Wealth inequality and environmental policy
(preliminary draft)

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Abstract

In this paper, we study the incidence of a pollution tax on wealth inequality through its effects on factor prices. We develop a general equilibrium model of a closed economy in which firms can produce a generic good by using two alternative technologies: dirty and clean. We focus on the case in which the dirty technology is relatively more capital intensive. We find the pollution tax elasticity of factor prices (wage and interest rate) and show contrasting results for two cases: (i) where firms use only one technology and (ii) where firms operate by using both technologies simultaneously. We show that the presence and utilization of an alternative clean technology leads to a decrease in relative price of capital to labor. By using our results on tax elasticity of factor prices, we find that the pollution tax is regressive in single technology case and progressive in two-technology case.

Keywords: wealth inequality, environmental policy, pollution tax, distributional impacts, firm behavior, household behavior, sources side

JEL-Classification: H23, Q52, Q58

1 Introduction

Who is willing to pay more for environmental protection? And what are the sources of differences among individuals that lead them to prefer different levels for environmental policy tools such as pollution taxes? In this study, we focus on these questions and particularly on the dimension of wealth inequality and its implications on the preferred pollution tax of the households. Environmental policies may affect the individuals with a higher wealth differently than the ones with

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a lower wealth because of the fact that they have more capital invested in the market and their consumption levels are not the same.[1]

Policies that aim to reduce the level of pollution can affect the firm decisions through their adjustment of factor demands and thus can have an important impact on the factor prices such as the wage and the interest rate. In a general equilibrium setting, the changes in factor prices affect the household revenues. Indeed, when the factors are unevenly distributed within the society, these impacts can lead to differences in the preferred pollution taxes for the households.

There has been many works that study the distributional impacts of environmental policies. Most of the studies consider a partial equilibrium framework by focusing only on the uses side of income, which means the impact of environmental policies on the commodity prices, and the common result is that the pollution taxes are regressive because the dirty commodities constitute a larger share of the poor households' expenditures. Besides, there is a growing literature that consider a general equilibrium framework and thus taking into account the sources side of income as well, which are close to our framework. For example, Fullerton and Heutel (2007) study the incidence of environmental taxes in a general equilibrium framework and they take into account general forms of substitution among the factors. They show the importance of the elasticity of substitution between dirty and clean goods in both production and consumption sides. Furthermore, using this framework, they identify the impact of a pollution tax on the factor prices as well as on the prices of the outputs. In more recent works, Rausch et al. (2011) and Dissou and Siddiqui (2014) show that the pollution tax can be progressive by considering the sources side of income by using a similar approach.

The incidence of environmental taxes can also be studied by considering the heterogenities among the households in terms of labor income, capital income, transfer income or time preferences. For example, Chiroleu-Assouline and Fodha (2014), Fullerton and Monti (2013) and Marsiliani and Rengström (2002) study the heterogeneity in terms of labor income, Fullerton and Heutel (2010) and Rausch et al. (2011) study the heterogeneity in terms of transfer income and Borissov et al. (2014) could be given as an example that study the heterogeneity in the discount rates of the households. In this paper we abstract from these and we consider only the case of heterogeneity in terms of capital endowment.

The income data of the U.S. economy from the 2007 Survey of Consumer Finances (SCF) show that the revenues from capital constitute 25% of the total overall income. Moreover, as shown by Fullerton and Heutel (2010), the fraction of income coming from capital is increasing over income deciles.[2] For example, the fraction of income that comes from capital is 5.7% for the lowest income decile, 7.8% for the fifth income decile and 45.6% for the highest income decile. Accordingly, neglecting the heterogeneity in capital revenues generates a significant gap in the theoretical analysis.

A number of previous studies consider wealth inequality in their analysis. For example, Rausch et al. (2011) and Dissou and Siddiqui (2014) consider it but they do not conduct in depth theoretical analysis of its implications on the households' preferred pollution taxes. One of the purposes of this study is to fill this gap in the literature. Furthermore, Kempf and Rossignol (2007) study the

[1] Throughout the text we treat capital ownership and wealth as identical terms. This equivalence relies on the assumption that all wealth owned by the households are lent to the firms in the economy and thus employed in production.

[2] With the exception that the lowest income decile has slightly higher share of capital in their income compared to the next decile.

relationship between wealth inequality and environmental protection in a theoretical framework and address the questions that are similar to ours. By using an endogenous growth model, they show that the richer households prefer a higher environmental tax and correspondingly inequality is harmful for the environment. But this result is based on the fact that the relative price of labor to capital is independent from the environmental tax since their model does not incorporate alternative cleaner production technologies. This dimension is indeed the main focus of our paper and it makes our framework, and thus our results, significantly different from theirs.

Our aim in this study is two folds. First, to investigate the effects of a pollution tax on the firm behavior and factor prices in the partial competitive equilibrium and identify the determinants of these effects. Second, in a general equilibrium setting, to relate these findings with the households' preferred pollution taxes and eventually identify the cases in which the pollution tax is regressive or progressive respect to the households' welfare.[3]

We develop a static general equilibrium model by taking into account households, firms and the government. Households have different wealth endowments and their utility depends on their consumption level and the level of environmental quality. The level of environmental quality depends negatively on the level of pollution. The production side of the model is inspired by the works of Harberger (1962), Copeland and Taylor (2004), Fullerton and Heutel (2007) and many others that apply the international trade framework of Heckscher-Ohlin. We study an economy with firms that produce a generic good by using two different technologies, namely dirty and clean, with each of them using capital, labor and pollution as an input to produce the final output.[4] The factor prices of capital and labor are determined endogenously in the equilibrium, the government determines the pollution tax and uses its revenues for government spending purposes.

Our results show that the impact of a pollution tax on the factor prices depends on the characteristics of the production technologies utilized by the firms in the economy. We find that the relative price of factor that is more intensively used in the dirty technology will decrease as a response to an increase in the pollution tax - which is a well-known result in the literature. Moreover, when we consider the dirty technology is more capital intensive than the clean one, the interest rate always decreases with the pollution tax. But, whether the wage increases or decreases depends on the comparison of the relative intensities of pollution and capital between the production technologies. In particular, we show that the wage increases when the relative pollution intensity respect to capital is higher in the dirty technology, and *vice versa*. These results, which we summarize in Table (1), differ from the many studies in the literature (for example Copeland and Taylor (2004), Fullerton and Heutel (2007)). These findings are based on the fact that in our setting, contrary to theirs, the clean technology also pollutes thus its pollution intensity matters.

On the household side, we investigate the household's decision about its preferred pollution tax and we identify the trade-off that they face between a higher consumption and a better environmental quality. At this point, this paper differs from the ones in the literature (such as Fullerton and Heutel (2007), Dissou and Siddiqui (2014) in two ways. First, we consider the utility of household depends also on the environmental quality that leads to the trade-off that we mentioned above. Second, this paper does not address the uses side effects of the pollution tax. The reason is that our model constitute a closed economy in which the firms produce a generic good by using alternative

[3]In this paper, we use the progressivity and regressivity terms always in terms of welfare.

[4]The use of pollution as an input in the production process is a well-established modeling approach in the environmental economics literature and the motivation behind is explained in Section 2.1.

technologies.[5] In this setting, pollution tax has no effect on the commodity prices. On the contrary, the models presented by those papers are consistent with a closed economy with two sectors. Therefore, the pollution tax increases the relative price of the dirty good to the clean one and thus causes the uses side effect.

Having only the sources side in the setting leads us to find the effect of wealth on a household's preferred pollution tax which depends on two opposite channels. We call the first one as the satiation effect. It says that the households with a higher wealth consume more and their marginal utility of consumption is lower, thus they would be more willing to sacrifice from their consumption for a better environmental quality. And we call the second channel as the cost of pollution tax effect. It says that the households with a higher wealth has larger capital investments in the market, thus when the return of capital falls their revenues are more reduced by the pollution tax. Accordingly, whether the pollution tax increases or decreases with wealth depends on which one of these effects dominates. We show that, in fact, it depends on the pollution tax elasticity of consumption which is determined by the pollution tax elasticities of the factor prices.

By linking the results of the production side, we show that if the firms are operating with a single production technology then the richer households prefer a higher pollution tax, hence the tax is regressive. On the contrary, if the firms are using the dirty and clean technologies simultaneously, the pollution tax leads to a reallocation of resources in the clean technology. In this case, when the dirty technology is more capital intensive, the richer households lose more from their consumption in percentage terms which means that they would prefer a lower pollution tax. In other words, when the economy operates on two technologies the tax is progressive.

The following section presents the model. In detail, Section 2.1 presents the firm decision and analyzes the impact of a pollution tax on the factor prices, Section 2.2 explains the role of the government and how the proceeds from the pollution tax are used, Section 2.3 presents the household decision, characterizes the general equilibrium for this economy and shows the conditions for the impact of the wealth on the preferred pollution tax of an household. Then Section 3 discusses the implications of the cases when some of the assumptions that we made are relaxed. Finally Section 4 concludes.

2 The Model

Within a static framework, we analyze a closed economy that consists of households, firms and the government. We consider a continuum of households indexed by $i \in (0, 1)$ with each of them supplying one unit of labor inelastically. Each household i has an initial capital (*wealth*) endowment k_i and it follows a distribution $\mathcal{G}(.)$ which determines the state of wealth inequality in the society. The total capital in the economy, therefore, is $\bar{K} = \int_0^1 k_i di$.

Household's utility $V(c, E)$ depends on consumption of the generic good (c) and the level of environmental quality (E) that decreases withthe level of pollution (z). The firms produce the generic good in a perfectly competitive market by using capital (k), labor (l) and pollution. The factor prices of capital and labor (r and w) are determined endogenously in the equilibrium. The

[5] Our model can also be interpreted as a small open economy with two sectors in which the production factors are mobile across sectors but immobile across countries. In this type of setting, the country engages in goods trade but has an isolated financial market. This setting is suitable for some of the developing countries today. (reference?)

government determines the unit price of pollution (τ) and uses the collected tax revenue for its expenditures.

In the following sections we explain the aims and the decision making processes of the firms, the government and the households in detail and study the outcome in a general equilibrium framework.

2.1 Production

The production of the generic good is a function of capital (k), labor (l) and pollution (z). In line with Siebert et al. (1980), Copeland and Taylor (1994), Copeland and Taylor (2004), Fullerton and Heutel (2007), we take into account pollution as an input in the production process. This approach for modeling production is usually called as "joint production technology". [6]

We assume functional separability between pollution and the physical inputs in the joint production technology. Hence, the production function is denoted as $F(z, G(k,l))$ where the first argument of $F(.,.)$ is pollution (z) and the second argument is the conjoint physical input of capital and labor ($G(k,l)$). This way of specification is similar to and more general than the one in Copeland and Taylor (2004).[7] Functional separation implicitly assumes that the relative factor demands are identical in both final good production process and the pollution abatement process. [8] As will be shown later on, this restriction is necessary to analyze the single production technology (Section 2.1.1) while it is not necessary for multiple production technologies (Section 2.1.2). We prefer to keep this form to maintain consistency throughout the text.

