Introducing asymmetric information in the credible bargain: an explanation of the real wage stickiness

Abstract

In this paper, we introduce the asymmetric information a la Kennan in the Hall-Milgrom framework so as to replicate the elasticity of the real wage with respect to labor productivity found by Hagedorn-Manovskii (0.5). We believe that the empirical real wage stickiness results from the rigidity of the two bounds of the bargaining set. The Hall-Milgrom model, by only making the lower bound less flexible, does not generate enough wage rigidity. At the same time, the Kennan's asymmetric information implies a less pro-cyclical upper bound. In introducing this imperfection in the Hall-Migrom framework, we make the two bounds of the bargaining set less flexible and find a real wage-labor productivity elasticity very closed to Hagedorn-Manovkii. Hence, our simple story provides an empirically realistic explanation of the real wage rigidity which is completely based on the bargaining theory.

1/ Introduction

The starting point of this paper is related to the Shimer's (2005) « unemployment volatility puzzle »: the DMP class of models (« DMP » for the initial contributions of Diamond (1982), Mortensen (1985) and Pissarides (1985)) would be successful in reproducing the observed unemployment rates in different countries, but completely unable to replicate the volatility of these unemployment rates following an aggregate productivity shock. Remember that the DMP framework modelizes in an intuitive and simple way the labor market imperfections responsible of some stylized facts (mainly the Beveridge curve and the different cyclical behaviors of labor productivity and real wages) that a walrasian representation of the labor market is not able to reproduce.

Shimer shows, for the 1951-2001 period in the United-States, that the unemployment rate implied by a canonical DMP model is 15.2 times less volatile than empirically observed for that period, the vacancy rate 11.4 times less volatile and the tightness 12.5. Hence, the DMP class of models would be unable to reproduce the volatility of the main variables on the labor market following productivity shocks of observed sizes, that is to say small sizes.

Shimer attributes this weak volatility to the high pro-cyclicity of the real wage in these models. This strong pro-cyclicity would result from the use of the Nash bargain as a mechanism of determining the real wage in these models. In the Nash bargain, the agreement reached by the two players is such that each obtains at least the amount of her threat point and the joint surplus is shared between the two players with respect to their bargaining power. The threat points, as usual, are the outside options of the players: the unemployment value for the worker (since there is no on-the-job search) and the value of a vacancy for the employer (that is to say zero since there is free entry in the good's market). The present value of the real wage that results from the Nash bargain is always located between two bounds. These two bounds are called "the bargaining set". For the real wage, the upper bound is the present value of the labor productivity (which is the reservation value of the employer) and the lower bound is the unemployment value (which is the reservation value of the worker). Shimer shows that when there is a shock on labor productivity, the two bounds fluctuate by the same magnitude. Since in usual calibrations the bargaining powers of the workers and the employers are approximately identical, the fluctuations of the real wage will be of the same order as those of labor productivity. As a result, when there is a positive shock on labor productivity, the real wage will increase by the same amount. Hence, the profit of the employer, equal to labor productivity minus the real wage, and consequently, the incitation to create vacancies, increase only marginally: this explains why the unemployment rate does not fluctuate so much in this class of models.

For Shimer, the only way to solve the unemployment volatility puzzle is to replace the Nash bargain by alternative mechanisms for the determination of the real wage. These alternative mechanisms have to exhibit a relatively high degree of stickiness for the real wage, for the profit to increase following a positive labor productivity shock.

Nevertheless, this argument raises an important question: one could reasonably wonder if the real wage is in fact sticky. Hagedorn and Manovskii (2006, 2008) will answer this important question. There are three key points in their empirical study. First, from aggregate data (BLS (1951:1 – 2004 – 4)) of the United-States, they find an elasticity of the real wage with respect to labor productivity of approximately 0.5: this means that when labor productivity increases by 1 percent, the real wage responds by an increase of only 0.5 percent. For their estimation, they use the BLS' seasonally adjusted real average output per person in the non-farm business sector as a measure of labor productivity and the labor share times labor productivity as a measure of the real wage. Second, in order to avoid the cyclical selection bias attributable to the entry of low wage workers into employment in booms and exit in recession, they also estimate the elasticity of the real wage with respect to labor productivity employing individual data from the PSID for the real wage. Once again, they find an elasticity of approximately 0.5. The third key point of their study is that this elasticity is the same for new matches as for ongoing matches: contrary to the results of some estimation in Pissarides (2007), the elasticity for news matches is not higher than the elasticity for ongoing matches.

As a result, a large part of the subsequent literature on the unemployment volatility puzzle introduces sticky real wages in the DMP class of models. For example, Hall (2005) replaces the Nash bargain by the Nash demand game for the determination of the real wage in new matches and considers that the wage resulting from this game will be independent of productivity shocks because of the existence of a "social norm". Hagedorn and Manovskii (2006, 2008) keep the Nash bargain but impose that the real wage-labor productivity elasticity estimated empirically be a target of their calibration strategy. They find that a bargaining power for the worker of 0.19 allows the Nash bargain to replicate the elasticity of 0.5 and use this value of the bargaining power to make the real wage sticky.

