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# Waiting for the Prince Charming: Fixed-Term Contracts as Stopgaps

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## Executive Summary

Over the last decades, fixed-term employment has raised in the European Union. Fixed-term employment provides flexibility to firms in front of workload change, but it is costly to the workers: fixed-term workers face a higher unemployment risk than open-ended workers. Moreover, fixed-term employment also causes an inefficiently high turnover. As a response, European governments have carried out many reforms of employment protection legislation. Prominent questions remain, though. In particular, how do fixed-term contracts impact unemployment and welfare? What are the main risks associated with the implementation of these reforms?

To tackle these issues, I extend the classic model of Mortensen and Pissarides to add open-ended and fixed-term contracts. On the job creation side, firms post vacancies while unemployed workers search for a job. Vacancies and unemployed workers meet according to a matching function. New firm-worker matches differ in productivity. They optimally choose between going back to search or starting production under a fixed-term contract or an open-ended contract. On the job destruction side, firms lay off open-ended workers that become unprofitable and pay firing costs. Fixed-term matches only split with an exogenous probability at zero cost.

The future value of a match is uncertain to searching firms and workers. Two polar cases arise. If the initial productivity of the match is low, the firms and the worker go back to search. On the contrary, if the initial productivity is high, the firm and the worker lock up the match to make the most out of it; the immediate gains overcome the potential firing costs in the future. The option of offering a fixed-term contract strikes a balance. It provides a production opportunity for the best rejected matches and a quick return to searching for a more productive match.

Two opposite mechanisms shape the equilibrium. On the one hand, higher firing costs encourage substitution towards fixed-term contracts. On the other hand, higher firing costs discourage the destruction of open-ended matches, which reduces unemployment and job creation of both open-ended and fixed-term contracts. As such, the response of unemployment and welfare is non-monotonous. I calibrate the model on the French labor market and find that small changes in firing costs cannot jointly enhance employment and welfare. The optimal reform consists in a large cut in firing costs and the ban of fixed-term contracts.

Post-reform dynamics are costly in the short run. Consider a 50 % cut in firing costs combined with the ban of fixed-term contracts. On impact, it leads to an increase in unemployment and decrease in open-ended employment. Unemployment remains above its initial value for at least one year before eventually reaching a lower value. Similarly, open-ended employment stays below its initial value in the short-run. Overall, an in-depth reform may improve open-ended employment and unemployment in the long-run at the expense of an adverse transition.

# Waiting for the Prince Charming: Fixed-Term Contracts as Stopgaps

Normann Rion\*

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## Abstract

In this paper, I build a simple Mortensen-Pissarides model embedding a dual labor market. I derive conditions for the existence of an equilibrium with coexisting strongly protected open-ended contracts and exogenously short fixed-term contracts. I also study dynamics after a reform on employment protection legislation. Temporary contracts play the role of fillers while permanent contracts are used to lock up high-productivity matches. High firing costs favor the emergence of a dual equilibrium. Employment protection legislation encourages the resort to temporary employment in job creation. This scheme is intertwined with a general-equilibrium effect: permanent contracts represent the bulk of employed workers and a more stringent employment protection reduces aggregate job destruction. This pushes down unemployment and in turn reduces job creation flows through temporary contracts. The model is calibrated to match the French labor market. Policy experiments demonstrate that there is no joint gain in employment and social welfare through reforms on firing costs around the baseline economy. The optimal policy consists in implementing a unique open-ended contract with a strong cut in firing costs. Increases in firing costs within a dual labor market lead to a sluggish adjustment, while large cuts in firing costs lead to a quick one. The adjustment time of the labor market is highly non-monotonous between these two extremes. Policy-related uncertainty significantly strengthens fixed-term employment on behalf of open-ended employment. Considering extensions, I draw conclusions on the inability of a large class of random-matching models to mimic the distribution of temporary contracts' duration while maintaining possible the expiring temporary contracts' conversion into permanent contracts.

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# 1 Introduction

At the end of the 80s, the economic downturn following the oil shocks conducted firms to ask for looser firing rules. Many European countries enforced policies with the aim of reducing labor market rigidity. Temporary contracts were thus introduced in France in 1979 and the legal scope of their use was progressively extended. The segmentation of the labor market, opposing strongly protected open-ended contracts and shorter and shorter fixed-term contracts, has dramatically augmented ever since; in 2018, temporary contracts represent more than 80 % of created jobs. These contracts lead to an excessive job destruction, which pushes up unemployment<sup>1</sup>. The impacted workers benefit less of training<sup>2</sup> as well as they experience a higher risk of work accident<sup>3</sup> and lower wages than permanent workers<sup>4</sup>. Moreover, fixed-term contracts constitute poor stepping-stones towards permanent employment<sup>5</sup>.

Should a dual labor market be reformed ? What post-reform dynamic behavior should we expect ? To address these questions, I extend the classic model of Mortensen and Pissarides (1994) [27] to include both open-ended and fixed-term contracts. As in the latter, firms and workers meet randomly according to a matching function. When a firm-worker pair forms, it draws a productivity from a given distribution and assesses accordingly whether an open-ended or a fixed-term contract is preferable. The latter differ only over one dimension; permanent contracts stipulate a red-tape firing cost, while temporary contracts have high exogenous job destruction rate. The model is calibrated to match moments of the French labor market.

My contribution includes theoretical and numerical insights. I design the model to be the simplest possible allowing an endogenous choice between open-ended contracts and fixed-term contracts at the hiring stage, which facilitates its integration into more complex frameworks, as is done in Rion (2019) [35]. Pushing the model to its theoretical limits, I find that a large class of random-search models is unable to match the actual distribution of fixed-term contracts' duration when expiring temporary contracts are convertible into permanent ones. In numerical terms, I analyze the dynamic response of the dual labor market following reforms on employment protection legislation, which, as far as I know, has not been done yet. Overall, the resulting model provides rich insights about the economic schemes at stake in a dual labor market and the desirability of employment protection legislation reforms in terms of welfare and employment.

At the equilibrium, when a firm-worker pair meets and draws a high productivity, the best manner to maximize benefits consists in locking up the pair through a permanent contract. Otherwise, agents can commit to a short productive relationship through a temporary contract or go back to searching for a better match on the labor market. The meeting process being costly, agents opt for a temporary contract if the productivity draw is not too disappointing. Thus, fixed-term contracts provide a productive interlude, while enabling to go back to seeking a high-productivity match before long. Temporary contracts act as fillers before meeting the

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<sup>1</sup>See Blanchard and Landier (2002) [10] and Cahuc et al (2016) [12]

<sup>2</sup>Relevant empirical studies include Arulampalam and Booth (2002) [5], Arulampalam (2004) [6], Albert et al (2005) [3] and Cutuli and Guetto (2013) [14]

<sup>3</sup>See Guadalupe (2003) [21] and Picchio and van Ours (2017) [31]

<sup>4</sup>See Hagen (2003) [22], Mertens et al. (2007) [26], Pfeifer (2012) [29] and Pfeifer (2014) [30] for estimates of the wage gap.

<sup>5</sup>See Fontaine and Malherbet (2016) [18]

Prince Charming<sup>6</sup>.

I find that two opposed effects mix up to shape the behavior of the labor market with respect to employment protection legislation. The first effect is a substitution effect towards temporary employment on behalf of permanent employment. Bolstering firing costs makes permanent contracts more rigid and fosters the resort to less productive temporary contracts for the sake of flexibility. Thus, higher firing costs provide an incentive to hire more temporary contracts and favor the emergence of an equilibrium with coexisting temporary and permanent contracts. The second effect is a general-equilibrium effect. Magnified employment protection legislation reduces permanent job destruction. Since most workers are placed under permanent contracts, aggregate job destruction reduces and so does unemployment. In turn, the depressed unemployment rate fuels the diminution of temporary job creation flows.

As for the social welfare and employment, I find that there is no free lunch when the dual nature of the labor market is preserved: a benevolent planner cannot both decrease unemployment and improve welfare around the baseline economy. The optimal policy consists in the removal of temporary contracts and a large cut in firing costs. Consequently, this model supports a transition towards a unique open-ended contract, a theme the literature has extensively tackled<sup>7</sup>.

In terms of dynamics, an increase in firing costs with respect to the baseline calibration leads to a several-year long adjustment. Indeed, flows are reduced on the permanent side of the labor market, which tends to slow down transitions. Conversely, a strong decrease in firing costs enhances permanent job creation and destruction flows, which reduces the duration of adjustment. Between these two endpoints, the behavior of the adjustment time after a moderate cut in firing posts is non-monotonous and intricate. Transitions toward a unique-contract equilibrium with a cut in firing costs end by 2 years. Taking account of frequent marginal employment protection legislation reforms - one every 7 months on average in France - I extend the model to include regulatory uncertainty with respect to firing costs. This policy risk significantly bolsters temporary employment on behalf of permanent employment and pushes up unemployment.

This paper is related to two strands in the literature. Firstly, it contributes to explain the dynamic interaction between temporary and permanent contracts as Saint-Paul (1996) [36] and Dolado et al. (2002) [17] do. Secondly, it delineates the mechanisms underpinning the choice between temporary and permanent contracts. Smith (2007) [39] first highlighted the role of temporary contracts as stopgaps. In his stock-flow matching model, firms hire poorly productive workers on limited duration to be ready to hire highly productive workers when they appear. His framework is very different, though; search is directed and labor market institutions are not explicitly modeled. In terms of ambitions, Cahuc et al (2016) [12] is the closest paper. To this extent, I refer to the excellent literature review it provides in the introduction of the paper. Cahuc et al (2016) builds a matching model where firm-worker pairs face heterogeneous frequencies of productivity shocks. Workload fluctuations drive the contractual decision ; temporary contracts are preferred when productivity shocks are frequent, whereas permanent contracts are beneficial when productivity shocks are unusual. The model is highly realistic. Expiring temporary contracts can be converted into permanent contracts as in

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<sup>6</sup>This all the more true if on-the-job search is an option. Potlivka (1996) [33] finds that most contingent workers do not search for a perennial job. The model thus delineates a lower bound in terms of temporary contracts' desirability at the hiring stage

<sup>7</sup>See Lepage-Saucier et al. (2013) [25] and Amable (2014) [4] among others

the French law. In addition, firm-worker pairs endogenously optimize hired temporary contracts' duration. However, as I show below, these findings critically rely on the assumption that hired matches start with fixed productivity. Relaxing this assumption leads to the conclusion that the shortest temporary contracts are the most frequently converted into permanent contracts, which is not the case in the data. Importantly, this issue cannot be solved by introducing heterogeneity across firms and workers' characteristics in a large class of random-search models.

The paper is structured as follows. Section 2 describes the model and derives the main theoretical results. In Section 3, I calibrate the model on French data, study employment protection legislation reforms, the associated post-reform dynamics and the impact of regulatory uncertainty. Section 4 discusses the extension of the model including temporary contracts' conversion to permanent contracts and the endogenous choice of temporary contracts' durations. Section 5 concludes.

## 2 The model

In this model a-la Mortensen Pissarides (1994) [27], there are two continua of firms and households and time is continuous. The interest rate is denoted  $r$ . Firms are numerous and can either employ one worker or maintain a vacancy opened. They face i.i.d idiosyncratic productivity shocks that occur with a probability  $\lambda$  per unit of time and are drawn from a log-Normal distribution  $\log \mathcal{N}(0, \sigma_z^2)$  with cumulative distribution function  $G$  and support  $(0, \infty)$ . Our model mainly departs from the classic Mortensen-Pissarides framework by assuming that workers are either employed through open-ended contracts or fixed-term contracts. I assume that temporary and permanent contracts share the labor market.

The matching function is standard, with constant returns to scale over the number of vacancies  $v$  and the number of job seekers  $e$ . The number of matches per unit of time is denoted  $m(e, v)$ , while the job market tightness  $\theta$  is classically set as the number of vacancies over the number of job seekers  $\theta = \frac{v}{e}$ . The constant-return-to-scale feature of the matching function entails that the job-meeting probability  $p = m(e, v)/e$  and the seeker-meeting probability  $q = m(e, v)/v$  only depend on the labor market tightness.  $q$  is a non-increasing function, whereas  $p$  is a non-decreasing one. Both  $q$  and  $p$  are convex. When paired with a worker, firms can hire through a temporary contract or hire through a permanent contract. They are also able to resume searching for a worker if they are not satisfied with the productivity of the match.

As is classic in the Mortensen-Pissarides model with Nash bargaining, the shape of decisions is entirely determined by the surplus of matches. Joint surpluses are defined as the sum of workers' and firms' surpluses. The surplus of firms for continuing permanent contracts is by assumption  $J^p(z) - (V - F) = J^p(z) - V + F$ , where  $J^p$  is the firm's surplus from a continuing permanent contract,  $V$  the firm's surplus from an unfilled vacancy and  $F$  is the firing cost. Interestingly, the firm's surplus for a new permanent match is  $J_0^p(z) - V$ , where  $J_0^p$  is the firm's surplus from a new permanent contract. Indeed, when a contact occurs between a firm and a worker, the firm's outside option does not include the payment of a firing cost if no contract is signed. The permanent and temporary workers' surpluses are standard. We denote  $W^p(z)$  the worker's surplus from a continuing permanent contract,  $W_0^p(z)$  the worker's surplus from a new permanent contract,  $W^f(z)$  the worker's surplus from a temporary contract and  $U$  the

unemployed's surplus.

$$\begin{aligned} S^f(z) &= (J^f(z) - V) + (W^f(z) - U) \\ S^p(z) &= (J^p(z) - [V - F]) + (W^p(z) - U) \\ S_0^p(z) &= (J_0^p(z) - V) + (W_0^p(z) - U) \end{aligned}$$

Wages are determined following a Nash-bargaining process. The workers' bargaining power is marked down as  $\eta$  and is common to both permanent and fixed-term contracts. Indeed, analyzing the role of employment protection legislation implies shutting down all differences between permanent and temporary contracts, with the exception of genuinely legal ones. Therefore,

$$J^p(z) - (V - F) = (1 - \eta) S^p(z) \quad (2.1)$$

$$J_0^p(z) - V = (1 - \eta) S_0^p(z) \quad (2.2)$$

$$J^f(z) - V = (1 - \eta) S^f(z) \quad (2.3)$$

Nash-bargaining makes endogenous separations as well as hiring decisions jointly efficient. In other words, there is no conflict over the hiring and firing choices between firms and workers. The formulas above enable the computation of the workers' rents

$$\begin{aligned} W^p(z) &= U + \frac{\eta}{1 - \eta} (J^p(z) - V) + \frac{\eta}{1 - \eta} F \\ W_0^p(z) &= U + \frac{\eta}{1 - \eta} (J_0^p(z) - V) \\ W^f(z) &= U + \frac{\eta}{1 - \eta} (J^f(z) - V) \end{aligned}$$

Workers appropriate a fraction of the firms' surplus because of the sunk hiring costs firms pawn. The matching procedure can be considered as a production function with two inputs : job seekers and vacancies. The involvement of firms through recruiting costs in this process before any production takes place generates a hold-up situation favoring workers at the moment of wage bargaining<sup>8</sup> and enables them to extract a rent. Moreover, continuing permanent workers benefit from a supplementary rent  $\frac{\eta}{1 - \eta} F$  when compared with temporary workers and new permanent workers. The firing cost pushes up the permanent workers' bargaining power by enhancing the threat of a costly separation. The firm will reward the worker through the wage for avoiding the separation and its associated cost  $F$ . Thus, the firing cost influences labor market outcomes through two channels : as a pure firing tax and through wages. The firing cost is not involved in Nash bargaining for a new permanent match since the worker is not yet an insider at the moment of the wage bargaining.

