"Financing Investment with Long-Term Debt and Uncertainty Shocks"

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Motivation: Long-Term Debt

Recent literature on quantitative corporate finance (Hennessy and Whited (2005)) considers only **short-term** debt

Largely due to computational reasons!

This is **not** a costless simplification:

1. No agency costs: bondholders know investment and debt when they lend
2. Built-in maturity mismatch and hence rollover risk
3. Hard to generate large credit spreads

Main effects:

1. Reduces leverage (as in Leland and Toft (1996)), generates more default, and higher credit spreads
2. Amplifies response of investment to **changes in credit spreads**

Motivation: Uncertainty Shocks

Introduce uncertainty shocks (Bloom (2009)) to replicate empirical results on Q-theory:

1. Tobin's Q is a *sufficient statistic* for investment (Abel (1979) and Hayashi (1982))
2. Doesn't work well empirically
3. Models appeal to measurement error (Erickson and Whited (2001), Eberly et al. (2008))
4. **Q-theory works better with bond prices or credit spreads** (Gilchrist and Zakrajsek (2008), Philippon (2009))

Why Do Uncertainty Shocks Help?

Shock to Productivity

1. $\nearrow$ in **productivity** $\Rightarrow \searrow$ in the probability of default, $\searrow$ credit spreads
2. $\nearrow$ in **productivity** $\Rightarrow \nearrow$ in investment, $\nearrow$ in Q

Generates: $Corr(I/K, Q) > 0$, $Corr(I/K, spread) < 0$

Shock to Volatility

1. $\nearrow$ in **volatility** $\Rightarrow \nearrow$ in the probability of default, $\nearrow$ credit spreads
2. $\nearrow$ in **volatility** $\Rightarrow \searrow$ in investment, $\nearrow$ in Q (growth option value vs assets in place)

Generates: $Corr(I/K, Q) < 0$, $Corr(I/K, spread) < 0$

Contribution

This paper:

1. Extends a standard neoclassical model of financing and investment to incorporate **long-term debt** and **stochastic volatility**
2. Explores the **quantitative impacts** of these new ingredients in a calibrated model

Findings:

Long-term debt and stochastic volatility lead to:

1. Lower and more volatile leverage
2. Higher probability of default, and higher *credit spreads*
3. An increase in the explanatory power of *credit spreads* on i/k
4. A decrease in the explanatory power of Tobin's Q on i/k

(compared to model with one-period debt and deterministic volatility of profits)

Environment

This model builds on Gomes and Schmid (2009)

Model Ingredients:

- Dynamic, partial equilibrium, exogenous pricing kernel
- Financial decisions: debt and equity issuance, default
- Real decision: investment

Departure from literature:

- Shocks to volatility of productivity
- Long-term debt

Environment

Time:

- Time is discrete
- Problem is infinite horizon

Uncertainty:

- Aggregate Shocks: productivity z_a
- Idiosyncratic Shocks: productivity z_i
- Idiosyncratic Shocks: volatility σ

$\Rightarrow$ Tomorrow's shock z_i' has volatility σ

$\Rightarrow$ Shock σ today has an impact only on **tomorrow's** realizations of z_i

Exogenous State Vector: $s \equiv (z_a, z_i, \sigma)$

Firm Problem

Firms:

- Produce: $\pi(k, s)$, using capital k
- Invest in capital k
- Irreversible investment ($i \geq 0$), and linear adjustment cost ϕ_+ for $i > 0$
- Long-term (exponentially decaying) debt: stock b
- Issue equity: $d < 0$
- Default if equity $V < 0$
- Taxes: Profits –net of interest expenses– are taxed at rate τ

Equity Value:

Firms maximize the expected discounted stream of dividends

$$V(k, b, s) = \max_{k', b'} \quad d + \mathbb{E}\left[M(s, s') \max\left(0, V(k', b', s')\right)\right]$$

Firm Problem

Budget constraint:

$$\tilde{d} = \underbrace{(1-\tau)\pi(k,s)}_{After-Tax\ \ Profits} + \underbrace{\tilde{q}\ell}_{New\ \ Loan} - \underbrace{\delta_b b}_{Debt\ \ Repayment} - \underbrace{i}_{Investment} - \underbrace{\phi_+\ i}_{Cost\ \ of\ \ Investment}$$

