

# Uninsured Entrepreneurial Risk and Public Debt Policies

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## Abstract

This paper builds a stylized economy with both entrepreneurial and non entrepreneurial households to assess the optimal level of public debt in a framework that better reproduces the U.S. wealth distribution. Households are subject to uninsurable idiosyncratic risk, entrepreneurs face uninsurable productivity risk and markets are incomplete. I find that the optimal level of debt is negative. The response of workers and entrepreneurs to variations of public debt is very different. Workers desire lower levels of debt because their overaccumulation behavior is reduced and replaced by the high propensity to save of entrepreneurs. Entrepreneurs want higher levels of debt because it help reduce the cost of insuring against the risk they face. In the end, because they are in greater numbers than entrepreneurs, households determine the optimal level of public debt in the economy. Still, the optimal level of public debt is lower in an economy with entrepreneurs than in an economy without.

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**Key words :** public debt, entrepreneurship, precautionary saving, borrowing constraints

**JEL classification:** E62, E21, E23, H30

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## 1 Introduction

Since Aiyagari and McGrattan (1998), it is known that public debt has opposing effects on an economy where agents bear uninsurable idiosyncratic risk on their labor productivity. On the one hand, crowding out of private capital and rising taxes have adverse effects on welfare. On the other hand, higher interest rates reduce the cost of precautionary saving what helps improve welfare. In an economy calibrated on the U.S., Aiyagari and McGrattan (1998) find that the latter effect dominates the others for positive levels of public debt and show that a debt over GDP ratio of  $2/3$  is optimal. This result relies on the strong saving behavior of workers in the economy as it is their only means of insuring against idiosyncratic risk. Although precautionary saving by workers is not disputed in the literature, empirical evidence suggest that there is a class of agents, made of entrepreneurial households, that have a strong influence on aggregate saving and output. Entrepreneurs are a small fraction of the economy, around 7.5% as reported by Cagetti and De Nardi (2006), but they earn about 22% of total income and own 40% of total wealth (Quadrini (2000)). Moreover, their portfolio is poorly diversified and is biased toward their private business assets. Moskowitz and Vissing-Jorgensen (2002) find that 45% of entrepreneurial wealth is invested in their business assets. However, Gentry and Hubbard (2004) argue that entrepreneurs may increase their nonbusiness assets as possible insurance against business risk (this is explored in Covas (2006)) or allocate more saving to liquid assets in anticipation of future business investment needs. But whether it is saving in business assets or nonbusiness assets, findings in Gentry and Hubbard (2004) show that entrepreneurs have much higher saving rates than non-entrepreneurs.

In this paper, I argue that entrepreneurs should be an important class of agents to consider for public debt policies for at least three reasons. First, although precautionary saving by workers is important, Aiyagari and McGrattan (1998) may have overestimated the optimal level of public debt because they leave workers alone to account for overall saving in the economy when empirical evidence suggest that entrepreneurs are the driving force behind national saving. Second, entrepreneurs are a distinct class of agents and as such their response to changes in public debt might not be the same as that of workers. On the one hand, as their income heavily relies on entrepreneurial production, they might be more sensible to crowding-out coming from higher interest rates. On the other hand, as it is clear from Covas (2006), precautionary saving to insure against idiosyncratic business productivity risk is an important behavior of entrepreneurs that could be helped by higher interest rates. Last, it is discussed in Ball and Mankiw (1995) and Floden (2001) that gains and losses from higher levels of public debt are shared very differently between poor and rich households. Thus, it is important to consider public debt policies in a framework where the wealth distribution is correctly reproduced and

entrepreneurs are key to achieve that goal.

This paper models a stylized economy with both entrepreneurial and non-entrepreneurial households subject to uninsurable risk. As it is traditional with Bewley-Huggett-Aiyagari class of models, I consider borrowing constraints and market incompleteness. The economy has both a corporate production sector and an entrepreneurial production sector. The government levies proportional taxes and issues public debt in order to finance public consumption. The benchmark calibration parameters are chosen to make comparison easy with the standard results of Aiyagari and McGrattan (1998). This setup yields a negative steady state level of optimal public debt and matches the U.S. wealth distribution and wealth Gini. The intuition as to why the optimal level of public debt is lower than in Aiyagari and McGrattan (1998) is the following. In this model, workers are not responsible for all of total saving in the economy as an important part of total saving is due to entrepreneurs. For a given level of the capital-output ratio, introducing some percentage of entrepreneurs in the economy dramatically reduce the overaccumulation behavior of workers as it is compensated by the higher propensity to save of entrepreneurs. When public debt increases, because workers accumulate less, they are more concerned by the adverse effects of crowding-out and rising taxes than the beneficial effect of the reduced cost on precautionary saving. Eventually, workers settle for a lower level of optimal public debt in this economy. However, the behavior of entrepreneurs is very different. Entrepreneurial households are better off with a level of debt higher than the level found in Aiyagari and McGrattan (1998). This is because in the presence of uninsurable production risk, entrepreneurs always tend to invest in the non-risky asset as a means of insurance. Still, in general equilibrium, this type of precautionary saving lowers the interest rate and the attractiveness of the risky investment is increased. Thus, a larger amount of public debt keeps interest rates at a higher level and helps entrepreneurs in their precautionary saving behavior by reducing the cost of postponing consumption. Entrepreneurs go with higher levels of debt until the beneficial effect of reduced cost on precautionary saving is lower than crowding-out of private capital and higher taxes. Because they endure substantial levels of risk, this happens for a high level of public debt. In the end, the optimal level of public debt in the economy is explained by the two opposing effects of debt on workers and entrepreneurs. Because workers are in larger proportions in the economy, the optimal level of public debt is negative and amounts to  $-110\%$  at the steady state.

