Family Policies: What Does The Standard Endogenous Fertility Model Tell Us?

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Abstract

There is a general consensus in the economic literature and in economic institutions about the legitimacy of policies subsidizing education. This legitimacy lies in the fact that education is a source of positive externalities. In a standard framework of endogenous fertility, the present paper shows that this result is still valid but that subsidizing education also requires taxing births. Education subsidies decrease the net cost of children such that parents can exhibit a too high fertility rate. Furthermore, when health is introduced as another source of externalities, the model shows that health expenditure should not always be subsidized. Indeed, the taxation of births plays the role of an indirect subsidy on health expenditure because it decreases the cost of health relative to the cost of children's quantity. When externalities on education are very high relative to positive externalities on health, the indirect subsidy on health can exceed the subsidy that is really needed. Then health expenditure has to be taxed.

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1 Introduction

There is a general consensus in the economic literature and in economic institutions about the legitimacy of policies subsidizing education. This legitimacy lies in the fact that education is a source of positive externalities [Hanushek & Welch (2006)]. In this paper, I use the standard framework of *endogenous fertility with a trade-off between quality and quantity.* I show that this result is still valid but that subsidizing education also requires taxing births[2] for a large set of Social Welfare functions including Benthamite and Millian[3] ones. Indeed, education subsidies decrease the net cost of children such that parents can exhibit a too high fertility rate. Following this result, health is introduced as another source of positive externalities reducing child mortality. As a result, despite its status of positive externality, health expenditure should not always be subsidized. Indeed, the taxation of births plays the role of an indirect subsidy on health expenditure because it decreases the cost of health relative to the cost of the quantity of children. In order to reach the same number of surviving children, parents tend to have fewer children in better health. When externalities on education are very high, the tax on births has also to be high. Hence, if the positive externalities on health are low, the indirect subsidy can exceed the subsidy that is really needed. Then health expenditure has to be taxed.

The "standard framework" of endogenous fertility comes from the seminal works of Becker *et al.* [1973,1976,1988]. It consists in a model where parents value the number of their offspring (quantity) as well as their future human capital (quality). They maximize their expected utility subject to a non linear budget constraint[4]. Then a trade-off between quality and quantity takes place. This fundamental contribution of Becker has been followed by the major improvements of Galor *et al.* [1999, 2002], De la Croix & Doepke [2003], Kalemli-Ozcan [2003], etc, resulting in a unified framework. Surprisingly, there are very few studies exploring the optimality properties of the trade-off between quality and quantity in this

[2] Notice that, in this paper, family policies are limited to tax - transfer policies. In reality, familiy policies include a large set of instrument like, for instance, coercive policies and preventive actions.

[3] A Millian social welfare function consists in the average utility of the representative agent while a Benthamite social welfare function value the total utility distributed in the economy.

[4] This non linearity is fundamental in models of trade-off between quality and quantity. Because quality is provided to each child (with or without equity), its cost crucially depends on the quantity choices. Then the parental budget constraint is no longer linear.

unified framework.

The question of optimal fertility has been studied in other frameworks. Samuelson [1975], Deardorff [1976] and Michel & Pestieau [1993] explore the question of the optimal population growth rate in an overlapping generation model with exogenous fertility. A model with endogenous fertility has been proposed by Michel & Wigniolle [2007] and generalized by Golosov *et al.* [2007]. Their interest focuses on the Pareto optimality of equilibria. However, they do not deal either with the quality-quantity trade-off or with the question of optimal family policies.

Boulding [1964] proposed implementing a market of tradable procreation rights. This idea is explored by De la Croix & Gosseries [2007]. It finally consists in a system of tax or subsidy on the quantity of children. However, they do not explain the reasons why governments are not satisfied with their national fertility[5].

Groezen *et al.* [2003] proposes a model of endogenous fertility and deals with the question of optimal family policies. He argues that, in the presence of a Pay As You Go (PAYG) pension system, children are a source of positive externalities because their marginal production will finance the pension system. It implies that the competitive fertility rate is too low, and so a child allowance has to be implemented[6]. However, if there is no PAYG pension system, the competitive fertility is optimal. Groezen *et al.* do not deal with the trade-off between quality and quantity which partly causes this last result.

Nerlove *et al* [1984] study the optimality of fertility behaviors in a framework where parents value both the quantity and the well being of their offspring by considering that parental utility positively depends on the consumption they allocate to their children[7]. In this framework, they find that taxing birth can be an optimal fiscal policy when the social welfare function is Millian. Conversely, when the Social Welfare function is Benthamite, they find that an optimal economic policy always consists in a child allowance. These results sensibely differ from mine because Nerlove *et al* do not really deal with the intergenerational inefficiencies of the parental quality quantity trade-off.

My paper is more closely related to the contributions of Spiegel [1993] and Balestrino *et*

[5] In that sense, the present paper has to be considered as a complement to this literature.

[6] Loupias & Wigniolle [2004] show that, in a closed framework, a generalized Allais-Samuelson-Diamond golden rule can be reached only if fertility is subsidized.

[7] They also propose alternative formulations which will be mentionned in the rest of the paper.

al. [2000]. They both deal with optimal fiscal schemes when there is a trade-off between quality and quantity. Their main conclusion is that a taxation of births can constitute an optimal family policy. This result crucially comes from the assumption that the social planner tries to reduce inequities[8]. In these models, a tax on births is an efficient instrument for reducing inequities.

In the present paper, a complementary approach is proposed. The existence of tax - transfer on births is not conditional either on the existence of differences between the government's objective and parental preferences or on a problem of unobservability of behaviors[9]. Indeed, the implementation of birth tax or child allowance comes from both the existence of externalities in the human capital accumulation process and from the non linearity of the parental budget constraint with regard to quality and quantity. These two features are common to the largest majority of papers dealing with the quality quantity trade-off. Departing from the Millian Welfare function (the Social Planer maximizes the utility of the representative agent (average utility)) or adding alternative externalities will enrich the analysis[10]. However, it will never affect my main mechanism because it results from the fundamental assumptions of the usual theory of endogenous fertility.

As mentionned precedently, the model's main assumption is the existence of externali-

[8]Balestrino *et al.* propose a model of optimal taxation where parents are heterogenous. Parental choices are all Pareto efficient. However, the government is characterized by a Benthamite function of Social Welfare, so it tries to reduce welfare inequalities between groups of parents. Moreover, this government faces a mimicking problem à la Stiglitz (the workforce participation is not observable). Fertility being observable, taxing births can help the government to differentiate parents who are really poor from mimickers. Another enlightening contribution comes from Cigno and Pettini [2002] who find a similar result without mimicking problems.

Spiegel [1993] proposes a model of trade-off between quality and quantity with a Rawlsian social planner. He shows that a poll tax on births enables the government to decentralize the social optimum of the economy. However, that instrument is a perfect substitute for a tax on the second period consumption.

[9]In Spiegel's framework, if the government does not value welfare inequalities, no tax on births is required. In Balestrino *et al*, even if the social planner dislikes welfare inequalities, the observability of abilities would make the individual indirect utilites observable. Then lump sum transfers would ensure an optimal redistribution of welfare. No tax on births would be necessary.