Dividends or Equity Issuance:

$$d = \left(1 + \underbrace{\lambda\, \mathbf{1}_{\{\tilde{d}<0\}}}_{Issuance\ \ Cost}\right) \tilde{d}$$

New Loan: (Sells for price q)

$$\ell = b' - (1-\delta_b)b$$

Lender Problem

Lenders: (q = Price of a \$1 loan)

$$
\begin{aligned}
q_t \quad = \quad & \mathbb{E}_t\Bigg[M_{t,t+1}\Bigg(\delta_b\ \mathbf{1}_{t+1} + \xi\ \frac{k_{t+1}}{b_{t+1}}\ (1-\mathbf{1}_{t+1})\Bigg)\Bigg] \\
& + \mathbb{E}_t\Bigg[M_{t,t+2}\Bigg(\underbrace{\delta_b(1-\delta_b)}_{Coupon}\ \mathbf{1}_{t+2} + \underbrace{\xi\ k_{t+2}}_{Default\ \ Payoff}\ \underbrace{\frac{(1-\delta_b)}{b_{t+2}}}_{Claim}\ \underbrace{\mathbf{1}_{t+1}(1-\mathbf{1}_{t+2})}_{Default\ \ Event}\Bigg)\Bigg] \\
& + \ldots
\end{aligned}
$$

As an infinite sum:

$$
\begin{aligned}
q_t \quad = \quad & \sum_{s=1}^{\infty}\ \mathbb{E}_t\Bigg[M_{t,t+s}\Bigg(\delta_b(1-\delta_b)^{s-1}\ \mathbf{1}_{t+s}\Bigg)\Bigg] \\
& + \sum_{s=1}^{\infty}\ \mathbb{E}_t\Bigg[M_{t,t+s}\Bigg(\xi\frac{k_{t+s}}{b_{t+s}}\ (1-\delta_b)^{s-1}\ \mathbf{1}_{t+s-1}(1-\mathbf{1}_{t+s})\Bigg)\Bigg]
\end{aligned}
$$

Lender Problem

Recursive Formulation:

Given firms' policies, $(k', b') = (g_k(k,b,s), g_b(k,b,s))$, the loan price satisfies,

$$q(k',b',s) = \mathbb{E}\left[M(s,s')\left(\delta_b + (1-\delta_b)\, q(k'',b'',s')\right) \mathbf{1}_{\{V'\geq 0\}}\right] + \mathbb{E}\left[M(s,s')\,(1-\delta_b)\,\xi \frac{k'}{b'}\,(1-\mathbf{1}_{\{V'\geq 0\}})\right]$$

Price Schedule Inclusive of Tax Subsidy: $\tilde{q} = \tilde{q}(q;\tau)$

$$\tilde{q} = \sum_{t=1}^{\infty}\left(\frac{1}{1+(1-\tau)c(q)}\right)^{t}\delta_b(1-\delta_b)^{t-1} = \frac{1}{1+(1-\tau)(q^{-1}-1)}$$

Recursive Formulation of the Firm Problem

Recursive Formulation of the Firm Problem:

Given the loan price schedule $q(k', b', s)$, firms solve the following program,

$$V(k,b,s) = \max_{k',b'} \quad d + \mathbb{E}\left[M(s,s')\max\left(0, V(k',b',s')\right)\right],$$

subject to,

$$\begin{aligned} d &= \left(1+\lambda \mathbf{1}_{\{\tilde{d}<0\}}\right)\left\{(1-\tau)\pi(k,s) + \tilde{q}(k',b',s)\ell - \delta_b b - i(1+\phi_+)\right\} \\ i &= k' - (1-\delta_k)k \geq 0 \\ \ell &= b' - (1-\delta_b)b \end{aligned}$$

Recursive Equilibrium

Recursive Competitive Equilibrium:

A recursive competitive equilibrium consists of a loan price schedule $q(k', b', s)$, a value function $V(k, b, s)$, and optimal decision rules $g_{k'}(k, b, s)$ and $g_{b'}(k, b, s)$, such that

1. **Firms:** The value function $V(k, b, s)$ solves the firm problem. The associated optimal decision rules for the firm are denoted by $k' = g_{k'}(k, b, s)$ and $b' = g_{b'}(k, b, s)$
2. **Lenders:** The loan price schedule $q(k', b', s)$ satisfy the lenders Euler equation

Computational Considerations

Solving the Model:

1. Inner loop: Given bond prices, solve firm problem by VFI (with PFI)
2. Outer loop: Update bond prices given firm's decisions

Computational Issues:

Time-consuming given large number of states

Hard to achieve full convergence with long-term debt (bc non convex constraint set)

- Chatterjee and Eyigungor (2011) provide an algorithm that performs well
- We extended their algorithm to incorporate *endogenous investment*
- Makes computation even slower!

