriber will then be given the excess value of the share ($P_{subscription}$) over and above his liability (K) to the Company (the overplus). In the case of the last instalment to be paid, $P_{subscription}$ equals $P_{fully\text{-}paid}$; that is, the subscriber gets $\max[P_{fully\text{-}paid} - K, 0]$ or, in other words, $P_{fully\text{-}paid} - K$ or 0, whichever is greater. This is, of course, the classic expression of the terminal value of a call option upon a fully-paid share for which the exercise price is K.[6] Regardless of whether the subscriber defaults or not, he will end up with $\max[P_{fully\text{-}paid} - K, 0]$. Thus the terminal value of a subscription share is the terminal value of a call option upon a fully-paid share. It follows that the present value of a subscription share should, from the payment of the penultimate instalment to the last instalment, be the present value of a call option (with a strike price of K) on a fully-paid share. □

We now extend this argument to consider the value of a subscription share when the next-to-last instalment payment is to be made.

Proposition 2 After n-2 instalments of K each have been paid, but before the penultimate instalment is paid, the subscription share is a call option (with strike price K) on the call option described in Proposition 1.

Proof Again, the subscriber will either default or not default. If he defaults, he gets either nothing or $P_{subscription} - K$, when the Company sells his forfeited subscription share for what it is worth. This is also what he has in terms of net asset value if he does not default and pays the due instalment of K. Regardless whether he defaults or not, the value of his subscription share (when the penultimate instalment is due) is therefore $\max[P_{subscription} - K, 0]$. If we imagine a call option on the subscription share whose exercise date is the same date as that of the due instalment and whose exercise price is K as well, the terminal value of that call option will also be either $P_{subscription} - K$ or 0. We would thus conclude that the present value of the subscription share, before the penultimate instalment was due and after the previous instalment was paid, would have to be the value of a call option on the subscription share (with exercise price K). Since the subscription share itself will become a call option upon a fully-paid share after the penultimate instalment is paid, the subscription share must be viewed, before that time, as a call option upon a call option. □

This argument can be extended recursively to apply to the values of the subscription share for any number of instalments due upon it. For n instalment payments, the subscription share would be equivalent to a call option on a call option on a call option . . . (n-1 times) on a call option on a fully-paid share. Although we may note that a subscriber would most likely default whenever an instalment (K) was due and whenever $P_{subscription} < K$, it will nevertheless be true that, regardless of whether he defaults or not, the net value (at the time of an instalment payment) of a subscription share will always be $\max[P_{subscription} - K, 0]$.

[6] This is well known to be true for non-dividend-paying shares, at least, but applies to South Sea subscription shares as well since they had the same dividend rights as had fully-paid shares. The Company declared that all subscription shares carried rights to the 10% midsummer dividend that was declared in April for fully-paid shares. As it turned out, this dividend resulted only in a 10% stock bonus.

Values of compound call options are not difficult to compute even though closed-form solutions for compound option values are difficult to derive. The first successful effort was Geske's (1979) solution for a call on a call. Good textbook treatments of compound options readily extend his solution to calls on puts, puts on puts and puts on calls (McDonald, 2003, p.441–4). To our knowledge, however, no one has presented a closed-form solution for a general *n*-fold compound call option of the type we are describing here. Examples of modern partly-paid shares that have a forfeiture sanction attached to them are not hard to find, but they usually have other features that make them difficult to value. After forfeiture, there typically is no requirement that an overplus be delivered to defaulters. Although it is unlikely an owner of a partly-paid share would ever default in a situation in which he could expect to get a positive overplus, it is true nevertheless that the promise of one is the only guarantee that the terminal value of a partly-paid share will be exactly the same as the terminal value of a call option on a fully-paid share. Modern partly-paid shares are more complex also because the schedules of instalments to be paid are typically not known. The South Sea Company published fixed schedules for instalment payments. It did change them occasionally, but the subscription shares were not subject to sudden surprise calls for instalments to be paid.

Matters were simpler in 1720 when it came to the definition of partly-paid shares and it took some time for partly-paid shares and related law to evolve into their modern forms. By the time we come to 1790s and beyond, descriptions of partly-paid share finance became regular features of incorporating Acts. In any Act for an infrastructure project, such as a canal, railway, bridge, gasworks or waterworks, can be found passages that deal with how finance for the firm can be raised with partly-paid shares. Restrictions on instalment schedules came to be specified and many Acts contained a mixture of provisions as to how forfeiture of shares would be handled.[7] Subscription equity finance, paid for in instalments, was such a common feature in 18th-century and 19th-century corporate financial life, issues of fully-paid shares were exceptional. In modern times the situation is quite the reverse; when new shares are issued, corporations typically issue very large numbers of nominally small, fully-paid shares.

4. Subscription share value computations

We use a simple algorithm to compute subscription share values as compound option values. The techniques we use can be found in most any intermediate

[7] Even then the principle of the subscriber's obligation to pay all instalments was far from completely established as it would eventually become in current company law. A typical early 19th-century incorporation Act, such *as 6 Geo.4, c.67* for the Ashton under Lyne Gas and Waterworks *(Local and Personal Acts*, 1825), gave that company an option to pursue defaulting subscribers with Actions on Debt, but would also allow the Company to take subscription shares in forfeit. DuBois (1938) is essential reading for the legal basis of corporate subscription finance in the 18th century.

textbook on derivatives pricing and here we adapt the theory, terminology, and notation presented by Cox and Rubinstein (1985, pp.171–8). There are a number of frameworks in which to present the fundamentals of option pricing theory, but the framework is determined largely by the way in which the stochastic character of share returns is modelled. The range of choice in such stochastic models is immense, but in this paper we use the simplest of them all, the binomial random walk. The binomial random walk is a simple, but is also a powerful means of describing the way in which prices, such as share and option prices, evolve through time. Using it can lead to a number of exact pricing formulae for options that serve as accurate approximations to formulae that can be obtained from other more complex stochastic models. Over some appropriately short interval of time, the price of a fully-paid share is assumed to take a random walk either 'up' or 'down'. The potential sizes of these respective steps are denoted u $(u > 1)$ and d $(0 < d < 1)$. So, starting with a fully-paid share price ($P_{fully\text{-}paid}$), the binomial random walk results in a price of either $u \times P_{fully\text{-}paid}$ or $d \times P_{fully\text{-}paid}$. If we now imagine an option contract that is written on the share, the value of that contract will also follow a random walk whose values we can respectively denote as $P_{option,u}$ and $P_{option,d}$.

We express the solution for the option's present value (P_{option}) in a standard way that uses Δ, the number of fully-paid shares that are held in a portfolio whose value will replicate the value of the option. When Δ shares in this so-called replicating portfolio are combined with an appropriate amount of borrowing (B), the possible future values of the portfolio will mimic the possible future values of the option and hence it must be the case that the present value of the portfolio will mimic the present value of the option. The exact expressions for Δ and B that define such a replicating portfolio are standard. These expressions are respectively:

$$\Delta = \frac{P_{option,\,u} - P_{option,\,d}}{(u-d)P_{fully-paid}} \tag{1}$$

and

$$B = \frac{u \times P_{option,\,d} - d \times P_{option,\,u}}{(u-d)r} \tag{2}$$

r stands 1 plus the interest rate at which risk-free borrowing or lending can proceed. The value of the replicating portfolio is $P_{fully\text{-}paid}\,\Delta + B$, which is also the value of the option, P_{option}.[8] These formulae can be used recursively to compute the value of an option back to the present period so that between the present date and the terminal date for the option a binomial random walk for option values can be constructed.