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<sup>8</sup>See Grout (1984) [20] for a thorough development of this question

## 2.1 The agents' value functions

**Vacancies and unemployed workers** A firm-worker contact occurs with probability  $q(\theta)$ . The cost of a vacancy is  $\gamma$  per unit of time regardless of the contract type. When the idiosyncratic productivity of the match reveals, the firm chooses between hiring the worker through a permanent contract or a temporary contract and letting the worker go back into the unemployed's pool.

$$rV = -\gamma + q(\theta) \int \max \left[ J_0^p(z) - V, J^f(z) - V, 0 \right] dG(z) \quad (2.4)$$

The firing costs only apply if the match is validated in the first place. The potentially ephemeral constitution of a match does not boil down to the payment of firing costs if immediate separation is preferable. Thus, the role of firing costs is not unrealistically exacerbated at the creation stage.

The unemployed workers' value function embeds the unemployment benefit  $b$  and the various possibilities stemming from the eventual contact with a firm, which occurs with probability  $p(\theta)$ .

$$rU = b + p(\theta) \int \max \left[ W_0^p(z') - U, W^f(z') - U, 0 \right] dG(z') \quad (2.5)$$

**Continuing open-ended contracts** The firm's capital value of a continuing permanent match consists in an immediate profit from production  $z$  net of the worker's wage  $w^p(z)$ . When the match exogenously separates with probability  $s$ , the firm goes back to searching a worker without paying the firing cost. In real life, this probability is associated with events such as retirement, death or resignations. In this case, the firing cost needs not to be paid. The firm may also face productivity shock with probability  $\lambda$  and assess whether it keeps or lays off the worker regarding the new idiosyncratic productivity of the match. In the latter case, the firing cost needs to be paid.

$$rJ^p(z) = z - w^p(z) + s(V - J^p(z)) + \lambda \int (\max [J^p(z'), V - F] - J^p(z)) dG(z') \quad (2.6)$$

$J^p$  being increasing in the idiosyncratic productivity of the match, there exists a job destruction margin for continuing permanent contracts  $z^p$  defined as

$$J^p(z^p) = V - F$$

The subsequent probability of separation for permanent contracts is

$$\xi = s + \lambda G(z^p) \quad (2.7)$$

The workers' value function when he is under a continuing permanent contract firstly consists in the wage  $w^p$ . The different alternatives mentioned above apply.

$$rW^p(z) = w^p(z) + \lambda \int (\max [W^p(z'), U] - W^p(z)) dG(z') + s(U - W^p(z)) \quad (2.8)$$



The value function for new permanent contracts are analogous. The only notable difference originates from a specific wage  $w_0^p$  and the transformation into a continuing permanent contract when a productivity shock occurs. The new productivity either entails a costly separation or a renegotiation of wages, which include the payment of a firing cost as a possible outcome of the negotiation. Consequently, the reassessed relationship corresponds to a continuing permanent contract.

$$rJ_0^p(z) = z - w_0^p(z) + \lambda \int (\max [J^p(z'), V - F] - J_0^p(z)) dG(z') + s(V - J_0^p(z)) \quad (2.9)$$

$$rW_0^p(z) = w_0^p(z) + \lambda \int \max ([W^p(z'), U] - W_0^p(z)) dG(z') + s(U - W_0^p(z)) \quad (2.10)$$

**Fixed-term contracts** Similarly to permanent contracts, the firm employing a fixed-term worker gets an immediate profit from a match with production  $z$ . It also pays a wage  $w^f(z)$ . The match may split if the stipulated termination date is reached, which happens with probability  $\delta$ . In this case, the firm gets back to the labor market and earns the net return  $V - J^f(z)$ . A productivity shock may also strike with probability  $\lambda$ .

$$rJ^f(z) = z - w^f(z) + \lambda \int (J^f(z') - J^f(z)) dG(z') + \delta (V - J^f(z)) \quad (2.11)$$

Two serious discrepancies between the model and the actual French labor code need to be highlighted. The desire for simplicity guides these two strong assumptions.

Firstly, the duration of temporary contracts is exogenous. Endogenous separations for temporary contracts are not possible. At first view, a certain lack of realism must be pointed at here. Indeed, according to the French legislation, a firm may fire a temporary worker before the stipulated date through the payment of a firing cost. Nevertheless, this makes the analysis vainly cumbersome. The main trade-off between temporary and permanent contracts is set between long-lasting rigid permanent contracts and very short temporary contracts. Consequently, the advantage of immediately paying the firing cost when temporary contracts are very short is thin when the firm can wait for the stipulated termination date to get rid of the worker. For the sake of simplicity, we assume that the firm always prefer to wait for the end of a temporary contract instead of paying the firing cost. The results barely change when this subtlety is accounted for. Here, the separation probability implicitly models the average duration of temporary contracts.

Secondly, the model does not embed the possibility of conversion into a permanent contract when the stipulated duration is reached, whereas this option exists in reality. French data demonstrates that temporary contracts seldom convert into permanent contracts. The quarterly probability of transition from temporary to permanent employment constitutes an upper bound of the conversion probability and amounts to 7.4 % according to Hairault et al (2015) [23]. Meanwhile, the distribution of the duration of temporary contracts has its mean at 1.5 months and a median located around 5 days as Dares Analyses (2018) [15] mentions. Thus, these 7.4 % more likely include multiple round trips between temporary employment and unemployment somehow ending into permanent employment rather than unique and direct temporary-to-

permanent-employment trajectories. Consequently, the conversion rate probably lies far below transition rates from temporary to permanent employment. Our calibration being conducted on a monthly basis, the value of the conversion probability becomes insignificant.

The temporary worker's value function embeds the wage  $w^f(z)$  and the expectations about next-period outcomes. If the match is not separated, the worker expects to get the average worker's value from a temporary contract. If a split occurs, the worker returns searching for a job.

$$rW^f(z) = w^f(z) + \lambda \int \left( W^f(z') - W^f(z) \right) dG(z') + \delta \left( U - W^f(z) \right) \quad (2.12)$$

## 2.2 Joint surpluses and wages

At the equilibrium, we assume that there is free-entry on the vacancy-side of the matching activity. The present discounted value of a vacancy is zero. Competition between firms depletes the profit opportunities from new jobs. In other words, the rent provided by the posting of vacancies attracts new entrants until its disappearance at the equilibrium.

$$V = 0$$

This condition enables the writing of a condition for job creation, which merely states that the firm's expected cost for a job-vacancy contact  $\gamma/q(\theta)$  equals the expected profit from the same contact.

$$\frac{\gamma}{q(\theta)} = \int \max \left[ J_0^p(z), J^f(z), 0 \right] dG(z) \quad (2.13)$$

Using the Nash-bargaining rules (2.1)–(2.3) and the job creation condition (2.13), I get

$$\frac{\gamma}{(1-\eta)q(\theta)} = \int \max \left[ S_0^p(z), S^f(z), 0 \right] dG(z) \quad (2.14)$$

Deriving the expression for surpluses using the proper value functions along with (2.13), I get

$$(r + s + \lambda)S^p(z) = z - rU + (r + s)F + \lambda \int \max \left[ S^p(z'), 0 \right] dG(z') \quad (2.15)$$

$$S_0^p(z) = S^p(z) - F \quad (2.16)$$

$$(r + \delta + \lambda)S^f(z) = z - rU + \lambda \int S^f(z') dG(z') \quad (2.17)$$

$$rU = b + \frac{\eta\gamma\theta}{1-\eta} \quad (2.18)$$

The unemployed value the benefit  $b$  and the rent they obtain from finding a job on the next-period. A common rent for all ex-post insiders is a share of the firms' recruitment cost.  $\gamma$  is lost for the firm at the very moment of agreement over the formation of a match and the subsequent wage bargaining. The asset value of being unemployed is also the employed's outside option.

Wages are computed thanks to the Nash-sharing rules with the associated value functions and the expressions of joint surpluses.

$$w^p(z) = \eta(z + (r + s)F + \gamma\theta) + (1 - \eta)b \quad (2.19)$$

$$w_0^p(z) = \eta(z - \lambda F + \gamma\theta) + (1 - \eta)b \quad (2.20)$$

$$w^f(z) = \eta(z + \gamma\theta) + (1 - \eta)b \quad (2.21)$$

These equations for the wages of the permanent jobs entail that  $w_0^p(z) = w^p(z) - \eta(r + s + \lambda)F$ . The wages of new permanent workers are lower than continuing permanent workers' ones. This reflects the lower bargaining power of outsiders. On one hand, at a given labor market tightness  $\theta$ , the new permanent workers' wages decrease with the firing cost. After the signature of the permanent contract, firms have to pay the firing cost in case of adverse productivity shock. Consequently, before the occurrence of any productivity shock, firms compensate the new worker's expected gain of bargaining power and the associated expected loss in profits by decreasing the current wage proposal, which does not entail the payment of a firing cost in case of disagreement. On the other hand, the continuing permanent workers benefit from the case of a costly separation case embedded in the firms' outside option. Thus, continuing permanent workers increase with firing costs. Beyond these specificities, wages increase with unemployment benefits and recruitment cost : their threat point in the Nash-bargaining process is enhanced. Similarly, a higher labor market tightness encompasses greater job-finding opportunities to the unemployed, which raises the latter's outside option : wages raise.

### 2.3 Job creation and job destruction

Joint surpluses of matches are enough to pinpoint the hiring and firing behavior in the economy. These joint surpluses are linear and increasing in the idiosyncratic productivity of the match. Consequently, I define the job destruction margin for continuing permanent contracts  $z^p$  as

$$S^p(z^p) = 0 \quad (2.22)$$

A permanent match separates when the pair prefers paying the firing cost instead of continuing. The productivity must be sufficiently low to fill this requirement ; the worker and the firm benefit from a separation if  $z < z^p$ . Using an integration by part in the definition of  $z^p$  (2.22) jointly with the definition of  $S^p$  (2.15), the job destruction condition for permanent contracts is

$$z^p - b + (r + s)F + \frac{\lambda}{r + s + \lambda} \int_{z^p}^{+\infty} (1 - G(z)) dz = \frac{\eta\gamma\theta}{1 - \eta} \quad (2.23)$$

This equation circumscribes the job destruction of permanent contracts and defines a positive relationship between  $\theta$  and  $z^p$ . The intuition behind this result is classic in the literature. A looser market tightness implies a higher job-finding probability for the unemployed, which makes their outside option stronger at the moment of wage bargaining: wages raise. This encourages firms to be more demanding in terms of the productivity of matches and to increase the job destruction margin. An enhanced firing cost diminishes the job destruction margin ; firms are

more reluctant to pay the firing cost and accept to maintain matches with worse productivity than before. Interestingly, a continuing permanent match behaves as if it held a bond with face value  $F$  and yield  $(r + s)$  that it used to pay the firing costs in case of an endogenous separation.

In the same manner, I define the threshold  $z^f$  as follows

$$S^f(z^f) = 0 \quad (2.24)$$

The interpretation of this threshold is twofold. As far as a contact between a vacancy and a job-seeker is concerned, it states whether a temporary contract is profitable. As for a continuing temporary contract, it defines whether the firm would like to fire or keep the worker if no constraint was enforced. In this model, the constraint is a legal prohibition to terminate a contract before its stipulated end date<sup>9</sup> The Nash-sharing rule entails that  $S^f(z^f) = 0$ , which means that the creation of a temporary contract is profitable for both parties as soon as  $z \geq z^f$ <sup>10</sup>. Conversely, when  $z < z^f$ , an existing match would have interest into splitting ; the firm and the worker would prefer to get back into searching. (2.24) jointly with the Nash-bargaining rule (2.3)

$$z^f = \left(1 + \frac{\lambda}{r + \delta}\right) \left(b + \frac{\eta\gamma\theta}{1 - \eta}\right) - \frac{\lambda}{r + \delta} E_z \quad (2.25)$$

where  $E_z = \int z dG(z)$ . As previously, a higher labor market tightness enlarges wages because of workers' augmented outside option. Temporary contracts are profitable on a thinner range and  $z^f$  increases. A higher average productivity  $E_z$  encourages the resort to temporary contracts: the profitability margin decreases as temporary contracts become more often beneficial.

An analogous threshold  $z^c$  can be defined for the desirability of new permanent contracts

$$S_0^p(z^c) = 0 \quad (2.26)$$

Meanwhile, since  $S^p(z^p) = 0$  and  $\partial S^p / \partial z = 1/(r + s + \lambda)$ , I can rewrite  $S^p$  as

$$S^p(z) = \frac{z - z^p}{r + s + \lambda} \quad (2.27)$$

(2.26), (2.27) and (2.26) enables a convenient writing for  $z^c$

$$z^c = z^p + (r + s + \lambda)F \quad (2.28)$$

Proceeding in the same manner for the joint surplus of temporary matches and new permanent matches

$$S^f(z) = \frac{z - z^f}{r + \delta + \lambda} \quad (2.29)$$

$$S_0^p(z) = \frac{z - z^c}{r + s + \lambda} \quad (2.30)$$

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<sup>9</sup>As I previously wrote, the French legislation enables the firing of a temporary worker with the payment of a firing tax. This specificity is ruled out as it has no implication on the core results.

<sup>10</sup>Of course, if a temporary contract is profitable, but still less beneficial than a permanent contract, then the hire takes place through a permanent contract.

Notice that  $\partial S_0^p / \partial z = 1/(r + s + \lambda) > 1/(r + \delta + \lambda) = \partial S^f / \partial z$  if and only if  $s < \delta$ . I assume the validity of the latter condition, which states that the destruction rate of temporary contract is higher than the exogenous separation rate of open-ended contracts. This is undoubtedly the case in the data as I demonstrate in the calibration section. As a result,  $S_0^p$  and  $S^f$  being increasing and linear in  $z$ , there exists  $z^*$  such that

$$S_0^p(z^*) = S^f(z^*)$$

For all  $z \geq z^*$ ,  $S_0^p(z) \geq S^f(z)$ . (2.29) and (2.30) can be used to circumscribe  $z^*$  jointly with (2.3).

$$\left( \frac{1}{r + s + \lambda} - \frac{1}{r + \delta + \lambda} \right) z^* = \frac{z^c}{r + s + \lambda} - \frac{z^f}{r + \delta + \lambda} \quad (2.31)$$

The behavior of the thresholds is characterized by the following proposition

**Proposition 1.** *These assertions are equivalent*

1.  $z^* > z^f$
2.  $z^* > z^c$
3.  $z^c > z^f$

*Proof.* See Appendix B □

In the same manner, the equality of two of the thresholds  $z^f$ ,  $z^c$  or  $z^*$  is equivalent to the equality between all of them  $z^f = z^c = z^*$ .