The rest of the paper is organized as follows. Next section describes the benchmark economy. Section 3 details the results. The last section concludes.

## 2 An Entrepreneurial Economy with Public Debt

The economy is populated by a continuum of infinitely lived households of measure one that can be either workers or entrepreneurs. A fixed fraction of the population are workers. They do not have access to the risky entrepreneurial technology, supply labor elastically to a corporate sector and face uninsurable income risk à la Aiyagari (1994). The remaining fraction of the population are entrepreneurs. They produce using a risky technology and are self-employed.

### 2.1 The production sector

In this economy one consumption good is produced by two production sectors. One sector is characterized by entrepreneurs that operate their own technology. The other is a corporate sector.

#### 2.1.1 Entrepreneurial sector

Entrepreneurs operate a small business. The risky technology employed by entrepreneurs is represented by<sup>1</sup>:

$$Y_t^e = \theta_t f(k_t)$$

$k_t$  is the capital stock in the risky investment,  $\theta_t$  denotes productivity.  $\theta_t$  follows a first-order Markov process. Capital depreciates at a fixed rate  $\delta$ .

#### 2.1.2 Corporate sector

In the corporate sector we assume that there is a continuum of firms which have a neoclassical production technology and behave competitively in product and factor markets. The corporate output is given by<sup>2</sup>:

$$Y_t^c = F(K_t, Z_t L_t)$$

There is no aggregate risk.  $K$  is aggregate capital used in the corporate sector,  $L$  the detrended non-entrepreneurial aggregate labor supply and  $Z$  the labor productivity.  $L$  grows at the exogenous rate  $g$ , so we can write that  $Z_t =$

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<sup>1</sup> This technology exhibits diminishing returns to scale.

<sup>2</sup> The function  $F$  exhibits constant returns to scale with respect to  $K$  and  $N$ , has positive and strictly diminishing marginal products, and satisfies the Inada conditions.

$(1+g)^t$ , given that the initial level of labor productivity is set to unity. Capital depreciates at the same constant rate  $\delta$  as in the entrepreneurial sector. Input markets are competitive and the wage rate  $w$  and the interest rate  $r$  verify:

$$\begin{aligned} r + \delta &= F_K(K_t, Z_t L_t) \\ w &= F_L(K_t, Z_t L_t) \end{aligned}$$

## 2.2 Households

Households can either be employed in the corporate sector or self-employed in the entrepreneurial sector.

### 2.2.1 Entrepreneurial households

Entrepreneurs can either invest in their small business and face an uninsurable idiosyncratic productivity shock or accumulate a safe asset yielding a non-stochastic income. Let  $k_{t+1}$  denote the resources allocated to the risky investment. The gross risky investment is given by:

$$i_t = k_{t+1} - (1 - \delta)k_t$$

Let  $a_{t+1}^e$  denote the resources allocated to the safe investment. The return on this investment is noted  $r_t$  in each period. Entrepreneurs can also borrow at the same rate. The borrowing constraint faced by entrepreneurs is noted  $\bar{a}^e$ . The budget constraint of the entrepreneur is the following:

$$\begin{aligned} c_t + k_{t+1} + a_{t+1}^e &= x_t \\ x_{t+1} &= \theta_{t+1} f(k_{t+1}) + (1 - \delta)k_{t+1} + Tr + (1 + r)a_{t+1}^e - \tau \xi_{t+1} \\ \text{with } \xi_{t+1} &= \theta_{t+1} f(k_{t+1}) - \delta k_{t+1} + r a_{t+1}^e \end{aligned}$$

where  $c_t$  denotes consumption,  $Tr$  lump-sum government transfers and  $\tau$  a tax rate.

Given that, the recursive formulation of the entrepreneurial household's problem is the following:

$$v^e(\theta, x) = \max_{c, k', a^{e'}} U^e(c) + \beta \mathbb{E}[v^{e'}(\theta', x') | \theta] \quad (1)$$

subject to

$$c + k' + a^{e'} = \theta f(k) + (1 - \delta)k + Tr + (1 + r)a^e - \tau \xi$$

$$k' \geq 0 \text{ and } a^{e'} \geq \bar{a}^e$$

where  $v^e(x, \theta)$  is the optimal value function of the entrepreneur,  $x$  the entrepreneurs' current period wealth,  $U^e$  an utility function and  $\beta$  the discount factor.

### 2.2.2 Non-entrepreneurial households

Non-entrepreneurial households supply labor elastically to the corporate sector and face uninsurable labor income risk. These households can accumulate a safe asset. Let  $a_{t+1}^{ne}$  denote the resources allocated to this safe asset and  $r_t$  the return on this saving in each period. The borrowing constraint faced by these households is noted  $\bar{a}^{ne}$ . The budget constraint of the non-entrepreneurial household is the following:

$$c_t + a_{t+1}^{ne} = y_t$$

$$y_{t+1} = (1 - \tau)w e_{t+1} l_{t+1} + Tr + (1 + (1 - \tau)r) a_{t+1}^{ne}$$

where  $c_t$  denotes consumption,  $w$  current period wage,  $l_t$  labor supply,  $Tr$  lump-sum government transfers and  $\tau$  a tax rate. The labor efficiency process  $e_t$  follows a first-order Markov process.