[10]For instance, I assume that there is no inequalities in term of wealth or abilities between agents. So, educational policies do not find their legitimacy in the governement willingness to reduce inequalities. In reality, the reduction of inequalities is a major motivation to implement educational policies. However, in the present framework, I do not need to introduce inequalities to obatin my results. Doing so would only enrich the model. Behaviors being observable, the presence of inequalities would necessitate to implement additional transfers between families.

ties in human capital accumulation[11]. When parents choose their optimal trade-off between quality and quantity, they do not consider that their education investment will improve the overall efficiency of the human capital accumulation process. It implies that, at the competitive equilibrium, they tend to under-invest in education. So, an optimal economic policy is the implementation of a subsidy on education spending. However, the budget constraint in the standard model of trade-off between quality and quantity is not linear. It implies that reducing the costs of quality also reduces the net cost of quantity. In consequence, when it is optimal to subsidize education, it is also optimal to tax births. This central result is robust to the introduction of a "natalist bias" in the social planner's preferences (the Benthamite utility function will be a special case of "natalist bias") and to the extension to endogenous child mortality.

The introduction of endogenous child mortality is an important extension of this framework because it changes the nature of the trade-off between quality and quantity. Indeed, now parents do not only have to decide how to allocate their spendings between quality and quantity, they also have to decide their optimal strategy to reach their desired number of children. In other words, they face an alternative trade-off between quality and quantity of surviving children in which their health expenditure will be a source of externalities.

In the extended model, higher parental health expenditure reduces child mortality. Furthermore, the average level of health spending has a negative impact on child mortality. The literature of development economics provides strong evidence that overall health quality is one of the main determinant of individual health quality. For instance, Dasgupta [1993] shows that 45 per cent of all deaths in developing countries can be imputed to infectious and parasitic disease. Private health expenditure helps reduce the probability of being infected when an agent is in contact with diseases. So a higher average level of health expenditure reduces death probabilities in all families. This positive externality implies that private health expenditure is too low at the competitive equilibrium.

In this extended framework which only considers the Millian case for simplicity, reaching optimality requires, once again, subsidizing education and taxing births. Now, the taxation of births plays the role of an indirect subsidy on health expenditure. Indeed, it increases the cost

[11] This is in line with Galor et Al [1999, 2002], De la Croix & Doepke [2003], Kalemli-Ozcan [2003] etc.

of quantity relative to the cost of health. Parents tend to increase their health expenditure and to decrease the number of births to reach the same number of surviving children. For strong externalities on health expenditure, the indirect tax will not be sufficient to reach optimal health expenditure at the competitive equilibrium. So private health expenditure has to be subsidized.

The recommendation to tax births in complement to subsidies for education and health, can be analyzed in the light of some empirical evidence from China and Sub-Saharan Africa. They both face a problem of overpopulation and implement alternative strategies to reduce fertility. My paper's results are obviously theoretical and cannot reproduce the very complex demographic, economic and politic situations of these countries. However, it provides some incentives to put into questions their strategies.

China is experimenting with a specific fiscal scheme on births that subsidizes the first birth and strongly taxes subsequent ones. However, empirical studies such as those of Kanbur & Zhang [2003] and Fan & Zhang [2000] show that investment in education and health is insufficient in China. The present paper proposes an alternative fiscal scheme that would reallocate public funds from the first birth subsidy to the promotion of education and health, without loss of efficiency in birth control.

Sub-Saharan African countries have implemented several family planning programs which strongly promote investment in health and education. However, a recent report of the World Bank [2007] shows that this policy has been inefficient in reducing the net fertility rate in a large majority of these countries. The paper argues that these policies have been inefficient because they did not increase the relative cost of quantity. It shows that more attention should be paid to the implementation of a fiscal scheme that would explicitly sanction births.

The rest of the paper is organized as follows. In Section 2, the benchmark model is presented. Its recommendations in terms of family policies are discussed. In Section 3, endogenous child mortality and public health expenditure are introduced. Section 4 discusses the paper's empirical implications for China and Sub-Saharan Africa. Section 5 concludes.

2 The Benchmark model

2.1 The Competitive Equilibrium

The model consists in an overlapping generation economy with L_t agents who live for two periods: childhood and adulthood. During childhood, an agent receives education from his parent and does not consume. When he becomes adult, he has to choose his consumption level C_t, the number of his children N_t and their education e_t. For simplicity, families are monoparental. Parents exhibit altruism for their children in the sense that they value their future human capital[12]. The parental utility function is denoted:

$$u_t = U\left(C_t, \xi N_t, h_{t+1}\right) \tag{1}$$

$U\left(.,.,.\right)$ is strictly increasing and concave in its arguments[13]. N_t denotes the number of children born in the family and $\xi \in]0, 1[$ the fraction of children who survive to age five. I assume that parents value the number of surviving children and not the number of children born. It implies that child mortality is a source of disutility. ξ is exogenous in this Section but will be endogenized thereafter. There is no uncertainty about the reproductive success of a family[14].

Finally, h_{t+1} represents the human capital in $t+1$ of an adult born in t[15]. Following De la Croix & Doepke [2003], parents finance a schooling time e_t and the average human capital of teachers equals the average human capital in the population. There is also an intrafamily transmission of human capital: the human capital of parents h_t positively influences the future human capital of children. Because parents do not decide their own human capital level, the transmission of human capital into the family is an externality. Moreover, the

[12] I do not consider altruism going from children to parents as, for instance, in Erhlich & Lui [199?] and Belan & Wigniolle [????]. Nerlove *et Al* [1984] argue that, in this case, children and capital markets are substitue because children's altruism ensures their parent to receive support in their old age. Then, a policy introducing more efficient capital markets can make fertility decreasing.

[13] Alternative representations of utility could have been chosen like: $U\left(C_t, \xi, N_t, h_{t+1}\right)$ or $U\left(C_t, \xi N_t h_{t+1}\right)$. As it will become clear in the following sections, these alternative specifications would lead to exactly the same results.

[14] So, unlike the models of Sah [1991] and Kalemli-Ozcan [2003] which assume uncertainty, parents will not overshoot their number of children to ensure the compliance of their optimal fertility rate. Because child death is assumed to occur before age five, parents can rapidly ensure the replacement of dead children.

[15] As in Becker [1976], Galor & Al [1999,2002], De la Croix & Doepke [2003] and Kalemli Ozcan [2003] the paper assumes that parents directly value the future human capital of their children. They do not value their future well being. In other words, altruism is limited to one generation.

average level of human capital in the population has a positive impact on the children's future human capital. This second externality represents the influence of the efficiency of the school system ($\overline{h_t}$ is the teachers' productivity) and the presence of peer effects. Human capital is accumulated through the following process[16]:

$$h_{t+1} = l\left(e_t, h_t, \overline{h_t}\right),\ l'_1 > 0, l''_{11} \leq 0, l'_2 > 0, l''_{22} < 0, l'_3 > 0, l''_{33} \leq 0, \tag{2}$$

The function l is strictly increasing and concave regarding educational investment. Note that, following equation (2), e_t can be expressed as a function of h_t, $\overline{h_t}$ and h_{t+1} such that: $e_t = e\left(h_{t+1}, h_t, \overline{h_t}\right)$ and $e'_1 > 0$, $e'_2 < 0$, $e'_3 < 0$.

The maximization of utility is subject to the following budget constraint:

$$C_t + \left[\frac{\sigma}{\xi} + \phi\right] w_t h_t X_t + \theta w_t \overline{h_t} \Omega\left(X_t\right) \cdot e_t = w_t h_t \tag{3}$$

$X_t \equiv \xi N_t$ represents the number of surviving children at the end of period t. Each child born takes a part $\sigma \in \left]0,1\right[$ of its parent's time allocation that is normalized to one. Moreover each surviving child consumes an extra part ϕ of this time[17]. So the quantity cost of a surviving child is greater than the cost of a non surviving child. The total cost of quantity is equal to $\left[\frac{\sigma}{\xi} + \phi\right] w_t h_t X_t$. It includes the ineffective costs engaged for non surviving children. Consequently it negatively depends on the child survival rate.

