Price Dynamics with Customer Markets*

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Abstract

We provide new evidence of the relationship between firm pricing and customer dynamics. We build and quantify a model where firms choose prices taking into account their effect on the evolution of their customer base, and customers face frictions to reallocate to other firms. The model accounts for salient features of retail price data, such as the high kurtosis of the price distribution and the low pass-through of cost shocks. We also study aggregate dynamics in our framework and show that microfounding customer reallocation leads to a countercyclical response of markups to demand shocks.

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1 Introduction

The customer base of a firm—the set of customers buying from it at a given point in time—is an important determinant of firm performance. Its effects are long lasting, as customer-supplier relationships are subject to a certain degree of stickiness (Hall (2008)). Starting with Phelps and Winter (1970), a large literature has stressed that the price is an important instrument to attract and retain customers, and firms seek to maintain and grow their customer base through their pricing decisions. However, our only evidence on the link between pricing and customer base dynamics rests on anecdotes or surveys (Blinder et al. (1998), and Fabiani et al. (2007)). Furthermore, little is known about the quantitative relevance of the implications of this link for pricing. The aim of this paper is to address both of these gaps.

We exploit novel data from the retail industry to provide direct evidence that firm prices influence households decision to stop buying from their current supplier. A natural implication of this result is that firms should take the effect on the customer base into account when they set prices. To understand whether this is an important mechanism, we build and estimate a microfounded model of firm pricing with customer markets. We find that the predictions of the model match well with the cross-sectional distribution of prices and the pass-through of cost shocks implied by our data. This suggests that customer markets play a relevant role in price formation, which is important to acknowledge given the recent interest of the macro literature on retail prices (see, for example, Burstein and Jaimovich (2012), Coibion et al. (2015), and Kaplan and Menzio (forthcoming,a)). Furthermore, our findings lend support to a growing literature that uses customer markets to explain a wide array of phenomena, from business cycle dynamics (Kaplan and Menzio (forthcoming,b)) to the movements in international relative prices (Drozd and Nosal (2012)), and firm performance (Gourio and Rudanko (2014)).

Our empirical analysis relies on scanner data documenting pricing and customer base evolution for a major U.S. retailer. The data contain information on grocery purchases for a large sample of customers between 2004 and 2006. Household-level scanner data are particularly well suited to study customer base dynamics. First, we observe a wealth of details on all the shopping trips each household makes to the chain (list of goods purchased, prices, quantities, etc.). More importantly, we can infer when customers leave the retailer by looking at prolonged spells without purchases at the chain. Hence, these data allow us to study the relation between a customer’s decision to abandon the firm and the price of the bundle of goods she consumes there. We show that an increase in the price significantly raises the probability that the household leaves the firm. This implies that the customer base is elastic to prices: a 1% change in the price of the customer’s typical basket of grocery
goods would raise the firm yearly customer turnover from 14% to 21%.

To assess the implications of customer markets for pricing, we setup a model where firms choose prices taking into account their effects on the dynamics of their customer base. Customers respond to price changes but their ability to reallocate across suppliers is impaired by the presence of search frictions. Modeling the market friction as search costs suits well our application since search costs have been found to importantly affect price dispersion in clustered retail markets (Sorensen (2000)) similar to those our data refer to. The distinctive feature of our setting is that it delivers endogenous customer dynamics, arising as a consequence of customers search and exit decisions.

Customers start each period in the customer base of the firm from which they bought in the previous period. Every period, firms draw a new idiosyncratic productivity level, and post a price. Then, customers can decide to pay an idiosyncratic search cost to observe the state of another randomly selected firm, compare it to that of their old supplier, and decide where to buy (extensive margin of demand). After these decisions have been made, each customer decides her purchased quantity of the good (intensive margin of demand). In this setting, firms face a trade-off between charging a higher price and extracting more surplus from customers, versus posting a lower price to extract a lower surplus but from a larger mass of customers.

While being tractable, the model provides a rich laboratory to study how the relationship between customer and price dynamics is shaped, in equilibrium, by idiosyncratic production and search costs. The price posted by the firm and its current level of productivity determine the value that customers obtain if they remain in the firm’s customer base. Customers of firms in the left tail of the distribution of such values are more likely to search and leave. As a result, firms have incentives to avoid being in the left tail of this distribution, which introduces an element of strategic complementarity in prices affecting the shape of the price distribution. The same mechanism also affects firms incentive to pass-through cost shocks.

We use the estimated price elasticity of the customer base, jointly with additional moments from our data, to identify the key objects of the model: the distribution of search costs and the productivity process. We use the estimated model to quantify the relevance of customer market in explaining the patterns we observe in the data. Our model delivers a yearly average turnover of 9%, nearly two-thirds of what we measure in the data (14%). This implies that customer dynamics triggered by variation in prices due to idiosyncratic cost shocks can explain a large fraction of the overall turnover. The relevance of customer markets in explaining salient features of real world pricing is also witnessed by two other distinctive predictions of the model: i) a price distribution with higher kurtosis and smaller dispersion than the distribution of production cost, ii) incomplete pass-through of cost shocks. Not only
do both of these features match well with the evidence from our pricing data but the high kurtosis of the price distribution has also been independently documented for homogeneous packaged good by Kaplan and Menzio (forthcoming,a).

Finally, we use our model to study the effect of an aggregate demand shock on markups. In ad-hoc models of customer markets firms react to strengthened demand by raising margins, generating procyclical markups. This result can be overturn by introducing habits into the utility function of consumers (Ravn et al. (2006)). We show that countercyclical behavior can also occur by endogenizing the evolution of the customer base. In our setting, a positive demand shock does not only make customers more eager to consume the good but it also pushes them to search harder for better sellers. Therefore, the demand shock increases the mass of customers who are potentially up for grab, motivating firms to lower their prices, and compress their markups, to retain them.

Related Literature. Our paper relates to the seminal work by Phelps and Winter (1970) who study the pricing problem of a firm facing customer retention concerns. In their paper, the response of the firm’s customer base to a change in the firm’s price is modeled with an ad hoc function. We instead endogenize customer dynamics which arise as the outcome of customers’ optimal search decisions in response to firms’ pricing. Fishman and Rob (2003), Alessandria (2004), and Menzio (2007) also study the firm price-setting problem in models where search costs prevent customers from freely moving to the lowest price supplier. Fishman and Rob (2003) study the implications of customer markets for firm dynamics. Alessandria (2004) shows that such a model can generate large and persistent deviations from the law of one price, consistent with the empirical evidence on international prices. Menzio (2007) looks at the role of asymmetric information and commitment in the optimal pricing decision of the firm. Differently from our paper, in these studies customers face a homogeneous search cost and, as a result, optimal pricing is such that no endogenous customer dynamics occur in equilibrium.

Unlike the literature cited above, we exploit micro data to discipline our model and provide a quantitative assessment of the relevance of customer markets for pricing. This relates our findings to contributions that aim at documenting empirical stylized facts on the behavior of prices and markups. Our evidence on the shape of the price distribution ties in to the recent empirical work by Kaplan and Menzio (forthcoming,a). While their focus is on customers and the price they pay for the same good (or bundle of goods), we are interested in the point of view of sellers and the price they charge.

Another set of related contributions uses customer markets to address questions different from the ones we study here. Gourio and Rudanko (2014) explore the relationship between
the firm’s effort to capture customers and its performance. They show that customer markets have nontrivial implications for the relationship between investment and Tobin’s q. Drozd and Nosal (2012) introduce in a standard international real business cycle model the notion that, when producers want to increase sales, they must exert effort to find new customers. This extension helps to rationalize a number of empirical findings on the dynamics of international prices and trade. Dinlersoz and Yorukoglu (2012) focus on the importance of customer markets for industry dynamics in a model where firms use advertising to disseminate information to uninformed customers. Shi (2011) studies a setting where firms cannot price discriminate across customers and use sales to attract new customers. Kleshchelski and Vincent (2009) examine the impact of customer markets on the pass-through of idiosyncratic cost shocks to prices but focus on a case with no heterogeneity in firm productivity and markups. Burdett and Coles (1997) study the role of firm size for pricing when firms use the price to attract new customers. Their work complements ours: price and customer dynamics in their setting are shaped by the heterogeneity in firm size (age). For us, the driving force is the heterogeneity in productivity.

Several other studies analyze the implications of product market frictions for business cycle fluctuations (Petrosky-Nadeau and Wasmer (forthcoming), Bai et al. (2012), and Kaplan and Menzio (forthcoming,b)). The industrial organization literature has also studied the implications of customer markets for a variety of subjects. For instance, Foster et al. (forthcoming) stress their role in affecting firm survival and Einav and Somaini (2013) and Cabral (2014) focus on their effect on the competitive environment.

Finally, we relate to the literature on deep habits where persistence in demand is due to the preferences of the agents rather than to costly search. The pricing and aggregate implications of this setup have been studied by Ravn et al. (2006), Ravn et al. (2010) and Nakamura and Steinsson (2011). Whereas in these models price variation has persistent effects on demand because it affects the habit formation at the level of each product and, therefore, the quantity purchased by each customer (the intensive margin), in our model the action takes place along the extensive margin of demand.

The rest of the paper is organized as follows. Section 3 presents the data and descriptive evidence of the relationship between customer dynamics and prices. In Section 2 we lay out the model and in Section 2.3 we characterize the equilibrium. In Section 2.4 we discuss identification and estimation of the model. In Section 4 we present some quantitative predictions of the model and compare them with empirical evidence from our data. In Section 5 we introduce an application of the model with the goal of studying the implications of customer markets for the dynamics of markups. Section 6 concludes.
2 The model

The economy is populated by a measure one of firms producing a homogeneous good and by a measure one of customers who consume it. The economy is in steady state and there are no foreseen aggregate shocks.

2.1 The problem of the firm

Firms produce the same homogeneous good. We assume a linear production technology \( y = z \ell \) where \( \ell \) is the production input, and \( z \) is the firm-specific productivity. Idiosyncratic productivity is distributed according to a conditional cumulative distribution function \( F(z'|z) \) with bounded support \([z, \bar{z}]\). We also assume that \( F(z'|z_h) \) first order stochastically dominates \( F(z'|z_l) \) for any \( z_h > z_l \) to induce persistence in firm productivity. The profit per customer accrued to the firm is given by \( \pi(p, z) \equiv d(p)(p - w/z) \), where \( p \) denotes the price, the constant \( w > 0 \) denotes the marginal cost of the input \( \ell \), and the function \( d(\cdot) \) is a downward sloping demand function.\(^1\) We assume that profits per customer are single-peaked in \( p \).

Firms differ not only in their idiosyncratic productivity but also in the mass of customers buying from them. In particular, we denote by \( m \) the firm’s customer base which is defined as the mass of customers who bought from that firm in the previous period, adjusted for an exogenous attrition rate \( \delta \). For given customer base \( m \), the mass of customers actually buying from the firm in the current period is determined in equilibrium and we conjecture, and later verify, that it is given by the function \( \mathcal{M}(m, p, z) \) depending on the price and productivity of the firm in the current period, as well as on the customer base.

We assume a constant probability \( \kappa \) of a firm exiting the market. Once a firm exits the market it looses all customers and its value is zero. An exiting firm is replaced by a new firm which starts with a given customer base \( m_0 \), and draws a productivity \( z_0 \) from the invariant productivity distribution \( f(z) \) associated to the conditional distribution \( F(z'|z) \).

We study a stationary Markov Perfect equilibrium where pricing strategies are a function of the current state. As there are no aggregate shocks, the aggregate state is constant and the relevant state for the firm problem in period \( t \) is the pair \( \{z, m\} \). The firm pricing problem

\(^1\)In Appendix E, we extend this framework adding a model of the labor market to endogenize the wage \( w \).
in its recursive form solves

$$\tilde{W}(z, m) = \max_p M(m, p, z) \pi(p, z) + \beta (1 - \kappa) \int_{\tilde{z}}^{z} \tilde{W}(z', m') \, dF(z' \mid z)$$  

$$\text{s.t.} \quad m' = (1 - \delta) M(m, p, z),$$

where $\tilde{W}(z, m)$ denotes the firm value at the optimal price. The price impacts firm value through two channels. First, it affects the level of profits per customer as in standard models of firm pricing. Given our assumption of single-peakedness of the profit function $\pi(p, z)$, there is a unique level of $p$ that maximizes the profits per customer. Second, the price $p$ affects the dynamics of the customer base. In fact, it influences the mass of customers buying from the firm in the current period, and, if there is persistence in the evolution of the customer base, the mass of customers buying from the firm in future periods. As a result, the pricing problem of the firm is dynamic in nature.

We study an environment where there is persistence in the customer base, as in Phelps and Winter (1970) and Rotemberg and Woodford (1999). In these models an ad-hoc functional form for the evolution of the customer base is assumed, where the mass of customers served by a firm is given by the product of its original customer base and a growth rate, which depends on its (relative) price. Our law of motion for customers preserves this standard structure and is given by:

$$M(m, p, z) \equiv m \Delta(p, z).$$

This similarity notwithstanding, there are two important innovations that we introduce. First, while the customer evolution is typically characterized with ad-hoc functional form assumptions, our $\Delta(\cdot)$ function is endogenous and results from the solution to a game between the firm and its customers. It depends on the equilibrium distribution of prices as well as on the distributions of productivity and search costs. Accounting for this dependence matters for the estimation where we will match micro moments obtained from customers’ decisions. Moreover, it has important implications when using the model for policy experiments, as we will illustrate with the application in Section 5.