Given that, the recursive formulation of the non-entrepreneurial household's problem is the following:

$$v^{ne}(e, y) = \max_{c, l, a^{ne'}} U^{ne}(c, l) + \beta \mathbb{E}[v^{ne'}(e', y') | e] \quad (2)$$

subject to

$$c + a^{ne'} = (1 - \tau)w e l + Tr + (1 + r(1 - \tau)) a^{ne}$$

$$a^{ne'} \geq \bar{a}^{ne}$$

where  $v^{ne}(e, y)$  is the optimal value function of the non-entrepreneurial household,  $y$  its current period wealth,  $U^{ne}$  an utility function and  $\beta$  the discount factor.

### 2.3 The government

The government issues public debt and levies taxes to finance public expenses. The revenues of entrepreneurial production, labor and returns on capital assets are taxed proportionally at an identical rate  $\tau$ . The government's budget constraint verifies

$$G + Tr + rB = B' - B + T$$

$G$  is the level of public expenses,  $Tr$  is the level of the lump-sum transfers to the households,  $B$  the level of public debt and  $T$  tax revenues. We assume that public expenses, lump-sum transfers, and public debt are a constant fraction of GDP in every period. We note  $\gamma$  the ratio of public expenses to GDP,  $\varphi$  the ratio of lump-sum transfers to GDP and  $b$  the ratio of public debt to GDP. Thus the tax rate has the following simple expression at the steady state:

$$\tau = \frac{\gamma + (r - g)b + \varphi}{1 - \delta k + rb} \quad (3)$$

where  $k$  is the capital-output ratio in the economy.  $A$  accounts for total wealth in the economy and thus includes physical capital and public debt such that:

$$A = K + B$$

Finally, we detrend all relevant variables by the exogenous growth rate  $g$  in order to stationarize the problem. Thus the detrended value of a variable  $x$  is noted  $\hat{x}$  and verifies:  $\hat{x}_t = \frac{x_t}{(1+g)^t}$ .

### 2.4 Equilibrium

The recursive detrended steady state equilibrium of this economy is a value function for the entrepreneur:  $\hat{v}^e(\theta, \hat{x})$  and for the non-entrepreneur:  $\hat{v}^{ne}(e, \hat{y})$ , policy functions for the entrepreneur:  $\hat{k}(\theta, \hat{x})$ ,  $\hat{a}^e(\theta, \hat{x})$ ,  $\hat{c}^e(\theta, \hat{x})$  and for the non-entrepreneur:  $\hat{a}^{ne}(e, \hat{y})$ ,  $\hat{c}^{ne}(e, \hat{y})$ , factor prices  $(r, \hat{w})$ , capital and labor demand from the corporate sector  $(\hat{K}, \hat{L})$ ; a constant cross sectional distribution of entrepreneurs' characteristics  $\Gamma^e(\theta, \hat{x})$  of mass  $\chi$ ; a constant cross sectional distribution of non-entrepreneurs' characteristics  $\Gamma^{ne}(e, \hat{y})$  of mass  $(1 - \chi)$ ; a tax rate  $\tau$  such that:

- (1) Given  $r$  and  $\tau$ , the entrepreneur's policy function solve the entrepreneur's decision problem (1)

- (2) Given  $r$ ,  $\hat{w}$  and  $\tau$ , the non-entrepreneur's policy function solve the non-entrepreneur's decision problem (2)
- (3) Aggregate capital  $\hat{K}$ , aggregate labor  $L$  and aggregate entrepreneurial capital  $\hat{K}^e$  are given by:

$$\begin{aligned}\hat{K} + \hat{B} &= \sum_{\theta \in \Theta} \int \hat{a}^e(\theta, \hat{x}) d\Gamma^e(\theta, \hat{x}) + \sum_{e \in E} \int \hat{a}^{ne}(e, \hat{y}) d\Gamma^{ne}(e, \hat{y}) \\ L &= \sum_{e \in E} \int e l(e, \hat{y}) d\Gamma^{ne}(e, \hat{y}) \\ \hat{K}^e &= \sum_{\theta \in \Theta} \int \hat{k}(\theta, \hat{x}) d\Gamma^e(\theta, \hat{x})\end{aligned}$$

- (4) Given  $\hat{K}$  and  $L$  the factor prices are:

$$\begin{aligned}r &= F_{\hat{K}}(\hat{K}, ZL) - \delta \\ \hat{w} &= F_L(\hat{K}, ZL)\end{aligned}$$

- (5) Given the policy functions of entrepreneurs and workers, the probability measure if entrepreneurs  $\Gamma^e$  and non-entrepreneurs  $\Gamma^{ne}$  are invariant.
- (6) Government budget constraint holds.

## 2.5 Calibration

For the sake of comparison, the model economy is calibrated to match certain observations in the U.S. data. We let one period in the model be one year in the data. We divide our parameters in two sets. The first set of parameters can be estimated from the data. To keep comparison with Aiyagari and McGrattan (1998), most of the parameters in this set are taken from their paper. Table 1 sums up the parameters in this set. The second set of parameters is used to match relevant moments of the data. Table 2 sums up the latter.