One ambition of this paper is to describe the endogenous choice between temporary contracts and permanent contracts at the hiring step. With the thresholds previously defined and the increasing character of  $S_0^p$  and  $S^f$ , the job creation condition (2.14) becomes

$$\frac{\gamma}{(1 - \eta)q(\theta)} = \int_{\max[z^c, z^*]}^{+\infty} S_0^p(z) dG(z) + \int_{z^f}^{\max[z^f, z^*]} S^f(z) dG(z) \quad (2.32)$$

Using two integrations by parts, the definitions of thresholds and proposition 1, the latter becomes

$$\frac{\gamma}{(1 - \eta)q(\theta)} = \frac{1}{r + s + \lambda} \int_{\max[z^c, z^*]}^{+\infty} (1 - G(z)) dz + \frac{1}{r + \delta + \lambda} \int_{z^f}^{\max[z^f, z^*]} (1 - G(z)) dz \quad (2.33)$$

The formal definition of a steady-state equilibrium in this model can be spelled out

**Definition 1.** *A steady-state equilibrium in this economy is characterized by the tuple  $(\theta, z^p, z^c, z^f, z^*)$  verifying equations (2.23), (2.25), (2.28), (2.31) and (2.33).*

This job creation condition (2.33) heavily depends on the distribution of idiosyncratic shocks and the subsequent value of thresholds. It is possible to have no permanent contracts, no temporary contracts as well as both contracts at the hiring stage. Thus, a formal definition of dual job creation is useful.

**Definition 2.** *Job creation is said to be dual if one kind of contracts is not systematically preferred to the other at the hiring stage.*

In this model, job creation obeys the following proposition

**Proposition 2.** *Considering an equilibrium  $(\theta, z^p, z^c, z^f, z^*)$ .*

- *Job creation only occurs through permanent contracts if and only if  $z^* \leq z^f \leq z^c$ . Permanent contracts are hired when  $z \in (\max[0, z^c], +\infty)$  as figure 1 displays.*

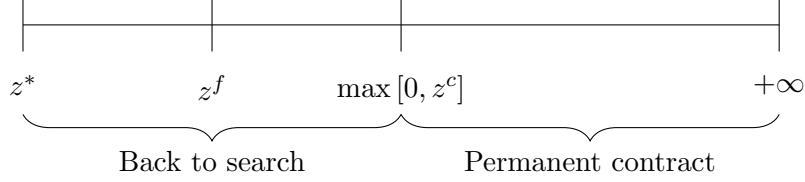


Figure 1: Hiring permanent contracts only

- *Job creation is dual if and only if  $\max[0, z^f] < z^*$ . Temporary contracts are hired when  $z \in (\max[0, z^f], z^*)$  and permanent contracts are hired when  $z \in (z^*, +\infty)$ .*

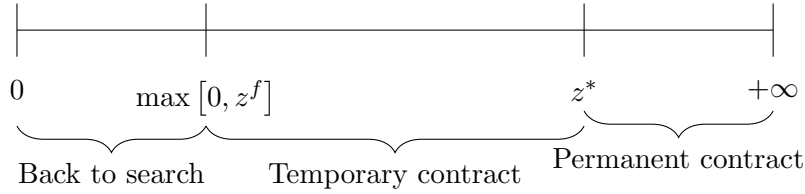


Figure 2: Dual job creation

*Proof.* See Appendix B □

The proposition above states that dual job creation necessitates the immediate gains of hiring a permanent contract in good times to overcome the losses due to future downturns and the eventual payment of firing costs. A firm benefiting from a high idiosyncratic shock at the moment of the match may want to take advantage of this opportunity in full. To this extent, the best way to make the most of the situation consists in hiring a permanent worker, which lasts longer and thus provides a higher surplus than a temporary worker. The match willingly locks itself through a permanent contract in order to maximize the expected surplus. Otherwise, when the firm-worker pair draws a low productivity, both worker and firms go back searching. Temporary contracts serve as a compromise between a rigid contract or no productive relationship at all. Hiring through a temporary contract appears as a median action: it generates a surplus through production while enabling to go back to searching for a better match before long. Consequently, in this setup, a downward shift of the productivity distribution expands the share of temporary contracts in job creation. In this case, temporary contracts are expedients to earn surplus while waiting for better days to come. When, finally, a high productivity shock arises from a firm-worker meeting, an open-ended contract is signed.

## 2.4 Aggregate flows and stocks on a dual labor market

The delineation of conditions for dual job creation enables the computation of labor market flows and stocks.

The share of permanent workers that moves into unemployment falls into exogenous separations, which occur with probability  $s$ , and endogenous separations, which occur when an adverse productivity shock occurs. Thus, the associated probability is  $\xi = s + \lambda G(z^p)$ . Unemployed workers' probability to become employed under open-ended contracts is  $\mu^p = p(\theta)(1 - G(\max[z^*, z^c]))$ . On the fixed-term side of the labor market, temporary matches come to their stipulated end date with probability  $\delta$ . An unemployed worker finds a temporary job if he contacts a firm and the productivity of the resulting match is in the proper region, which occurs with probability  $\mu^f = p(\theta)(G(\max[z^*, z^f]) - G(z^f))$ . Denoting  $u$  the measure of unemployed workers,  $n^p$  the measure of permanent workers and  $n^f$  the measure of temporary workers, temporary and permanent employments evolve according to the following system

$$\dot{n}^p = -\xi n^p + \mu^p u \quad (2.34)$$

$$\dot{n}^f = -\delta n^f + \mu^f u \quad (2.35)$$

Normalizing the workers' population to 1 leads to the following expressions for steady-state permanent employment, temporary employment and unemployment.

$$n^p = \frac{\mu^p \delta}{\mu^p \delta + \xi \delta + \mu^f \xi} \quad (2.36)$$

$$n^f = \frac{\mu^f \xi}{\mu^p \delta + \xi \delta + \mu^f \xi} \quad (2.37)$$

$$u = \frac{\xi \delta}{\mu^p \delta + \xi \delta + \mu^f \xi} \quad (2.38)$$

As previously, an increase in  $\theta$  has ambiguous consequences. On one hand, it strengthens the workers' outside option, which in turn raises wages, encourages the destruction of permanent jobs and makes firms more demanding at the hiring stage in terms of productivity. On the other hand, job creation is bolstered by the enlarged probability of contact. Therefore, the impact on labor market stocks is ambiguous.

## 2.5 Comparative statics

In this subsection, I carry out comparative-statics exercises to describe the steady-state behavior of the model.

Figure 3 diagrammatically sums up the movements of the different loci equations (2.23), (2.25), (2.28), (2.31) and (2.33) describe after an increase in firing costs.

Enlarged firing costs decrease the job destruction margin for a given labor market tightness. The job destruction curve  $(JD^p)$  moves downward to reach  $(JD^p)'$ . Firm-worker pairs compensate their losses in the expected surplus of a continuing match by demanding more productive newcomers:  $(z^*)$  shifts upwards to  $(z^*)'$ . The locus for profitability of the temporary contracts

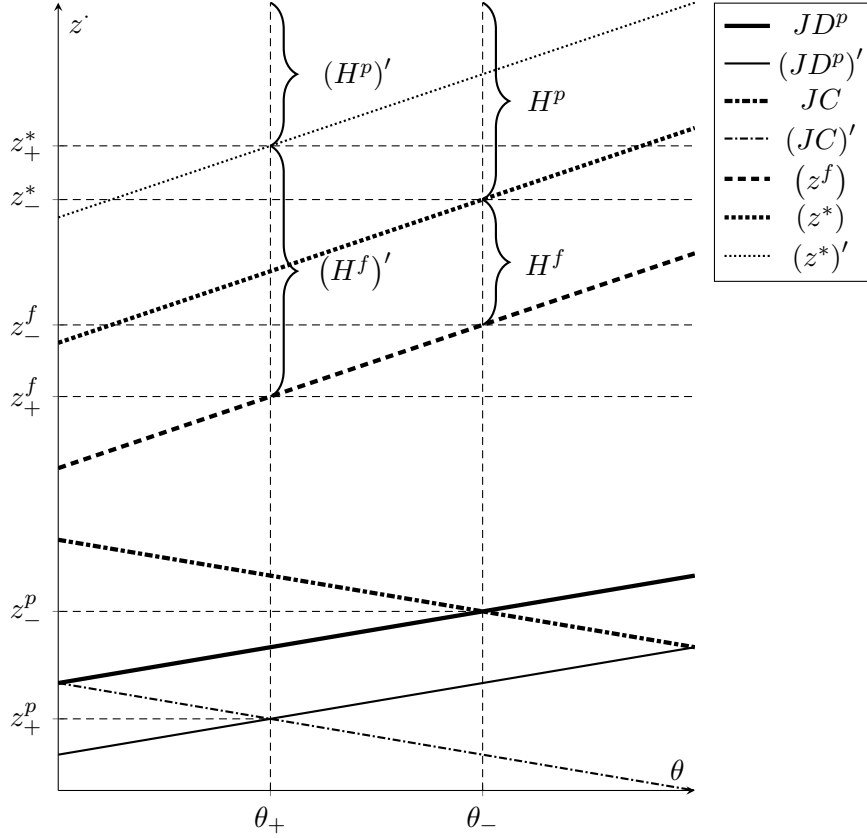


Figure 3: An increase in  $F$  in the dual Mortensen-Pissarides model

The plus + and minus - subscripts respectively denote the equilibrium values before and after the change in firing costs.  $H^p$  and  $(H^p)'$  respectively name the hiring region under permanent contracts before and after the change in firing costs.  $H^f$  and  $(H^f)'$  are their counterparts for temporary contracts.

$(z^f)$ , as for it, remains unchanged. Consequently, magnified firing costs necessarily increase the share of temporary contracts in job creation. This phenomenon causes a shift of the job creation condition downward: at a given  $\theta$ , the expected profit from a firm-worker contact decreases as the signatures of permanent contracts dwindle and are replaced by temporary contracts at the margin.  $(JC)$  shifts downward to reach its new position  $(JC)'$ . The job creation curve shifts further than the job destruction curve and the equilibrium labor market tightness decreases. Notice that the subsequent reduction in the labor market tightness is weaker than in the classic case. The replacement of permanent hires with temporary ones at the margin mitigates the fall in the expected surplus from a contact and abates the fall in the posting of vacancies. Indeed, at the margin, matches with marginal productivity lower than  $z^c$  delivered no opportunities of a positive profit, whereas the matches with marginal productivity slightly lower than  $z^*$  deliver a positive profit through temporary employment.

Overall, the evolution of permanent employment is ambiguous : job creation reduces as well as job destruction. As for temporary employment, job creation probability increases but the evolution of unemployment is unclear. Therefore, the temporary-job creation flow is indeterminate.

An important alternative situation where the reasoning above is no longer valid consists



in the insensibility of the steady-state equilibrium to firing costs. This happens if endogenous destructions of permanent jobs do not take place. Proposition 3 recapitulates the previous results

**Proposition 3.** *At the steady-state equilibrium,*

- *with endogenous job destruction of permanent contracts*

$$\frac{\partial \theta}{\partial F} < 0, \frac{\partial z^p}{\partial F} < 0, \frac{\partial z^c}{\partial F} > 0, \frac{\partial z^f}{\partial F} < 0, \frac{\partial z^*}{\partial F} > 0$$

- *otherwise*

$$\frac{\partial \theta}{\partial F} = \frac{\partial z^c}{\partial F} = \frac{\partial z_i^f}{\partial F} = \frac{\partial z^*}{\partial F} = 0$$

*Proof.* See Appendix B □

An increase in firing costs entails a substitution towards temporary employment on behalf of permanent employment, while the classic result of an ambiguous response of unemployment remains. The enhanced relative flexibility of temporary contracts makes them significantly more attractive than permanent contracts, which are progressively replaced by temporary contracts at the margin. When there is no endogenous destruction of permanent jobs, the equilibrium becomes insensitive to firing costs.

This exercise shows that the relative desirability of each contract at the hiring stage heavily depends on firing costs. Intuitively, sufficiently low firing costs lead to a complete shutdown of temporary employment, the corner case being firing costs such that  $z^c = z^f = z^*$  as proposition 2 suggests. Similarly, prohibitively high firing costs should drive permanent employment to zero through the utter disappearance of job creation through open-ended contracts, which seems pretty unrealistic. This may occur if the distribution of idiosyncratic shocks is bounded upwards. In our case, an intermediate case arises. When firing costs are high enough, endogenous permanent job destruction vanishes. This makes the equilibrium insensitive to further increases in firing costs. The knife-edge value of firing costs solution corresponds to the equilibrium where  $z^p = 0$  when referring to proposition 2. The following proposition formalizes this intuition.

**Proposition 4.** *There exists  $\hat{F}$  and  $\tilde{F}$  such that the steady-state equilibria verify*

- *If  $F \leq \hat{F}$ , there are only permanent contracts*
- *If  $\hat{F} < F < \tilde{F}$ , permanent and temporary contracts coexist*
- *If  $\tilde{F} \leq F$ , there are no endogenous job destruction of permanent contracts. The equilibrium becomes insensitive to  $F$*

Figure 4 sums up the different sort of equilibria in terms of contractual composition depending on firing costs.

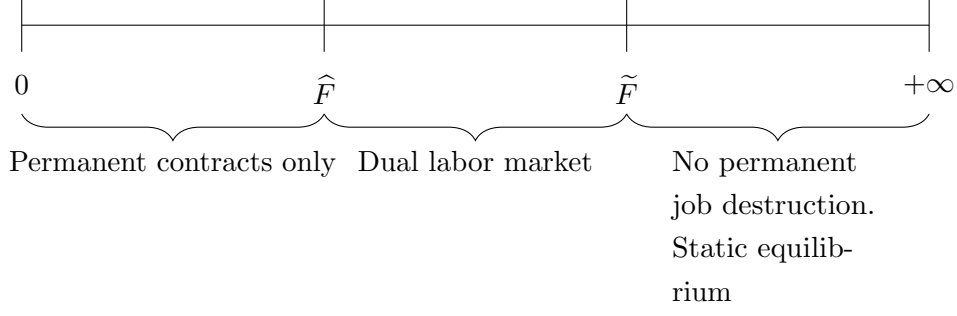


Figure 4: Firing costs and equilibrium employment

*Proof.* See Appendix B. □

This result demonstrates the adaptability of our approach. The productivity-flexibility trade-off may deliver a classic labor market with a rigid side only, a dual labor market or a fixed-term-oriented labor market. This is ideal to study transitions and steady-state outcomes related large-scale policies, such as the ban of temporary contracts or large cuts in firing costs. The insensitive equilibrium associated with  $F \geq \tilde{F}$  is interesting too. The data suggests that only 30% of permanent job destruction is currently endogenous, which is not so far from the 0% of our extreme case. To this extent, the economic schemes at stake in the region close to  $\tilde{F}$  might be insightful.

### 3 Numerical analyses

As we saw in the comparative statics section, quantitative analyses are necessary to circumscribe the behavior of employment and unemployment with respect to shocks on parameters.

#### 3.1 Calibration

In this section, we calibrate a model to mimic the behavior of the French labor market. Availability of data concerning dualism on the labor market guided this choice. Moreover, the extent of the subsequent analyses can be widened to many Western Europe countries, which share quite similar labor market institutions even though the strength of employment protection legislation may differ. One may quote Germany, Italy, Spain or Portugal among others<sup>11</sup>.