Second, we generalize the law of motion so that it can depend not only on the price the firm sets but also on its productivity. This extension allows to use our model to study the equilibrium price pass-through of idiosyncratic cost shocks. It also proves useful when we bring the model to the data, since having heterogeneity in productivity helps us matching the cross-sectional variation in prices. Our formulation does, however, share an important feature with classic customer market models: the growth rate of the customer base does not depend on the initial mass of customers. This property allows for a substantial simplification
of the firm’s problem. In particular, it can be obtained that the value function of a firm is homogeneous of degree one in $m$, i.e. $\tilde{W}(z,m) = m \tilde{W}(z,1) \equiv m W(z)$, where using equation (1) and $M(m,p,z) = m \Delta(p,z)$, it is immediate to show that $W(z)$ solves\(^2\)

$$W(z) = \max_p \Delta(p,z) \pi(p,z) + \Delta(p,z) \beta (1-q) \int_z^{\bar{z}} W(z') dF(z' | z), \quad (3)$$

where $q \equiv \kappa + \delta - \kappa \delta$ is the probability of exogenous dissolution of the firm-customer match due to either firm or customer random exit. The relevant state to the firm pricing problem is its productivity, as the level of the customer base affects multiplicatively the firm value. The solution to the firm problem in equation (3) gives an optimal pricing strategy that depends on productivity and we denote by $\hat{p}(z)$.

We emphasize that, while the initial level of the customer base does not affect the optimal price, its evolution does. This follows as a change in the price affects the growth rate of the customer base, i.e., the value of $\Delta(p,z)$, and given the persistence of the customer base, it affects the firm value in the current period as well as in future periods. Our framework is well suited to capture the relationship between firm prices and customer dynamics when this is driven by variation in idiosyncratic productivity; extending it to encompass how firm size affects this relationship is an interesting direction for future research.\(^3\)

The objective of the firm maximization problem can be expressed as the product of two terms, $W(z) \equiv \Delta(\hat{p}(z), z) \Pi(\hat{p}(z), z)$, where $\Pi(p,z)$ denotes the expected present discounted value of each customer to the firm. Under the assumption that the functions $\Delta(p,z)$ and $\pi(p,z)$ are differentiable in $p$, the first order condition to the firm problem is given by

$$\frac{\partial \Pi(p,z)}{\partial p} \frac{p}{\Pi(p,z)} = - \frac{p}{\Delta(p,z)} \frac{\partial \Delta(p,z)}{\partial p}, \quad (4)$$

where $\varepsilon_m(p,z) \equiv \partial \log(\Delta(p,z))/\partial \log(p)$ denotes the extensive margin elasticity of demand.

We will discuss conditions under which equation (4) is necessary and sufficient in Section 2.3. The function $\Pi(p,z)$ is maximized at the static profit maximizing price,

$$p^*(z) \equiv \frac{\varepsilon_d(p)}{\varepsilon_d(p) - 1} \frac{w}{z}, \quad (5)$$

where $\varepsilon_d(p) \equiv \partial \log(d(p))/\partial \log(p)$ denotes the intensive margin elasticity of demand. Equa-

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\(^2\)Under the assumption that the discount rate $\beta$ is low enough so that the maximization operator in equation (3) is a contraction, by the contraction mapping theorem we can conclude that our conjecture about homogeneity of $\tilde{W}(z,m)$ is verified.

\(^3\)Incidentally, our setup lends itself well to the data we use for the estimation where we observe prices from several firms but the customer base of only one chain. See section 3 for details.
tion (4) implies however that, due to concerns about customer dynamics, the optimal price is in general different from the one that maximizes static profits.

If the growth in the customer base is non-decreasing in the price, equation (4) implies that setting a price above the static profit maximizing price is never optimal. Hence, \( \hat{p}(z) \leq p^*(z) \) for all \( z \). Moreover, if the growth in the customer base is strictly decreasing in the price in a neighborhood of the static profit-maximizing price \( p^*(z) \), the optimal price is distorted downwards with respect to it, i.e. \( \hat{p}(z) < p^*(z) \). The first order condition in equation (4) illustrates the trade-off the firm faces when setting the price in a region where customer retention is a concern. When \( \hat{p}(z) < p^*(z) \), the optimal price balances the marginal benefit of an increase in price (more profit per customer) with the cost (decrease in the customer base). The requirement that the solution to the firm problem must satisfy the first order condition implies that we study equilibria where the firm objective, and in particular \( \Delta(p, z) \), is differentiable in \( p \). In Section 2.3, we will derive the necessary equilibrium properties that guarantee that these properties are satisfied.

### 2.2 The problem of the customer

Customers value the good sold by the firms described in the previous section according to the function \( v(p) \), denoting the customer surplus associated to the demand function \( d(p) \). We assume that \( v(p) \) is continuously differentiable with \( v'(p) < 0 \), and bounded above with \( \lim_{p \to 0^+} v(p) < \infty \). These properties are satisfied in standard models of consumer demand.

Each customer starts the period in the customer base of the firm she bought from in previous period. At the beginning of every period, a customer can be randomly reallocated to a new entrant because either the firm she was matched with exited (with probability \( \kappa \)) or the customer herself leaves for random reasons (for instance she moved to a different city). We allow for random exit to acknowledge that price dynamics, the object we study in detail in this paper, is unlikely to account for all the exits observed in the data. Conditioning on a firm surviving, random exit is i.i.d. across customers of that firm.

After random relocation has taken place, the customer observes perfectly the state of the firm she is matched to, in particular she observes its productivity. Given the equilibrium pricing function of the firm, this allows her to assess the probability distribution of the path of prices of that firm. After observing the state of her current match, the customer decides whether she wants to pay a search cost to draw another firm. The search cost \( \psi \geq 0 \) is measured in units of customer surplus, it is idiosyncratic to each customer and it is drawn each period from a cumulative distribution \( G(\psi) \), with an associated density \( g(\psi) \). For tractability, we restrict our attention to density functions that are continuous on all the support. Heterogeneity, albeit transitory, in search costs makes the customer base a
Figure 1: The problem of a customer matched to a firm with productivity $z$

The customer can search at most once per period. Search is random, with the probability of drawing a particular firm being proportional to its customer base $m$. As in Fishman and Rob (2005), this assumption captures the idea that consumers search new suppliers not by randomly sampling firms but by randomly sampling other consumers. On the technical side, this is the key assumption that will allow us to solve for an equilibrium where the value of a firm scales up multiplicatively with its customer base. Conditional on searching, the customer observes the state of the new match and takes then another decision concerning whether to exit the customer base of her initial firm and match to the new firm. In particular, the customer compares the distribution of the path of current and future prices at the two firms and buys from the firm offering higher expected value. Finally, we assume that a customer cannot recall a particular firm once she exits its customer base. Figure 1 summarizes timing and payoffs of the problem of the customer.

We next characterize the customer problem. Let $V(p, z, \psi)$ denote the value function of a customer $i$ who has drawn a search cost $\psi$ and is matched to firm $j$, which has current productivity $z$ and posted price $p$. This value function solves the following problem,

$$V(p, z, \psi) = \max \left\{ \bar{V}(p, z), \hat{V}(p, z) - \psi \right\}, \tag{6}$$

where $\bar{V}(p, z)$ is the customer’s value if she does not search, and $\hat{V}(p, z) - \psi$ is the value if she does search. The value in the case of not searching is
\[
\hat{V}(p, z) = v(p) + \beta (1 - q) \mathbb{E}_G \left[ \int_z^z V(\hat{p}(x), x, \psi') dF(x | z) \right] + \beta q \mathbb{E}_G \left[ \int_z^z V(\hat{p}(x), x, \psi') df(x) \right].
\]

We notice that the state of the firm problem depends on the on the productivity \(z\) because the pricing function \(\hat{p}(\cdot)\) mapping future productivity into prices in the Markov equilibrium makes productivity \(z\) a sufficient statistic for the distribution of future prices at the firm. We also notice that the state of the firm problem includes the current price \(p\), despite in equilibrium productivity is enough to determine the current price, as this notation is needed to study the game between the firm and its customers where the firm could, in principle, deviate from the equilibrium price. Finally, the expectation operator \(\mathbb{E}_G[\cdot]\) refers to the realization of future search costs which are drawn from the i.i.d. distribution \(G\). The value function \(\hat{V}(p, z)\) is strictly decreasing in \(p\) and increasing in \(z\).

Given the specifics of the search technology, the value to the customer if searching is given by

\[
\hat{V}(p, z) = \int_{-\infty}^{+\infty} \max \left\{ \hat{V}(p, z) , x \right\} dH(x),
\]

where the customer takes expectations over all possible draws of potential new firms, and where \(H(\cdot)\) is the equilibrium cumulative distribution of continuation values from which the firm draws a new potential match when searching.

We are now ready to describe the customer’s optimal search and exit policy rules. Such policies are characterized by simple cut-off rules. The customer matched to a firm with productivity \(z\) charging price \(p\) searches if she draws a search cost \(\psi \leq \hat{\psi}(p, z)\), where \(\hat{\psi}(p, z) \equiv \int_{\hat{V}(p, z)}^{\infty} (x - \hat{V}(p, z)) dH(x) \geq 0\) is the threshold to search. Conditional on searching, the customer exits if she draws a new firm promising a continuation value \(\hat{V}^{\text{new}}\) larger than the current match, i.e. \(\hat{V}^{\text{new}} \geq \hat{V}(p, z)\). Notice that the threshold \(\hat{\psi}(p, z)\) is strictly increasing in \(p\). The dependence on the price is straightforward, following from its effect on the surplus \(v(p)\) that the customer can attain in the current period. The intuition behind the dependence on the firm’s productivity is that, as searching is a costly activity, the decision of which firm to patronize is a dynamic one, and involves comparing the value of remaining in the customer base of the current firm with the value of searching. Because of the Markovian structure of prices, the customer’s expectation about future prices is completely determined by the firm’s current productivity. We notice that if the continuation value is increasing in \(z\) (a sufficient condition is that \(\hat{p}(z)\) is decreasing) then the threshold \(\hat{\psi}(p, z)\) is increasing in \(z\).
2.3 Equilibrium

In this section we define an equilibrium, discuss its existence, and characterize its general properties. We start by defining the type of equilibrium we study.

We start by deriving the equilibrium dynamics of the customer base as a function of price and productivity, given the optimal search and exit strategy of the customers. Given customers’ optimal decision rule, the mass of customers buying from a firm with productivity \( z \) and charging price \( p \) is given by \( M(m, p, z) = m \Delta(p, z) \), with

\[
\Delta(p, z) \equiv 1 - G\left(\hat{\psi}(p, z)\right)\left(1 - H(\bar{V}(p, z))\right) + Q\left(\bar{V}(p, z)\right),
\]

where \( G(\hat{\psi}(p, z)) \) is the fraction of customers searching from the firm customer base, a fraction \( 1 - H(\bar{V}(p, z)) \) of which actually finds a better match and exits the customer base of the firm. The mass \( m \) is the probability that searching customers in the whole economy draw the firm as a potential match. The function \( Q(\bar{V}(p, z)) \) denotes the equilibrium mass of searching customers currently matched to a firm with continuation value smaller than \( \bar{V}(p, z) \). It is multiplied by \( m \) which is equal to the probability that searching customers in the whole economy draw this firm as a potential match. Therefore, the product \( mQ(\bar{V}(p, z)) \) amounts to the mass of new customers entering the customer base. Equation (8) verifies the conjecture about the equilibrium customer dynamics made in Section 2.1.

We are now ready to define and discuss the equilibrium. We study equilibria where the continuation value to customers is non-decreasing in productivity, implying that customers’ rank of firms coincides with their productivity. This is a natural outcome as more productive firms are better positioned to offer lower prices and therefore higher values to customers.

**Definition 1** Let \( V(z) \equiv \bar{V}(\hat{\psi}(z), z) \) and \( p^*(z) \) be given by equation (5). We study stationary Markovian equilibria where \( V(z) \) is non-decreasing in \( z \). A stationary equilibrium is then

(i) search and exit strategies that solve the customer problem in equations (6)-(7);

(ii) a firm pricing strategy \( \hat{p}(z) \) that solves equation (4) for each \( z \);

(iii) a customer base for new entrant firms \( m_0 = q/\kappa \), with \( q = \kappa + \delta \);

(iv) a dynamic of the customer base at a surviving firm with productivity \( z \) given by \( m' = (1 - \delta) \Delta(\hat{p}(z), z)m \), where \( \Delta(\cdot) \) is given by equation (8);
(v) an invariant distribution of customers $K(\cdot)$ over productivities, that for each $z$ solves

$$K(z) = (1 - q) \int_{\bar{z}}^{z} \int_{\bar{z}}^{\hat{z}} \Delta(\hat{p}(x), x) dF(s|x) dK(x) + q \int_{\bar{z}}^{z} dF(s) ;$$

(vi) two invariant distributions, $H(\cdot)$ and $Q(\cdot)$, that solve

$$H(x) = K(\hat{z}(x)) \quad \text{and} \quad Q(x) = \int_{\bar{z}}^{\hat{z}(x)} G(\hat{\psi}(\hat{p}(z), z)) dK(z) ,$$

for each $x \in [\mathcal{V}(\hat{z}), \mathcal{V}(\bar{z})]$, where $\hat{z}(x) = \max\{z \in [\hat{z}, \bar{z}] : \mathcal{V}(z) \leq x\}.$

The next proposition states conditions under which the equilibrium that we evaluate exists and characterizes some of its properties.