This nested structure for production function captures the fact that the physical inputs for production (capital and labor) are having a bilateral elasticity of substitution between them and pollute to operate the production process. Moreover, the conjoint physical input of capital and labor has an elasticity of substitution with pollution. The shapes of $F(.,.)$ and $G(.,.)$ determines the substitutability (or complementarity) of each input respect to the others. We assume the following properties for the production function:

[6] One way of motivating this is to think about two production processes: the first one is the production of the final good and the second one is the abatement of pollution. The first production process uses capital and labor as inputs and produces the final good as well as pollution as a by-product. The second one also employs capital and labor to produce equipment which are used to reduce the level of pollution that is generated by the first production process. These two production processes can be transformed into a joint production technology. Jouvet et al. (2005) also show a similar exercise of this transformation and conclude by obtaining a production function homogenous of degree one of capital, labor and pollution.

[7] Copeland and Taylor (2004) assumes that the production function is Cobb-Douglas in pollution and conjoint physical input of capital and labor, that is $x = z^{\alpha}(F(K_x, L_x))^{1-\alpha}$.

[8] See Appendix E for details. Note that this certain assumption is necessary just for this motivation of the production function and it does not have any role in our results.

Assumption 1. *The production function satisfies the following properties:*[9,10]

(i) $F(.,.)$ *and* $G(.,.)$ *are homogenous of degree one.*
(ii) $F_1(.,.) > 0$, $F_{11}(.,.) < 0$, $F_2(.,.) > 0$, $F_{22}(.,.) < 0$, $F_{12}(.,.) > 0$
(iii) $G_1(.,.) > 0$, $G_{11}(.,.) < 0$, $G_2(.,.) > 0$, $G_{22}(.,.) < 0$, $G_{12}(.,.) > 0$

Assumption 1 means that the production technology embodies constant returns to scale. It also implies that each factor's marginal productivity is positive and decreasing in its amount and is increasing in other factors' amounts. [11]

We proceed step by step for the decision making process of the firms. Our aim is to analyze the effect of a change in the pollution tax on the prices of capital and labor and on the allocation of resources in the economy. We first investigate a simple case in which there is only a single production technology available. Then we study the case in which there are two alternative production technologies with different factor intensities. We will show that these two cases may have contrasting results depending on the characteristics of the production technologies.

2.1.1 Single production technology

In this framework there is only one production technology available. The firms take the prices of input factors as given and minimize their cost by deciding on their factor demands $(\alpha_z, \alpha_k, \alpha_l)$ for producing one unit of the output. The problem of the representative firm is:

$$\min_{\{\alpha_z,\alpha_k,\alpha_l\}} \{\tau\alpha_z + r\alpha_k + w\alpha_l\} \tag{1}$$
$$\text{subject to } F(\alpha_z, G(\alpha_k, \alpha_l)) = 1 \tag{2}$$
$$\text{and } 0 \leq \alpha_j \text{ for } j \in \{z, k, l\}$$

where r, w and τ denote the interest rate, wage and unit pollution tax respectively. The cost minimization problem in (1) yields the following first order conditions:

$$F_1(\alpha_z, G(\alpha_k, \alpha_l)) = \tau \tag{3}$$
$$F_2(\alpha_z, G(\alpha_k, \alpha_l))G_1(\alpha_k, \alpha_l) = r \tag{4}$$
$$F_2(\alpha_z, G(\alpha_k, \alpha_l))G_2(\alpha_k, \alpha_l) = w \tag{5}$$

[9] Throughout the text we use the following notations for a derivative of a function: $f'(x) = \partial f/\partial x$, $f''(x) = \partial^2 f/\partial x^2$, $f_i(x,y) = \partial f/\partial i$ and $f_{ij}(x,y) = \partial^2 f/\partial i \partial j$ where i and j denote the order of the arguments of f. For example, $f_1(x,y) = \partial f/\partial x$, $f_2(x,y) = \partial f/\partial y$, $f_{11}(x,y) = \partial^2 f/\partial x^2$ and $f_{12}(x,y) = \partial^2 f/\partial x \partial y$.

[10] These assumptions on the production function are satisfied by most commonly used production functions such as Cobb-Douglas and CES. We consider to proceed on the analysis by using the general form in order to cover a larger family of functional forms.

[11] The assumptions on capital and labor are straightforward and standard, however, the ones on pollution still need to be justified. Total output increases if we increase pollution keeping the amount of capital and labor constant ($F_1(.,.) > 0$). One can think that in this case the amount of capital and labor allocated for abatement activities are reallocated in the production of the final good. Therefore pollution will increase due to decreased abatement and total output will increase due to higher amount of capital and labor employed in the final good production process. Of course a technology is more *dirty* if it needs more amount of capital and labor relocated from final good production to the pollution abatement for having a unitary decrease in pollution.

Since marginal productivity of each factor is always positive and we assume perfect competition among the firms, capital and labor will be employed at their highest quantities ($\bar{K}$ and $\bar{L}$) in the equilibrium. Constant returns to scale property of the production function implies that the relative intensity of capital to labor is fixed by the factor endowment in the economy.

$$\frac{\alpha_k}{\alpha_l} = \frac{\bar{K}}{\bar{L}} \tag{6}$$

Equations (2 to 6) allow us to obtain factor intensities and the prices of capital and labor as a function of the pollution tax ($\alpha_z(\tau), \alpha_k(\tau), \alpha_l(\tau), w(\tau), r(\tau)$). Furthermore, by taking into account the fact that $\bar{K} = \alpha_k(\tau)F(\alpha_z(\tau), G(\alpha_k(\tau), \alpha_l(\tau)))$ or $\bar{L} = \alpha_l(\tau)F(\alpha_z(\tau), G(\alpha_k(\tau), \alpha_l(\tau)))$ we can determine the equilibrium level of output.

It is easy to show that an increase in pollution tax decreases the pollution intensity of production ($\alpha_z'(\tau) < 0$). A lower pollution intensity reduces the marginal productivity (and hence the price) of conjoint physical input ($F_2(., G(.))$). Moreover, the relative price of capital and labor will not change since the relative intensity of capital to labor is fixed by the total endowment (eq. (6)). As a result, the prices of labor and capital will decrease at the same rate.

Proposition 1. *When the economy operates using a single production technology, the wage and the interest rate are decreasing in the pollution tax ($w'(\tau) < 0$, $r'(\tau) < 0$). Moreover, both has the same elasticity respect to the pollution tax, $\epsilon_{w,\tau} = \epsilon_{r,\tau} < \epsilon_{R,\tau} < 0$.[12] where R denotes the gross interest rate.*[13]

Proof. See Appendix A1. □

To summarize, in this basic framework the interest rate and the wage decreases with the same elasticity as a response to an increase in the pollution tax. This result relies on the following assumptions: (*i*) only one technology is available in the economy, (*ii*) the production function is constant returns to scale and it is separable between pollution and conjoint physical input of capital and labor, (*iii*) the endowment of capital and labor is fixed in the economy, (*iv*) labor supply is inelastic.

In the following section, we will relax the first assumption and we investigate how the results will change. More specifically, we will investigate how the responses of factor prices to an increase in pollution tax will change when an alternative production technology is available to use.

2.1.2 Two production technologies: dirty and clean

In this framework, we consider that the generic good can be produced by using two different technologies: dirty (X) and clean (Y).[14] The two technologies both require the use of capital (k), labor (l) and pollution (z) and they are denoted as $X = F^X(z_x, G^X(k_x, l_x))$ and $Y = F^Y(z_y, G^Y(k_y, l_y))$. The functions $F^i(.)$ and $G^i(.)$ for $i \in \{X, Y\}$ satisfy the properties given in Assumption 1.

[12]The term $\epsilon_{x,y}$ denotes the elasticity of x respect to y ($\frac{\partial x/\partial y}{x/y}$)

[13]Here we also report the differences respect to the elasticity of gross capital return because they will be useful for the analysis of the household's problem.

[14]Studying only two technologies case is not too restrictive because even if we had taken into account an economy with n technologies, in this framework, the firms would utilize maximum two of them. This assertion is valid in the case where $F^i(.)$ and $G^i(.)$ for $i \in \{1, ..., n\}$ are homogenous of degree one. See Appendix D for details.

The representative firm takes the factor prices as given and minimizes its unit cost of production for each technology with the following programme:

$$\min_{\{\alpha_z^X,\alpha_k^X,\alpha_l^X,\alpha_z^Y,\alpha_k^Y,\alpha_l^Y\}} \left\{\tau(\alpha_z^X + \alpha_z^Y) + r(\alpha_k^X + \alpha_k^Y) + w(\alpha_l^X + \alpha_l^Y)\right\} \tag{7}$$

$$\text{subject to } F^i(\alpha_z^i, G^i(\alpha_k^i, \alpha_l^i)) = 1 \text{ for } i \in \{X, Y\} \tag{8}$$

$$\text{and } 0 \leq \alpha_j^i \text{ for } i \in \{X, Y\} \text{ and } j \in \{z, k, l\}$$

The cost minimization problem leads to the following first order conditions:

$$F_1^X(\alpha_z^X, G^X(\alpha_k^X, \alpha_l^X)) = F_1^Y(\alpha_z^Y, G^Y(\alpha_k^Y, \alpha_l^Y)) = \tau \tag{9}$$

$$F_2^X(\alpha_z^X, G^X(\alpha_k^X, \alpha_l^X))G_1^X(\alpha_k^X, \alpha_l^X) = F_2^Y(\alpha_z^Y, G^Y(\alpha_k^Y, \alpha_l^Y))G_1^Y(\alpha_k^Y, \alpha_l^Y) = r \tag{10}$$

$$F_2^X(\alpha_z^X, G^X(\alpha_k^X, \alpha_l^X))G_2^X(\alpha_k^X, \alpha_l^X) = F_2^Y(\alpha_z^Y, G^Y(\alpha_k^Y, \alpha_l^Y))G_2^Y(\alpha_k^Y, \alpha_l^Y) = w \tag{11}$$

where $\left\{\alpha_z^i, \alpha_k^i, \alpha_l^i\right\}$ for $i \in \{X, Y\}$ are the derived demands of pollution, capital and labor, respectively, for producing one unit of output by using technology i. The six equations in (9 - 11) together with the two equations in (8) yield a system of eight equations with eight variables, that is, intensities of all factors in each sector and prices of capital and labor. And the solution of this system will give each of the variables as a function of pollution tax. Note that contrary to the single technology framework, factor intensities, wage and interest rate are independent from the total resource endowment ($\bar{K}$ and $\bar{L}$).

We define the technology with higher pollution intensity as the dirty one and we assume no factor intensity reversal to ensure that the dirty technology, according to this definition, always remains as the dirty one. Moreover, for the current analysis, we assume that the dirty technology is more capital intensive as well.[15] Formally:

Assumption 2. *The dirty technology (X) is assumed to be more capital intensive than the clean technology (Y):*

$$\alpha_z^X > \alpha_z^Y, \alpha_k^X > \alpha_k^Y, \alpha_l^X < \alpha_l^Y.$$

Note that in Assumption 2 we compare the factor intensities ($\frac{z_x}{X} > \frac{z_y}{Y}, \frac{k_x}{X} > \frac{k_y}{Y}, \frac{l_x}{X} < \frac{l_y}{Y}$) between the technologies to define the type of production technology. This approach is equivalent to the comparison of factor shares in production ($\frac{\tau z_x}{X} > \frac{\tau z_y}{Y}, \frac{rk_x}{X} > \frac{rk_y}{Y}, \frac{wl_x}{X} < \frac{wl_y}{Y}$).