[8] The reader might notice here that we are ignoring the complicating effects of value dilution on the price of the option. In the process of paying instalments on South Sea subscription shares, there well may have been a depressing, or diluting effect, on South Sea fully-paid share values. Such effects however can be safely ignored, as argued in the last paragraph of supplementary Appendix 1.

A small numerical example will illustrate the principle of the replicating portfolio. Imagine a share that is currently worth £500 ($P_{fully\text{-}paid}$). From now to the next period imagine that possible share price appreciation is 50% ($u = 1.5$) and possible share price depreciation is also 50% ($d = 0.5$). Imagine also there is a call option on the share that will expire in the next period if it is not exercised then. This option we assume has an exercise price of £400 (K). In the next period the share price either goes up to £750 or down to £250. The call option will thus either become worth £750 – £400 = £350 or will become worth nothing, because it will not be exercised. The replicating portfolio will mimic the possible values of the option depending on what happens in $P_{fully\text{-}paid}$'s random walk. Equation 1 says that

$$\Delta = \frac{£350 - £0}{(1.5 - 0.5)£500} = 0.7$$

and, assuming that the risk-free rate of interest is 5%, Eq. 2 says that

$$B = \frac{1.5 \times £0 - 0.5 \times £350}{(1.5 - 0.5)1.05} = -\frac{£175}{1.05} = -£166 \;\&\; 2/3$$

The replicating portfolio is defined as containing 0.7 share and a loan liability in the next period of £175. Why it is called a replicating portfolio should now be clear; if the share price goes up to £750, the portfolio will be worth 0.7×£750 – £175 = £350 or, if the share price goes down to £250, the portfolio will be worth 0.7×£250—£175 = £0. The portfolio perfectly replicates the possible future values of the call option and therefore the portfolio must be worth as much as the call option in the present; the portfolio and the option are worth 0.7 × £500 – £166 & 2/3 = £183 & 1/3.

With a binomial random walk for option values defined, there is nothing to prevent us from valuing another option on the option in the same manner. This computational algorithm can then be nested within itself to compute the value of an *n*-fold compound option. Further details of how we implement this algorithm and our resulting programs are described in our final supplementary Appendix 4.

5. Data and distributions of fully-paid share returns in 1720

In this section we provide some more details of the data that we will analyse, why we have selected the data we have and its special limitations. Secondly, we will also study the distribution of share returns contained in this data. This will provide guidance for how we should calibrate the programs we use to calculate theoretical subscription share values.

If we look at the fully-paid share values depicted in Figs 1 and 2, we see that there are two substantial periods for which there are no depicted values. These were the periods in which the Company's transfer ledgers were closed and trade in fully-paid shares could only be carried on in a forward market for delivery of shares. This does not mean that there were no share value numbers reported for these periods. We have such numbers in plenty, but it is not easy to understand what they represent actually. The primary sources for value data in 1720 are the two price courants, Freke's, *Prices of stocks* and Castaing's *Course of the exchange*.[9] During the midsummer period (23 June–22 August) the Company's ledgers were closed for making up the midsummer dividend. It was common practice amongst companies such as the South Sea Company to close their ledgers for such purposes over lengthy periods of time. In this case, both price courants indicate that reported share values were values for forward delivery 'at the opening', i.e. on or about 22 August. No further information is reported about the forward contracts to which these values were attached. What were their size and what kind of securities was embedded into these contracts that would ensure performance by the two contracting parties? This kind of information, that is, everything that must be known in order to locate an underlying spot value for a fully-paid share, is simply not revealed by the publishers of the price courants. The second closing was a sudden closing of the ledgers on 31 August, when it was also announced that the closing would last until 22 September. The transfer ledgers were, however, re-opened on 12 September. We know of this second closing only because discussion of it can be found in Company records and in a court case that was a consequence of the Company's sudden decision to re-open the ledgers.[10] But no mention of this can be found in the price courants and so we have no word from them about the nature of the value numbers they reported for the 31 August–12 September period. They were very probably forward delivery values as well 'for the opening', but that is only a reasonable speculation. The best evidence we have so far for forward premia in private financial contracting in 1720 is that such premia could be very large. We conclude that, for the time being at least, it is very difficult to locate approximate spot values for fully-paid shares in those periods in which transfer ledgers were closed.[11]

Figures 1 and 2 also depict subscription share values and these values can be found in either Freke's or Castaing's publication. We can depict values for these shares throughout the periods in which the share ledgers were closed because it appears that spot trade in subscription shares was carried out by a trade in

[9] The genesis of the British financial press and the relative standing of Freke's and Castaing's reportage are discussed by Neal (1990, ch.2).

[10] *Add. MS* 25,499, *Court Minutes*, 25 August 1720. The closing was thought opportune while the further management of subscriptions was discussed. The transfer books were ordered (11 September) to be re-opened on the next day. The forward delivery contract in dispute in Maber and Thornton (1722) was undertaken 7 September for 'the next opening'. On 12 September Maber failed to take delivery of the stock, which precipitated Thornton's successful suit.

[11] The evidence concerning size of forward premia in 1720 is summarized by Shea (2007, section III). Dale *et al*. (2005) implicitly assumed that the forward values reported in the price courants for the midsummer closing are what today would be called 'zero-premium' forward values—forward values that differ from the underlying spot value by only the small interest-cost of carrying the stock forward. It was under this highly counterfactual assumption they imputed spot values from the reported midsummer forward values.

endorsable subscription receipts. By late June it was certainly the case that such trade was possible and legal because its existence was acknowledged and ratified in the Bubble Act (*6 Geo.1, c.18*). But still that does not mean that we know how Freke and Castaing got their subscription value data. Although it is a reasonable presumption that they were conducting surveys of values in spot trade of subscription receipts, they do not say so anywhere in their publications. This is a concern because we know that only in the case of the 1st subscription were receipts delivered out to subscribers soon after the subscription was taken in (14 April). Even in the case of the 2nd subscription (taken on 20 April) receipts were apparently not ordered to be delivered until July and throughout July.[12] So we might well wonder just how transactions in subscriptions were actually carried out, especially in the early trade of the 1st and 2nd subscriptions, and what precisely were the numbers that Freke and Castaing were reporting. Some of the early subscription values reported by Freke and Castaing splice nicely with some special transactions records of sales of 1st and 2nd subscriptions by Company directors through brokers.[13] There is thus the possibility that some of the early values reported by price courants were the results of sales by directors of places on the subscription lists. In other words, some of the early reported values could have been the result of primary sales rather than secondary trade in subscription receipts.

