

# The Fiscal Multiplier\*

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## Abstract

We measure the size of the fiscal multiplier using a heterogeneous-agent model with incomplete markets, capital, and rigid prices and wages. The environment encompasses the essential elements necessary for a quantitative analysis of fiscal policy. First, output is partially demand-determined due to pricing frictions in product and labor markets, so that a fiscal stimulus increases aggregate demand. Second, incomplete markets deliver a realistic distribution of dynamic consumption and investment responses to stimulus policies across the population. These elements give rise to the standard textbook Keynesian-cross logic, which, unlike what conventional wisdom would suggest, is significantly reinforced in our dynamic forward-looking model.

The analysis reveals that market incompleteness is key to determining the size of the fiscal multiplier, which is uniquely determined in our model for any combination of fiscal and monetary policies of interest. The main findings are as follows. First, the impact multiplier strongly depends on the type of financing and is 1.34 if deficit-financed and 0.61 if contemporaneously tax-financed for a pegged nominal interest rate. The cumulative multiplier is close to 0.5, implying strong crowding out for both financing schemes. Second, the size of the multiplier remains similar in a liquidity trap, implying very little state-dependence. Third, the multiplier is strongly affected by the monetary policy response, e.g., using a Taylor rule, the impact multipliers drop to 0.66 and 0.54, respectively. We elucidate the importance of market incompleteness for our results and contrast them to models featuring complete markets or hand-to-mouth consumers.

**Keywords:** Fiscal Multiplier, Incomplete Markets, Sticky Prices, Liquidity Trap

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# 1 Introduction

In an attempt to stabilize the economy during the Great Recession, monetary authorities lowered nominal interest rates to nearly zero and fixed them at that level for several years. Having reached the limit of traditional monetary policy, U.S. legislators stepped in with the largest fiscal stimulus since the 1930s. Although attempts to stabilize the economy through spending occur in virtually every recession, the questions of how much and through which channels an increase in government spending affects output, employment and investment are far from being answered.

The traditional logic describing the effects of these policies is well known. A government spending stimulus increases aggregate demand, which leads to higher labor demand and thus higher employment and wages. Higher labor income then stimulates consumption, in particular of poor households, which leads to even higher aggregate demand, and, thus, higher employment, higher income, more consumption and so on. The equilibrium impact of an initial government spending of \$1 on output - the fiscal multiplier - is then the sum of the impacts of the initial increase in government spending and of the induced private consumption response.

This simple argument is based on two essential elements. The first element is that output is demand determined, which ensures that the increase in government spending stimulates aggregate demand in the first place. The typical underlying assumption is that prices are rigid so that firms adjust quantities and not only prices in response to increased government demand. Firms increase production to satisfy this demand by raising employment, capital, and wages, which leads to higher household income. The second essential element of the argument is a significant deviation from the permanent income hypothesis, such that households have a high marginal propensity to consume (MPC) out of the transitory increase in income induced by the stimulus. Higher private consumption directly induced by government spending then leads to further increase in labor demand, income, consumption and so on.

A quantitative assessment of a stimulus policy requires both elements to be disciplined by the empirical behavior of households and firms. This requires a model that, first, features the right amount of nominal rigidities, so that the aggregate demand channel is as in the data. Second, it requires incorporating observed marginal propensities to consume that imply a substantial deviation from the permanent income hypothesis. However, this is not the path

traditionally followed in the literature. Instead, one strand of the literature (e.g., Baxter and King, 1993; Heathcote, 2005; Brinca et al., 2016; Ferriere and Navarro, 2018) assumes flexible prices. This framework is limited in its ability to provide a full assessment as only the supply but not the demand channel is operative—that is, the first essential element is not present. Accordingly, this strand has been criticized for building in strong crowding out of private consumption by assumption. Another strand of the literature uses Representative Agent New Keynesian (RANK) models with sticky prices and wages e.g., Christiano et al. (2011). Nominal rigidities provide a role for the demand channel but now the second essential element is missing because in these models households are assumed to be representative agents who behave exactly like permanent-income ones and respond little to a temporary income shock. This stands in the face of the findings of a large empirical literature that has documented substantial MPC heterogeneity and large consumption responses to transitory income and transfer payments.

In this paper we measure the size of the fiscal multiplier in a dynamic equilibrium model featuring these two elements disciplined by the observed behavior in micro data. Specifically, we extend the standard Bewley-Imrohorglu-Huggett-Aiyagari model to include New-Keynesian style nominal price and wage rigidities.<sup>1</sup> Introducing incomplete markets allows the model to match the rich joint distribution of income, earnings and wealth. Such heterogeneity is crucial in generating a realistic distribution of MPCs and, more generally, for assessing the effects of policies that induce redistribution. The nominal rigidities allow for the model to have a meaningful demand channel operating. In addition, the government budget constraint in the model is specified, as in the data, in nominal terms. This lets us exploit the result in Hagedorn (2016, 2018) that incomplete market models with (partially) nominal fiscal policy imply price level determinacy. This allows us to measure the size of the fiscal multiplier for arbitrary combinations of monetary and fiscal policies of practical interest, including a constant nominal interest rate, without having to impose restrictions on policy rules to ensure

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<sup>1</sup>See Kaplan and Violante (2018) for a recent review of this emerging literature combining incomplete-markets models with nominal rigidities. Additional references include, among others, Oh and Reis (2012), Guerrieri and Lorenzoni (2017), Gornemann et al. (2012), Kaplan et al. (2018), Auclert (2016), Lütticke (2015), McKay et al. (2016), Bayer et al. (2019), Ravn and Sterk (2017), Den Haan et al. (2017), Bhandari et al. (2017), Auclert and Rognlie (2017), Hagedorn et al. (2019a) and Hagedorn et al. (2019b). Dupor et al. (2018) build on our insights to quantify the size of the fiscal multiplier in the Great Recession in the US exploiting cross-regional evidence and a multi-region HANK model.

determinacy.

To assess the effects of fiscal policies, it is also necessary to include capital accumulation into the model. A fiscal stimulus affects demand for both consumption and investment, and the two channels interact significantly in equilibrium. Moreover, if the financing of fiscal spending involves changes in government debt, it may imply a crowding out of investment, the consequences of which need to be taken into account in policy evaluation. Indeed, crowding out of investment has been found to be particularly important (Barro and Redlick, 2011). Of course, the equilibrium response of investment depends on the elasticity of interest rates and wages as those prices determine firms' substitution between labor and capital. This is why we extend previous work on Heterogeneous Agent New Keynesian (HANK) models and introduce wage rigidities—to match the empirical properties of wages in the data. This allows the model capture the correct response of investment, the dynamics of labor incomes necessary to reproduce the dynamics of demand, and the response of firms' profits.

Importantly, households in our model respond to both current and future income changes, making the logic underlying the size of the multiplier dynamic. An increase in fiscal spending leads to higher current income, part of which is saved to be spent in the future, which raises private consumption demand not only today but also in all subsequent periods. This higher path of demand leads to a higher income path, relaxing precautionary savings motives and allowing households to consume even more today. Indeed, we find that future (rather than current) income increases drive the contemporaneous consumption response. Moreover, agents in our model anticipate and act on their expectations of future policies, e.g. how and when the stimulus will be repaid, enabling us to evaluate the entire policy path of interest.<sup>2</sup>

After calibrating the model, we first establish that it closely replicates the evidence on households' consumption responses to current and future income transfers as well as the size of the fiscal multiplier measured in specific experiments in Ramey and Zubairy (2018). To replicate these experiments in the model we must also measure in the data and incorporate in

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<sup>2</sup>This is in contrast to the theoretically elegant Two Agent New Keynesian (TANK) model where one fraction of households is hand-to-mouth and the other fraction behaves according to the permanent income hypothesis (Bilbiie, 2008, 2017). Models in this class can feature a high MPC and the demand channel, but they exhibit the same determinacy problem as the RANK model and lack propagation and anticipation effects. In particular, a TANK model delivers a much smaller multiplier even for the same MPC - relating current income to current consumption - than our model. The reason is that hand-to-mouth consumers spend the full increase in current income but do not respond to increases in future income.

the model the fiscal and monetary policy responses to the stimulus. This is crucial because “the fiscal multiplier” is not a single number – its size crucially depends on how it is financed (debt, distortionary taxation, reduction of transfers), how monetary policy responds, what the state of the economy is, what households and firms expect about future policy changes, and whether spending is increased or transfers are directed to low-income households. These important details are difficult to control for in empirical studies (see Ramey, 2011, 2019, for surveys). As a consequence, these studies can reveal the consequences of a particular configuration of policies but are less informative on the implications of alternative combinations of policy choices implemented given specific economic conditions.

Having established the soundness of the quantitative laboratory that we developed, we proceed to fill some gaps where the empirical literature to date is less informative. Specifically, we quantify the size of the multiplier associated with the changes in government spending or transfers for different financing schemes, for different monetary policy responses, and in different economic states, such as the liquidity trap. We also compare our findings to the implications of key alternative models in the literature, in particular, the standard model with complete markets.<sup>3</sup> Even qualitatively the comparison is ex-ante unclear. On the one hand, the theoretical findings in Hagedorn (2016, 2018) suggest that the response of real interest rate to changes in government spending is smaller in incomplete markets models. Kaplan et al. (2018) show that a given change in real interest rate has smaller effects in incomplete markets models. This suggests that the intertemporal substitution channel is weaker in our model, pointing to a smaller multiplier than if markets were complete. On the other hand, government spending and other policies induces changes in prices, incomes, taxes, etc. which redistribute resources across households. This redistributive channel is larger in incomplete markets models (as it is absent in complete markets), suggesting that the multiplier could be larger than in complete market models. In addition, these two channels interact in general equilibrium as

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<sup>3</sup>In contemporaneous and complementary work, Auclert et al. (2018) also study the size of the multiplier in a heterogeneous-agent model with nominal rigidities. They characterize theoretically the constant-real-rate multiplier in a version of the model with no redistribution across households. Further, they develop a general equilibrium sufficient-statistics approach based on a matrix of “iMPCs” - intertemporal marginal propensities to consume. In contrast to our paper, their approach requires more restrictive assumptions on monetary and fiscal policy, and focuses on first-order deviations from steady-state, whereas we study a broader class of monetary and fiscal policies and investigate the importance of state dependence in a liquidity trap. However, our broad conclusions regarding the effect of different forms on financing on the size of the multiplier and the differences between heterogeneous-agent and two-agent models are congruous.

changes in the real interest rate have distributional effects, and redistribution itself affects the equilibrium real interest rate. In terms of investment, the incompleteness of markets implies that Ricardian equivalence does not hold such that a deficit-financed government spending increase crowds-out private investment, as in the framework of Barro and Redlick (2011). At the same time higher aggregate demand leads to higher investment—a channel absent in the neoclassical setting described in Barro and King (1984) that guides the analysis in Barro and Redlick (2011). We use our model to inform us on the magnitude of the two effects.

To understand the empirical response of monetary policy to a fiscal stimulus, we use the data and methodology in Ramey and Zubairy (2018) and find that the nominal interest rate in the data responds neither to military spending announcements nor to the government spending shocks of Blanchard and Perotti (2002). Armed with this evidence, we assume in our benchmark experiments that monetary policy does not respond to a fiscal stimulus and keep the nominal interest fixed. Our main quantitative findings are as follows. First, the impact multiplier of an increase in government spending is equal to 0.61 if spending is financed by a contemporaneous increase in taxes and 1.34 if it is deficit-financed. The multiplier is high on impact but dies out quite quickly so that the cumulative multiplier, which is the discounted average multiplier over time, falls to 0.43 and 0.55 for tax- and deficit-financed spending, respectively, implying that the magnitude of crowding out is sizeable and basically independent of the type of financing. Second, if we consider instead monetary policy described by an interest rate feedback rule, a fiscal stimulus becomes considerably less effective, with tax- and deficit-financed impact multipliers falling to 0.54 and 0.66, respectively. Third, we assess the size of the fiscal multiplier in a liquidity trap, a question that has received renewed interest in the continuing aftermath of the Great Recession. We engineer a liquidity trap in the model where the natural real interest rate falls below zero and is consistent with salient aggregate dynamics during the Great Recession. The results from the benchmark analysis are relatively little changed. For example, the impact multiplier is now about 0.73 for tax-financed and 1.39 for deficit-financed spending, implying that the multiplier is only mildly state-dependent in our non-linear model for the identical response of monetary policy (i.e., a constant nominal interest rate). If monetary policy in normal times is described by a Taylor rule, the multiplier in a liquidity trap is substantially larger, but this is a consequence of a different monetary policy response, not state dependence.

Two stark differences to the complete markets case warrant an explanation. First, the size of the multiplier depends on how it is financed, specifically deficit-financed spending is more effective in stimulating the economy than tax-financed, which is not surprising in models where Ricardian equivalence is violated. Increasing spending and taxes at the same time first stimulates demand but then offsets it through raising taxes that particularly affects high-MPC households. In contrast, with deficit financing, the newly issued debt is mainly bought by low-MPC households whereas high-MPC households consume a large fraction of their additional income. Deficit financing thus implicitly redistributes from asset-rich households with low MPCs who finance their consumption more from asset income to low-asset households with high MPCs who rely more on labor income so that the aggregate MPC increases. The effect is partially offset due to the crowding out of investment by the expansion in government debt.

Second, the reason why we find smaller, when compared to complete markets, multipliers in a liquidity trap is that the response of the real interest rate is smaller in our model.<sup>4</sup> As a result, the magnitude of the intertemporal channel is also smaller while the sizes of both intertemporal substitution and redistribution channels are similar in and outside of the liquidity trap. Our decomposition reveals that the intertemporal channel contributes 0.97 (1.1) and the redistributive channel contributes 0.37 (−0.5) to the deficit- (tax-) financed multiplier of 1.34 (0.61), with a similar decomposition in a liquidity trap.

Our quantitative analysis shows that the multiplier puzzles that have been documented for RANK models do not carry over to our model.<sup>5</sup> In RANK models, the multiplier increases if prices become more flexible and is unbounded when price rigidities vanish implying a discontinuity at fully flexible prices where the multiplier is smaller than one. The reason is that the inflation response is larger when prices are more flexible and that the private consumption response is one-to-one related to the inflation rate in RANK models where only the intertemporal substitution channel is operating. In contrast, we find that the multiplier in a liquidity trap becomes smaller as prices become more flexible and that the discontinuity at

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<sup>4</sup>Standard RANK models are linear and do not feature state dependence of the size of the multiplier. Instead, the large differences in the size of the multipliers in and outside a liquidity trap arise only because of different monetary policies.

<sup>5</sup>Thus, our model overcomes a Catch-22 of HANK models with fiscal policy fully specified in real terms. As Bilbiie (2018) shows, some calibrations of these models can resolve the puzzles but then the multiplier is small. Other calibrations can generate large multipliers but then the puzzles are even aggravated, a Catch-22. In contrast, government bonds are nominal in our HANK model, which allows us to resolve the puzzles and at the same time multipliers can be, depending on the distributional channel and the type of financing, large.

fully flexible prices disappears. The muted intertemporal substitution channel in combination with different responses of inflation and real interest rates explains this finding. In particular, we do not find large deflations as observed in liquidity traps in RANK models.

The paper is organized as follows. Section 2 presents our incomplete markets model. In Section 3 we calibrate the model economy and contrast its implications to the data. In Section 4 we quantitatively study the size and properties of the fiscal multiplier for various combinations of fiscal and monetary policies in and outside the liquidity trap. Section 5 concludes.

## 2 Model

The model is a standard New Keynesian model with capital with one important modification: Markets are incomplete as in Huggett (1993); İmrohoroğlu (1989, 1992); Aiyagari (1994, 1995). Price setting is constrained by costly prices adjustments as in Rotemberg (1982) leading to price rigidities. As is standard in the New Keynesian literature, final output is produced in several intermediate steps. Final good producers combine the intermediate goods to produce and sell their output in a competitive goods market. Intermediate goods producers are monopolistically competitive. They set the price they charge to the final good producer to maximize profits taking into account the price adjustment costs they face. The intermediate goods producer rent inputs, capital and a composite of differentiated labor, in competitive factor markets. We also allow for sticky wages and assume that differentiated labor is monopolistically supplied as well.

### 2.1 Households

The economy consists of a continuum of agents normalized to measure 1 who are ex-ante heterogeneous with respect to their subjective discount factors, have CRRA preferences over consumption and additively separable preferences for leisure:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t),$$

where

$$u(c, h) = \begin{cases} \frac{c^{1-\sigma}-1}{1-\sigma} - v(h) & \text{if } \sigma \neq 1 \\ \log(c) - v(h) & \text{if } \sigma = 1, \end{cases}$$



$\beta \in (0, 1)$  is the household-specific subjective discount factor and  $v(h)$  is the disutility of labor.

Agents' labor productivity  $\{s_t\}_{t=0}^{\infty}$  is stochastic and is characterized by an  $N$ -state Markov chain that can take on values  $s_t \in \mathcal{S} = \{s_1, \dots, s_N\}$  with transition probability characterized by  $\gamma(s_{t+1}|s_t)$  and  $\int s = 1$ . Agents rent their labor services,  $h_t s_t$ , to firms for a real wage  $w_t$  and their nominal assets  $a_t$  to the asset market for a nominal rent  $i_t^a$  and a real return  $(1 + r_t^a) = \frac{1+i_t^a}{1+\pi_t}$ , where  $1 + \pi_t = \frac{P_t}{P_{t-1}}$  is the inflation rate ( $P_t$  is the price of the final good). The nominal return on bonds is  $i_t$  with a real return  $(1 + r_t) = \frac{1+i_t}{1+\pi_t}$ . There are two classes of assets, bonds and capital with potentially different returns, but households can invest in one asset,  $A$ , that a mutual fund (described below) collects and allocates to bonds and capital.

To allow for sticky wages in our heterogeneous worker economy we need to extend the literature which models wage rigidities in representative agent models (Erceg et al., 2000). We follow the representative agent literature and assume that each household  $i$  provides differentiated labor services which are transformed by a representative, competitive labor recruiting firm into an aggregate effective labor input,  $H_t$ , using the following technology:

$$H_t = \left( \int_0^1 s_{it} (h_{it})^{\frac{\epsilon_w - 1}{\epsilon_w}} di \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \quad (1)$$

where  $\epsilon_w$  is the elasticity of substitution across differentiated labor.

A middleman firm (e.g. a union) sells households labor services  $s_{it} h_{it}$  at the wage  $W_{it}$  to the labor recruiter, which, given aggregate labor demand  $H_t$  by the intermediate goods sector, minimizes costs

$$\int_0^1 W_{it} s_{it} h_{it} di, \quad (2)$$

implying a demand for the labor services of household  $i$ :

$$h_{it} = h(W_{it}; W_t, H_t) = \left( \frac{W_{it}}{W_t} \right)^{-\epsilon_w} H_t, \quad (3)$$

where  $W_t$  is the (equilibrium) nominal wage which can be expressed as

$$W_t = \left( \int_0^1 s_{it} W_{it}^{1-\epsilon_w} di \right)^{\frac{1}{1-\epsilon_w}}.$$

The union sets a nominal wage  $\hat{W}_t$  for an effective unit of labor (so that  $W_{it} = \hat{W}_t$ ) to maximize profits subject to wage adjustment costs modeled similarly to the price adjustment

costs in Rotemberg (1982). These adjustment costs are proportional to idiosyncratic productivity  $s_{it}$ , are measured in units of aggregate output, and are given by a quadratic function of the change in wages above and beyond steady state wage inflation  $\bar{\Pi}^w$ ,

$$\Theta \left( s_{it}, W_{it} = \hat{W}_t, W_{it-1} = \hat{W}_{t-1}; H_t \right) = s_{it} \frac{\theta_w}{2} \left( \frac{W_{it}}{W_{it-1}} - \bar{\Pi}^w \right)^2 H_t = s_{it} \frac{\theta_w}{2} \left( \frac{\hat{W}_t}{\hat{W}_{t-1}} - \bar{\Pi}^w \right)^2 H_t.$$

Our extension to an heterogeneous agent economy requires to make assumptions on the aggregation of heterogeneous workers. There is not a unique way to do so and we choose one that is tractable and delivers the representative agent outcome when all heterogeneity is shut down. The union's wage setting problem is assumed to maximize<sup>6</sup>

$$\begin{aligned} V_t^w \left( \hat{W}_{t-1} \right) &\equiv \max_{\hat{W}_t} \int \left( \frac{s_{it}(1-\tau_t)\hat{W}_t}{P_t} h(\hat{W}_t; W_t, H_t) - \frac{v(h(\hat{W}_t; W_t, H_t))}{u'(C_t)} \right) di \\ &- \int s_{it} \frac{\theta_w}{2} \left( \frac{\hat{W}_t}{\hat{W}_{t-1}} - \bar{\Pi}^w \right)^2 H_t di + \frac{1}{1+r_t} V_{t+1}^w \left( \hat{W}_t \right), \end{aligned} \quad (4)$$

where  $C_t$  is aggregate consumption and  $\tau_t$  is a proportional tax on labor income.<sup>7</sup>

Some algebra (see Appendix III) yields, using  $h_{it} = H_t$  and  $\hat{W}_t = W_t$  and defining the real wage  $w_t = \frac{W_t}{P_t}$ , the wage inflation equation

$$\theta_w \left( \pi_t^w - \bar{\Pi}^w \right) \pi_t^w = (1-\tau_t)(1-\epsilon_w)w_t + \epsilon_w \frac{v'(h(\hat{W}_t; W_t, H_t))}{u'(C_t)} + \frac{1}{1+r_t} \theta_w \left( \pi_{t+1}^w - \bar{\Pi}^w \right) \pi_{t+1}^w \frac{H_{t+1}}{H_t}. \quad (5)$$

Thus, at time  $t$  an agent faces the following budget constraint:

$$P_t c_t + a_{t+1} = (1+i_t^a)a_t + (1-\tau_t)P_t w_t h_t s_t + T_t,$$

where  $T_t$  is a nominal lump sum transfer. Households take prices as well as wages and hours from the middleman's wage setting problem as given. Thus, we can rewrite the agent's problem

<sup>6</sup>Equivalently one can think of a continuum of unions each setting the wage for a representative part of the population with  $\int s = 1$  at all times.