As we stated before, the factor intensities and the factor prices are independent from the aggregate level of capital and labor. However, the allocation of resources between the two technologies will depend on the total resources. The total demand for factor j in technology a can be computed by multiplying the unit demand for that factor and the total production of that technology. Therefore, the total resource constraint implies the following:

$$X\alpha_k^X(\tau) + Y\alpha_k^Y(\tau) = \bar{K} \tag{12}$$

$$X\alpha_l^X(\tau) + Y\alpha_l^Y(\tau) = \bar{L} \tag{13}$$

[15] In Section 3 we will discuss the implications of the case in which the dirty technology is more labor intensive.

where X and Y represent total production by the dirty and clean technology respectively. Solving these two equations for total output of each technology (X and Y) yields to the following relations:

$$F^X(z_x, G^X(k_x, l_x)) = X(\tau) = \frac{\alpha_l^Y(\tau)\bar{K} - \alpha_k^Y(\tau)\bar{L}}{\alpha_k^X(\tau)\alpha_l^Y(\tau) - \alpha_l^X(\tau)\alpha_k^Y(\tau)} \tag{14}$$

$$F^Y(z_y, G^Y(k_y, l_y)) = Y(\tau) = \frac{\alpha_l^X(\tau)\bar{K} - \alpha_k^X(\tau)\bar{L}}{\alpha_k^Y(\tau)\alpha_l^X(\tau) - \alpha_l^Y(\tau)\alpha_k^X(\tau)} \tag{15}$$

Using equations (14) and (15) we can obtain the allocation of each factor between the technologies, that is, $z_x(\tau) = X(\tau)\alpha_z^X(\tau)$, $k_x(\tau) = X(\tau)\alpha_k^X(\tau)$, $l_x(\tau) = X(\tau)\alpha_l^X(\tau)$, $z_y(\tau) = Y(\tau)x_y(\tau)$, $k_y(\tau) = Y(\tau)\alpha_k^Y(\tau)$, $l_y(\tau) = Y(\tau)\alpha_l^Y(\tau)$.

Now that we obtained all the factor intensities, the factor prices, the amounts of each factor employed in each technology and the total amounts of production made by using each technology, we can characterize the partial competitive equilibrium:

Definition 1. *For a given pollution tax (τ), the unique partial competitive equilibrium for this economy is characterized by the vector of factor intensities in each technology $\{\alpha_z^X, \alpha_k^X, \alpha_l^X, \alpha_z^Y, \alpha_k^Y, \alpha_l^Y\}$, the vector of labor and capital prices $\{w, r\}$, the vector of the factors amounts employed in each technology $\{z_x, k_x, l_x, z_y, k_y, l_y\}$ and the the total production in each technology $\{X, Y\}$ such that:*

(i) *The firms minimize their costs, thus the eight equations in (8 to 11) hold.*

(ii) *The markets clear, thus the resource constraints (equations (14 and 15)) hold.*

By using the definition above, we determine the level of total output and allocation of factors between the two technologies, as well as the factor intensities and the factor prices at the equilibrium as a function of the pollution tax. So how does the pollution tax affects these variables, in particular the prices of capital and labor?

Basically, an increase in the pollution tax makes pollution more expensive as an input. Hence both sectors will use pollution less intensively which causes an adverse effect on the productivities of labor and capital. Since the dirty technology is more pollution intensive, an increase in the tax affects the use of this technology at most. It will be more profitable for the firms to use the clean technology, thus, some of the resources that are used in the dirty technology will be reallocated in the clean one. Consequently, the share of the clean technology, which is more labor intensive, will increase in aggregate production. This leads to an increase in relative productivity of labor respect to capital.

Accordingly, a rise in the pollution tax affects the factor prices from two channels: (i) a decline in pollution intensity and (ii) reallocation of capital and labor from the dirty technology to the clean one. Both channels impose a negative impact on the interest rate while they push the wage in two opposite directions. On the one hand, less pollution intensity pushes the wage downward, and on the other hand, factor reallocation from capital intensive technology to the labor intensive one pushes it upward. Whether the wage increases or decreases depends on which one of these effects dominates. In the following proposition we show that in fact it depends on the relative intensity of pollution and capital between the two technologies:

Proposition 2. *When the economy operates using both technologies, the interest rate decreases in the pollution tax ($r'(\tau) < 0$). The change in the wage depends on the technologies' relative* *pollution intensities respect to capital.*

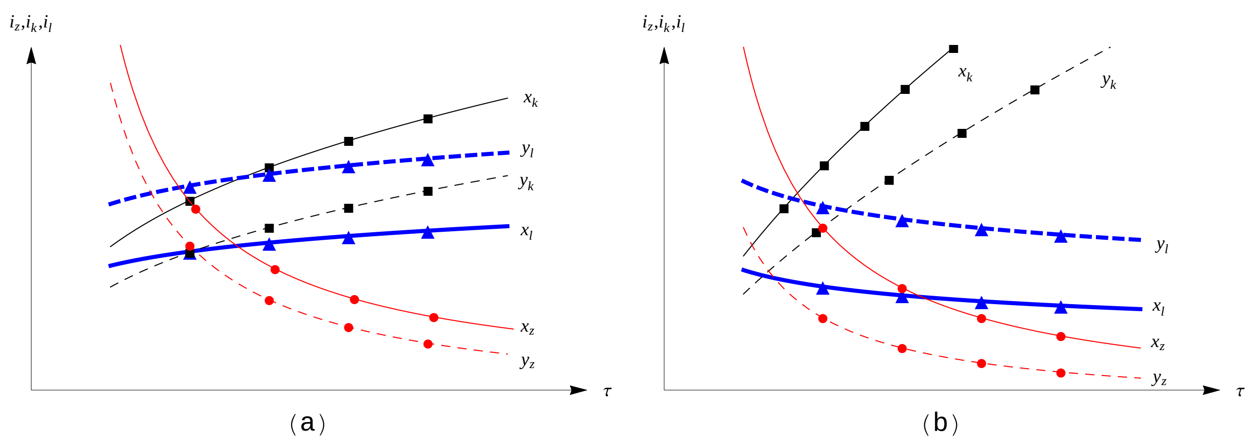
(*i*) *if* $\frac{\alpha_z^X}{\alpha_k^X} < \frac{\alpha_z^Y}{\alpha_k^Y}$ *then* $r'(\tau) < w'(\tau) < 0$ *and* $\epsilon_{r,\tau} < \epsilon_{R,\tau} < \epsilon_{w,\tau} < 0$

(*ii*) *if* $\frac{\alpha_z^X}{\alpha_k^X} = \frac{\alpha_z^Y}{\alpha_k^Y}$ *then* $r'(\tau) < w'(\tau) = 0$ *and* $\epsilon_{r,\tau} < \epsilon_{R,\tau} < \epsilon_{w,\tau} = 0$

(*iii*) *if* $\frac{\alpha_z^X}{\alpha_k^X} > \frac{\alpha_z^Y}{\alpha_k^Y}$ *then* $r'(\tau) < 0 < w'(\tau)$ *and* $\epsilon_{r,\tau} < \epsilon_{R,\tau} < 0 < \epsilon_{w,\tau}$

Proof. See Appendix A2. □

The comparison of the two cases ((*i*) and (*iii*)) in Proposition 2 is illustrated in fig.(1). As it is clear from the figure, pollution will be used less intensively in both technologies when the pollution tax increases. Besides, as Proposition 2 asserts, the interest rate declines in both cases as a response to an increase in the pollution tax. This makes the firms to use capital more intensively in both technologies. However, the wage can increase or decrease once the pollution tax rises. When the relative pollution intensity of the dirty technology to the clean one ($\frac{\alpha_z^X}{\alpha_z^Y}$) is lower than the relative capital intensity ($\frac{\alpha_k^X}{\alpha_k^Y}$) then the wage decreases and so labor is employed more intensively in both technologies. (fig.(1,a)). In the contrary case ($\frac{\alpha_z^X}{\alpha_z^Y}$) > ($\frac{\alpha_k^X}{\alpha_k^Y}$), higher pollution tax leads to an increase in the wage, therefore more environmental protection leads to a decline in labor intensities of both technologies. fig.(1,b).



Note: Panel (a) illustrates the case where $\frac{\alpha_z^X}{\alpha_k^X} < \frac{\alpha_z^Y}{\alpha_k^Y}$ and panel (b) illustrates the case where $\frac{\alpha_z^X}{\alpha_k^X} > \frac{\alpha_z^Y}{\alpha_k^Y}$. The solid curves are for the dirty technology (X) and the dashed curves are for the clean one (Y). The squares, triangles and circles mark the unit factor demand curves for capital, labor and pollution respectively.

Figure 1: Example unit factor demands respect to the pollution tax

Whether the relative intensity of capital to labor increases or decreases in the two technologies depends on how their relative price changes with the pollution tax. Proposition 2 implies that the relative price of capital to labor will decrease as a response to higher pollution tax. Therefore, more environmental protection makes the firms to use capital more intensively. This leads us to the following proposition:

Proposition 3. *If the economy operates using both technologies, and if Assumption 1 and Assumption 2 hold, then higher pollution tax will increase relative intensity of capital to labor in both technologies.*

$$\frac{d(\alpha_k^i(\tau)/\alpha_l^i(\tau))}{d\tau} > 0 \quad \textit{for } i \in \{X, Y\} \tag{16}$$

where, α_j^i is the unit-demand for factor j in technology i.

Proof. See Appendix A3.

□

As it can be seen in Appendix A2, functional separability between pollution and physical inputs is not necessary for Proposition 2. But, Proposition 3 is conditional on that assumption.

Proposition 3 implies two extreme cases: In one extreme case, when the pollution tax is sufficiently high, all the resources will be allocated only in the clean technology and at this point capital/labor ratio in the clean technology equals to the ratio between total capital and total labor in the economy. As the tax decreases, the resources will be reallocated in the dirty technology and both technologies will become more labor intensive. In the other extreme case, the tax will be low enough such that all resources will be allocated only in the dirty technology. Obviously, in this case the capital/labor ratio in the dirty technology equals to the ratio of their total endowments in the economy.

Accordingly, we can define two thresholds for the pollution tax: *(i)* the dirty threshold and *(ii)* the clean threshold. In the case where the pollution tax is lower than the dirty threshold only the dirty technology is used and if it is greater than the clean threshold the firms operate by using only the clean technology. When the tax is between these thresholds, the firms will operate by using both of the technologies simultaneously in production.

Proposition 4. *If τ_{dirty} and τ_{clean} satisfy $\frac{\alpha_k^X(\tau_{dirty})}{\alpha_l^X(\tau_{dirty})} = \frac{\bar{K}}{\bar{L}}$ and $\frac{\alpha_k^Y(\tau_{clean})}{\alpha_l^Y(\tau_{clean})} = \frac{\bar{K}}{\bar{L}}$, then:*

(i) if $\tau \leq \tau_{dirty}$ then the firms use only dirty technology, $k_x = \bar{K}, l_x = \bar{L}, k_y = 0, l_y = 0$.
(ii) if $\tau_{dirty} < \tau < \tau_{clean}$ then the firms use dirty and clean technologies simultaneously,
$k_x > 0, l_x > 0, k_y > 0, l_y > 0$ with $k_x + k_y = \bar{K}, l_x + l_y = \bar{L}$
(iii) if $\tau \geq \tau_{clean}$ then the firms use only clean technology, $k_x = 0, l_x = 0, k_y = \bar{K}, l_y = \bar{L}$.