Algorithm

Transforming the model:

1. Add **small**, **continuous** i.i.d. shock to profits

   $m \sim \textit{truncated } \mathcal{N}(0, \sigma_m^2), \quad \text{with } \sigma_m = 0.04$

2. Add a **small** dividend smoothing motive: Firms maximize PDV of

   $h(d) = d - \kappa d^2, \quad \text{with } \kappa = 0.01$

Algorithm:

1. Requires **exact computation** of default thresholds
2. Use **very slow relaxation** for bond price updates,

   $q^{k+1} = \zeta q^k + (1 - \zeta) q^{new}, \quad \text{with } \zeta = 0.95$

Modified Firm Problem

Modified Firm Problem:

Given the loan price schedule $q(k', b', s)$, firms solve,

$$V(k, b, s) = \max_{k', b'} \quad h(d) + \mathbb{E}\left[M(s, s') \max\left(0, V(k', b', s')\right)\right],$$

subject to,

$$d \quad = \quad \left(1 + \lambda \mathbf{1}_{\{\tilde{d}<0\}}\right)\left\{(1-\tau)(\pi(k, s) + m) + \tilde{q}(k', b', s)\ell - \delta_b b - i(1+\phi_+)\right\}$$

where m is the i.i.d. cash flow shock

Numerical details

Practical implementation:

1. State Space: (k, b, z_a, z_i, σ) with (96*96*4*16*2) $= 1.2m$ grid points
2. Implementation: CUDA code run on NVIDIA Fermi card

   Typical run is ≈ 5 *hours* (**Speed up 500**x)

Monte Carlo Simulations:

1. Simulate a panel of 10,000 firms for 200 periods (drop first 5 periods)
2. Compute statistics/run regressions with simulated data

Calibration: Aggregate Exogenous States

Productivity Process: Follows an AR(1) process

$$\log z_a' = \rho_a \log z_a + \sigma_a \epsilon_a'$$

Discretized as a Markov Chain, with $\rho_a = 0.85$, $\sigma_a = 0.02$

Stochastic Discount factor:

$$M(z_a, z_a') = \beta e^{-\gamma_0 \left(\log z_a' - \rho_a \log z_a\right)}$$

Set $\gamma_0 = 15$

Note that $\mathbb{E}_{s'|s}[M(s, s')] = \beta$, so term structure is flat

Calibration: Idiosyncratic Exogenous States

Idiosyncratic Productivity Process: Follows an AR(1) process

$$\log z_i' = \rho_i \log z_i - \sigma^2/2 + \sigma \epsilon_i'$$

Discretized as a Markov Chain, with $\rho_i = 0.9$

Idiosyncratic Volatility Process: Follows a Markov chain with 2 states

$$\sigma \in \{\sigma_L, \sigma_H\}$$

Set $\sigma_L = 0.10$, $\sigma_H = 0.25$, with transition matrix $\Gamma_{\sigma\sigma'}$ given by

$$\Gamma = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$$

Calibration: Real Side

Parameters chosen to match means of the data: Tobin's Q, i/k, and π/k

Profits:

$$\pi(k, s) = z_a z_i k^{\alpha} - f$$

Set $\alpha = 0.4$, $f = 0.92$, $\delta_k = 0.14$

Adjustment Cost:

$$\phi(i, k) = \phi_+ \; i \qquad \text{for } i > 0$$

Set $\phi_+ = 0.05$

Parameters

	Parameter	Model	Description
Preference	β	0.98	Subjective discount rate
Technology	α	0.4	Production parameter
	ϕ_+	0.05	Cost of positive investment
	f	0.92	Fixed cost of operation
	δ_k	0.14	Capital depreciation rate
	δ_b	0.2	Exponential decay for debt
Institution	λ	0.25	Linear cost of issuing equity
	ξ	0.80	Recovery rate in bankruptcy
	τ	0.20	Average corporate tax rate
Uncertainty	ρ_a	0.85	Autocorrelation of z_a
	σ_a	0.02	Volatility of z_a
	ρ_i	0.90	Autocorrelation of z_i
	σ_L	0.10	Low Volatility of z_i
	σ_H	0.25	High Volatility of z_i