### 2.5.1 Technology

We choose the corporate production function to be Cobb-Douglas:

$$Y_t^c = F(K_t, Z_t L_t) = K_t^\alpha (Z_t L_t)^{1-\alpha} \quad 0 < \alpha < 1$$

Capital share of output  $\alpha$  is set to 0.3 as in Aiyagari and McGrattan (1998). Capital depreciates at a rate  $\delta$  that we set to 0.075.

The entrepreneur's risky technology is given by:

$$Y_t^e = \theta_t k^\nu$$

The entrepreneurial productivity process follows a two-states first-order Markov process. The choice of parameters  $\nu$ ,  $\theta \in \Theta$  and the transition probabilities for the Markov process are discussed later on. The fraction of entrepreneurial households  $\chi$  is fixed to 0.0755% as in Cagetti and De Nardi (2006). The economy grows at the exogenous rate  $g$  fixed to 0.0185 as in Aiyagari and McGrattan (1998).

### 2.5.2 Households

The utility function of the entrepreneurial household is of the CRRA type and writes:

$$U^e(c_t) = \frac{c_t^{1-\mu}}{1-\mu}$$

The non-entrepreneurial household values leisure and has an utility of the following form:

$$U^{ne}(c_t, l_t) = \frac{(c_t^\eta l_t^{1-\eta})^{1-\mu}}{1-\mu}$$

The risk aversion parameter  $\mu$  is set to 1.5 and the parameter controlling labor elasticity  $\eta$  is set to 0.328 as in Aiyagari and McGrattan (1998). We suppose that the non-entrepreneurial household can not borrow so that  $\bar{a}^{ne}$  is 0. Entrepreneurs can borrow and their borrowing constraint  $\bar{a}^e$  is set to  $-4.0^3$ .

The labor efficiency follows a first-order autoregressive process:

$$e_{t+1} = \rho^{ne} e_t + \varepsilon_{t+1}^{ne}$$

For comparison we follow Aiyagari and McGrattan (1998) and set  $\rho^{ne}$  to 0.6 and  $\varepsilon^{ne}$  to 0.24. This process is approximated by a seven states discrete process using the methodology of Tauchen and Hussey (1991).

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<sup>3</sup> Huggett (1993) suggest a credit limit of one year's average endowment. In the data, individuals can borrow much more than that. The value chosen here is roughly three times the annual net income of an average entrepreneur. I conduct robustness test on this value later on.

### 2.5.3 Government

We assume that the ratio of government purchases to GDP  $\gamma$  is 0.217 and the ratio of lump-sum transfers to GDP  $\varphi$  is 0.082 as specified in Aiyagari and McGrattan (1998). In our benchmark economy, the debt over GDP ratio, noted  $b$ , is initially set to  $\frac{2}{3}$ , as it is the average level reported by Aiyagari and McGrattan (1998) in the U.S. for the postwar period.

Table 1

Parameter	Value	Source
$\alpha$	0.3	Aiyagari and McGrattan (1998)
$\delta$	0.075	Aiyagari and McGrattan (1998)
$\chi$	0.0755	Cagetti and De Nardi (2006)
$g$	0.0185	Aiyagari and McGrattan (1998)
$b$	$\frac{2}{3}$	Aiyagari and McGrattan (1998)
$\mu$	1.5	Aiyagari and McGrattan (1998)
$\eta$	0.328	Aiyagari and McGrattan (1998)
$\bar{a}^{ne}$	0	Aiyagari and McGrattan (1998)
$\bar{a}^e$	-4	See text and robustness
$\rho^{ne}$	0.9	Aiyagari and McGrattan (1998)
$\varepsilon^{ne}$	0.21	Aiyagari and McGrattan (1998)
$\gamma$	0.217	Aiyagari and McGrattan (1998)
$\varphi$	0.082	Aiyagari and McGrattan (1998)

### 2.5.4 Reproducing data moments

After the definition of the first set of parameters, we have to choose the remaining six parameters so as to closely match our target economy. The first calibration target is the capital-output ratio. The value of the capital-output ratio varies with the definition of capital and can range from 2.0 to over 3.0. Here we target a value of 2.5 for the sake of comparison with Aiyagari and McGrattan (1998). We explore an alternative target for the capital-output ratio in the robustness section. Next we have to match the U.S. wealth Gini and the U.S. wealth distribution as reported by Quadrini (2000). Thus we set the remaining six parameters as follows. We choose the discount factor to be 0.978. The entrepreneurial ability  $\theta$  can take only two values. The low value  $\theta_{low}$  is 0.3, the high value  $\theta_{high}$  is 1.11. This implies that the transition matrix  $\pi$  for entrepreneurial ability is a 2-by-2 matrix with each row adding to unity. Thus

this gives two parameters to pin down. We fix  $\pi_{11} = P(\theta' = \theta_{low} | \theta = \theta_{low})$  to 0.988 and  $\pi_{21} = P(\theta' = \theta_{low} | \theta = \theta_{high})$  to 0.115. Finally we have to choose a value for  $\nu$ , the degree of decreasing returns to scale of the entrepreneurial technology. We fix this parameter to 0.6. Thus we have six calibrated parameters to match six observed statistics: the capital output-ratio, the wealth Gini, and the top four percentile of the distribution of wealth. The results of this calibration and comparison with U.S. data and closely related models are reported in Table 3 and discussed in the results section.