Proof. See Appendix A4.

□

As it is shown in Proposition 4, τ_{dirty} and τ_{clean} depend only on the relative endowment of capital and labor in the economy. Using the definition of these thresholds and equation (16), we can show that both of the thresholds are increasing in $\frac{\bar{K}}{\bar{L}}$. For a given amount of labor force, the more capital endowed in the economy is the more profitable the dirty technology would be compared to the clean one, hence, it would require higher pollution taxes to induce the firms to use the cleaner technology.

Corollary 1. *τ_{dirty} and τ_{clean} are both increasing the the ratio of total capital and labor in the economy, $\partial\tau_{dirty}/\partial(\bar{K}/\bar{L}) > 0$ and $\partial\tau_{clean}/\partial(\bar{K}/\bar{L}) > 0$.*

Proof. See Appendix A5.

□

Table (1) summarizes the results of Proposition 1 and Proposition 2 which show the impact of an increase in the pollution tax on the prices of capital and labor.

	Single technology	Dirty&clean technologies	
		$\alpha_z^X/\alpha_k^X < \alpha_z^Y/\alpha_k^Y$	$\alpha_z^X/\alpha_k^X > \alpha_z^Y/\alpha_k^Y$
Interest rate	$r'(\tau) < 0$	$r'(\tau) < 0$	$r'(\tau) < 0$
Wage	$w'(\tau) < 0$	$w'(\tau) < 0$	$w'(\tau) > 0$
Elasticities	$\epsilon_{r,\tau} = \epsilon_{w,\tau} < \epsilon_{R,\tau} < 0$	$\epsilon_{r,\tau} < \epsilon_{R,\tau} < \epsilon_{w,\tau} < 0$	$\epsilon_{r,\tau} < \epsilon_{R,\tau} < 0 < \epsilon_{w,\tau}$

Table 1: Impact of an increase in pollution tax on factor prices and their tax elasticities

We can conclude the analysis of production side by stating that the effects of an increase in the pollution tax on factor prices depend on the characteristics of the production technologies available and utilized by the firms in the economy. When the production technologies satisfy the properties given in Assumption 1 and Assumption 2, meaning that the technologies embody constant returns to scale and the dirty technology is more capital intensive than the clean one, the impact of an increase in the pollution tax on the factor prices will be as shown in Table (1) in the equilibrium.

2.2 Government

Government collects the pollution tax and use it to finance its expenditure. The way government expends this revenue indeed plays crucial role for inequality. As we will explain in the next section, our focus in this paper is to investigate the effect of pollution tax on inequality through the change in factor incomes. To abstract from revenue-recycling approach, in line with Harberger (1962), Chiroleu-Assouline and Fodha (2006), Fullerton and Heutel (2007) and others, we consider that the government uses the collected tax revenues to buy the goods from the market ($G = \tau Z$) which has no effect on the households' utility.

In this context, the government revenue from pollution tax will not include any kind of redistribution neither in the form of public services nor in the form of a transfer to the households. This assumption allows us to keep our focus on the trade-off between consumption and environmental quality abstracted from redistributional effects. Besides, it provides analytical tractability and convenience.

Consequently, the households will consume (C) only the revenues that are obtained from labor and capital rented to the firms. Since the model is static, there is neither public nor private investment and saving. This means that the total consumption (public and private) equals the total production. Formally:

$$Y(\tau) + X(\tau) = C(\tau) + G(\tau) = ((1 + r(\tau))\bar{K} + w(\tau)\bar{L}) + \tau Z(\tau) \tag{17}$$

The left hand side of equation (17) is the aggregate production in terms of numeraire price and the right hand side denotes total private and public consumption. Now we can investigate the effect of pollution tax on the aggregate production, private and public consumption by looking at the derivative of equation (17) with respect to the pollution tax:[16]

$$Y'(\tau) + X'(\tau) = (r'(\tau)\bar{K} + w'(\tau)\bar{L}) + Z(\tau) + \tau Z'(\tau) = \tau Z'(\tau) < 0 \tag{18}$$

An increase in the pollution tax decreases the total private consumption due to the decrease in the factor revenues. This holds true even in the case where the wage increases in the pollution tax because the effect of the decrease in the interest rate on total private consumption dominates the gains from the increase in the wage.[17] Moreover, the aggregate production is also decreasing in the pollution tax ($\tau(\tau)Z'(\tau) < 0$). Hence there is no room for double dividend in this model. The effect on government spending remains ambiguous since an increase in pollution tax leads to a decrease in the tax base.

2.3 Households

In this section our aim is to study the preferred pollution tax of the households and investigate how it changes according to their wealth endowment. Note that we focus only on the dimension of wealth inequality by assuming identical wage rate for each household, therefore we will not study the case of wage inequality.

Household i's utility $V(c_i, E)$ depends on its level of consumption (c_i) and the level of environmental quality (E).[18] We impose the following assumptions for the utility function:

Assumption 3. *The utility function $V(c_i, E)$ is additively separable in c_i and E ($V_{cE}(.)$=0), increasing and concave in c ($V_c(.) > 0$ and $V_{cc}(.) < 0$) and increasing and concave in E ($V_E(.) > 0$ and $V_{EE}(.) < 0$). We assume that:*

$$V(c_i, E) = v(c_i) + h(E) \tag{19}$$

These assumptions about the effects of consumption and environmental quality on utility are standard and widely used in the literature. However, the assumption on the additive separability is rather restrictive. In Section 3, we study the impact of relaxing this assumption but, for the rest of this section, we abstract from the cross relationship between consumption and environmental quality in the household's utility. This leads us to have a more clear analytic resolution.

Since environmental quality is a decreasing function pollution ($E(z)$ with $E'(z) < 0$), we can rewrite the utility function as $V(c_i, E(z)) = U(c_i, z)$ where $U(.)$ is increasing and concave in c_i ($U_c(.) > 0$ and $U_{cc}(.) < 0$) and decreasing and concave in z ($U_z(.) < 0$ and $U_{zz}(.) < 0$). Thereafter we will use the utility function $U(.)$ in our analysis.

Due to the static nature of our framework, the households that maximize their utility will consume all of their revenue which consists of the wage income from their labor supply and the gross return from their capital supply. In Section 2.1, we showed that the wage and the interest rate are determined by the pollution tax in the partial competitive equilibrium. Therefore, in the general

[16] See Appendix B for the proof.

[17] Note that in the case where the wage is increasing in the pollution tax, there may exist some households with a very low wealth such that their consumption increases in the pollution tax. Total consumption of the households, however, is always decreasing in pollution tax. See Appendix B.

[18] See Michel and Rotillon (1995) and Weitzman (2010) for a detailed discussion of this type of preferences.

equilibrium, the consumption level of the household i will depend on the pollution tax and its wealth, that is

$$c_i(\tau, k_i) = w(\tau) + (1 + r(\tau))k_i \tag{20}$$

Now we can characterize the general equilibrium in this economy:

Definition 2. *For a given pollution tax (τ), the unique general equilibrium for this economy is characterized by the vector of factor intensities in each technology $\{\alpha_z^X, \alpha_k^X, \alpha_l^X, \alpha_z^Y, \alpha_k^Y, \alpha_l^Y\}$, the vector of labor and capital prices $\{w, r\}$, the vector of the factors amounts employed in each technology $\{z_x, k_x, l_x, z_y, k_y, l_y\}$, the total production in each technology $\{X, Y\}$, the government spending $\{G\}$, the consumption level of each household $\{c_i\}_{i=0}^{1}$ and the total consumption $\{C = \int c_i\}$ such that:*

(i) *The firms minimize their costs, thus the eight equations in (8 to 11) hold.*

(ii) *The markets clear, thus the resource constraints (equations (14 and 15)) hold.*

(iii) *The government budget is balanced ($G = \tau(z_x + z_y)$) hold.*

(iv) *All households maximize their utility (equation (20) holds for each i)*

To find the preferred pollution tax of a household we consider the following maximization programme:

$$\max_{\{\tau\}} \{U(c_i(\tau, k_i), z(\tau))\} \tag{21}$$

which leads to the following first order condition:

$$\frac{\partial U(c_i(\tau_i^\star, k_i), z(\tau_i^\star))}{\partial \tau_i^\star} = U_c(.)\frac{\partial c_i(\tau_i^\star, k_i)}{\partial \tau_i^\star} + U_z(.)\frac{\partial z(\tau_i^\star)}{\partial \tau_i^\star} = 0 \tag{22}$$

Condition (22) clearly reflects the trade-off between higher consumption and better environmental quality. On the one hand, the pollution tax has an adverse effect on consumption due to its impact on factor prices which decreases the revenue of the household (the first term in the RHS of eq. (22)). This effect indeed has a negative impact on the household's utility. On the other hand, it decreases the level of pollution hence has a positive effect on the utility from the environmental well-being channel (the second term in the RHS of eq. (22)). Therefore one may expect that there is a preferred pollution tax for a household that balances these opposite effects.

In Proposition 2 we showed that when the firms are operating by using dirty and clean technologies, we may have a case such that the wage is increasing in the pollution tax ($w'(\tau) > 0$). In this case, the pollution tax may increase the total revenues of some households which have a low wealth because the increase in wage may dominate the loss from their gross capital return. Thus, the pollution tax will not impose a trade-off as in equation (22) for these households and their utility will obviously increase in the tax. However, as shown in Proposition 4, there exists a threshold for pollution tax above which only the clean technology is used. Above this threshold, independent of their wealth, the trade-off in equation (22) will be valid for all households because when the firms are operating by using a single technology the wage decreases in pollution tax ($w'(\tau) < 0$) as shown in Proposition 1.

To proceed further, we assume the following:

Assumption 4. *Once $\tau_i^\star$ exists for household i, its marginal utility is decreasing respect to the pollution tax (τ) at this tax level, that is*

$$\frac{\partial^2 U(c_i(\tau_i^\star, k_i), z(\tau_i^\star))}{\partial {\tau_i^\star}^2} < 0 \tag{23}$$

This assumption implies that the utility of household reaches a peak when the equation (22) holds. Equation (23) implies

$$U_{cc}(.)(c_1(\tau, k_i))^2 + U_c(.)c_{11}(\tau, k_i) + U_z(.)z''(\tau) + U_{zz}(.)(z'(\tau))^2 < 0 \tag{24}$$

which is the second order condition for the maximization programme in (21). Equation (22) shows that household's preferred pollution tax depends on its wealth. To investigate the effect of an increase in the household's wealth on its preferred pollution tax, we take the derivative of equation (22) and solve it for $\partial\tau_i^\star/\partial k_i$ which yields the following result:[19]

$$\text{sign}(\frac{\partial\tau_i^\star(k_i)}{\partial k_i}) = \text{sign}(\underbrace{U_{cc}(.)\frac{\partial c_i(\tau_i^\star, k_i)}{\partial k_i}\frac{\partial c_i(\tau_i^\star, k_i)}{\partial \tau_i^\star}}_{\text{>0 ; Satiation effect}} + \underbrace{U_c(.)\frac{\partial^2 c_i(\tau_i^\star, k_i)}{\partial\tau_i^\star\partial k_i}}_{\text{<0 ; Cost of pollution tax effect}}) \tag{25}$$

The first term in the RHS of equation (25), which has a positive sign, can be called as the *satiation effect*. When a household is richer, its level of consumption is relatively higher and thus its marginal utility of consumption is lower. This results as a lower marginal rate of substitution between consumption and environmental quality. In other words, since they are more fulfilled, the richer households care less about the loss from their consumption due to the pollution tax. Therefore, through this channel the richer households would prefer a higher pollution tax.