The unit time period is set to one month aligned with the average duration of 1.5 months for temporary contracts in 2017 documented by Dares Analyses (2018)<sup>12</sup>. Interestingly, the median duration associated with temporary contracts is much lower and amounts to 5 days. The interest rate is set to 5 % annually, which boils down to 0.4 % on a monthly basis. The matching function is specified as a Cobb-Douglas function  $m(u, v) = mu^\sigma v^{1-\sigma}$ . The elasticity of the matching function with respect to unemployment  $\sigma$  is set to 0.6, which stands in the middle of the range 0.5-0.7 estimated as reasonable by Burda and Wyplosz (1994) [11] for Western Europe economies. In order to avoid the effect of congestion externalities and the complexity of their interlacing

<sup>11</sup>OECD (2013) [2] and EPLex [1] detail the employment protection legislations of many countries.

<sup>12</sup>I essentially rely on this note to target moments characterizing the French labor market. It is based on the *Enquête sur les mouvements de main d'œuvre* carried out by Dares

with the labor market institutions, we set the bargaining power  $\eta$  to 0.6<sup>13</sup>. As I mention in the theoretical discussion above, one important question is the distribution of idiosyncratic shocks. Uniform distributions are used in seminal papers<sup>14</sup> and include important benefits in terms of tractability. However, Tejada (2017) [40] shows that log-normal distributions replicate well the distribution of wages in dual labor market. Results are not qualitatively different across different distributions of idiosyncratic shocks. Table 1 sums up the choice of parameters.

$r$	$\sigma$	$\eta$
0.4 %	0.6	0.6

Table 1: Parameters

$F/\overline{w^p}$	$\mu^f / (\mu^p + \mu^f)$	$n^f/n$	$s/\xi$	$u$	$q(\theta)$
1.33	0.83	0.12	0.70	0.26	0.33

Table 2: Targets for a calibration of the French labor market

$F$	$b$	$s$	$\delta$	$\sigma_z$	$m$	$\gamma$
1.30	1.00	0.013	0.67	0.05	0.53	$3.78 \cdot 10^{-3}$

Table 3: Calibrated parameters

Table 2 outlines the labor markets moments we intend to target. We assume that the unemployed workers include the inactive ones, as Fontaine (2016) [19] suggests that the latter significantly contribute to the labor market flows. As a result, we target an unemployment rate of 26 %, which corresponds to the 2016’s French inactivity rate<sup>15</sup>. The steady state share of temporary contracts on the labor market is set to 12 %. The average ratio of temporary contracts in job creation is set to 83 %<sup>16</sup>. I consider that exogenous splits of permanent matches constitute 70 % of separations. As for the calibration of the French firing costs, we rely on Kramarz and Michaud (2010) [24]. Individual lay-offs marginally cost 4 months of the median wage, while the marginal cost of lay-off within a collective-termination plan represents 12 months of the median wage<sup>17</sup>. The former being the most frequent case, we reckon that total firing costs represent 4 months of the permanent workers’ average wage. As in Bentolila et al. (2012) [9] and Cahuc (2016) [12], we assume that red-tape costs actually embodied by  $F$  only represent one third of total firing costs for the firm. Thus, we target a ratio of 4/3 for  $F$  with respect to the monthly permanent workers’ average wage. We also target the quarterly vacancy-filling rate to 70 %, which is equivalent to a monthly rate of 33 %. As for the occurrence probability of a productivity shock  $\lambda$ , there is no consensus stemming from the empirical literature. I assume a yearly average frequency and carry out three calibrations with three different average

<sup>13</sup>The Hosios condition is still valid in our framework.

<sup>14</sup>Among others, Mortensen and Pissarides (1994), Mortensen and Pissarides (1999) [28], Cahuc (2002)

<sup>15</sup>Source: *Recensement de la population 2016 - Insee*

<sup>16</sup>Average from Q1-2000 to Q2-2019 computed with data from *Acos - Urssaf (Déclarations préalables à l’embauche)*

<sup>17</sup>To be accurate, Kramarz and Michaud (2010) assess that firms with more than 50 employees face a marginal cost of 97,727 FFfr (Table 1b), which represents 14 months of the workers’ median wage. Consequently, the associated median wage of fired workers is 6980 FFfr. Thus, Table 2 shows that individual terminations cost 27,389 FFfr, which amounts to 4 months of the fired workers’ median wage, while the termination within a collective firing plan marginally costs 81,850 FFfr, which equals 12 months of the median wage.

durations between shocks: a month, a year and the mid-length of a business cycle, namely 3 years. The following results are not qualitatively modified<sup>18</sup>. In the same manner, the red-tape cost associated with an endogenous job destruction is difficult to assess because of their legal nature. I find that the results below are qualitatively similar with a zero share of exogenous splits<sup>19</sup>. Changing the distribution of shocks to a uniform one does not alter the results either. The calibration results in the determination of  $(\sigma_z, F, b, s, \delta, m, \gamma)$ , whose values are detailed in table 3.

### 3.2 Steady-state employment and welfare

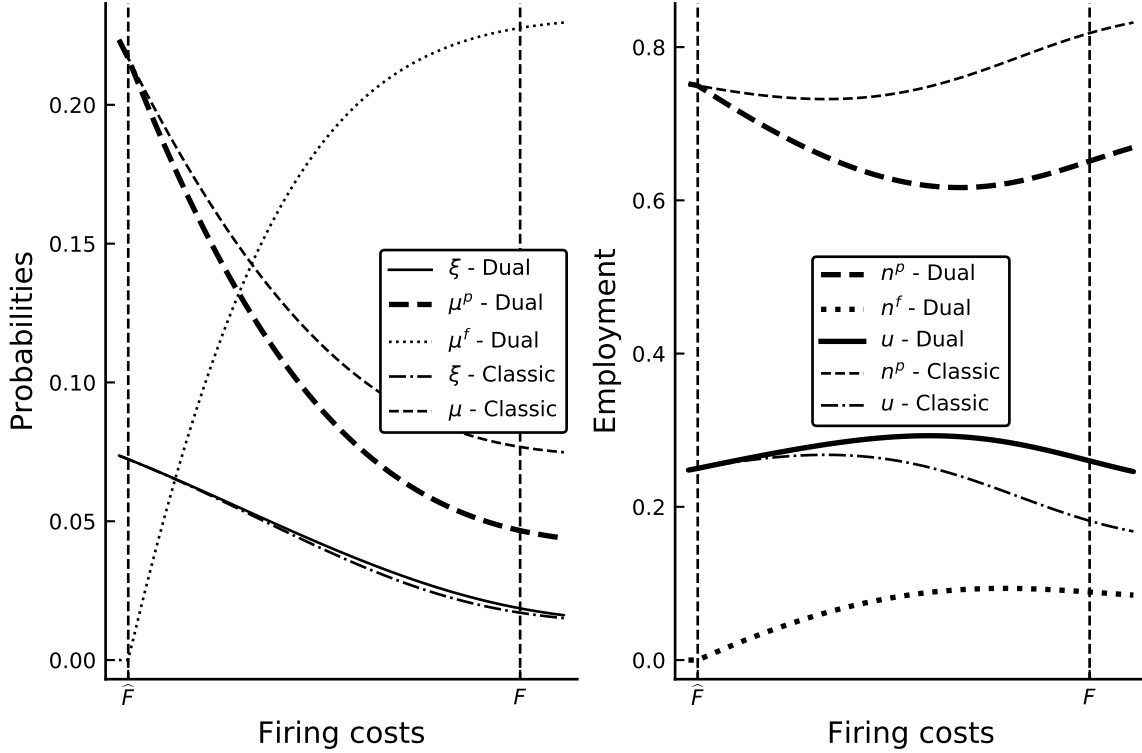


Figure 5: Steady-state transition probabilities and employments and firing costs

On the x-axis,  $\hat{F}$  is the threshold beyond which firing costs generate an equilibrium with dual job creation and is defined in Proposition 4.  $F$  denotes the value of firing costs as specified in the baseline calibration (see Table 3).

Left-hand side of Figure 5 displays the steady-state transition probabilities and their evolution with respect to firing costs. As firing costs increase, the separation probability of permanent contracts decreases. In the same manner, the permanent-contract job-finding rate decreases with firing costs while the temporary-contract job-finding rate increases. Magnified firing costs encourage the substitution towards temporary contracts. This substitution effect is responsible for the lower permanent job-finding rate in a dual labor market compared with a classic labor market when  $F \geq \hat{F}$ . Relatively to a traditional labor market, employees' outside option is bolstered by the possibility to find a job through a temporary contract for a given labor market

<sup>18</sup>The interested reader can visualize the impact of different probabilities of productivity-shock occurrence on MyBinder [34]

<sup>19</sup>All the robustness checks are available in Appendix A

tightness. Accordingly, the permanent job destruction margin is higher in the dual than in the classic case and so is the permanent job destruction probability.

The right-hand side of Figure 5 displays the evolution of employment rates in a classic and a dual Mortensen-Pissarides model. On the left-hand part of the graph, firing costs are lower than  $\hat{F}$ . Job creation only happens through permanent contracts, which entails no temporary employment at all on the steady state. Obviously, the dual and the classic Mortensen-Pissarides models coincide on this interval. Theoretically, firing costs have a relatively ambiguous effect on employment. On one hand, firing costs discourage lay-offs because of a magnified separation cost. On the other hand, the workers' enhanced threat point in the wage-bargaining process discourages hires. As  $F$  goes beyond  $\hat{F}$ , an equilibrium with dual job creation becomes possible. The classic model delivers lower and lower employment levels. At one point, the increase in firing costs ends up jeopardizing hiring incentives more than it discourages lay-offs: unemployment reaches a minimum level and begins increasing.

This phenomenon also happens for overall employment but the outcome differs. This difference between both models comes from the emergence of an additional mechanism in the dual case. Indeed, when dual job creation is an equilibrium outcome, augmented firing costs encourage the substitution of permanent employment towards temporary employment. Starting from  $\hat{F}$ , an increase in firing costs widens the spectrum of situations where temporary contracts are profitable. As temporary workers occupy a larger share of the jobs, the average job destruction rate increases. In our calibration, the widened possibilities of hires brought in by temporary contracts decrease less unemployment than magnified job destruction pushes it up : unemployment increases up to a maximum value of 28 %. This substitution effect is quantitatively substantial. Permanent employment drops to a minimum of 62 % of the whole workers' population, while temporary employment increases from 0 % to a maximum of 13 %.

Interestingly, when  $F$  takes values beyond these extrema, employment behaviors reverts. Permanent employment increases from 62 % to 65%, while temporary employment reduces from 13 % to 12 %. At first view, however, temporary employment should still increase according to the substitution effect we previously described. A general-equilibrium effect is responsible for this behavior. Since permanent employment represents the bulk of workers, unemployment tends to diminish as the permanent job destruction probability reduces. Even though higher firing costs push up the temporary jobs' hiring probability, the latter reduction in unemployment is strong enough to push down the job creation flow into temporary employment. Since temporary job destruction rates are constant, temporary employment shrinks. In the French data, a minor share of permanent matches' separations induce the payment of a firing cost. The calibrated economy is close to the corner equilibrium with no endogenous permanent job destruction. This sets the French economy in the region where the general-equilibrium effects of an increase in firing costs surpasses the substitution effect.

Beyond employment, important considerations include the impact of firing costs on welfare. The comparative statics carried out earlier as well as proposition 3 provide preliminary results. The unemployed would prefer the highest possible value for  $\theta$ , which is associated with  $F = 0$ . Indeed, the unemployed workers value high job-finding probabilities. As for the temporary and permanent matches, the results are theoretically ambiguous and a numerical analysis is necessary. Social welfare  $SW$  is defined as the sum of production from permanent matches, production

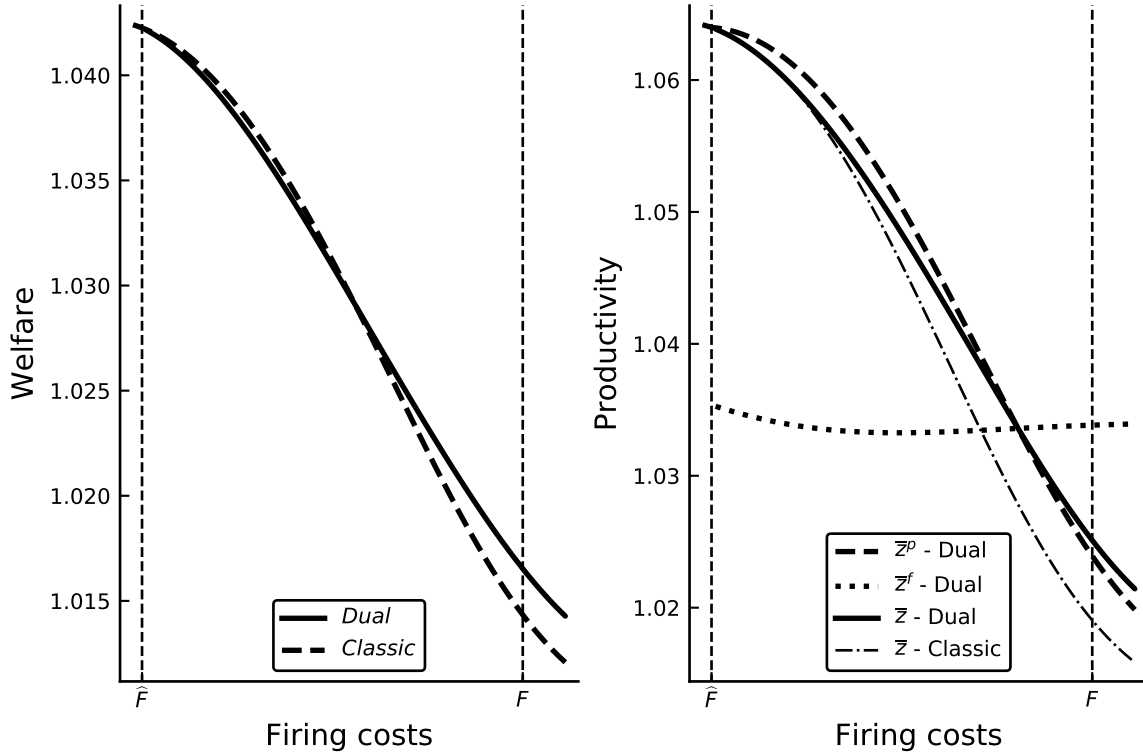


Figure 6: Evolution of social welfare and productivities with respect to firing costs

$\bar{z}^p$  and  $\bar{z}^f$  are the average productivities of permanent and temporary matches respectively.  $\bar{z}$  is the overall average productivity in the economy.

from temporary matches, the unemployed's home production net of the vacancy costs. Therefore, the steady-state social welfare verifies  $rSW = n^p \bar{z}^p + n^f \bar{z}^f + bu - \gamma v$ , where  $\bar{z}^p$  and  $\bar{z}^f$  are the average productivities of permanent and temporary matches respectively. Figure 6 displays steady-state social welfare on the left panel and productivities on the right panel as firing costs vary.

Many competing mechanisms intervene in the variation of social welfare with respect to firing costs. One may divide them in two categories: employment effects and productivity effects. The vacancy cost being tiny, one may neglect its quantitative effect in the comparative-statics analysis of welfare. As employment variations with firing costs are studied above, I now consider the evolution of productivities.