Definition: Variables

Real Policies:

Tobin's Q $$Q = \frac{V(k,b,s)+b'}{k'}$$

Investment Rate $$\frac{i}{k} = \frac{k'-(1-\delta_k)k}{k}$$

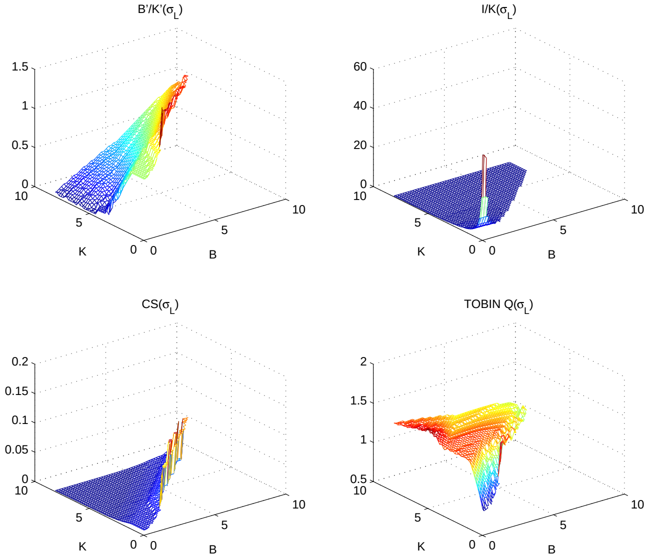
Profitability $$\frac{\pi}{k} = \frac{zk^{\alpha}-f+m}{k}$$

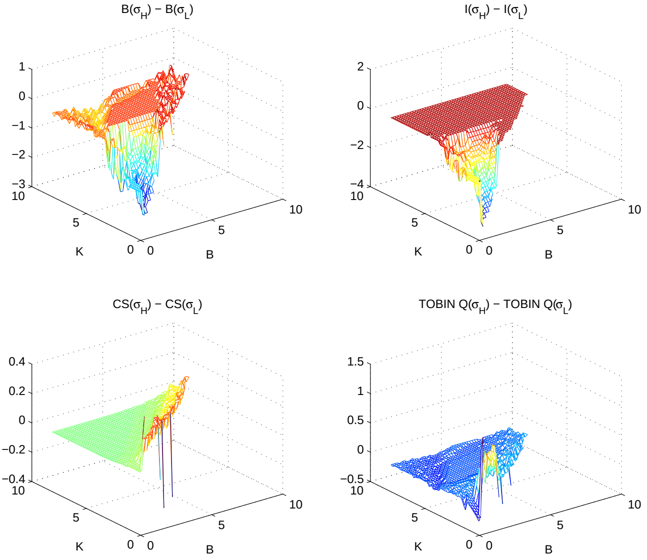
Financial Policies:

Leverage $$\frac{b'}{k'}$$

Credit Spreads $$CS = \delta_b \; q(k', b', s)^{-1} - \beta^{-1} + 1 - \delta_b$$

Default $$I^{DF} = \mathbf{1}_{\{V(k,b,s)<0\}}$$


B'/K'(σ_L)
1.5
1
0.5
0
10
5
0
K
0
5
10
B
I/K(σ_L)
60
40
20
0
CS(σ_L)
0.2
0.15
0.1
0.05
0
TOBIN Q(σ_L)
2
1.5
1
0.5


B(σ_H) − B(σ_L)
I(σ_H) − I(σ_L)
CS(σ_H) − CS(σ_L)
TOBIN Q(σ_H) − TOBIN Q(σ_L)
K
B

Simulation Results: Summary Statistics

Model Specification		Data	(4)
Debt			**5 period**
Volatility			**Stochastic**
Real Policies:			
Tobin's Q	$E(Q)$	1.30	2.51
	$\sigma(Q)$	0.63	0.55
Investment Rate	$E(i/k)$	0.15	0.15
	$\sigma(i/k)$	0.06	0.25
Profitability	$E(\pi/k)$	0.17	0.18
	$\sigma(\pi/k)$	0.08	0.18
Financing Policies:			
Leverage	$E(b/k)$	0.35	0.39
	$\sigma(b/k)$	0.09	0.30
Credit Spreads (%)	$E(c - R^f)$	1.09	1.26
	$\sigma(c - R^f)$	0.41	3.14
Default (%)	$E(I^{DF})$	0.40	1.02