<sup>7</sup>Any decision problem in a heterogeneous group requires assumptions on the aggregation of individual needs and this wage setting problem is no exception. Fortunately, our choices here have virtually no effect on our findings. We divide  $v(h(\hat{W}_t; W_t, H_t))$  by  $u'(C_t)$  but using individual consumption and dividing by  $u'(C_{it})$  instead has virtually no effect. We discount with  $\frac{1}{1+r_t}$  but discounting using  $\frac{1}{1+r_t^a}$  or  $\beta$  instead also has negligible effect on our findings.

recursively as follows:

$$\begin{aligned}
V(a, s, \beta; \Omega) &= \max_{c \geq 0, a' \geq 0} u(c, h) + \beta \sum_{s \in \mathcal{S}} \gamma(s'|s) V(a', s', \beta; \Omega') & (6) \\
\text{subj. to } & Pc + a' = (1 + i^a)a + P(1 - \tau)whs + T \\
& \Omega' = \Upsilon(\Omega),
\end{aligned}$$

where  $\Omega(a, s, \beta)$  is the distribution on the space  $X = \mathcal{A} \times \mathcal{S} \times \mathcal{B}$  of agents' asset holdings  $a \in \mathcal{A}$ , labor productivity  $s \in \mathcal{S}$  and discount factor  $\beta \in \mathcal{B}$ , across the population, which will, together with the policy variables, determine the equilibrium prices. Let  $\mathbb{B}(X) = \mathcal{P}(\mathcal{A}) \times \mathcal{P}(\mathcal{S}) \times \mathcal{P}(\mathcal{B})$  be the  $\sigma$ -algebra over  $X$ , defined as the Cartesian product over the Borel  $\sigma$ -algebra on  $\mathcal{A}$  and the power sets of  $\mathcal{S}$  and  $\mathcal{B}$ . Define our space  $M = (X, \mathbb{B}(X))$ , and let  $\mathcal{M}$  be the set of probability measures over  $M$ .  $\Upsilon$  is an equilibrium object that specifies the evolution of the distribution  $\Omega$ .

## 2.2 Production

### 2.2.1 Final Good Producer

A competitive representative final goods producer aggregates a continuum of intermediate goods  $y_{jt}$  indexed by  $j \in [0, 1]$  and with prices  $p_{jt}$ :

$$Y_t = \left( \int_0^1 y_{jt}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.$$

where  $\epsilon$  is the elasticity of substitution across goods. Given a level of aggregate demand  $Y_t$ , cost minimization for the final goods producer implies that the demand for the intermediate good  $j$  is given by

$$y_{jt} = y(p_{jt}; P_t, Y_t) = \left( \frac{p_{jt}}{P_t} \right)^{-\epsilon} Y_t, \quad (7)$$

where  $P$  is the (equilibrium) price of the final good which can be expressed as

$$P_t = \left( \int_0^1 p_{jt}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}.$$

### 2.2.2 Intermediate-Goods Firms

Each intermediate good  $j$  is produced by a monopolistically competitive producer using the technology:

$$Y_{jt} = \begin{cases} K_{jt}^\alpha H_{jt}^{1-\alpha} - F & \text{if } \geq 0 \\ 0 & \text{otherwise} \end{cases}, \quad (8)$$

where  $0 < \alpha < 1$ ,  $K_{jt}$  is capital services rented,  $H_{jt}$  is labor services rented and the fixed cost of production is denoted  $F > 0$ .

Intermediate-goods firms rent capital and labor in perfectly competitive factor markets. A firm's real marginal cost is  $mc_{jt} = \partial \Xi_t(Y_{jt}) / \partial Y_{jt}$ , where

$$\Xi_t(Y_{jt}) = \min_{K_{jt}, H_{jt}} r_t^k K_{jt} + w_t H_{jt}, \text{ and } Y_{jt} \text{ is given by (8)}. \quad (9)$$

Given our functional forms, we have

$$mc_t = \left(\frac{1}{\alpha}\right)^\alpha \left(\frac{1}{1-\alpha}\right)^{1-\alpha} (r_t^k)^\alpha (w_t)^{1-\alpha} \quad (10)$$

and

$$\frac{K_{jt}}{H_{jt}} = \frac{\alpha w_t}{(1-\alpha)r_t^k}. \quad (11)$$

Prices are sticky as intermediate-goods firms face Rotemberg (1982) price adjustment costs. Given last period's individual price  $p_{jt-1}$  and the aggregate state  $(P_t, Y_t, Z_t, w_t, r_t)$ , the firm chooses this period's price  $p_{jt}$  to maximize the present discounted value of future profits, satisfying all demand. The intermediate-goods firm's pricing problem is

$$V_t^{IGF}(p_{jt-1}) \equiv \max_{p_{jt}} \frac{p_{jt}}{P_t} y(p_{jt}; P_t, Y_t) - \Xi(y(p_{jt}; P_t, Y_t)) - \frac{\theta}{2} \left( \frac{p_{jt}}{p_{jt-1}} - \bar{\Pi} \right)^2 Y_t - F + \frac{1}{1+r_t} V_{t+1}^{IGF}(p_{jt}).$$

Some algebra (see Appendix III) yields the New Keynesian Phillips Curve

$$(1 - \epsilon) + \epsilon mc_t - \theta (\pi_t - \bar{\Pi}) \pi_t + \frac{1}{1+r_t} \theta (\pi_{t+1} - \bar{\Pi}) \pi_{t+1} \frac{Y_{t+1}}{Y_t} = 0.$$

The equilibrium real profit of each intermediate goods firm, fully taxed by the government, is

$$d_t = Y_t - F - \Xi(Y_t).$$

## 2.3 Mutual Fund

The mutual fund collects households savings  $A_{t+1}/P_{t+1} \equiv \int a_{it}/P_{t+1} di$ , pays a real return  $\tilde{r}_t^a$ , and invests them in real bonds  $B_{t+1}/P_{t+1}$  and capital  $K_{t+1}$ . It maximizes

$$V^{MF}(K_t) \equiv \max_{K_{t+1}, \frac{B_{t+1}}{P_{t+1}}} (1 + r_{t+1}^k - \delta)K_{t+1} + (1 + r_{t+1})B_{t+1}/P_{t+1} - (1 + \tilde{r}_{t+1}^a)(A_{t+1}/P_{t+1}) + \frac{V^{MF}(K_{t+1})}{(1 + \tilde{r}_{t+2}^a)}$$

subj. to  $A_{t+1}/P_{t+1} = K_{t+1} + B_{t+1}/P_{t+1} + \Phi(K_{t+1}, K_t)$

for adjustment costs  $\Phi(K_{t+1}, K_t)$  and taking  $K_t$  and  $K_{t+2}$  as given. Note that we drop expectation operators for simplicity since we consider perfect foresight experiments only. The equilibrium first-order conditions are

$$r_{t+1} = \tilde{r}_{t+1}^a, \tag{12}$$

$$1 + r_{t+1}^k - \delta = (1 + \tilde{r}_{t+1}^a)(1 + \Phi_1(K_{t+1}, K_t)) + \Phi_2(K_{t+2}, K_{t+1}), \tag{13}$$

$$A_{t+1}/P_{t+1} = K_{t+1} + B_{t+1}/P_{t+1} + \Phi(K_{t+1}, K_t). \tag{14}$$

The total dividends of the fund are

$$D_{t+1}^{MF} = (1 + r_{t+1}^k - \delta)K_{t+1} + (1 + r_{t+1})B_{t+1}/P_{t+1} - (1 + \tilde{r}_{t+1}^a)(A_{t+1}/P_{t+1}),$$

and after-tax per unit of investment they are  $d_{t+1}^{MF} = (1 - \tau_k)D_{t+1}^{MF}/(A_{t+1}/P_{t+1})$ . Households therefore receive (or have to pay)  $d_{t+1}^{MF}A_{t+1}/P_{t+1}$  in period  $t + 1$  per unit invested such that households' real return equals

$$1 + r_{t+1}^a = 1 + \tilde{r}_{t+1}^a + d_{t+1}^{MF}$$

and corresponding nominal return is

$$1 + i_{t+1}^a = (1 + r_{t+1}^a) \frac{P_{t+1}}{P_t}.$$

## 2.4 Government

**Fiscal Policy** The government obtains revenue from taxing labor income, profits and dividends and issuing bonds. Household labor income is taxed progressively with a nominal

lump-sum transfer  $T$  and a proportional tax  $\tau$ :

$$\tilde{T}(wsh, T) = -T + \tau Pwsh.$$

The government issues nominal bonds denoted by  $B^g$ , with negative values denoting government asset holdings and fully taxes away profits of intermediate goods firms, obtaining nominal revenue  $Pd$ . The government also taxes dividend income at the rate  $\tau_k$ . The government uses the revenue to finance exogenous nominal government expenditures,  $G_t$ , interest payments on bonds and transfers to households. The government budget constraint is therefore given by:

$$B_{t+1}^g = (1 + i_t)B_t^g + G_t - P_t d_t - \tau_k P_t D_t^{MF} - \int \tilde{T}(w_t s_t h_t, T_t) d\Omega. \quad (15)$$

**Monetary Policy** The monetary authority follows a Taylor rule, which sets the nominal interest rate according to:

$$i_{t+1} = \max(X_{t+1}, 0), \quad (16)$$

where

$$X_{t+1} = \left(\frac{1}{\zeta}\right) \left(\frac{P_t}{P_{ss}}\right)^{\phi_1(1-\rho_R)} \left(\frac{Y_t}{Y_{ss}}\right)^{\phi_2(1-\rho_R)} [\zeta(1 + i_t)]^{\rho_R} - 1.$$

## 2.5 Equilibrium

Market clearing requires that the labor demanded by firms is equal to the aggregate labor supplied by households, that the demand for bonds issued by the government and for capital equal their supplies and that the amount of assets provided by households equals the demand for them by the mutual fund:

$$H_t = \int H_{jt} dj = H_{jt} = \int h_{it} di = h_{it} \quad (17)$$

$$B_t = B_t^g \quad (18)$$

$$K_t = \int K_{jt} dj \quad (19)$$

$$K_{t+1} + B_{t+1}/P_{t+1} + \Phi(K_{t+1}, K_t) = A_{t+1}/P_{t+1} = \int \frac{a_{t+1}(a_t, s_t, \beta)}{P_{t+1}} d\Omega_t, \quad (20)$$

where  $a_{t+1}(a_t, s_t, \beta)$  is the asset choice of an agent with asset level  $a_t$ , labor productivity  $s_t$  and discount factor  $\beta$ .

The aggregate resource constraint is given by

$$Y_t = K_t^\alpha H_t^{1-\alpha} = \int c_t(a_t, s_t, \beta) d\Omega_t + \frac{G_t}{P_t} + F + K_{t+1} - (1 - \delta)K_t + \Phi(K_{t+1}, K_t). \quad (21)$$

It does not include price and wage adjustment costs because we follow the preferred interpretation of those costs in Rotemberg (1982) as being virtual—they affect optimal choices but do not cause real resources to be expended. This modeling choice allows to avoid these adjustment costs becoming a non-trivial fraction of output in e.g., the liquidity trap. In a liquidity trap prices fall, leading to large price adjustment costs, which, if they were actual resource costs, would lead to a boom in the price adjustment cost industry and may imply an increase in aggregate output (see Hagedorn et al., 2018, for details). As explained in Eggertsson and Singh (2019), assuming that price adjustment costs are as-if avoids these outcomes and moves our model closer to one with price setting à la Calvo.

**Definition:** A monetary competitive equilibrium is a sequence of tax rates  $\tau_t$  and  $\tau_k$ , nominal transfers  $T_t$ , nominal government spending  $G_t$ , supply of government bonds  $B_t^g$ , value functions  $V_t: X \times \mathcal{M} \rightarrow \mathcal{R}$  with policy functions  $a_t: X \times \mathcal{M} \rightarrow \mathcal{R}_+$  and  $c_t: X \times \mathcal{M} \rightarrow \mathcal{R}_+$ , hours choices  $H_t, H_{jt}, h_{it}: \mathcal{M} \rightarrow \mathcal{R}_+$ , capital decisions  $K_t, K_{jt}: \mathcal{M} \rightarrow \mathcal{R}_+$ , bond choices  $B_t: \mathcal{M} \rightarrow \mathcal{R}_+$ , price levels  $P_t: \mathcal{M} \rightarrow \mathcal{R}_+$ , pricing functions  $r_t, r_t^a, r_t^k, \tilde{r}_t^a: \mathcal{M} \rightarrow \mathcal{R}$  and  $w_t: \mathcal{M} \rightarrow \mathcal{R}_+$ , and a law of motion  $\Upsilon: \mathcal{M} \rightarrow \mathcal{M}$ , such that:

1.  $V_t$  satisfies the Bellman equation with corresponding policy functions  $a_t$  and  $c_t$  given price sequences  $r_t^a, w_t$  and hours  $h_t$ .
2. Firms maximize profits taking prices  $P_t, r_t^k, w_t$  as given.
3. Wages are set optimally by middlemen.
4. The mutual fund maximizes profits taking prices as given.
5. For all  $\Omega \in \mathcal{M}$  market clearing conditions (17) - (20) and the resource constraint (21) are satisfied.
6. Aggregate law of motion  $\Upsilon$  generated by  $a'$  and  $\gamma$ .

### 3 The Fiscal Multiplier

In this section we calculate the fiscal multiplier in our model with incomplete markets, conducting the following experiment. Assume that the economy is in steady state with nominal bonds  $B_{ss}$ , government spending  $G_{ss}$ , transfers  $T_{ss}$ , labor tax rate  $\tau_{ss}$ , dividend tax  $\tau_{k,ss}$ , and where the price level is  $P_{ss}$ . The real value of bonds is then  $B_{ss}/P_{ss}$ , the real value of government expenditure is  $G_{ss}/P_{ss}$  and so on. We then consider an M.I.T. (unexpected and never-again-occurring) shock to government expenditures and compute the impulse response to this persistent innovation in  $G$ . Agents are assumed to have perfect foresight after the shock. Eventually the economy will reach the new steady state characterized by government spending  $G_{ss}^{new} = G_{ss}$ , government bonds  $B_{ss}^{new} = B_{ss}$ , transfers  $T_{ss}^{new} = T_{ss}$ , tax rates  $\tau_{ss}^{new} = \tau_{ss}$  and  $\tau_{k,ss}^{new} = \tau_{k,ss}$ , and the price level  $P_{ss}^{new}$ .

#### 3.1 The Fiscal Multiplier in Incomplete Market Models

In our model the fiscal multiplier affects both aggregate consumption and investment. In terms of consumption, the stimulus operates through two interdependent channels — intertemporal substitution and redistribution. The intertemporal substitution channel describes how government spending changes real interest rates and how this changes private consumption. The distributional channel describes how government spending changes prices, income, taxes, etc., the redistribution induced by these changes and the resulting impact on private consumption. In terms of investment, the multiplier operates through changes in real interest rates, through crowding-out of capital when deficit financing is used and through stimulating demand which makes firms hire not only more labor but also more capital. In addition to capturing the relevant transmission mechanisms for fiscal policy, our HANK framework also delivers price level determinacy (Hagedorn, 2016, 2018). This allows us to study arbitrary combinations of monetary and fiscal policies of interest, in particular a constant nominal interest rate as prevails at the zero lower bound (ZLB). We do not face the indeterminacy issues with the representative-agent New Keynesian model at the ZLB raised by Cochrane (2017) and have a uniquely determined fiscal multiplier at the ZLB, one of the cases where we are most interested in knowing its size.

We now explain the role of the two consumption channels in shaping the fiscal multiplier,



how they interact with price level determinacy, what determines their strengths, and explain the differences to complete markets. After that we turn to the investment channel. Finally, we formally define the fiscal multiplier in our environment as well as the corresponding multipliers that would arise in RANK and TANK models faced with same dynamics of prices as arising in our model. This will be helpful in illustrating the role of various features of our model that lead to quite different conclusions from the ones implied by those prominent alternative frameworks.

### 3.1.1 Intertemporal Substitution Channel

To understand the workings of the intertemporal substitution channel in our model it is instructive to start with the complete markets case where this is the only channel operating. We then move to incomplete markets to elucidate the differences.

With complete markets, the size of the multiplier  $m$  is determined by the response of the real interest rates only. The consumption Euler equation for our utility function,  $\frac{C_1^{1-\sigma}}{1-\sigma} + \dots$ , is

$$C_t^{-\sigma} = \beta(1 + r_{t+1})C_{t+1}^{-\sigma}. \quad (22)$$

Iterating this equation and assuming that consumption is back to the steady-state level at time  $T$ ,  $C_T = C_{ss}$ , we obtain for consumption at time  $t = 1$  when spending is increased,

$$C_1^{-\sigma} = \left( \prod_{t=1}^{T-1} \beta(1 + r_{t+1}) \right) C_T^{-\sigma}, \quad (23)$$

so that the initial percentage increase in consumption equals

$$\frac{C_1}{C_{ss}} = \left( \prod_{t=1}^{T-1} \beta(1 + r_{t+1}) \right)^{\frac{-1}{\sigma}} = \left( \prod_{t=1}^{T-1} \frac{1 + r_{t+1}}{1 + r_{ss}} \right)^{\frac{-1}{\sigma}}, \quad (24)$$

where we have used that  $\beta(1 + r_{ss}) = 1$  in a complete markets steady state. The fiscal multiplier  $m$  - the dollar change in output for each dollar increase in  $g$  - is one-to-one related to the percentage change in private consumption

$$m = 1 + \left( \frac{C_1}{C_{ss}} - 1 \right) \frac{C_{ss}}{\Delta g} \quad (25)$$

and is thus one-to-one related to the accumulated response of the real interest rate which is

induced by the fiscal stimulus,

$$m = 1 + \frac{C_{ss}}{\Delta g} \left( \left( \prod_{t=1}^{T-1} \frac{1+r_{t+1}}{1+r_{ss}} \right)^{\frac{-1}{\sigma}} - 1 \right). \quad (26)$$

The multiplier is then proportional to the consumption response

$$\log\left(\frac{C_1}{C_{ss}}\right) = \underbrace{\frac{1}{\sigma}}_{\text{Intertemporal Substitution}} \sum_{t=1}^{T-1} \underbrace{(\log(1+r_{ss}) - \log(1+r_{t+1}))}_{\text{Change of real interest rates}}, \quad (27)$$

which can be decomposed in the change in the real interest rate,  $\approx r_t - r_{ss}$ , and the effect of this change on consumption, whose strength is governed by the IES,  $\frac{1}{\sigma}$ .

Both components of the intertemporal substitution channel tend to be weaker in incomplete markets models. The effect of the real interest rate on consumption is smaller since some households are credit constrained and thus not on their Euler equation, breaking the tight link between consumption and real interest rates. In addition, the change in real interest rates is smaller. To understand the difference, assume for simplicity that the nominal interest rate is fixed at  $i_{ss}$  and that the steady state is reached after  $T$  periods so that

$$\prod_{t=1}^{T-1} (1+r_{t+1}) = \prod_{t=1}^{T-1} \left( \frac{1+i_{ss}}{1+\pi_{t+1}} \right) = \prod_{t=1}^{T-1} (1+i_{ss}) \frac{P_T}{P_1} = \prod_{t=1}^{T-1} (1+i_{ss}) \frac{P_{ss}^{new}}{P_1}, \quad (28)$$

i.e., the response of  $\prod_{t=1}^{T-1} (1+r_{t+1})$  is one-to-one related to the response of  $P_{ss}^{new}/P_1$ . This response is quite large in complete market models (Christiano et al., 2011) but small here.

This is a consequence of the result that incomplete markets combined with fiscal policy specified partially in nominal terms delivers a globally determined price level independently of how monetary policy is specified. We refer the reader to Hagedorn (2016, 2018) for details and only provide the intuition for the key result that  $P_{ss}^{new} = P_{ss}$ . Define households' real steady-state asset demand as  $S$ . The asset demand  $S$ , the real stock of capital,  $K_{ss}$ , and the amount of nominal bonds is the same,  $B_{ss}^{new} = B_{ss}$ , in both steady states and therefore both price levels solve the same asset market clearing condition<sup>8</sup>

$$S(1+r_{ss}, \dots) = K_{ss} + \frac{B_{ss}}{P_{ss}} = K_{ss} + \frac{B_{ss}^{new}}{P_{ss}} = K_{ss} + \frac{B_{ss}^{new}}{P_{ss}^{new}}. \quad (29)$$

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<sup>8</sup>Price level determinacy is a consequence of only two empirically well-grounded assumptions, incomplete markets and nominal government bonds, and in particular does not require any further selection criteria, such as a price or inflation targeting rule.

Together these arguments imply that the intertemporal substitution channel is weaker in our incomplete markets model than in the corresponding complete markets model, where  $P_{ss}^{new} > P_{ss}$ . Note that since  $P_1$  typically increases in response to a stimulus,  $P_{ss}^{new}/P_1 < 1$ , and thus  $\prod_{t=1}^{T-1}(1+r_{t+1}) < \prod_{t=1}^{T-1}(1+i_{ss})$  i.e., the intertemporal substitution channel by itself implies a multiplier smaller than one,  $m < 1$ . The multiplier here is smaller than one whereas it is equal to one in Woodford (2011) since the real interest rate is constant in the small-open economy experiment considered in Woodford (2011) but endogenous and time-varying here.<sup>9</sup>

### 3.1.2 Distributional Consequences of a Stimulus

An increase in spending, the necessary adjustments in taxes and transfers, and the resulting responses of prices and hours induce redistribution across economic agents. For example, changes in the tax code naturally deliver winners and losers. An increase in the price level and of labor income leads to a redistribution from households who finance their consumption more from asset income to households who rely more on labor income. Changes in interest rates redistribute between debtors (the government) and lenders (Erosa and Ventura, 2002; Doepke and Schneider, 2006).