The second term in the RHS of equation (25), which has negative sign, can be called as the *cost of pollution tax effect*. It reflects the fact that the richer households pay more (lose more from their consumption due to an increase in the pollution tax) for the environmental protection in absolute terms. This is because of the fact that an increase in the pollution tax reduces the interest rate. Since the richer households have a greater amount of capital invested in the market, they are the ones whose consumption levels are more affected by the pollution tax. Consequently, through this channel the richer households will prefer a lower pollution tax.

Accordingly, whether the richer households would prefer a higher or a lower pollution tax will depend on which one of these two effects dominates. In fact, if we assume that $v(c)$ in the household utility has the logarithmic form (as in Kempf and Rossignol (2007)), we can analytically show that the dominating effect depends only on the pollution tax elasticity of consumption.[20]

Proposition 5. *If the household's utility satisfies the properties given in Assumption (3) and assumption (4), and moreover $v(c_i) = log(c_i)$, then the preferred pollution tax of a household is increasing in its wealth if and only if the pollution tax elasticity of consumption is increasing in wealth. Formally:*

$$\text{sign}(\frac{\partial\tau_i^\star(k_i)}{\partial k_i}) = \text{sign}(\frac{\partial\epsilon_{c_i,\tau}}{\partial k_i}) \tag{26}$$

[19]See Appendix C1.

[20]In Section 3 we will study the important implications of a more general case in which the household utility from consumption has the CRRA form.

Proof. See Appendix C2. □

Note that the sign of $\epsilon_{c_i,\tau}$ is negative, hence an increase in this elasticity will mean that the percentage loss in consumption due to an increase in the pollution tax is lower. Proposition 5 shows that the richer households want a better environmental protection if and only if their percentage loss in consumption due to the pollution tax is lower than the poorer households. Thus, the households identify their choices only through the economic incentives and independent of the status of the environment. It is plain that this result relies on the two following assumptions: (*i*) additive separability of the utility function with respect to consumption and environmental well-being and (*ii*) logarithmic form of utility of consumption. Above we discussed the implications of relaxing the first assumption and in Section 3 we will discuss the implications of relaxing the second one.

Proposition 5 raises a further question: what does the sign of pollution tax elasticity of consumption depends on? Since our framework is static and the households consumes all and only the revenues from their factor supplies, the pollution tax elasticity of consumption is increasing in wealth if and only if the pollution tax elasticity of the gross interest rate (R) is greater (less negative) than the one of the wage (w). More specifically, if the percentage decline in the gross interest rate due to the pollution tax is lower than the one of the wage then the richer households will experience a lower percentage loss from their consumption due to an increase in the tax compared to the poorer households. This fact combined with the assertion in Proposition 5 leads to the following result:

Proposition 6. *If the household's utility satisfies the properties given in Assumption (3) and assumption (4), and moreover $v(c_i) = log(c_i)$, the preferred pollution tax is increasing in the household's wealth if and only if the pollution tax elasticity of gross interest rate is greater (less negative) than the one of the wage. Formally:*

$$\text{sign}(\frac{\partial \tau_i^\star(k_i)}{\partial k_i}) = \text{sign}(\epsilon_{R,\tau} - \epsilon_{w,\tau}) \tag{27}$$

Proof. See Appendix C3. □

Taking into account Proposition 6 together with the main results of Section 2.1 which are summarized in Table 1 lead us to the central claims of this section.

Proposition 7. *When the firms in the economy operate by using a single production technology, the preferred pollution tax of an household is increasing in its wealth and the tax is regressive,* $\frac{\partial \tau_i^\star(k_i)}{\partial k_i} > 0$.

Proof. Direct conclusion of Proposition 1 and Proposition 6. □

Proposition 8. *When the firms in the economy operate by simultaneously using dirty and clean production technologies which satisfy the properties in Assumption 2, the preferred pollution tax of an household is decreasing in its wealth and the tax is progressive,* $\frac{\partial \tau_i^\star(k_i)}{\partial k_i} < 0$.

Proof. Direct conclusion of Proposition 2 and Proposition 6. □

3 Discussion

3.1 Case of non-separable utility function

Additive separability of the utility function with respect to consumption and environmental quality implies two things: (*i*) the marginal utility of consumption does not depend on pollution and (*ii*) the marginal utility of environmental quality is independent from the level of consumption. To evaluate the effect of $U_{cE}(.)$ on the household's preferred pollution tax and, hence, on progressiveness of the pollution tax, we rewrite equation (25) for the case in which $U_{cE}(.) \neq 0$:

$$\text{sign}(\frac{\partial \tau_i^\star(k_i)}{\partial k_i}) = \text{sign}(\underbrace{U_{cc}(.)\frac{\partial c_i(\tau_i^\star, k_i)}{\partial k_i}\frac{\partial c_i(\tau_i^\star, k_i)}{\partial \tau_i^\star}}_{>0\text{ ; Satiation effect}} + \underbrace{U_c(.)\frac{\partial^2 c_i(\tau_i^\star, k_i)}{\partial \tau_i^\star \partial k_i}}_{<0\text{ ; Cost of pollution tax effect}} + U_{cz}(.)\underbrace{\frac{\partial c_i(\tau_i^\star, k_i)}{\partial k_i}}_{>0}\underbrace{Z'(\tau)}_{<0}) \tag{28}$$

In the case where the marginal utility of consumption decreases in the level of pollution ($U_{cz}(.) < 0$), the richer households will be more affected by pollution. This makes them to prefer relatively higher pollution tax which is embodied in the fact that the last term in the RHS of equation (28) is positive. Therefore, assuming $U_{cz}(.) < 0$ makes the pollution tax more regressive. On the contrary case, when $U_{cz}(.) > 0$, the pollution tax is more progressive.

3.2 Case of CRRA utility of consumption

In our model, we assumed that the household's utility of consumption is in the logarithmic form. Here, we will study a more general case in which the utility function is in the CRRA form, that is:

$$U(c_i, z) = \frac{c_i^{1-\theta} - 1}{1-\theta} - d(z) \tag{29}$$

The form given in equation (29) satisfies the properties of Assumption 3. In this case, we can show that:

$$U_{cc}(.)\frac{\partial c_i(\tau_i^\star, k_i)}{\partial k_i}\frac{\partial c_i(\tau_i^\star, k_i)}{\partial \tau_i^\star} + U_c(.)\frac{\partial^2 c_i(\tau_i^\star, k_i)}{\partial \tau_i^\star \partial k_i} = c_i^{-\theta-1}(\frac{wR}{\tau}(\epsilon_{R,\tau} - \epsilon_{w,\tau}) + (1-\theta)R(w' + r'k_i)) \tag{30}$$

Recall that, as in equation (25), the LHS of equation (30) determines the sign of $\frac{\partial \tau_i^\star}{\partial k_i}$. When $\theta = 1$ (logarithmic utility of consumption), equation (30) is identical to equation (A.35) which implies Propositions (5, 6, 7, 8).

In the other extreme case, in which the households are risk neutral ($\theta = 0$), the RHS of equation (30) is reduced to $c_i^{-\theta-1}r'(\tau)$. This implies that the pollution tax is always progressive even if the percentage decline in the gross interest rate due to the pollution tax is lower than that of the wage (the case of single technology). This is because of the fact that, in this case, there would be no satiation effect. Therefore, the rich households prefer a lower pollution tax compared to the poor ones since they always pay more for the pollution tax.

When the relative risk aversion coefficient is between zero and one, ($0 < \theta < 1$), the last term in the RHS of equation (30) has a negative sign (except for one special case which will be explained below). This implies that, in this case, the pollution tax is more progressive than what our model suggests in Section 2.3. For example, Proposition 7 asserts that when the firms are operating by using a single technology the pollution tax is regressive. But, when $0 < \theta < 1$, the pollution tax can be progressive for relatively very rich households. Therefore, the household's preferred pollution tax will have an inverted-U shape with respect to its wealth (k_i).

In a special case where the wage increases in the pollution tax ($w'(\tau) > 0$), the last term in the RHS of equation (30) has a positive sign for the households with too low wealth endowment (k_i). However, since $Rw' = \frac{wR}{\tau}\epsilon_{w,\tau}$, even for the very poor households, the RHS of equation (30) will never turn to positive and, consequently, the pollution tax remains always progressive.

To sum-up, except for the case in which $w'(\tau) > 0$ and k_i is very low, the lower the relative risk aversion is, the higher would be the degree of progressiveness of the pollution tax. Besides, assuming $0 < \theta < 1$, may lead to an inverted-U shaped relation between the household's preferred tax and its wealth if the firms are using only one technology. However, the same assumption will not change the results of Proposition 8 and thus the pollution tax will remain always progressive when firms use two technologies in production.

4 Concluding remarks

We showed that the households with uneven wealth endowments prefer different levels of pollution tax. This is due to the fact that wealth inequality implies two distinctions between the rich and the poor households: (*i*) their consumption levels are not the same and (*ii*) the amounts of capital that they invest in the market are different. In fact, these differences correspond to the channels that we identified as the determinant of the household's preferred pollution taxes which we called as the satiation effect and the cost of pollution tax effect. The satiation effect means that the marginal utility of consumption is lower for the richer households, henceforth, they are more willing to sacrifice from their consumption for a better environmental quality. The cost of pollution tax effect refers to the fact that the revenue of the rich is more reduced by the pollution tax due to their higher capital investment in the market. Furthermore, we showed that the effect that dominates depends on the pollution tax elasticity of consumption. This means that the effect of household's wealth on its preferred pollution tax depends on its percentage and not on its absolute loss from consumption due to the tax. Moreover, the tax elasticity of consumption obviously depends on how the revenues of the households are affected by the increase in the pollution tax.

By using a general equilibrium framework, we showed that the impact of the pollution tax on the household revenue (which comes from the wage and the interest rate) depends on the characteristics of the production technologies employed by the firms. We identified the cases in which the wage and the interest rate move in the same or different direction as a response to an increase in the pollution tax. When the firms operate by using only one production technology, the pollution tax elasticity of the wage and the interest rate are identical which makes the rich to lose less than the poor from their consumption in percentage terms. Thus, in this case, the rich prefer a higher pollution tax and the tax is regressive. This result changes when the firms operate by using two technologies: (*i*) dirty and more capital intensive and (*ii*) cleaner and more labor intensive. In this case, an increase in the pollution tax leads to a reallocation of factors from the dirty technology to the clean one. This reallocation leads to a relatively higher decrease in the returns of capital. Consequently, in this case, the rich loses more than the poor from their consumption in percentage terms and thus they prefer a lower pollution tax and the tax is progressive.