The cost of one unit of education is not affected by the variations in the child mortality rate. Indeed, no educational investment is engaged until a child reaches age five. The total cost of education is concave in X_t, one unit of education can benefit more than one child. $\theta w_t \overline{h_t} \Omega\left(X_t\right) \cdot e_t$ denotes the cost of giving e_t units of education to X_t children with $\Omega'(X_t) \geq 0$ and $\Omega''(X_t) \leq 0$. If education is a pure public good in the family ($\Omega(X_t) = 1$), providing e_t units of education to one child implies the same cost as providing e_t units to X_t children. If education is a pure private good in the family ($\Omega(X_t) = X_t$), one unit of education benefits only one child. Then the total cost of education equals the unitarian cost of education times the number of surviving children.

[16]Notice that for all function $\Gamma(\alpha_1, \alpha_2, ..., \alpha_n, ...)$, Γ'_n represents the partial derivative of Γ with regard to α_n.

[17]Note that $\frac{\sigma}{\xi} + \phi < 1$. $\theta > 0$ is a scalar that allows the relative education costs to vary.

The price of the final good is normalized to one. It is produced in quantity Y_t, following a linear technology:

$$Y_t = AH_t \tag{4}$$

A is a productivity factor and H_t is the total amount of human capital in the workforce. At the labor market's equilibrium, H_t is:

$$H_t = \left[1 - \left(\frac{\sigma}{\xi} + \phi\right) X_t - \theta e_t \Omega\left(X_t\right)\right] h_t L_t \tag{5}$$

Notice that, ex-post, at the equilibrium of the labor market, $\overline{h_t} = h_t$. By assumption, there is no inequality of human capital. The workforce participation of a parent consists in his remaining time after childbearing, and teachers do not participate in the production of the final good. Furthermore, as the labor market is competitive, the wage equals the workers' marginal productivity:

$$w_t = A \tag{6}$$

A parent born in $t-1$ determines his optimal demands $\left(C_t^*, X_t^*, h_{t+1}^*\right)$ by maximizing $u_t = U\left(C_t, X_t, h_{t+1}\right)$ with respect to C_t, X_t and h_{t+1}[18] subject to (2) and (3). This problem can be solved by maximizing the objective function $V_t\left(X_t, h_{t+1}\right)$ with respect to X_t and h_{t+1} :

$$V_t\left(X_t, h_{t+1}\right) \equiv U\left(w_t h_t - \left[\frac{\sigma}{\xi} + \phi\right] w_t h_t X_t - \theta w_t \overline{h_t} \Omega\left(X_t\right) e\left(h_{t+1}, h_t, \overline{h_t}\right), X_t, h_{t+1}\right) \tag{7}$$

To ensure global concavity of the problem, its Hessian Matrix is assumed to be negative semi-definite. The competitive equilibrium is described by the set $\left\{C_t^*, X_t^*, e_t^*, h_t^*, \overline{h_t^*}, h_{t+1}^*, H_t^*, Y_t^*, w_t^*\right\}$ satisfying equations $(2), (3), (4), (5), (6)$ and the following First Order Conditions:

$$U_X' = \left(\frac{\sigma}{\xi} + \phi + \theta \Omega'\left(X_t^*\right) e\left(h_{t+1}^*, h_t^*, h_t^*\right)\right) A h_t^* U_C' \tag{8}$$

$$\frac{U_{h_{t+1}}'}{U_C'} = \theta A h_t^* \Omega\left(X_t^*\right) e_1'\left(h_{t+1}^*, h_t^*, h_t^*\right) \tag{9}$$

The existence of externalities on human capital accumulation implies that the competitive equilibrium cannot be optimal. The next sections derive the social optimum of the economy and compare it to the competitive equilibrium.

[18] Note that, h_{t+1} depends on the family's human capital, the average human capital and the educational choices of parents. As parents know the level of h_t and $\overline{h_t}$ when they determine e_t; choosing e_t is equivalent to choosing h_{t+1}.

2.2 The Social Optimum

The presence of externalities makes private choices on education inefficient. Parents do not consider the positive effect of their educational investment on the overall efficiency of human capital accumulation. Consequently, they naturally tend to under invest in education. Intuitively, the implementation of a subsidy on education should be sufficient to correct this distortion. Equilibrium would be ensured by the existence of a lump sum transfer. However, doing so implicitly assumes that education is a pure public good within the family and that the objective of the social planner is the same as the objective of the representative agent.

Defining the social planner's objective function is not straightforward. The crucial point lies in his preference for the size of populations. A large set of papers dealing with optimality in endogenous fertility models attribute a Millian objective to the Social Planner[19]. In this representation, the social planner tries to maximize the utility of the representative agent $U(C, X, h)$ at the steady state. Doing so implies that he is interested in the well-being of the representative agent without taking into account the size of the population enjoying $U(C, X, h)$ and inequities matters. This representation of social preferences is often opposed to the Benthamite social welfare functions. With a Benthamite utility, the Social Planner tries to maximize the total utility distributed in the economy.

In the present paper, I propose a general social welfare function which include the Millian and Benthamite cases as well as intermediary ones[20]. To do so, a "natalist bias" denoted $f(X)$ is introduced in the Social Planner's preferences. The Social Welfare function, at the steady state, is then[21]:

$$SU = f(X)\, U(C, X, h) \tag{10}$$

This formulation is a generalization of the Millian case where $f(X) = 1$. $f(X)$ represents

[19] See Groezen *et al.* [2003], Wigniolle & Loupias [2004], Zhang [2003], Zhang & Zhang [2007], etc. This formulation can also be included in the A-Efficiency problems from Golosov *et al.* [2007].

[20] Notice that Rawlsian objectives are not included in this simple formulation.

[21] Notice that, as in Groezen *et al.* [2003], Wigniolle & Loupias [2004], Zhang [2003], Zhang & Zhang [2007], etc., I only focus on the social welfare function at the steady state. The decentralization of the dynamical optimal path would require to define the Social Planner objective as follows:

$$SU = L_0 \sum_{t=0}^{+\infty} \left(\frac{1}{1+\beta}\right)^t \left[\left(\prod_{s=0}^{t-1} X_s\right) u\left(C_t, X_t, h_{t+1}\right)\right]$$

the "social planner's Natalist Bias". Following Nerlove et Al [1984], when $f(X) = 1 + X$, the maximand of a stationary Benthamite social welfare function reduces to the Welfare social function in (10). More generally, for a given X, a higher value of $f(X)$ means that the Social planner exhibits a higher natalist bias. In other words, *ceteris paribus*, he prefers larger generations. $f(X)$ is assumed to be strictly increasing and concave in X[22][23].