These distributional effects matter due to the endogenous heterogeneity in the MPCs in the data that is replicated in the model. This heterogeneity together with the properties of the redistribution determine the aggregate consumption response, and, since output is demand determined due to price rigidities, also affect output. Individual household's consumption  $c_t$  at time  $t$  depends on the sequence of transfers  $T$ , tax rates  $\tau$ , labor income  $wh$ , prices  $P$  and nominal interest rates  $i^a$ , so that aggregate private consumption satisfies

$$C_t(\{T_l, \tau_l, w_l h_l, P_l, i_l^a\}_{l \geq 0}) = \int c_t(a, s, \beta; \{T_l, \tau_l, w_l h_l, P_l, i_l^a\}_{l \geq 0}) d\Omega_t. \quad (30)$$

In our model hours are a household choice variable but they are demand determined as well. Of course, consumption and hours worked are jointly determined in equilibrium but to understand

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<sup>9</sup>We also overcome an indeterminacy problem in Woodford (2011). He assumes that after a fiscal stimulus consumption converges back to its pre-stimulus level, ruling out other belief driven steady states. Since the real interest rate is constant, this implies that consumption is equal to its steady-state level in all periods, implying a multiplier equal to one. If instead households believe income to change permanently by  $x\%$ , then consumption demand increases by  $x\%$  as well, confirming the initial belief as an equilibrium outcome. There is a steady state for each  $x \neq 0$ , assumed away in Woodford (2011) who focuses instead on the  $x = 0$  equilibrium. Such assumptions to ensure determinacy are not needed here.

the demand response of the fiscal stimulus it turns out to be useful to consider  $wh$  as exogenous for consumption decisions here. In particular it allows us to distinguish between the initial impact, “first round” demand impulse due to the policy change and “second, third, ... round” due to equilibrium responses. Those arise in our model since an initial policy-induced demand stimulus leads to more employment by firms, and thus higher labor income which in turn implies more consumption demand, which again leads to more employment and so on until an equilibrium is reached where all variables are mutually consistent. Denoting pre-stimulus variables by a bar, we can now decompose the aggregate consumption response,

$$(\Delta C)_t = C_t(\{T_l, \tau_l, w_l h_l, P_l, i_l^a\}_{l \geq 0}) - C_t(\{\bar{T}, \bar{\tau}, \bar{w} \bar{h}, \bar{P}, \bar{i}^a\}_{l \geq 0}), \quad (31)$$

into its different channels:

$$(\Delta C)_t = \underbrace{C_t(\{T_l, \tau_l, \bar{w} \bar{h}, \bar{P}, \bar{i}^a\}_{l \geq 0}) - C_t(\{\bar{T}, \bar{\tau}, \bar{w} \bar{h}, \bar{P}, \bar{i}^a\}_{l \geq 0})}_{\text{Indirect Impact of Transfers and Taxes}} \quad (32)$$

$$+ \underbrace{C_t(\{T_l, \tau_l, w_l h_l, \bar{P}, \bar{i}^a\}_{l \geq 0}) - C_t(\{T_l, \tau_l, \bar{w} \bar{h}, \bar{P}, \bar{i}^a\}_{l \geq 0})}_{\text{Indirect Equilibrium Effect: Labor Income}} \quad (33)$$

$$+ \underbrace{C_t(\{T_l, \tau_l, w_l h_l, P_l, i_l^a\}_{l \geq 0}) - C_t(\{T_l, \tau_l, w_l h_l, \bar{P}, \bar{i}^a\}_{l \geq 0})}_{\text{Price and Interest Rate Adjustment}}. \quad (34)$$

Total demand is the sum of private consumption demand  $C$  and real government consumption  $g = G/P$ , which both determine output. The private consumption response does not directly depend on  $G/P$  but it does indirectly. First, transfers  $T$  and taxes  $\tau$  have to adjust to balance the intertemporal government budget constraint. Second, increases in  $G/P$  translate one-for-one into increases in demand. On impact an increase by  $\Delta g$  increases demand by  $\Delta g$  and thus hours worked from  $\bar{h}$  to  $\bar{h} + \Delta h^g$ , where  $\Delta h^g$  is the amount of hours needed to produce  $\Delta g$  while keeping the capital stock unchanged. As before, this increase in labor income stimulates private demand which in turn leads to higher employment, then again higher consumption and so on until convergence. We therefore decompose the total demand effect  $\Delta D$  of an increase

in government spending by  $\Delta g$  as

$$(\Delta D)_t = \underbrace{(\Delta g)_t}_{\text{Direct Govt' Spending Response}} + \underbrace{(\Delta C)_t}_{\text{Private Consumption Response}} \quad (35)$$

$$= (\Delta g)_t + \underbrace{C_t(\{\bar{T}, \bar{\tau}, \bar{w}(\bar{h} + \Delta h^g), \bar{P}, \bar{i}^a\}_{l \geq 0}) - C_t(\{\bar{T}, \bar{\tau}, \bar{w}\bar{h}, \bar{P}, \bar{i}^a\}_{l \geq 0})}_{\text{Direct Impact on Private Consumption}} \quad (36)$$

$$+ \underbrace{C_t(\{T_l, \tau_l, w_l h_l, P_l, i_l^a\}_{l \geq 0}) - C_t(\{\bar{T}, \bar{\tau}, \bar{w}(\bar{h} + \Delta h^g), \bar{P}, \bar{i}^a\}_{l \geq 0})}_{\text{Indirect Private Consumption Response}}. \quad (37)$$

A fiscal stimulus, in addition to the immediate impact on government demand, also leads to higher employment and labor income and thus stimulates private consumption, the *Direct Impact on Private Consumption*. The remainder of the private consumption is as above the sum of the direct impact of transfers and taxes, the indirect equilibrium effects of labor income and price and interest rate adjustment, such that the full decomposition of the total demand effect  $(\Delta D)_t$  is

$$(\Delta D)_t = \underbrace{(\Delta g)_t}_{\text{Direct G Impact}} \quad (38)$$

$$+ \underbrace{C_t(\{\bar{T}, \bar{\tau}, \bar{w}(\bar{h} + \Delta h^g), \bar{P}, \bar{i}^a\}_{l \geq 0}) - C_t(\{\bar{T}, \bar{\tau}, \bar{w}\bar{h}, \bar{P}, \bar{i}^a\}_{l \geq 0})}_{\text{Direct G Impact on C}} \quad (39)$$

$$+ \underbrace{C_t(\{T_l, \tau_l, \bar{w}(\bar{h} + \Delta h^g), \bar{P}, \bar{i}^a\}_{l \geq 0}) - C_t(\{\bar{T}, \bar{\tau}, \bar{w}(\bar{h} + \Delta h^g), \bar{P}, \bar{i}^a\}_{l \geq 0})}_{\text{Indirect Tax/Transfer Impact}} \quad (40)$$

$$+ \underbrace{C_t(\{T_l, \tau_l, w_l h_l, \bar{P}, \bar{i}^a\}_{l \geq 0}) - C_t(\{T_l, \tau_l, \bar{w}(\bar{h} + \Delta h^g), \bar{P}, \bar{i}^a\}_{l \geq 0})}_{\text{Indirect Labor Income Impact}} \quad (41)$$

$$+ \underbrace{C_t(\{T_l, \tau_l, w_l h_l, P_l, i_l^a\}_{l \geq 0}) - C_t(\{T_l, \tau_l, w_l h_l, \bar{P}, \bar{i}^a\}_{l \geq 0})}_{\text{Indirect Price and Interest Impact}}. \quad (42)$$

### 3.1.3 Investment Channel

Investment demand is another component of aggregate demand and the strength of this channel depends both on the cost of investment – the real interest rate  $r_t^k$  – and the demand for intermediate goods. Intermediate goods firms set prices subject to Rotemberg adjustment costs and have to satisfy the resulting demand  $Y_{jt}$  for their product through hiring labor  $H_{jt}$

or buying capital goods  $K_{jt}$ , which leads to the cost minimization condition (11):

$$\frac{K_{jt}}{H_{jt}} = \frac{\alpha w_t}{(1-\alpha)r_t^k}.$$

Given prices  $w_t$  and  $r_t^k$ , a higher demand  $Y_{jt}$  leads to an increase both in capital and employment, the demand channel of investment. Higher capital costs  $r_t^k$  dampen investment demand but only if they increase more than wages. Since firms have to satisfy demand, the relative costs of the factor inputs matter. As an example, suppose that  $r_t^k$  increases by 1% but that wages increase by 2%, so that  $\frac{\alpha w_t}{(1-\alpha)r_t^k}$  increases. In this case, firms would substitute from labor to capital although capital costs have increased just because the costs of the other input factor, wages, has increased even more.<sup>10</sup> Two features of our calibration below make this scenario less likely. High capital adjustment costs imply a strong response of adjustment costs and thus of  $r_t^k$  to capital changes and wage rigidities dampen the movement of wages in response to demand fluctuations. In addition to these partial equilibrium considerations, general equilibrium requires asset market clearing, that is the real interest rate received by households has to be such that they are willing to absorb all assets supplied, bonds and capital,

$$S_{t+1}(\{T_l, \tau_l, w_l h_l, P_l, i_l^a\}_{l \geq 0}) = \int \frac{a_{t+1}(a, s; \{T_l, \tau_l, w_l h_l, P_l, i_l^a\}_{l \geq 0})}{P_{t+1}} d\Omega_t \quad (43)$$

$$= K_{t+1} + B_{t+1}/P_{t+1} + \Phi(K_{t+1}, K_t), \quad (44)$$

such that the change in aggregate savings equals the sum of the change in capital, real bonds and adjustment costs,

$$(\Delta S)_{t+1} = (\Delta K_{t+1}) + (\Delta \frac{B_{t+1}}{P_{t+1}}) + (\Delta \Phi(K_{t+1}, K_t)). \quad (45)$$

This asset market clearing condition implies that, if the stimulus is financed by increasing government debt, capital could be partially crowded out since the increase in savings is partially absorbed by higher government debt. To assess the magnitude of this channel we compute the sequence of real interest rates  $r^{a, Crowding}$  and capital stocks  $K^{Crowd}$  which would clear the asset market if bonds were fixed at their steady-state level  $B_{ss}$ ,

$$S_{t+1}(\{T_l, \tau_l, w_l h_l, P_l, r_l^{a, Crowding}\}_{l \geq 0}) = K_{t+1}^{Crowd} + B_{ss}/P_{t+1} + \Phi(K_{t+1}^{Crowd}, K_t^{Crowd}), \quad (46)$$

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<sup>10</sup>The findings of Rupert and Šustek (2019), derived in a complete markets New Keynesian model with capital, flexible wages and low adjustment costs of capital, suggest that this is not only a theoretical possibility.

such that the difference in capital stocks,

$$K_{t+1} - K_{t+1}^{Crowd}, \quad (47)$$

is the effect of crowding out.

The asset market clearing condition also has a supply side. Households have to be willing to increase their savings. We decompose the change in aggregate savings into two channels. The same redistributive forces that affect consumption behavior in turn affect the savings behavior of households, the redistributive channel of savings. The second channel describes the effect of higher real interest rates on savings. The decomposition of aggregate savings is

$$(\Delta S)_{t+1} = S_{t+1}(\{T_l, \tau_l, w_l h_l, P_l, r_l^a\}_{l \geq 0}) - S_{t+1}(\{\bar{T}, \bar{\tau}, \bar{w} \bar{h}, \bar{P}, \bar{r}^a\}_{l \geq 0}) \quad (48)$$

$$= \underbrace{S_{t+1}(\{T_l, \tau_l, w_l h_l, P_l, \bar{r}^a\}_{l \geq 0}) - S_{t+1}(\{\bar{T}, \bar{\tau}, \bar{w} \bar{h}, \bar{P}, \bar{r}^a\}_{l \geq 0})}_{\text{Distributional Impact}} \quad (49)$$

$$+ \underbrace{S_{t+1}(\{T_l, \tau_l, w_l h_l, P_l, r_l^a\}_{l \geq 0}) - S_{t+1}(\{T_l, \tau_l, w_l h_l, P_l, \bar{r}^a\}_{l \geq 0})}_{\text{Interest Rate Impact}}. \quad (50)$$

### 3.1.4 Multiplier: Definition

We now follow Farhi and Werning (2016) in computing the response of the economy to a fiscal stimulus. Concretely, we compute the response of the economy to an unexpected increase in the path of nominal government spending to  $G_0, G_1, G_2, \dots, G_t, \dots, G_{ss}$ , where  $G_{ss}$  is the steady-state nominal spending level and  $G_t \geq G_{ss}$ .

We summarize the effects of spending on output in several ways. The government spending elasticity of date  $t$  real output equals

$$\frac{\frac{Y_t - Y_{ss}}{Y_{ss}}}{\frac{\frac{G_0 - G_{ss}}{P_0} - \frac{G_{ss}}{P_{ss}}}{\frac{G_{ss}}{P_{ss}}}}, \quad (51)$$

where  $P_{ss}, G_{ss}, Y_{ss}$  are the steady state price level, nominal spending and real output respectively and  $\frac{G_0}{P_0}$  is real government spending at date  $t = 0$ .

We multiply this elasticity by  $\frac{Y_{ss}}{G_{ss}/P_{ss}}$  to convert it to dollar equivalents, which yields the

path of the incomplete markets multipliers as the sequence of

$$m_t^{IM} = \frac{\frac{Y_t}{Y_{ss}} - 1}{\frac{G_0 P_{ss}}{P_0 G_{ss}} - 1} \frac{Y_{ss}}{G_{ss}/P_{ss}}. \quad (52)$$

We also compute the present value multipliers for a discount factor  $\tilde{\beta}$  as

$$m_t^{PV} = \frac{\sum_{k=0}^t \tilde{\beta}^k \left( \frac{Y_k}{Y_{ss}} - 1 \right) \frac{Y_{ss}}{G_{ss}/P_{ss}}}{\sum_{k=0}^t \tilde{\beta}^k \left( \frac{G_k P_{ss}}{P_k G_{ss}} - 1 \right) \frac{Y_{ss}}{G_{ss}/P_{ss}}}, \quad (53)$$

where the two statistics coincide at  $t = 0$  and represent the impact multiplier. A useful statistic is then the cumulative multiplier (Mountford and Uhlig, 2009), which represents the discounted percentage change in real output relative to the discounted percentage change in real government spending for any path of government spending:

$$\bar{M} = m_\infty^{PV} = \frac{\sum_{t=0}^{\infty} \tilde{\beta}^t \left( \frac{Y_t}{Y_{ss}} - 1 \right) \frac{Y_{ss}}{G_{ss}/P_{ss}}}{\sum_{t=0}^{\infty} \tilde{\beta}^t \left( \frac{G_t P_{ss}}{P_t G_{ss}} - 1 \right) \frac{Y_{ss}}{G_{ss}/P_{ss}}}. \quad (54)$$

For comparison with the complete markets case, we also compute the as-if dynamic complete markets multiplier,  $m_t^{CM}$ . The objective is not to compute the equilibrium of the complete markets model, but to use the paths of  $r^a$ ,  $w_t$  and  $r_t$  obtained from our incomplete markets model to derive the full consumption and investment paths that would have arisen in a complete-markets model given those prices. That allows us to isolate the intertemporal substitution effect on the multiplier as the distribution effect is not present in the complete markets model. Computing the as-if consumption path is straightforward since intertemporal substitution is the only channel operating as we discussed in Section 3.1.1 and we use the incomplete markets path of real interest rates. Following the line of arguments in that section and iterating the consumption Euler equation using the real return  $1 + r_{t+1}^a$  on households' assets yields the as-if percentage response of aggregate consumption,

$$\frac{C_t^{CM}}{C_{ss}} - 1 = \prod_{s=t}^{\infty} \frac{1 + r_{ss}^a}{1 + r_{t+1}^a} - 1.$$

To compute the as-if investment path we use the as-if path of consumption, wages  $w_t$  and the real return on bonds  $1 + r_{t+1}$  as inputs to solve for the paths of  $K_{t+1}$  as well as of  $1 + r_t^k$  and  $H_t$ . To compute the paths requires solving a system of three equations in each time period: the cost-minimization condition (11), the mutual fund first-order condition (13) and the resource



constraint (21). Jointly solving this system of equations delivers a path of investment,  $I_t^{CM}$ , hours,  $H_t^{CM}$  and the return on capital  $r_t^k$ .

Since the multiplier is in terms of units of consumption and not in percentages, adjusting for the magnitudes of steady state consumption, output and government spending,

$$\begin{aligned} m_t^{CM} &= \frac{\frac{C_t^{CM} - C_{ss}}{C_{ss}} \frac{C_{ss}}{Y_{ss}} + \frac{I_t^{CM} - I_{ss}}{I_{ss}} \frac{I_{ss}}{Y_{ss}} + \frac{G_t/P_t - G_{ss}/P_{ss}}{Y_{ss}}}{\frac{G_0 P_{ss}}{P_0 G_{ss}} - 1} \frac{Y_{ss}}{G_{ss}/P_{ss}} \\ &= \frac{\frac{C_t^{CM} - C_{ss}}{C_{ss}}}{\frac{G_0 P_{ss}}{P_0 G_{ss}} - 1} \frac{C_{ss}}{G_{ss}/P_{ss}} + \frac{\frac{I_t^{CM} - I_{ss}}{I_{ss}}}{\frac{G_0 P_{ss}}{P_0 G_{ss}} - 1} \frac{I_{ss}}{G_{ss}/P_{ss}} + \frac{G_t/P_t - G_{ss}/P_{ss}}{G_0/P_0 - G_{ss}/P_{ss}}. \end{aligned}$$

The as-if complete markets multiplier also allows us to decompose the incomplete markets multiplier. The incomplete markets multiplier combines three channels – the investment, the intertemporal substitution and the distributional one – while the as-if complete markets multiplier captures only the first two channels but shuts down the distributional one. This allows us to interpret the difference between these two multipliers as the contribution of the distributional channel,

$$m_t^{Distribution} = m_t^{IM} - m_t^{CM}, \quad (55)$$

and the as-if multiplier as the contribution of the intertemporal substitution and the investment channel.

We proceed similarly for a comparison with a Two Agent New Keynesian (TANK) model, that combines a permanent income household with a hand-to-mouth household. The permanent income household behaves like a representative agent so that the full consumption path can be computed as above using the consumption Euler equation. The consumption of hand-to-mouth households equals  $(1 - \tau)w_t h_t + T_t$ , where we import  $w_t$  and  $T_t$  from our incomplete markets model. We assume that the demand for hours  $h_t$  is the same for all households. The paths of investment and hours are computed as described above in the representative agent economy. Combining consumption of both groups yields an aggregate consumption path which we use to compute the as-if TANK multiplier path,  $m_t^{TANK}$ .

## 3.2 Calibration

To quantitatively assess the size of the fiscal multiplier, we now calibrate the model.

**Preferences.** Households have separable preferences over labor and constant relative risk aversion preferences for consumption. We set the risk-aversion parameter,  $\sigma$ , equal to 1. Following Krueger et al. (2016), we assume permanent discount factor heterogeneity across agents. We allow for two values of the discount factor (0.999 and 0.993), which we choose along with the relative proportions (0.2 and 0.8, resp.) to match the Gini of net worth net of home equity, the ratio of median and 30th percentile of net worth net of home equity in the 2013 SCF, and the ratio of aggregate savings net of home equity to quarterly GDP of 11.46.<sup>11</sup> This allows us to capture the overall level of wealth in the economy and important distributional moments. We assume the functional form for the disutility of labor  $v$ :

$$v(h) = \psi \frac{h^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}. \quad (56)$$

We set the Frisch elasticity,  $\varphi = 0.5$ , following micro estimates. We choose  $\psi = 0.6$  so that in steady state  $h = 1/3$ .

**Productivity Process.** Krueger et al. (2016) use data from the Panel Study of Income Dynamics to estimate a stochastic process for labor productivity. They find that log income consists of a persistent and a transitory component. They estimate that the persistent shock has an annual persistence of 0.9695 and a variance of innovations of 0.0384. The transitory shock is estimated to have variance 0.0522. We follow Krueger et al. (2016) in converting these annual estimates into a quarterly process. We discretize the persistent shock into a seven state Markov chain using the Rouwenhorst method and integrate over the transitory shock using Gauss-Hermite quadrature with three nodes.

**Production Technology.** We set the capital share  $\alpha = 0.36$ . We choose the quarterly depreciation rate  $\delta = 0.032$  to generate the same real return on capital net of depreciation and on bonds when the capital output ratio is 10.26. We assume the functional form for  $\Phi$ :

$$\Phi(K', K) = \frac{\phi_k}{2} \left( \frac{K' - K}{K} \right)^2 K, \quad (57)$$

and set  $\phi_k = 17$  to match estimates of the elasticity of investment to Tobin's  $q$  from Eberly et al. (2008). We choose the elasticity of substitution between intermediate goods,  $\epsilon = 10$ , to

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<sup>11</sup>We calibrate to a capital to quarterly output ratio of 10.26, and government debt to quarterly GDP ratio of 1.2.

match an average markup of 10%. The adjustment cost parameter on prices and wages is set to  $\theta = \theta_w = 300$  to match the slope of the NK Philips curve,  $\epsilon/\theta = 0.03$ , in Christiano et al. (2011). We set the firm operating cost  $F$  equal to the steady state markup such that steady state profits equal zero (Basu and Fernald, 1997). Profits, which are nonzero only off steady state, are fully taxed and are distributed to households as lump-sum transfers.

**Government.** We set the proportional labor income tax  $\tau$  equal to 25%, and the dividend tax  $\tau_k$  equal to 36% (Trabandt and Uhlig, 2011). We set nominal government spending  $G$  in steady state equal to 6% of output (Brinca et al., 2016). The value of lump-sum transfers  $T$  is set to 8.55% of output such that roughly 40% of households receive a net transfer from the government (Kaplan et al., 2018). This generates a government debt to quarterly GDP ratio of 1.2 in steady state.

**Steady State Model Fit.** In the calibrated model 3% of agents are at the borrowing constraint with 0 wealth, and 10% of agents have less than \$1,000. The wealth to annual income ratio is 4.9. The annual MPC out of transitory income equals 0.4, which is in the middle range of empirical estimates 0.2-0.6 (e.g., Johnson et al., 2006).<sup>12</sup>

### 3.3 Households' Demand and Marginal Propensity to Consume

Before moving to the full general equilibrium analysis, we provide several partial equilibrium consumption results to build intuition for our general equilibrium findings and to show that our incomplete markets model is closer to the empirical findings in the micro consumption literature than standard frameworks. In all of the following experiments, we consider standard household consumption/savings problems for prices fixed at their steady-state values. To illustrate the properties of the MPC in our model, the experiments differ in the timing and the amount of transfers households receive.

Each of the four panels in Figure 1 plots four separate experiments, where each line corresponds to the aggregate consumption path in response to finding out at date 0 that all households will receive a transfer either at date 0, or at date 4, or at date 8, or at date 12 without an obligation to ever repay. The four panels differ in the size of the transfer received, either \$10, \$100, \$1,000 or \$10,000.

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<sup>12</sup>We compute the annual MPC using the quarterly MPC via the formula:  $MPC_a = 1 - (1 - MPC_q)^4$ . The quarterly MPC in the model is 0.12.