On the permanent side of the labor market, the higher the firing costs, the lower the rate of permanent job destruction. As firing costs increase, continuing permanent contracts become less and less productive. In contrast, augmented firing costs push up the job creation threshold  $\max[z^c, z^*]$ . New permanent contracts become more productive as firing costs enlarge. Overall, though, the stock of continuing permanent matches is bigger than the stock of new permanent matches and the former effect prevails. The average productivity of permanent contracts  $\bar{z}^p$  in the dual model (respectively  $\bar{z}$  in the classic model) decreases. The productivity of permanent contracts is higher in the dual model than in the classic model. This essentially comes from the fact that temporary employment replaces permanent employment as firing costs increases beyond  $\hat{F}$ .  $z^*$  increases faster with firing costs in the dual equilibrium than  $z^c$  does in the classic

case.

On the temporary side of the labor market, an increase in firing costs has two opposite effects. On one hand, the higher the firing cost, the lower the profitability threshold  $z^f$ . This tends to harm temporary jobs' productivity. On the other hand, higher firing costs encourage the substitution towards temporary contracts on behalf of permanent contracts in the neighborhood of  $z^*$ . This makes temporary contracts more productive on average. The right-hand-side graph of figure 6 shows that those two effects cancel out over the chosen interval of firing costs.

The overall behavior of welfare results from the intertwined productivity-related and employment-related mechanisms. For low values of firing costs, the lower unemployment rate in the classic labor market overwhelms the productivity gains the substitution towards temporary contracts provides. As firing costs come closer to their baseline value, the converse becomes true. Temporary contracts constitute an efficient cure against the fall of productivity of now immutable permanent contracts.

Referring to right panel of Figure 5 and the left panel of Figure 6, a benevolent social planner willing to reduce unemployment and improving welfare faces a dilemma if it wants to keep the dual structure of the labor market at minimum political cost. Indeed, starting from the baseline firing costs, there is no free lunch for small changes in firing costs. In the neighborhood of  $F$ , increasing firing costs decreases unemployment but reduces welfare, while decreasing firing costs harms unemployment but enlarges welfare. A both welfare and employment improving reform needs a large cut in firing costs. Once enforced, firing costs end up close to  $\hat{F}$ . In this region, a classic labor market is preferable to a dual one in terms of welfare and employment. Thus, large welfare and employment gains come at high political cost since the optimal reform consists in strongly cutting firing costs and banning temporary contracts. Interestingly, the shape of the latter reform roughly resembles a unique-contract reform as dealt with in the literature.

As previously mentioned, there is a diversity of views about the value of the frequency of shock occurrence  $\lambda$ . In Appendix A, I show the same graphs as above for calibrations with significantly higher and lower  $\lambda$ . Overall, a higher frequency of productivity shocks magnifies the variability of steady-state values with respect to firing costs. This makes sense because the payment of the firing cost is more likely. The differences between the dual and classic labor markets enlarge and curves are emphasized. In quantitative terms, changes in firing cost with a similar size have a stronger impact in terms of employment and welfare. Overall, the risk in the economy significantly impacts the results of reforms.

### 3.3 Dynamics

In this section, we study dynamics after a change in firing costs as well as transitions between dual and classic labor markets. The mathematical aspect of these dynamics is discussed in Appendix C. I also study the consequences of regulatory uncertainty.

**Post-reform adjustment speed** The speed of adjustment of an economy after a reform constitutes an important factor from the policy maker's point of view. The labor market tightness and the thresholds pinpointing job destruction and job creation are forward-looking variables. Therefore, they jump immediately to the new steady-state values after an unexpected change in

the parameters<sup>20</sup>. This result is valid for both the classic and dual Mortensen-Pissarides model. Figure 7 displays the time at which 99 % of the adjustment is completed.

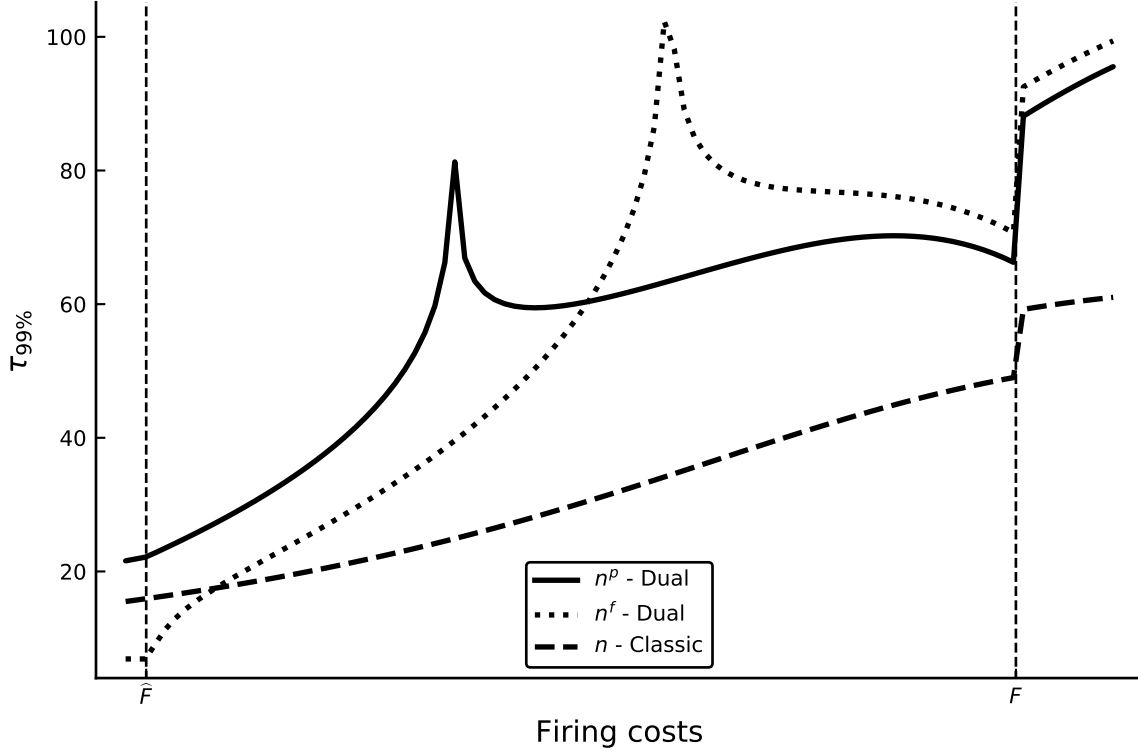


Figure 7:  $\tau_{99\%}$  of different reforms with respect to the post-reform firing costs in months

The pre-reform firing costs are those of the baseline French economy. The y-axis represents the time by which 99% of the adjustment is done when considering steady-state pre-reform and post-reform values.

Starting from the baseline value, an increase in firing costs reduces the transition probabilities on the permanent side of the labor market. Since permanent matches represent the bulk of employees in the baseline calibration, the new value of unemployment depends on permanent employment to reach its new steady-state value. The sluggish motion of permanent employment slows down the adjustment of unemployment. This reflects on the adjustment of temporary employment, which essentially relies on job creation. A 1-per-cent increase in firing costs take 90 months, which represents 7.5 years.

Conversely, a cut in firing costs increases flows on the permanent side of the labor market. This tends to speed up the adjustment. Why does Figure 7 display non-monotonicities ? In the right-hand side of the  $(F, \hat{F})$  interval, as post-reform firing cost decreases, adjustment time of temporary employment increases to a peak. This spike corresponds to the highest level of temporary employment, while the probability of temporary job creation decreased compared to its baseline value. The acceleration of flows on the permanent side of the labor market is not strong enough to exceed the negative impact of the lowered temporary job creation probability. The spike in permanent employment adjustment time stems from the impact response of permanent employment. A cut in firing costs immediately increases the permanent job destruction margin  $z^p$ . Consequently, the least productive continuing permanent matches

<sup>20</sup>This result is extensively developed in Pissarides (2000) [32] page 59-63.



now deliver a negative surplus and split. Permanent employment decreases and unemployment increases on impact. As long as the post-reform steady-state permanent employment is lower than the baseline value, the impact reaction goes in the right direction. This is the case on the right-hand of the  $(F, \hat{F})$  interval. In contrast, for sufficiently low post-reform firing costs, the new steady-state permanent employment is higher than its baseline values of firing costs, while permanent employment slumps on impact. This initial dive in permanent employment can be large enough to exceed the speed gains stemming from the increased transition probabilities on the permanent side of the labor market and a spike appears. Note that this spike is not present in the classic model: the larger job creation and job destruction flows prevent the latter effect from prevailing. These bigger flows also account for the smaller adjustment durations of the classic labor market after a reform

In the neighborhood of  $\hat{F}$ , the flows in and out of permanent employment are large enough to quickly revert the economic schemes pointed out above. The adjustment is over by roughly 2 years.

Overall, the policy maker faces a high uncertainty with respect to adjustment time when it implements a reform. For example, an approximate knowledge of the probability of shock occurrence dramatically impacts the position of the two spikes described above as figures 15 and 18 show. In the following paragraph, we describe the post-reform dynamics for two specific cases.

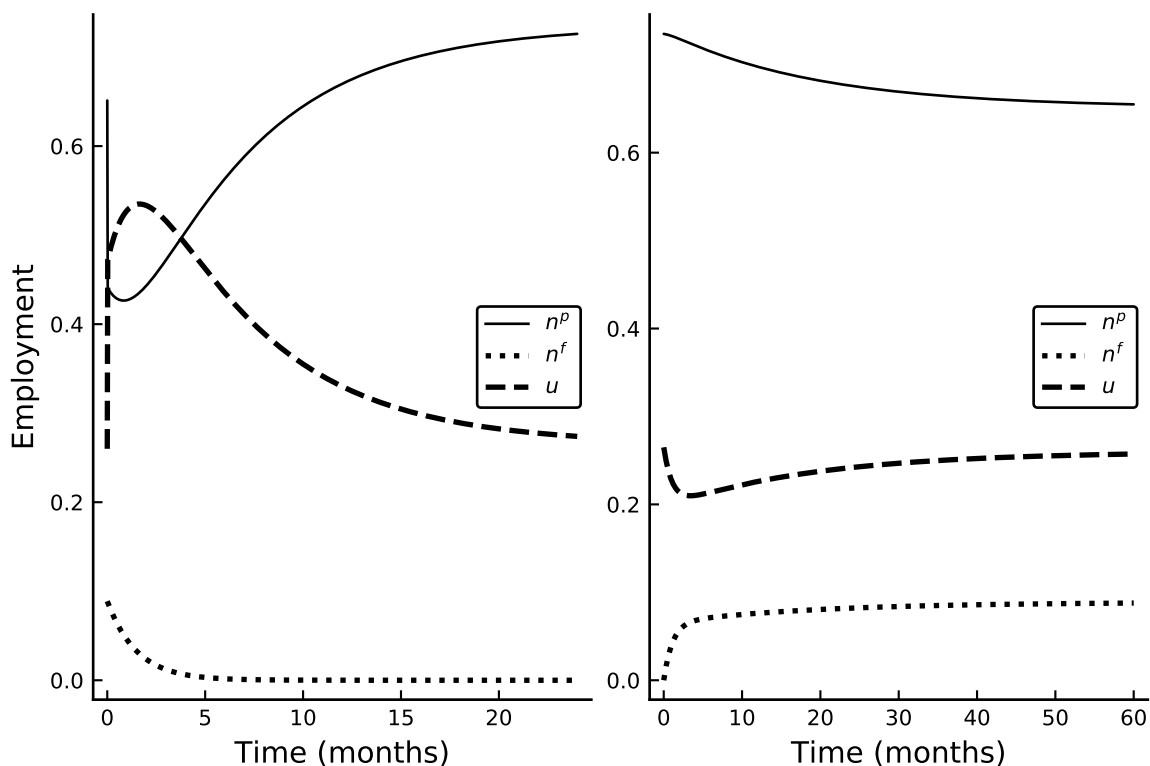


Figure 8: Post-reform dynamics

On the left, transitions of employments and unemployment following a unique-contract reform. On the right, transitions of employments and unemployment following the introduction of temporary contract

**Post-reform dynamics** Left-hand side of figure 8 shows the transition from the baseline dual labor market to a classic labor market with half of the baseline firing costs. This experiment corresponds to the implementation of the optimal unique-contract reform as delineated above. The labor market fully adjusts by roughly 24 months. It reaches a new equilibrium with higher permanent employment, no temporary employment and a roughly similar unemployment. Temporary employment vanishes by 10 months. Unemployment first increases for two reasons. Firstly, the aggregate job-finding rate drops ; some of the unemployed who initially found a temporary job now remain unemployed. Secondly, the drop in firing costs pushes up the permanent job destruction margin  $z^p$ , which causes the split of the least productive permanent matches. Importantly, the impact reaction of permanent employment is at odds with the final steady-state increase in the latter. The policy maker has to deal with a magnified unemployment rate as well as a depleted permanent employment on the first year of the transition. The political cost of such a transition is prominent.

Right-hand side figure 8 represents the transition from a classic to a dual labor market. The adjustment is much slower and takes around 5 years to be 99 % completed. Temporary employment quickly weighs in job creation as the main force, but the temporary job creation flows remain small because of the initial reduction in unemployment. The thin job destruction flow from permanent employment is the only force that may drive up unemployment and the substitution of permanent employment towards temporary employment ends up being slow. These sluggish dynamics after the introduction of temporary employment might bring an explanation to the expansion of temporary employment in the Western economies over the last decades. In the 70s, temporary contracts were introduced with a restricted scope and unemployment was low. Progressively, the legal constraints on the employability of temporary contracts loosened. In our model, an equivalent situation would embed an exogenous probability to accept or refuse a temporary match<sup>21</sup>. Initially, temporary matches are introduced, but most of them are refused. Imagine that the acceptance rate of temporary contracts is gradually extended. Considering the 5-year convergence duration as a trustworthy figure, temporary employment would take years to reach its equilibrium value if it had to adjust to each of the reforms on the acceptance rate.

**Regulatory uncertainty** If studying one-and-for-all in-depth reforms is relevant from a policy point of view, it remains a purely theoretical view. According to Fontaine and Malherbet (2016) [18], reforms are actually frequent and often marginal in Western and Southern Europe. Between 2005 and 2013, they count 17 employment protection legislation reforms in France, 49 in Italy, 38 in Spain, 23 in Greece and 17 in Portugal. Thus, a natural question to address concerns the impact of regulatory uncertainty on the labor market equilibrium.

Let me assume that firing costs undergo i.i.d shocks with probability  $\epsilon$  per unit of time. Firing costs follow a uniform distribution over  $\{F_1, F_2\}$  such that  $F_1 < F_2$ . I describe the computational details of the model in Appendix D. For the sake of comparability with respect to the case without regulatory uncertainty, I consider the case where the average firing costs equals the baseline firing cost in the calibration above. I set  $\epsilon$  to match the French and Portuguese score of 17 reforms in 9 years, which seems to be a lower bound in Western and Southern

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<sup>21</sup>Cahuc (2002) [13], Sala and Silva (2009) [37], Bentolila et al (2012) [8] and Sala et al (2012) [38] made this assumption.