Nevertheless, neither of these two contributions explains in a satisfactory way why there is a real wage rigidity. In both cases, as in other papers, this rigidity is set exogenously, in an ad-hoc manner. The real wage stickiness is much postulated than explained. The Hall-Milgrom (2008) paper is, to our knowledge and to our opinion, the first attempt that successfully provides an explanation of this rigidity that is micro-founded, based on the bargaining theory. As we will show in the next section, Hall and Milgrom (HM for the rest of the paper) replace the axiomatic Nash bargain by the strategic alternating offers bargaining of Rubinstein (1982). They demonstrate that the joint surplus of a match is such that the threat, for the employer as for the worker, to leave the negotiation before they reach agreement is not credible: the outside options of the two players are not credible threat points. The main implication is that the lower bound of the bargaining set is no longer the unemployment value but the costs of delaying the agreement, whereas the upper bound is, as in the Nash bargain, the labor productivity. The costs of delay are constant and not affected by productivity shocks. As a consequence, the real wage in the HM model is sticky because the lower bound of the bargaining set is itself sticky following a labor productivity shock. They find an elasticity of the real wage with respect to labor productivity in this model of 0.7. Hence, there is an important part of the wage rigidity that this model, with a realistic calibration, is not able to capture.

In this paper, we add to the HM model another source of real wage rigidity, so as to replicate the Hagedorn-Manovskii real wage-labor productivity elasticity. This additional source of rigidity results from the following intuition: we think that when there is an aggregate shock to labor productivity, not only the lower but also the upper bound of the bargaining set are less pro-cyclical than labor productivity. We do not think that the wage rigidity comes from an exogenous static wage (because of a social norm for instance) whereas the two bounds fluctuate. We do not either think that this rigidity results from a very low value of the bargaining power of the worker, nor from a contra-cyclical behavior of this power. We do believe that the real wage is sticky because its two bounds are sticky. Then, we add to the HM model a mechanism that makes the upper bound of the bargaining set less pro-cyclical. This mechanism has to be based on the bargaining theory and intuitively convincing. The asymmetric information of Kennan (2009) respects all these conditions.

In the Kennan model, the labor productivity has two components: an aggregate one, observable by both the worker and the employer, and an idiosyncratic one, observable only by the employer. The idiosyncratic component could take two values, low and high. Kennan demonstrates that the optimal behavior of the worker is to be prudent in response to the asymmetric information relative to idiosyncratic productivity. This implies that the upper bound of the bargaining set is always the low value of this productivity, less pro-cyclical than the productivity effectively realized.

In this paper we introduce the asymmetric information of the Kennan model in the HM framework, to have the two bounds sticky. With the same calibration as HM, we obtain a real wage-labor productivity elasticity of approximately 0.55, very closed to the Hagedorn-Manovskii estimation. Hence, we provide an empirically realistic explanation of the real wage rigidity which is completely based on the bargaining theory. Our simple story identifies all the mechanisms that make the real wage less pro-cyclical when there is an aggregate productivity shock. We are then able to explain this rigidity only with the behavior and the strategies of the agents, without having to introduce exogenous sources.

The rest of this paper is organized as follows. In the next section, we briefly present the HM and the Kennan models, focusing on the wage bargain and the implications for the wage stickiness. In section 3, we introduce the asymmetric information "a la Kennan" in the HM framework and comment our results. We show that when we keep the Kennan assumption relative to the behavior of the idiosyncratic productivity, our result applies only for news matches. Then, we make an alternative assumption and find that our elasticity now applies for both new and ongoing matches, in line with the estimation of Hagedorn-Manovskii. We conclude in section 4.

2/ Hall-Milgrom (2008) and Kennan (2009) contributions

2.1/ The Hall-Milgrom framework

Before turning to the HM contribution, we briefly return, in a formal way, on the implications of the Nash bargain for the real wage, as typically used in the DMP class of models. We take here the notations and the Bellman equations employed in HM. We wrote in introduction that the real wage resulting from the Nash bargain was always located inside the bargaining set in the following manner:

$$W_i = \beta P_i + (1 - \beta)(U_i - V_i)$$

with $$U_i = z + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [\varphi(\theta_i)(W_{i'} + V_{i'}) + (1 - \varphi(\theta_i))U_{i'}]$$

$$V_i = \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [sU_{i'} + (1 - s)V_{i'}]$$

$$P_i = p_i + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} (1 - s)P_{i'}$$

with i the state of the economy at the current period, U_i the unemployment value, V_i the value an employed worker enjoys for the rest of her career, z the flow value received per period by a job-seeker (composed of the unemployment benefits, the value of leisure and sometimes the value of the home production), θ_i the labor market tightness, $\varphi(\theta_i)$ the job-finding rate, β the bargaining power of the worker, W_i the present value of the wage, r the discount rate (equal to the interest rate), s the exogenous separation rate, pi the flow value marginal product of labor, P_i the present value of the output produced over the course of the job, c the per period cost to post a vacancy and $\pi_{i,i'}$ the transition matrix. Workers and employers are risk neutral. $(U_i – V_i)$ is the opportunity cost of a match or the reservation value of a worker.