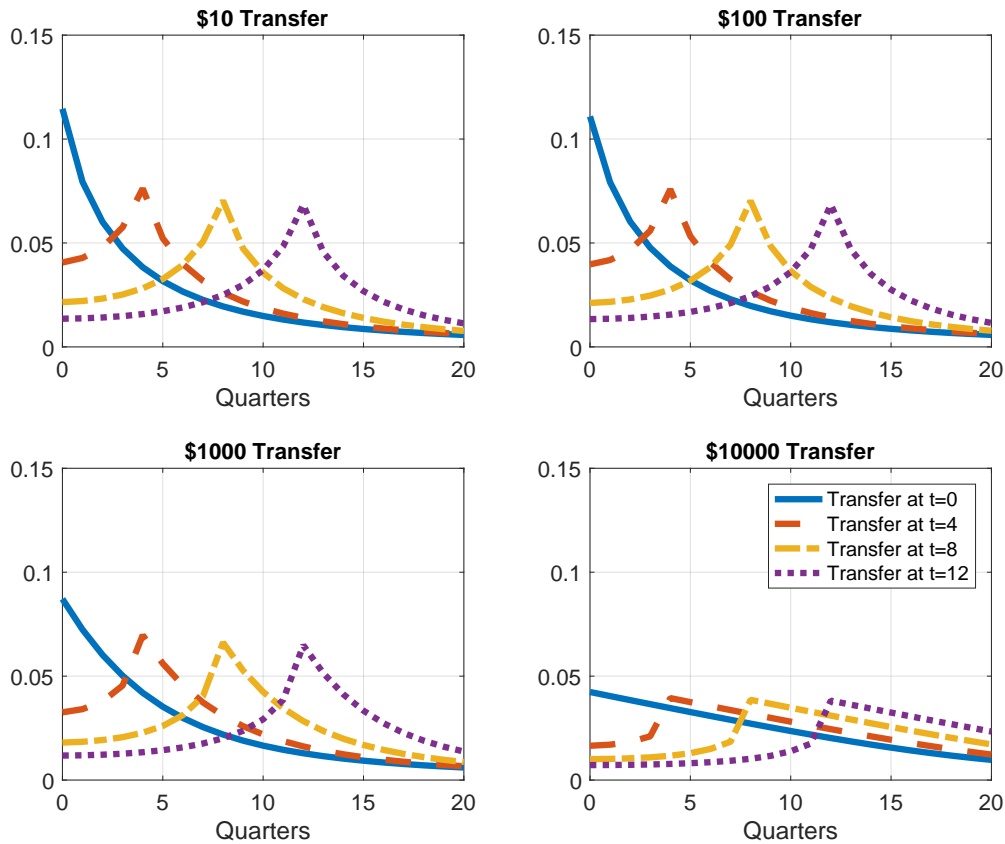
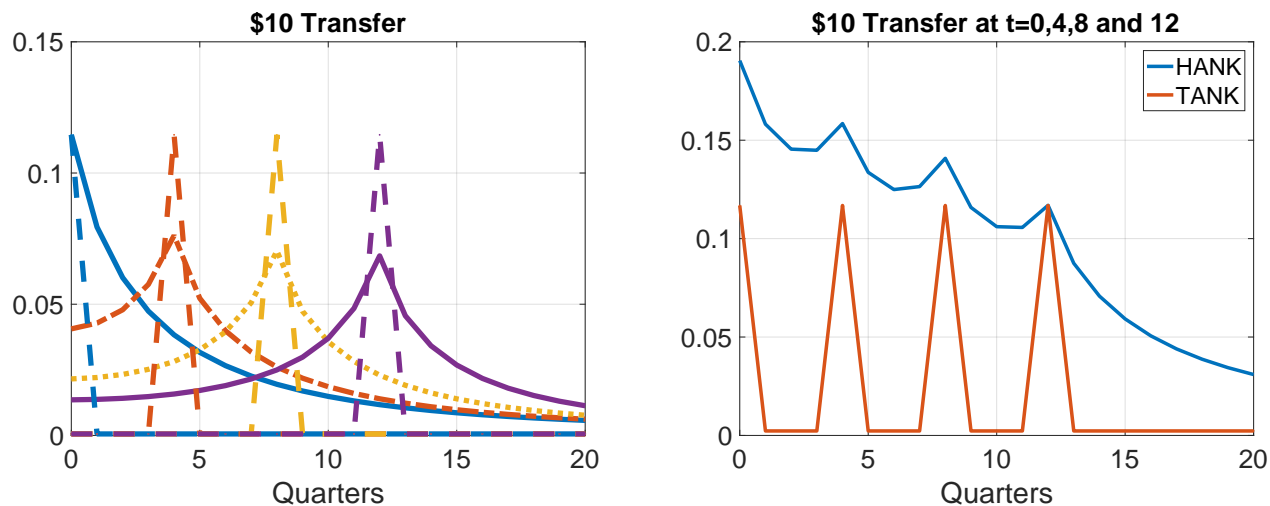


Figure 1: Propensity to Consume for Transfers of Size \$10, \$100 \$1,000 and \$10,000.

Consider first the experiment of giving a household \$10 today. A permanent income household would save basically all of the money and consume a small fraction. In our model households face idiosyncratic income risk, inducing a desire to smooth income but also credit constraints, preventing perfect smoothing. A borrowing constrained household would consume the full \$10. Unconstrained, but low asset households will also consume a large fraction of the transfer because it relaxes precautionary savings motives. These arguments together imply an initial MPC significantly larger than in complete markets models. The fraction of the transfers not spent in the initial period is spent in the following periods at a decaying rate. If households receive larger transfers, the initial MPC falls, mainly because larger transfers are more likely to relax the credit constraints. For example, a \$10,000 transfer is sufficient to relax all households' credit constraints, so that even the borrowing constrained household will not consume the full transfer. Now, suppose households do not receive the transfer today but only learn today that they will receive a transfer in a future period, at date 4, 8, or 12. The credit constrained households cannot respond until the transfer arrives. The unconstrained



(a) Propensity to Consume for Transfers of Size \$10 that are Received in Periods 0, 4, 8 OR 12.

(b) Propensity to Consume for Transfers of Size \$10 that are Received in Periods 0, 4, 8 AND 12.

Figure 2: HANK vs TANK

households are able to smooth consumption, but their MPC is lower, so the initial rise in consumption is smaller than before.

These model properties are consistent with the data but inconsistent with a complete markets RANK model.<sup>13</sup> They are also inconsistent with a TANK model which deviates from the RANK model by introducing two types of agents – a permanent income household and a hand-to-mouth household. The TANK model can match high MPCs and is theoretically tractable. However, as shown in Panel (a) of Figure 2, when we replicate the same experiment in a TANK model, instead of tent-shaped impulse responses we obtain in our HANK model, the TANK model delivers spiky responses. The TANK model also misses out on the sensitivity to the size of the transfer and all of the dynamic anticipation and propagation effects because the response of permanent income households is minimal and consumption of hand-to-mouth households responds to current income only. To understand why this is important we need to think about how the general equilibrium Keynesian cross multiplier logic works.

As we saw, if a household receives a transfer in the first period, it is spent over all future periods. In general equilibrium, that would mean an increase in aggregate demand and also in income not only today but also in all future periods. Consumption today increases not only because of the increase in income today but also because of the increases in future income by relaxing precautionary savings motives. To illustrate this point we now combine

<sup>13</sup>For a discussion on the empirical evidence on intertemporal MPCs see Auclert et al. (2018).

the four previous experiments, so that households learn in period 0 that they will receive a transfer at *all* dates 0, 4, 8 and 12. Panel (b) of Figure 2 shows that now the impact response of consumption is nearly twice as high, even though in period 0 the same transfer is received. This is a partial equilibrium example, but in general equilibrium the demand and income increases at different times reinforce each other, generating what Auclert et al. (2018) have coined as an intertemporal keynesian cross. The TANK economy fails to reproduce the dynamic multiplier path because the anticipation and propagation effects in that model are basically zero.

### 3.4 Fiscal Multipliers in the Data and in the Model

While the previous section has established that our model is consistent with micro behavior of households in response to a fiscal stimulus, we now show that our model also replicates the aggregate response reported in Ramey and Zubairy (2018). They use well-identified government spending shocks to calculate the 2- and 4-year cumulative multipliers,

$$m^{2years} = \frac{\sum_{k=0}^{2years} Y_k - Y_{ss}}{\sum_{k=0}^{2years} g_k - g_{ss}} \quad \text{and} \quad m^{4years} = \frac{\sum_{k=0}^{4years} Y_k - Y_{ss}}{\sum_{k=0}^{4years} g_k - g_{ss}}, \quad (58)$$

as the sum of output responses within the first two or four years divided by the corresponding sum of government spending responses.

The upper panel “Empirical Results” of Table I reports their findings for post WWII data, the sample which fits our modeling assumptions best, while the bottom panel “Model Results” reports the corresponding findings from our model. The size of the fiscal multiplier depends on how it is financed and on the response of monetary policy. Thus, to conduct this experiment in the model, we feed in the impulse response to government spending shock of taxes and deficit – found for this sample by Ramey and Zubairy (2018)<sup>14</sup> – and of the nominal interest rate, which we document below in this section. Considering the full sample, Column (1) of Table I indicates that the model matches well the size of the fiscal multiplier estimated in the data. However, this empirical estimate of the multiplier corresponds to a particular sequence of monetary and fiscal responses (e.g., taxes, deficits) coincident with the fiscal stimulus. Thus it does not identify the consequences of selecting a different mix of monetary and fiscal policies that policymakers could have alternatively implemented. Our model allows us to disentangle

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<sup>14</sup>See Figure 5 in Supplementary Appendix to their article.

Table I: Fiscal Multiplier: Empirical Results and Model

	Full Sample	High Unemployment	Low Unemployment
<i>Empirical Results</i>			
2-year cum. multiplier	0.75	-1.63	0.80
4-year cum. multiplier	0.51	-2.77	0.49
<i>Model Results</i>			
2-year cum. multiplier	0.73	0.70	0.71
4-year cum. multiplier	0.58	0.54	0.55

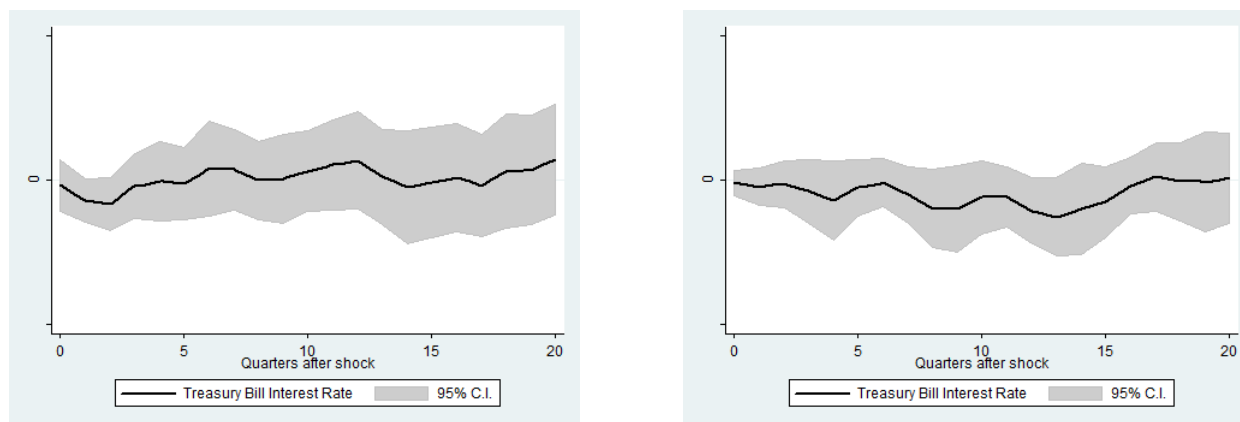
these effects and consider the consequences of alternative policy choices through counterfactual experiments. We will turn to this task in the next section.

In addition to this baseline analysis, Ramey and Zubairy (2018) also assess the state-dependence of multipliers by computing them in states of high and low unemployment, reported in Columns (2) and (3) of Table I, respectively. They argue that the evidence for spending multipliers during recessions—that is times of high unemployment—is fragile in post-WWII data because the variation in the data is not rich enough and recessions are too short to deliver reliable results. As reported in the lower panel “Model Results” of Table I, for both the full post-WWII sample and the one restricted to only low unemployment times our model results are close to those found in the data.<sup>15</sup> For high unemployment times we find similar multipliers as for periods of low unemployment, implying no state-dependence.<sup>16</sup> We conclude that for the experiments with robust corresponding empirical findings, our theoretical model replicates the results in the data and thus represents a good laboratory. We can then use the model to calculate the fiscal multiplier for those experiments where the data are less informative but the results are of keen interest to policymakers e.g., during high unemployment times or when the economy is in a liquidity trap.

Computing the fiscal multiplier in the model also requires to specify the response of monetary policy to a fiscal stimulus. We calculate this response using the experiments and

<sup>15</sup>To generate the equivalent high and low unemployment states we feed in positive and negative TFP shocks.

<sup>16</sup>This is consistent with the conclusion of Ramey and Zubairy (2018) based on historical data dating back to 1889.



(a) Ramey-Zubairy Shock

(b) Blanchard-Perotti Shock

Figure 3: Monetary Policy Response to a Fiscal Stimulus

data accompanying Ramey and Zubairy (2018). In Panel (a) of Figure 3 we plot the response of the three-month Treasury bill interest rate to the same fiscal stimulus used in Ramey and Zubairy (2018) and also used to calculate the empirical results shown in the upper panel of Table I. We find that the interest rate is not significantly affected, which is consistent with monetary policy not responding to the fiscal stimulus. In Panel (b) of Figure 3 we show that this conclusion is robust to considering the response to government spending shocks of Blanchard and Perotti (2002). The flat response holds for both post WWII data and the larger historical sample starting in 1889. One could interpret the flat response as monetary policy supporting fiscal policy. Indeed, we will see in Section 4.1.3 that when we allow for a Taylor rule, which implies that the nominal interest rate increases when fiscal policy is expansionary, the multiplier is significantly lower.

## 4 The Fiscal Multiplier: Controlled Measurement

Balancing the government budget when government spending or transfers are increased requires taxes, debt or both to adjust. To the extent that Ricardian equivalence does not hold, different paths of taxes and debt will have different implications for the path of aggregate consumption and therefore prices and output. Moreover, the size of the fiscal multiplier will depend on how monetary policy chooses to respond to a fiscal stimulus. It may also exhibit dependence on the other shocks affecting the economy, e.g., a large demand shock triggering a liquidity trap. Unfortunately, it is not possible to conduct randomized experiments in the data



to isolate the pure effect of additional spending on the economy from the consequences of other shocks and policy responses. We can, however, use our quantitative model for this purpose. Thus, our objective in this section is to quantify how different shocks, financing schemes and policy responses shape the size of the fiscal multiplier.

To be able to compare the results across the experiments we put all of them on the same footing by considering the effect of the same change in government spending (and later transfers). Specifically, we compute the equilibrium responses of prices, employment, output and consumption to a persistent increase in nominal government spending for 20 quarters, consistent with the empirical duration of elevated spending in Ramey and Zubairy (2018). We assume that spending increases by one percent in period  $t$  and after that the amount of additional spending decays at rate  $1 - \rho_g$  per quarter for the subsequent 19 quarters. We set  $\rho_g = 0.9$  in the baseline experiments (implying a half-life of 7 quarters) and vary this parameter in Appendix I.3. For balancing the government budget when government spending is increased, we consider two benchmark scenarios:<sup>17</sup>

1. Transfer are adjusted period by period to keep nominal debt constant.
2. Deficit financing and delayed future reduction of transfers to eventually pay back the debt.

In both scenarios we keep the labor and dividend tax rates,  $\tau$  and  $\tau_k$  fixed. For each experiment we report the dynamic response of hours, consumption, output, prices, tax revenue and debt as well as the paths of the incomplete markets multiplier  $m_t^{IM}$ , of the as-if complete markets multiplier  $m_t^{CM}$  and of the as-if TANK multiplier  $m_t^{TANK}$ . In Section 4.1 we study the size of the multiplier in “Normal Times,” which is in contrast to the “Liquidity Trap” studied in Section 4.2. The cumulative multiplier  $\bar{M}$  and various additional statistics across the experiments we conduct in the body of the paper are collected in Table II.

## 4.1 Fiscal Multiplier in Normal Times

### 4.1.1 Tax Financing: Constant Nominal Debt

Under the first financing scheme, we assume that the government adjusts lump-sum transfers period by period to keep the level of nominal debt constant. The four panels of Figure 4 show

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<sup>17</sup>In Appendix I.4 we also consider a third scenario: Deficit financing and delayed cuts in government expenditure after 40 quarters.

Table II: Main Results: Consumption, Investment and Multipliers

Experiment:	Normal Times					Liquidity Trap		
	Benchmark		Taylor Rule		Transfer	Benchmark		Transfer
	Tax	Deficit	Tax	Deficit	Deficit	Tax	Deficit	Deficit
Financing:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Impact Mult.	0.61	1.34	0.54	0.66	0.66	0.73	1.39	0.86
Cumul Mult.	0.43	0.55	0.40	0.29	-0.3	0.48	0.51	-0.11
$100 \times \Delta C_0$	-2.7	1.4	-2.9	-0.4	5.2	-2.5	1.3	4.8
$100 \times \Delta I_0$	0.3	0.6	-0.02	-1.6	0.4	0.2	0.5	0.4
Decomposition of Consumption ( $100 \times \Delta$ )								
Direct G on C	1.2	1.2	1.2	1.2	0.0	1.2	1.2	3.9
Tax/Transfers	-3.1	0.5	-3.1	-0.2	4.6	-2.9	0.5	0.4
Indirect Income	-0.7	0.2	-0.8	-0.7	1.1	-0.6	0.0	0.8
Prices	-0.1	-0.5	-0.2	-0.8	-0.5	-0.2	-0.5	-0.4

Note - The table contains the impact and the cumulative multiplier  $\bar{M}$  (using definition (60) for the last column and (A1) otherwise) as well as the initial consumption and investment responses,  $\Delta C_0$  and  $\Delta I_0$  (as a % of output). The last four rows show the decomposition of the initial aggregate consumption response (also multiplied by 100) into the direct  $G$  impact on  $C$  (Eq. 39), the effect of taxes/transfers (Eq. 40), indirect income effects (Eq. 41) and the price and interest rate effects (Eq. 42).

the results for the private consumption and output response, the different multipliers, the decomposition of private consumption, and the evolution of government bonds.

The bottom right panel plotting the evolution of fiscal policy illustrates that the level of nominal government bonds is unchanged since the stimulus is tax-financed. The top left panel shows that on impact  $G$  increases by 1% (0.06% of output) and consumption decreases by 0.027% of output. As illustrated in the top right panel, this leads to an impact multiplier of 0.61. The incomplete markets multiplier converges to zero since the consumption response, although negative, slowly dies out and becomes small relative to the initial government spending increase. The decomposition of the consumption response in the bottom left panel reveals the quantitative importance of the direct, the indirect and the price effects. The stimulus of 0.06% directly increases households' labor supply by the same amount, leading to a aggregate consumption increase of 0.012% of output (Eq. 39). The

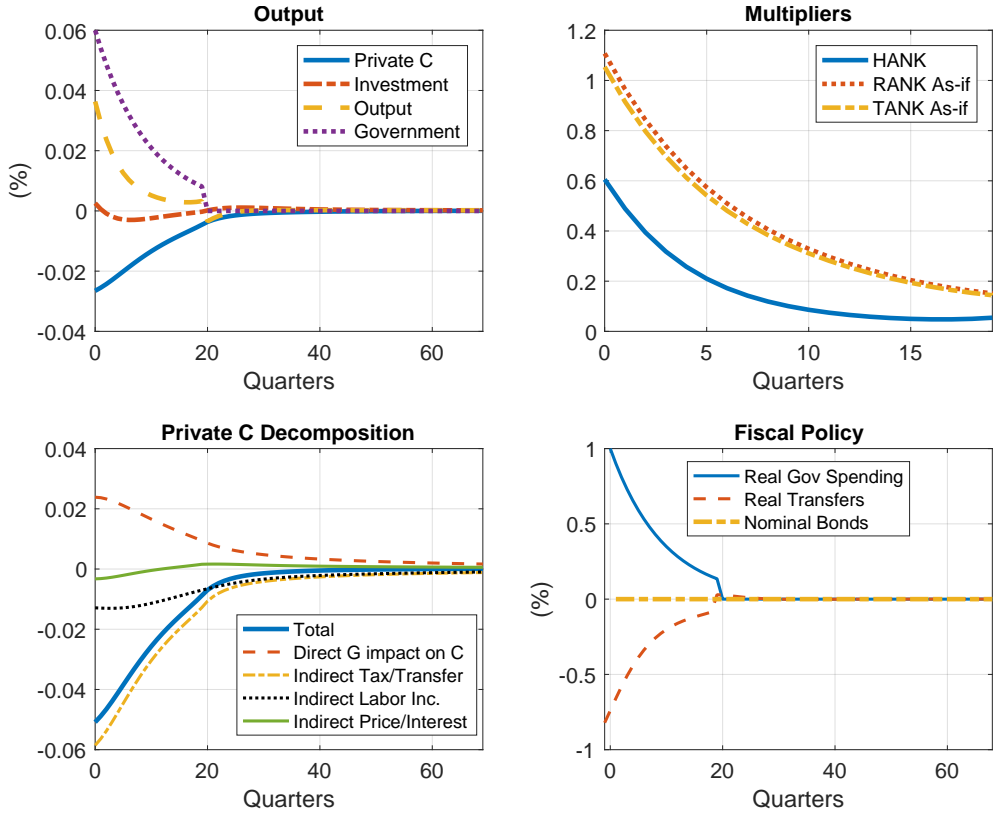


Figure 4: Fiscal Multiplier and Aggregate Consumption: Tax Financing

contemporaneous cut in transfers lowers aggregate consumption by  $-0.031\%$  on impact (Eq. 40), implying a total initial negative effect of  $-0.018\%$ . This effect is negative since an increase in government spending increases household income in proportion to their productivity and thus predominantly benefits high-income households. In contrast, the transfer cut is uniform across all income groups and thus negatively affects high MPC households. This decrease leads to lower consumption demand, which in turn leads to lower labor demand, lower labor income and again lower consumption demand until an equilibrium is reached. These indirect income effects sum up to  $-0.007\%$  (Eq. 41) further lowering the consumption response. Finally, the decomposition shows that the price increase and (the unchanged) interest rate effects are small (Eq. 42). The investment channel is also contributing very little.

Whereas the incomplete markets multiplier is significantly below one,  $m_0^{IM} = 0.61$ , the as-if multiplier in the complete markets model is,  $m_0^{CM} = 1.11$ , so that the contribution of the distributional channel (Eq. 55),  $m_0^{Distribution} = -0.5$ . The results is unsurprising given the foregoing discussion, as the permanent income representative household responds little to the cut in transfers. We find similar results for the TANK economy with  $m_0^{TANK} = 1.05$  and

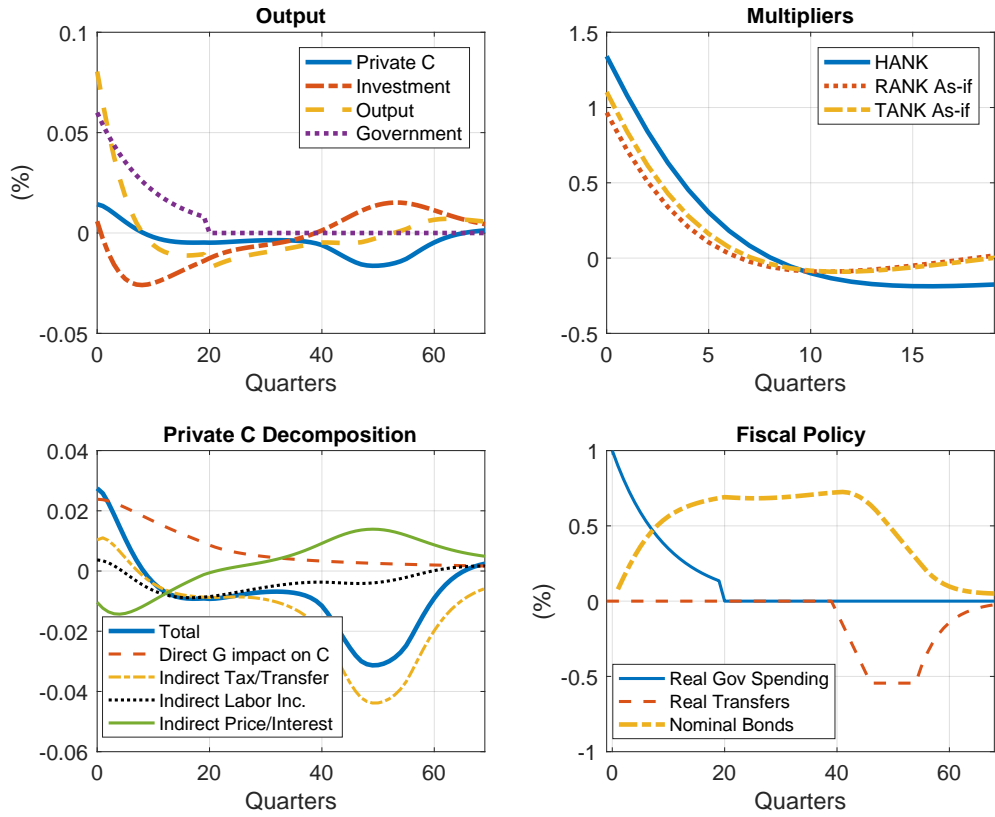


Figure 5: Fiscal Multiplier and Aggregate Consumption: Deficit Financing

a negative distributional channel of  $-0.44$ . The as-if multiplier decreases slightly, due to the higher average MPC in the TANK economy relative to RANK. However, while the TANK and our HANK economies feature the same quarterly MPC, the large difference between them is due to dynamic anticipation effects arising in the HANK model that are absent in TANK. The hand-to-mouth agents in TANK respond only to the contemporaneous decrease in transfers, but not to the anticipated cut in future transfers (as occurs in our HANK economy), leading to a much smaller drop in private consumption demand. The larger drop in aggregate household consumption demand in the incomplete markets model then translates through the Keynesian cross logic into a smaller general equilibrium multiplier of 0.61 relative to 1.05.