Both Effects: Long-Term Debt + Stochastic Volatility

Model Specification		Data	(1)	(4)
Debt			**1 period**	**5 period**
Volatility			**Deterministic**	**Stochastic**
Real Policies:				
Tobin's Q	$E(Q)$	1.30	**2.61**	**2.51**
	$\sigma(Q)$	0.63	**0.36**	**0.55**
Investment Rate	$E(i/k)$	0.15	0.15	0.15
	$\sigma(i/k)$	0.06	0.19	0.25
Profitability	$E(\pi/k)$	0.17	0.17	0.18
	$\sigma(\pi/k)$	0.08	0.14	0.18
Financing Policies:				
Leverage	$E(b/k)$	0.35	**0.76**	**0.39**
	$\sigma(b/k)$	0.09	0.27	0.30
Credit Spreads (%)	$E(c - R^f)$	1.09	**0.008**	**1.26**
	$\sigma(c - R^f)$	0.41	0.03	3.13
Default (%)	$E(I^{DF})$	0.40	**0.007**	**1.02**

Both Effects: Long-Term Debt + Stochastic Volatility

Model Specification	(1)	(4)
Debt	**1 period**	**5 period**
Volatility	**Deter.**	**Stoch.**
Correlations:		
Corr(i/k,Tobin's Q)	0.31	0.36
Corr(i/k,Credit Spreads)	**-0.01**	**-0.17**

Effect of Stochastic Volatility

Model Specification		Data	(1)	(2)
Debt			1 period	1 period
Volatility			**Deter.**	**Stoch.**
Real Policies:				
Tobin's Q	$E(Q)$	1.30	**2.61**	**2.46**
	$\sigma(Q)$	0.63	**0.36**	**0.58**
Investment Rate	$E(i/k)$	0.15	0.15	0.15
	$\sigma(i/k)$	0.06	0.19	0.26
Profitability	$E(\pi/k)$	0.17	0.17	0.17
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Effect of Long-Term Debt

Model Specification		Data	(2)	(4)
Debt			**1 period**	**5 period**
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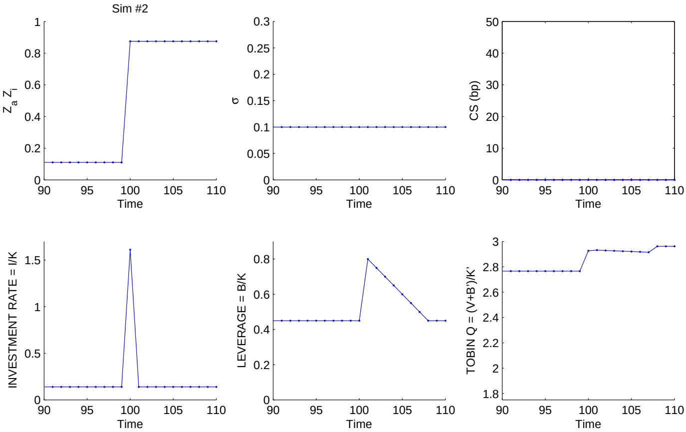
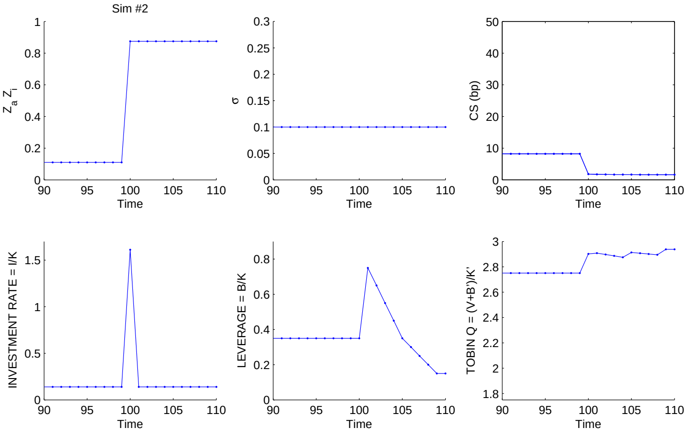
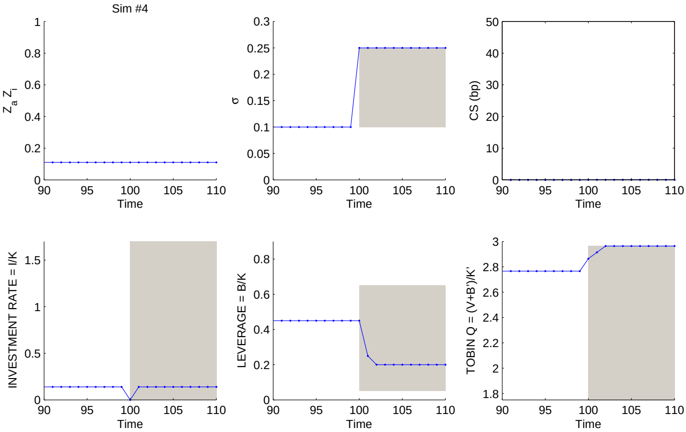
Impulse Response: *z* shock, 1 period debt



