Idiosyncratic risk, insurance, and aggregate consumption dynamics: a likelihood perspective

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Microeconomic evidence on insurance

- Consumption responds to idiosyncratic income changes.
- Consumption responds to anticipated income changes.
- Large literature on models of incomplete markets.
- Most incomplete markets models allow only self insurance.
- Some aspects of data point to households having more insurance (Blundell et al., 2008; Heathcote et al., 2012).
- Other aspects suggest households have less (Kaplan and Violante, 2011).
- How do these issues relate to the aggregate consumption data?

Incomplete markets and the aggregate data

- Incomplete markets models:
  - → Attractive micro-foundations given evidence above.
  - → But not in the standard toolkit of empirical macroeconomics.
- Representative agent models: formal interpretation of time series data.
  - → Many aggregate shocks give rich covariance structure.
  - → Judge the model on full range of empirical implications (An and Schorfheide, 2007).

This paper: take incomplete markets models to the data using same techniques as for rep. agent framework.

Results

- Standard incomplete markets model fits the data much better than representative agent model.

- Allowing for partial insurance against skill shocks leads to even better fit.

- Extending the model to match the response of consumption to fiscal stimulus payments does not improve fit.

Model: preferences and insurance

Unit mass of households with preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{i,t}^{1-\chi}}{1-\chi}.$$

Budget constraint:

$$\begin{aligned} a' + c &= y_t(e,s) + (1+r)a \\ a' &\geq 0. \end{aligned}$$

Income-pooling insurance scheme:

$$y_t(e,s) = \frac{[e + b^u(1-e)]s^{1-b^s}}{\int [e_{j,t} + b^u(1-e_{j,t})]s_{j,t}^{1-b^s}\,dj} \int e_{j,t} w_t s_{j,t}\,dj,$$

$b^s = b^u = 0$: no insurance $y_t(e,s) = w_t e s$.
$b^s = b^u = 1$: full insurance $y_t(e,s) =$ aggregate income.

Model: aggregate shocks

Aggregate wage process:

Log wage:	$\log w_t = z_t + A_t + \epsilon_t^T,$
Trend growth:	$z_t = z_{t-1} + g,$
Persistent shock:	$A_t = \rho^A A_{t-1} + \epsilon_t^A,$
Transitory shock:	$\epsilon_t^T.$

Aggregate employment conditions:

Job finding rate:	$\lambda_t = (1 - \rho^\lambda)\bar{\lambda} + \rho^q \lambda_{t-1} + \epsilon_t^\lambda.$
Job separation rate:	$\zeta_t = (1 - \rho^\zeta)\bar{\zeta} + \rho^\zeta \zeta_{t-1} + \epsilon_t^\zeta.$

All aggregate shocks are Gaussian with mean zero.

Model: idiosyncratic shocks

- Constant transition matrix across three skill levels.
- Income process calibrated to match (Domeij and Heathcote, 2004):
  - → autocorrelation and dispersion of wages in PSID
  - → realistic distribution of wealth: Gini and Lorenz(0.4).
- Unemployment risk correlated with skill: $\zeta_{t,s} = \zeta_t + \zeta^s$.
- Dispersion in unemployment risk calibrated using unemployment by education.

Methods: overview

- Solve the model using Reiter (2009) algorithm
  - → large-scale linear state-space representation of aggregate economy.
- Reduce model dimension using balanced truncation
  - → medium-scale linear state-space representation of aggregate economy.
- Proceed with standard techniques used on representative agent models:
  - → Kalman filter computes likelihood of data.
  - → Easily calculate moments, impulse responses, spectral density matrices.

Methods: solving the model

Solve the model using Reiter (2009) algorithm:

- discretize distribution of wealth with a histogram with many bins,
- discretize savings policy rules with splines with many knots,
- express equilibrium conditions as a system of equations ($> 3,600$ in all),

$$F(X_t, X_{t+1}, \eta_{t+1}, \epsilon_{t+1}) = 0,$$

- linearize around stationary equilibrium using automatic differentiation (normalized by trend, no aggregate shocks),
- solve linear rational expectations model with standard methods

$$X_{t+1} = \Psi_X X_t + \Psi_\epsilon \epsilon_{t+1}.$$

- X contains aggregate variables of interest: use an observation matrix, H, to select them.