The impulse response of the remaining variables to a 1% innovation in government spending are plotted in Figure A-5 in the appendix. The cumulative multiplier, in Table II, is 0.43.

#### 4.1.2 Deficit Financing

Under this scenario we assume that real transfers are kept constant after the innovation to government spending. Eventually, the debt must be repaid. We assume that the repayment

occurs sufficiently far into the future - after 40 quarters- from the onset of stimulus and the reduction of transfers is gradual for computational convenience.<sup>18</sup> Variations in the way the eventual repayment is modeled is inconsequential for our substantive findings. The four panels of Figure 5 summarize the evolution of the key variables of interest.

Deficit instead of tax financing increases the initial multiplier  $m_0^{CM}$  from 0.61 to 1.34 and the initial aggregate consumption response from  $-0.027\%$  to  $0.014\%$ . The decomposition of the consumption response makes clear why. The direct impact of the spending stimulus is identical ( $0.012\%$ ) but now there is no initial offsetting effect from contemporaneously higher taxes/lower transfers. The indirect income effects now accumulate to  $0.002\%$  and the deficit financing leads to an increase in government bonds and the consumption response becomes negative only from period 9 onwards. However, the increase in government spending is ultimately financed through a future reduction in transfers, which results in a contraction in future output. Thus, the cumulative multiplier, reported in Table II is only 0.55, significantly smaller than the impact multiplier. The impulse responses of the remaining variables are plotted in Figure A-6 in the appendix.

Since the as-if multipliers  $m_0^{CM} = 0.97$  and  $m_0^{TANK} = 1.10$  are still close to one, the contribution of the distributional channel, is now positive  $m_0^{Distribution} = 0.37$  for complete markets and 0.24 for the TANK model. The fiscal expansion increases employment and thus labor income but because of deficit financing there is no offsetting effect through higher taxes. This benefits high MPC households, who primarily rely on labor income and not asset income, and allows them to increase consumption demand. The additional government debt on the other hand is bought by low MPC households which reduce their consumption. In total, this redistribution towards high MPC households increases aggregate consumption demand, implying  $m_0^{Distribution} > 0$ .

Another difference between deficit and tax financing is a negative impact of investment demand on the multiplier with deficit financing. Figure 6 illustrates why this is the case. The right panel shows that  $r^k/w$  increases, making firms substitute from capital towards labor.

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<sup>18</sup>Specifically, we assume that real transfers are kept constant during the first 40 quarters after the innovation to government spending. Then, they are reduced linearly over eight quarters, kept constant for another eight quarters, and then revert back to the real steady state level with an autocorrelation parameter of 0.8. Thus, under this timing scheme, the government chooses only the level of adjustment to transfers to guarantee that nominal government debt returns to its original level.

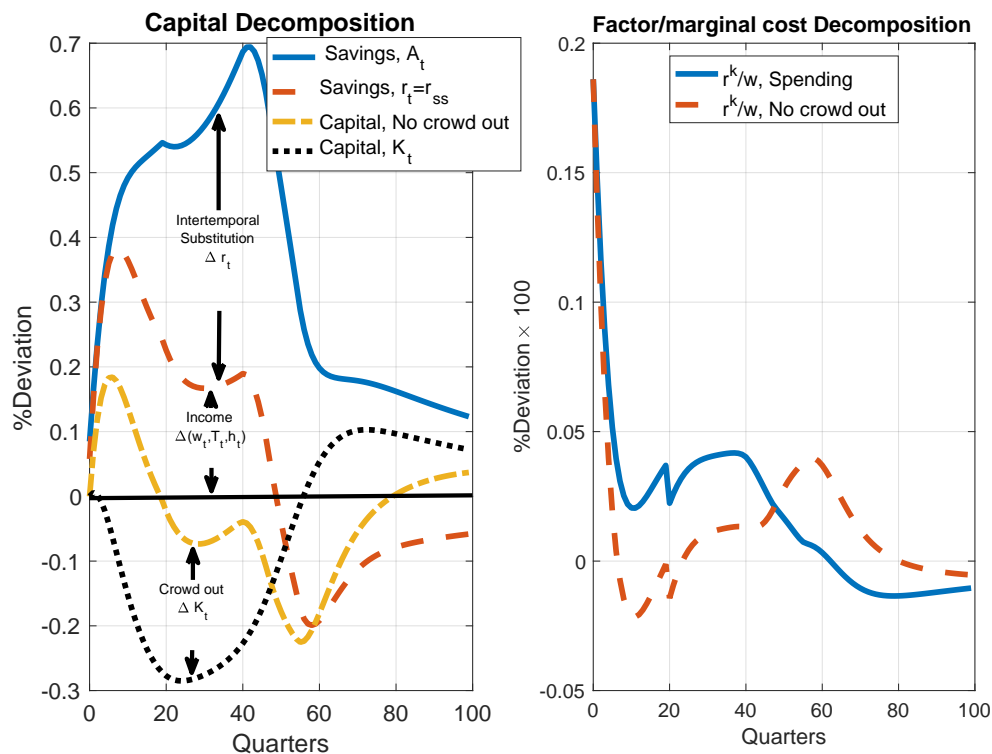


Figure 6: Fiscal Multiplier and Aggregate Investment: Deficit Financing

The left panel shows that total savings increase (the blue line “Savings,  $A_t$ ”) by more than they would have increased if the real interest rate  $r^a$  were to remain constant (the red line “Savings,  $r_t = r_{ss}$ ”), reflecting the “Interest Rate Impact” on savings (Eq. 50). The red line is however not zero, indicating that savings would have increased due to the distributional effects even if the real interest rate did not change. This “Distributional Impact” on savings (Eq. 49) is reflected by the difference between the red line and the solid black horizontal line through zero.

But while total Savings increase, capital still falls below its steady-state level since the increase in savings is smaller than the increase in additional government debt,  $K - K^{Crowd} < 0$  (Eq. 47). This crowding out of capital (the difference between the yellow “Capital, No crowd out” and the black dotted line “Capital,  $K_t$ ”) with deficit financing explains the drop in investment and its negative contribution to aggregate demand and the multiplier. The right panel also shows that  $r^k/w$  in the counterfactual crowding out experiment where debt is held constant (Eq. 46) is lower than in the benchmark, since aggregate savings have to increase less as no additional debt needs to be absorbed in this experiment.

### 4.1.3 Taylor Rule

The empirical analysis in Section 3.4 implies that monetary policy in the data accommodates fiscal policy by keeping interest rates unchanged. For purposes of comparison in this section we measure the fiscal multiplier when monetary policy follows a Taylor rule instead of an interest rate peg. Specifically, we consider the monetary authority which sets the nominal interest rate according to:

$$i_{t+1} = \max(X_{t+1}, 0), \quad (59)$$

where

$$X_{t+1} = \left(\frac{1}{\zeta}\right) \left(\frac{P_t}{P_{ss}}\right)^{\phi_1(1-\rho_R)} \left(\frac{Y_t}{Y_{ss}}\right)^{\phi_2(1-\rho_R)} [\zeta(1+i_t)]^{\rho_R} - 1.$$

We follow the literature in setting  $\rho_R = 0.8$ ,  $\phi_1 = 0.5$ ,  $\phi_2 = 0.25$  and  $\zeta = 1/(1+r_{ss})$ .

We find that if monetary policy follows a Taylor rule, the impact multiplier with deficit financing drops to 0.66 and the cumulative multiplier to 0.29 (see Figures 7(a) and A-7). The lower multiplier is a result of a drop in output due to tighter monetary policy. A higher nominal interest rate contracts consumption demand and thus reduces income, which in turn reduces savings and investment. Lower investment reduces income and thus contracts consumption which again reduces income and so on. The consumption and investment responses reinforce each other and together determine an effect of monetary policy on output in our experiment: a 25 base point increase in the real rate (i.e. annually 100 base points) leads on impact to a 1% drop in output. This is an expected magnitude for the drop in output in response to an increase in the real return on bonds and is in line with the findings in e.g., Kaplan et al. (2018). Combining the expansionary output effects of the fiscal stimulus and the contractionary effects of monetary policy yields a multiplier of 0.66.

The same arguments apply to tax instead of deficit financing, in which case the multiplier falls to 0.54 (see Figures 7(b) and A-8). The Taylor rule induced decline in the size of the multiplier is smaller with tax than with deficit financing not because monetary policy is less effective but because monetary policy responds less if tax financing is used. Indeed, the real interest rate elasticity of output is unchanged but the smaller price responses with tax financing require a much smaller increase in interest rates.

As a result, while the impact multipliers are very different between financing schemes with fixed nominal interest rates, they become quite similar when a Taylor rule is used. With

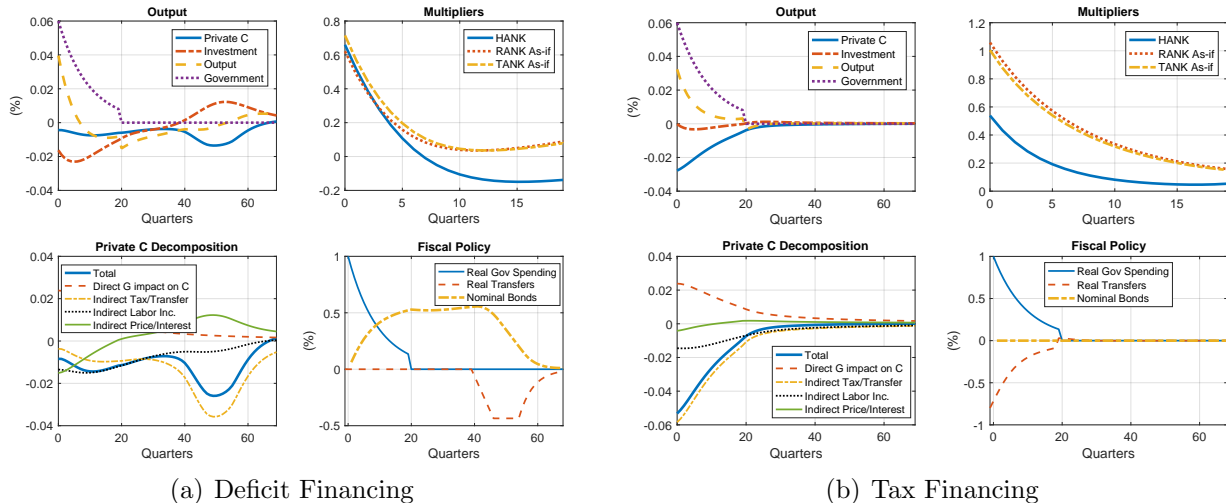


Figure 7: Fiscal Multiplier and Aggregate Consumption: Taylor Rule

deficit financing, fiscal stimulus leads to a strong response of monetary policy, which largely undoes the stimulus. With tax financing, fiscal policy is not very stimulative to begin with and monetary policy remains largely unchanged.

#### 4.1.4 Transfer Multiplier

In this section we consider the multiplier in response to a one percent increase in government *transfers*, again for 20 periods and with persistence  $\rho_T = 0.9$ . We assume that nominal government spending adjusts to keep real government spending constant in response to the innovation in transfers. We allow the government to deficit-finance the increase in transfers following the same financing scheme as above by first increasing government debt, then by decreasing transfers starting 40 quarters later to pay back the debt.

The impulse responses plotted in Figure A-9 are qualitatively and quantitatively similar to the impulse responses to a deficit-financed increase in government spending. Private consumption rises more, however, when transfers increase than when spending increases. This can be understood because, an increase in transfers benefits low income, high MPC households since their disposable income increases more (in percentage terms) than that of high income, low MPC households. This implies an impact multiplier of 0.66 using the definition of  $m_0^{IM}$  (Eq. 52) but with  $G$  replaced by  $T$ . Aggregate consumption, the sum of private and government consumption, however, increases less when transfers increase than when spending increases, simply because government spending is unchanged in the former case. As a result, income



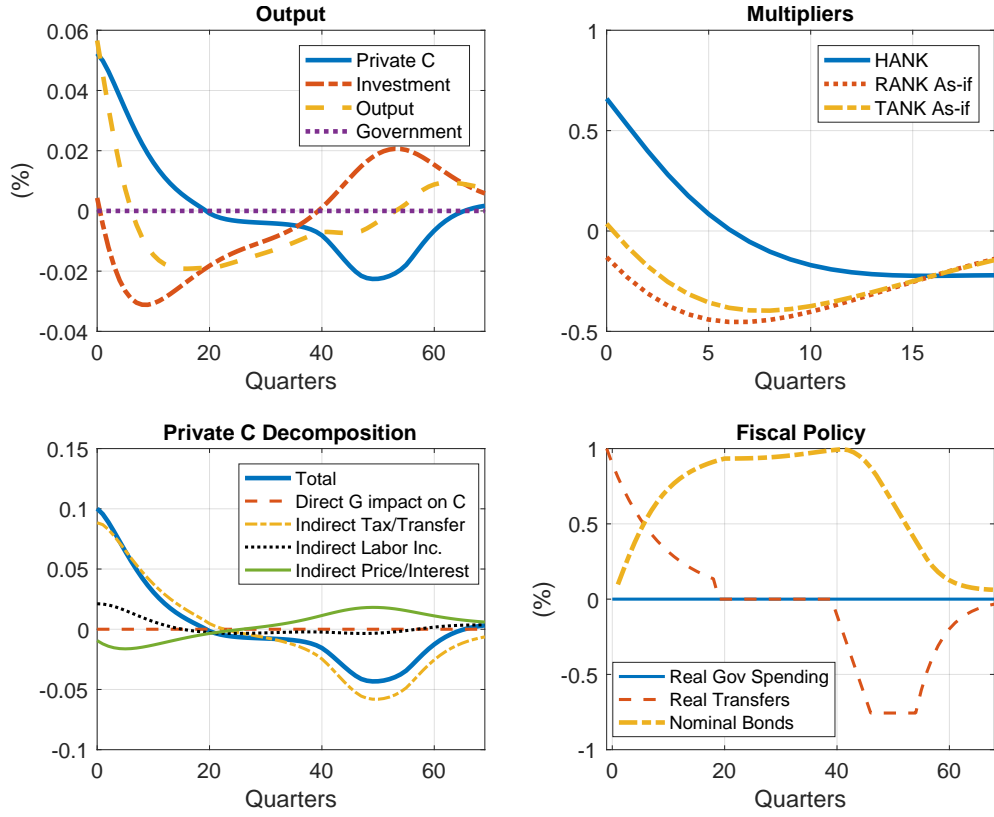


Figure 8: Transfer Multipliers (Deficit Financed)

increases by more in the latter case, implying higher savings and investment such that the output increase is larger.

The cumulative multiplier is now defined as

$$\bar{M} = m_{\infty}^{PV} = \frac{\sum_{t=0}^{\infty} \tilde{\beta}^t \left( \frac{Y_t}{Y_{ss}} - 1 \right) Y_{ss}}{\sum_{t=0}^{19} \tilde{\beta}^t \left( \frac{T_t P_{ss}}{P_t T_{ss}} - 1 \right) T_{ss} / P_{ss}}, \quad (60)$$

where in the denominator we sum only over the first 20 periods since summing to infinity would take the repayment period into account and thus would render the denominator equal or close to zero. Note that this definition does not change the numbers for the cumulative multiplier reported for other experiments. The cumulative multiplier ends up being quite small around 0.1 because the future decrease in transfers needed to return nominal government debt to its steady state level is sufficiently contractionary to almost offset the contemporaneous gains. Measuring the welfare implications of this policy, we find that welfare falls by  $-0.00067\%$  in consumption equivalents because the welfare gain in the initial periods is eventually (slightly)

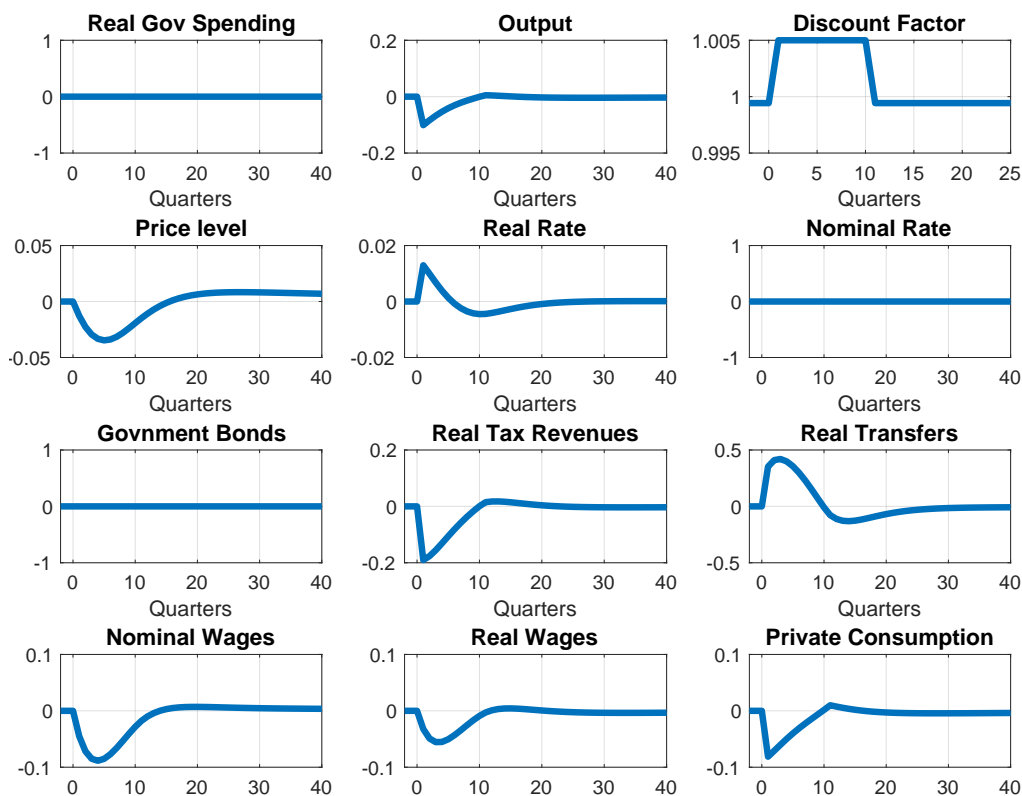


Figure 9: Economy in a Liquidity Trap

outweighed by the later welfare losses.<sup>19</sup>

Increasing transfers is purely redistributive and thus has no direct effects in the RANK model. The as-if RANK multiplier is, however, negative due to an increase in real interest rates. The TANK model features a redistributive channel, implying a larger multiplier than in the RANK case. The TANK multiplier is much smaller than our incomplete markets model's one since the TANK model lacks the anticipation and propagation effects operating in a HANK model as we explained in Section 3.3. Figure 8 shows the results.

## 4.2 Fiscal Multiplier in a Liquidity Trap

In this section we explore the extent to which the size of the multiplier may vary with other shocks hitting the economy. In particular, we measure the government spending multiplier when the economy suffers a large demand shock. Therefore, we generate a liquidity trap

<sup>19</sup>For completeness we report the welfare losses for an increase in government spending:  $-0.0065\%$  for deficit financing and  $-0.0063\%$  for tax financing. Note that these numbers set the welfare gains from higher government consumption to zero and one should therefore be cautious about using these numbers to assess the welfare consequences of a fiscal stimulus.

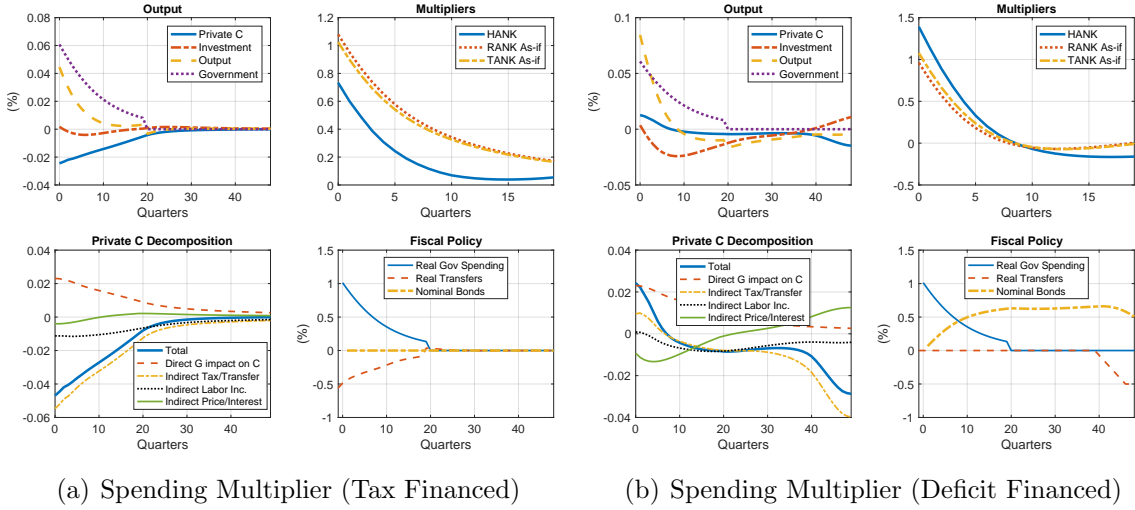


Figure 10: Fiscal Multipliers in a Liquidity Trap

in the model, where the ZLB on nominal interest rates is binding. In doing so we follow Cochrane (2017) and increase the quarterly discount factor  $\{\beta_t\}_{t=0}^9$  by 50 basis points.<sup>20</sup> We follow Christiano et al. (2011) in setting the duration of this shock to 10 quarters and also in introducing an investment wedge,  $\tau_t^I$ , to the mutual fund problem, set to 3.6% for 10 quarters.<sup>21</sup> All other parameters are unchanged.

We then feed the  $\{\beta_t, \tau_t^I\}_{t=0}^9$  series into our model and calculate the response of the economy, which is shown in Figure 9. The resulting recession is quite large as output initially drops by about 7 percent. We assume that the government keeps the real value of government spending and the nominal value of government debt constant and adjusts lump-sum transfers period-by-period to satisfy its budget constraint.