Our results suggest that the pollution tax always decreases the wealth inequality in the economy since the rich always loses more from their consumption in absolute terms. This is due to the fact that we abstracted from the wage inequality and the redistributional effects of the pollution tax. Further research could include these dimensions. For example, the labor supply side of the model can be improved to allow heterogenities in labor income and the government transfers that are not neutral can be considered.

We discussed about relaxing a few of the assumptions that we made throughout the text and in the discussion section. Relaxing the other key assumptions of this framework can lead to further research on this subject. For example, transforming the model into the dynamic framework will allow to investigate intertemporal effects of environmental policies in the existence of wealth inequality. Introducing consumer preferences towards dirty and clean products will allow to study both the sources and the uses sides of income. Moreover, the extension of the model for multiple countries will provide benefits that are many fold. A simple model of two countries with different wealth distributions, factor endowments and production technologies would allow to analyze concepts such as pollution havens as well as to identify patterns of factors in response to environmental policies. Furthermore, imperfections in capital mobility and labor mobility can result with different implications.

Moreover, our setting is more compatible with the reality compared to other studies in the literature since we consider that the clean technology also pollutes. Using this framework will allow to have more robust results in the empirical research on this subject. Finally, this study provided a potential benchmark for further analysis in political economics research concerning environmental policies and wealth inequalities.

Appendix A

A1: Proof of Proposition 1

We use the first order conditions given in (3 to 5). First we use (3) to obtain:

$$z(\tau) = F_1^{-1}(\tau; G(\bar{K}, \bar{L})) \tag{A.1}$$

Note that since $G(\bar{K}, \bar{L})$ is given and constant, it affects $z(\tau)$ as a parameter. By using the properties of the production function given in Assumption 1, we know that $F_{11}^{-1}(.,.) < 0$ hence

$$z'(\tau) < 0 \tag{A.2}$$

Now that we have $z(\tau)$, we replace it in equations (4 and 5) to get the following:

$$F_2(z(\tau), G(\bar{K}, \bar{L}))G_1(\bar{K}, \bar{L}) = r \tag{A.3}$$

$$F_2(z(\tau), G(\bar{K}, \bar{L}))G_2(\bar{K}, \bar{L}) = w \tag{A.4}$$

We can now compute the wage and interest rate as a function of pollution tax and how they change according to that.

$$r'(\tau) = z'(\tau)F_{21}(z(\tau), G(\bar{K}, \bar{L}))G_1(\bar{K}, \bar{L}) < 0 \tag{A.5}$$

$$w'(\tau) = z'(\tau)F_{21}(z(\tau), G(\bar{K}, \bar{L}))G_2(\bar{K}, \bar{L}) < 0 \tag{A.6}$$

since $z'(.) < 0$, $F_{21}(.) > 0$, $G_1(.) > 0$ and $G_2(.) > 0$ which completes the first part of the proof.

The elasticities of wage and interest rate respect to the pollution tax are:

$$\epsilon_{r,\tau} = \frac{r'(\tau)}{r(\tau)/\tau} = \frac{z'(\tau)F_{21}(z(\tau), G(\bar{K}, \bar{L}))G_1(\bar{K}, \bar{L})\tau}{F_2(z(\tau), G(\bar{K}, \bar{L}))G_1(\bar{K}, \bar{L})} = z'(\tau)\frac{F_{21}(z(\tau), G(\bar{K}, \bar{L}))\tau}{F_2(z(\tau), G(\bar{K}, \bar{L}))} < 0 \tag{A.7}$$

$$\epsilon_{w,\tau} = \frac{w'(\tau)}{w(\tau)/\tau} = \frac{z'(\tau)F_{21}(z(\tau), G(\bar{K}, \bar{L}))G_2(\bar{K}, \bar{L})\tau}{F_2(z(\tau), G(\bar{K}, \bar{L}))G_2(\bar{K}, \bar{L})} = z'(\tau)\frac{F_{21}(z(\tau), G(\bar{K}, \bar{L}))\tau}{F_2(z(\tau), G(\bar{K}, \bar{L}))} = \epsilon_{r,\tau} < 0 \tag{A.8}$$

$$\epsilon_{R,\tau} = \frac{R'(\tau)}{R(\tau)/\tau} = \frac{r(\tau)}{R(\tau)}\frac{r'(\tau)}{r(\tau)/\tau} = \frac{r(\tau)}{1 + r(\tau)}\epsilon_{r,\tau} \tag{A.9}$$

which completes the second part of the proof.

Note that this property implies the following relationships:

$$\frac{w(\tau)}{r(\tau)} = \frac{w'(\tau)}{r'(\tau)} = \frac{w''(\tau)}{r''(\tau)} \tag{A.10}$$

Equation (A.10) can be obtained as follows:

$$\frac{r'(\tau)}{r(\tau)} = \frac{w'(\tau)}{w(\tau)} \tag{A.11}$$

$$\Rightarrow Log(r'(\tau)) - Log(r(\tau)) = Log(w'(\tau)) - Log(w(\tau)) \tag{A.12}$$

$$\Rightarrow \frac{r''(\tau)}{r'(\tau)} - \frac{r'(\tau)}{r(\tau)} = \frac{w''(\tau)}{w'(\tau)} - \frac{w'(\tau)}{w(\tau)} \tag{A.13}$$

$$\Rightarrow \frac{w''(\tau)}{r''(\tau)} = \frac{w'(\tau)}{r'(\tau)} = \frac{w(\tau)}{r(\tau)} \tag{A.14}$$

A2: Proof of Proposition 2

We use the first order conditions (9 to 11) of the cost minimization problem in (7) to obtain the derived unit=production demands for factors in both of the two technologies. For the dirty technology we have $\left\{\alpha_z^X(\tau), \alpha_k^X(\tau), \alpha_l^X(\tau)\right\}$ and for the clean technology we have $\left\{\alpha_z^Y(\tau), \alpha_k^Y(\tau), \alpha_l^Y(\tau)\right\}$. From now on we will drop functional arguments (τ) for notational simplicity.

Let $\eta_x = \alpha_k^X/\alpha_l^X$, $\eta_y = \alpha_k^Y/\alpha_l^Y$, $\zeta_x = \alpha_z^X/\alpha_l^X$ and $\zeta_y = \alpha_z^Y/\alpha_l^Y$. By Definition 1 ($\alpha_z^X > \alpha_z^Y$, $\alpha_k^X > \alpha_k^Y$ and $\alpha_l^X < \alpha_l^Y$) we have $\eta_x > \eta_y$ and $\zeta_x > \zeta_y$. Perfect competition implies:

$$\tau\alpha_z^X + (1 + r(\tau))\alpha_k^X + w(\tau)\alpha_l^X = \bar{p} \tag{A.15}$$

$$\tau\alpha_z^Y + (1 + r(\tau))\alpha_k^Y + w(\tau)\alpha_l^Y = \bar{p} \tag{A.16}$$

where $\bar{p}$ is the price of the generic good and we take is as numeraire hence $\bar{p} = 1$. Now we will compute how the unit cost changes with the pollution tax. For that we take the derivative of equations (A.15 and A.16) respect to τ. Note that all the derived demands depend on the pollution tax, however, they are obtained from the cost minimization problem which means that when we apply the envelope theorem we will have $\tau a_z'(\tau) + r(\tau)a_k'(\tau) + w(\tau)a_l'(\tau) = 0$ for $i \in \{x, y\}$. Applying this to the derivative of equations (A.15 and A.16):

$$\alpha_z^X + r'(\tau)\alpha_k^X + w'(\tau)\alpha_l^X = 0 \tag{A.17}$$

$$\alpha_z^Y + r'(\tau)\alpha_k^Y + w'(\tau)\alpha_l^Y = 0 \tag{A.18}$$

We divide (A.17) by α_l^X and (A.18) by α_l^Y to obtain:

$$\zeta_x + r'(\tau)\eta_x + w'(\tau) = 0 \tag{A.19}$$

$$\zeta_y + r'(\tau)\eta_y + w'(\tau) = 0 \tag{A.20}$$

Subtracting (A.20) from (A.19) gives:

$$r'(\tau) = -\frac{\zeta_x - \zeta_y}{\eta_x - \eta_y} < 0 \text{ by Definition 1} \tag{A.21}$$

Furthermore, we multiply (A.20) by η_x/η_y and subtract the resulting equation from (A.19) to obtain:

$$w'(\tau) = \frac{\zeta_x\eta_y - \zeta_y\eta_x}{\eta_x - \eta_y} \tag{A.22}$$

The sign of $w'(\tau)$ depends on the relative factor intensities between the two technologies. We have:

$$w'(\tau) > 0 \text{ if } \frac{\zeta_x}{\eta_x} > \frac{\zeta_y}{\eta_y} \Leftrightarrow \frac{\alpha_z^X}{\alpha_k^X} > \frac{\alpha_z^Y}{\alpha_k^Y} \tag{A.23}$$

$$w'(\tau) = 0 \text{ if } \frac{\zeta_x}{\eta_x} = \frac{\zeta_y}{\eta_y} \Leftrightarrow \frac{\alpha_z^X}{\alpha_k^X} = \frac{\alpha_z^Y}{\alpha_k^Y} \tag{A.24}$$

$$w'(\tau) < 0 \text{ if } \frac{\zeta_x}{\eta_x} < \frac{\zeta_y}{\eta_y} \Leftrightarrow \frac{\alpha_z^X}{\alpha_k^X} < \frac{\alpha_z^Y}{\alpha_k^Y} \tag{A.25}$$

which completes the first part of the proof. For the elasticities, we can rewrite equations (A.15) and (A.16) as follows:

$$\zeta_x\tau + \eta_x(1 + r(\tau)) + w(\tau) = \frac{\bar{p}}{\alpha_l^X} \tag{A.26}$$

$$\zeta_y\tau + \eta_y(1 + r(\tau)) + w(\tau) = \frac{\bar{p}}{\alpha_l^Y} \tag{A.27}$$

Multiplying equation (A.26) by ζ_y and equation (A.27) by ζ_x and subtracting the latter from the former, we get:

$$(1 + r(\tau))(\zeta_y\eta_x - \zeta_y\eta_x) + w(\tau)(\zeta_y - \zeta_x) = \bar{p}\left(\frac{\zeta_y}{\alpha_l^X} - \frac{\zeta_x}{\alpha_l^Y}\right) = \frac{\bar{p}}{\alpha_l^X\alpha_l^Y}(\alpha_z^Y - \alpha_z^X) < 0 \tag{A.28}$$

Dividing LHS of inequality (A.28) by $(\eta_x - \eta_y)$ and using equations (A.21) and (A.22), we can show:

$$-w'(\tau)(1 + r(\tau)) + w(\tau)r'(\tau) < 0 \tag{A.29}$$

Therefore:

$$\frac{r'(\tau)}{1 + r(\tau)} < \frac{w'(\tau)}{w(\tau)} \Leftrightarrow \epsilon_{R,\tau} < \epsilon_{w,\tau} \tag{A.30}$$

Moreover, since $r' < 0$,we can conclude that: $\epsilon_{r,\tau} < \epsilon_{R,\tau} < 0$.
Finally, equation (A.23) define the conditions for the sign of $\epsilon_{w,\tau}$ and it completes the second part of the proof.