**Proposition 1** Let productivity be i.i.d. with $F(z'|z_1) = F(z'|z_2)$ continuous and differentiable for any $z'$ and any pair $(z_1, z_2) \in [\hat{z}, \bar{z}]$, and let $G(\psi)$ be differentiable for all $\psi \in [0, \infty)$, with $G(\cdot)$ differentiable and not degenerate at $\psi = 0$. There exists an equilibrium as in Definition 1 where $\hat{p}(z)$ satisfies equation (4), and

(i) $\hat{p}(z)$ is strictly decreasing in $z$, with $\hat{p}(\bar{z}) = p^*(\bar{z})$ and $\hat{p}(\bar{z}) < \hat{p}(z) < p^*(z)$ for $z < \bar{z}$, implying that $\mathcal{V}(z)$ is strictly increasing. Moreover, the optimal markups are given by

$$\mu(p, z) = \frac{p}{w/z} = \frac{\varepsilon_d(p)}{\varepsilon_d(p) - 1 + \varepsilon_m(p, z) \Pi(p, z)/(d(p) p)} ,$$

where $p = \hat{p}(z)$ for each $z$.

(ii) $\hat{\psi}(\hat{p}(z), z)$ is strictly increasing in $z$, with $\hat{\psi}(\hat{p}(\bar{z}), \bar{z}) = 0$ and $\hat{\psi}(\hat{p}(z), z) > 0$ for $z < \bar{z}$, implying that $\Delta(\hat{p}(z), z)$ is strictly increasing, with $\Delta(\hat{p}(\bar{z}), \bar{z}) > 1$ and $\Delta(\hat{p}(z), z) < 1$.

The proof of the proposition can be found in Appendix B. Here we just point out that, while the results of Proposition 1 refer to the case of i.i.d. productivity shocks, numerical results in Section 4 show they hold even in the case of persistent productivity processes.

We now comment on the properties of the equilibrium highlighted in the proposition. The equilibrium is characterized by price dispersion: this is important, as price dispersion is what motivates customers to search. Price dispersion is driven by heterogeneity in firm productivity, as in Reinganum (1979), and by the level and dispersion of search frictions.\footnote{For tractability we will abstract from (possible) equilibria where symmetric firms charge different prices.}
More productive firms charge lower prices and, therefore, offer higher continuation value to customers. If all the firms had the same productivity, Proposition 1 would imply a unique equilibrium where the price is that maximizing static profits, \( p^*(\tilde{z}) \), and as a result the customer base of every firm would be constant.\(^5\) The equilibrium is also characterized by dispersion in customer base growth: more productive firms grow faster, and there is a positive mass of lower productivity firms that have a shrinking customer base and a positive mass of higher productivity firms that are expanding their customer base.

Optimal markups in equation (9) depend on three distinct terms: \( \epsilon_d(p) \), \( \epsilon_m(p, z) \), and \( \bar{\pi}(p, z) = \Pi(p, z)/(d(p) p) \). The terms \( \epsilon_d(p) \) and \( \epsilon_m(p, z) \) represent the price elasticities of quantity purchased (per-customer) and of customer growth, respectively. We notice that the elasticity of total firm demand to the price, i.e. \( m \Delta(p, z) d(p) \), is given by \( \epsilon_d(p) + \epsilon_m(p, z) \). An increase in price reduces total current demand both because it reduces quantity per customer (intensive margin effect) and because it reduces the number of customers (extensive margin effect). Moreover, the optimal markup solves a dynamic problem as a loss in customers has persistent consequences for future demand due to the inertia in the customer base. This dynamic effect is captured by the term \( \bar{\pi}(p, z) \), which measures the firm present discounted value of a customer scaled by the current revenues. It follows that active customer markets are associated with a strictly lower markup than the one that maximizes static profit; the lower, the larger the product \( \epsilon_m(p, z) \bar{\pi}(p, z) \).

Finally, it is useful to discuss two interesting limiting cases of our model (see Appendix C for more details). The first limiting case concerns the equilibrium when we let search costs diverge to infinity. The model then reduces to one where customer base concerns are not present. Because the customer base is unresponsive to prices, the firm problem reverts to a standard price-setting problem commonly studied in the macroeconomics literature: the firm sets the price \( p \), taking into account only its impact on static demand \( d(p) \). In equilibrium, optimal prices maximize static profits, i.e. \( \hat{p}(z) = p^*(z) \) for all \( z \in [\underline{z}, \bar{z}] \). There is price dispersion, and there is no search in equilibrium. The second limiting case concerns the equilibrium when search costs become arbitrarily small.\(^6\) In this case, as the scale of search costs becomes arbitrarily small, equilibrium prices approach the static profit maximizing price of the most productive firm, \( \hat{p}(\bar{z}) \). As a result, there is no price dispersion and customers do not search.

\(^5\)This special case is useful to understand our relation to Diamond’s (1971) results. Our model delivers equilibrium price dispersion as a result of heterogeneity in productivity. If productivity was homogeneous as in Diamond (1971) the monopoly price would be the only equilibrium price.

\(^6\)We restrict attention to the model that satisfies the assumptions of Proposition 1, so that the first order condition presented in equation (4) is necessary for optimality.
2.4 Parametrization of the model

To quantify our model, we need to make parametric assumptions and pick parameter values. In Section 3, we present data which we use to guide our parameter choice. Here, we present our functional form assumptions and discuss the calibration of parameters on which our data provide no information.

To mirror the frequency of our data, we assume that a period in the model corresponds to a week. We fix the firm discount rate to $\beta = 0.9992$ corresponding to an annual discount factor of about 4%. We set the firm exit rate $\kappa = 0.0009$, corresponding to a yearly exit rate of 4.5% and consistent to estimates reported by Evans (1987) for U.S. firms. We assume that customers have logarithmic utility in consumption. Consumption is defined as a composite of two types of goods $c \equiv \left[ d^{\frac{1}{\theta - 1}} + n^{\frac{1}{\theta - 1}} \right]^{\frac{\theta}{\theta - 1}}$, with $\theta > 1$ governing the demand elasticity. The first good (that we label $d$) is supplied by firms facing product market frictions as described in this section; the other good ($n$) acts as a numeraire and it is sold in a frictionless centralized market. The customer budget constraint is given by $pd + n = I$, where $I$ is the agent’s nominal income, which we normalize to one.\(^7\) We set the nominal wage equal to the price of the numeraire good, so that $w = 1$.\(^8\) As a result, we obtain a standard downward sloping demand function $d(p) = I/P (p/P)^{-\theta}$ where $P = (p^{1-\theta} + 1)^{1-\theta}$ is the price of the consumption basket. We set $\theta$ so that the average elasticity of demand (including both extensive and intensive margins) is equal to 4, in the range of values standard in the macro literature (see Burstein and Hellwig (2007) for a discussion) and similar to the average across the product categories reported in Chevalier et al. (2003) who, like us, analyze grocery products.\(^9\)

We assume that the logarithm of idiosyncratic firm productivity evolves according to an AR(1) process, $\log(z') = \rho \log(z) + \varepsilon$, where $\varepsilon$ is i.i.d. normally distributed, $\varepsilon \sim N(0, \sigma)$. Finally, we assume that customers draw their search cost from a Gamma distribution with shape parameter $\zeta$, and scale parameter $\lambda$. The Gamma is a flexible distribution and fits the assumptions we made over the $G$ function in the specification of the model. In particular, for $\zeta > 1$, we obtain that the distribution of search costs is differentiable at $\psi = 0$.\(^10\) The parameters governing the productivity process, $\rho$ and $\sigma$, the Gamma distribution, $\zeta$ and $\lambda$, are picked to match statistics from micro data on customer and price dynamics described in

\(^7\)In Appendix E we show that $I$ can be derived based on a model of the labor market.

\(^8\) This is equivalent to assume that the numeraire good $n$ is produced by a competitive representative firm with linear production function and unitary labor productivity. See Appendix E for details.

\(^9\)The average elasticity of demand is obtaining by summing over the intensive and extensive margins at the firm level, and then aggregating over firms: $\int_{\mathbb{Z}} [\varepsilon_m(p(\hat{\varphi}(z), z) + \varepsilon_d(p(\hat{\varphi}(z)))] dK(z)$.

\(^10\)In the estimation procedure we will discuss later we do not impose any constraints on the values the parameter $\zeta$ can take. Our unconstrained point estimate lies in the desired region.
the next section. Finally, we set the exogenous customer attrition rate $\delta$ so that the overall customer attrition rate in the model (summing random exits and those driven by prices) matches the fraction of the households in our data that experience an exit from the customer base of the supermarket chain discussed in the next section (22% on a yearly basis).

3 Data and estimation

We complement the theoretical analysis with an empirical investigation that relies on scanner data from a large U.S. supermarket chain. In this section we document that the price posted by a firm influences its customers’ decision to leave the firm, therefore supporting the mechanism at the heart of our model. In Section 2.4, we will also use a measure of the size of this effect to estimate the model and quantify the importance of customer markets in shaping firm price setting.

3.1 Data sources and variable construction

The empirical analysis exploits two main pieces of data. The first dataset (henceforth, “retailer price data”) has been previously used and documented by Eichenbaum et al. (2011). It consists of store level weekly\textsuperscript{11} revenues and quantities for the full set of UPCs purchasable at the stores of a large US supermarket chain between the years 2004 and 2006. The chain is a major grocery retailer operating over a thousand outlets across 10 states. The data only cover a subset of the stores but we observe at least one store for each of the price areas setup by the retailer. The products sold by the chain are mostly food (packaged and non packaged) and household supplies (detergents, personal hygiene products, etc.). The data contain also a measure of cost provided by the retailer for each UPC-store-week. This represents the replacement cost for the chain, i.e. the cost for the retailer of restocking the product. It includes the wholesale price but also other costs associated with logistics (delivery to the store, etc.). Eichenbaum et al. (2011) treat this measure as a good approximation of the retailer’s marginal cost.

The second and most crucial source of information we exploit (henceforth, “retailer consumers panel”) is a companion dataset consisting of cashier register records on purchases by a panel of households carrying a loyalty card of the same chain providing the price data.\textsuperscript{12} For

\textsuperscript{11} The retailer changes the price of the UPCs at most once per week, hence the frequency of the retailer price data captures the entire time variation.

\textsuperscript{12} The chain is able to associate the loyalty cards belonging to different members of a same family to a single household identifying number, which is the unit of observation in our data. Therefore, in the analysis we use the terms “customer” and “household” interchangeably. The household identifier also allows to track members of a same household when they lose (and replace) their individual loyalty card.
every trip made at the chain between June 2004 and June 2006 by customers in the sample, we have information on the date of the trip, store visited, and list of all goods purchased (as identified by their Universal Product Code, UPC) at any store of the chain, as well as quantity and price paid. The customers in our sample make an average of 150 shopping trips at the chain over the two years; if those trips were uniformly distributed, that would imply visiting a store of the chain six times per month. The average expenditure per trip is $69 for the average household. There is a great deal of variation (the 10th percentile is $29; the 90th is $118) explained, among other things, by income and family size of the different households.\footnote{Our data do not include information on purchases by customers not carrying a loyalty card of the chain. However, the chain pushes for high penetration of the loyalty card (only card holders can enjoy the price promotions) and most households use it regularly (Einav et al. (2010)). Moreover, the focus of this study, is on “regular” customers who can be meaningfully said to be part of the customer base of a firm. This seems to be more the case for individuals who sign up for a loyalty card than for occasional shoppers who do not bother to.}

We use these two databases to construct the variables needed to measure the comovement between the customer’s decision to exit the customer base and the price of her typical basket of goods posted at the chain: (i) an indicator signaling when the household is exiting the chain’s customer base, and (ii) the price of the household basket. Below we briefly describe the procedure followed to obtain them; the details are left to Appendix A.

We consider every customer shopping at the retailer in a given week as belonging to the chain’s customer base in that week. We assume that a household has exited the customer base when she has not shopped at the chain for eight or more consecutive weeks. The decision to exit is imputed to the last time the customer visited the chain. Although brief spells without purchases can be justified with alternative explanations (e.g. consuming inventory or going on vacation), the typical customer is unlikely to experience an eight-week spell without shopping for reasons other than having switched to a different chain. In fact, for the average household in our sample, four days elapse between consecutive grocery trips and the 99th percentile of this statistic is 28 days, half the length of the absence we require before inferring that a household is buying its groceries at a competing chain. This suggests that the eight-week window is a conservative choice.

We construct the price of the basket of grocery goods usually purchased by the households in the following fashion. We identify the collection of UPC’s belonging to a household’s basket ($K_i$) using the retailer consumers panel and build the index as a weighted average of the price of such goods, with price information taken from the retailer price data. The price of the basket of customer $i$, shopping at store $j$ in week $t$ is then
\[ p_{ijt} = \sum_{k \in K_i} \omega_{ik} p_{kjt}, \quad \omega_{ik} = \frac{\sum_t E_{ikt}}{\sum_{k \in K_i} \sum_t E_{ikt}}, \]

where \( p_{kjt} \) is the price of UPC \( k \) in week \( t \) at the store \( j \) where customer \( i \) shops, and \( E_{ikt} \) is the expenditure (in dollars) by customer \( i \) in UPC \( k \) in week \( t \). Note that the price of the basket is household specific because households differ in their choice of grocery products \( (K_i) \) and in the weight such goods have in their budget \( (\omega_{ik}) \).