In the next section we study the pricing of 1st and 2nd subscriptions shares relative to fully-paid share values in two very different periods: (i) before the midsummer closing of the Company ledgers when share values were high and rising quickly and (ii) after the second closing when prices were much lower and declining quickly. The former period was dominated by positive price changes and the latter period was dominated by negative price changes, but daily price changes in both of these periods appear to be nearly bimodal, as Figs 3 and 4 attest. The difference in the two periods' distributions of returns might be easier to see over a longer returns' horizon. We therefore have run a Monte Carlo experiment to obtain an idea of the possible range of returns over a longer one-month horizon. With 10,000 independent 30-day random samplings from the discrete distributions represented in Figs 3 and 4, we obtained the simulated 30-day cumulated returns distributions illustrated in Figs 5 and 6.

[12] The Duke of Portland had to wait until the end of May before he received the receipts for his purchases of the first subscription (*Pw B* 165, p.13–4). According to the Anonymous *An Account of the Subscriptions* (Anon, 1722, p.15–6), when the 3rd subscription lists were being made out (17–21 June), people were trying still to obtain receipts for the earlier subscriptions. In the *Minute Book of the Committee of the Treasury* (*HLRO*, B64, p.19), it was not until 8 June that receipts for the 2nd subscription were ordered to be delivered. On 22 July (p.24) the Committee of Treasury called for further expedition from the Cashier in the delivery of those receipts. In one of his many newsletters to the Duke of Portland, Pheasant Crisp wrote how the Company was making an effort to issue receipts for the 2nd subscription as late as 25 July 1720 (*Pw B* 8, 10).

[13] The Committee of Secrecy made its reports to the House of Commons in 1721 and left behind a valuable, but little used collection of papers (*HLRO*, Box 158).

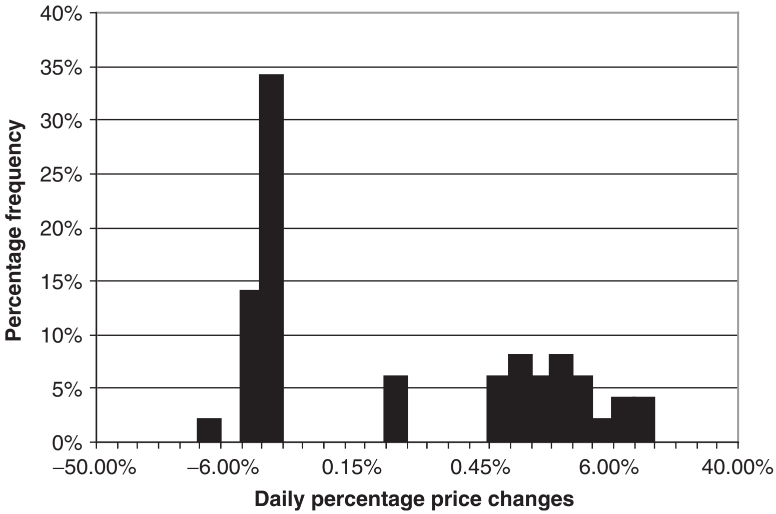

Fig. 3. Discrete distribution of daily South Sea original share price changes, 14 Apr–22 Jun 1720 (55 observations)

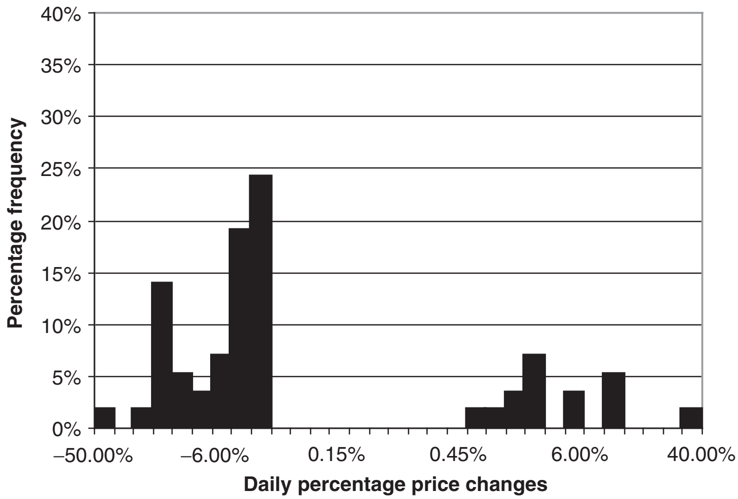

Fig. 4. Discrete distribution of daily South Sea original share price changes, 13 Sep–18 Nov 1720 (58 observations)

If we look at the distributions of the simulated 30-day returns, we can obtain a better idea of the likely range in returns' variation. For example, in the 14 April–22 June period, the middle 90% of the distribution of monthly returns in Fig. 5 lies between +160% and +10%. Although negative monthly returns are possible in the simulated distributions, they are extremely unlikely, whereas very

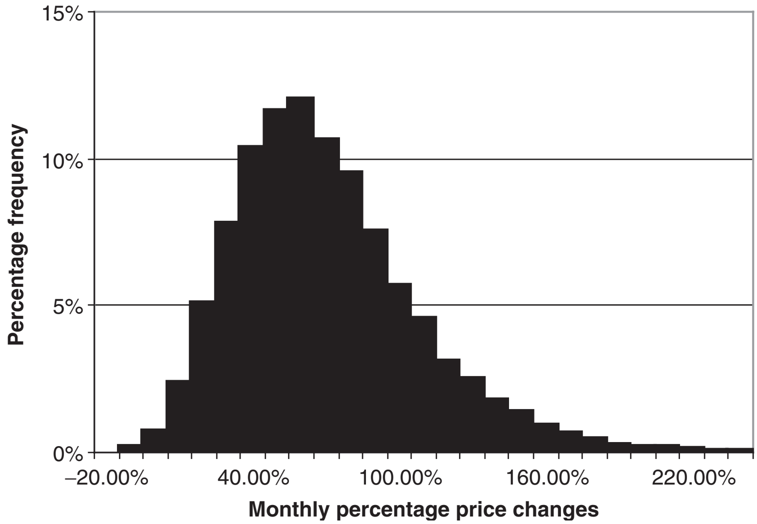

Fig. 5. Discrete distribution of 30-day cumulated daily price changes, 10,000 Monte Carlo replications, 14 Apr–22 Jun 1720

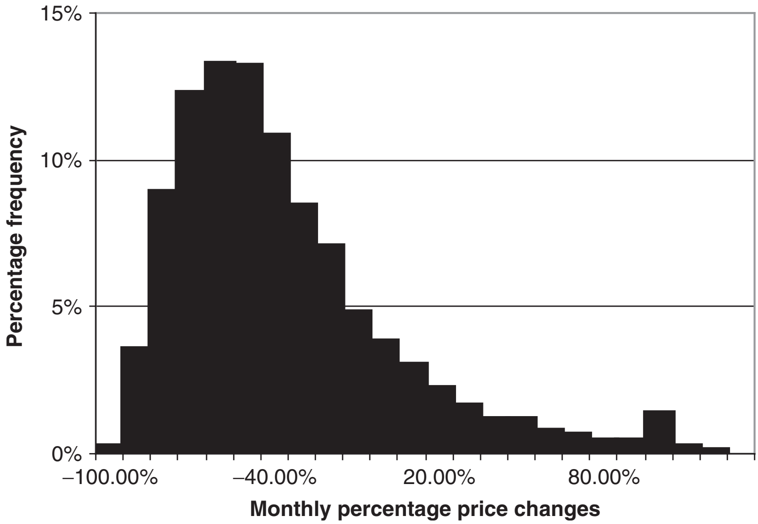

Fig. 6. Discrete distribution of 30-day cumulated daily price changes, 10,000 Monte Carlo replications, 13 Sep–18 Nov 1720

large positive monthly returns are highly likely. This suggests that very large values of u and only very small negative values for d-1 will be good choices for calibrating our model in the early period. In Fig. 6 we see a very different distribution. The middle 90% of that distribution lies in the monthly return range of +55%

to −85%, suggesting that a very different set of estimates for *u* and *d* are appropriate for the later period. In the next section we see how our choices for *u* and *d* affect the empirical performance of our pricing theory.