Under this scenario, we compute the effect of a simultaneous (at the same time as the liquidity trap starts) 1% increase in nominal government spending under the same balanced-budget and deficit financed schemes outside the liquidity trap. Thus, we can compute the fiscal multiplier as the percent increase in output under this scenario, relative to the benchmark with no increase in spending, divided by the relative percent differences in government spending. The multipliers are plotted in Figure 10. Figure 11 shows the transfer multiplier, where only deficit financing is meaningful. The corresponding impulse responses are reported in Figures

<sup>20</sup>In a linearized complete-markets model this would generate a fall in the natural real rate of interest — the real interest rate in a world with flexible prices and wages — of 2 pp (annualized) for 10 quarters and then a return to zero afterwards.

<sup>21</sup>In the liquidity trap if the mutual fund invests in  $K_{t+1}$  units of capital it will receive  $(1 + (1 - \tau_{t+1}^I)(r_{t+1}^k - \delta))K_{t+1}$ .

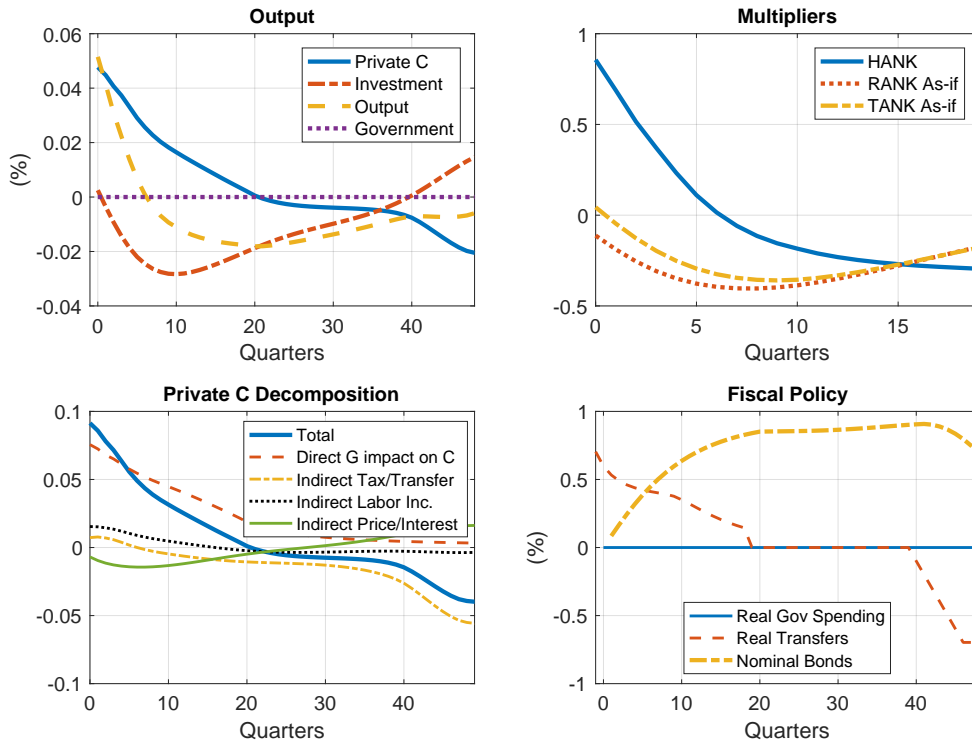


Figure 11: Transfer Multiplier in a Liquidity Trap (Deficit Financed)

A-12 through A-14.

Both the spending and the transfer multipliers in a liquidity trap are not very different from their counterparts in the benchmark experiments outside the liquidity trap when we keep the nominal interest rate constant. For deficit financing, the impact spending multiplier,  $m_0^{IM} = 1.39$  (1.34 in the benchmark) and the cumulative one  $\bar{M} = 0.51$  (0.55 in benchmark). For tax financing, the impact multiplier,  $m_0^{IM} = 0.73$  (0.61 in the benchmark) and the cumulative one  $\bar{M} = 0.48$  (0.43 in the benchmark). The transfer multiplier,  $m_0^{IM} = 0.86$  (0.66 in the benchmark) and the cumulative one  $\bar{M} = -0.11$  ( $-0.3$  in benchmark). The welfare loss of the transfer stimulus remains quite low,  $-0.0058\%$  ( $-0.00067$  in benchmark) in consumption equivalents.

Since values are close in a demand-induced liquidity trap and outside a liquidity trap and monetary policy is the same across the two experiments, our findings imply that there is not much state-dependence in the multiplier. However, when outside the liquidity trap a Taylor rule is implemented, we find a much smaller multiplier for deficit financing, on impact  $m_0^{IM} = 0.66$  and cumulative  $\bar{M} = 0.29$  (see Section 4.1.3). But if one contrasts these multipliers to those obtained in the liquidity trap, this substantial difference is due to a different monetary

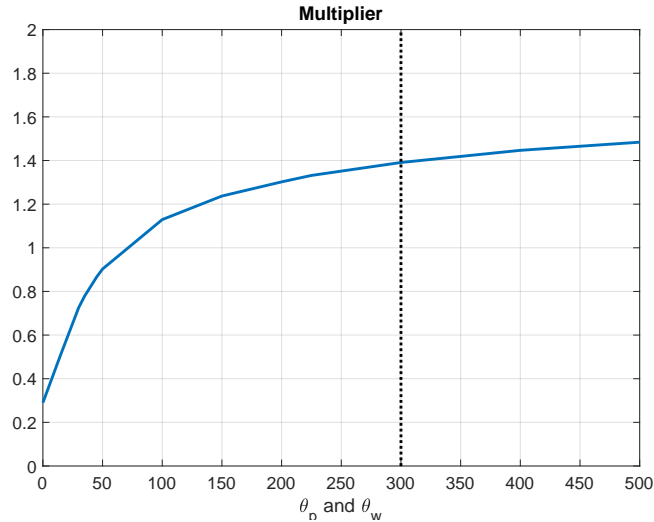


Figure 12: Liquidity Trap and Degree of Rigidities  $\theta_p = \theta_w$ : Spending Multiplier at  $t = 0$ .

policy and not due to model non-linearities that could imply state-dependence.

#### 4.2.1 The Degree of Price and Wage Rigidities

In New Keynesian complete markets models, the size of the multiplier increases if prices become more flexible. There is, however, a discontinuity at fully flexible prices. The multiplier is smaller than one if prices are fully flexible, but is arbitrarily large if the degree of rigidity is close to but not equal to zero. The reason is the response of inflation and thus real interest rates when prices are rigid. The more flexible prices are, the larger is the deflation in a liquidity trap and the larger is the inflation increase in response to the stimulus. Since the inflation response is one-to-one related to the output gain, the multiplier is decreasing and the inflation response is increasing in  $\theta_p = \theta_w$ .

In contrast, with fully flexible prices, an increase in the discount factor stimulates savings and thus implies a fall in the real interest rate or equivalently an increase in inflation. A fiscal stimulus raises the real interest rate in a fully flexible price economy that is the inflation rate falls and the multiplier is smaller than one.

Figure 12 shows the multiplier in our HANK economy when we vary  $\theta_p = \theta_w$  between 0 (flexible prices and wages) and 500 (very rigid wages and prices). Two stark differences to the complete markets version emerge. The multiplier is now decreasing instead of increasing when rigidities are reduced, i.e.  $\theta_p = \theta_w$  falls. And the multiplier converges to the flexible price outcome when rigidities vanish, i.e.  $\theta_p = \theta_w$  converges to zero. The discontinuity and

the associated exploding multipliers which characterize the standard complete markets model disappear. The reason for these different findings is price level determinacy in our model which implies that for small but non-zero rigidities the real interest rate and inflation responses are close to the flexible price outcome.

## 5 Conclusions

This paper develops a quantitative laboratory that contains the essential elements needed to evaluate the effects of fiscal stimulus on the economy. At its core is a dynamic Heterogeneous Agent New Keynesian (HANK) model with incomplete markets, capital, price and wage rigidities, and the government budget constraint specified at least partially in nominal terms. These features interact in meaningful ways, but their primary roles are as follows.

1. **Market incompleteness** gives rise to large and heterogeneous across households MPCs as observed in the data. As Ricardian equivalence does not hold, government policies have distributional consequences, which are essential in determining their impact. Moreover, the incomplete-markets model allows conducting a meaningful analysis of transfer multipliers, which is vital because many stimulus policies take the form of transfers rather than an increase in spending.
2. Matching the empirically relevant degree of **price rigidities** ensures that the model captures the stimulative effect of additional spending on aggregate demand.
3. The model is **fully dynamic** so that demand is also governed by intertemporal motives, which has significant consequences for assessing the equilibrium impact of policies. For example, households save some of the additional income induced by the current fiscal stimulus for future consumption. This implies higher aggregate demand and thus higher income in the future, which relaxes households' precautionary savings motive and allows them to consume even more today. The dynamic model not only captures the equilibrium magnitude of these effects but also allows households to take into account that, e.g., a deficit-financed spending stimulus leads to either lower transfers, higher taxes or lower government expenditure in the future.
4. Introducing **nominal government bonds** ensures that the model delivers a uniquely

determined price level (Hagedorn, 2016, 2018), implying a unique fiscal multiplier for arbitrary combinations of monetary and fiscal policies of interest, including an interest rate peg. This significantly expands the set of policies that can be evaluated.

5. Modeling **capital accumulation** is important to capture an investment channel of fiscal stimulus which complements the consumption demand channel. A stimulus increases demand for both employment and investment. A higher investment demand stimulates the production of investment goods which leads to more demand for employment and investment and so on until prices adjust and an equilibrium is reached. Having capital in the model also allows to capture the consequences of crowding-out of investment through higher government debt under deficit-financing (Barro and Redlick, 2011).
6. Introducing **wage rigidities** into the model is important for three reasons. First, it allows us to match the empirically relevant dynamics of labor incomes, important for measuring the dynamics of demand. Second, rigid wages constraint the volatility of firms' profits, the distribution of which has first-order effects. Finally, the equilibrium increase in investment depends on the elasticity of interest rates and wages as those prices determine firms' substitution between labor and capital.

We establish that the model replicates the size of fiscal multipliers that can be robustly estimated in the data when empirical properties of how the spending is financed and the response of monetary policy are replicated in the model. Having verified that the model represents a good quantitative laboratory, we use it to conduct several controlled experiments that reveal the size of the fiscal multiplier for various configurations of policies and economic states, which are difficult to measure directly in the data. For every experiment we conduct using our model, we also perform a parallel experiment using a Representative Agent and Two Agent New Keynesian (RANK and TANK) models to clarify why the richness of our framework leads to often very different conclusions. The key lessons that we learned about fiscal stimulus from our experiments are as follows.

The timing of expected repayment plays an essential role in determining the size of the multiplier: the deficit-financed government spending multiplier is significantly larger than the tax-financed one, 1.34 vs. 0.61, respectively. Distributional effects are also crucial as they contribute 0.37 to the 1.34 multiplier when spending is deficit-financed and contribute a

negative number,  $-0.5$ , to the multiplier of  $0.61$  in the case of tax financing. The response of monetary policy is crucial for the effectiveness of a fiscal stimulus. When we consider monetary policy following a Taylor rule, instead of a nominal interest rate peg, and thus translating output and price increases induced by a fiscal stimulus outside of the liquidity trap into higher nominal and real rates, the effectiveness of fiscal stimulus falls considerably, with multipliers of  $0.54$  and  $0.66$  in cases of tax and deficit financing, respectively. This happens because higher rates contract demand through the intertemporal substitution channel and because they redistribute towards asset-rich low MPC households, implying a further contraction in aggregate demand.

In the liquidity trap, the tax and deficit-financed multipliers,  $1.39$ , and  $0.73$ , respectively, are similar to those outside of the liquidity trap for the same monetary policy response (fixed nominal interest rate), implying the lack of strong state-dependence. Thus, if monetary policy is described by a Taylor rule, the difference in the size of the multiplier in and outside of the liquidity trap is driven largely by the different response of the monetary policy.

We also find that our model does not feature liquidity trap puzzles documented in RANK models. Hagedorn et al. (2018) find that these puzzles carry over to the fully nonlinear RANK model as well as to a HANK model with real debt. We conclude that it is not the nonlinearity of our model that overcomes the puzzles, but it is the combination of nominal government debt and market incompleteness. Specifically, we show that in contrast to RANK models, the multiplier in a liquidity trap becomes smaller as prices become more flexible, and there is no discontinuity at fully flexible prices.

Considering transfer multipliers, which are a standard policy tool but received only scant attention in the literature, we find that a deficit-financed increase in lump-sum transfers by 1 dollar increases output on impact by 66 cents. Temporary increases in transfer payments generate winners and losers, and we find that the transfer policy leads to a small welfare loss since the initial gains in welfare are outweighed by the losses later on. The same transfer experiment in complete markets does not alter welfare due to Ricardian equivalence.

Some additional analysis of fiscal multipliers is relegated to Appendix I. It reveals that, in contrast to RANK models, a pre-announced future spending increase is less effective than an unexpected stimulus. Moreover, the multiplier is decreasing in the size of the spending stimulus. Finally, we document the intertemporal trade-off of a stimulus. Measuring the



effectiveness of the stimulus using the impact and the cumulative multipliers does not always lead to the same conclusions. For example, while the impact multiplier increases from 1.06 to 1.65 when we increase the persistence of stimulus spending from 0.1 to 1, the cumulative multiplier falls from 1.18 to 0.33. This difference reflects the short-run/long-run trade-off of a stimulus, which are absent from the standard Keynesian cross logic but which are present and can be measured in our dynamic model. A short-run multiplier larger than one suggests that a fiscal stimulus could be an effective policy tool in a recession, and the long-run results suggest that this policy is costly but that these costs only occur when the economy has already recovered.

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## I Further Analysis

In this Appendix we assess how the size of the fiscal multiplier depends on the timing of spending (“forward spending”), on the scale of spending and on the persistence of the stimulus in Sections I.1, I.2 and I.3. In Section I.4 we assess the consequences of cutting government spending  $G$  instead of transfers to pay back the debt.

The results of this analysis yield three main conclusions:

1. **A pre-announced future spending increase is less effective than an unexpected stimulus.** This is in contrast to RANK models, in which the further the spending is in the future the larger is its impact today.
2. **The multiplier is decreasing in the size of the spending stimulus.** One should be cautious about proposals that for government spending to be effective it has to be large. Interpreting higher effectiveness as a higher multiplier, we find that scaling up the stimulus decreases its effectiveness.
3. **The intertemporal trade-off of a stimulus: Potentially large initial gains come with potentially larger cost later on.** Measuring the effectiveness of spending using the impact multiplier, a standard statistic reported in the literature, or the cumulative multiplier, i.e., the discounted average multiplier, does not always point in the same direction. For example, while the impact multiplier increases from 1.06 to 1.65 when we increase the persistence of stimulus spending from 0.1 to 1, the cumulative multiplier falls from 1.18 to 0.33. This difference reflects the short-run/long-run trade-off of a stimulus, which are absent from the standard Keynesian cross logic but which are present and can be measured in our dynamic model.

### I.1 Forward Spending

Farhi and Werning (2016) show that in complete markets New Keynesian models the further the spending is in the future the larger is the impact, suggesting that “forward-spending” is an effective fiscal policy tool. In contrast, our analysis implies that the multiplier becomes smaller if the spending is pre-announced to occur at a future date. For concreteness, in Figure A-1 we assess the effects of stimulus pre-announced to occur 4 quarters ahead (the corresponding impulse responses are plotted in Figure A-10 in the appendix). The additional spending is deficit financed. The price level now increases gradually in anticipation of the future increase in government spending implying that initially output and consumption fall before increasing

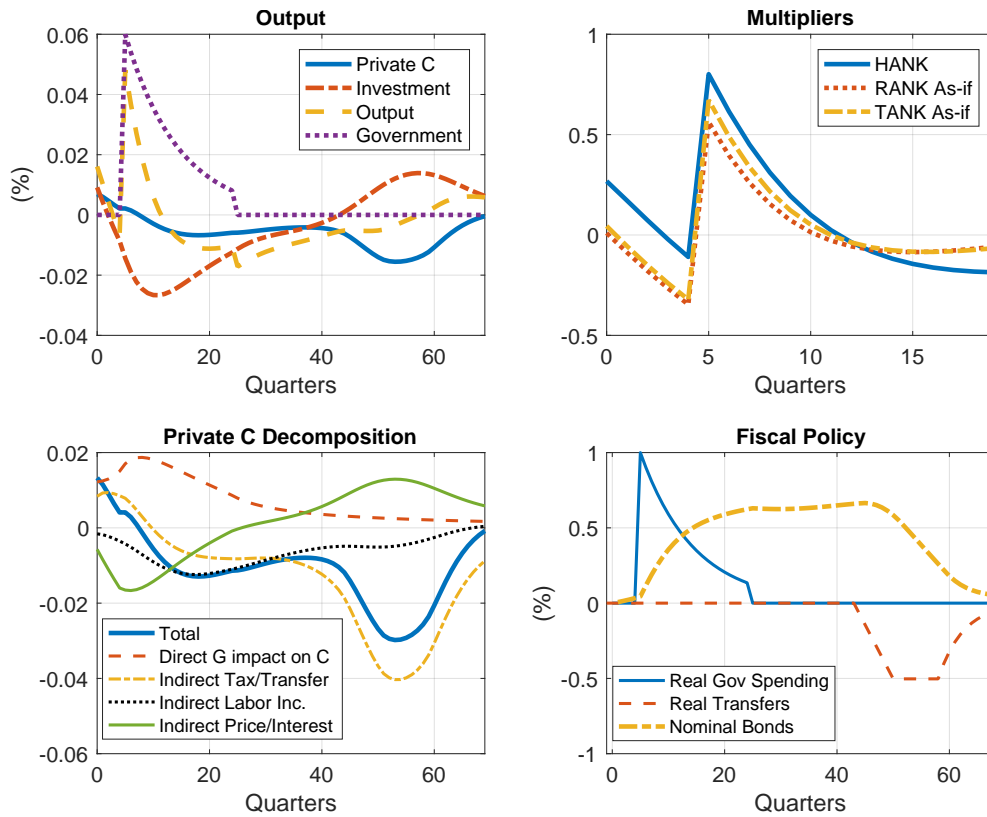


Figure A-1: Future (4 Quarters Ahead) Spending: Deficit Financing

at the time of the spending increase 4 quarters in the future. However, the initial drop in output makes households shift consumption to these earlier periods which dampens the fall in consumption initially but at the same time lowers their demand at the time of the actual spending increase in quarter 4. As a result, the increase in consumption as well as the multiplier at that time are smaller than the corresponding multiplier in the case when the stimulus occurs immediately and is deficit financed. The investment response is, however, larger since debt only starts to increase when spending actually happens, implying that there is no crowding out and thus higher investment in the first periods with future spending.

## I.2 Scale Effects

We now assess the dependence of the size of the multiplier on the scale of the government spending or transfer stimulus. Panel (a) of Figure A-2 shows the government spending multiplier for one percent intervals between 1% and 10% increase. Panel (b) of Figure A-2 shows the same for the transfer multiplier again for one percent intervals between 1% and 10% increases.

The fall in the multiplier as we increase the scale of spending or transfers is expected in light of our results in Figure 1 in Section 3.3. Those partial equilibrium experiments showed

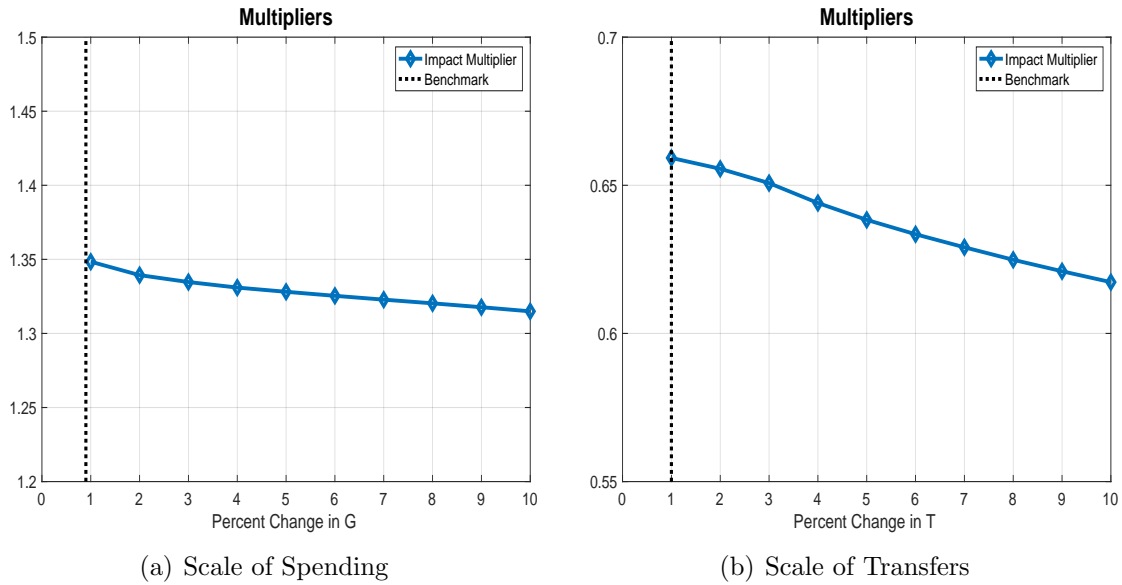


Figure A-2: Multiplier as a Function of Scale of Spending/Transfers (Deficit Financing)

that the propensity to consume falls in the size of the transfer households receive, implying that a larger scale of spending or transfer leads to lower MPCs. The Keynesian cross logic implies that in equilibrium these lower MPCs translate into a lower multiplier.

### I.3 More Persistent Spending

In this section we study how the persistence of government spending affects the multipliers. As before, we consider a persistent increase in nominal government consumption by one percent that decays over the subsequent 19 quarters at varying rates of  $1 - \rho_g$ , where  $\rho_g \in [0, 1]$ , per quarter and then reverts to its steady-state level. Thus, if  $\rho_g = 0$ , stimulus is purely transitory and takes place in one period only, while if  $\rho_g = 1$ , nominal government spending is increased by the same amount for all 20 quarters. Regardless of the persistence of spending, the policy is deficit financed according to the same scheme as before with repayment starting 40 quarters after the initial increase in spending.