A3: Proof of Proposition 3

From equations (10) and (11), we have:

$$r = F_2^a(\alpha_z^X, G^a(\alpha_k^X, \alpha_l^X))G_1^a(\alpha_k^X, \alpha_l^X) \tag{A.31}$$

$$w = F_2^a(\alpha_z^X, G^a(\alpha_k^X, \alpha_l^X))G_2^a(\alpha_k^X, \alpha_l^X) \quad \text{for } a \in \{x, y\} \tag{A.32}$$

Dividing equation (A.31) by (A.32) we get:

$$\frac{r}{w} = \frac{G_1^a(\alpha_k^X, \alpha_l^X)}{G_2^a(\alpha_k^X, \alpha_l^X)} \tag{A.33}$$

Proposition 3 implies that $\frac{d(r/w)}{d\tau} < 0$ and so:

$$\frac{d(\frac{G_1^a(\alpha_k^X, \alpha_l^X)}{G_2^a(\alpha_k^X, \alpha_l^X)})}{d\tau} < 0 \Leftrightarrow \frac{d(\frac{a_k}{a_l})}{d\tau} > 0 \quad \text{for } a \in \{x, y\} \tag{A.34}$$

A4: Proof of Proposition 4

Resource Constraints for capital and labor imply that:

$$X\alpha_k^X + Y\alpha_k^Y = \bar{K} \tag{A.35}$$

$$X\alpha_l^X + Y\alpha_l^Y = \bar{L} \tag{A.36}$$

Solving equations (A.35) and (A.36) for X and Y will result in the followings:

$$X = \frac{\alpha_l^Y \bar{K} - \alpha_k^Y \bar{L}}{\alpha_k^X \alpha_l^Y - \alpha_l^X \alpha_k^Y} \tag{A.37}$$

$$Y = \frac{\alpha_l^X \bar{K} - \alpha_k^X \bar{L}}{\alpha_l^X \alpha_k^Y - \alpha_k^X \alpha_l^Y} \tag{A.38}$$

Therefore:

$$X = 0 \Leftrightarrow \frac{\alpha_k^Y(\tau_{clean})}{\alpha_l^Y(\tau_{clean})} = \frac{\bar{K}}{\bar{L}} \tag{A.39}$$

$$Y = 0 \Leftrightarrow \frac{\alpha_k^X(\tau_{dirty})}{\alpha_l^X(\tau_{dirty})} = \frac{\bar{K}}{\bar{L}} \tag{A.40}$$

The denominator in RHS of equation (A.39) is positive. Since $\frac{d(\frac{a_k}{a_l})}{d\tau} > 0 \quad for$ for $a \in \{x, y\}$, if pollution tax is higher than τ_{clean}, then the production in dirty technology will be negative which is not possible. Therefore, for pollution tax higher than τ_{clean}, economy will use only the clean technology. With the same method, it is easy to show that for pollution tax lower than τ_{dirty}, the economy will operate only by the dirty technology.

A5: Proof of Corollary 1

From equations (A.39) and (A.40), we know that:

$$\frac{d(\frac{\alpha_k^Y(\tau_{clean})}{\alpha_l^Y(\tau_{clean})})}{d(\frac{\bar{K}}{\bar{L}})} = 1 > 0 \tag{A.41}$$

$$\frac{d(\frac{\alpha_k^X(\tau_{dirty})}{\alpha_l^X(\tau_{dirty})})}{d(\frac{\bar{K}}{\bar{L}})} = 1 > 0 \tag{A.42}$$

And from Proposition 4 we know that $\frac{d(a_k(\tau)/a_l(\tau))}{d\tau} > 0$ for $a \in \{x, y\}$. Therefore:

$$\frac{d(\tau_{clean})}{d(\frac{\bar{K}}{\bar{L}})} > 0 \tag{A.43}$$

$$\frac{d(\tau_{dirty})}{d(\frac{\bar{K}}{\bar{L}})} > 0 \tag{A.44}$$

Appendix B

Multiplying equation (A.17) by total production of the dirty technology, (X), and Multiplying equation (A.18) by total production of the clean technology, (Y), results in the followings:

$$Z_x + r'(\tau)k_x + w'(\tau)l_x = 0 \tag{B.1}$$

$$Z_y + r'(\tau)k_y + w'(\tau)l_y = 0 \tag{B.2}$$

By adding the two last equations, we have:

$$Z = -(r'(\tau)\bar{K} + w'(\tau)\bar{L} = -C'(\tau) \tag{B.3}$$

Using equation (B.3) in the RHS of the first equality in equation (18), will leads to the second equality of that equation. Moreover, since $Z > 0$, total private consumption is decreasing in pollution tax.

Appendix C

Appendix C1: Proof for equation (25)

We start from the first order condition resulted from household's maximization programme given in equation (21):

$$\frac{\partial U_i(c_i(\tau_i^\star, k_i), z(\tau_i^\star))}{\partial \tau_i^\star} = U_c(.)\frac{\partial c_i(\tau_i^\star, k_i)}{\partial \tau_i^\star} + U_z(.)\frac{\partial z(\tau_i^\star)}{\partial \tau_i^\star} = 0 \tag{C.1}$$

To find $\frac{\partial\tau_i^\star(k_i)}{\partial k_i}$ we take the derivative of (C.1) with respect to k_i at $\tau_i^\star(k_i)$:

$$U_{cc}(.)\frac{\partial c}{\partial k_i}c_1(\tau,k_i) + U_{cz}(.)z'(\tau)\frac{\partial\tau^\star}{\partial k_i}c_1(\tau,k_i) + U_{cc}(.)(c_1(\tau,k_i))^2\frac{\partial\tau^\star}{\partial k_i} + U_c(.)c_{11}(\tau,k_i)\frac{\partial\tau^\star}{\partial k_i} + U_c(.)c_{12}(\tau,k_i)$$
$$+ U_{cz}(.)c_2(\tau,k_i)z'(\tau) + U_{cz}(.)c_1(\tau,k_i)\frac{\partial\tau^\star}{\partial k_i}z'(\tau) + U_{zz}(.)(z'(\tau))^2\frac{\partial\tau^\star}{\partial k_i} + U_z(.)z''(\tau)\frac{\partial\tau^\star}{\partial k_i} \tag{C.2}$$

Setting $U_{cz}(.) = 0$ (by Assumption 3) and collecting $\frac{\partial\tau^\star}{\partial k_i}$ we obtain:

$$\frac{\partial\tau^\star(k_i)}{\partial k_i} = -\frac{S_1}{S_2} \tag{C.3}$$
$$\text{where } S_1 = U_{cc}(.)c_2(\tau,k_i)c_1(\tau,k_i) + U_c(.)c_{12}(\tau,k_i) \tag{C.4}$$
$$S_2 = U_{cc}(.)(c_1(\tau,k_i))^2 + U_c(.)c_{11}(\tau,k_i) + U_z(.)z''(\tau) + U_{zz}(.)(z'(\tau))^2 \tag{C.5}$$

Equation (C.5), S_2, corresponds to the second order condition and it is negative ($S_2 < 0$) by Assumption 4 . Therefore S_1 determines the sign of $\frac{\partial\tau^\star}{\partial k_i}$.

Appendix C2: Proof for Proposition 5

If $v(c_i) = log(c_i)$, then, $U_c(.) = \frac{1}{c_i}$ and $U_{cc}(.) = \frac{-1}{c_i^2}$. By replacing these two equations in equation (C.4), we will have:

$$s_1 = -\frac{1}{c_i^2}\frac{\partial c_i}{\partial k_i}\frac{\partial c_i}{\partial\tau} + \frac{1}{c_i}\frac{\partial^2 c_i}{\partial k_i\partial\tau} \tag{C.6}$$

And equivalently:

$$s_1 = \frac{\partial(\frac{1}{c_i}\frac{\partial c_i}{\partial\tau})}{\partial k_i} = \frac{1}{\tau}\frac{\partial\epsilon_{c_i,\tau}}{\partial k_i} \tag{C.7}$$

Appendix C3: Proof for Proposition 6

$$\epsilon_{c_i,\tau} = \frac{\partial c_i}{\partial\tau}\frac{\tau}{c_i} = \frac{r'(\tau)k_i + w'(\tau)}{(1+r(\tau))k_i + w(\tau)}\tau \tag{C.8}$$

Therefore:

$$\frac{\partial\epsilon_{c_i,\tau}}{\partial k_i} = \frac{r'(\tau)c_i - (1+r(\tau))c_i'}{c_i^2}\tau = \frac{\tau}{c_i}(r'(\tau)w(\tau) - (1+r(\tau))w'(\tau)) = \frac{(1+r(\tau))w(\tau)}{c_i^2}(\epsilon_{R,\tau} - \epsilon_{w,\tau}) \tag{C.9}$$

Using equation C.9 and equation C.7, we can get:

$$s1 = \frac{(1+r(\tau))w(\tau)}{\tau c_i^2}(\epsilon_{R,\tau} - \epsilon_{w,\tau}) \tag{C.10}$$

Which establishes the prove for the proposition 7.

Appendix D: The case of n technologies

We claim that in our framework, where the economy is open and operating in n-sectors (thus, prices in all the sectors are fixed), or equivalently, where the economy is closed but producing and consuming only one generic good with n-technologies, the economy will operate using maximum two sectors/technologies.

We have endowment constraints:

$$\sum_{i=1}^{n} k^i = \bar{K} \tag{D.1}$$

$$\sum_{i=1}^{n} l^i = \bar{L} \tag{D.2}$$

For each sector i, we have: $Q_i = F^i(z^i, G^i(k^i, l^i))$ which has a market price p^i_Q that is exogenously given. The prices of capital and labor (r and w) are endogenously determined, however, the price of z (τ) is exogenously given (by the government). The firms solve the following problem:

$$\max_{\{z_i,k_i,l_i\}} \left\{ \sum_{i=1}^{n} (p^i_Q F^i(z^i, G^i(k^i, l^i)) - rk^i - wl^i - \tau z^i) \right\}$$

subject to (D.1), (D.2) and $z^i \geq 0\ \forall i$

First order conditions for an interior solution are:

$$p^i_Q F^i_1(z^i, G^i(k^i, l^i)) = \tau \tag{D.3}$$

$$p^i_Q F^i_2(z^i, G^i(k^i, l^i)) G^i_1(k^i, l^i) = r \tag{D.4}$$

$$p^i_Q F^i_2(z^i, G^i(k^i, l^i)) G^i_2(k^i, l^i) = w \tag{D.5}$$

Therefore, we have:
$\{D.1, D.2, D.3, D.4, D.5\} \Rightarrow 3n + 2$ equations and $\left\{ \left\{ k^i, l^i, z^i \right\}, r, w \right\} \Rightarrow 3n + 2$ variables.