This brief description highlights how our data are well suited to study customer base dynamics. Not only are they rich in detail about households’ grocery consumption (products purchased, expenditure, prices, etc.) but they also include a panel dimension which is crucial to observe the evolution of the customer base of the supermarket chain. Furthermore, given the significant market share held by the chain, its wide geographical spread and the fact that it offers all the mainstream grocery packaged good products, the behavior of its customers provides insights that are likely to generalize to the population of retail shoppers at large. At the same time, exploiting data from a single retail chain has limitations preventing us from investigating other relevant questions related to the topic of this study. The most obvious one is that we no longer track customers once they leave the chain. Hence, we cannot provide any empirical insights on their post-exit behavior or on the characteristics of the firms they join.

Finally, it is also worth addressing that the setting analyzed in the model and our application differ in some respects. In fact, in the theoretical model we studied the behavior of customers buying from firms producing a single homogeneous good; our application documents the exit decisions of customers from supermarket stores where they buy bundles of goods.\(^{14}\) However, under the assumption that customers’ behavior depends on the price of the whole basket of goods they typically buy at the supermarket, we can focus on the resulting price index of the customer basket in order to retrieve the response of the customer base to variation in prices. Since customers baskets are in large majority composed of package goods, which are standardized products, the assumption that the basket is a homogenous good is not unwarranted.

### 3.2 Evidence on customer base dynamics

In Figure 2 we plot the survival function for our sample of customers, that is the probability of remaining in the customer base of the firm as a function of the length of the household

\(^{14}\)The choice of focusing on the customer base of the store rather than that of one of the branded product it sells is data driven. With data from a single chain we cannot track the evolution of the customer base of a single brand. In fact, if we observed customers no longer buying a particular brand we could only infer that they are not buying it at the chain we analyze, but we could not rule out that they are buying it elsewhere.
spell as customer. In the plot, we explore the sensitivity of customer base evolution to our
definition of exit by displaying three survival functions. The continuous line refers to our
baseline definition; the lines in dashed patterns originate from reducing the absence spell
required to determine that the customer has exited to one month or to extend it to three
months. The first thing to notice is that, regardless of the definition of exit we adopt, the odds
of exiting the customer base evolve smoothly. The second noteworthy fact emerging from
the plot is that the probability of staying as a customer is very high even after a relatively
long permanence in the customer base. Though using our less demanding definition of exit
the survival probability declines steeply, it is still close to 60% after 90 weeks spent in the
customer base. The figure when requiring two or three months to identify an exit is almost as
high as 80%. In other words, exits are rare events and the customer base is quite sticky: the
fraction of households staying in the customer base of the chain all throughout the sample
span is 53% with the one-month absence rule and 78% and 87% for two and three months of
absence, respectively.

To sharpen our understanding of the determinants of a customer’s decision to exit the
customer base of the firm she is currently shopping from, we estimate a linear probability
model where the dependent variable is an indicator for whether the customer has left the
customer base of the chain in a particular week. Our aim is to capture the effect of the price
posted by the chain for the basket of goods purchased by the customer on her decision to
exit. In Table 1, we report results of regressions of the following form,

\[ Exit_{it} = b_0 + b_1 \log(p_{it}) + b_2 \log(p_{mkt}^{it}) + b_3 tenure_{it} + X_i'c + \varepsilon_{it}. \] (11)

Our main interest is on the coefficient of the retailer price of the basket, \( b_1 \). Existing
theories on the link between prices and customer dynamics (Phelps and Winter (1970))
stress that a firm’s ability to retain its customers should be influenced by its idiosyncratic
price variations but not from aggregate shocks that move the competitors’ prices as well. To
isolate idiosyncratic price variations, we control for the prices posted by the competitors in
the same market of the chain using information from the IRI Marketing data set.\footnote{A detailed description of the data can be found in Bronnenberg et al. (2008). All estimates and analyses
in this paper based on Information Resources Inc. data are by the authors and not by Information Resources
Inc. We provide additional details on the IRI data and on the construction of the price index for the
competitors of the chain in Appendix A.} This data
allows us to compute the average price of the basket in a market for every customer
(\( p_{mkt}^{it} \)).

To further control for sources of aggregate variation, we include in the regression year-week
fixed effects that account for time-varying drivers of the decision of exiting the customer base
common across households (e.g., disappearances due to travel during holiday season).
Notes: The figure plots the survival function for our sample of households, where failure is defined as exit from the customer base. We report three survival functions for different criteria to determine whether the customer has exited: one consecutive month without shopping at the chain (dashed line), two consecutive months without shopping at the chain (our baseline definition, continuous line), and three consecutive months without shopping at the chain (dash-dotted line). To ensure that all individuals have similar potential length for their spells, we only consider the first spell as customer for those having multiple ones and we only retain households who started being customers of the chain within the first 40 weeks of our data.

The coefficient $b_1$ is then identified by $UPC$-chain specific shocks as those triggered, for example, by the expiration of a contract between the chain and the manufacturer of a UPC. We also observe the price of a same good moving differently in different stores within the chain, for instance due to variation in the cost of supplying the store linked to logistics (e.g. distance from the warehouse). Since these shocks can hit differently goods with different intensity in delivery cost (e.g. refrigerated vs. nonrefrigerated goods), $UPC$-store specific shocks also contribute to our identification. We do not need to assume that such shocks will make a supermarket uniformly more expensive than the competition. Shocks that affect the convenience of a chain with respect to a subset of goods suffice to induce the customers who particularly care about those goods to leave. Kaplan and Menzio (forthcoming, a) use independent scanner data to provide ample evidence for this source of variation. They report that the bulk of price dispersion arises not from the difference from high-price and low-price stores but from dispersion in the price of a particular good (or product category) even
among stores with similar overall price level. Since the retailer price in equations (11) can be endogenous if the chain conditions to variables unobserved to the econometrician that also influence the customer’s decision to leave, we instrument it with the cost of the basket. Similarly to what we did to obtain the price of the basket, we calculate the cost of a basket obtained as the weighted average of the replacement cost of the UPCs included in it.

In our empirical specification we acknowledge that, unlike posited in the model, customers are heterogenous in more dimensions than their cost to search. The limited number of exits occurring in our sample implies that the within unit variation in the dependent variable is low. Therefore, we cannot control nonparametrically for cross-household heterogeneity using household or store fixed effects. Instead, we include in our specification a rich set of covariates that control for the major characteristics affecting store choice: demographics, location, market characteristics, and tenure. The demographic variables (age, income, and education) are matched from Census 2000. We calculate using data on grocery shop location by Reference US both the distance (in miles) between a household’s residence and the closest store of the chain and that to the closest supermarket of a competing brand. We account for market structure by controlling for the total number of supermarket stores in the zip code of residence of the customer. To pick up the heterogeneity in the type of goods different customers include in their basket, we control for the price volatility of the customer-specific basket and for its price in the first week in the sample, as a scaling factor. Finally, we calculate customer tenure, defined as the number of consecutive weeks the customer has spent in the customer base of the chain, and include it in the regression to account for the fact that long-term customers of the chain may be less willing to leave it ceteris paribus.

The main specification in column (1) shows that the basket price posted by the retailer significantly impacts the probability of leaving. The effect is also quantitatively important. The average probability of exiting the customer base (0.3% weekly) implies a yearly turnover of 14%; if the retailer’s prices were 1% higher, its yearly turnover would jump to 21%. The coefficient on the competitors’ price, which we would expect to enter with a negative sign, is not significant. This may be due to the fact that the IRI data only allow us to imperfectly capture competitors’ behavior. In fact, the IRI dataset contains price information only on a subset of the goods included in a customer’s basket, although it arguably covers all the major product categories. Furthermore, the IRI data do not contain detailed information on the location of the outlets. This introduces measurement error in our construction of the set of stores a customer considers as options for her shopping. The negative coefficient on tenure confirms the intuition that the longer the relationship between a firm and a customer, the less likely they are to be interrupted. Among the several individual characteristics we control for, it is worth mentioning that distance from stores of the chain and distance from
Table 1: Effect of the price of the basket on the probability of exiting the customer base

<table>
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<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \log(p) )</td>
<td>0.14**</td>
<td>0.01</td>
<td>0.16**</td>
<td>0.15**</td>
</tr>
<tr>
<td></td>
<td>(0.066)</td>
<td>(0.04)</td>
<td>(0.080)</td>
<td>(0.064)</td>
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<tr>
<td>( \log(p)^* \text{Walmart entry} )</td>
<td></td>
<td></td>
<td>0.018**</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(0.009)</td>
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</tr>
<tr>
<td>( \log(p_{mkt}) )</td>
<td>0.001</td>
<td>0.000</td>
<td>0.001</td>
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<td></td>
<td>(0.001)</td>
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<tr>
<td>Tenure</td>
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<td>-0.003***</td>
<td>-0.004***</td>
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<tr>
<td></td>
<td>(0.001)</td>
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<tr>
<td>Observations</td>
<td>52,670</td>
<td>52,670</td>
<td>66,182</td>
<td>52,101</td>
</tr>
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**Notes:** An observation is a household-week pair. The results reported are calculated through two-stages least squares where we use the logarithm of the cost of the basket (constructed based on the replacement cost provided for each UPC by the retailer) as instrument for the logarithm of the price of the basket. In column (2), the price of the household basket is substituted with a price index for the store overall. In column (4), the exit of the customer is assumed to have occurred in the first week of absence in the eight (or more) weeks spell without purchase at the chain rather than the week of the last shopping trip before the hiatus. We trim from the sample households in the top and bottom 1% in the distribution of the number of trips over the two years. Coefficients on a series of variables are not reported for brevity: demographic controls matched from Census 2000 (ethnicity, family status, age, income, education, and time spent commuting) as well as distance from the closest outlet of the supermarket chain and distance from the closest competing supermarket (provided by the retailer). The logarithm of the price of the household basket in the first week in the sample and the standard deviation of changes in the log-price of the household basket over the sample period are included as a controls in all specifications. Week-year fixed effects are also always included. Standard errors are in parenthesis and account for within-household correlation through a two-steps feasible-GLS estimator. ***: Significant at 1% **: Significant at 5% *: Significant at 10%.

the closest competing store enter with the expected sign. Customers living in proximity of a store of the chain are less likely to leave it, and those living closer to competitors’ stores are more inclined to do so.

Documenting that changes in the price of the goods included in their regular consumption basket affect customers’ decision to abandon the chain not only provides a compelling motivation to our decision to model the link between customer base and pricing policy but also lends support to the central tenet of the growing literature on customer markets. Pre-existing evidence of this relationship is based on survey data where firms report concerns about customer retention as the main reason for their reluctance to adjust prices (see Blinder et al. (1998), and Fabiani et al. (2007)). To the best of our knowledge, we are the first to document this fact using micro data based on actual customers’ decisions. In the other columns of Table 1 we assess the robustness of this finding.

In columns (2)-(4), we replacing the price of the individual basket with a price index for
the store basket, defined as the average of the prices of the UPC’s sold by the store where the customer buys, weighted for their sales. Such price is, by construction, equal for all the customers shopping in the same store. Column (2) shows that this results in a price coefficient that is negative and not significant, confirming the importance of being able to construct individual specific baskets in order to make inference on customer’s behavior. In column (3), we experiment with an alternative way to control for the effect of competition: we exploit episodes of entry by Walmart, a major retailer with which our chain is in direct competition. We use data from Holmes (2011) to identify the date of entry by a Walmart supercenter, i.e. a store selling groceries on top of general discount goods, in a zip code where the retailer we study also operates a supermarket. The resulting event study allows us to measure the effect of the retailer price on the probability of exit controlling for the most relevant change in the competitive environment. The coefficient obtained falls in the same ballpark as the estimate in the main specification, which is reassuring on the effectiveness of the IRI price in measuring the competitors’ behavior. In column (4), we change the assumption on the imputation of the date of exit. Rather than assuming that the customer left on the occasion of her last trip to the store, we posit that the exit occurred in the first week of her absence. Even in this case, the main result stays unaffected. Finally, we performed a placebo test to investigate whether it is possible to obtain results with the same level of significance of our main specification out of pure chance. We estimated our main specification 1,000 times each time with a different dependent variable where exits from the customer base, while kept constant in number, are randomly assigned to customers. We find that only in 2.8% of the cases the simulation yields a price coefficient that is positive and significant at 5%.

3.3 Estimation

Our data allow us to estimate the key parameters of the model -those characterizing the idiosyncratic productivity process and the search cost distribution- using a minimum-distance estimator. The productivity process influences the variability of prices, which is necessary for customers to obtain any benefit from search. The parameters of the search cost distribution, on the other hand, directly determine how costly it is to search. Below we discuss the moments in the data that allow to pin down those parameters. The discussion is provided only for the sake of intuition; given the nonlinearity of the model, all the moments contribute to the identification of all the parameters.

Our theoretical model describes how persistence and volatility of productivity (\( \rho \) and \( \sigma \), respectively) determine autocorrelation and volatility of the resulting firm prices. We therefore estimate \( \rho \) and \( \sigma \) by matching the autocorrelation and the volatility of the logarithm
of firm prices observed in the data. As before, we analyze the price of a composite bundles of goods to accommodate the multiproduct nature of grocery retailing (Smith (2004)). We construct for each store the price index of the goods sold at the establishment and estimate an AR(1) process, controlling for time and store fixed effects. We find that the autocorrelation of log-prices in the data is equal to 0.61. For the volatility of the process, we target the unconditional weekly standard deviation of the residuals, which gives us an estimate of the volatility of the store price index (0.027).