Methods: reducing the model

Reduce model dimension using balanced truncation:

- most of X_t is not needed for calculating the dynamics of our objects of interest with high accuracy
  - $\rightarrow$ dimensions in which X_t varies little.
  - $\rightarrow$ dimensions in which variation has small effect on HX_t $\forall t$.
- Balanced truncation eliminates states that are not needed for these reasons.
- Large literature on reduction of linear systems (Antoulas, 2009).
- Explicit bounds on accuracy of reduced system.
- Steps above can be implemented easily with Matlab Control System Toolbox.

Methods: taking model to data

Proceed with standard techniques used on representative agent models:

- Likelihood function
  - → shape of the likelihood is the basis for maximum likelihood and Bayesian estimation.
  - → computed with the Kalman filter.
- Watson's (1993) measure of fit
  - → find the smallest measurement error to reconcile model and data autocovariances
  - → report measurement error variance relative to data variance
  - → similar to $1 - R^2$ from linear regression
  - → computed frequency by frequency from spectral density matrices.

Methods: advantages and disadvantages

Advantages:

- Reiter method easily extends to many aggregate states.
  - → Allows for many persistent aggregate shocks as is common in empirical DSGEs.
- Reiter method easily extends to rich aggregate features.
  - → General equilibrium.
  - → Nominal rigidities (McKay and Reis, 2013).
- Resulting linear state-space representation facilitates statistical analysis.

Disadvantages:

- Solution may not be accurate if shocks move the economy far from steady state.
- Will discuss accuracy checks after results.

Data

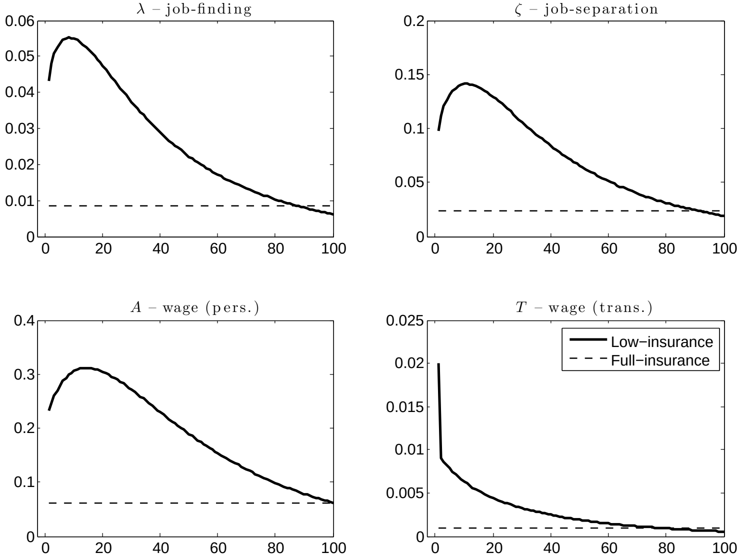
Aggregate data from 1966:I to 2012:III:

- consumption of non-durables and services,
- labor income net of taxes and government transfers,
- a measure of short-term unemployment,
- a measure of long-term unemployment.

Consumption and income are real, per capita, $100 \times \Delta \log(\cdot)$.

Symbol	Parameter	Value	Target/Prior
Panel A. Objects of interest			
b^u	Unemployment insurance	0.3	
b^s	Skill insurance	0	
Panel B. Calibrated for each specification			
β	Discount factor	0.971	Aggregate assets 5$\times$ annual income.
Panel C. Calibrated on balanced growth path			
χ	Risk aversion	2	
r	Interest rate	0.0075	3% annual interest rate.
$\bar{\lambda}$	Avg. job finding rate	0.679	Mean long-term unemployment.
$\bar{\zeta}$	Avg. high-skill job separation rate	0.037	Mean short-term unemployment.
Panel D. Estimated driving processes			
g	Trend income growth	0.004	Uniform[0,1].
ρ^A	Autoregressive coefficient of A	0.951	Beta: mn. = 0.5, var. = 0.04
ρ^λ	Autoregressive coefficient of λ	0.920	Beta: mn. = 0.5, var. = 0.04
ρ^ζ	Autoregressive coefficient of ζ	0.924	Beta: mn. = 0.5, var. = 0.04
σ^A	Standard deviation of ϵ^A	1.040	Inverse Gamma: mn. = 1, var. = 4
σ^λ	Standard deviation of ϵ^λ	2.591	Inverse Gamma: mn. = 1, var. = 4
σ^ζ	Standard deviation of ϵ^ζ	0.432	Inverse Gamma: mn. = 1, var. = 4
σ^T	Standard deviation of ϵ^T	0.290	Inverse Gamma: mn. = 1, var. = 4

TABLE: Parameter values, targets and priors for the low-insurance economy.