6. Empirical results

How well does the model of compound call option pricing fit the empirical data on subscription share values? To answer this question we do not fit an econometric model, as such. What are reported in this section are the calculated theoretical values of subscription shares that come from our model when the model is calibrated, or tuned we might say, to calculate these values for a given set of volatilities, or (*u-1*,*d-1*)-pairs. The questions concerning empirical performance can be framed basically as the following: (i) Can empirically reasonable volatilities result in theoretical subscription values that are close to empirical values? and (ii) Are very unreasonable volatilities just as capable of producing or are even required in order to produce good theoretical subscription share values?

The reader can understand the results in Tables 1 through 4 by keeping the following scheme in mind. Imagine three different volatilities: (u'-1,*d-1*), (*u-1*,*d-1*), and (*u-1*,d-$1'$) with $u' > u > 1$ and $1 > d > d' > 0$. For example, the two pairs (u'-1,*d-1*) and (*u-1*,*d-1*) can represent what we might call a volatility range, in which the two random walks differ only in their upward potential. Now imagine that we calculate two theoretical values for a subscription share using the two different volatilities that define a range. If an empirical subscription share value can be found that falls between the two theoretical values, then we can conclude that our model is capable of duplicating the empirical subscription share value precisely using some volatility that falls within the volatility range (u'-1,*d-1*):(*u-1*,*d-1*). Rather than solving for such volatilities precisely (and there is no reason that they have to be unique), the tables are a simple visual presentation of the model's capability of capturing empirical subscription share values when using some reasonable (or unreasonable) volatility ranges. Each table shows a selection of 1720 dates and fully-paid share values taken from Freke (1720). The minimum and maximum subscription share values reported are taken from all available sources, but they come primarily come from either Freke (1720) or Castaing (1720). The reason we wish to use such extreme values is as follows. The interday and intraday variations in share values, even from the same source, could be quite large. The two price courants were more likely to disagree upon 1st subscription share values than they were to disagree upon fully-paid share values or values for the 2nd subscription series. The publishers did not indicate any precise times of day when their reported data were collected, so we cannot be sure that subscription share values were even collected at the same time of day as fully-paid share values were collected. We strongly suspect that on some days reported subscription share values were even stale, left over values that were observed either earlier in the day or even on the day before. We are concerned therefore

Table 1 Simulated 1st subscription share values, 20 April–22 June 1720

Date	Fully-paid share value	Minimum observed subscription value	Ranges of monthly volatility				Maximum observed subscription value
			160% (*u*-1) to -1% (*d*-1)	160% (*u*-1) to -10% (*d*-1)	160% (*u*-1) to -20% (*d*-1)	20% (*u*-1) to -20% (*d*-1)	
20/04/20	£336	£60	£87	£123	£153	£114	£65
29/04/20	£337	£67	£88	£122	£151	£115	£75
07/05/20	£334	£70	£84	£118	£147	£111	£70
16/05/20	£352	£79	£102	£129	£157	£126	£80
23/05/20	£487	£220	£237	£254	£274	£258	£220
25/05/20	£480	£215	_**£230**_*	_**£248**_*	£267	£250	£230
26/05/20	£470	£230	_**£220**_*	_**£238**_*	£258	£241	£230
27/05/20	£474	£230	_**£224**_*	_**£242**_*	£261	£244	£230
28/05/20	£510	£270	_**£260**_*	_**£277**_*	£294	£280	£300
30/05/20	£555	£320	_**£305**_*	_**£322**_*	£336	£325	£340
31/05/20	£600	£376	£350	_**£367**_*	_**£379**_*	_**£370**_*	£395
01/06/20	£675	£400	_**£425**_**	_**£442**_**	_**£452**_**	_**£445**_**	£525
02/06/20	£700	£540	£450	£467	£476	£470	£600
03/06/20	£750	£540	£500	£517	£525	£520	£550
04/06/20	£770	£510	£520	_**£537**_*	_**£544**_*	_**£540**_*	£540
06/06/20	£770	£500	_**£519**_*	_**£537**_*	£543	£540	£520
07/06/20	£760	£500	_**£509**_*	_**£527**_*	£533	£530	£525
08/06/20	£760	£470	£509	£527	£533	£530	£500
09/06/20	£740	£480	_**£489**_*	_**£507**_*	£513	£510	£500
10/06/20	£740	£500	_**£489**_*	_**£507**_*	£513	£510	£500
11/06/20	£750	£470	_**£499**_*	_**£517**_*	£523	£520	£510
13/06/20	£740	£465	_**£489**_*	_**£507**_*	£513	£510	£490
14/06/20	£715	£440	£464	_**£482**_*	_**£488**_*	_**£485**_*	£485
15/06/20	£710	£480	£459	_**£477**_*	_**£483**_*	_**£480**_*	£500

(Continued)

Table 1. Continued

Date	Fully-paid share value	Minimum observed subscription value	Ranges of monthly volatility				Maximum observed subscription value
			160% (u-1) to -1% (d-1)	160% (u-1) to -10% (d-1)	160% (u-1) to -20% (d-1)	20% (u-1) to -20% (d-1)	
16/06/20	£755	£450	***£504***[**]	***£522***[**]	***£527***[**]	***£524***[**]	£540
17/06/20	£750	£480	£499	***£517***[*]	***£522***[*]	***£519***[*]	£520
18/06/20	£745	£505	***£494***[*]	***£512***[*]	£517	£514	£510
20/06/20	£750	£500	***£499***[*]	***£517***[*]	£522	£519	£510
21/06/20	£760	£500	***£509***[*]	***£527***[*]	£532	£529	£520
22/06/20	£790	£520	***£539***[*]	***£557***[*]	£562	£559	£550
			15 captures out of 30[*]				
				5 captures out of 30[*]			
					5 captures out of 30[*]		
			2 encompassings[**]				
			8 complete misses				

only that our model can produce roughly correct magnitudes for subscription share values with reasonable volatilities. And since we model volatility as fixed through time, it is better that the reader concentrate upon reading the tables along each row and across columns rather than comparing rows through time.