To identify the parameters of the search cost distribution we exploit the estimates of the relationship between the price and the probability of exiting the customer base discussed in the previous section. The parameter $\zeta$ measures the inverse of the coefficient of variation of the search cost distribution, and governs the elasticity of customer exit to a change in the price. In particular, we estimate the parameter $\zeta$ by matching the average effect of log-prices on the exit probability predicted by the model to its counterpart in the data, measured by the estimate of the parameter $b_1$ reported in column (1) of Table 1. It is important to notice that this parameter was estimated controlling for a number of household-specific characteristics (demographics, distance from the store, etc.) which affect the decision on the store where to shop.

The parameter $\lambda$ governs the scale of the search cost distribution. When the scale of the search cost increases, firms pass-through idiosyncratic cost shocks more into their prices because the extensive margin of demand is less sensitive to price variation. Thus, we can identify $\lambda$ using information on the price pass-through of cost shocks. To this end, we estimate pass-through using the retailer price data by regressing the log-price index of each store in a given week on its log-cost index. We obtain a pass-through of 17% which we use as target for the estimation (see Appendix F for details on the estimation procedure).

Finally, we set the exogenous customer attrition rate $\delta$ so that the overall customer attrition rate in the model (summing random exits and those driven by prices) matches the fraction of the households in our sample that experience an exit from the customer base of the supermarket chain (0.0044 on a weekly basis).

We define $\Omega \equiv [\zeta \ \lambda \ \rho \ \sigma]'$ as the vector of parameters of interest and estimate it with a minimum-distance estimator. Denote by $v(\Omega)$ the vector of the moments predicted by the

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16This index is computed in a fashion analogous to the customer price index. It is the average of the price of all the UPCs sold in store $j$, weighted by the share of revenues they represent.

17Our finding is not inconsistent with evidence of complete pass-through presented by Eichenbaum et al. (2011) using the same data. First, they measure pass-through conditional on price adjustment; whereas we look at the unconditional correlation between prices and costs. Second, they deal with UPC-level pass-through while we measure pass-through of a basket of goods. If retailers play strategically with the pricing of different products, for example lowering margins on some UPC to compensate the cost increase they experienced on others, we can obtain both high UPC-level pass-through and low basket-level pass-through.
model as a function of parameters in $\Omega$, and by $v_d$ the vector of their empirical counterparts. The $n^{th}$ iteration of the estimation procedure unfolds according to the following steps:

1. Pick values for the parameters $\rho_n$, $\sigma_n$, $\lambda_n$ and $\zeta_n$ from a given grid,

2. Solve the model and obtain the vector $v(\Omega_n)$,

3. Evaluate the objective function $(v_d - v(\Omega_n))'\Sigma(v_d - v(\Omega_n))$. Where $\Sigma$ is a weighting matrix that we assume to be the identity matrix.

We select as estimates the parameter values from the proposed grid that minimize the objective function. Implementing step 2 requires solving a fixed point problem in equilibrium prices. In particular, given our definition of equilibrium and the results of Proposition 1, we look for equilibria where prices are in the interval $[p^*(\bar{z}), p^*(\bar{z})]$. In principle, our model could have multiple equilibria; however, numerically we always converge to the same equilibrium despite starting from different initial conditions. More details are provided in Appendix D.

The estimation results are summarized in Table 2. Persistence and volatility of the productivity process are easy to interpret. To provide context for the parameters of the search cost distribution, we perform a back-of-envelope calculation of the dollar equivalent value of the search costs the estimated distribution implies. If we were to follow one customer over time, we would find that her unconditional expected utility loss from searching is equivalent to a 0.02% permanent reduction in weekly income.
4 The distribution of prices with customer markets

In this section we use the parameter estimates reported in Table 2 to solve numerically our model (henceforth “baseline economy”) and characterize its quantitative properties.\(^{18}\) We show that the model is able to produce predictions in line with recent empirical evidence on the distribution of price levels. Moreover, we perform counterfactual exercises to highlight the role of customer dynamics for equilibrium prices. In particular, we vary the size of the search costs directly affecting the extensive margin elasticity of demand.

Panel (a) of Figure 3 reports the distribution of (log-) price levels implied by the model. The distribution of prices shows a high concentration of prices around the mean, and relatively fat tails. As a result excess kurtosis is 5.3. These characteristics are similar to those documented by empirical studies looking at the distribution of prices. In particular, Kaplan and Menzio (forthcoming,a) report a shape for the price distribution of homogeneous goods in the grocery sector similar to the one delivered by our model. The empirical distribution is characterized by a high concentration of prices around the mean, much higher than what implied, for instance, by a normal distribution.

To understand the properties of the price distribution, we need to look at the relationship between the pricing function and the extensive margin of demand. The price distribution depends on the equilibrium pricing function, whose shape can be explained by the behavior of extensive margin of demand.\(^{19}\) We report all of these objects in additional panels of Figure 3. The pricing function (panel (b)) is almost flat at intermediate levels of productivity thus explaining the high concentration of prices. At low and high levels of productivity, instead, the pricing function is steeper implying a higher pass-through of productivity shocks to prices. This accounts for the fat tails of the price distribution.

The pricing function is in turn shaped by the extensive margin of demand. In particular, if the extensive margin elasticity of demand reduces when productivity increases, then the pass-through of productivity shocks to prices will be incomplete. Under this scenario, as can be seen in equation (9), an increase in production cost is at least partially offset by a reduction in markups. This effect is more marked, the stronger is the relationship between productivity and demand elasticity. Panel (c) of Figure 3 shows that, indeed, the extensive margin elasticity strongly decreases with productivity at intermediate levels of productivity, explaining the flatness of the pricing function in that region.

Starting with the most productive firms, we next discuss how the mechanism explaining

\(^{18}\)See Appendix D for details on the numerical solution of the model.

\(^{19}\)Equation (9) implies that optimal prices also depend on the intensive margin elasticity (\(\varepsilon_d\)) and the value of a customer (\(\bar{\pi}\)). In Appendix G we investigate their role and we find it to be quantitatively small. Therefore, here we concentrate only on the role of the extensive margin of demand.
Figure 3: Equilibrium price dynamics

(a) Price Distribution

(b) Pricing function

(c) Extensive Margin: Elasticity

(d) Distribution of Search Costs

Notes: In panel (a), we plot the histogram of prices grouped into bins of size equal to 0.02 log-points. In panel (b), we plot the optimal log-prices as a function of productivity. In panel (c), we plot the extensive margin elasticity. In panel (d) we plot the density of search costs. All the objects refer to the baseline model whose parameters are reported in Table 2.

the shape of the extensive margin elasticity as a function of productivity. The most productive firms face low risk than customers will leave since they promise high expected value to their customers relatively to the average firm. As a result, they have low exit rates: only customers drawing both tiny search costs and an even better match will end up exiting. This
Notes: In panel (a), we plot the threshold for the search cost, below which customers search as a function of productivity. In panel (b) we plot the fraction of customers exiting the customer base of a firm. All the objects refer to the baseline model whose parameters are reported in Table 2.

combination is a low probability event, roughly insensitive to small variations in productivity because the estimated density of search costs has little mass close to the origin (panel (d)) meaning that variations in the threshold to search are associated to small variations in the mass of customers searching.

As productivity decreases, the threshold to search as well as the probability of drawing a better match increase. Moreover, the decision to search and exit become more sensitive to changes in productivity because the estimated density of search costs increases, implying that variations in the threshold to search are associated to significant variations in the mass of customers searching. Finally, as productivity approaches the left tail of the distribution, the extensive margin elasticity flattens again. This happens because customers paying higher prices at lower productivity firms substitute towards the numeraire good (good n). Therefore, everything else being equal, variations in the price of the good with product market friction (good d) have less of an impact on the utility of these customers.\textsuperscript{20}

To sum up, the incomplete pass-through of idiosyncratic cost shocks to prices and high clustering of prices around the mean are tightly linked in our model to variation in the

\textsuperscript{20}Notice that the threshold to search at low productivity firms is in the increasing part of the density of search cost (Figure 4), so the flattening of the extensive margin elasticity at low productivity is not due to a smaller mass of customers exiting at the margin.
elasticity of demand the different firms face. In particular, variation in the elasticity of the extensive margin of demand is what truly drives those results. This is a distinctive characteristic of our model relatively to alternative models of endogenous markups where demand elasticity varies because of the interaction of variation in market share with the intensive margin of demand (see for instance Atkeson and Burstein (2008)). As we discussed above, in our model the variation in the elasticity of demand may not be necessarily associated to the variation in market share. In fact, firms at intermediate levels of productivity, and therefore with intermediate market shares on average, face steeper variation in elasticity than the least productive firms. This feature seems to find indirect confirmation in the data, since it is key to obtain excess kurtosis of the distribution of prices.

To further illustrate how customer markets play a key role in generating the distribution of prices we obtain, we perform a counterfactual exercise varying their strength. In Figure 5 we report the price distributions (left panel) and the pricing functions (right panel) from three economies identical but for the level of the search cost. The first one is our baseline economy, which we compare to an alternative one where we increase the search costs by a factor of two and another one where we increase the search costs by a factor of five.

As we increase the scale of the search costs (increasing $\lambda$), it is harder for customers to exit, therefore competition for customers reduces. This results into a price distribution characterized by a higher mean, a higher standard deviation and a lower kurtosis. All of these effects are due to the impact of higher search costs on the optimal pricing function. When the scale of the search cost grows, firms charge higher price for each level of productivity and pass-through productivity shocks more. This reflects that the extensive margin elasticity becomes less sensitive to productivity changes when customers search costs are higher. In fact, as the search cost grows large, the model converges to monopolistic competition where the extensive margin elasticity is constant.

Finally, in Figure 4 we illustrate the predictions of our model regarding customer dynamics. We plot the search cost thresholds (left panel) and the fraction of customers exiting the customer base (right panel) as a function of the productivity $z$ of the firms the customer is matched to. There is substantial variation in exit rates across firms. Firms above the median in terms of productivity lose less than 0.25% of their customers every week but low productivity ones can see their customer base erode significantly. Recent literature has emphasized the role of the customer base as an important and persistent determinant of the level of firm demand (Foster et al. (2008), Foster et al. (forthcoming)). Our findings are fully consistent with this view. In fact, Table 3 reports in the top row the probability that a firm in the top 25% of the distribution of demand is still in the same quartile after 1 month, 1 quarter and 1 year, respectively. The bottom row shows the same statistics for the bottom
Figure 5: Comparative statics: the role of search costs

Notes: In the left panel, we plot the distribution of log-prices and in the right panel the optimal pricing function as a function of productivity. The blue solid line and histogram refer to our baseline economy with parameters reported in Table 2. The black and red objects refer to two counterfactuals where we increase the size of search costs ($\lambda = 1.65$ & $\lambda = 0.7$ respectively).

Table 3: Persistence of demand

<table>
<thead>
<tr>
<th></th>
<th>Total demand: $d \times m$</th>
<th>Demand per customer: $d$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 month</td>
<td>1 quarter</td>
</tr>
<tr>
<td>Top 25%</td>
<td>0.84</td>
<td>0.81</td>
</tr>
<tr>
<td>Bottom 25%</td>
<td>0.90</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Notes: The top row reports the probability that a firm in the top quartile of the sales distribution stays there after 1 month, 1 quarter and 1 year. The bottom row displays the same statistics for the bottom quartile of the distribution. We report separately the results for the distribution of total demand (left panel) and for the distribution of demand per customer (right panel). The statistics are obtained by simulating our baseline economy with parameters reported in Table 2.

25% of the distribution. We look at two different outcomes: total demand (left panel) and demand per customer (right panel). The latter can be linked to the intensive margin: each customer expands or contracts her demand for the customer market goods depending on the price she faces. The total demand for the firm, however, is the sum of the demands of
all of its customers. Even firms with declining productivity, which will post higher prices and, therefore, see their demand per customer shrink, can have strong total demand if they have a large installed base of customers. Our statistics do show that firm level demand is very persistent, much more so than the underlying exogenous process of firm productivity. For instance, a firm in the top 25% of the demand distribution is more that 80% likely to stay in the same group after a quarter; whereas a firm in the top 25% of the distribution of productivity (and therefore demand per customer) only has a 26% chance of staying in the same group after one quarter, which is almost the unconditional probability. We also find asymmetry in persistence between the top and bottom quartiles of the distribution of demand: persistence at the bottom is stronger than persistence at the top. This happens because low ranked sellers have on average low productivity and lose customers at a faster rate than that at which high productivity firms, i.e. top sellers, gain them.

5 Demand shocks and markups cyclicality

Starting with Phelps and Winter (1970), there has been an active debate on the role of customer markets in shaping aggregate markup dynamics, in particular for the propagation of transitory demand shocks (Rotemberg and Woodford (1991, 1999)). Although it was commonly conjectured that customer markets could be a source of countercyclical markups, this result proved hard to achieve. In ad-hoc models with customer markets, in fact, a positive demand shock only makes customers more keen to buy in the current period relative to future periods, leading to a pro-cyclical response of markups. In this section we show that once we endogenize customer dynamics, demand shocks generate countercyclical aggregate markups. In our setting a positive demand shock not only increases current demand, but also affects customers incentives to search and hence firms’ probability of losing customers. This increases competition for customers, providing incentives to lower markups.  