When there is a transitory productivity shock, P increases, which induces a direct jump of W weighted by β. This direct impact of P on W involves an increase of U, because U depends on the wages offers in other jobs. U also rises since the job-finding rate rises. V increases but less than U, since the shock is transitory. Consequently, P and (U – V) rise by the same amount. Since the value of β usually obtained approximate 0.5, the increases of P and (U – V) have the same impact on W. Then, W and P jump by the same amount and the elasticity of W with respect to P equals 0.96.

In their paper, HM replace the Nash bargain with the alternating offers model of Rubinstein (1982) with outside options. In this framework, the worker (who is unemployed, since there is no job-to-job flows) and the employer alternate in making proposals until they reach an agreement. After a proposer makes an offer, the responding party has three options: accept the current proposal; reject it, perceive her reservation utility during this period and make a counter-offer next period; abandon the negotiation and take her higher outside option. Here, the reservation utilities are the flow values received by the players during the negotiation, that is to say z for the worker (z is a gain) and $-\gamma$ for the employer (γ is fixed cost for the employer since there is no production during the bargain). The best outside option for the worker is U and zero for the employer, as in usual versions of the Nash bargain. HM introduces, on top of the outside options, an important parameter in the Rubinstein model: the probability δ that the bargain ends after a proposal was made but before reaching an agreement. In that case the players obtain their outside options. This probability is completely exogenous and is different from the case that a player unilaterally leaves the bargain to take her outside option. HM then examines two polar cases, the other cases being linear combinations of the polar ones. These cases are $\delta = 1$ and $\delta = 0$.

The case $\delta = 1$ is such that that player selected to make the first proposal makes a "take-it-or-leave-it" one. Remember that $\delta = 1$ means that the bargain is bound to interrupt just after this first proposal. The player who offers this proposal makes it in a way that this offer is just acceptable for the responding party, that is to say marginally higher than the outside option of this party, since she would not be able to make a counter-proposal if she decided to reject this first proposal. HM make the assumption that it is always the employer that makes the first offer. In that case, the proposal just acceptable for the worker is $W_i + V_i = U_i$. If the worker had made the first offer, the proposal just acceptable for the employer would have been $P_i - W_i = 0$. With $\delta = 1$, the threat points are the outside options, as in the usual versions of the Nash bargain. Note that the player that makes the first proposal receives all the joint surplus.

However, this case is of "secondary importance" for HM, since empirically δ is closed to zero. Hence, it is on the other polar case, $\delta= 0$, that HM focus their analysis. This case fully corresponds to the Rubinstein (1982) model with outside options.

To make the exposition easier, first suppose that neither the worker nor the employer has an outside option. HM then define W and W' as, respectively, the lower wage that a worker will accept to perceive and the higher wage the employer will accept to pay. In other words, W is the reservation wage of the worker and W' the reservation wage of the employer, with obviously W' > W. When the employer makes the first offer, it is optimal to propose W, the proposal just acceptable for the worker. As in the Rubinstein model, W is such that the worker is indifferent between accepting this proposal this period and refusing it (receiving z this period) to make the counter-proposal W' next period. Consequently, at the unique perfect equilibrium of this game, we have:

$$W_i + V_i = z + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [W'_{i'} + V_{i'}]$$

If the worker had made the first proposal, it would have been optimal to offer P – W', just acceptable for the employer. Once again, P – W' is such that the employer is indifferent between accepting this proposal this period and refusing it (loosing γ this period) to make the counter-proposal W next period. Consequently, at the unique perfect equilibrium of this game, we would have:

$P_i - W'_i = -\gamma + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [P_{i'} - W_{i'}]$

It is the impatience of the players that leads them to make just acceptable offers at the first proposal. Hence, the bargaining process is only virtual: it is not optimal for the players to waste time bargaining since they perfectly know the unique solution of the game.

Consider now what happens when we introduce outside options U_i and zero. **HM demonstrate, and this is the point of the paper, that these outside options do not impact the result of the bargain.** They use the proposition 6 of Binmore, Rubinstein and Wolinsky (1986): as long as the value of the outside options of the two players are less than the payments obtained by these players at the unique perfect equilibrium of the Rubinstein game without outside options, the unique perfect equilibrium of the Rubinstein game with outside options will be the same as the unique perfect equilibrium without these options. In that case, the threat to leave the bargain is not credible. HM take this case as the reference. Since the employer always make the first proposal, we show above that at the unique perfect equilibrium without outside options the employer gains $P_i - W_i$ and the worker $W_i + V_i$. Since we are in a DMP framework in which unemployed workers and employers with vacancies cannot meet instantaneously, we always have $\varphi(\theta_i) < 1$. HM also assume that $P_i > W_i$ to justify a recruiting effort by employers and a positive level of employment at equilibrium. Then, under these assumptions and even if we do not prove it again here, we always have $P_i - W_i > 0$ and $W_i + V_i > U_i$: **the introduction of outside options, and especially $U_{i,}$, has no impact on the equilibrium of the game. The outside options are not credible threats since the two players always perceive at the conclusion of the bargain a payment that is higher than the outside options.**

The intermediate cases for δ between 0 and 1 are only linear combinations of the two polar cases. When we multiply by δ the equations obtained for $\delta = 1$ and by $(1 - \delta)$ those for $\delta = 0$, we have:

$W_i + V_i = \delta U_i + (1 - \delta) [z + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [W'_{i'} + V_{i'}]]$