Now we will show that if the functions $F(.)$ and $G(.)$ are homogeneous of degree 1 then these equations are not independent when $n > 2$. Therefore the solution for $n > 2$ does not exist. In other words, it is not possible that the economy operates with more than two technologies. To show that, we define:

$$\eta^i = \frac{k^i}{l^i} \tag{D.6}$$

$$\zeta^i = \frac{z^i}{l^i} \tag{D.7}$$

Using the property of homogenous of degree 1 for $F(.)$ and $G(.)$, we can rewrite equations (D.3) to

(D.5) as follows:

$$p_Q^i F_1^i(\frac{\zeta^i}{G^i(\eta^i,1)},1) = \tau \tag{D.8}$$

$$p_Q^i F_2^i(\frac{\zeta^i}{G^i(\eta^i,1)},1) G_1^i(\eta^i,1) = r \tag{D.9}$$

$$p_Q^i F_2^i(\frac{\zeta^i}{G^i(\eta^i,1)},1) G_2^i(\eta^i,1) = w \tag{D.10}$$

For n sectors, we have $\left\{\left\{\eta^i, \zeta^i, k^i, l^i, z^i\right\}, r, w\right\} \Rightarrow 5n+2$ variables and ($D.1, D.2, D.6, D.7, D.8, D.9, D.10$) $\Rightarrow 5n+2$ equations. At this point, the number of equations equals the number of variables and, thus, the system of equations seems to have a solution. However, a subset of this equation system, equations ($D.8, D.9, D.10$) contain $3n$ equations with $2n+2$ variables. Therefore, if $n > 2$ then the number of equations is greater than the number of variables. This fact concludes that the system of equations are not independent. Hence there is no solution for $n > 2$ when all of the n-technologies are being operated by the economy. In other words, the economy will use maximum two technologies for a given τ.

In fact, we can generalize the results above. Consider an economy with n-technologies (sectors) where all of the technologies are homogenous of degree 1 and they use m factors as inputs. In the case where the prices of s factors are given, meaning that $m-s$ factors' prices are determined endogenously (and their total amount must be constrained by endowment or ceiling constraints), we can conclude that maximum $m-s$ technologies will be operated by the economy.

Appendix E: An alternative setting: pollution as a byproduct

In this alternative setting the firms are involved in two processes. In the first process, they hire capital and labor (k^P, l^P) to produce the final good. Pollution (z) is byproduct of this process. Since we assume that the pollution is taxed (τ), the firms will get involved in the abatement activities in which they use capital and labor (k^A, l^A) to produce equipment that is used to reduce pollution. Therefore, in this alternative setting, pollution is a function of final good production $(H(k^P, l^P))$ and abatement process $(B(k^A, l^A))$:

$$z = \Phi(H(k^P, l^P), B(k_k^A, l^A))$$

$$\text{Where:} \quad \Phi_1(.) > 0, \Phi_2(.) < 0, \Phi_{11}(.) > 0, \Phi_{22}(.) > 0$$

where j^P and j^A are demands of factor j for production of final good and for pollution abatement respectively. Since factor prices and pollution tax are given to the firms, their cost-minimization problem for producing one unit of final good is as follows:

$$\min_{\{a_z, a_k, a_l\}} \left\{(a_k^P + a_k^A)r + (a_L^P + a_L^A)w + \Phi(H(a_k^P, a_L^P), B(a_k^A, a_L^A))\tau)\right\} \tag{E.1}$$

$$\text{subject to: } H(a_k^P, a_L^P) = 1 \tag{E.2}$$

Here, a_j^P and a_j^A are demand of factor j for unit production of final good and for corresponding pollution abatement respectively. Factor demands in our main setting a_z, a_k, a_l can be translated to this setting as follows:

$$a_z = \Phi(H(a_k^P, a_L^P), B(a_k^A, a_L^A)) \tag{E.3}$$

$$a_k = a_k^P + a_k^A \tag{E.4}$$

$$a_l = a_L^P + a_L^A \tag{E.5}$$

Constant returns to scale form assumption for $F(.)$ and $G(.)$ in our main setting can be translated to constant returns to scale property of $H(.)$, $B(.)$ and $\Phi(.)$ in this alternative setting. Firms' minimization problem leads to the following first order conditions:

$$r = H_1(a_k^P, a_L^P)(1 - \Phi_1(H(a_k^P, a_L^P), B(a_k^A, a_L^A))\tau) = \tau\Phi_2(H(a_k^P, a_L^P), B(a_k^A, a_L^A))B_1(a_k^A, a_L^A) \tag{E.6}$$

$$w = H_2(a_k^P, a_L^P)(1 - \Phi_1(H(a_k^P, a_L^P), B(a_k^A, a_L^A))\tau) = \tau\Phi_2(H(a_k^P, a_L^P), B(a_k^A, a_L^A))B_2(a_k^A, a_L^A) \tag{E.7}$$

Besides, resource constrains imply:

$$Y(a_k^P + a_k^A) = \bar{K} \tag{E.8}$$

$$Y(a_l^P + a_l^A) = \bar{L} \tag{E.9}$$

$$\text{Where:} Y = H(\frac{a_k^P \bar{K}}{a_k^P a_k^A}, \frac{a_l^P \bar{L}}{a_l^P a_l^A}) \tag{E.10}$$

Equations (E.6) to (E.10) provides seven equations and seven variables: $\left\{a_k^P, a_L^P, a_k^A, a_L^A, w, r, Y\right\}$. Therefore, factor demands and input prices can be found as a function of pollution tax (τ). For the sake of notation simplicity, in the following, we don't write (τ) knowing that all these variables are function of this variable.

As we explained in Section 2.1, the assumption of functional separability directly implies that, once there is only one technology used in the economy, wage and interest rate will have the identical pollution tax elasticity. Now, we can investigate the implication of this result in this alternative setting. Below, we will prove that, in this alternative setting, relative price of wage to interest rate remains unchanged, if and only if, production process and pollution abatement process have identical relative factor intensity.

Dividing equation (E.6) by equation (E.7) results in:

$$\frac{r}{w} = \frac{H_1(a_k^P, a_L^P)}{H_2(a_k^P, a_L^P)} = \frac{B_1(a_k^A, a_L^A)}{B_2(a_k^A, a_L^A)} \tag{E.11}$$

Thus:

$$\frac{d(\frac{r}{w})}{d\tau} = 0 \Rightarrow \begin{cases} \frac{d(\frac{H_1(a_k^P, a_l^P)}{H_2(a_k^P, a_l^P)})}{d\tau} = 0 \\ \frac{d(\frac{B_1(a_k^A, a_l^A)}{B_2(a_k^A, a_l^A)})}{d\tau} = 0 \end{cases} \Rightarrow \begin{cases} \frac{d(\frac{a_k^P}{a_l^P})}{d\tau} = 0 \\ \frac{d(\frac{a_k^A}{a_l^A})}{d\tau} = 0 \end{cases} \tag{E.12}$$

Since $H(.)$ is constant returns to scale and by definition $H(a_k^P, a_L^P) = 1$, the first equality in equation (E.12) implies that a_k^P and a_l^P are constant. Therefore:

$$a_k^{P'} = a_l^{P'} = 0 \tag{E.13}$$

Moreover, resource constraint and CRS property of production function implies the following:

$$\frac{a_k^P + a_k^A}{a_l^P + a_l^A} = \frac{\bar{K}}{\bar{L}} \tag{E.14}$$

Making derivative from equation (E.14) and applying equation (E.13) leads to the following:

$$a_k^{A\prime}(a_l^P + a_l^A) = a_l^{P\prime}(a_k^P + a_k^A) \underbrace{\Rightarrow}_{\text{by eq. (E.12)}} a_k^A a_l^P = a_l^A a_k^P \Rightarrow \frac{a_k^A}{a_l^A} = \frac{a_k^P}{a_l^P} \tag{E.15}$$

The intuition behind this observation is that if pollution tax increases, firms will hire more capital and labor for abatement process. In overall, hence, the input hired in production process will decrease while that hired in pollution abatement process will increase. Consequently, if, compared to the former process, the latter uses one factor relatively more intensively than the other one, the price of that factor will increase relatively. Hence, relative price of factors will remain constant only if both process employ the factors with the same relative intensity.

Finally, we can investigate what dirty and clean technology mean when our main setting is transformed to this alternative one: If two production technologies, $(H^d(.), H^c(.))$,are available, $H^d(.)$ is dirty if and only if the pollution it generates to produce one unit of final good is more than the pollution that $H^c(.)$ generates for producing the same amount of final good.

References

Borissov, K., T. Bréchet, and S. Lambrecht (2014). Environmental policy in a dynamic model with heterogeneous agents and voting. In E. Moser, W. Semmler, G. Tragler, and V. M. Veliov (Eds.), *Dynamic Optimization in Environmental Economics*, Volume 15 of *Dynamic Modeling and Econometrics in Economics and Finance*. Springer Berlin Heidelberg.

Chiroleu-Assouline, M. and M. Fodha (2006). Double dividend hypothesis, golden rule and welfare distribution. *Journal of Environmental Economics and Management 51*(3), 323–335.

Chiroleu-Assouline, M. and M. Fodha (2014). From regressive pollution taxes to progressive environmental tax reforms. *European Economic Review 69*, 126 – 142.

Copeland, B. R. and M. S. Taylor (1994). North-south trade and the environment. *The Quarterly Journal of Economics 109*(3), 755–787.

Copeland, B. R. and M. S. Taylor (2004). Trade, growth, and the environment. *Journal of Economic Literature 42*(1), 7–71.

Dissou, Y. and M. S. Siddiqui (2014). Can carbon taxes be progressive? *Energy Economics 42*, 88 – 100.

Fullerton, D. and G. Heutel (2007). The general equilibrium incidence of environmental taxes. *Journal of Public Economics 91*(3-4), 571–591.

Fullerton, D. and G. Heutel (2010). Analytical general equilibrium effects of energy policy on output and factor prices. *The B.E. Journal of Economic Analysis & Policy 10*(2).

Fullerton, D. and H. Monti (2013). Can pollution tax rebates protect low-wage earners? *Journal of Environmental Economics and Management 66*(3), 539 – 553.

Harberger, A. C. (1962). The incidence of the corporation income tax. *Journal of Political Economy 70*(3), 215–240.

Jouvet, P.-A., P. Michel, and G. Rotillon (2005). Optimal growth with pollution: how to use pollution permits? *Journal of Economic Dynamics and Control 29*(9), 1597–1609.

Kempf, H. and S. Rossignol (2007). Is inequality harmful for the environment in a growing economy? *Economics & Politics 19*(1), 53 – 71.

Marsiliani, L. and T. I. Rengström (2002). Inequality, environmental protection and growth. W. Allen Wallis Institute of Political Economy Working Paper No: 35/2002, University of Rochester.

Michel, P. and G. Rotillon (1995). Disutility of pollution and endogenous growth. *Environmental and Resource Economics 6*(3), 279 – 300.

Rausch, S., G. E. Metcalf, and J. M. Reilly (2011). Distributional impacts of carbon pricing: A general equilibrium approach with micro-data for households. *Energy Economics 33, Supplement 1*, S20 – S33. Supplemental Issue: Fourth Atlantic Workshop in Energy and Environmental Economics.

Siebert, H., J. Eichberger, R. Gronych, and R. Pethig (1980). *Trade and Environment: A Theoretical Inquiry*. Amsterdam: Elsevier/North Holland.

Weitzman, M. L. (2010). What is the damages function for global warming - and what difference might it make? *Climate Change Economics 1*(01), 57 – 69.