[TO BE COMPLETED]

6 Concluding remarks

This paper provides an assessment of the importance of customer dynamics in shaping firm pricing strategies. We exploit novel data to document the premise of the customer market literature that prices influence customers’ decision to exit the customer base of their firm. We then develop and estimate a rich yet tractable model to assess the role that customer

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21 The habit formation literature (Ravn et al. (2006)) provides another example of countercyclical markups arising from individual behavior.
Figure 6: Impulse responses to an aggregate demand shock, in % deviations from s.s

- Markup
- Price
- Consumption
- Price dispersion
- Preference Shock
- Search
- Extensive margin
- Customer value
- Intensive margin

Baseline vs. Counterfactual: Constant Extensive Margin Elasticity
markets play in determining price setting. We find that this mechanism is important to explain salient features of the data. These results bring new evidence on the forces involved in price formation which should be of great interest for the increasing number of both micro and macroeconomists analyzing price data for an assortment of purposes. Finally, we use our framework to study aggregate dynamics and show that the conventional wisdom according to which customer market models cannot generate countercyclical markups in response to demand shocks relates only to ad-hoc model of customer base evolution. Our microfoundation linking customer dynamics to search and exit decision by individual customers can results in a countercyclical response of markup to a temporary demand shock.

Our study relies on a number of simplifying assumptions, whose relaxation seems of interest for future research. First, for tractability we refrain from explicitly modeling persistent heterogeneity in customers search/opportunity costs (although we control for these factors in the empirical analysis) and we do not allow for price discrimination. The presence of customers heterogeneity in shopping behavior is well documented (Aguiar and Hurst (2007)), which makes studying its implications for optimal pricing and customer dynamics an important topic. Due to lack of data, we do not consider the role of advertising in generating demand dynamics (Hall (2014)). While our conjecture is that the analysis of the pricing incentives presented in this paper would still apply, we think that extending the analysis to advertising, as well as to other strategies to attract and retain customers, and confront the results with direct firm level evidence, could provide new insights about firms behavior.

References


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Appendix - Not for publication

A Data sources and variables construction

A.1 Data and selection of the sample

The empirical evidence presented in Section 3 is based on two data sources provided by a large supermarket chain that operates over 1,500 stores across the United States. This implies that we can observe our agents behavior only when they shop with the chain; on the other hand, cash register data contain significantly less measurement error than databases relying on home scanning (Einav et al. (2010)).

The main data source contains information on grocery purchases at the chain between June 2004 and June 2006 for a panel of over 11,000 households. For each grocery trip made by a household, we observe date and store where the trip occurred, the collection of all the UPCs purchased with quantity and price paid. The data include information on the presence and size of price discounts but do not generally report redemption of manufacturer coupons. Data are collected through usage of the loyalty card; purchases made without using the card are not recorded. However, the chain ensures that the loyalty card has a high penetration by keeping to a minimum the effort needed to register for one. Furthermore, nearly all promotional discount are tied to ownership of a loyalty card, which provides a strong incentive to use it.

Household-level scanner data report information on the price paid conditional on a certain item having been bought by the customer. Therefore, if we do not observe at least one household in our sample buying a given item in a store in a week, we would not be able to infer the price of the item in that store-week. This has important implications as our definition of basket requires us to be able to attach a price to each of the item composing it in every week, even when the customer does not shop. The issue can be solved using another dataset with information on weekly store revenues and quantities between January 2004 and December 2006 for a panel of over 200 stores. For each good (identified by its UPC) carried by the stores in those weeks, the data report total amount grossed and quantity sold. Exploiting this information, we can calculate unit value prices every week for every item in stock in a given store, whether or not that particular UPC was bought by one of the households in our main data. Unit value prices are computed using data on revenues and quantities sold as

$$ UVP_{stu} = \frac{TR_{stu}}{Q_{stu}}, $$

36
where $TR$ represent total revenues and $Q$ the total number of units sold of good $u$ in week $t$ in store $s$.

As explained in Eichenbaum et al. (2011), this only allows us to recover an average price for goods that were on promotion. In fact the same good will be sold to loyalty card carrying customers at the promotional price and at full price to customers who do not have or use a loyalty card. Without information on the fraction of these two types of customers it is not possible to recover the two prices separately. Furthermore, since prices are constructed based on information on revenues, missing values can originate even in this case if no unit of a specific item is sold in a given store in a week. This is, however, an unfrequent circumstance and involves only rarely purchased UPCs, which are unlikely to represent important shares of the basket for any of the households in the sample. For the analysis, we only retain UPCs with at most two nonconsecutive missing price observations and impute price for the missing observation interpolating the prices of the contiguous weeks.

On top of reporting revenues and quantities for each store-week-UPC triplet, the store-level data also contain a measure of cost. This variable is constructed on the basis of the estimated markup imputed by the retailer for each item and includes more than the simple wholesale cost of the item (the share of transportation cost, etc.). Eichenbaum et al. (2011) suggest to think about it as a measure of replacement cost, i.e. the cost of placing an item on the shelf to replace an analogous one just grabbed by a consumer. We use this measure to construct our instrument of the basket price.

It is important to notice that the retail chain sets different prices for the same UPC in different geographic areas, called “price areas.” The retailer supplied store-level information for 270 stores, ensuring that we have data for at least one store for each price area. In order to use unit value prices calculated from store-level data to compute the price of the basket of a specific household, we need to determine to which price area the store(s) at which she regularly shops belong. This information is not supplied by the retailer that kept the exact definition of the price areas confidential. A possible solution is to infer in which price areas the store(s) visited by a household are located by comparing the prices contained in the household panel with those in the store data. In principle the household data should give information on enough UPC prices in a given week to identify the price area representative store whose pricing they are matching. However, even though two stores belonging in the same price area should have the same prices, they may not have the same unit value prices if the share of shoppers using the loyalty card differs in the two stores. Therefore, we choose to restrict our analysis to the set of customers shopping predominantly (over 80% of their grocery expenditure at the chain) in one of the 270 stores for which the chain provided complete store-level data. This choice is costly in terms of sample size: only
1,336 households (or 12% of the original sample) shop at one of the 270 stores for which we have store-level price data. However, since the 270 representative stores were randomly chosen, the resulting subsample of households should not be subject to any selection bias.

A final piece of the data is represented by the IRI-Symphony database. We use store-level data on quantities and revenues for each UPC in 30 major product categories for a large sample of stores (including small and mom & pop ones) in 50 Metropolitan Statistical Areas in the United States. The data allow to construct unit value prices for all the stores competing with the chain who provided the main dataset. However, the coarse geographic information prevents us from matching each customer with the stores closer to her location (in the same zip code, for instance) and forces us to adopt the MSA as our definition of a market.

A.2 Variables construction

Exit from customer base. The dependent variable in the regression presented in equation (11) is an indicator for whether a customer is exiting the customer base of the chain. With data on grocery purchases at a single retail chain it is hard to definitively assess whether a household has abandoned the retailer to shop elsewhere or is simply not purchasing groceries in a particular week, for instance because she is just consuming its inventory. In fact, we observe households when they buy groceries at the chain but do not have any information on their shopping at competing grocers. Our choice is to assume that a customer is shopping at some other store when she has not visited any supermarket store of the chain for at least eight consecutive weeks. The \textit{Exit} dummy is then constructed so that it takes value of one in correspondence to the last visit at the chain before a spell of eight or more weeks without shopping there. Table 4 summarizes shopping behavior for households in our sample. It is immediate to notice that an eight-week spell without purchase is unusual, as customers tend to show up frequently at the stores. This strengthens our confidence that customers missing for an eight-week period have indeed switched to a different retailer.

Composition of the household basket and basket price. The household scanner data deliver information on all the UPCs a household has bought through the sample span. We assume that all of them are part of the household basket and, therefore, the household should care about all of those prices. Some of the items in the household’s basket are bought regularly, whereas others are purchased less frequently. We take this into account when constructing the price of the basket by weighting the price of each item by its expenditure share in the household budget. The price of household $i$’s basket purchased at store $j$ in week $t$ is computed as:
Table 4: Descriptive statistics on customer shopping behavior

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std.dev.</th>
<th>25th pctile</th>
<th>75th pctile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of trips</td>
<td>150</td>
<td>127</td>
<td>66</td>
<td>200</td>
</tr>
<tr>
<td>Days elapsed between consecutive trips</td>
<td>4.2</td>
<td>7.5</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Expenditure per trip ($)</td>
<td>69</td>
<td>40</td>
<td>40</td>
<td>87</td>
</tr>
<tr>
<td>Frequency of exits</td>
<td>0.003</td>
<td>0.065</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ p_{ijt} = \sum_{k \in K_i} \omega_{ik} p_{kjt}, \quad \omega_{ik} = \frac{\sum_{t} E_{ikt}}{\sum_{k} \sum_{t} E_{ikt}}, \]

where \( K^i \) is the set of all the UPCs \((k)\) purchased by household \( i \) during the sample period, \( p_{kjt} \) is the price of a given UPC \( k \) in week \( t \) at the store \( j \) where the customer shops. \( E_{ikt} \) represents expenditure by customer \( i \) in UPC \( k \) in week \( t \) and the \( \omega_{ik} \)'s are a set of household-UPC specific weights. There is the practical problem that the composition of the consumer basket cannot vary through time; otherwise basket prices for the same customer in different weeks would not be comparable. This requires that we drop from the basket all UPCs for which we do not have price information for every week in the sample. However, the price information is missing only in instances where the UPC registered no sales in a particular week. It follows that only low market-share UPCs will have missing values and, therefore, the UPCs entering the basket computation will represent the bulk of each customer’s grocery expenditure. The construction of the cost of the basket follows the same procedure where we substitute the unit value price with the measure of replacement cost provided by the retailer.

We choose to calculate the weights using the total expenditure in the UPC by the household over the two years in the sample. This can lead to some inaccuracy in identifying the goods the customer cares for at a given point in time. For example, if a customer bought only Coke during the first year and only Pepsi during the second year of data, our procedure would have us give equal weight to the price of Coke and Pepsi throughout the sample period. If we used a shorter time interval, for example using the expenditure share in the month, we would correctly recognize that she only cares about Coke in the first twelve months and only about Pepsi in the final 12 months. However, weights computed on short time intervals are more prone to bias induced by pricing. For example, a two-weeks promotion of a particular UPC may induce the customer to buy it just because of the temporary convenience; this would
give the UPC a high weight in the month. The effect of promotion is instead smoothed when we compute weights using expenditure over the entire sample period.

The construction of the price of the competitors occurs in two steps. First, we use the IRI data and the same procedure described above to obtain a price for the basket of each consumer at every store located in her same MSA. Next, we average those prices across stores to obtain the average market price of the consumer basket. In particular, the price is computed as:

\[ p_{mkt}^{it} = \sum_{z \in M_i} s_z \sum_{k \in K_i} \omega_{ik} p_{kzt}, \quad \omega_{ik} = \frac{\sum_t E_{ikt}}{\sum_k \sum_t E_{ikt}}, \quad s_z = \frac{\sum_t R_{zt}}{\sum_{z' \in M} \sum_t R_{z't}} \]

where \( M_i \) is the MSA of residence for customer \( i \) and \( R_{zt} \) represents revenues of store \( z \) in week \( t \). In other words, in the construction of the competitors’ price index, stores with higher (revenue-based) market shares weight more.

**Composition of the store basket and basket price.** The construction of the price (and cost) index for the store is conceptually analogous to that described above for the household basket. In principle, we would want to compute the store price index including all the UPCs sold at a store throughout the sample period, weighted by the share of revenues they generated. However, to keep the composition of the store basket constant through time, we must restrict ourselves to the UPCs for which we have no missing price information in any of the weeks in the sample span. This severely reduces the size of the store basket. At the same time, goods without missing information are the best sellers, which are those the store is likely to care more about.

## B Proof of Proposition 1

The following lemma discusses some key properties of the optimal price useful to prove Proposition 1.

**Lemma 1** Let \( \Delta(p, z) \) be continuous in \( p \), and let \( \varepsilon_m(p, z) \equiv \partial \log(\Delta(p, z))/\partial \log(p) \). If a price \( \bar{p}(z) \) exists such that \( \varepsilon_m(p, z) > 0 \) for all \( p > \bar{p}(z) \), and \( \varepsilon_m(p, z) = 0 \) for all \( p \leq \bar{p}(z) \), then we have \( \hat{p}(z) \in [\bar{p}(z), p^*(z)] \) if \( \bar{p}(z) < p^*(z) \), and \( \hat{p}(z) = p^*(z) \) otherwise.

The proof of the lemma is an immediate implication of equation (4). We next prove the results of Proposition 1.