Then, the social planner maximizes (10) subject to the following resource constraint[24]:

$$C = \left[1 - \left(\frac{\sigma}{\xi} + \phi\right) X - \theta\Omega(X) e\right] Ah \tag{11}$$

The optimal steady state is described by the set $\left\{\widehat{C}, \widehat{X}, \widehat{h}\right\}$ satisfying equation (11) and the following First Order Conditions:

$$U'_X = -\frac{f'\left(\widehat{X}\right)}{f\left(\widehat{X}\right)} U\left(\widehat{C}, \widehat{X}, \widehat{h}\right) + \left(\frac{\sigma}{\xi} + \phi + \theta\Omega'\left(\widehat{X}\right) e\left(\widehat{h}, \widehat{h}, \widehat{h}\right)\right) A\widehat{h} U'_C \tag{12}$$

$$\frac{U'_{h_{t+1}}}{U'_C} = A\left(\widehat{X}\left[\frac{\sigma}{\xi} + \phi\right] + \theta\Omega\left(\widehat{X}\right)\left[e\left(\widehat{h}, \widehat{h}, \widehat{h}\right) + \widehat{h}\left(\widehat{e}'_1 + \widehat{e}'_2 + \widehat{e}'_3\right)\right] - 1\right) \tag{13}$$

Obviously, at the optimal steady state, all the existing externalities are taken into account. In this economy, externalities concern the accumulation of human capital. When parents invest in education, they improve the future human capital of their children, such that, in turn, they improve both the future average level of human capital in the whole economy and their dynasty's level of human capital. However, parents do not take into account their positive impact on the future efficiency of the accumulation process. It implies that they tend to underinvest in education.

Furthermore, the preferences of parents differ from the preferences of the social planner. Parents are not concerned with pro-natalism or equity between generations. Consequently, they could have too few children. However the externalities on education increase their fertility rate because quality and quantity are substitutes. The competitive equilibrium can

[22] $f'(X) > 0$ simply means that distributing $\overline{U}$ to one agent is less valuable than distributing $\overline{U}$ to $X > 1$ agents. $f''(X) < 0$ ensures the existence of the trade-off between the utility distributed to the representative agent and the size of the generation enjoying it.

[23] Notice that no other externality on fertility is introduced. Doing so would enrich the model of alternative mechanisms but will not change its main results. Pestieau (a voir avec pollution et espace vital) et autres comme Galor avec impact positif de la pop sur le niveau technologique dans les etats malthusiens.

[24] To ensure global concavity of the problem, its Hessian matrix is assumed to be negative semi-definite

then be characterized by over or under fertility. The implementation of an economic policy is required.

2.3 The Optimal Tax-Transfer Policy

In order to decentralize the social optimum, the government has to implement a public policy which makes the competitive steady state[25] converge to the optimal one. An optimal policy makes the set $\{C^*, X^*, h^*\}$ identical to the set $\left\{\widehat{C}, \widehat{X}, \widehat{h}\right\}$. The following sub-Sections discuss the optimal tax-transfer policies[26] in the general case $(\Omega(X) \neq 1)$ and in the specific case where education is a pure public good inside the family $(\Omega(X) = 1)$.

I assume that the government can observe the agent's behaviors, it allows to decentralize the first order optimum. This assumption is strong but fundamental because it shows that the standard problem of trade-off between quality and quantity (externalities on human capital and non linearity of the parental budget constraint) structurally generates inoptimal laissez faire equilibria *regarding both education and fertility choices.* Adding alternative externalities on fertility or studying second order optima will interestingly contrast this result.

To summarize, education choices are not optimal because there is an externality on education investment. A subsidy on education spending has to be implemented to correct this externality. Fertility choices are not optimal for two reasons. First, the social planner does not exhibit the same preferences for quantity as individuals. Secondly, when the cost structure is not linear $(\Omega(X) \neq 1)$, the implementation of the education subsidy decreases the total quantity costs. A tax or a subsidy on births has to be implemented. Obviously, such a family policy will not be required in the specific case where the social planner exhibits no Fertility Bias $(f(X) = 1)$ *and* education is a pure public good in the family $(\Omega(X) = 1)$.

2.3.1 Optimal Tax-Transfer Policy in the general case

Proposition 1 *Whatever the intensity of the social planner's natalist bias, a policy of education subsidies is optimal when it is combined with a family policy that can be either a*

[25] At the competitive steady state, $h^*_{t+1} = h^*_t = \overline{h^*_t}$.

[26] Notice that, as in Nerlove *et al* [1984], I only focus on non coercive policies and on linear taxation. Without inqualities, focusing on linear taxation is not a strong assumption because redistribution is not a matter of concern.

tax or a subsidy on births. The government budget constraint has to be balanced by the implementation of a lump sum tax on each family.

Proof. The economic policy described in Proposition 1 leads to the following competitive steady state:

$$U'_X = \left(\frac{\sigma-\Lambda}{\xi}+\phi+\Omega'(X^*)\,\theta(1-\kappa)\,e(h^*,h^*,h^*)\right)Ah^*U'_C \tag{14}$$

$$\frac{U'_h}{U'_C} = \theta Ah^*\Omega(X^*)(1-\kappa)\,e'_1(h^*,h^*,h^*) \tag{15}$$

$$C^* = \left[1-\left(\frac{\sigma-\Lambda}{\xi}+\phi\right)X^*+\theta(1-\kappa)\,e(h^*,h^*,h^*)\,\Omega(X^*)\right]Ah^*-t \tag{16}$$

$$t = \kappa\theta e(h^*,h^*,h^*)\,\Omega(X^*)\,Ah^*+\frac{\Lambda}{\xi}X^*Ah^* \tag{17}$$

$\Lambda>0$ (resp $\Lambda<0$) represents a subsidy (resp a tax) on each child birth. $\kappa>0$ (resp $\kappa<0$) denotes a subsidy (resp a tax) on educational investment. When parents invest in one unit of education, they only pay a part $1-\kappa$ of this investment. t is the lump sum transfer making the government budget constraint balanced. Equation (17) represents the government budget constraint; equations (14) and (15) are just the expression of equations (8) and (9) when the economic policy is implemented.

Observing systems $\{11,12,13\}$ and $\{14,15,16,17\}$, any policy making the sub-systems $\{14,15\}$ and $\{12,13\}$ identical, decentralizes the social optimum. Indeed, (16) and (17) imply that (11) is satisfied. It follows that[27]:

$$\widehat{\kappa} = \frac{1-\widehat{X}\left[\frac{\sigma}{\xi}+\phi\right]-\theta\Omega\left(\widehat{X}\right)\left[\widehat{e}+h\left(\widehat{e}'_2+\widehat{e}'_3\right)\right]}{\theta\Omega\left(\widehat{X}\right)\widehat{h}\widehat{e}'_1} \tag{18}$$

$$\frac{\widehat{\Lambda}}{\xi} = \frac{f'(\widehat{X})}{f(\widehat{X})}\frac{\widehat{U}}{A\widehat{h}\widehat{U}'_C}-\frac{\Omega'(\widehat{X})\widehat{e}}{\Omega(\widehat{X})\widehat{h}\widehat{e}'_1}\left(1-\widehat{X}\left[\frac{\sigma}{\xi}+\phi\right]-\theta\Omega\left(\widehat{X}\right)\left[\widehat{e}+\widehat{h}\left(\widehat{e}'_2+\widehat{e}'_3\right)\right]\right) \tag{19}$$

$$\widehat{t} = \varepsilon_{\widehat{X}}^{f(\widehat{X})}\frac{\widehat{U}}{\widehat{U}'_C}+\frac{A\widehat{e}}{\widehat{e}'_1}\left(1-\varepsilon_{\widehat{X}}^{\Omega(\widehat{X})}\right)\left(1-\widehat{X}\left[\frac{\sigma}{\xi}+\phi\right]-\theta\Omega\left(\widehat{X}\right)\left[\widehat{e}+\widehat{h}\left(\widehat{e}'_2+\widehat{e}'_3\right)\right]\right) \tag{20}$$

By (11), $\widehat{\kappa}$ can be expressed as:

$$\widehat{\kappa} = \frac{\frac{\widehat{C}}{A\widehat{h}}-\theta\Omega\left(\widehat{X}\right)h\left(\widehat{e}'_2+\widehat{e}'_3\right)}{\theta\Omega\left(\widehat{X}\right)\widehat{h}\widehat{e}'_1}$$

[27] Notice that $\widehat{U}\equiv U\left(\widehat{C},\widehat{X},\widehat{h}\right)$ and $\widehat{e}=e\left(\widehat{h},\widehat{h},\widehat{h}\right)$.