The results in Table 1 for the 1st subscription shares reflect quite well on the compound option model. Even before the finer detail of the table is examined, the eye is struck by the ability of the model to generate simulated share values that are not only the right order of magnitude, but in many cases are but a small percentage distance from actual empirical values. When we look at the ability of the model to precisely simulate empirical values, we note that most empirical values are captured when we employ volatilities with very large *d*'s and large *u*'s.[14] The simulated distribution of monthly returns on fully-paid shares before midsummer 1720 (Fig. 5) has a very small tail in the negative range (only about 1% of the simulated monthly returns were slightly negative) and its top decile begins at about +160%. The volatility range in Table 1 that best corresponds to these simulated returns volatilities generates 15 captures out of a possible 30. If we hold *u* fixed and allow *d* to approach 1, we see that the model's ability to simulate actual subscription share values falls off markedly; any volatility range that allows for a monthly return on fully-paid shares of less than -10% permits the model to capture no more than five actual subscription share values in the 30 instances it could do so. The model fails to effect an exact capture in two instances in which we can say that it still has some success. On two dates, 1 June and 16 June, the ranges of reported subscription share values were quite large enough to encompass all the simulated values that the model is capable of producing. There may well have been trade at subscription values between the extremes that the model could have precisely captured using reasonable volatilities if we could have but observed those trading values. This outcome we shall call an 'encompassing'. The outright failure of the model to generate any value within the range of the extreme empirical values we call a 'complete miss' and there are eight of them out of the 30 instances in Table 1. Even so, we must repeat that no complete miss is an order-of-magnitude miss and three of them (23 May, 3 June, and 8 June) are very nearly captures. Where the model fails most noticeably (four complete misses) is in April and early May, in which it appears incapable of generating sufficiently low theoretical subscription values. One reason that the model overprices very early subscription values could be that this was a period in which real trade in subscriptions was more costly and difficult because the trade in endorsable subscription receipts had not yet been made legal. Perhaps the small supply of subscription receipts themselves was putting the trade at a disadvantage. Or perhaps this was when markets were having their first look at the new shares and they were not regarded as sufficiently seasoned to avoid being especially discounted.

[14] In the tables 'captures' are indicated by boldfaced and italicised (with an asterix) simulated values that straddle one of the extreme empirical subscription values.

How well does the success of the model carry over into post-August period? In this period the reportage of subscription share values becomes more uniform amongst our sources. Within-day variation in values becomes very infrequently reported, as well. The measures of success in the model's performance will therefore not include encompassings, but will be measured only by complete captures. In Table 2 are the results for the 1st subscription shares for this period. The table reports success in the sense that it generally shows that empirical subscription values are best captured by volatilities with very large downward potential. Reading carefully across most rows will also confirm that the subscription values not only positively respond to volatility (as any option value should), but are more sensitive to downward volatility than upward volatility. The model is most likely to capture empirical subscription values with positive and negative volatilities that are more extreme, but still consistent within the range of simulated monthly returns pictured in Fig. 6. We have identified already that the monthly-return range +55% to -85% captures 90% of this distribution, but our model best generates captures with a volatility range wider than +70% to -95%. That range corresponds with about the middle 97% of the distribution depicted in Fig. 6. So we find, as we found in Table 1, that in order to generate captures, we have to relax *d* to the smallest values that are still consistent with our simulated ranges of monthly returns on fully-paid shares. So far therefore, our model tends to understate, by a bit, subscription values per unit of volatility, especially downward volatility.

The captures reported in Table 2 (out of 52 possible) are almost entirely what could be called unique captures—captures accomplished with only one of the volatility ranges displayed in the table. The model is a bit more capable in this period of avoiding complete misses. There are only 12 out of a possible maximum of 52. The majority of the captures (27) are accomplished by using the volatility range with the most extreme potential downward volatility, but 16 further captures are accomplished with the use of other, not very different volatility ranges. One might still ask, however, if there are other reasonable volatility ranges equally capable of generating captures. For example, do we have to use actually volatility ranges that are consistent with potential monthly returns on shares as low as -90%? The answer to that is, probably, yes. To take an example, in Table 2 we have generated a capture of the subscription price for 21 September (£280) by using the volatility range, in terms of monthly returns on fully-paid shares, of (+60%,-95%):(+60%,-90%). If we were to consider a volatility range with a downward potential a bit smaller than that, say between -90% and -85% per month, what would have to happen to upward potential volatility to continue to generate captures? Clearly, it would have to increase and, in fact, it would have to increase quite a bit. The volatility range (+95%,-90%):(+95%,-85%) generates a capture of the subscription price of £280 with the resulting simulated value pair of (£281,£265). So, even though a -85% potential negative return on fully-paid shares seems to be a perfectly reasonable one to use from the monthly returns distribution shown in Fig. 6, it would have to be coupled with what is, we believe, an unreasonably high upward potential monthly return (+95%) for

Table 2 Simulated 1st subscription share values, 14 September–15 November 1720

Date	Fully-paid share value	Minimum observed subscription value	Ranges of monthly volatility: 70% (*u*-1) to -99% (*d*-1)	70% (*u*-1) to -95% (*d*-1)	60% (*u*-1) to -95% (*d*-1)	60% (*u*-1) to -90% (*d*-1)	Maximum observed subscription value
14/09/20	£570	£460	***£492***[*]	***£460***[*]	***£453***[*]	£434	£460
15/09/20	£570	£460	***£491***[*]	***£460***[*]	***£453***[*]	£434	£460
16/09/20	£550	£440	£472	***£441***[*]	***£434***[*]	£415	£440
17/09/20	£480	£370	£404	***£374***[*]	***£369***[*]	£349	£370
19/09/20	£450	£280	£375	£346	£340	£321	£280
20/09/20	£440	£280	£365	£336	£331	£311	£280
21/09/20	£390	£280	£318	£290	***£284***[*]	***£265***[*]	£280
22/09/20	£390	£270	£314	£287	***£281***[*]	***£264***[*]	£270
23/09/20	£375	£270	£314	£287	***£280***[*]	***£264***[*]	£270
24/09/20	£350	£270	£314	£287	***£280***[*]	***£263***[*]	£270
26/09/20	£280	£170	£209	£185	***£179***[*]	***£166***[*]	£170
27/09/20	£290	£170	£209	£185	***£179***[*]	***£165***[*]	£170
30/09/20	£200	£200	£134	£113	£108	£94	£200
01/10/20	£300	£180	£227	£202	£196	£182	£180
03/10/20	£295	£170	£221	£197	£191	£177	£170
04/10/20	£260	£170	***£188***[*]	***£165***[*]	£159	£146	£170
05/10/20	£250	£160	***£179***[*]	***£156***[*]	£149	£137	£160
06/10/20	£270	£160	£197	£174	***£167***[*]	***£154***[*]	£160
07/10/20	£260	£160	£187	***£164***[*]	***£158***[*]	£145	£160
08/10/20	£260	£160	£187	***£164***[*]	***£158***[*]	£145	£160
10/10/20	£260	£160	***£181***[*]	***£158***[*]	£152	£138	£160
11/10/20	£240	£150	***£163***[*]	***£140***[*]	£134	£120	£150
12/10/20	£240	£140	***£163***[*]	***£140***[*]	***£134***[*]	£120	£140
13/10/20	£200	£140	£129	£106	£100	£85	£140
14/10/20	£190	£130	£120	£97	£92	£78	£130
15/10/20	£190	£125	£120	£97	£92	£77	£125
17/10/20	£210	£130	***£136***[*]	***£113***[*]	£107	£93	£130