Table 2

Parameter	Value
$\beta$	0.978
$\theta_{low}$	0.3
$\theta_{high}$	1.11
$\pi_{11}$	0.988
$\pi_{21}$	0.115
$\nu$	0.6

### 3 Results

This section presents the results obtained with the benchmark economy. A first section reports the overall behavior of the model. Next, I report wealth distribution statistics for the benchmark economy and several similar models. Then I examine the long-run welfare effects of public debt in this stylized economy. Eventually, I conduct experiments to assess the robustness of the model to parameter values.

#### 3.1 *Public debt in a stylized setting with entrepreneurs*

The benchmark model yields an equilibrium interest rate of 5.9%<sup>4</sup>, on average entrepreneurs invest 45% of their wealth in business equity (Moskowitz and Vissing-Jorgensen (2002) find an empirical equivalent of 45%) and they own

<sup>4</sup> Aiyagari and McGrattan (1998) set the interest rate to 4.5% through the calibration of the capital-output ratio. Here the immediate link between the capital-output ratio and the interest rate is broken because of the entrepreneurs. Cagetti and De Nardi (2006) find an equilibrium interest rate of 6.5% in a model with both workers and entrepreneurs.

49% of total wealth (Quadrini (2000) find an empirical equivalent of 40%). Figure 1 report general properties of the benchmark model. Increasing the level of public debt raises the supply of safe assets in the economy. Consequently, the after tax and before tax interest rates increase and the wage rate decreases as shown in the first and second quarter of Figure 1. Public debt has a crowding out effect on corporate capital. The increase in the after-tax interest rate reduces the gap between the after-tax interest rate and the rate of time preference. The cost of postponing consumption to build up a buffer stock of saving is then reduced. Households choose to hold more assets at the steady state equilibrium. Also, because the repayment of debt interests increases, the income tax rate generally increases. But the tax rate does not change monotonically. This property is similar to the result in Aiyagari and McGrattan (1998). This is due to changes in the tax base. Hours worked steadily decrease because of the diminishing wage rate.

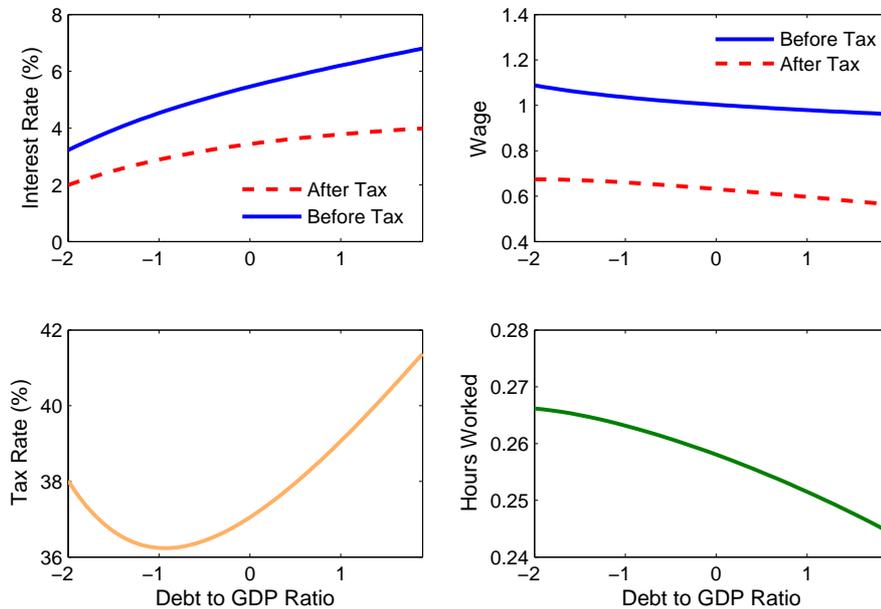


Fig. 1. Interest rates, wage rates, tax rate and hours worked

Figure 2 report entrepreneurial saving behavior in the risky and the non-risky assets. The lower half of Figure 2 report entrepreneurial investment in the safe asset. There is a steady increase of the safe investment as the debt over GDP ratio rises. As reported by Covas (2006), in the presence of uninsurable production risk, entrepreneurs always prefer investing in the safe asset. However, this behavior tend to decrease the interest rate in general equilibrium and raise the attractiveness of the risky investment. Public debt by increasing the supply of safe assets increases the interest rate. Thus, entrepreneurs accumulate more of the safe investment as the level of debt rises and as the cost of precautionary saving is decreased. Interestingly, the upper half of Figure 2 shows that the accumulation of risky entrepreneurial capital is non-monotonic as the level of

debt increases. For low values of the interest rate, raising the level of public debt crowds-in entrepreneurial capital and it is crowded-out the rest of the time. In his seminal paper, Woodford (1990) shows how higher levels of public debt can crowd-in private capital. Woodford (1990) argues that public debt can crowd-in private capital when households are liquidity constrained and face interesting risky investment opportunities only at some periods and not all the time. Here, entrepreneurial households can be liquidity constrained and although they face investment opportunities at all times, some opportunities are more interesting than others because of the uninsurable production risk. Thus, crowding-in à la Woodford (1990) can occur in this economy but are completed by other effects. First, note that the profile of the risky investment when debt increases is extremely flat, notably a lot flatter than the safe investment profile. Note also that the tax profile is decreasing for some levels of public debt when entrepreneurial capital is crowded-in. For a given increase in public debt, the interest rate rises, entrepreneurs accumulate more safe assets and the financial income of entrepreneurs increases. But as long as the expected return on the risky asset is higher than the return on the safe asset, entrepreneurs can use their higher financial income to invest both in the risky asset and the safe asset and entrepreneurial capital can be crowded-in. When the expected return on the risky asset is lower than the return on the safe asset, entrepreneurial capital is crowded-out.