$P_i - W'_i = (1 - \delta)[-\gamma + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [P_{i'} - W_{i'}]]$

These two equations replace the Nash bargain in a typical DMP model:

$W_i = \beta P_i + (1 - \beta)(U_i - V_i)$

A way to understand the implications of the HM model for the bargaining set is to calculate the solution for the average, which is hardly different from W.

$$\frac{1}{2}(W_i + W'_i) = \frac{1}{2}[P_i + (I - \frac{1-\delta}{1+r}\pi)^{-1}[\delta U_i + (1-\delta)(z+\gamma)] - V_i]$$

To understand what happens, return to the study of the polar cases. When $\delta = 1$, the average wage is between (U – V) and P, as in the usual Nash bargain. When $\delta = 0$, U is replaced by the present value of $(z + \gamma)$, with z and γ independent of P. Consequently, when there is a positive shock on P, only the upper bound of the bargaining set rises, implying a lower flexibility of the real wage than in the Nash bargain.

When we look at the intermediate cases for δ, the equation of the average wage is such that the impact of U is limited by the probability δ. This results from the fact that U only intervenes when the bargain is interrupted by an exogenous event, even if the players would have preferred to continue the bargain, since, as we saw, the payments at the conclusion of this bargain are higher than the outside options. The credible bargain of HM gives however the same role to P and V as does the Nash bargain. The lower impact of U on the wage then involves a less pro-cyclical wage since the lower bound is less pro-cyclical because composed, on top of U and V, of the present value of z and γ. The elasticity of W with respect to P is 0.70, well below the one obtained for the Nash bargain (0.96).

Nevertheless, this elasticity is well above the one observed empirically by Hagedorn-Manovskii (0.5). Hence, the HM model, with a realistic calibration, does not capture an important part of the wage rigidity following a labor productivity shock. We think that this weakness comes from the fact that in the HM framework, only the lower bound of the bargaining set is sticky but the upper bound is as flexible as in the Nash bargain. We are to see that the asymmetric information a la Kennan is a promising way to make the upper bound less pro-cyclical.

2.2/ Kennan (2009)

As we said in introduction, Kennan assumes that total labor productivity of each firm is the sum of two components. The first one is the aggregate labor productivity, common to all firms. As in the HM model, the flow value of this aggregate component is p_i and follows a Markov process, this time with two states (i = 1 in the bad state and i = 2 in the good one). The present value of this aggregate productivity is P_i and this value is known by both the workers and the employers at any time. The second component of total labor productivity is an idiosyncratic one, which could randomly take two flow values: a low one, y_L and a high one y_H. For each state i, a firm has a probability τ_i to realize y_H. It is assumed that this probability is higher in the good state than in the bad state: $\tau_2 > \tau_1$. In other words, the covariance between P and τ is positive. τ_i could also be thought as the proportion of firms realizing y_H at the state i. Kennan assumes that there is no possibility of switching from low to high, nor from high to low value of the idiosyncratic productivity, once the match has been made. Hence, we note Y_L the present value of the low idiosyncratic component and Y_H the present value of the high idiosyncratic component over the course of the job:

$$Y_L = y_L + \frac{1}{1+r} (1 – s) Y'_L$$

$$Y_H = y_H + \frac{1}{1+r} (1 – s) Y'_H$$

Kennan also assumes that only the employer could observe the value of the realized idiosyncratic productivity. The worker observes p_i and knows y_H, y_L and τ_i, but she is unable to observe if the realized value of the idiosyncratic productivity is y_H or y_L.

Now we turn to the determination of the wage in the Kennan paper. Since the joint surplus of the match is no longer common knowledge, we could not directly use the Nash bargain. Kennan employs the Neutral Bargaining Solution of Myerson (1984), roughly a generalization of the Nash bargain in the case of imperfect information. Kennan underlines that the negotiation lasts only one round. If the two players are unable to agree at the end of this round, the bargain fails and each perceives its outside option. Hence, this corresponds to the polar case $\delta = 1$ in HM. The employer has a probability v to make the first proposal. As the worker has no "right to respond" to this first proposal, the employer offers a wage that is just equal to the outside option of the worker. Consequently, $W_i = U_i$. Symmetrically, the worker, with a probability 1- v, proposes a wage equal to the total productivity of the firm. Nevertheless, the worker does not know exactly the value of this total productivity, since she is unable to observe the realized value of y. Hence, the worker has two possible strategies when she makes the unique offer. If she is "prudent", she always proposes $W'_{iL} = P_i + Y_L$ and the employer always accepts it, whatever the realization of y. On the contrary, if the worker is "offensive", she always proposes $W'_{iH} = P_i + Y_H$ and the employer accepts it only with a probability τ_i. Kennan shows that there are values τ_i* and $(y_H – y_L)$* under which the expected gain for the worker with the prudent strategy $P_i + Y_L$ is higher than the expected gain with the offensive strategy, $\tau_i(P_i + Y_H) + (1 - \tau_i)U_i$. Particularly, with his calibration, τ_i*= 0.56. Kennan chooses the values for τ_i and $(y_H - y_L)$ so that the prudent strategy is always optimal for the worker. As a result, when v = 0.5, the average wage is:

$$\frac{1}{2} (W_{iL} + W'_{iL}) = [P_i + Y_L + (U_i – V_i)]$$

When there is a positive shock on the aggregate productivity (p jumps from p_1 to p_2), τ_i increases from τ_1 to τ_2. Hence, there are a higher number of firms that realize y_H and the (present value) average total productivity - $\tau_i((P_i + Y_H) + (1 - \tau_i) (P_i + Y_L)$ - rises because both P_i and τ_i increase. Nevertheless, the wage is only impacted by the rise of P from P_1 to P_2, since the upper bound of the bargaining set is $(P_i + Y_L)$: the wage increase is not impacted by the fact that a higher number of firms realize y_H in the good aggregate state. Consequently, the average total productivity increases much than the wage when there is a positive shock on aggregate productivity, which creates a rigidity of the real wage with respect to the average total productivity. Note here the key role of the positive covariance between P and τ.

The stickiness of W in this model then results from an upper bound ($P_i + Y_L$) less flexible than the upper bound of the traditional Nash bargain without asymmetric information, which would be $\tau_i((P_i + Y_H) + (1 - \tau_i) (P_i + Y_L)$. However, as in the usual Nash bargain, the lower bound in the Kennan model is still U_i.

3/ Introduction of the asymmetric information a la Kennan in the HM framework

In this paper, we introduce the Kennan's imperfect information in the HM framework to replicate the Hagedorn-Manovskii elasticity of the real wage with respect to labor productivity. The HM model alone is not able, with a realistic calibration, to generate a sufficient amount of wage rigidity. As we said in introduction, we do this integration because we think that when there is a shock on labor productivity, not only the lower bound of the bargaining set (as in HM) but also the upper bound (as in Kennan) are less flexible than labor productivity.

On top of the fact that these two models make the real wage sticky with solid micro-foundations, we believe they are convincing from an intuitive point of view. We think, like HM, that currently the labor markets are so depressed that the real threat an agent can exert during the wage bargain is not to leave the bargain but instead to delay it. We also think, like Kennan, that the worker is not able to observe completely the productivity of the firm for which she works, especially when she bargains over the wage for the first time.

Remember that Hagedorn-Manovskii (2006, 2008) find an elasticity of the real wage with respect to labor productivity of 0.5 from both aggregate and individual data. Most importantly, they find that this elasticity is the same for new matches as for ongoing matches.

In what follows, we proceed in two steps. First, we show that when we introduce the Kennan's imperfect information in the HM framework and keep the initial assumption of Kennan that the idiosyncratic component of productivity does not switch during the course of the job, we find an elasticity of the real wage with respect to labor productivity of 0.55 for new matches. Second, we only replace this assumption and suppose instead that the idiosyncratic component of productivity can switch from the low to the high value, but never from the high to the low one during the course of the job. This time, we find an elasticity of 0.55 for both new and ongoing matches.

3.1/ The model with the Kennan's initial assumption

3.1.1/ The model

The basic assumptions of our model are those of the DMP class of models:

$$U_i = z + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [\varphi(\theta_i)(W_{i'} + V_{i'}) + (1 - \varphi(\theta_i))U_{i'}]$$

$$V_i = \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [sU_{i'} + (1 - s)V_{i'}]$$

As in Kennan, total labor productivity has two components: an aggregate and an idiosyncratic. The aggregate labor productivity flow is still p_i. The present value of this aggregate productivity over the course of the job is P_i with:

$$P_i = p_i + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} (1 - s)P_{i'}$$

We assume, with HM, that there are five states in this economy (N = 5). The idiosyncratic productivity can take two flow values, y_L and y_H, with a probability τ_i to realize y_H and a positive covariance between P and τ.

In a first time, we keep the Kennan assumption that the idiosyncratic component of productivity does not switch during the course of the job. Since in our model we are in the HM framework, the wage bargain could last infinitely. In that case, the Kennan assumption means that the flow value of the idiosyncratic component that stays constant over the course of the job is not the flow value realized the period the worker and the employer meet but instead the flow value realized the period they agree on the wage and the match becomes productive. Hence, if the firm realizes the low flow value when they reach an agreement, the present value of the idiosyncratic productivity for the course of the job is:

$$Y_L = y_L + \frac{1}{1+r} (1 - s) Y'_L$$

Alternatively, if the firm realizes the high value when they agree, the present value is:

$$Y_H = y_H + \frac{1}{1+r} (1 - s) Y'_H$$

We also use the free-entry condition:

$$q(\theta_i) [\tau_i(P_i + Y_H) + (1 - \tau_i) (P_i + Y_L) - W_i] = c$$

with $q(\theta_i)$ the probability of a firm to fill a vacancy each period and c the flow cost of posting a vacancy. Since there is free-entry on the good market, the expected profit from initiating the recruitment of a new worker by opening a vacancy is zero.