European countries. Consequently, our results will understate the actual impact of regulatory uncertainty in the mentioned countries. Numerically speaking, I leave unchanged all the other parameters of the previous calibration. The resulting equilibrium is described in detail in Appendix D. Interestingly, a steady-state no longer exists: the system is now a Markov Jump Non-Linear System<sup>22</sup> Figure 9 shows the paths of temporary and permanent employment for a given realization of policy shocks.

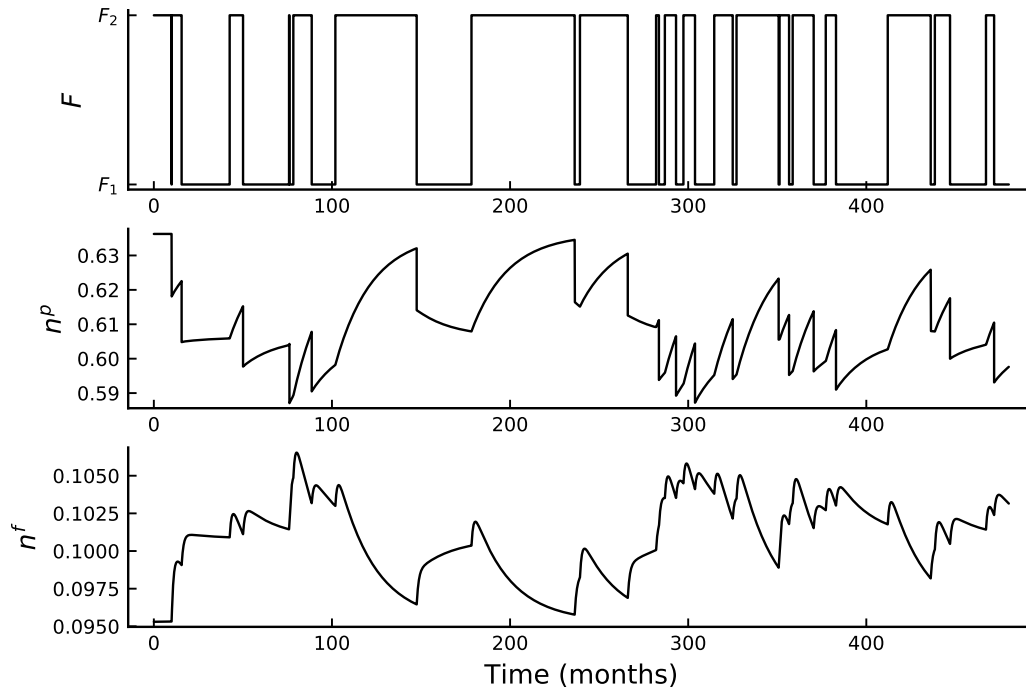


Figure 9: Employment dynamics with a given path of policy shocks

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<sup>22</sup>Costa et al (2012) [16] is an approachable introduction to the linear case.

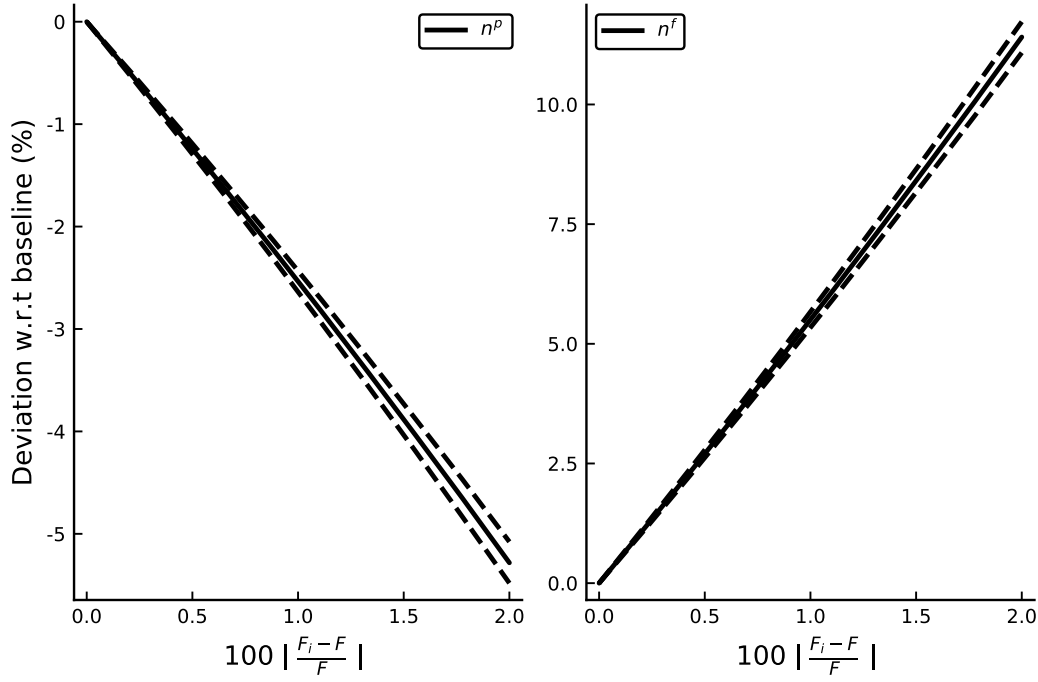


Figure 10: Extend of regulatory uncertainty and employment

For different values of  $|\frac{F_1 - F}{F}| = |\frac{F_2 - F}{F}|$ , I simulate 10,000 paths of policy shocks with a duration of 5,000 months. The bootstrap average and 90-per-cent confidence interval respectively appear as plain and dashed lines

Intuitively, uncertainty in firing costs should discourage hires through permanent contracts. When  $F = F_2$ , matches with a productivity just over the job destruction cap split if policy shock leads to  $F = F_1$ . When  $F = F_1$ , matches fear a shock magnifying firing costs, which encourages substitution towards temporary contracts at the hiring step. To measure the numerical extent of regulatory uncertainty, I consider different values for the range of firing costs between states 1 and 2. For each of them, I simulate 10,000 paths of policy shocks over 5,000 months randomly starting from one of the steady-states with  $F = F_1$  or  $F = F_2$ . Figure 10 shows the resulting average temporary and permanent employment along with their bootstrap 90-per-cent confidence interval represented with dashes. Policy uncertainty significantly fuels substitution towards temporary employment on behalf of permanent employment. 2-per-cent fluctuations of firing costs around the baseline value lead to a 5-per-cent decrease of average permanent employment, while temporary employment increases by more than 10 %. Consequently, unemployment, which is not displayed above, rises by nearly 10 %.

## 4 Discussion

In this section, I relax some assumptions ; the duration of temporary contracts is endogenously chosen to maximize the joint surplus when a worker-firm pair forms and conversions into permanent contracts become possible when temporary contracts end. I demonstrate that the productivity of the match cannot properly fit the actual distribution of durations of temporary contracts for theoretical reasons. Overall, the aim of this section is to determine the missing

ingredients of a class of random-search models to match data.

In this framework, as we shall demonstrate later, the possibility to convert expiring temporary contracts into permanent ones makes permanent hires irrelevant. To this extent, I introduce a fixed hiring and transformation cost  $c$ , which corresponds to the administrative cost of hiring or transforming an expired fixed-term contract into an open-ended one. Consequently, while the surplus of continuing open-ended contracts remains unchanged, the surplus of a new permanent contract with productivity  $z$  becomes

$$S_0^p(z) = S^p(z) - F - c$$

The surplus associated with a job creation through a  $z$ -productivity temporary contract denoted as  $S_0^f(z)$  now includes the choice of the instantaneous expiration probability  $\delta$  and verifies

$$S_0^f(z) = \sup_{\delta \geq 0} S^f(z, \delta) - c$$

where  $S^f(z, \delta)$  denotes the joint surplus of a continuing temporary contract with productivity  $z$  and expiration probability  $\delta$ .

Firm's and worker's surpluses of a temporary contract become

$$\begin{aligned} rJ^f(z, \delta) &= z - w^f(z) + \lambda \int \left( J^f(z', \delta) - J^f(z, \delta) \right) dG(z') + \delta \left( \max[J_0^p(z), V] - J^f(z, \delta) \right) \\ rW^f(z, \delta) &= w^f(z) + \lambda \int \left( W^f(z', \delta) - W^f(z, \delta) \right) dG(z') + \delta \left( \max[W_0^p(z), U] - W^f(z, \delta) \right) \end{aligned}$$

The Nash-bargaining assumption and the definition of temporary contracts' surpluses entail

$$rS^f(z, \delta) = z - rU + \lambda \int \left( S^f(z', \delta) - S^f(z, \delta) \right) dG(z') + \delta \left( \max[S_0^p(z), 0] - S^f(z, \delta) \right)$$

Integrating the latter equation over  $z$  delivers the expression of  $\int S^f(z, \delta) dG(z)$ , which leads to the following expression of the temporary contracts' surplus.

$$(r + \lambda + \delta)S^f(z, \delta) = z - rU + \delta S_0^p(z)^+ + \frac{\lambda}{r + \delta} \left( Ez - rU + \delta \int S_0^p(z')^+ dG(z') \right) \quad (4.1)$$

Using the profit-sharing rules stemming from the Nash-bargaining assumption, the present discounted value of a vacancy  $V$  verifies

$$rV = -\gamma + (1 - \eta)q(\theta) \int \max[S_0^p(z), S_0^f(z), 0] dG(z)$$

The free entry condition  $V = 0$  along with the previous equation delivers the corresponding job creation equation.

The following proposition characterizes the optimal duration of temporary contracts.

**Proposition 5.** *Let  $z$  be in the support of  $G$ .*

- *If  $Ez/r < \alpha \equiv U + \int S_0^p(z')^+ dG(z')$ , the optimal expiration rate  $\delta^*$  verifies*

$$\delta^*(z) = \begin{cases} +\infty & \text{if } x(z) \leq 0 \\ \lambda \frac{1+\sqrt{1+x(z)}}{x(z)} - r & \text{if } 0 < x(z) < \frac{\lambda}{r} \left(2 + \frac{\lambda}{r}\right) \\ 0 & \text{otherwise} \end{cases}$$

$$\text{where } x(z) = \frac{z - r\alpha - (r + \lambda) \left( S_0^p(z)^+ - \int S_0^p(z')^+ dG(z') \right)}{r\alpha - Ez}$$

- *Otherwise,  $\delta^*(z) \in \{0, +\infty\}$*

*Proof.* See Appendix B □

$Ez/r - \alpha$  is the expected surplus of a temporary contract with zero probability of destruction after a productivity shock. The gain is the present discounted value of expected production  $Ez/r$ , while the losses are the present discounted values of a return into unemployment  $U$  and the expected value of a conversion into a permanent contract  $\int S_0^p(z')^+ dG(z')$ . These two events do not occur under a zero probability of temporary job destruction, which explains why they appear as losses. Conversely, a productivity shock has a probability one to hit such an eternal temporary contract, which explains the role of the *expected* surplus as a hiring criterion. If  $Ez/r \geq \alpha$ , a no-end temporary contract is expected to be profitable considering the impact of productivity shocks throughout its existence. Therefore, if the immediate surplus of a new match - namely the surplus before any productivity shock hits - is not too low, a zero probability of destruction is optimal. Otherwise, an immediate destruction is preferable and an infinite probability of destruction is chosen. Conversely, if  $Ez/r < \alpha$ , a no-end temporary contract has an expected negative surplus after a productivity shock. This encourages firms to shorten the stipulated durations of temporary contracts in order to avoid productivity shocks. Intuitively, if the productivity of the match is neither too low nor too high to opt for an infinite or zero probability of job destruction, there is room for optimization in terms of durations. The contract must be long enough to benefit from the current level of productivity but short enough to avoid losses associated with a productivity shock, which is expected to be detrimental. In that respect, an increase in the probability of shock occurrence pushes up the destruction probability for a given  $z$ . Moreover, the probability of destruction decreases with the productivity of the firm-worker pair. Importantly, proposition 5 and its ramifications are still valid if the match chooses the duration instead of the destruction probability of a temporary contract as is the case in Cahuc et al. (2016).

Figure 6 of Benghalem et al. (2016) [7] shows that the distribution of durations for fixed-term contracts is not limited to the set  $\{0, +\infty\} \subset \mathbb{R}$ . Thus, a first assumption is necessary to fit the data.

**Assumption 1.**  $Ez/r < \alpha$

The following proposition states the optimal choice between temporary and permanent contracts in function of the productivity of the match.

**Proposition 6.** *Under Assumption 1,*

- If  $c = 0$ , job creation only occurs through temporary contracts
- If  $0 < c < \frac{\lambda}{r} \left( \alpha - \frac{Ez}{r} \right)$ , job creation is dual if and only if  $x(z^c) < x^* \equiv (2 + \beta) \beta$ , where  $\beta = \sqrt{\frac{c\lambda}{r\alpha - Ez}}$ . Otherwise, job creation only occurs through temporary contracts.

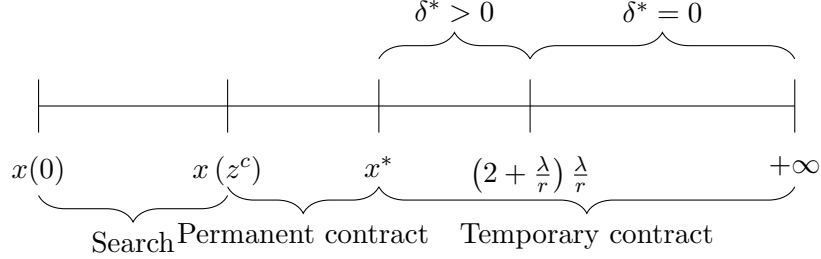


Figure 11: Dual job creation when  $0 < c < \frac{\lambda}{r} \left( \alpha - \frac{Ez}{r} \right)$

- If  $c \geq \frac{\lambda}{r} \left( \alpha - \frac{Ez}{r} \right)$ , job creation is dual if and only if  $x(z^c) < x^* \equiv \frac{\lambda}{r} + \frac{c(r+\lambda)}{r\alpha - Ez}$ . Otherwise, job creation only occurs through temporary contracts. Temporary contracts have a zero probability of destruction.

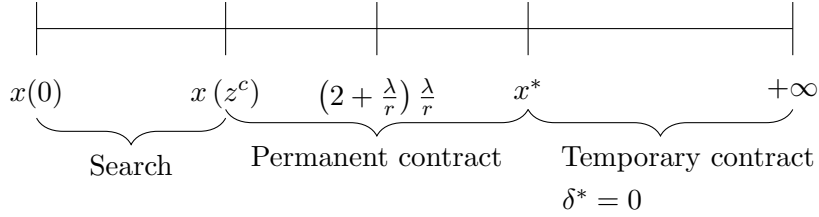


Figure 12: Dual job creation when  $c \geq \frac{\lambda}{r} \left( \alpha - \frac{Ez}{r} \right)$

*Proof.* See Appendix B □

When the hiring cost is null, a new permanent contract is equivalent to a new temporary contract with zero duration. Consequently, temporary contracts at least weakly dominate permanent contracts at the hiring stage. When the hiring cost is positive the job creation scheme is reversed with respect to the previous model. The endogenous choice of temporary contracts' job-destruction probability is not responsible for this plot twist. The possibility to convert an expiring temporary contract into a permanent one accounts for this inversion. Indeed, on one hand, the flexibility provided by temporary contracts has expanded, as it now enables long-term relationships through both long temporary contracts and conversion into permanent contracts. Consequently, avoiding the supplementary contracting cost constitutes the only motivation to directly hire through permanent contracts instead of converting a temporary contract. The firing cost is implicitly taken into account in the program of temporary matches because of the possibility of conversion to a permanent contract at expiry. Consequently, temporary contracts cease to constitute waiting devices in opposition to productive permanent contracts. The possibility to convert temporary contracts into permanent ones makes the former sufficiently

more flexible than the latter to cope with both high and low productivities at the hiring stage. The only pitfall of temporary contracts with a finite duration is a superior administrative costs. In case of conversion, the contracting cost is paid twice. Firms and workers no longer strike a balance between productivity and flexibility as in the previous model. The sole compromise takes place between flexibility and hiring costs.