Figure A-3 shows the impact and the cumulative multiplier for various degrees of persistence. As our previous analysis of household demand in Section 3.3 suggests, the impact multiplier  $m_0^{IM}$  is increasing in  $\rho_G$ . A higher persistence means higher government spending and thus higher income in future periods. This relaxes households' precautionary savings motives and increases their period 0 consumption demand, implying a higher initial multiplier,  $m_0^{IM}$ . When the spending becomes temporary,  $\rho_G \rightarrow 0$ , these dynamic interactions are minimized and the multiplier approaches 1. The dynamic effects are maximized when the spending becomes almost permanent,  $\rho_G \rightarrow 1$ , and the multiplier reaches 1.65. Note,



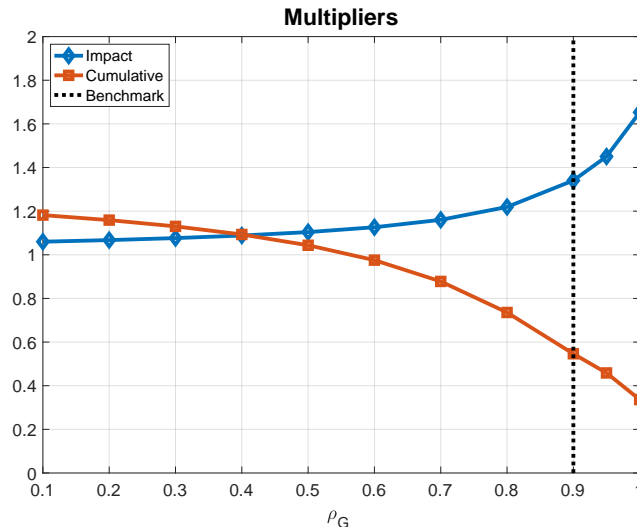


Figure A-3: Multipliers: Persistence  $\rho_G$  of Spending

however, that the cumulative multiplier moves in the opposite direction as the persistence of spending increases. As the spending becomes almost permanent, the cumulative multiplier falls significantly to a value of less than 0.4. This finding of a decreasing cumulative multiplier is a combination of the two previous results. First, our analysis in Sections 3.3 and I.2 shows that the multiplier is falling in the scale of government spending. As a higher  $\rho_G$  translates into a larger increase in total government spending, the multiplier decreases. Second, a higher  $\rho_G$  means more spending in future periods while spending in the initial period is unchanged. As Section I.1 shows, future spending is less effective than current spending, making the multiplier smaller again. This effect is reinforced here since a spending increase in, say, period 20 is already paid back starting in period 40. That is the repayment starts 20 periods after the stimulus whereas it starts 40 periods after the stimulus in the benchmark and in Section I.1. Since an earlier repayment is more contractionary, the multiplier falls.

## I.4 Repayment through $G$

In this section we consider the multiplier in the deficit-financed case, but instead of cutting transfers in order to bring nominal debt back to its steady-state value, we instead cut government spending  $G$  following the same scheme that was previously used for transfers.

The results of this experiment are plotted in Figure A-4. They are qualitatively and quantitatively similar to those obtained in Section 4.1.2 for when the deficit-financed increase in government spending is paid back by eventually cutting transfers. Output rises more, however, when spending is used to repay the debt than when transfers are used. The reason is that repayment through  $G$  or  $T$  has different distributional consequences. A reduction in  $G$  reduces labor income and thus hurts workers proportionally to their productivity

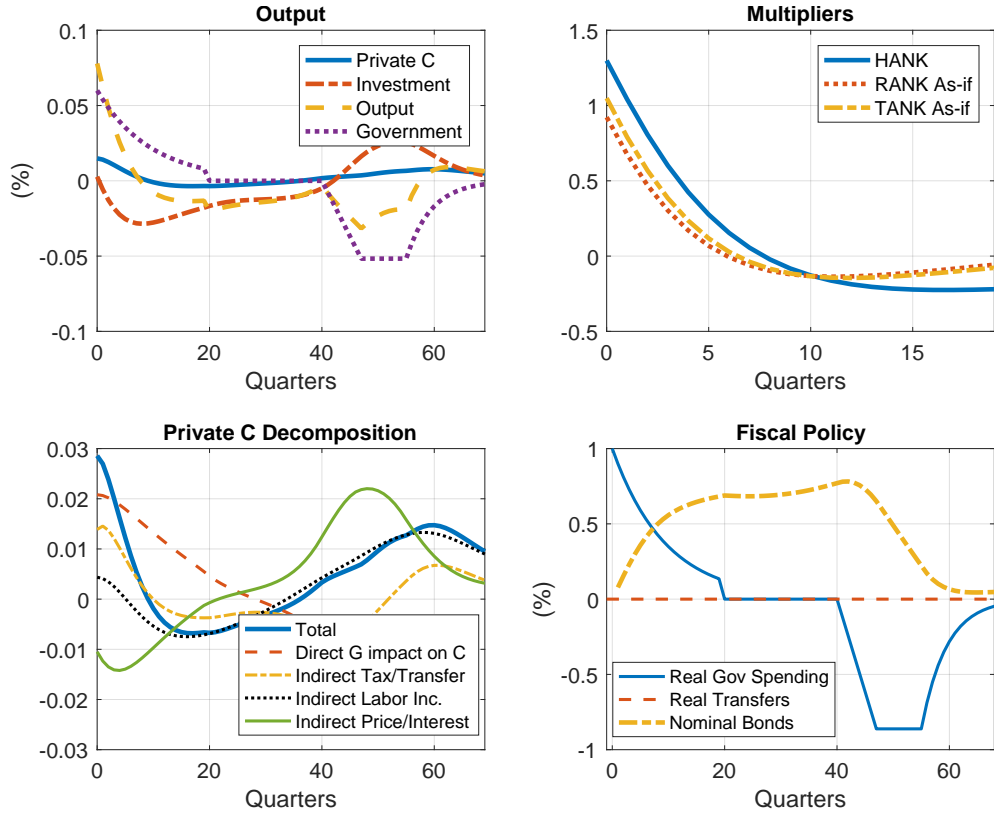


Figure A-4: Spending Multipliers (Deficit Repaid Using  $G$ )

whereas a reduction in transfers hurts workers independently of their productivity. Since high productivity workers have high income and low MPCs while low productivity workers have low income and high MPCs, a reduction in  $T$  hurts high MPC households more than a reduction in  $G$  does. At the time when the repayment starts, these arguments imply that consumption is higher with repayment through  $G$  than through  $T$ . The flip side is that the low MPC households are the savers and fewer resources in their hands means a fall in savings and investment at the time when repayment through  $G$  instead of  $T$  starts. Since a reduction in  $G$  reduces aggregate demand one-for-one, the negative output effect is much larger when repayment is through  $G$ . The differences between the two financing schemes at the time of repayment feed back into earlier periods. The same arguments explain why consumption is again higher but investment is again lower with repayment through  $G$ . Combining the effects on consumption and investment leads to a slightly smaller impact multiplier when the repayment is through  $G$ . Figure A-11 shows the impulse responses.

Analogously to transfer multipliers, the cumulative multiplier is defined as

$$\bar{M} = m_{\infty}^{PV} = \frac{\sum_{t=0}^{\infty} \tilde{\beta}^t \left( \frac{Y_t}{Y_{ss}} - 1 \right)}{\sum_{t=0}^{19} \tilde{\beta}^t \left( \frac{G_t P_{ss}}{P_t G_{ss}} - 1 \right)} \frac{Y_{ss}}{G_{ss}/P_{ss}}. \quad (\text{A1})$$

where in the denominator we again sum only over the first 20 periods. The larger output drop at the time of repayment explains why the cumulative multiplier is negative  $-0.15$  when repayment is through  $G$  while it is positive for repayment through  $T$ .

## II Figures

### II.1 Impulse Responses: Baseline Experiments, Section 4.

Figure A-5: Impulse Response to a 1% Increase in Nominal Government Spending.  
Tax Financing (Constant Nominal Debt).

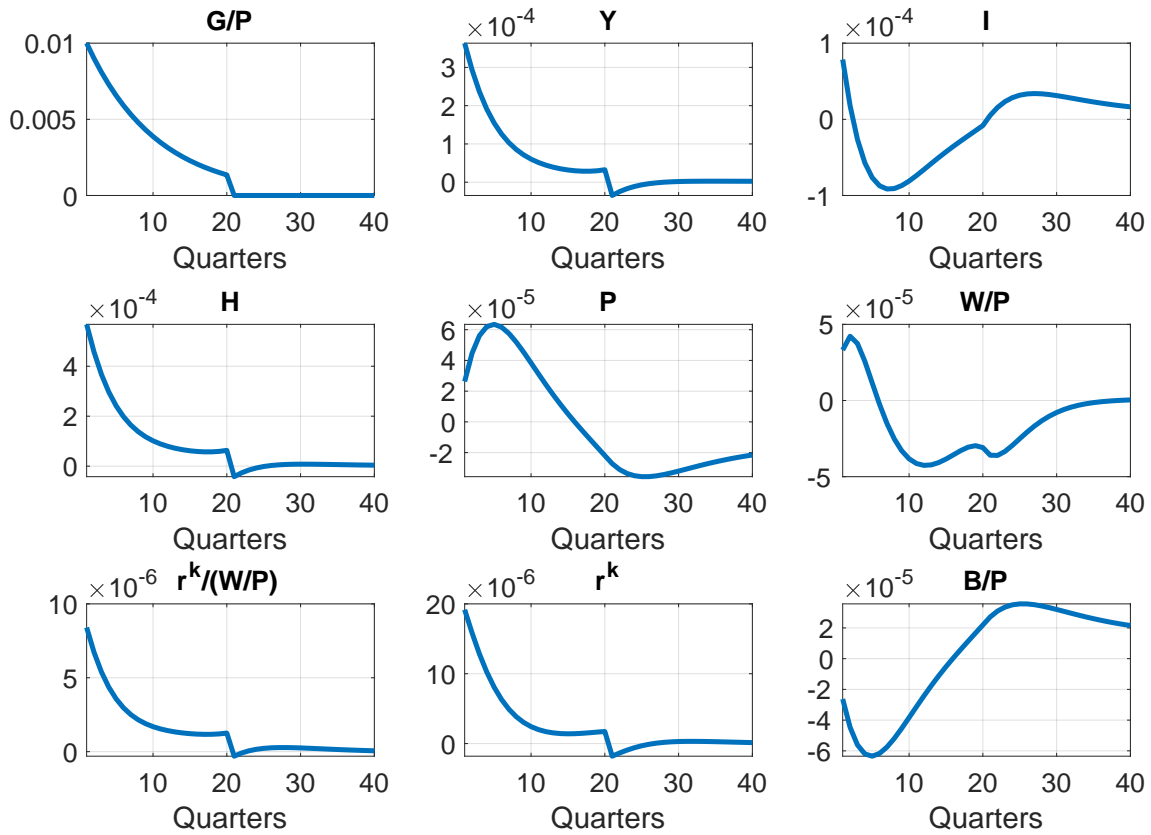
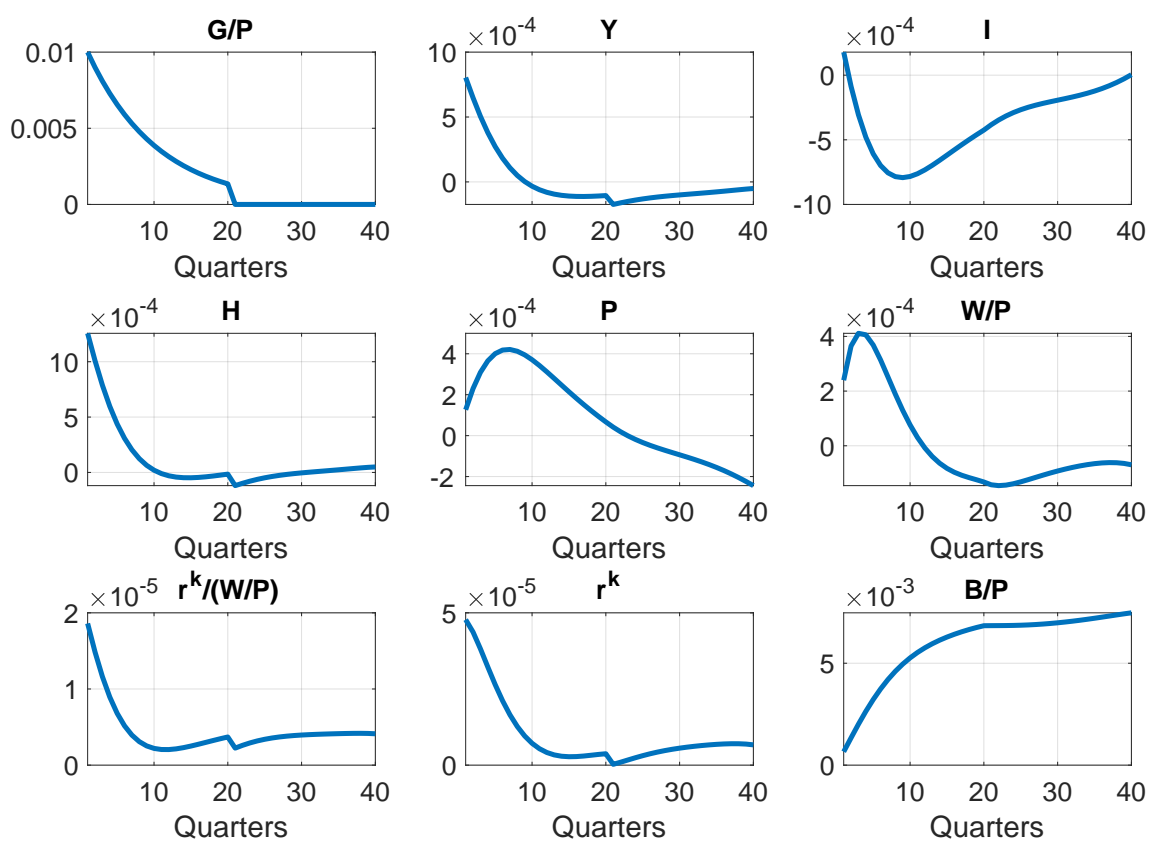


Figure A-6: Impulse Response to a 1% Increase in Nominal Government Spending.  
**Deficit Financing.**



## II.2 Impulse Responses: Taylor Rule, Section 4.1.3.

Figure A-7: Impulse Response to a 1% Increase in Nominal Government Spending.  
Taylor Rule, Deficit Financing.

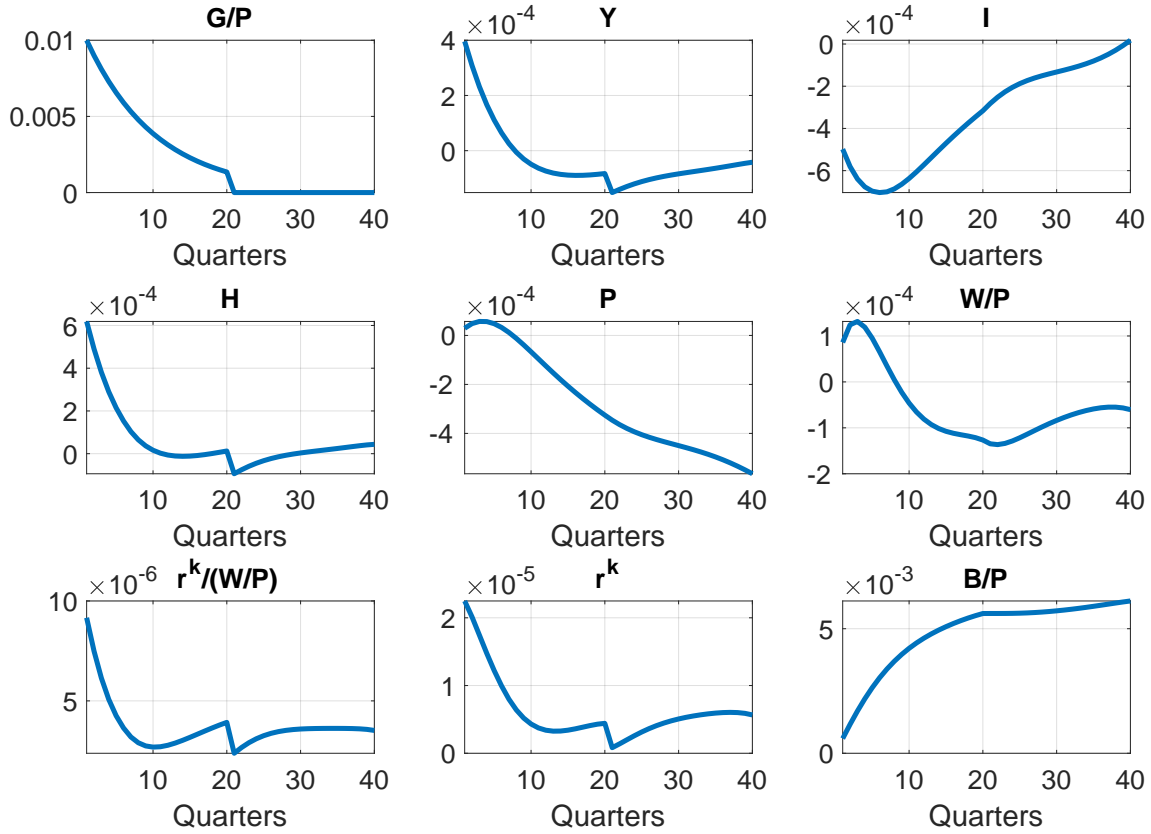
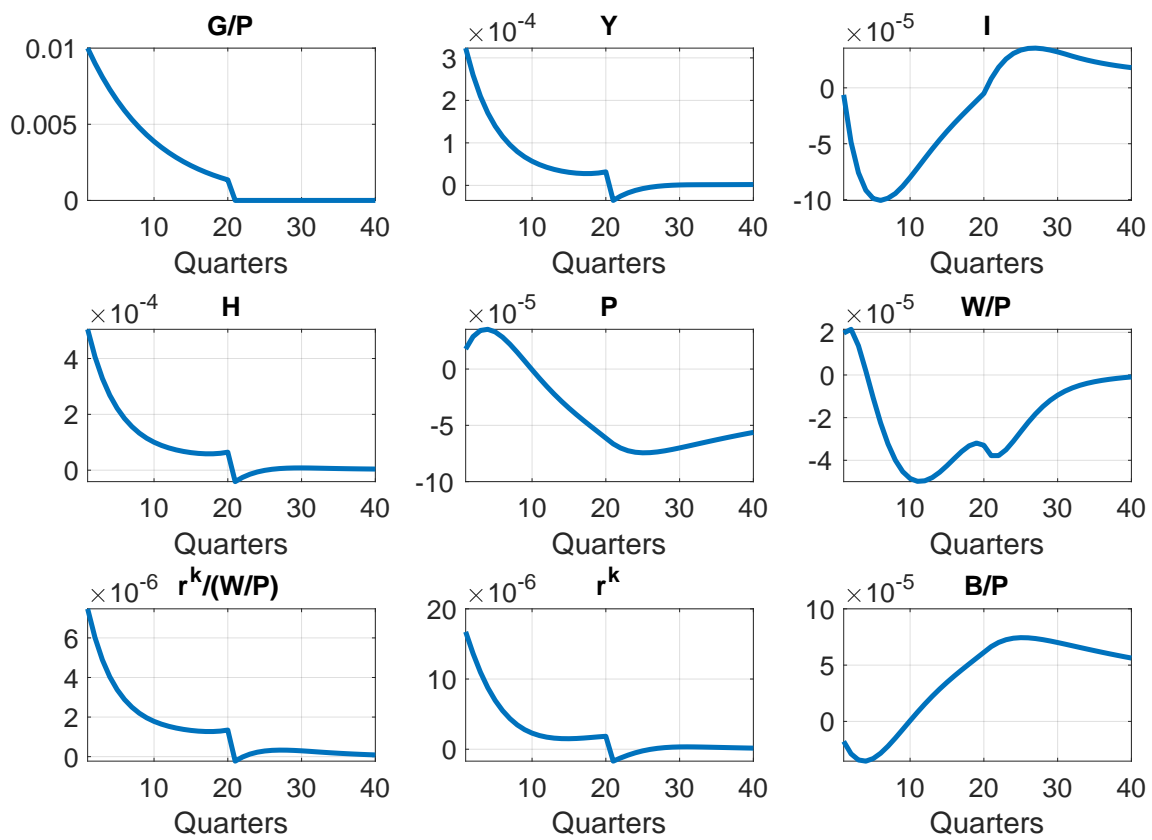
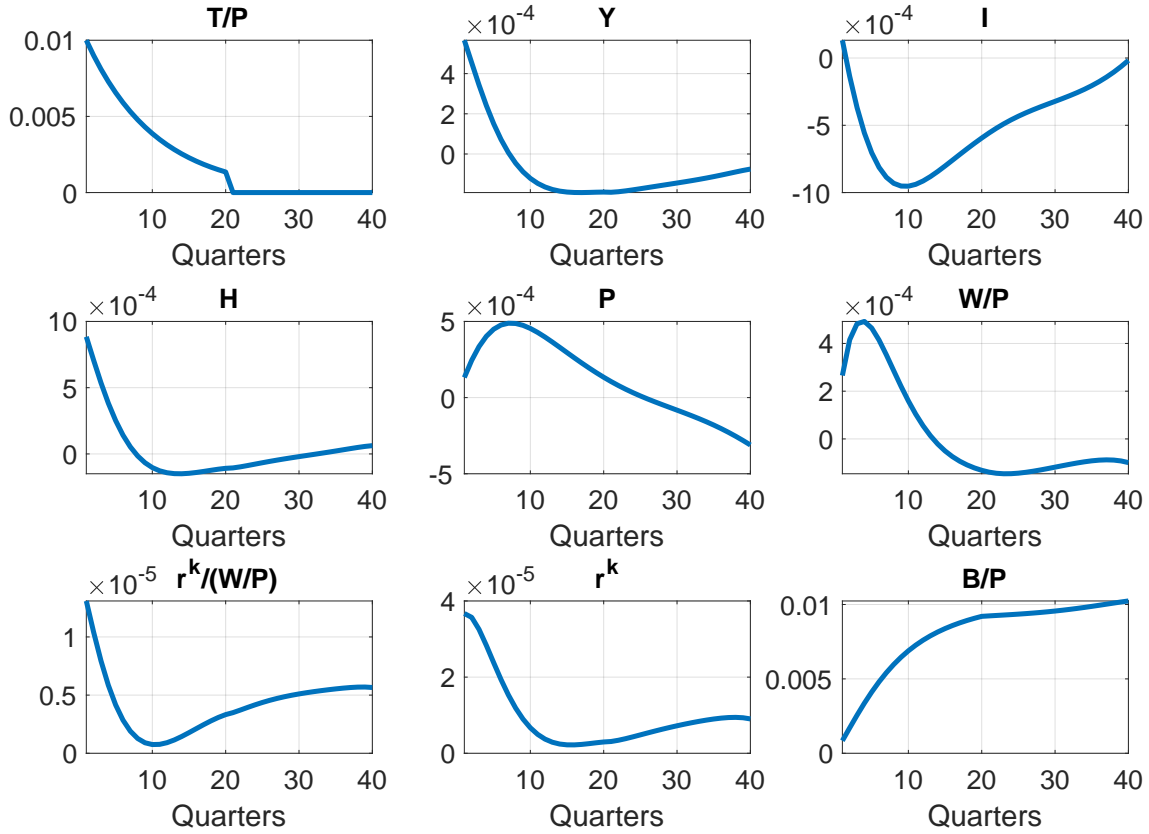


Figure A-8: Impulse Response to a 1% Increase in Nominal Government Spending.  
**Taylor Rule, Tax Financing** (Constant Nominal Debt).