We now turn to the wage bargain. Remember that the two key equations of the HM model were:

$$W_i + V_i = \delta U_i + (1 - \delta) [z + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [W'_{i'} + V_{i'}]]$$

$$P_i - W'_i = (1 - \delta)[-\gamma + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [P_{i'} - W_{i'}]]$$

with the average wage equal to:

$$\frac{1}{2}(W_i + W'_i) = \frac{1}{2}[P_i + (I - \frac{1-\delta}{1+r}\pi)^{-1}[\delta U_i + (1 - \delta)(z + \gamma)] - V_i]$$

We introduce in the HM framework the asymmetric information a la Kennan. Both the worker and the employer could observe the aggregate productivity but only the employer is able to observe the idiosyncratic productivity. The worker knows the values y_H and y_L and the probability τ_i but she is unable to observe what value the firm realizes at any point in time.

We note W_{iL} (respectively W_{iH}) the reservation wage of the worker if the firm realized y_L (respect.y_H) and W'_{iL} (respect.W'_{iH}) the reservation wage of the employer if the firm realized y_L (respect.y_H).

Once again, the worker has two possible strategies: being “prudent” or “offensive”.

We begin by considering the “prudent” strategy of the worker. Remember that in the HM framework, there are two polar cases, $\delta = 1$ and $\delta = 0$, the other cases being linear combinations of the polar ones.

Suppose first that $\delta = 1$: the bargain lasts only one round and we are exactly in the case examined by Kennan. We saw that if the worker is prudent and makes the unique offer, she always proposes $W'_{iL} = P_i + Y_L$ and the employer always accepts it. The employer is “opportunist”, since if y_H realizes, she obtains a rent equal to $Y_H - Y_L$. With this strategy, the worker is sure that the bargain never fails. We also showed that if the employer makes the unique proposal, she always offers U_i and the worker always accepts it.

Suppose now that $\delta = 0$: the bargain could last infinitely, as in Rubinstein (1982). If the worker is prudent, she always proposes W'_{iL} and always accepts W_{iL}. The best strategy for the employer is to be opportunist, in always accepting W'_{iL} and in always proposing W_{iL}. W_{iL} and W'_{iL} are such that:

$$W_{iL} + V_i = z + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [W'_{i'L} + V_{i'}]$$

$$P_i + Y_L - W'_{iL} = -\gamma + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [P_{i'} + Y_L - W_{i'L}]$$

Recall that both the employer and the worker are impatient. So, the huge advantage of this strategy for the worker is the same as in the HM model: the worker is sure that the agreement is reached at the first round.

When we take into account all the intermediate cases between $\delta = 1$ and $\delta = 0$, we have:

$$W_{iL} + V_i = \delta U_i + (1 - \delta) [z + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [W'_{i'L} + V_{i'}]]$$

$P_i + Y_L – W'_{iL} = (1 – \delta)[-\gamma + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [P_{i'} + Y_L - W_{i'L}]]$

And the average wage is, for the "prudent-opportunist" strategies:

$\frac{1}{2} (W_{iL} + W'_{iL}) = \frac{1}{2} [P_i + Y_L + (I – \frac{1-\delta}{1+r} \pi)^{-1} [\delta U_i + (1 – \delta)(z + \gamma)] – V_i]$

If we had introduced in the HM model the idiosyncratic component of productivity without asymmetric information, the average wage would have been:

$\frac{1}{2} (W_i + W'_i) = \frac{1}{2} [\tau_i(P_i + Y_H) + (1 - \tau_i) (P_i + Y_L) + (I – \frac{1-\delta}{1+r} \pi)^{-1} [\delta U_i + (1 – \delta)(z + \gamma)] – V_i]$

It is then clear that in our model, the upper bound of the bargaining set - $P_i + Y_L$ - is less flexible than the upper bound of the original HM model - $\tau_i((P_i + Y_H) + (1 - \tau_i) (P_i + Y_L)$ – following an aggregate productivity shock, since the covariance is positive between P_i and τ_i. **Hence, the real wage rigidity in our models results from both an upper and lower bounds less pro-cyclical than labor productivity.**

We now consider the "offensive" strategy of the worker.

Remember that the case $\delta = 1$ is exactly the same as Kennan. When the worker is offensive and makes the unique offer, she always proposes $W'_{iH} = P_i + Y_H$. The employer accepts this proposal only when she realizes y_H. The offer of the worker is then accepted only with a probability τ_i. When the employer makes the unique proposal, she always offers U_i, which is always accepted by the worker.

Now turn to the case $\delta = 0$. If the worker is offensive, she always proposes W'_{iH} and only accepts W_{iH}. The best strategy for the employer is then to "tell the truth", that is to say to proposes W_{iL} (and accepts W'_{iL}) when the firm realizes y_L and to proposes W_{iH} (and accepts W'_{iH}) when the firm realizes y_H. As a result, the worker accepts the proposal of the employer only with a probability τ_i and the employer accepts the proposal of the worker only with the same probability: each period, there is only a probability τ_i to reach an agreement. If the firm never realizes y_H, the agreement is never reached and the bargain lasts infinitely, whereas with the prudent strategy, the agreement is reached at the first round. W_{iH} and W'_{iH} are defined such that (remember that the idiosyncratic productivity does not change once the match becomes productive):

$W_{iH} + V_i = z + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [W'_{i'H} + V_{i'}]$

$P_i + Y_H - W'_{iH} = -\gamma + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [P_{i'} + Y_H - W_{i'H}]$

And the average wage is:

$\frac{1}{2} (W_{iH} + W'_{iH}) = \frac{1}{2} [P_i + Y_H + (I - \frac{1-\delta}{1+r} \pi)^{-1} [\delta U_i + (1 - \delta)(z + \gamma)] - V_i]$

To determine which strategy she will apply, the worker compares the expected gains of the prudent strategy with those of the offensive strategy. Nevertheless, we encounter the same problem as Kennan, because there are available data neither for idiosyncratic productivity nor for the probability τ_i. Or, these variables are crucial for the comparison of the expected gains of the two strategies. When τ_i and (y_H - y_L) are small, the prudent strategy dominates the offensive one. In our calibration strategy, we will choose small values for these variables to be sure that it is optimal for the worker to be in the prudent strategy, as Kennan did.