When productivity is moderate, the match can opt for a short temporary contract. On one hand, this provides a flexibility gain: if an adverse productivity shock occurs, the contract will end up quickly. The shorter the contract, the thinner this advantage. On the other hand, if a permanent contract is beneficial, a temporary match which is converted when it runs out pays twice the contracting cost. The shorter the contract, the heavier this drawback. Consequently, permanent contracts tend to be preferred to very short temporary contracts whenever the former are beneficial. Conversely, when productivity is high, the hiring costs become small compared to the flexibility gains a longer temporary contract provides. As productivity converges towards infinity, a no-term temporary contract is even better than a permanent contract. Somehow, the rigid permanent contract is more flexible than a no-end temporary contract, which constitutes a better device to lock a firm-worker pair with a high productivity as there is a zero probability of separation.

The dichotomy with respect to the value of the contracting cost arises from these phenomena. When the hiring cost is high, the scope for short temporary contracts is reduced as permanent contracts turn out to be more attractive. The limit point is the expected difference in surplus a finite temporary contract provides when a productivity shock strikes compared to a no-end temporary contract, namely  $\lambda(\alpha - Ez/r)/r$ . This situation is not realistic : the support of the distribution of durations is not limited to  $\{0\}$ . Therefore, a high contracting cost is incompatible with a proper fit of the data, hence the following additional assumption.

**Assumption 2.**  $0 < c < \frac{\lambda}{r}(\alpha - \frac{Ez}{r})$

The necessary theoretical foundations are elaborated enough to demonstrate the inadequacy of such a model to match data. Under assumptions 1 and 2, if job creation is dual, the resulting equilibrium is such that the shortest temporary contracts are the most likely to be transformed, which is at odds with the data. The only way to remedy this problem is to consider equilibria with job creation through temporary contracts exclusively. Permanent job creation solely occurs through conversion of temporary contracts. This is not relevant considering the ridiculous empirical probabilities of the latter event. Interestingly, notice that introducing heterogeneous firms in terms of shock arrival rate  $\lambda$  does not change these findings. To this extent, the model of Cahuc et al. (2016)[12] critically relies on the hypothesis that new jobs have the maximum productivity on the support of a uniform distribution. In this manner, a one-to-one link can be established between the distribution of  $\lambda$  and the distribution of temporary matches' durations. Dropping this assumption leads to the model presented in this section.

Why does the model fail ? The heterogeneity of firms or workers does not constitute a sufficient answer. Indeed, heterogeneity with respect to the shock arrival rate, firms' and workers' productivities, bargaining power or utility while unemployed does not impact the fact that the shortest temporary contracts are the most likely to be converted into permanent contracts given the firm-worker set of characteristics. On-the-job search does not help either. I leave for future



research the possible cures to this issue.

## 5 Conclusion

In this paper, I have built a simple matching model with both fixed-term and permanent contracts. The model provides a theoretical rationale to explain the contractual choice at the hiring step: temporary contracts act as stopgaps, offering both production and a possibility to return quickly on the labor market to fall onto a high-productivity match. In terms of policy, the removal of temporary contracts and a strong cut in firing costs leads to a gain in both welfare and employment within 18 months. Frequent marginal changes in policy act pushes up unemployment and temporary employment and weakens permanent employment.

The simplicity of the model also allows the review of assumptions considered as canonical since Mortensen and Pissarides (1994) [27]. For example, assuming that new jobs begin at a fixed and high productivity is not innocuous when enforced in a random-search model with a dual labor market. The possibility to convert expiring temporary contracts into permanent ones and to optimize the duration of the former leads to equilibria where the shortest temporary contracts are the most frequently converted. The introduction of heterogeneity does not solve this problem either. This sheds light on the abilities of random matching models to explain thoroughly the choice between temporary and permanent contracts and, thus, restricts the set of possible actions for future research.

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## A Robustness Checks

### A.1 $\lambda = 1$

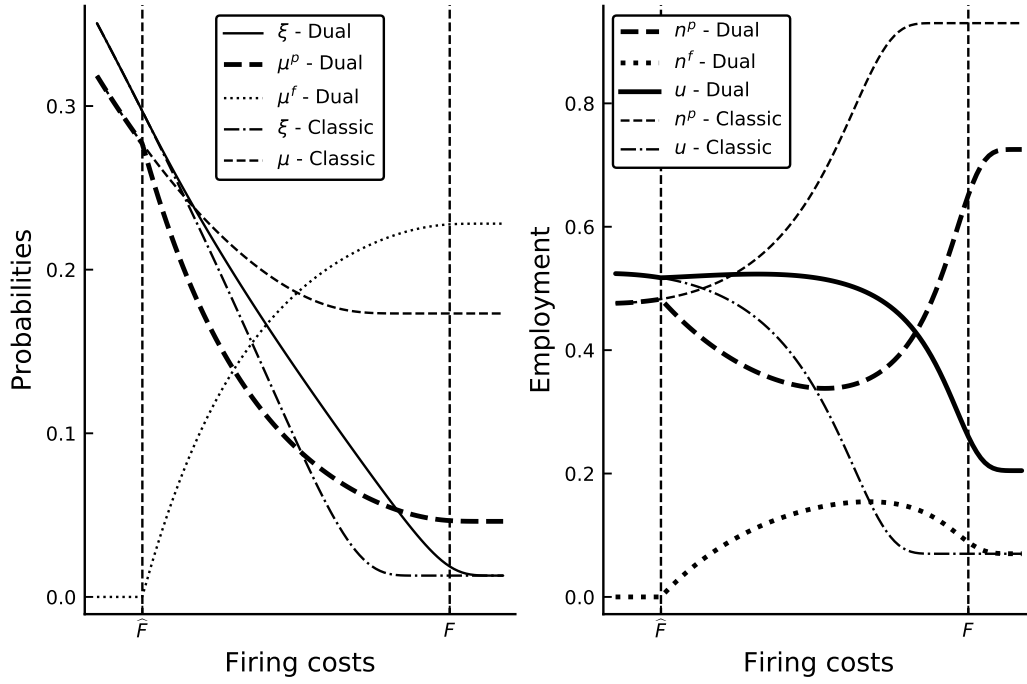


Figure 13: Steady-state employment values and firing costs

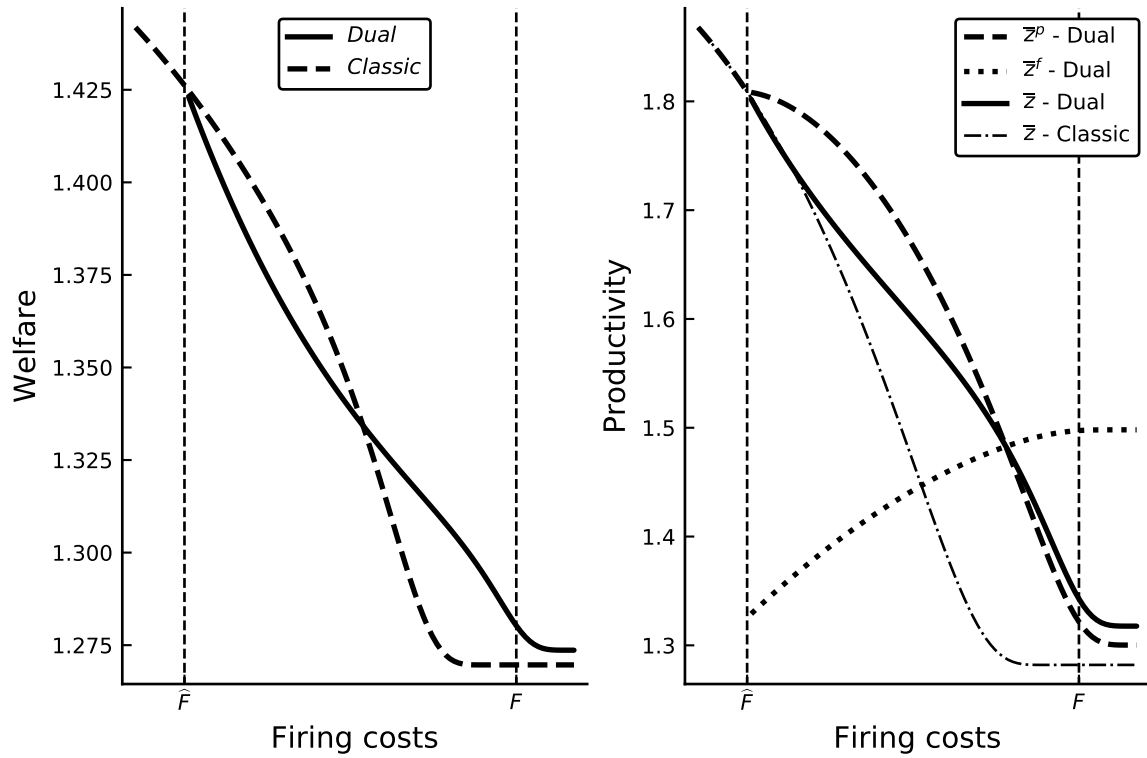


Figure 14: Social welfare and firing costs

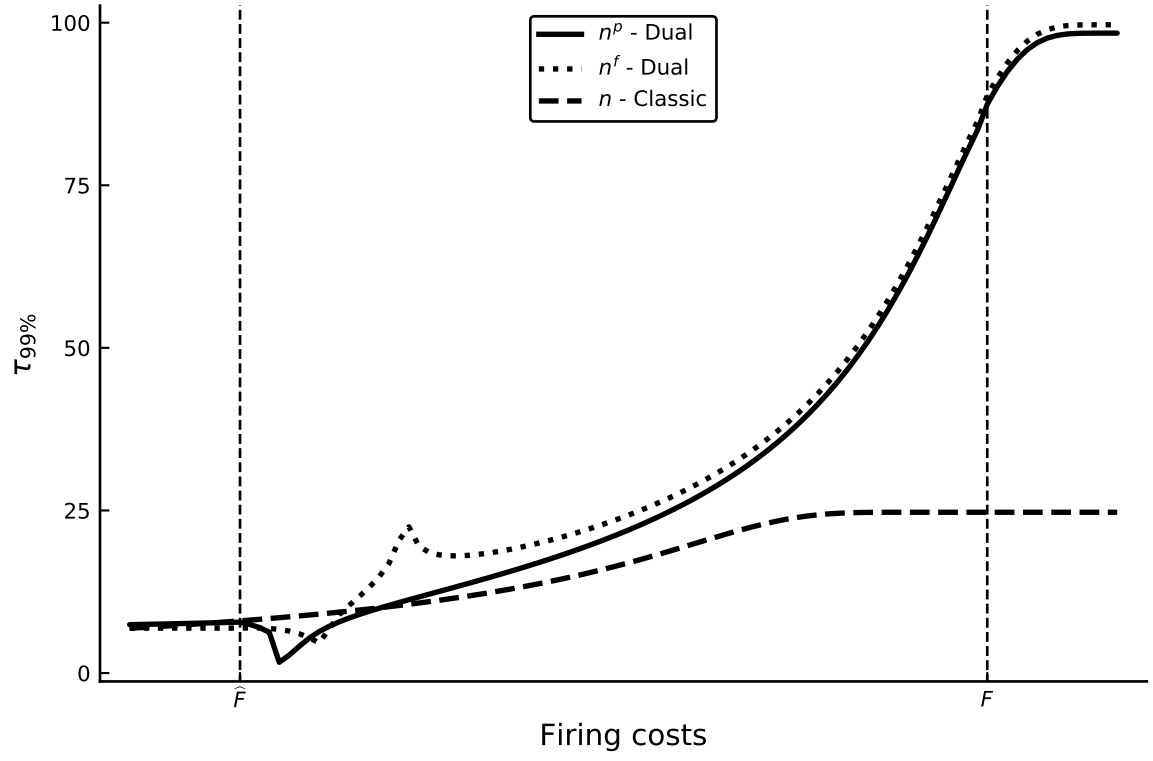


Figure 15:  $\tau_{99\%}$  of different reforms with respect to the post-reform firing costs in months