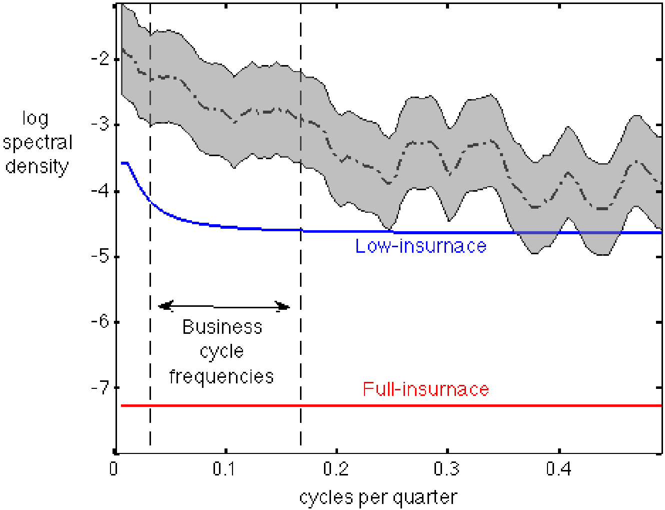
Notes: 100× log change in response to one standard deviation shock. The plot for ζ shows a negative shock to ζ.

A. Standard deviation	ΔC_t	ΔY_t	u_t^{short}	u_t^{long}
Data	0.535	1.029	0.921	1.143
Low-insurance	0.262	1.231	0.939	0.948
Full-insurance	0.066	1.231	0.939	0.948

B. Correlation of ΔC_t with	ΔY_t	u^{short}	u^{long}
Data	0.271	-0.339	0.064
Low-insurance	0.800	-0.038	-0.012
Full-insurance	0.789	0.001	-0.001

C. Autocorrelation of ΔC_t Lags	1	2	3	4
Data	0.407	0.199	0.130	0.062
Low-insurance	0.096	0.083	0.073	0.066
Full-insurance	-0.001	0.000	0.001	0.005

Model and data spectral densities



Watson's measure of fit

Ratio of residual variance to data variance:

	ΔC_t	ΔY_t	u_t^{short}	u_t^{long}
Low-insurance	0.591	0.127	0.225	0.266
Full-insurance	0.882	0.118	0.218	0.264

Likelihood of the data: stochastic singularity

- Stochastic singularity occurs when the model-implied covariance matrix for observables is singular.
- Not obviously the case here because four shocks and four observables.
- But no shock directly explains independent movements in consumption growth.
- Add i.i.d. measurement error to consumption growth.

Likelihood of the data

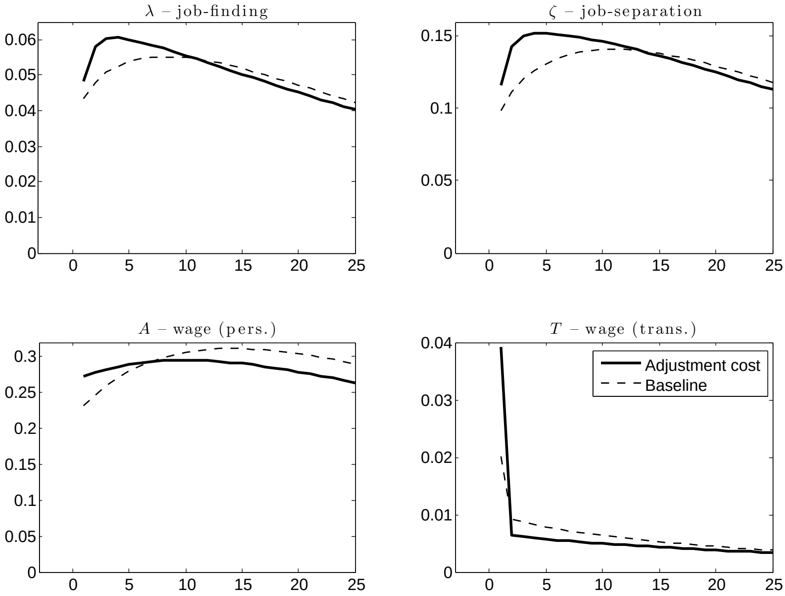
σ^v	Low-insurance	Full-insurance	Difference
0.1	-2324.1	-2639.9	315.8
0.2	-875.6	-894.7	19.1
0.3	-612.7	-614.8	2.1
0.4	-543.1	-542.7	-0.4
0.5	-526.8	-526.1	-0.7
0.52714	-526.3		
0.52722		-525.6	