$\widehat{e}_2' + \widehat{e}_3' < 0$ implies that $\widehat{\kappa}$ is always positive. The optimal education policy is always a subsidy. $f'(\widehat{X})$ and $\Omega'\left(\widehat{X}\right)$ being different from zero, $\widehat{\Lambda}$ and $\widehat{t}$ are also different from zero: a tax-transfer policy on births and a lump sum tax are effectively required to reach the optimal steady state. ■

An education subsidy has to be implemented because the human capital accumulation process is affected by externalities. Parents do not internalize all the returns of their investment in education. Then their laissez faire investment in children's human capital is slower than the optimal one. The optimal fiscal policy on births has two determinants. The first one is the social planner's natalist bias. If the social planner exhibits a strong preference for large populations, the laissez faire fertility rate tend to be smaller than at the optimum. The second determinant of the optimal policy on births is the optimal education policy itself. The non linearity of the parental budget constraint implies that a reduction in the education costs decreases the total net cost of a surviving child. Then parents tend to have more children at the laissez faire equilibrium than at the optimum. One main issue of that paper is to determine the conditions where births have to be taxed[28].

Proposition 2 *For low intensities of the social planner's natalist bias such that* $0 < \varepsilon_{\widehat{X}}^{f(\widehat{X})} < \widetilde{\varepsilon}$, *to tax births is an optimal family policy.*

Proof. After some calculus on (19), the following condition can be obtained:

$$\frac{\widehat{\Lambda}}{\xi} < 0 \Leftrightarrow \varepsilon_X^{f(X)} < \frac{\varepsilon_C^{U(C,X,h)} \cdot \varepsilon_X^{\Omega(X)}}{\varepsilon_{h_{t+1}}^e} \left(1 - \frac{\theta \Omega(X) A h e}{C} \left[\varepsilon_{h_t}^e + \varepsilon_{\overline{h_t}}^e\right]\right) \equiv \widetilde{\varepsilon} \tag{21}$$

$\varepsilon_{h_t}^e + \varepsilon_{\overline{h_t}}^e < 0$ implies that $\widetilde{\varepsilon} > 0$. ■

The value of $\widetilde{\varepsilon}$ is determined by the model's key variables. When the elasticity of utility to consumption ($\varepsilon_C^{U(C,X,h)}$) is high, parents consume a large part of their income and have few children. Therefore, all other things being equal, the competitive fertility rate is low and the tax level has not to be very high and could even become a subsidy.

When the private returns of investment in human capital are high (low values of $\varepsilon_{h_{t+1}}^e$) relative to its social returns ($\varepsilon_{h_{t+1}}^e - \left[\varepsilon_{h_t}^e + \varepsilon_{\overline{h_t}}^e\right]$), the tax will be low. Indeed, this implies that

[28] Let $\varepsilon_m^{p(m)}$ denote the elasticity of $p(m)$ with regard to m. So $\varepsilon_m^{p(m)} \equiv \frac{\frac{\partial p(m)}{\partial m} \cdot m}{p(m)}$ $\forall m$ and $\forall p(\cdot)$ being twice differentiable.

the distortions on educational choices are low, so the educational subsidy is low. Because the tax on births corrects the distortion provoked by the subsidy on education, its level will be low too.

Corollary 3 *When the Social Welfare function is Millian ($\varepsilon_X^{f(X)} = 0$), the optimal tax transfer policy for fertility is necessarily a tax on births.*

Proof. If $\varepsilon_X^{f(X)} = 0$, (21) is always satisfied. ■

Indeed, when the social planner has the same preferences as parents, initially, at the competitive steady state, fertility behaviors are optimal. However, when the social planner implements subsidies on educational investment, the cost of quantity also decreases. Then over fertility appears and a tax on births has to be implemented.

This result is crucial for models of trade-off between quality and quantity. It implies that implementing generous education policies could require restrictive family policies when education is not a pure public good in the family. The following sub-section explores the preceding optimal fiscal scheme in the specific case where education is a pure public good inside the family.

2.3.2 Optimal Policy when education is a pure public good in the family

In this case, the cost of providing e_t units of education to one child is the same as the cost of providing e_t units of education to an infinite number of children. It implies that the preceding results are modified.

Proposition 4 *When education is a pure public good in the family, taxing births is never necessary to decentralize the optimal steady state. Furthermore, if the social planner does not exhibit a natalist bias, no tax or transfer on fertility is required to reach the optimal steady state.*

Proof. If $\Omega'(X) = 0$, the fiscal scheme decentralizing the optimal steady state is the expression of system $\{18, 19, 20\}$ with $\Omega(X) = 1$ and $\Omega'(X) = 0$:

$$\widehat{\kappa} = \frac{1 - \widehat{X}\left[\frac{\sigma}{\xi} + \phi\right] - \theta\left[\widehat{e} + h\left(\widehat{e}_2' + \widehat{e}_3'\right)\right]}{\theta\widehat{h}\widehat{e}_1'} \tag{22}$$

$$\frac{\widehat{\Lambda}}{\xi} = \frac{f'(\widehat{X})}{f(\widehat{X})}\frac{U}{A\widehat{h}U_C'} \tag{23}$$

$$\widehat{t} = \varepsilon_{\widehat{X}}^{f(\widehat{X})}\frac{U}{U_C'} + \frac{A\widehat{e}}{\widehat{e}_1'}\left(1 - \widehat{X}\left[\frac{\sigma}{\xi} + \phi\right] - \theta\Omega\left(\widehat{X}\right)\left[\widehat{e} + \widehat{h}\left(\widehat{e}_2' + \widehat{e}_3'\right)\right]\right) \tag{24}$$

By the proof of Proposition 3, $\widehat{\kappa} > 0$. Education has to be subsidized. It is straightforward that $\widehat{\Lambda} \geq 0$. When $f'(X) = 0$, it follows from (23) that $\widehat{\Lambda} = 0$, $\widehat{\kappa} > 0$ and $\widehat{t} > 0$. ■

The fundamental results of the model have not really changed. Equation (19) is still satisfied. However, education policies no longer distort fertility behaviors. Indeed, as education is a pure public good into the family, total costs of education are not influenced by the number of children enjoying the educational investment. So only the distance between the social planner's preferences and the household's preferences can make the fertility behavior non optimal. Without this bias, competitive fertility choices are optimal and no tax or transfer on fertility is required.

As a first major result, in a standard model of trade-off between quality and quantity, a tax-transfer policy on fertility is always required to reach the optimal steady state if education is not a pure public good. In other words, without the implementation of a tax or a subsidy on births, an education policy is not completely effective.

In the following Section, the model is extended to include private health expenditure. Despite the modification of the nature of the trade-off between quality and quantity, the need to tax births will not be canceled by the introduction of health expenditure .