3.1.2/ Calibration and results

To compare our results to those of HM, we take the same calibration for the common parameters and for the aggregate productivity process. We set s = 0.001, r = 0.000192, δ = 0.0055, z = 0.71, γ = 0.23 and φ_O = 0.5. The transition matrix and the shocks on aggregate productivity are given in table 1:

TABLE 1: TIME-SERIES PROCESS FOR PRODUCTIVITY

Category	*Productivity deviation, percent*	*Daily transition probability to new category*				
		1	2	3	4	5
1	-2.69	0.9944	0.0046	0.0005	0.0000	0.0005
2	-0.95	0.0057	0.9859	0.0073	0.0005	0.0005
3	0.07	0.0000	0.0083	0.9849	0.0063	0.0005
4	1.21	0.0000	0.0010	0.0068	0.9854	0.0068
5	2.43	0.0000	0.0000	0.0005	0.0077	0.9918

As we said above, for the variables $(y_H - y_L)$ and τ_i, we take arbitrary values since there are no available data. However, we take very small values to be sure that the prudent strategy is optimal for the worker.

Our main result is the following: when the economy jumps from state 2 to state 4, the elasticity of W_{iH} with respect to $\tau_i((P_i + Y_H) + (1 - \tau_i) (P_i + Y_L)$ is equal to 0.55, very closed to the Hagedorn-Manovskii value (0.5). To check the robustness of this result, we take alternative values for $(y_H - y_L)$ and τ_i, always small to keep the optimality of the prudent strategy. We then find an elasticity always between 0.52 and 0.57, which tend to confirm the robustness of our first result.

3.2/ The model with a growing idiosyncratic productivity

In introduction, we said that the elasticity of Hagedorn-Manovskii applies for both new and ongoing matches. In the previous sub-section, we presented a satisfactory explanation of the real wage rigidity following an aggregate productivity shock. However, the elasticity we found only concerned new matches. To understand why, let us consider the next example.

Suppose that the economy was born in state 2. Since there are no ongoing matches, the (present value) average total productivity of this economy is $\tau_2(P_2 + Y_H) + (1 - \tau_2) (P_2 + Y_L)$. Some periods later, the economy jumps to state 4. At this state, new matches appear and coexist with the remaining ongoing matches. The average total productivity for the new matches is $\tau_4(P_4 + Y_H) + (1 - \tau_4) (P_4 + Y_L)$. Since, with the Kennan assumption the idiosyncratic productivity does not switch, the average total productivity for the ongoing matches is $\tau_2(P_4 + Y_H) + (1 - \tau_2) (P_4 + Y_L)$. If the proportion of these ongoing matches is α., the average total productivity of this economy at state 4 is: $(1 - \alpha)[\tau_4(P_4 + Y_H) + (1 - \tau_4) (P_4 + Y_L)] + \alpha[\tau_2(P_4 + Y_H) + (1 - \tau_2) (P_4 + Y_L)]$.

In the last sub-section, we calculated the elasticity of the real wage with respect to the average total productivity, when the economy jumps from state 2 to state 4, the following way:

$$\frac{W_{4L} - W_{2L}}{\tau_4(P_4 + Y_H) + (1 - \tau_4) (P_4 + Y_L) - \tau_2(P_2 + Y_H) - (1 - \tau_2) (P_2 + Y_L)}$$

Hence this elasticity applies only for new matches. A simple way to generalize this elasticity to ongoing matches is to make the following assumption. Suppose that the idiosyncratic productivity can jump from low to high flow value during the course of the job but never from high to low. Hence, at each state, the proportion of new matches that realize y_H is the same as the proportion of ongoing matches realizing y_H. In the example, when the economy jumps to state 4, all the ongoing matches that realized y_H in state 2 keep on realizing this value in state 4. Since $\tau_4 > \tau_2$, a positive part of the ongoing matches that realized y_L in state 2 now realize

y_H. The average total productivity of ongoing matches in state 4 is, under this assumption, $\tau_4(P_4 + Y_H) + (1 - \tau_4)(P_4 + Y_L)$. Since the average total productivity of new matches in state 4 is also $\tau_4(P_4 + Y_H) + (1 - \tau_4)(P_4 + Y_L)$, the average total productivity of this economy in state 4 is $\tau_4(P_4 + Y_H) + (1 - \tau_4)(P_4 + Y_L)$ and the calculation of the elasticity just above applies for both new and ongoing matches.