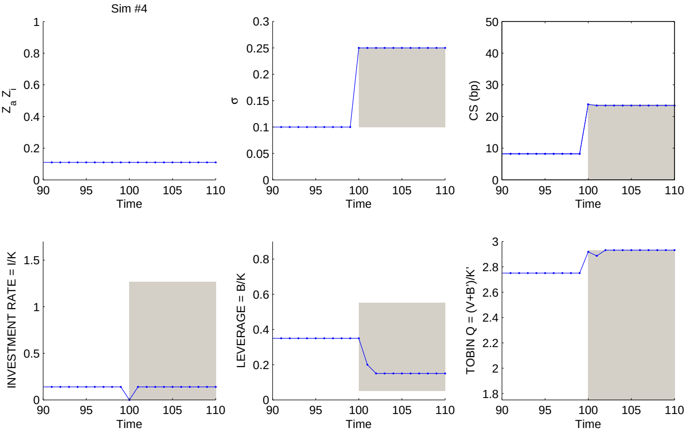
Impulse Response: *z* shock, 5 period debt



Impulse Response: σ shock, 1 period debt



Impulse Response: σ shock, 5 period debt



Using Regressions

Regression:

$$\left(\frac{i}{k}\right)_{jt} = \beta_0 + \beta_1 \log(c_{jt}) + \beta_2 \log(Q_{jt}) + \varepsilon_{jt}, \quad \text{for all firm } j \text{, and time } t$$

Data: (From Gilchrist and Zakrajsek)

Firm-level dataset on individual bond issues (period 1983-2006, 800 firms)

	$\log(c)$	$\log(Q)$	R^2
Data	-0.035		0.054
	(0.005)		
		0.051	0.064
		(0.016)	
	-0.034	0.002	0.062
	(0.005)	(0.002)	

Simulation Results: Regression results

Model Specification	log(c)	log(Q)	R^2
Data	-0.035		0.054
		0.051	0.064
	-0.034	0.002	0.062
(1) Deterministic σ	-0.105		0.000
1 period		0.362	0.088
	0.237	0.364	0.089
(2) Stochastic σ	-0.087		0.025
1 period		0.167	0.065
	0.044	0.207	0.068
(4) Stochastic σ	-0.108		0.041
5 period		0.222	0.086
	0.017	0.240	0.087

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Where is the Effect Stronger?

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5 period		0.222	0.086
	0.017	0.240	0.087
Far from default:	**0.304**	0.782	0.135
Close to default:	-**0.034**	0.098	0.092

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Far from default:	0.304	**0.782**	0.135
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Conclusion

We propose a neoclassical investment model with **stochastic volatility** in firms' productivity shocks and **long-term** defaultable debt

In our calibrated model, we find that these new ingredients:

1. Reduce the mean leverage, increase the probability of default
2. Increases the explanatory power of *credit spreads* on i/k
3. Decreases the explanatory power of Tobin's Q on i/k

Model extensions:

1. Experiment with idiosyncratic 'disaster' shocks (compare to stochastic volatility)
2. Use model to measure agency costs of debt (induced by multi-period maturity)

Questions.