### II.3 Impulse Responses: Transfer Multiplier, Section 4.1.4.

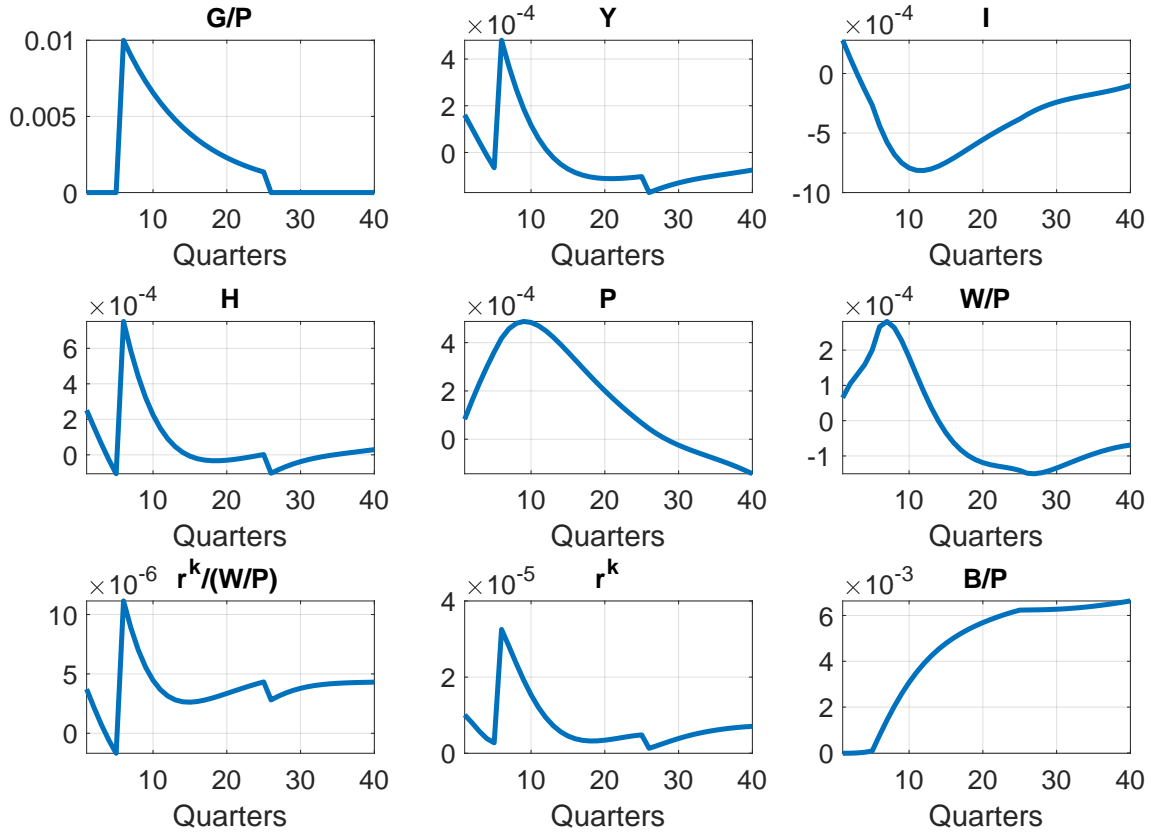
Figure A-9: Impulse Response to a 1% Increase in Nominal Government *Transfers*.  
Deficit Financing.





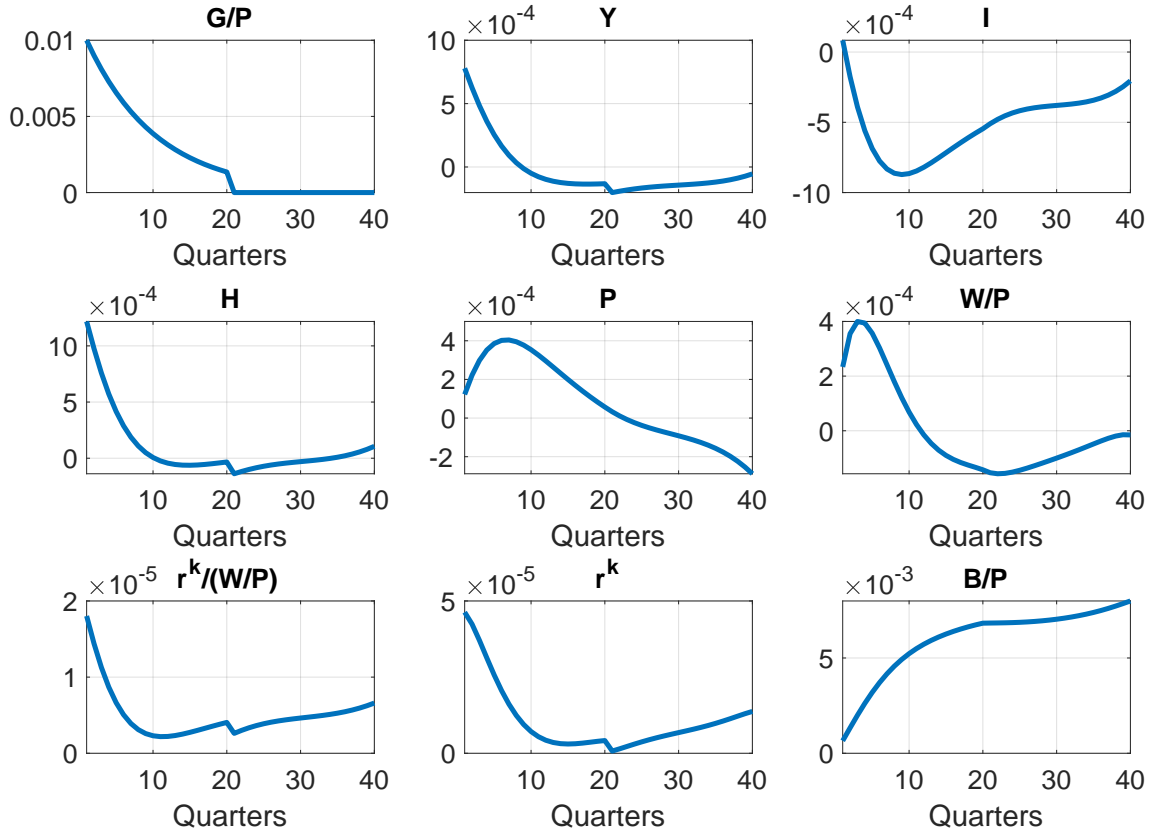
## II.4 Impulse Responses: Forward Spending, Section I.1.

Figure A-10: Impulse Response to a *Future* (4 Quarters Ahead) 1% Increase in Nominal Government Spending. **Deficit Financing.**



## II.5 Impulse Responses: Deficit Repaid using $G$ , Section I.4.

Figure A-11: Impulse Response to a 1% Increase in Nominal Government Spending. Deficit Financing, Deficit Repaid using  $G$ .



## II.6 Impulse Responses: Liquidity Trap, Section 4.2.

Figure A-12: Impulse Response to a 1% Increase in Nominal Government Spending. Liquidity Trap, Tax Financing (Constant Nominal Debt).

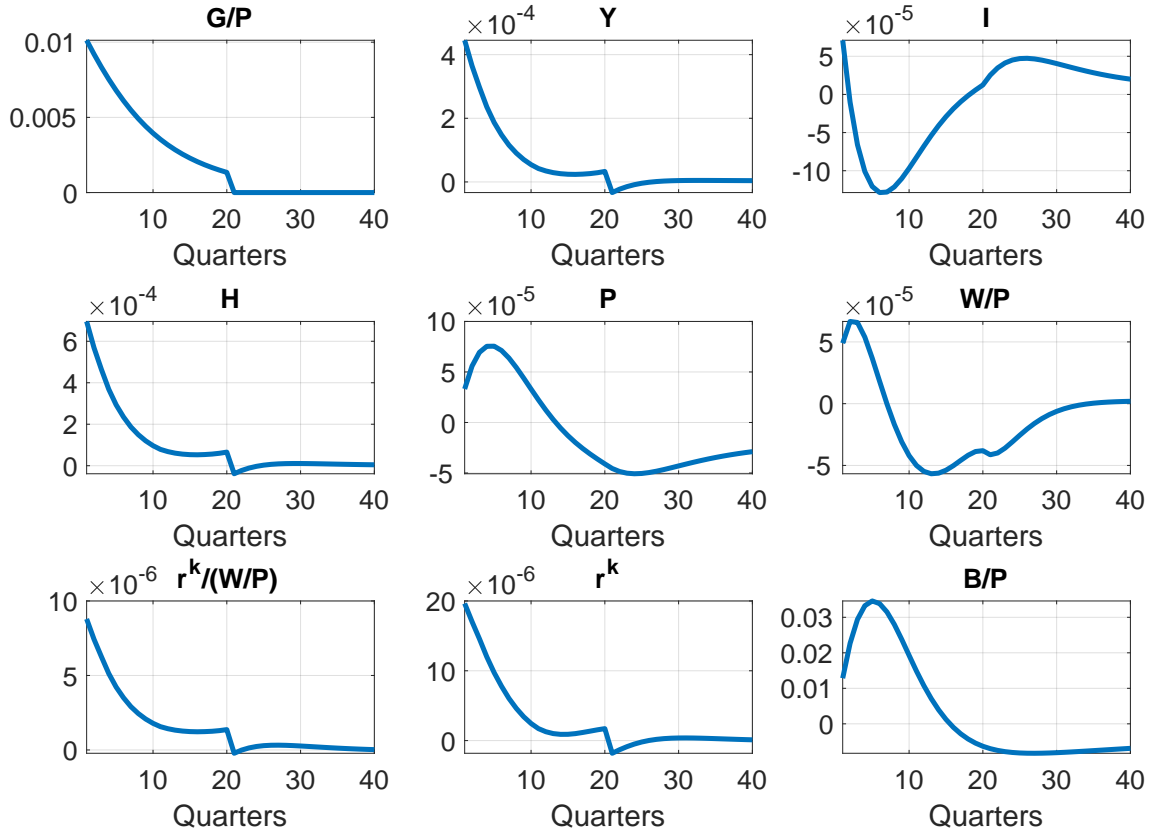


Figure A-13: Impulse Response to a 1% Increase in Nominal Government Spending.  
**Liquidity Trap, Deficit Financing.**

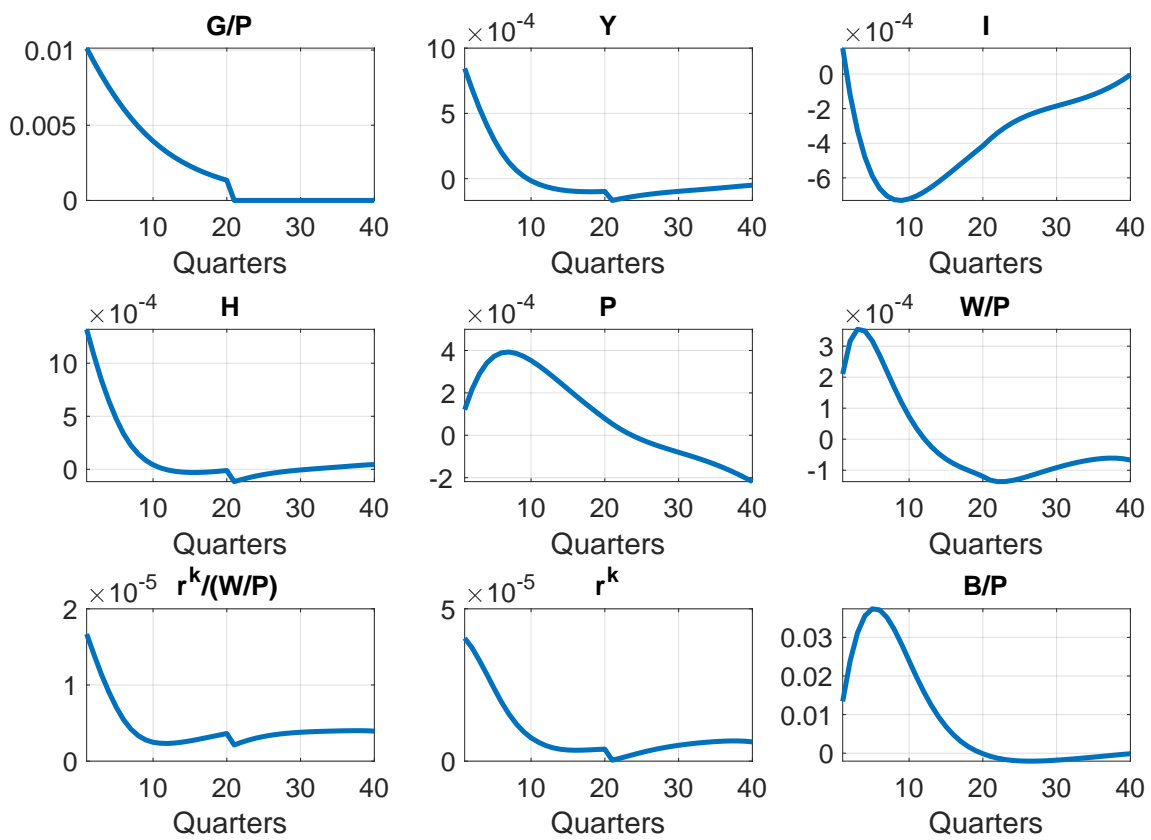
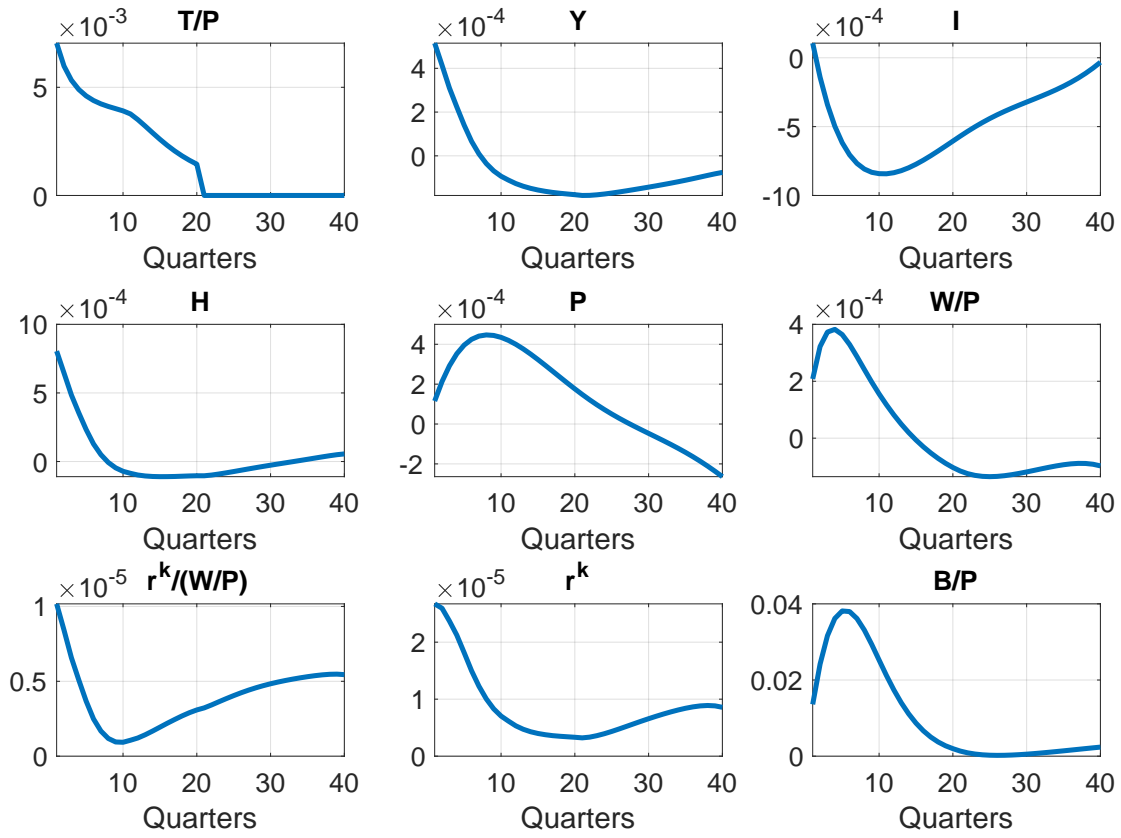


Figure A-14: Impulse Response to a 1% Increase in Nominal Government *Transfers*.  
**Liquidity Trap, Deficit Financing.**



### III Derivations

#### III.1 Derivation Pricing Equation

The firm's pricing problem is

$$V_t(p_{jt-1}) \equiv \max_{p_{jt}} \frac{p_{jt}}{P_t} y(p_{jt}; P_t, Y_t) - \Xi_t(Y_{jt}) - \frac{\theta}{2} \left( \frac{p_{jt}}{p_{jt-1}} - \bar{\Pi} \right)^2 Y_t + \frac{1}{1+r_t} V_{t+1}(p_{jt}),$$

subject to the constraints  $y_{jt} = Z_t K_{jt}^\alpha H_{jt}^{1-\alpha}$  and  $y(p_{jt}; P_t, Y_t) = \left( \frac{p_{jt}}{P_t} \right)^{-\epsilon} Y_t$ .

Equivalently,

$$V_t(p_{jt-1}) \equiv \max_{p_{jt}} \frac{p_{jt}}{P_t} \left( \frac{p_{jt}}{P_t} \right)^{-\epsilon} Y_t - \Xi_t(Y_{jt}) - \frac{\theta}{2} \left( \frac{p_{jt}}{p_{jt-1}} - \bar{\Pi} \right)^2 Y_t + \frac{1}{1+r_t} V_{t+1}(p_{jt}).$$

The FOC w.r.t  $p_{jt}$  is

$$(1 - \epsilon) \left( \frac{p_{jt}}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} + \epsilon m c_{jt} - \theta \left( \frac{p_{jt}}{p_{jt-1}} - \bar{\Pi} \right) \frac{Y_t}{p_{jt-1}} + \frac{1}{1+r_t} V'_{t+1}(p_{jt}) = 0 \quad (\text{A2})$$

and the envelope condition is

$$V'_{t+1} = \theta \left( \frac{p_{jt+1}}{p_{jt}} - \bar{\Pi} \right) \frac{p_{jt+1}}{p_{jt}} \frac{Y_{t+1}}{p_{jt}}. \quad (\text{A3})$$

Combining the FOC and the envelope condition,

$$\begin{aligned} & (1 - \epsilon) \left( \frac{p_{jt}}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} + \epsilon m c_{jt} \\ & - \theta \left( \frac{p_{jt}}{p_{jt-1}} - \bar{\Pi} \right) \frac{Y_t}{p_{jt-1}} + \frac{1}{1+r_t} \theta \left( \frac{p_{jt+1}}{p_{jt}} - \bar{\Pi} \right) \frac{p_{jt+1}}{p_{jt}} \frac{Y_{t+1}}{p_{jt}} = 0. \end{aligned} \quad (\text{A4})$$

Using that all firms choose the same price in equilibrium,

$$(1 - \epsilon) + \epsilon m c_t - \theta (\pi_t - \bar{\Pi}) \pi_t + \frac{1}{1+r_t} \theta (\pi_{t+1} - \bar{\Pi}) \pi_{t+1} \frac{Y_{t+1}}{Y_t} = 0. \quad (\text{A5})$$

#### III.2 Derivation Wage Equation

$$\Theta(s_{it}, W_{it}, W_{it-1}; Y_t) = s_{it} \frac{\theta_w}{2} \left( \frac{\hat{W}_t}{\hat{W}_{t-1}} - \bar{\Pi}^w \right)^2 H_t.$$

The middleman's wage setting problem is to maximize

$$\begin{aligned}
& V_t^w(\hat{W}_{t-1}) \\
\equiv & \max_{\hat{W}_t} \int \left( \frac{s_{it}(1-\tau_t)\hat{W}_t}{P_t} h(\hat{W}_t; W_t, H_t) - \frac{v(h(\hat{W}_t; W_t, H_t))}{u'(C_t)} \right) di - \int s_{it} \frac{\theta_w}{2} \left( \frac{\hat{W}_t}{\hat{W}_{t-1}} - \bar{\Pi}^w \right)^2 H_t di \\
& + \frac{1}{1+r_t} V_{t+1}^w(\hat{W}_t), \tag{A6}
\end{aligned}$$

where  $h_{it} = h(W_{it}; W_t, H_t) = \left( \frac{W_{it}}{W_t} \right)^{-\epsilon_w} H_t$ .

The FOC w.r.t  $\hat{W}_t$  is

$$\begin{aligned}
(1-\tau_t)(1-\epsilon_w) \left( \frac{\hat{W}_t}{W_t} \right)^{-\epsilon_w} \frac{H_t}{P_t} + \epsilon_w \frac{v'(h(\hat{W}_t; W_t, H_t))}{u'(C_t)} \left( \frac{\hat{W}_t}{W_t} \right)^{-\epsilon_w-1} \frac{H_t}{W_t} \\
- \theta_w \left( \frac{\hat{W}_t}{\hat{W}_{t-1}} - \bar{\Pi}^w \right) \frac{H_t}{\hat{W}_{t-1}} + \frac{1}{1+r_t} V'_{t+1}(\hat{W}_t) = 0 \tag{A7}
\end{aligned}$$

and the envelope condition is

$$V'_{t+1} = \theta_w \left( \frac{\hat{W}_{t+1}}{\hat{W}_t} - \bar{\Pi}^w \right) \frac{\hat{W}_{t+1}}{\hat{W}_t} \frac{H_{t+1}}{\hat{W}_t}, \tag{A8}$$

where we have used that  $\int s = 1$ .

Combining the FOC and the envelope condition,

$$\begin{aligned}
(1-\tau_t)(1-\epsilon_w) \left( \frac{\hat{W}_t}{W_t} \right)^{-\epsilon_w} \frac{H_t}{P_t} + \epsilon_w \frac{v'(h(\hat{W}_t; W_t, H_t))}{u'(C_t)} \left( \frac{\hat{W}_t}{W_t} \right)^{-\epsilon_w-1} \frac{H_t}{W_t} \\
- \theta_w \left( \frac{\hat{W}_t}{\hat{W}_{t-1}} - \bar{\Pi}^w \right) \frac{H_t}{\hat{W}_{t-1}} + \frac{1}{1+r_t} \theta_w \left( \frac{\hat{W}_{t+1}}{\hat{W}_t} - \bar{\Pi}^w \right) \frac{\hat{W}_{t+1}}{\hat{W}_t} \frac{H_{t+1}}{\hat{W}_t} = 0. \tag{A9}
\end{aligned}$$

Using that  $\hat{W}_t = W_t$ ,  $\pi_t^w = \frac{W_t}{W_{t-1}} = \frac{\hat{W}_t}{\hat{W}_{t-1}}$  and  $h_{it} = H_t$ :

$$\begin{aligned}
(1-\tau_t)(1-\epsilon_w) \frac{W_t}{P_t} + \epsilon_w \frac{v'(h(\hat{W}_t; W_t, H_t))}{u'(C_t)} \\
- \theta_w (\pi_t^w - \bar{\Pi}^w) \pi_t^w + \frac{1}{1+r_t} \theta_w (\pi_{t+1}^w - \bar{\Pi}^w) \pi_{t+1}^w \frac{H_{t+1}}{H_t} = 0. \tag{A10}
\end{aligned}$$