3 Optimal Tax-Transfer policy with health expenditure

The child survival probability is now endogenous. Parents can engage in health expenditure in order to reduce their children's mortality rate. In line with Shakraborty [2004], the child survival probability ξ_t is now:

$$\xi_t \equiv \xi\left(s_t, \overline{s_t}\right) \tag{25}$$

Parental expenditure on health has a strictly positive and concave influence on the children's survival probability, so $\xi_1' \equiv \frac{\delta\xi(s_t,\overline{s_t})}{\delta s_t} > 0$ and $\xi_{11}'' \equiv \frac{\delta^2\xi(s_t,\overline{s_t})}{\delta {s_t}^2} < 0$. This expenditure represents the health care provided by parents to children. Parental health care covers a large set of expenditure such as hygiene, sanitation improvements and efficient nutrition. $\overline{s_t}$ denotes the average health expenditure in the economy. In line with Dasgupta [1993], $\xi_2' \equiv \frac{\delta\xi(s_t,\overline{s_t})}{\delta \overline{s_t}} > 0$ and $\xi_{22}'' \equiv \frac{\delta^2\xi(s_t,\overline{s_t})}{\delta \overline{s_t}^2} < 0$.

The introduction of an externality on health expenditure implies that the parental choices on s_t will not be efficient at the competitive equilibrium. Intuitively, one can expect that the competitive level of health expenditure will be inferior to its optimal level. However, the existence of educational inefficiency could alter this result because, as previously shown, it decreases the total cost of quantity.

3.1 The Competitive Equilibrium

Parents now have to determine health expenditure for their children. In other words, they choose X_t and s_t. The addition of an externality on health spending implies that private health investment will not be optimal. Then the government introduces a subsidy r_t on health expenditure in complement to the previous fiscal system. The government budget constraint is now[29]:

$$t_t = \kappa_t \theta e\left(h_{t+1}, h_t, h_t\right) X_t w_t h_t + \frac{\Lambda_t w_t h_t}{\xi\left(s_t, \overline{s_t}\right)} X_t + r_t s_t \tag{26}$$

When the fiscal scheme is implemented, the familial budget constraint is:

$$C_t + \left(1 - r_t\right) s_t + \left[\frac{\sigma - \Lambda_t}{\xi\left(s_t, \overline{s_t}\right)} + \phi\right] w_t h_t X_t + \left(1 - \kappa_t\right) \theta w_t \overline{h_t} X_t \cdot e_t = w_t h_t \tag{27}$$

Now the final good can either be consumed or invested in health. Parents have to maximize the objective function $U\left(C_t, X_t, h_{t+1}\right)$ with regard to C_t, X_t and h_{t+1} and with respect to (27). As health expenditure does not enter the objective function, parents determine their

[29] To simplify the results, $\Omega'\left(X\right) = 1$. Education is a pure private good.

optimal health expenditure by minimizing $(1-r_t)\, s_t + \frac{\sigma-\Lambda_t}{\xi(s_t,\overline{s_t})} w_t h_t X_t$. It follows that, at the competitive steady state:

$$1-r = \frac{[\sigma-\Lambda]\, \xi_1^{*\prime}}{[\xi\,(s^*,s^*)]^2} X^* w h^* \tag{28}$$

Parents equalize the marginal return and the marginal cost of health expenditure $(1-r)$. The marginal benefit of health expenditure $\left(\frac{[\sigma-\Lambda]\xi_1^{*\prime}}{[\xi(s^*,s^*)]^2} X^* w h^*\right)$ consists in the reduction of the total cost of quantity[30]. In other words, equation (28) determines the optimal parental spending on health to have X_t surviving children.

The competitive steady state is now described by the set $\{C^*, X^*, s^*, e^*, h^*, \overline{h^*}, H^*, Y^*, w^*\}$ satisfying equations $(5), (6), (27), (28)$ and the following first order conditions with regard to X and h:

$$\frac{U'_X}{U'_C} = \frac{(\sigma-\Lambda+[\phi+(1-\kappa)\,\theta e\,(h^*,h^*,h^*)]\,\xi\,(s^*,s^*))}{\xi\,(s^*,s^*)} w h^* \tag{29}$$

$$\frac{U'_{h_{t+1}}}{U'_C} = (1-\kappa)\, X^* \theta w h^* e'_1(h^*,h^*,h^*) \tag{30}$$

Equations (28) and (29) show that the taxation of births increases the marginal cost of quantity and increases the marginal benefits of health expenditure.

3.2 The Social Optimum

For simplicity, $f(X)=1$ is assumed. The social planner maximizes a Millian Social Welfare function $SU = U\,(C,X,h)$. He holds a new maximization instrument s and he faces a new resource constraint:

$$C+s = \left[1-\left(\frac{\sigma}{\xi\,(s,\overline{s})}+\phi+\theta e\right) X\right] Ah \tag{31}$$

At the steady state $s=\overline{s}$. The social planner determines the optimal health expenditure by minimizing $\frac{\sigma}{\xi(s,s)} X A h + s$ with regard to s. Doing so, he equalizes the marginal social cost of health spending (equal to one) to its marginal social return. Obviously, the marginal social benefit of health spending is higher than the marginal private benefit (calculated in equation (28)). Formally, the optimal decision rule for s is:

$$1 = \frac{\sigma\left[\widehat{\xi}'_1+\widehat{\xi}'_2\right]}{[\xi\,(\widehat{s},\widehat{s})]^2} \widehat{X} A \widehat{h} \tag{32}$$

[30] As mentionned in the Benchmark model, a higher child survival rate decreases the cost of quantity.

The optimal equilibrium now results from the maximization of the following objective function with regard to X and h :

$$SU = U\left(\left[1-\left(\frac{\sigma}{\xi\left(\widehat{s},\widehat{s}\right)}+\phi+\theta e\right)X\right]Ah-\widehat{s},X,h\right)$$

Then the Social Optimum is described by the set $\left\{\widehat{C},\widehat{X},\widehat{h},\widehat{s}\right\}$ satisfying the equation $(31),(32)$ and the following conditions:

$$\frac{U'_X}{U'_C} = \frac{\left(\sigma+\left[\phi+\theta e\left(\widehat{h},\widehat{h},\widehat{h}\right)\right]\xi\left(\widehat{s},\widehat{s}\right)\right)}{\xi\left(\widehat{s},\widehat{s}\right)}A\widehat{h} \tag{33}$$

$$\frac{U'_{h_{t+1}}}{U'_C} = A\left(\widehat{X}\left[\frac{\sigma}{\xi\left(\widehat{s},\widehat{s}\right)}+\phi+\theta e\left(\widehat{h},\widehat{h},\widehat{h}\right)+\theta\widehat{h}\left(\widehat{e}'_1+\widehat{e}'_2+\widehat{e}'_3\right)\right]-1\right) \tag{34}$$