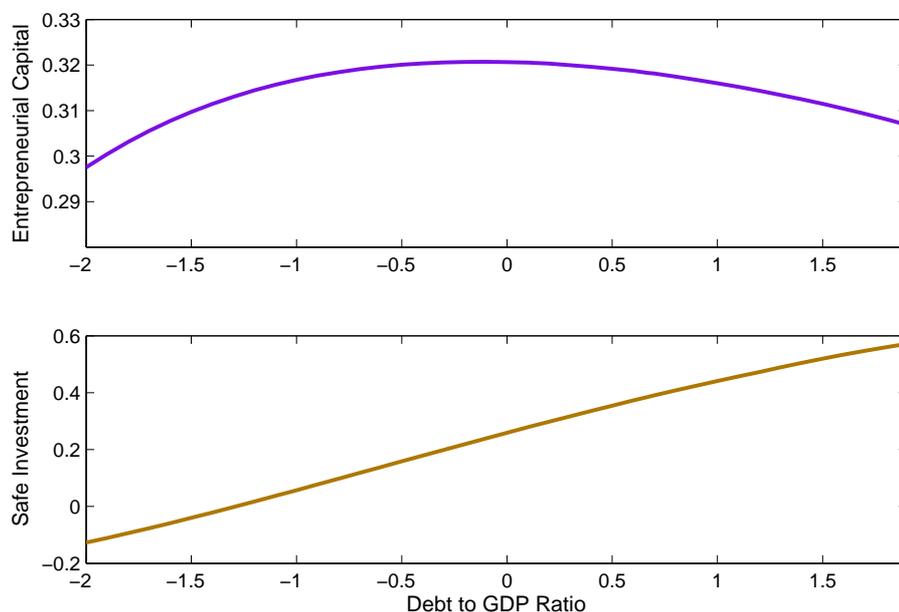


Fig. 2. Entrepreneurial risky and non-risky assets

### 3.2 Wealth distribution

As reported in Table 3, the benchmark economy yields an improved reproduction of the U.S. wealth distribution and wealth Gini. For the sake of comparison the table shows several related models. First we reproduce the model in Aiyagari and McGrattan (1998) and present their wealth distribution and Gini. Both the Gini and the skewness of the wealth distribution is not reproduced by this model. Next we extend this model with the labor productivity process found in Floden (2001), which is more consistent with the results found in the literature of a more persistent income process (see for instance Storesletten et al. (2004)), and we adjust the discount factor to match a capital-output ratio of 2.5. Although the statistics are slightly better, this is still a poor reproduction of the data. Next, we report our benchmark model but without entrepreneurial households. The statistics displayed are similar to those of the model in Aiyagari and McGrattan (1998) as the labor productivity process is the same. Finally, the next to last row displays the wealth distribution statistics found with the benchmark model where both entrepreneurial and non-entrepreneurial households are taken into account. The statistics are fairly close to the data<sup>5</sup>. The benchmark model reproduces the thick right tail found in the U.S. distribution. Most notably, the top 1% of the population hold 30% of total wealth in the economy and the Gini coefficient is 0.78. Thus, to the best of my knowledge, this model is the best setting to assess the steady state long-run optimal level of public debt with regards to wealth statistics so far. Reproducing the wealth distribution is not neutral when assessing public debt. As it is discussed in Floden (2001) or Ball and Mankiw (1995), public debt mostly benefit wealthy households because they gain from higher return on capital when poorer people suffer from higher taxes and lower wages.

Table 3  
U.S. and models wealth distribution comparison

Model	Capital-output ratio	Wealth Gini	Percentage wealth held by top			
			1%	5%	10%	20%
Aiyagari and McGrattan (1998)	2.5	0.42	4	15	26	44
<i>Floden</i> (2001)* (see text)	2.5	0.63	6	25	41	64
Baseline model without entrepreneurs	2.1	0.43	4	15	27	45
Benchmark model with entrepreneurs	2.5	0.78	30.8	55.5	63.5	75.2
U.S. Data	2.5	0.78	29.5	53.5	66.1	79.5

<sup>5</sup> I used as a reference the wealth distribution computed from the Survey of Consumer Finances 1992 as reported by Quadrini (2000).

### 3.3 Steady State optimal level of public debt

The result of the introduction of public debt on welfare is *a priori* undetermined because of several opposing effects. First, the crowding out of physical capital clearly reduces consumption and then welfare. Moreover the increase in the income tax rate tends to amplify the negative impact of public debt on welfare. Then the increase in the after-tax interest rate makes it less costly to accumulate precautionary saving in order to smooth consumption as the interest rate gets closer to the time preference rate. This last effect is welfare enhancing. It is difficult to predict which effect overcomes the others analytically.