**Monotonicity of prices.** Monotonicity of optimal prices follows from an application of Topkis’ theorem. In order to apply the theorem to the firm problem in equation (3) we need
to establish increasing differences of the firm objective \( \Delta(p, z) \Pi(p, z) \) in \((p, -z)\). Under the standard assumptions we stated on \( \pi(p, z) \), it is easy to show that \( \Pi(p, z) \) satisfies this property. The customer base growth function does not in general verify the increasing difference property. However, let \( \bar{p}(z) \) denote the price \( p \) that solves \( V(p, z) = \mathcal{V}(z) \). We have that \( \Delta(p, z) \) is continuous, strictly decreasing in \( p \) for all \( p > \bar{p}(z) \), and constant for all \( p \leq \bar{p}(z) \). Under the assumption of i.i.d. productivity, \( \Delta(p, z) \) is independent of \( z \), which is sufficient to obtain the result. We first show that optimal prices \( \hat{p}(z) \) are non-increasing in \( z \). Given, that productivity is i.i.d. and that we look for equilibria where \( \hat{p}(z) \geq p^*(z) \), we have that \( \bar{p}(z) = p^*(z) \) for each \( z \). From Lemma 1 we know that, for a given \( z \), the optimal price \( \hat{p}(z) \) belongs to the set \([p^*(z), p^*(z)]\). Over this set, the objective function of the firm,

\[
W(p, z) = \Delta(p, z) (\pi(p, z) + \beta \text{constant}) ,
\]

is supermodular in \((p, -z)\). Notice the i.i.d. assumption implies that future profits of the firm do not depend on current productivity as future productivity, and therefore profits, are independent from it. Similarly, \( \Delta(p, z) \) does not depend on \( z \), as the expected future value to the customer does not depend on the productivity of the current match as future productivity is independent from it. Abusing notation, we replace \( \Delta(p, z) \) by \( \Delta(p) \). To show that \( W(p, z) \) is supermodular in \((p, -z)\) consider two generic prices \( p_1, p_2 \) with \( p_2 > p_1 > 0 \) and productivities \( z_1, z_2 \in [\underline{z}, \bar{z}] \) with \( -z_2 > -z_1 \). We have that \( W(p_2, z_2) - W(p_1, z_2) \leq W(p_2, z_1) - W(p_1, z_1) \) if and only if

\[
\Delta(p_2) d(p_2)(p_2 - w/z_2) - \Delta(p_1) d(p_1)(p_1 - w/z_2) \leq \Delta(p_2) d(p_2)(p_2 - w/z_1) - \Delta(p_1) d(p_1)(p_1 - w/z_1),
\]

which, since \( \Delta(p_2) d(p_2) < \Delta(p_1) d(p_1) \) as \( d(p) \) is strictly decreasing and \( \Delta(p) \) is non-increasing, is indeed satisfied if and only if \( z_2 < z_1 \). Thus, \( W(p, z) \) is supermodular in \((p, -z)\). By application of the Topkis Theorem we readily obtain that \( \hat{p}(z) \) is non-increasing in \( z \).

**Existence of equilibrium.** Next we prove existence of an equilibrium. The fixed point problem is a mapping from candidate functions of equilibrium prices, \( \hat{p}(z) \), to the firm’s optimal pricing strategy, \( \hat{p}(z) \). Notice that \( W(p, z) \) in equation (12) is continuous in \((p, z)\). By the theorem of maximum, \( \hat{p}(z) \) is upper hemi-continuous and \( W(\hat{p}(z), z) \) is continuous in \( z \). Given that \( \hat{p}(z) \) is non-increasing in \( z \) it follows that \( \hat{p}(z) \) has a countably many discontinuity points. We thus proceed as follows. Let \( \hat{P}(z) \) be the set of prices that maximize the firm problem. Whenever a discontinuity arises at some \( \bar{z} \) (so that \( \hat{P}(\bar{z}) \) is not a singleton), we modify the optimal pricing rule of the firm and consider the convex hull of the \( \hat{P}(\bar{z}) \) as the set of possible prices chosen by the firm with productivity \( \bar{z} \). The constructed mapping
from $\mathcal{P}(z)$ to $\hat{\mathcal{P}}(z)$ is then upper-hemicontinuous, compact and convex valued. We then apply Kakutani’s fixed point theorem to this operator and obtain a fixed point. Finally, notice that since the convexification procedure described above has to be applied only a countable number of times, the set of convexified prices has measure zero with respect to the density of $z$. Hence, they do not affect the fixed point.

It is important to point out that differentiability of the distribution of productivity $F$ is not needed for the existence of an equilibrium. We assume it to ensure that $H(\cdot)$ and $Q(\cdot)$ are almost everywhere differentiable so that equation (4) is a necessary condition for optimal prices (see below). However, even when $F$ is not differentiable and the first order condition cannot be used to characterize the equilibrium, an equilibrium with the properties of Proposition 1 exists where $\hat{p}(z)$ and $\hat{\psi}(\hat{p}(z), z)$ are monotonic in $z$ but not necessarily strictly monotonic for all $z$.

**Necessity of the first order condition.** We show that $Q$ and $H$ are almost everywhere differentiable, so that Lemma 1 implies that equation (4) is necessary for an optimum. We guess that $\hat{p}(z)$ is strictly decreasing and almost everywhere differentiable. It immediately follows that $\mathcal{V}(z)$ is strictly increasing in $z$ and almost everywhere differentiable. Then, given the assumption that $F$ is differentiable, we have that $K$ is differentiable. From $H(x) = K(\mathcal{V}^{-1}(x))$ it follows that $H$ is also almost everywhere differentiable. Given that $G$ and $H$ are differentiable, so is $Q$. Then the first order condition in equation (4) is necessary for an optimum, which indeed implies that $\hat{p}(z)$ is strictly decreasing and differentiable in $z$ in any neighborhood of the first order condition. Finally, given that $\hat{p}(z)$ has a countably many discontinuity points, it has countably many points where it is not differentiable, and the first order condition does not apply at those points, but applies everywhere else. These points have measure zero with respect to the density of $z$ and therefore $\hat{p}(z)$ is almost everywhere differentiable.

**Proof of Point (i).** We already proved that $\hat{p}(z)$ is non-increasing in $z$. The proof that $\hat{p}(z)$ is strictly decreasing follows by contradiction. Consider that $\hat{p}(z_1) = \hat{p}(z_2) = \bar{p}$ for some $z_1, z_2 \in [\underline{z}, \overline{z}]$. Also, without loss of generality, assume that $z_1 < z_2$. Given that we already established the necessity of the first order condition presented in equation (4) when prices are monotonic, suppose that it is satisfied at the pair $\{z_2, \bar{p}\}$. Notice that, because of the assumed i.i.d. structure of productivity shocks together with $\pi_z(p, z) < 0$, it is not possible that the first order condition is also satisfied at the pair $\{z_1, \bar{p}\}$. Moreover, because the first order condition is necessary and we already established that $\hat{p}(z)$ cannot be increasing at any $z$, we conclude that the optimal price at $z_1$ is strictly larger than at $z_2$. That is, $\hat{p}(z_1) > \hat{p}(z_2)$.  

42
Notice that this verifies the conjecture used to prove the necessity of the first order condition, which in turn validates the use of equation (4) here.\textsuperscript{22}

Notice that, because $\hat{p}(z)$ is strictly decreasing in $z$, the fact that $v'(p) < 0$ together with i.i.d. productivity, implies, through an application of the contraction mapping theorem, that $V(z) = V(\hat{p}(z), z)$ is increasing in $z$.

**Proof of Point (ii).** \( \hat{\psi}(p, z) \geq 0 \) immediately follows its definition. The fact that $V(z)$ is strictly increasing in $z$ implies that $\hat{\psi}(\hat{p}(\bar{z}), \bar{z}) = 0$ and that $\hat{\psi}(\hat{p}(z), z)$ and $\Delta(\hat{p}(z), z)$ are strictly increasing in $z$. Because of price dispersion, some customers are searching, which guarantees that $\Delta(\hat{p}(\bar{z}), \bar{z}) > 1$. Likewise, $\Delta(\hat{p}(z), z) < 1$.

\section{Two polar cases}

The next remark explores two interesting limiting cases of our model and showcases the effect of search frictions on price dispersion.

**Remark 1** Let search costs be scaled as $\psi \equiv n \tilde{\psi}$, where $n > 0$. That is, let the value function in equation (6) be

\[ V(p, z, \psi) = \max \left\{ \bar{V}(p, z), \hat{V}(p, z) - n\tilde{\psi} \right\}. \]

**Two limiting cases of the equilibrium stated in Definition 1:**

(1) Let $n \to \infty$. Then, in equilibrium: (i) the optimal price maximizes static profits, i.e. $\hat{p}(z) = p^*(z)$ for all $z \in [\underline{z}, \bar{z}]$, and (ii) there is no search in equilibrium.

(2) Let $\pi(p^*(\bar{z}), \bar{z}) > 0$ and let the assumptions of Proposition 1 be satisfied. Then, $\hat{p}(\bar{z}) = p^*(\bar{z})$ and $\max\{\hat{p}(z)\} = \hat{p}(\bar{z})$ approaches $p^*(\bar{z})$ as $n \to 0$. As a result, in the limit, there is no price dispersion in equilibrium and customers do not search.

**Proof.** Part (1): Start by noticing that, because the mean of $G(\psi)$ is positive, the expected value of searching diverges to $-\infty$ as $n$ diverges to infinity. Because prices are finite for all

\textsuperscript{22}If prices are not strictly decreasing, this argument cannot be used as the first order condition is not necessary. However, it is possible to prove that $\hat{p}(z)$ is strictly decreasing in $z$ for some region of $z$. The argument follows by contradiction. Suppose that $\hat{p}(z)$ is everywhere constant in $z$ at some $\hat{p}$. Then $\hat{p}(z) = \hat{p}$ for all $z$. If $\hat{p} > p^*(\bar{z})$, then $\hat{p}$ would not be optimal for firm with productivity $\bar{z}$, which would choose a lower price. If $\hat{p} = p^*(\bar{z})$, then continuous differentiability of $G$ together with $H = G = Q = 0$ at the conjectured constant equilibrium price imply that the first order condition is locally necessary for an optimum, and a firm with productivity $z < \bar{z}$ would have an incentive to deviate according to equation (4), and set a strictly higher price than $\hat{p}$. Finally, the result that $\hat{p}(z) < p^*(\bar{z})$ for all $z < \bar{z}$ and that $\hat{p}(\bar{z}) = p^*(\bar{z})$ follows from applying Lemma 1, and using that $\hat{p}(z) \geq \hat{p}(\bar{z})$ and $\hat{p}(z) = \hat{p}(\bar{z})$ for all $z$. 43
Because \( LHS \rightarrow 0 \), the value of not searching is bounded. As a result, customers do not search so that firms do not face customer base concerns. Formally, \( \bar{p}(z) \rightarrow \infty \) for all \( z \in [z, \bar{z}] \). Because \( p^*(z) \) is finite for all \( z \in [z, \bar{z}] \), it follows immediately that \( p^*(z) < \bar{p}(z) \) for all \( z \in [z, \bar{z}] \). Then, using Lemma 1 we obtain that \( \hat{p}(z) = p^*(z) \) for all \( z \in [z, \bar{z}] \).

Part (2): From Proposition 1 we have that, in equilibrium, the highest price is \( \hat{p}(z) \). Moreover, under the assumptions of Proposition 1, the first order condition is a necessary condition for optimality of prices. We use this to show that, as \( n \) approaches zero, \( \hat{p}(z) \) has to approach \( \hat{p}(z) = p^*(z) \). In equilibrium, it is possible to rewrite equation (4), evaluated at \( \{\hat{p}(z), z\} \), as \( LHS(\hat{p}(z), n) = RHS(\hat{p}(z), n) \), where

\[
LHS(\hat{p}(z), n) \equiv G' \left( \frac{\hat{\psi}(\hat{p}(z), z)}{n} \right) \hat{\psi}_p(\hat{p}(z), z) + \left( G \left( \frac{\hat{\psi}(\hat{p}(z), z)}{n} \right) H'(V(\hat{p}(z), z)) + Q'(V(\hat{p}(z), z)) \right) \hat{V}_p(\hat{p}(z), z),
\]

\[
RHS(\hat{p}(z), n) \equiv -\frac{\pi_p(\hat{p}(z), z)}{\Pi(\hat{p}(z), z)} \left( 1 - G \left( \frac{\hat{\psi}(\hat{p}(z), z)}{n} \right) \right),
\]

given that \( H(V(\hat{p}(z), z)) = Q(V(\hat{p}(z), z)) = 0 \). Suppose that as \( n \downarrow 0 \), \( \hat{\psi}(\hat{p}(z), z) \) does not converge to zero. Then, \( G \left( \frac{\hat{\psi}(\hat{p}(z), z)}{n} \right) \uparrow 1 \) as \( n \downarrow 0 \). This implies that \( \lim_{n \downarrow 0} RHS(\hat{p}(z), n) > 0 \). Consider now the function \( LHS(\hat{p}(z), n) \). Again, suppose that as \( n \downarrow 0 \), \( \hat{\psi}(\hat{p}(z), z) \) does not converge to zero. Notice that the second term of the function approaches a finite number as \( \hat{V}_p(\hat{p}(z), z) \) is bounded by assumptions on \( v(p) \) and \( H'(V(\hat{p}(z), z)) \) and \( Q'(V(\hat{p}(z), z)) \) being bounded as a result of Proposition 1. Moreover, as long as \( \hat{p}(z) > \bar{p}(z) = p^*(\bar{z}) \), we have that \( \hat{\psi}_p(\hat{p}(z), z) > 0 \) so that \( \hat{\psi}_p(\hat{p}(z), z)/n \) diverges as \( n \) approaches zero. This means that \( G' \left( \frac{\hat{\psi}(\hat{p}(z), z)}{n} \right) \hat{\psi}_p(\hat{p}(z), z) \) is divergent, and therefore the first order condition cannot be satisfied.