Partial insurance

		Watson's measure of fit				Std. dev.	$\log \mathcal{L}$
b^s	b^u	ΔC_t	ΔY_t	u_t^{short}	u_t^{long}	ΔC_t	$(\sigma^v = 0.1)$
0	0.3	0.591	0.127	0.225	0.266	0.262	-2324
0	0.6	0.606	0.127	0.224	0.266	0.258	-2335
0.5	0.3	0.548	0.128	0.227	0.265	0.305	-2180
0.5	0.6	0.574	0.128	0.226	0.265	0.297	-2230
0.9	0.3	0.761	0.120	0.220	0.265	0.133	-2474
0.9	0.6	0.783	0.120	0.220	0.265	0.124	-2515
1	1	0.882	0.118	0.218	0.264	0.066	-2640

Matching evidence on response to fiscal stimulus payments

- Kaplan and Violante (2011) criticize the standard incomplete markets model for failing to match the way consumption responds to fiscal stimulus payments.
- Their solution: illiquid assets are less useful for smoothing consumption.
- Incorporate their idea with a quadratic adjustment cost on household asset positions.
- Calibrate the adjustment cost to match regression estimates from Johnson et al. (2006) in simulated data.
    - $\rightarrow$ Rebate coefficient = 0.25; Johnson et al. find 0.2 to 0.4.
    - $\rightarrow$ MPC out of unanticipated transitory income fluctuations = 0.20.


λ – job-finding
ζ – job-separation
A – wage (pers.)
T – wage (trans.)
Adjustment cost
Baseline
0.06
0.05
0.04
0.03
0.02
0.01
0
0.15
0.1
0.05
0.3
0.25
0.2
0.15
0.1
0.05
0.04
0.03
0.02
0.01
0
5
10
15
20
25

Matching evidence on response to fiscal stimulus payments

	Watson's measure of fit				Std. dev.	$\log \mathcal{L}$
	ΔC_t	ΔY_t	u_t^{short}	u_t^{long}	ΔC_t	$(\sigma^v = 0.1)$
Baseline low-insurance	0.591	0.127	0.225	0.266	0.262	-2324
Asset adjustment cost	0.589	0.130	0.227	0.266	0.309	-2466

Log-likelihood disagrees with other metrics here.

Accuracy

- Compare solution from Reiter method (with and without model reduction) to fully non-linear solution.
- To apply standard non-linear methods:
  - → assume λ_t and ζ_t are perfectly negatively correlated,
  - → ignore transitory wage shock,
  - → simplify the income process.
- Approximate aggregate shocks with Rouwenhorst (1995) algorithm.
- Find policy rules with endogenous grid method (Carroll, 2006).
- Simulate all three solutions with same shock sequence.

A. Mean relative to trend (×100)	ΔC_t	ΔY_t	u_t^{short}	u_t^{long}
Non-linear	0.000	-0.001	6.293	3.180
Reiter	0.000	-0.001	6.325	3.003
Reiter-reduced	0.000	-0.001	6.325	3.003

B. Standard deviation (×100)	ΔC_t	ΔY_t	u_t^{short}	u_t^{long}
Non-linear	0.233	1.229	0.867	1.312
Reiter	0.263	1.223	0.869	1.272
Reiter-reduced	0.263	1.223	0.869	1.272

C. Correlation of ΔC_t with	ΔY_t	u^{short}	u^{long}
Non-linear	0.764	-0.059	-0.050
Reiter	0.773	-0.037	-0.028
Reiter-reduced	0.773	-0.037	-0.028

D. First-order autocorrelation	ΔC_t	ΔY_t	u_t^{short}	u_t^{long}
Non-linear	0.133	0.033	0.893	0.968
Reiter	0.096	0.028	0.893	0.969
Reiter-reduced	0.096	0.028	0.893	0.969

Summary

- Full-information analysis of incomplete markets models now possible.
- Incomplete markets fit data much better than complete markets.
- Partial insurance against skill shocks fit the aggregate data best as has found in panel data.
- Micro evidence on consumption response to transitory income shocks need not invalidate standard incomplete markets model as a model of consumption dynamics in general
  - → but this evidence is important for how aggregate consumption responds to transitory shocks.

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