3.3 The Optimal Tax-Transfer Policy

An optimal policy has to make identical systems $\{(32),(33),(34)\}$ and $\{(28),(29),(30)\}$. In consequence, the optimal fiscal scheme is:

$$\widehat{\kappa} = \frac{1-\widehat{X}\left[\frac{\sigma}{\xi(\widehat{s},\widehat{s})}+\phi+\theta\widehat{e}+\theta\widehat{h}\left[\widehat{e}'_2+\widehat{e}'_3\right]\right]}{\theta\widehat{X}\widehat{h}\widehat{e}'_1} \tag{35}$$

$$\frac{\widehat{\Lambda}}{\xi} = \frac{\widehat{e}}{\widehat{X}h\widehat{e}'_1}\left(\widehat{X}\left[\frac{\sigma}{\xi\left(\widehat{s},\widehat{s}\right)}+\phi+\theta\widehat{e}\left(1+\varepsilon^e_h+\varepsilon^e_{\overline{h}}\right)\right]-1\right) \tag{36}$$

$$\widehat{r} = 1-\frac{\varepsilon_s^{\xi(s,\overline{s})}}{\varepsilon_s^{\xi(s,\overline{s})}+\varepsilon_{\overline{s}}^{\xi(s,\overline{s})}}\left[1+\frac{\widehat{e}\left(1-\widehat{X}\left[\frac{\sigma}{\xi(\widehat{s},\widehat{s})}+\phi+\theta\widehat{e}\left(1+\varepsilon^e_h+\varepsilon^e_{\overline{h}}\right)\right]\right)}{\widehat{X}h\widehat{e}'_1}\right] \tag{37}$$

$$\widehat{t} = \varepsilon_{\overline{s}}^{\xi(s,\overline{s})}\frac{\sigma A\widehat{h}}{\xi\left(\widehat{s},\widehat{s}\right)}\widehat{X}-\frac{Ah\varepsilon_s^{\xi(s,\overline{s})}\left(1-\widehat{X}\left[\frac{\sigma}{\xi(\widehat{s},\widehat{s})}+\phi+\theta\widehat{e}\left(1+\varepsilon^e_h+\varepsilon^e_{\overline{h}}\right)\right]\right)}{\varepsilon^e_{h+1}} \tag{38}$$

Optimal values of $\widehat{\kappa}$ and $\widehat{\Lambda}$ are the same as in the previous Section (given that the optimal values of $\widehat{C},\widehat{X}$ and $\widehat{h}$ have changed). It implies that Proposition 1 still applies. In other words, whatever the intensity of the social planner's natalist bias, a policy of education and health subsidies is optimal when it is combined with a tax-transfer policy on births. Here, because the social planner exhibits no natalist bias, the optimal family policy always consists in a tax on births. The government budget constraint still has to be balanced by the implementation of a lump sum tax on each family.

Proposition 5 *When the externality on health expenditure is strong such that* $\varepsilon_{\overline{s}}^{\xi(s,\overline{s})} > \overline{\varepsilon}$, *the optimal health policy consists in a subsidy.*

Proof. It is straightforward to show that parental health expenditure is not optimal at the competitive steady state.

At the competitive steady state (without taxation), (28) and (29) imply $s^* = \varepsilon_s^{\xi(s,\overline{s})}\sigma AhN$. At the optimal steady state, (32) and (33) imply $\widehat{s} = \left[\varepsilon_s^{\xi(s,\overline{s})} + \varepsilon_{\overline{s}}^{\xi(s,\overline{s})}\right]\sigma AhN$. It follows that $s^* < \widehat{s}$. However $s^* < \widehat{s}$ does not ensure that health expenditure should always be subsidized. (32) and (33) indicates that the optimal value of health subsidies is:

$$\widehat{r} = 1 - \frac{\varepsilon_s^{\xi(\widehat{s},\widehat{s})}}{\varepsilon_s^{\xi(\widehat{s},\widehat{s})} + \varepsilon_{\overline{s}}^{\xi(\widehat{s},\widehat{s})}}\left(1 - \frac{\widehat{\Lambda}}{\sigma}\right)$$

Then, $\widehat{r}$ is positive if the following condition holds:

$$\varepsilon_{\overline{s}}^{\xi(\widehat{s},\widehat{s})} > -\varepsilon_s^{\xi(\widehat{s},\widehat{s})}\frac{\widehat{\Lambda}}{\sigma} \equiv \overline{\varepsilon}$$

■

When the externality on $\overline{s}$ is strong such that $\varepsilon_{\overline{s}}^{\xi(\widehat{s},\widehat{s})} > \overline{\varepsilon}$, health expenditure has to be subsidized because parents tend to largely underinvest in health. Conversely, externalities on education are strong such that $\overline{\varepsilon} > \varepsilon_{\overline{s}}^{\xi(s,\overline{s})}$, health expenditure has to be taxed[31]. This result comes from the non linearity of the costs structure. Indeed, the existence of an externality on health expenditure implies that parents do not internalize all the returns on their investment in children's health. The comparison of (28) with $\Lambda = r = 0$ and (32) indicates that health expenditure at the competitive steady state is lower than at the optimal steady state. However, when education is subsidized, a tax on births has to be implemented. Doing so, the cost of quantity is increased relative to the cost of health, so parents tend to increase their health expenditure. The tax on births plays the role of an indirect subsidy on health. Finally, the sign of $\widehat{r}$ is determined by the difference between the intensity of the externality on health expenditure and the size of the indirect subsidy. If the externality on health is relatively strong ($\varepsilon_{\overline{s}}^{\xi(\widehat{s},\widehat{s})} > \overline{\varepsilon}$), the indirect subsidy will not be sufficient to reach $\widehat{s}$, so $\widehat{r}$ will be positive. Conversely, if the externality on health is relatively weak ($\varepsilon_{\overline{s}}^{\xi(\widehat{s},\widehat{s})} < \overline{\varepsilon}$), the indirect subsidy exceeds the health subsidy that is really needed. So $\widehat{r}$ will be negative: health expenditure will be taxed.

[31] $\overline{\varepsilon}$ positively depends on $\widehat{\Lambda}$ which, in turn, positively depends on $\varepsilon_{h_{t+1}}^{e}$.

To summarize, the present paper provides two results. First, whenever it is optimal to subsidize education *and* health, it is optimal to implement a tax -transfer policy on births. This policy always consists in a tax on births when the social planner has no natalist bias. Second, when the social returns on health expenditure are not sufficiently high, the optimal family planning program of the economy consists in the promotion of education financed by the taxation of health and births and a lump sum tax. Conversely, when the social returns on health expenditure are high, the optimal family planning program of the economy consists in the promotion of education and health financed by the taxation of births and a lump sum tax[32].This optimal policy has, in fact, two main objectives. The first one is to modify the parental trade-off between quality and quantity. More precisely, the government has to incite parents to transfer a part of their spending on fertility toward education investment. The second objective is to modify the parental trade-off between fertility and health. In order to reach the same number of surviving children, parents are incited to make less children in better health.

4 Some Empirical Issues At Stake

In this section, I discuss the main theoretical conclusion of the model in the light of some empirical evidence. This discussion shows that the model's theoretical conclusions could enrich the set of family policies that are implemented in countries facing over population problems. Obviously, the strong simplicity of the model does not allow to reproduce the very complex demographic puzzles of these countries. It imposes to limit the discussion on general statements.

Countries which face over-population problems implement active policies to slacken their population growth rate. Two examples are particularly illuminating: China and Sub-Saharan Africa. Although these two regions both face overpopulation problems, their family policies have been noticeably different. In the light of the theoretical fiscal scheme proposed in this paper, this Section provides a brief reflection on the improvements that could be made to the current policies implemented in these countries.

[32]Following Dasgupta [1994], the social returns on health expenditure are high. Then, the promotion of education and health financed by the taxation of births is a more realistic conclusion.

A recent report of the World Bank [2007] underlines that 31 of the 35 countries with the highest fertility rates come from Sub-Saharan Africa. For the majority of them, fertility rates have not changed over the last decades and are all greater than six children per woman. However, the vast majority of these countries have implemented family planning programs in collaboration with international organizations such as the World Bank.

The World Bank's report [2007] emphasizes that the main factor in the high fertility rates is the persistent high level of the desired number of children. In other words, the too high fertility rates in Sub-Saharan Africa do not come from the lack of family planning programs available. It argues that efforts have to be made to reduce the desired fertility. To do so, it recommends improving education and health programs at the local level. However, education indicators have all been increasing since the sixties. More recently, the net primary school enrolment rate increased from 50 to 70 percent between 1990 and 2006. In the same period, the youth and adult literacy rates increased[33]. This noticeable improvement in education rates has not been sufficient to reduce fertility rates.