Three remarks are worth noting. First, the strategies of the worker and the employer are not affected by this new assumption. The equations for the prudent strategy – our privileged case - are still:

$$W_{iL} + V_i = \delta U_i + (1 - \delta)\left[z + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [W'_{i'L} + V_{i'}]\right]$$

$$P_i + Y_L - W'_{iL} = (1 - \delta)\left[-\gamma + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} [P_{i'} + Y_L - W_{i'L}]\right]$$

and $Y_L = y_L + \frac{1}{1+r}(1 - s)Y'_L$

with Y_L unchanged since the prudence implies to determine the wage as if y_L realized each period.

Second, this assumption allows to have the same average total productivity for new and ongoing matches, and so the same elasticity for the real wage, only for positive shocks to aggregate productivity. However, this result is not so unrealistic given the secular growth of labor productivity. Third, since there is a possible switch from low to high idiosyncratic productivity, Y_L for the calculation of the effective present value of the average total productivity of the economy is now equal to:

$$Y_{iL} = y_L + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} (1 - s)[\tau_i Y_H + (1 - \tau_i) Y_{i'L}]$$

with $Y_H = y_H + \frac{1}{1+r}(1 - s)Y'_H$

The elasticity of the real wage with respect to average total productivity when the economy jumps from state 2 to state 4 is:

$$\frac{W_{4L} - W_{2L}}{\tau_4(P_4 + Y_H) + (1 - \tau_4)(P_4 + Y_{4L}) - \tau_2(P_2 + Y_H) - (1 - \tau_2)(P_2 + Y_{2L})}$$

We find again an elasticity belonging approximately between 0.53 and 0.57. Our model, with this alternative assumption, is then able to replicate both the elasticity for new and ongoing matches. Nevertheless, there is still the same problem of calibration and this elasticity applies only for positive aggregate shocks.

4/ Conclusion

In this paper, we have shown that introducing the asymmetric information a la Kennan in the HM framework generates an empirically realistic real wage rigidity, by making the two bounds of the bargaining set less pro-cyclical than labor productivity. We are then able to explain this rigidity only with the behavior and the strategies of the agents, without having to introduce exogenous sources. This result could be extended to the New-Keynesian literature, which in its last developments most of the time postulate this rigidity (see Blanchard-Gali (2005)).

However, our model is affected by the same important weakness as Kennan: the calibration, which is, for some variables, arbitrary. Nevertheless, we think that the robustness of our results is an encouraging sign: our explanation cannot be disqualified by this calibration problem.

Appendix

We said that the Kennan assumption means that the flow value of the idiosyncratic component that stays constant over the course of the job is not the flow value realized the period the worker and the employer meet but instead the flow value realized the period they agree on the wage and the match becomes productive. This means that the worker has no interest to prolong the bargain to learn about the realized flow value since this flow value could change as long as they do not reach an agreement.

Suppose instead that it is the flow value realized the period they meet that stays constant. In this case, the prudent strategy gives the same results as before. However, we could imagine another way of thinking the offensive strategy with this alternative assumption. Suppose that the offensive strategy consists in only proposing W'_{iH} (respectively only accepting W_{iH}) the first round of the bargain and accepting W_{iL} (respect. proposing W'_{iL}) the second round. In this case, if the employer tells the truth, they reached an agreement the first round if the firm realizes y_H and the worker obtains W_{iH} or W'_{iH}. If the firm realizes y_L, they reached an agreement the second round and the worker earns W_{iL} or W'_{iL}. At first sight, this strategy seems better than the prudent one, since the worker obtains at least the same payment from the second round, with a possibility to get W_{iH} or W'_{iH} at the first round. Nevertheless, the best response to this strategy for the employer is not to tell the truth but instead to be opportunist by only proposing W_{iL} and only accepting W'_{iL} each period, since the employer knows that the worker will limit her proposition and acceptance to W'_{iL} or W_{iL} the second round. If the employer is opportunist, the best response for the worker is to be prudent, to get W'_{iL} or W_{iL} from the first round. The best response of the employer to the prudent strategy is to be opportunist. Hence, the couple "offensive-tell the truth" is not an equilibrium. The only stable equilibrium in this case is "prudence-opportunism".

References

Blanchard O, Gali J. "Real Wage Rigidities and the New Keynesian Model" Federal Reserve bank of Boston WP No.05-14

Binmore K, Rubinstein A, Wolinsky A. "The Nash bargaining solution in economic modelling" Rand Journal of Economics Vol.17, No.2, Summer 1986

Diamond P. "Aggregate Demand Management in Search Equilibrium" Journal of Political economy Vol.82, No.5, 1982

Hall R. "Employment Fluctuations with Equilibrium Wage Stickiness" American Economic Review Vol.95, No.1, March 2005

Hall R, Milgrom P. "The limited Influence of Unemployment on the Wage Bargain" American Economic Review Vol.98, No.3, June 2008

Kennan J. "Private Information, Wage Bargaining and Employment Fluctuations" Review of Economic Studies, March 2009

Mortensen D. "Property Rights and Efficiency in Mating, Racing, and Related Games" American Economic Review Vol.72, No.5, 1982

Pissarides C. "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages" American Economic Review Vol.74, No.4, 1985

Pissarides C. "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?" WP November 15 2007

Shimer R. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies" AmericanEconomic review Vol.95, No.1, March 2005