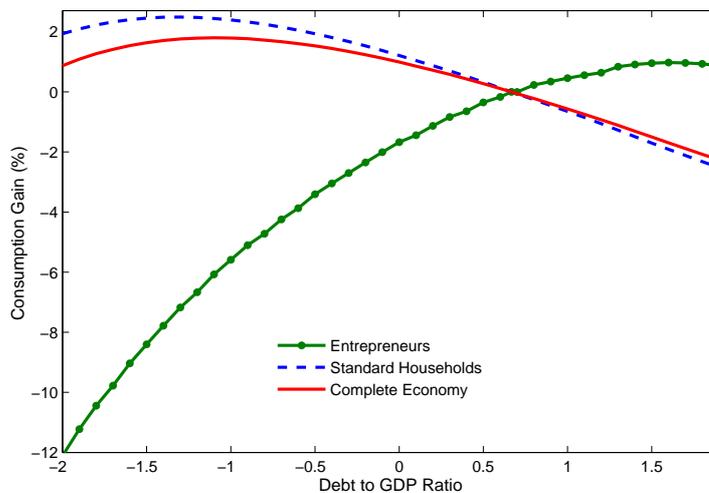


Fig. 3. Consumption gain versus debt/GDP ratio

In a stylized setting with both entrepreneurial and non-entrepreneurial households and reproducing key statistics of the U.S. wealth distribution, I find that the annual optimal level of public debt is negative and amounts to  $-110\%$  of GDP. This is to oppose to the result in Aiyagari and McGrattan (1998) where a positive level of debt of  $66\%$  is found. The welfare gain of being at the optimum level instead of the benchmark level amounts to  $1.8\%$  of consumption. The intuition behind this result is the following. Although entrepreneurial households constitute a small fraction of the population, they have a higher propensity to save. For a given level of public debt and a given set of parameters, the equilibrium capital-output ratio is always higher in an economy where at least a fraction of the population are entrepreneurs. As a result, workers in this model need to accumulate a lot less than in Aiyagari and McGrattan (1998). Overaccumulation by workers is replaced by the empirically relevant accumulation of entrepreneurs. This is why the economy discounts time by according more importance to the present than in Aiyagari and McGrattan (1998). Thus the discount factor is set to  $0.978$  in the benchmark model and is

much lower than the value in Aiyagari and McGrattan (1998) where this parameter is 0.991. This adjustment explains an important part of the change in the optimal level of public debt. The discount factor plays a major role in determining how much consumption smoothing an agent needs and thus impacts the balance between crowding out and the reduced precautionary saving cost. The higher the discount rate, the higher the need for consumption smoothing and the higher the optimal level of public debt. My computations show that a setting with only non-entrepreneurial households subject to the same calibration and most notably to the same discount rate as in the benchmark economy yield an optimal level of public debt of  $-80\%$ .

Scaling the discount factor to the targeted capital-output ratio explains an important part of the adjustment in the optimal level of debt. But interestingly, this level of debt is still lower with entrepreneurs than without even when we control for the adjustment in the discount factor. The exact same economy as in the benchmark model but without entrepreneurs yield an optimal level of public debt of  $-80\%$ . When even a small fraction of the population are entrepreneurs, the optimal level of debt is lower than this latter value. This is the result of two opposing effects. First, as shown by the bulleted line in Figure 3, the optimal level of debt for entrepreneurs alone is higher than the benchmark level. In fact, the optimal level of debt for entrepreneurs would be 160% of GDP and entrepreneurs would gain as much as 0.98% of consumption if public debt was at this level instead of the benchmark level. In the presence of uninsurable production risk, entrepreneurs always prefer the non-risky asset. However, in general equilibrium, this type of precautionary saving tend to lower the interest rate and the attractiveness of the risky investment is increased. This result is reported in Covas (2006). Without public debt, precautionary saving would lower interest rates. Here, a positive amount of public debt, keep interest rates at a higher level and help entrepreneurs to smooth consumption. Entrepreneurs go for higher levels of public debt until the positive impact of reduced precautionary saving cost is balanced by the adverse effects of increasing taxes and crowding out. Second, as shown by the dashed line in Figure 3, the optimal level of public debt for non-entrepreneurial households alone is  $-130\%$  and they would gain 2.5% of consumption if public debt was at this level instead of the benchmark level. This is mainly because the tax rate is higher in an economy with entrepreneurs. For a given level of public debt, introducing entrepreneurial households has two general equilibrium effects that can impact the tax rate. On the one hand, the interest rate is lower because the precautionary saving motive of entrepreneurial households is higher than that of their non-entrepreneurial counterparts. As shown by Equation 3, the effect of the interest rate on the tax rate is ambiguous. On the other hand, the capital-output ratio is higher in an economy with entrepreneurs. We can see in Equation 3 that a higher capital-output ratio unambiguously raises the steady state tax rate. My computations shows that

this last effect combined with the tax rate increasing effect of the interest rate dominate the tax rate decreasing effect of the interest rate. Eventually, for non entrepreneurial households, the adverse effects on welfare of higher taxes and crowding out of capital balances the welfare increasing effect of reduced precautionary saving cost for a lower level of public debt when there are even a slight fraction of entrepreneurs in the economy. Here, as they are the most numerous type of households and because the adverse effects of public debt is higher for them than for entrepreneurs, non-entrepreneurs drive the impact on the overall level of welfare in the economy. In the end, as shown by the straight line in Figure 3, overall consumption gain is maximum for a level of debt of  $-110\%$  and the whole economy would gain as much as  $1.80\%$  of consumption if public debt was at this level instead of the benchmark level. For this level of public debt, entrepreneurs alone would lose as much as  $6.0\%$  of consumption whereas non-entrepreneurs alone would gain as much as  $2.4\%$  of consumption.