This analysis concluded that, if \( \hat{\psi}(\hat{p}(z), z) \) does not converge to zero as \( n \) becomes arbitrarily small, the first order condition, i.e. equation (4), cannot be satisfied. This occurs because \( LHS(\hat{p}(z), n) \) would diverge to infinity, while \( RHS(\hat{p}(z), n) \) would remain finite. It then follows that, as \( n \) approaches zero, a necessary condition is that \( \hat{\psi}(\hat{p}(z), z) \) also approaches zero. This condition can be restated as requiring that \( \hat{p}(z) \) approaches \( \bar{p}(z) \) as \( n \) approaches zero. Moreover, given the assumptions of Proposition 1, \( \hat{p}(z) = \hat{p}(z) = p^*(\bar{z}) \).

In the end, if \( \hat{p}(z) \) approaches \( p^*(\bar{z}) \) as \( n \) becomes arbitrarily small (so that \( \hat{\psi}(\hat{p}(z), z) \rightarrow 0 \) and \( \hat{\psi}_p(\hat{p}(z), z) \rightarrow 0 \)), we have that \( \lim_{n \downarrow 0} LHS(\hat{p}(z), n) < \infty \) and \( \lim_{n \downarrow 0} RHS(\hat{p}(z), n) < \infty \) as \( \pi_p(p^*(\bar{z}), z) \) is bounded as \( \pi(p^*(\bar{z}), z) > 0 \). However, if \( \hat{p}(z) \) does not approach \( p^*(\bar{z}) \) as \( n \) becomes arbitrarily small, we have that \( LHS(\hat{p}(z), n) \) diverges as \( n \) approaches zero, while
LHS(\hat{p}(\tilde{z}), n) remains finite. As the first order condition has to be satisfied in equilibrium, a necessary condition is that, as \(n\) approaches zero, the highest price in the economy, i.e. \(\hat{p}(\tilde{z})\), has to approach the lowest price in the economy, i.e. \(p^*(\bar{z})\).

### D Numerical solution of the model

In order to solve the model, we start by setting the parameters. The parameters \(\beta, w, q,\) and \(I\) are constant throughout the numerical exercises. For the set of estimated parameters \(\Omega_n = [\lambda_n, \zeta_n, \rho_n, \sigma_n]^t\), we set a search grid. The grid is different for each parameter, as they differ both in their levels and in the sensitivity of the statistics of interest to their variation. We consider a grid with an interval of 0.01 for \(\sigma\), 0.05 for \(\rho\), 0.5 for \(\zeta\), and 0.01 for \(\lambda\). Each \(\Omega_n\) corresponds to a particular combination of parameters among these grids. For each \(\Omega_n\) we set \(\theta\) to obtain \(E[\varepsilon_d(z)] = 7\).

We next describe how we solve for the equilibrium of the model for a given combination of parameters. We start by discretizing the AR(1) process for productivity to a Markov chain featuring \(N = 25\) different productivity values. We then conjecture an equilibrium function \(\hat{p}(z)\). Given our definition of equilibrium and the results of Proposition 1, we look for equilibria where \(\hat{p}(z) \in [p^*(\bar{z}), p^*(\tilde{z})]\) for each \(z\), and \(\hat{p}(z)\) is decreasing in \(z\). Our initial guess for \(\hat{p}(z)\) is given by \(p^*(z)\) for all \(z\). We experiment with different initial guesses and found that the algorithm always converges to the same equilibrium.

Given the guess for \(\hat{p}(z)\), we can compute the continuation value of each customer as a function of the current price and productivity, i.e. \(\bar{V}(p, z)\), and solve for the optimal search and exit thresholds. Given \(\hat{p}(z)\) and the customers’ search and exit thresholds we can solve for the distributions of customers \(Q(\cdot)\) and \(H(\cdot)\) as defined in Definition 1. Notice that the latter also amounts to solve for a fixed point in the space of functions. Here, standard arguments for the existence of a solution to invariant distribution for Markov chains apply. Therefore, the assumption that \(F(z'|z) > 0\) and \(\Delta(\hat{p}(z), z) > 0\) ensure the existence of a unique \(K(z)\). Finally, given \(Q(\cdot), H(\cdot), \hat{p}(z)\) and \(\bar{V}(p, z)\), we solve the firm problem and the obtain optimal firm prices given by the function \(\hat{p}(z)\). We use \(\hat{p}(z)\) to update our conjecture about equilibrium prices \(\hat{p}(z)\), and iterate this procedure until convergence to a fixed point where \(\hat{p}(z) = \hat{p}(z)\) for all \(z \in [\underline{z}, \bar{z}]\).

Once we have solved for the equilibrium of the model at given parameter values, we construct the statistics to be matched to their data counterpart as follows.

- Log-price dispersion:
  \[
  \hat{\sigma}_p \equiv \sqrt{\sum_j K(z_j)(\log(\hat{p}(z_j)) - M_p)^2}
  \]
\[ M_p = \sum_j K(z_j) \log(\hat{p}(z_j)) \] and \( K(z_j) \) is the equilibrium fraction of customers buying from firms with productivity \( z_j \).

- Average comovement between the probability of exiting the customer base and the price:
  \[ \hat{b}_1 = \frac{\text{Cov}(E(z), \log(\hat{p}(z)))}{(\sigma_p)^2} \]
  where \( E(z) \equiv G(\hat{\psi}(\hat{p}(z), z))(1-H(\hat{V}(\hat{p}(z), z))) \), and \( \text{Cov}(E(z), \log(\hat{p}(z))) = \sum_i K(z_j)(\log(\hat{p}(z_j)) - M_p)(E(z_j) - M_E) \) and \( M_E = \sum_j K(z_j) E(z_j) \).

- Dispersion in the marginal effect of the price on the probability of exiting the customer base:
  \[ \hat{\sigma}_{b1} = \sqrt{\sum_j K(z_j)(\hat{b}_1(z_j) - M_{b1})^2} \]
  where \( \hat{b}_1(z_j) = G'(\hat{\psi}(\hat{p}(z), z))/G(\hat{\psi}(\hat{p}(z), z))(1-H(\hat{V}(\hat{p}(z), z))) \) and \( M_{b1} = \sum_i K(z_j) \hat{b}_1(z_j) \).

The autocorrelation of prices, \( \hat{\rho}_p \) coincides with the parameter \( \rho \) governing the persistence and autocorrelation of productivity. Thus the model-predicted statistics used to estimate the parameters are given by the vector \( v(\Omega_n) = [\hat{\rho}_p, \sigma_p, \hat{b}_1, \hat{\sigma}_{b1}]' \). We then evaluate the objective function \( (v_d - v(\Omega_n))'\Sigma(v_d - v(\Omega_n)) \) at each iteration. We assume the weighting matrix \( \Sigma \) to be the identity matrix. We select as estimates the parameter values from the proposed grid that minimize the objective function and check that the optimum in the interior of the assumed grid.

### E  A simple model of the labor market

In this appendix we provide details on how the model can be extended to use it to evaluate the role of aggregate shocks.

We assume that each household is divided into a mass one of shoppers/customers and a representative worker. The preferences of the household are given by

\[ E_t \left[ \int_0^1 V_t(p_t^i, z_t^i, \psi_t^i) \, di - J_t \right], \tag{13} \]

where \( V_t(p_t^i, z_t^i, \psi_t^i) \) is defined as in equation (6) and it is the value function that solves the customer problem in Section 2.2. We denote the disutility from the sequence of labor \( \ell_T \) as \( J_t \equiv \phi \sum_{T=t}^{\infty} \beta^{T-t} \ell_T \) with \( \phi > 0 \). The aggregate state for the household includes the distribution of prices, the distribution of customers over the different firms and the levels of income, the wage, and their laws of motion. Given that we allow for aggregate shocks, we
have to consider the possibility that the aggregate state varies over time. We index dynamics in the aggregate state through the time subscript \( t \) for the value function.

The worker chooses the path of \( \ell_t \) that maximizes household preferences in equation (13). The search problem of each customer is as described in Section 2.2. As for the consumption decision, each customer allocates her income across consumption of the good sold in the local market, the demand of which we denote by \( d \), and another supplied in a centralized market by a perfectly competitive firms, the demand of which we denote by \( n \), to solve the following problem

\[
v_t(p_t) = \max_{d, n} \left( \frac{d^{\theta-1} + n^{\theta-1}}{1 - \gamma} \right)^{1/(1-\gamma)}
\]

s.t. \( p_t d + q_t n \leq I_t \),

where \( \theta > 1 \) and \( I_t \equiv (w_t \ell_t + D_t) \) is nominal income, which the customer takes as given. Nominal income depends on the household labor income \((w_t \ell_t)\) and dividends from firms ownership \((D_t)\). The first order condition to the problem in equations (14)-(15) delivers the following standard downward sloping demand function for variety \( d \)

\[
d_t(p_t) = \frac{I_t}{P_t} \left( \frac{p_t}{P_t} \right)^{-\theta} .
\]

where \( P_t = ((p_t)^{1-\theta} + (q_t)^{1-\theta})^{1/(1-\theta)} \) is the price of the consumption basket. Without loss of generality we use the price \( q_t \) as the numeraire of the economy. From the first order conditions for the household problem, we obtain that the stochastic discount factor is given by \( \beta \Lambda_{t+1}/\Lambda_t \), where \( \Lambda_{t+s} = \int_0^1 (c_{i+s}^{1-\gamma}) / P_{t+s} \) is the household marginal increase in utility with respect to nominal income; \( c_{i+s}^{1-\gamma} \) denotes customer \( i \)’s consumption basket in period \( t+s \).

The production technology of the perfectly competitively sold good (good \( n \)) is linear in labor, so that its supply is given by \( y_t^n = Z_t \ell_t^n \), where \( Z_t \) is aggregate productivity, and \( \ell_t^n \) is labor demand by this firm. The production technology of the other good (good \( d \)) is also linear in labor, so that its supply is given by \( y_t^j = Z_t z_t^j \ell_t^j \), where \( Z_t \) is aggregate productivity, and \( \ell_t^j \) is labor demand by this firm, where \( j \) indexes one particular producer. Perfect competition in the market for variety \( n \) and in the labor market implies that workers are paid a wage equal to the marginal productivity of labor so that \( w_t = q_t Z_t \). Equilibrium in the labor markets requires \( \ell_t = \ell_t^n + \int_0^1 \ell_t^j \) dj.

There are two exogenous driving processes in our economy: aggregate productivity \( Z \) and the numeraire \( q \). We consider an economy in steady state at period \( t_0 \) where expectations are such that \( Z_t = 1 \) and \( q_t = 1 \) for all \( t \geq t_0 \). Notice that in this case the economy coincides
with the economy described in Section 2.
F Pass-through

To avoid inflating the short-term (weekly) pass-through due to the persistence of both price and cost variables, we include in the specification lagged values of the independent variable. We experiment with an alternative way to deal with the persistence of the dependent variable by measuring the short-term pass-through using first differences. Finally, we include time and market fixed effects to control for aggregate trend (e.g. demand shocks) that can move prices independently from cost shifts.

Table 5: Pass-through of idiosyncratic cost shocks

<table>
<thead>
<tr>
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<td>Dep. variable</td>
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<td>$\log(p_{jt})$</td>
<td>$\Delta \log(p_{jt})$</td>
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<td>0.24***</td>
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<td>$\log(cost_{j,t-1})$</td>
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<td>(0.04)</td>
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<tr>
<td>$\log(cost_{j,t-2})$</td>
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<td>0.02</td>
<td></td>
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<tr>
<td></td>
<td>(0.05)</td>
<td>(0.07)</td>
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<tr>
<td>$\log(cost_{j,t-3})$</td>
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<td>0.05*</td>
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<tr>
<td>$\log(cost_{j,t-4})$</td>
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<tr>
<td></td>
<td>(0.05)</td>
<td>(0.05)</td>
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</tr>
<tr>
<td>$\Delta \log(cost_{jt})$</td>
<td></td>
<td></td>
<td>0.13*</td>
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<tr>
<td>Time f.e.</td>
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<td>Yes</td>
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</table>

Notes: An observation is a store($j$)-week($t$) pair. The dependent variable is the price index of the store and the independent variables are the cost index of the store and its lags. Standard errors are in parenthesis and are clustered at the store level. ***: Significant at 1% **: Significant at 5% *: Significant at 10%.
The determinants of optimal prices

We take the equilibrium values of these components at our baseline estimates and replace them, one at the time, with the corresponding average across firms. For instance, in the left plot we report how the pricing function would look like if the intensive margin elasticity of demand were constant at $\tilde{\varepsilon}_d = E[\varepsilon_d(\hat{p}(z))]$. Then we use equation (9) to compute the counterfactual prices where the functions $\varepsilon_m(\cdot)$ and $\tilde{\pi}(\cdot)$ of the baseline economy. In the middle and right panels, we instead fix the customer value and the extensive margin elasticity to their cross-sectional averages respectively.
Figure 7: Determinants of optimal prices

**Notes:** In each panel we plot the optimal price function from our baseline model, and we contrast it with that from a counterfactual exercise. In the left panel, we fix the intensive margin elasticity ($\epsilon_d$) to a constant equal to its average across firms in the baseline economy. In the central panel, we fix the customer value ($\bar{\pi}$) to a constant equal to its average across firms in the baseline economy. In the right panel, we fix the extensive margin elasticity ($\epsilon_m$) to a constant equal to its average across firms in the baseline economy.