The present paper does not recommend increasing the amount spent on the family planning programs. It proposes complementing family planning programs with taxes on births helping to finance education and health. Without taxing births, these programs reduce the net cost of the children's quantity, implying the persistence of a high desired number of children.

Obviously, the Sub-Saharan African population puzzle cannot be reduced to a simple model of fertility. More complex problems of political instability, starvation and HIV pandemy that are well beyond the scope of this paper, have a direct and significant effect on fertility and education behavior. The possibility of implementing taxes on births in a population that is largely engaged in an informal economy is particularly questionable. However, the increase of quantity costs has to be contemplated as an instrument of future family planning programs.

China also implements a family policy to reduce its population growth rate. However, its strategy differs from the strategy of family planning programs in Sub-Saharan Africa. Since 1980, China has implemented a One-Child policy which strongly constrains families'

[33]In Sub-Saharan Africa, the youth literacy rate was 64% in 1990 and 73% in 2006. The adult literacy rate was 54% in 1990 and 61% in 2006. See Appendix 1 for a more complete description.

fertility. It consists in a system which provides generous subsidies for the first birth and imposes very high taxes on the subsequent births. If parents decide to have a second child without being allowed to do so, they lose a large part of their retirement pension, their child care allowance and other social advantages. Furthermore, some physical sanctions have been implemented in rural areas. This fiscal scheme is relatively different from the one proposed in this paper. The Chinese policy does not tax all the births at the same rate. The first birth is subsidized whereas the subsequent births are heavily taxed. The high level of the tax on subsequent births is a very efficient incentive to have only one child. Then the large majority of families are subsidized to reach the target of one child per family. It implies that the Chinese One-Child Policy is a very costly family policy. It cannot finance education and health policies. So, nothing ensures that the relative costs of education and health reach their optimal value. Indeed, a large literature stresses the insufficiency of public expenditure on health and education in Chinese rural areas where the large majority of the population is concentrated (for example, see Kanbur & Zhang [2003] and Fan & Zhang [2000]).

The results of this paper indicate that some marginal changes in the One-Child policy could improve the overall efficiency of the Chinese family planning policy. It proposes taxing all births such that the family policy does not imply effective costs. The amount saved by the Chinese government could be invested in more ambitious education and health policies reducing the large inequalities existing between urban and rural areas. *Theoretically*, this system would not increase the overall cost of the Chinese family planning program and would lead to the same fertility rates. However, it would increase health and education investment. Furthermore, the Chinese family policy is coercive while the economic policy proposed in this paper is non coercive. Then, if the two policies are equally efficient, the non coecive policy is always welfare improving.

5 Conclusion

The present paper analyses optimal family policies in the standard model of trade-off between quality and quantity. Given the non linearity of the parental budget constraint, to subsidize education and health will be optimal if a tax (or a subsidy) on births is also implemented. Indeed, a subsidy on education reduces both the cost of educational investment and the total cost of fertility. This result applies for a large set of social welfare functions like the Millian and the Benthamite ones. Obviously, the model concludes that taxing births without financing education and health is not optimal either.

Finally, the fiscal scheme proposed in this model is quite simple: education and health expenditure are promoted by the taxation of births and lump sum transfers. The implementation of this scheme could improve the overall efficiency of the current family policies implemented in China and Sub-Saharan Africa. The main objective of the present investigation was to explore the family policy recommendations of the standard endogenous fertility model. As a natural extension of this work, future research should integrate countries' specificities to make quantitative propositions of economic policy and to propose a more precise discussion of empirical evidences.

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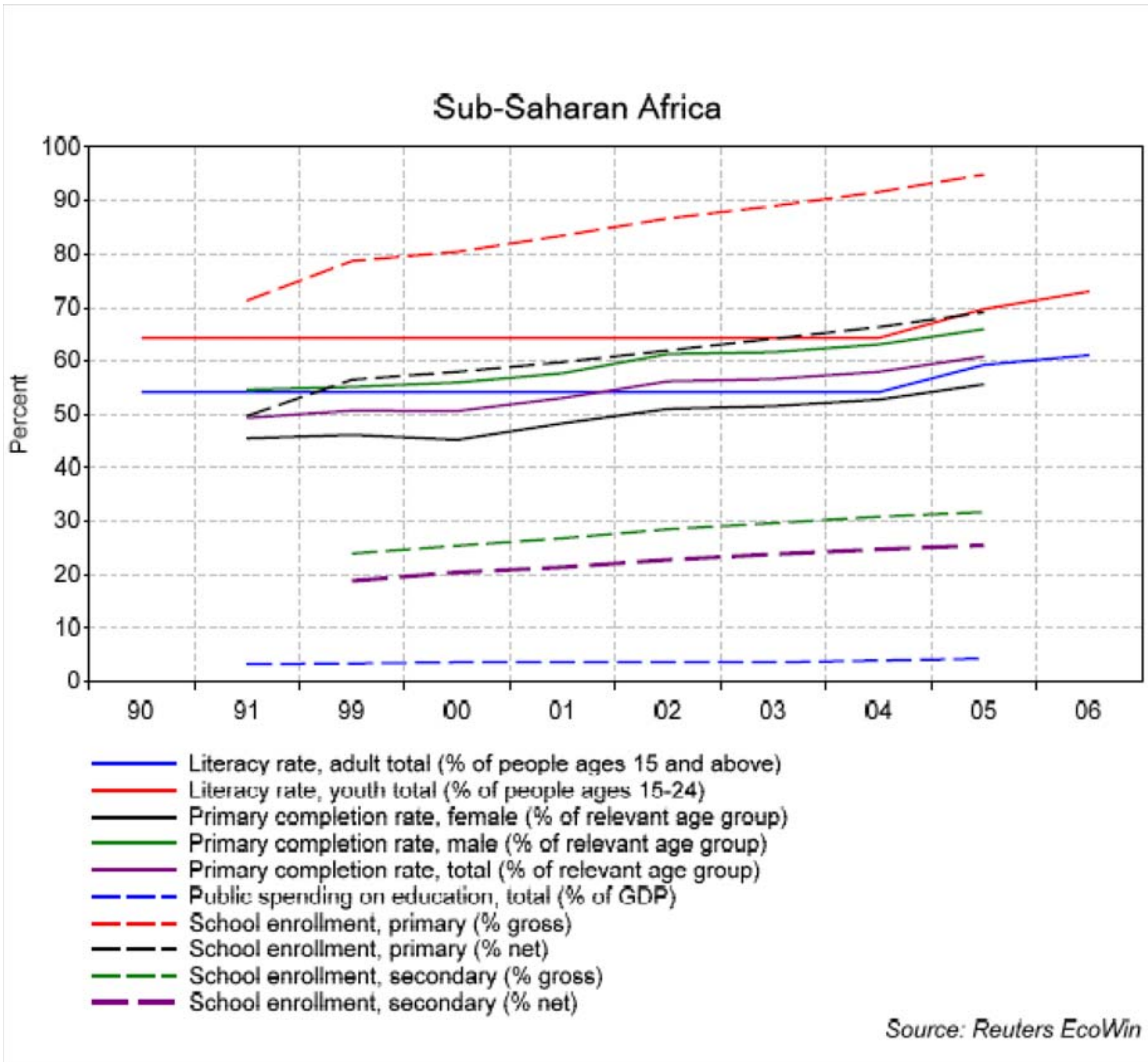
Appendix



Education in Sub-Saharan AfricaSince 1990