(Continued)

Table 2. Continued

Date	Fully-paid share value	Minimum observed subscription value	Ranges of monthly volatility: 70% (*u*-1) to -99% (*d*-1)	70% (*u*-1) to -95% (*d*-1)	60% (*u*-1) to -95% (*d*-1)	60% (*u*-1) to -90% (*d*-1)	Maximum observed subscription value
18/10/20	£210	£130	£136*	£113*	£107	£92	£130
19/10/20	£210	£150	£136	£112	£107	£92	£150
20/10/20	£240	£130	£161	£137	£132*	£118*	£130
21/10/20	£210	£130	£135*	£112*	£106	£92	£153
22/10/20	£215	£130	£136*	£112*	£107	£92	£130
24/10/20	£215	£130	£136*	£112*	£107	£91	£130
25/10/20	£215	£130	£136*	£111*	£106	£91	£130
26/10/20	£212	£130	£133*	£109*	£104	£88	£130
27/10/20	£210	£120	£131*	£107*	£102	£87	£120
28/10/20	£210	£120	£131*	£107*	£102	£86	£120
29/10/20	£208	£125	£129*	£105*	£100	£85	£125
31/10/20	£222	£120	£136*	£115*	£109	£94	£120
01/11/20	£210	£120	£125*	£105*	£99	£85	£120
02/11/20	£205	£120	£120*	£100*	£94	£81	£120
03/11/20	£209	£120	£124*	£103*	£97	£84	£120
04/11/20	£212	£120	£126*	£106*	£100	£86	£130
05/11/20	£220	£120	£133*	£112*	£106	£91	£120
07/11/20	£210	£120	£123*	£103*	£97	£84	£120
08/11/20	£214	£120	£127*	£106*	£100	£86	£120
09/11/20	£209	£100	£122	£102*	£96*	£82	£100
10/11/20	£211	£120	£124*	£103*	£97	£84	£120
11/11/20	£216	£120	£128*	£107*	£101	£87	£120
12/11/20	£210	£100	£122	£102*	£96*	£82	£100
14/11/20	£210	£125	£122	£102	£96	£82	£125
15/11/20	£190	£125	£112	£93	£86	£72	£125

27 captures out of 52 (70% (*u*-1) to -99% (*d*-1))

8 captures out of 52 (70% (*u*-1) to -95% (*d*-1))

8 captures out of 52 (60% (*u*-1) to -95% (*d*-1) / 60% (*u*-1) to -90% (*d*-1))

12 complete misses

the autumn of 1720. In experiments of this type we have explored the model's capabilities over a wider selection of volatility ranges than are reported in the tables. In experiments like this, however, since it is impossible to give a statistical characterization of our model's powers, it is essential that the interested reader is able to duplicate and to extend our results. To that end, the programs that generate our results are provided and instructions for their use are provided in Appendix 4. We are fairly sure that the interested reader can confirm that the volatility ranges displayed in Table 2 are the only reasonable volatility ranges that our model can use to generate captures of actual subscription share values.

As far as the 1st series of subscription shares are concerned, our model appears to be generally successful. As a general rule, it manages to simulate empirical subscription share values when it is calibrated with empirically defensible returns volatilities for fully-paid shares. How well this success extends to the case of the 2nd series of subscription shares is the story told by Table 3 and 4, but before we turn to these results, the reader should recall that the 2nd series differs from the 1st series in two important respects. It had a higher issue price (£400 per share, as opposed to £300 per share) and thus should *ceteris paribus* have been less valuable than the 1st series. On the other had, its instalments were spread over a much longer period of time than were the instalments of the 1st series (see the relevant tables in supplementary Appendix 3; which *ceteris paribus* should have made the 2nd series more valuable than the 1st. There were brief times in 1720 when the 2nd series subscription shares appeared to be as valuable and perhaps even more valuable than were the 1st series shares. Because the 2nd series shares had approximately a year longer duration than did the first series, it would be surprising indeed if the volatilities that best explained their values were quite as extreme as were the volatilities that best explained the 1st series shares' values. After all, in April-June 1720 it might have been felt that negative returns on South Sea shares were highly unlikely in the medium term, but it probably was not case that people felt that such lopsided volatility was likely to persist for as long as two years. Likewise, in the dark days of autumn 1720 people could probably have been more optimistic about the return of upside volatility in the long term than they could feel optimistic about its reappearance in the short term. Our calculations of subscription values, however, are quite crude in the sense that a fixed set of volatilities is employed to calculate long-term as well as short-term option values. It would therefore not be surprising to find that the volatility ranges that best result in captures of 2nd series subscription values will not be as extreme as they were for the 1st series subscription shares. That is indeed the case and we consider it one of the more notable successes of our theory and computations. In the April-June period, volatility ranges that are consistent with more frequent negative returns and smaller and less frequent positive returns better account for the 2nd series values than they do for 1st series values (comparing Table 3 to Table 1). Similarly in autumn 1720, volatility ranges that are consistent with less frequent and smaller negative returns better account for the 2nd series values than they do for 1st series values (comparing Table 4 to Table 2).

Table 3 Simulated 2nd subscription share values, 20 April–22 June 1720

Date	Fully-paid share value	Minimum observed subscription value	Ranges of monthly volatility: 160% (*u*-1) to -1% (*d*-1)	160% (*u*-1) to -10% (*d*-1)	160% (*u*-1) to -20% (*d*-1)	20% (*u*-1) to -20% (*d*-1)	Maximum observed subscription value
03/05/20	£337	£15	£21	£108	£168	£74	£20
04/05/20	£337	£18	£21	£108	£168	£74	£18
05/05/20	£336	£18	£21	£107	£167	£73	£19
06/05/20	£336	£16	£21	£107	£167	£73	£20
07/05/20	£334	£22	£21*	£106*	£165	£72	£22
09/05/20	£346	£30	£22*	£114*	£174	£79	£32
11/05/20	£351	£30	£22*	£117*	£177	£81	£40
18/05/20	£351	£50	£22*	£116*	£176	£81	£60
19/05/20	£370	£55	£36*	£129*	£190	£93	£75
20/05/20	£381	£90	£47*	£137*	£198	£102	£90
24/05/20	£476	£215	£142	£205*	£275*	£177*	£220
25/05/20	£480	£220	£146	£207*	£279*	£181*	£220
26/05/20	£470	£220	£136	£200*	£270*	£172*	£220
27/05/20	£474	£235	£140	£203*	£273*	£176*	£235
28/05/20	£510	£275	£176	£223*	£295*	£196*	£290
30/05/20	£555	£220	£221	£231	£303*	£205*	£325
31/05/20	£600	£300	£266	£290*	£364*	£268*	£420
01/06/20	£675	£400	£341	£315	£389	£296	£500
02/06/20	£700	£540	£366	£398	£468	£380	£600
03/06/20	£750	£530	£378	£445	£513	£427	£560
04/06/20	£770	£520	£436	£464*	£531*	£446*	£530
06/06/20	£770	£520	£436	£464*	£531*	£446*	£545
07/06/20	£760	£520	£426	£464*	£531*	£446*	£540
08/06/20	£760	£490	£426	£464*	£531*	£446*	£525