### *3.4 Robustness to the credit constraint*

In this section, we experiment the effect on the optimal level of public debt of a change in the borrowing constraint of entrepreneurs. In the benchmark economy, this constraint is set to a value that represent roughly three times the annual net income of an average entrepreneur. It is difficult to pin down an exact value for this parameter. Huggett (1993) suggest a credit limit of one year's average endowment. Covas (2006) loosen up this value by one year and report that in the data individuals can often borrow much more. Generally speaking, Cagetti and De Nardi (2006) argue that the borrowing constraint is related to the level of the entrepreneur's wealth<sup>6</sup>. How much the entrepreneur can borrow is an important aspect of his decisions and saving behavior that will have an effect on his interaction with public debt. Thus in my experiment, I alternatively loosen and tighten the borrowing constraint of the entrepreneur. The results of this experiment is reported in Figure 4.

In the benchmark economy, the optimal level of public debt is  $-110\%$ . If the borrowing constraint of entrepreneurs is loosened to a value that is about 1.5 times the benchmark value (this means a borrowing constraint set to  $-6$ ), the optimal level of public debt is  $-130\%$ . Symmetrically, if the borrowing constraint is tightened to a value of 0.75 times the benchmark value (this means a borrowing constraint set to  $-3$ ), the optimal level of debt is  $-100\%$ . If it tightened even more, to a value of 0.25 times the benchmark value (this means a borrowing constraint set to  $-1$ ), the optimal level of debt would be slightly over  $-90\%$ . The main result of this experiment is that a tighter borrowing constraint produce a higher optimal level of public debt. This is not surpris-

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<sup>6</sup> For a survey on entrepreneurship and borrowing constraints see Cagetti and De Nardi (2006).

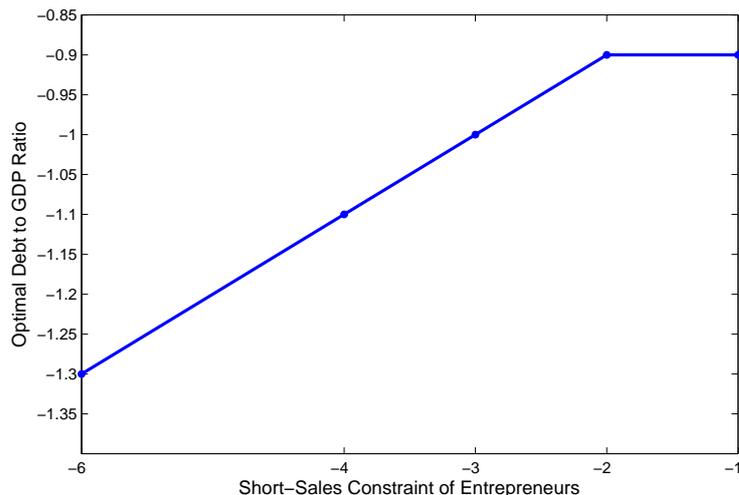


Fig. 4. Optimal level of public debt and the borrowing constraint of entrepreneurs and is correlated with the results in Covas (2006). A tighter borrowing constraint make it more difficult for entrepreneurs to smooth consumption to insure against adverse shocks. As the level of risk faced by entrepreneurs is independent of the borrowing constraint, a tighter borrowing constraint gives them a stronger incentive to self-insure. As a result, the tighter the borrowing constraint, the higher is the need of a greater level of public debt for entrepreneurs.

#### 4 Conclusion

I model a stylized economy with entrepreneurial and non-entrepreneurial households in order to assess the optimal level of public debt. The setting of this paper reproduce key statistics of the U.S. wealth distribution and yield an optimal level of public debt of  $-110\%$ . This value is lower than the result in Aiyagari and McGrattan (1998). I decompose the effects that produce this level of public debt. First, entrepreneurs always want highly positive levels of public debt. This is because they are subject to idiosyncratic productivity risk that increase their need for precautionary saving. On the contrary, non-entrepreneurial households are better off with lower levels of debt because part of their overaccumulation behavior has been transferred to entrepreneurs and because the tax rate is higher in an economy with entrepreneurs. But as entrepreneurs are less numerous, their impact on the overall welfare level in the economy is not as important as non-entrepreneurs. The important qualitative result of this paper is that entrepreneurs reduce the overaccumulation behavior of non-entrepreneurs and thus that optimal level of public debt is lower in an economy with entrepreneurs than in an economy without.

## Appendix

### A Computational strategy

The model is solved using the following steps:

- (1) We guess values for the equilibrium interest rate, the equilibrium amount of hours worked along with values for the aggregate level of risky assets and production of entrepreneurs. Those guesses yield the steady state tax rate.
- (2) Given prices and the tax rate, we solve the problem of the entrepreneur and the non-entrepreneur by iterating on each agent's consumption policy function.
- (3) We iterate on the distribution of agents until a stationary distribution is found.
- (4) Aggregate variables obtained with the stationary distribution are used to update our guesses. The process is repeated until a fixed point for each guessed element is found.

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