(Continued)

Table 3. Continued

Date	Fully-paid share value	Minimum observed subscription value	Ranges of monthly volatility: 160% (u-1) to -1% (d-1)	Ranges of monthly volatility: 160% (u-1) to -10% (d-1)	Ranges of monthly volatility: 160% (u-1) to -20% (d-1)	Ranges of monthly volatility: 20% (u-1) to -20% (d-1)	Maximum observed subscription value
09/06/20	£740	£510	£406	***£464****	***£530****	***£446****	£515
10/06/20	£740	£525	£406	£435	£503	£417	£525
11/06/20	£750	£480	£416	***£445****	***£512****	***£426****	£525
13/06/20	£740	£475	£378	***£435****	***£502****	***£417****	£500
14/06/20	£715	£455	£381	***£411****	***£479****	***£393****	£510
15/06/20	£710	£495	£376	£406	£474	£388	£520
16/06/20	£755	£465	£421	***£468****	***£533****	***£450****	£525
17/06/20	£750	£490	£416	***£444****	***£510****	***£426****	£515
18/06/20	£745	£520	£411	£439	£506	£421	£525
20/06/20	£750	£520	£416	£443	£510	£426	£525
21/06/20	£760	£510	£426	***£453****	***£519****	***£435****	£535
22/06/20	£790	£530	£456	***£481****	***£546****	***£464****	£565

6 captures out of 36

18 captures out of 36

18 captures out of 36

11 complete misses

Table 4 Simulated 2nd subscription share values, 14 September–18 November 1720

Date	Fully-paid share value	Minimum observed subscription value	Ranges of monthly volatility: 60% (u-1) to -70% (d-1)	60% (u-1) to -50% (d-1)	20% (u-1) to -50% (d-1)	20% (u-1) to -20% (d-1)	Maximum observed subscription value
14/09/20	£570	£310	£455	£379	_**£317**_*	_**£252**_*	£310
16/09/20	£550	£290	£431	£378	_**£297**_*	_**£233**_*	£290
17/09/20	£480	£210	£365	£315	_**£239**_*	_**£171**_*	£210
19/09/20	£450	£210	£338	£288	_**£215**_*	_**£146**_*	£210
20/09/20	£440	£140	£329	£279	_**£206**_*	_**£137**_*	£200
21/09/20	£390	£130	£284	£234	_**£166**_*	_**£97**_*	£130
23/09/20	£375	£110	£270	£221	_**£154**_*	_**£86**_*	£120
24/09/20	£350	£110	£248	£199	_**£134**_*	_**£67**_*	£110
26/09/20	£280	£60	£185	£139	_**£78**_*	_**£29**_*	£60
27/09/20	£290	£60	£194	£147	_**£86**_*	_**£34**_*	£70
30/09/20	£200	£50	_**£116**_*	_**£83**_*	_**£34**_*	£4	£100
01/10/20	£300	£60	£202	£155	_**£93**_*	_**£38**_*	£60
03/10/20	£295	£60	£197	£150	_**£89**_*	_**£36**_*	£60
04/10/20	£260	£50	£166	£122	_**£62**_*	_**£19**_*	£50
05/10/20	£250	£60	£156	£115	_**£56**_*	_**£13**_*	£60
06/10/20	£270	£60	£173	£129	_**£66**_*	_**£22**_*	£60
07/10/20	£260	£50	£164	£122	_**£61**_*	_**£17**_*	£60
08/10/20	£260	£60	£164	£121	_**£61**_*	_**£17**_*	£60
10/10/20	£260	£60	£157	_**£119**_*	_**£60**_*	£15	£60
11/10/20	£240	£50	£139	_**£104**_*	_**£49**_*	£9	£55
12/10/20	£240	£45	£139	£103	_**£49**_*	_**£9**_*	£45
13/10/20	£200	£50	£105	_**£73**_*	_**£29**_*	£0	£50
14/10/20	£190	£40	£96	_**£65**_*	_**£24**_*	£0	£50
19/10/20	£210	£46	£113	_**£80**_*	_**£33**_*	£2	£46

(Continued)

Table 4. Continued

Date	Fully-paid share value	Minimum observed subscription value	Ranges of monthly volatility: 60% (*u*-1) to -70% (*d*-1)	60% (*u*-1) to -50% (*d*-1)	20% (*u*-1) to -50% (*d*-1)	20% (*u*-1) to -20% (*d*-1)	Maximum observed subscription value
20/10/20	£240	£40	£138	£102	£48*	£9*	£40
21/10/20	£210	£50	£112	£79*	£33*	£2	£50
22/10/20	£215	£45	£116	£83*	£35*	£3	£45
26/10/20	£212	£50	£113	£80*	£34*	£2	£50
28/10/20	£210	£45	£111	£78*	£32*	£2	£55
29/10/20	£208	£50	£109	£77*	£31*	£1	£50
31/10/20	£222	£50	£121	£87*	£38*	£4	£50
01/11/20	£210	£45	£111	£78*	£32*	£2	£50
02/11/20	£205	£45	£106	£74*	£29*	£1	£45
03/11/20	£209	£45	£110	£77*	£31*	£1	£45
04/11/20	£212	£45	£108	£73*	£31*	£1	£50
05/11/20	£220	£50	£115	£79*	£35*	£3	£50
07/11/20	£210	£45	£106	£72*	£29*	£1	£45
08/11/20	£214	£40	£110	£74*	£31*	£1	£45
09/11/20	£209	£45	£105	£71*	£29*	£0	£45
10/11/20	£211	£40	£107	£72*	£30*	£1	£40
11/11/20	£216	£40	£111	£75*	£32*	£2	£40
14/11/20	£210	£40	£102	£64*	£23*	£0	£40
15/11/20	£190	£25	£87	£54*	£11*	£0	£40
18/11/20	£175	£15	£75	£52	£15*	£6*	£15
			1 capture out of 44	24 captures out of 44	20 captures out of 44		

We conclude this section by noting that even amongst our failures in generating captures the model is still usually capable of producing one simulated share value that comes close to one of the extreme empirical subscription values. 1st subscription values were as valuable as £540 and worth as little as £130 in 1720. Yet these extreme values and most all values between them can be explained by our model using only the concurrent value of a fully-paid share and a reasonable set of returns volatilities. Similarly, the even wider range of 2nd series values (£540 to £15) in 1720 can be explained when using the same fully-paid share values and nearly the same volatilities that were used to explain the 1st series values. This is a rather robust performance for a model that has to explain subscription values in both the heady days of early summer 1720 and in the gloomy days of autumn 1720. These results were obtained despite the use of a very crude model for share returns and share return volatilities. The distributions of fully-paid share returns were assumed to be constant over the life of our compound options, regardless of whether they were short-term or long-term options. We now present our final conclusions and suggestions for further research.

7. Conclusions and suggested further research

The South Sea subscription shares of 1720 were compound call options. They were the creation of a law, which when examined closely enough, clearly suggests how the shares' option-like nature can be given precise expression in a theory. From the theory it is but a short step to defining a computational method for theoretical subscription share values that can be compared to their historical values. As crude as our resulting model was, it was still capable of producing the approximate values for subscription shares that were quite close to their values in history. The model performed well for early 1720 and it performed equally well for periods after the bursting of the South Sea Bubble. Further research can go in several directions. We should next turn our attention to the small amounts of data we have that pertain to the 3rd and 4th subscriptions to see if they conform to the theory presented in this paper. Then there are some other South Sea data questions on which some progress might be made with the help of data from financial derivatives markets. Finding the actual path of the South Sea Bubble itself should be a priority. Locating spot values for fully-paid shares in the crucial midsummer period will not be easy, but perhaps data on financial derivatives values will be of some help. Financial derivatives textbooks, for example, tell us how synthetic forward contracts can be constructed in portfolios that contain positions in derivatives and other assets. It would be straightforward to modify the exercise to create or estimate synthetic spot values for shares from data on their forward and option contract values. Another thing that can be done with financial derivatives is to use their values in estimating implied volatilities of share returns. A time series of time-changing volatilities in returns could be just as useful way of measuring the progress of the South Sea Bubble as would be looking at a time path of fully-paid share values. Now that we know finally what the South Sea subscription shares actually were, more refined applications of option pricing theory may help solve some of the outstanding puzzles and help create new puzzles concerning the South Sea Bubble.

We must remember, however, that what has been presented in this paper is a model that only explains the values of subscription shares relative to the values of fully-paid shares; it is not necessarily, nor is it likely to be, a good model of market participants' thought processes as they carried on trade in shares. The data we have examined here pre-date by two and a half centuries some of the theoretical developments that underpin our simulation model. So, it is quite proper to ask the question, 'how did they do it?'. How much option theory did a person in 1720 have to know to do a good job in valuing the South Sea subscription shares? For that matter, how much modern option theory does one have to know today in order to reasonably value options?

What distinguishes modern option pricing theory from what was generally understood before is that option theorists today are capable of deriving closed-form mathematical expressions for option values. Such capabilities are not, however, strictly necessary to value options. Many of the most important elements of modern theories have been known for centuries, such as: (i) the higher an option's exercise cost, the lower should be the option's value; (ii) the longer the term of the option, the more valuable it is; and (iii) the riskier the asset on which an option is written, the more valuable the option. This much has been known about options in the Netherlands and England since the 17th century. There is good evidence that informed opinion on options did exist and was acted upon, although the historical record is almost empty of overt expressions of such informed opinion. Murphy (2006) discusses these matters in a close study of the existing records of a London financial broker of the 1690s, Charles Blunt. Blunt and some of his clients, clearly understood the three elementary facts about option values listed above. They understood much else besides, such as the principle of put-call parity.[15] At the same time knowledgeable investors would have been used to seeing and pricing subscription shares. In the 1694 creation of the Bank of England itself the first public offer of equity was by subscription for partly-paid shares, which were actively traded in the nearly two years that passed before they became fully paid. Further issues were made in 1709, again, in the form of partly-paid shares. Alongside the South Sea Company in 1720, other firms such as the Royal African Company and the York Buildings Company sought to issue new shares on a subscription, partly-paid basis. It is highly likely that, in the more

[15] There are examples in Blunt's ledgers of put and call options that share the same (at-the-money) exercise price (Murphy, 2006, Fig. 2). Put-call parity would dictate that in such circumstances the put should be worth only slightly less than the call and that is precisely what can be seen in the Blunt ledgers. The Blunt evidence also contains an example of a client (one Thomas Estcourt) who clearly knew how to construct a synthetic call option—an operation that would make sense to only a person who understands put-call parity.

than 20 years of options and subscription share pricing and trading in London before 1720, experience was concentrated amongst a small cadre of investors and financial professionals. If irrationality, ineptitude, and frenzy were rampant in the financial markets of 1720, they were never so dominant so as to make a nonsense of the pricing relationships between South Sea subscription shares and fully-paid shares.

The last observation above prompts us to turn to the matter of what light our study sheds on the South Sea Bubble itself. Values of both fully-paid and partly-paid South Sea shares rose and fell in 1720, driven by a fundamental that itself may or may not have been mispriced. We have already alluded to the probable complexity of that fundamental. The speculation about the fundamental value of the South Sea Company assets, as well as speculation about the Company's eventual capital structure, was intense. Further research into the capital structure and the proper relative values of its components needs to be performed before we better approach the question of the South Sea Company's asset fundamental.

Another line of research is suggested by more recent theories of financial market mispricings that depend upon certain notions of market inefficiencies. If markets are inefficient in the sense that there are significant barriers to arbitrage, such barriers might allow a class of investors to trade, on the basis of intrinsically useless information (noise) and in sufficient numbers and with sufficient persistence, so as to drive market prices away from fundamental values. These are the so-called noise-trader models and are well surveyed by Shleifer and Summers (1990). They have at their heart a kind of sociological theory of financial market behaviour that resonates with historical accounts of certain famous financial market bubbles. White (1990), for example, points out that the relatively naïve retail investor's presence was more strongly detectable in the booming markets of the 1920s prior to the Great Crash of 1929 than was previously so. Such investors were the probable source of the surge in demand for credit for buying stock. Although the terms of credit for stock purchases were not cheap, the mechanism at least was there to accommodate such investors' demands. So it too might have been in 1720. Shea (2007) documents some evidence that the South Sea subscription shares might have been especially attractive to the naïve and small investor and also served as a mechanism by which an enthusiastic, but credit-constrained, buying public could get cheaply into the stock market. Although in this paper we have made the case that subscription share markets were not themselves inefficient, that does not mean that other parts of the stock market were necessarily efficient. Perhaps there were other inefficiencies that permitted the mispricing of the fundamental value of the South Sea scheme itself. Such mispricing, if it existed, could very well have been practiced by the type of investor who was in the first instance attracted to the stock markets by the subscription shares. These are plausible ideas, but confirming the presence of noise traders and the special risks their presence posed to knowledgeable and rational investors will be a considerable challenge in further

South Sea studies. Many of the kinds of records that would help us to identify investors, traders, and their dealings simply do not exist for this period. Carlos and Neal (2006) so far are the only investigators who have studied such records and they limit their study to Bank of England securities and their investors.

The evidence coming from 1720 will not give up its secrets easily, but now that we know there was a functioning financial derivatives market that operated alongside the market for fully-paid South Sea shares, there is more hope that some progress will be made in our understanding. We can hope that the South Sea Bubble will attract the interest of more financial economists, as well as financial economic historians, as not only a very curious episode in financial economic history, but also as a scientific problem that someday might actually be solved.

Supplementary material

Supplementary material (Appendices 1, 2, 3, and 4) is available online at the OUP website.

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