## A.2 $\lambda = 1/36$

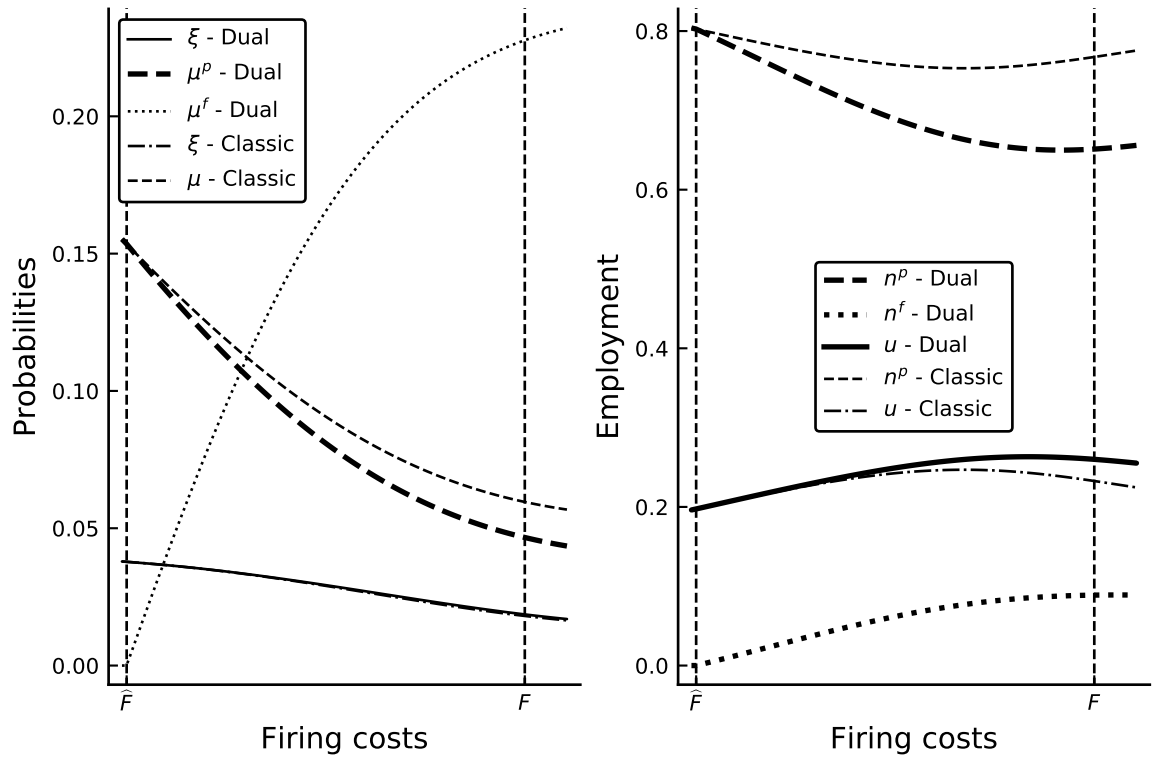


Figure 16: Steady-state employment values and firing costs

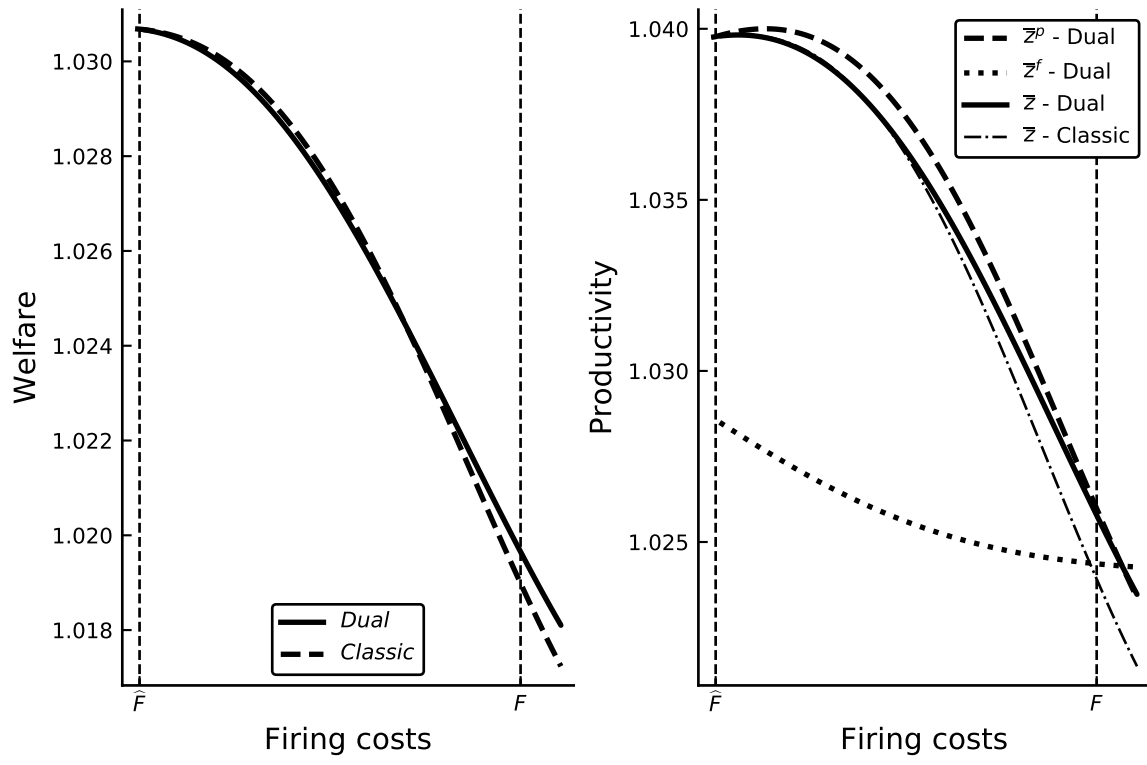


Figure 17: Social welfare and firing costs

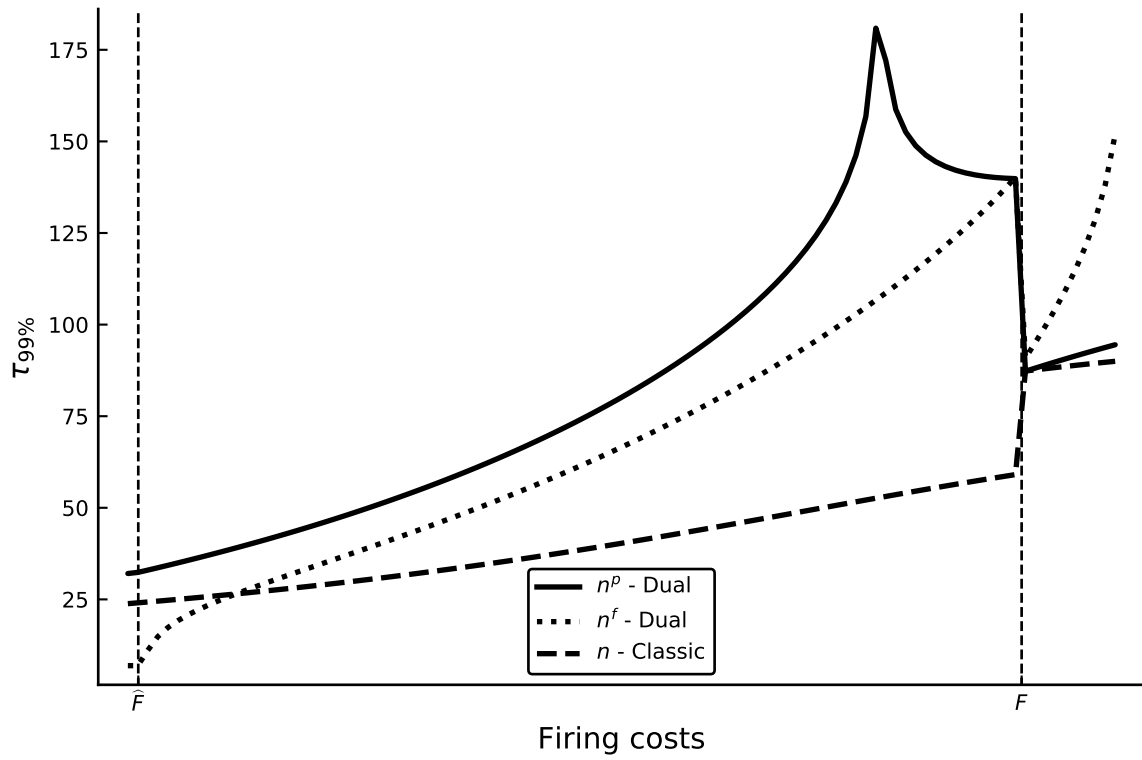


Figure 18:  $\tau_{99\%}$  of different reforms with respect to the post-reform firing costs in months

### A.3 $s/\xi = 0$

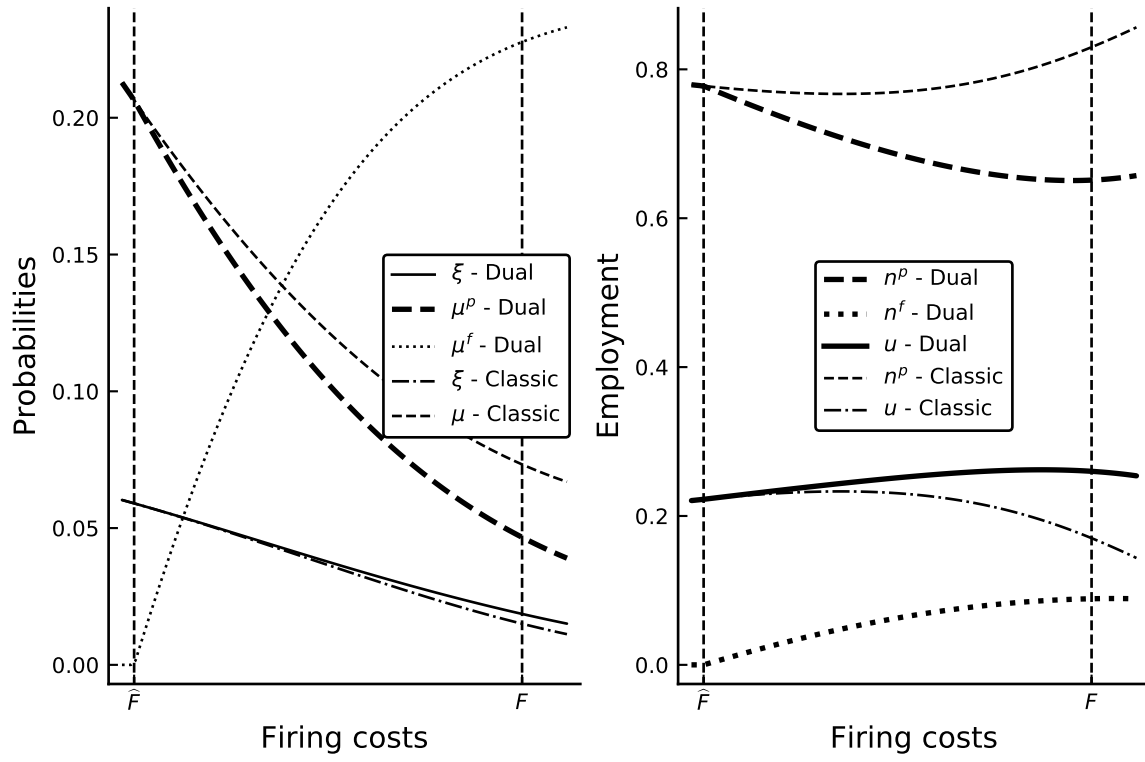


Figure 19: Steady-state employment values and firing costs

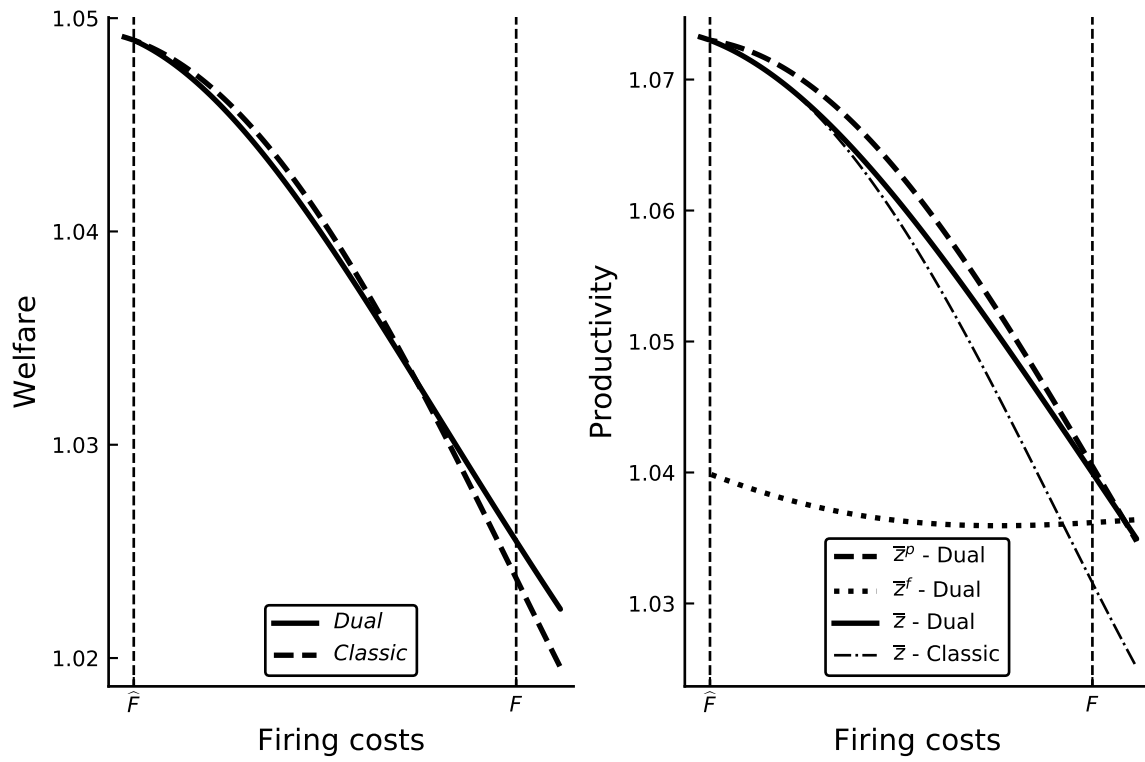


Figure 20: Social welfare and firing costs



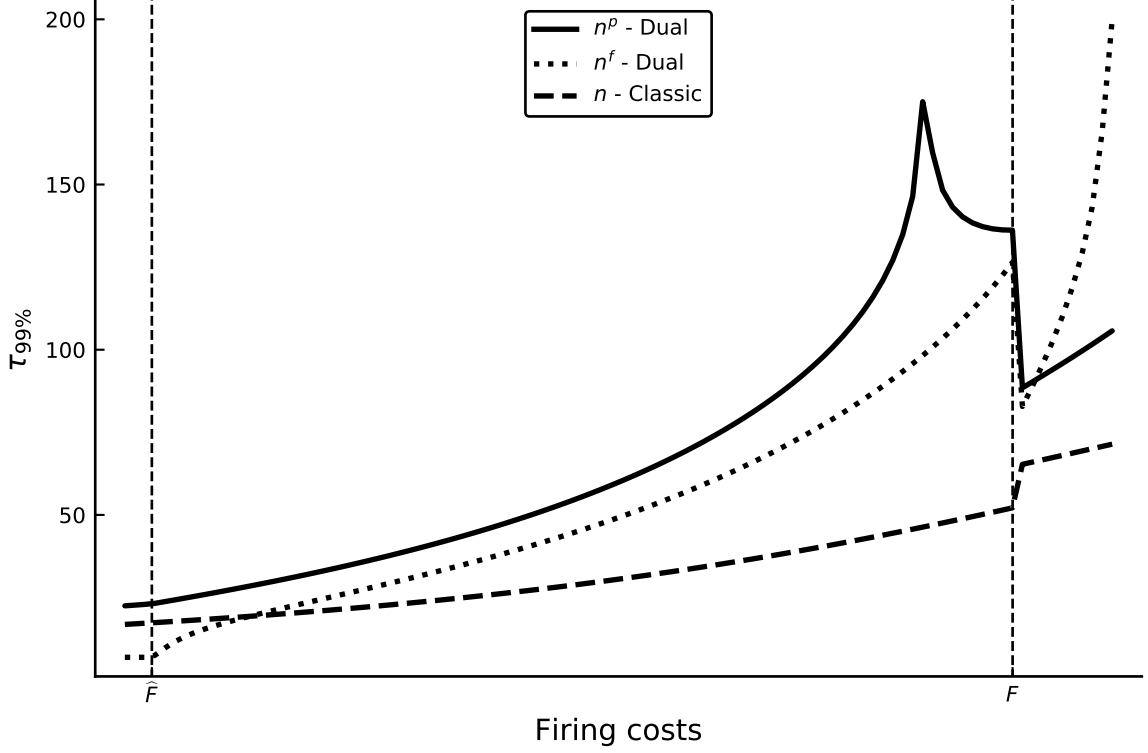


Figure 21:  $\tau_{99\%}$  of different reforms with respect to the post-reform firing costs in months

## B Proofs

**Proposition 1** I denote  $\rho^p = 1/(r + s + \lambda)$  and  $\rho^f = 1/(r + \delta + \lambda)$ . As mentioned above,  $\rho^p > \rho^f$ .

- Assume that  $z^* > z^f$ . (2.31) implies that  $\rho^p z^* = (\rho^p - \rho^f) z^* + \rho^f z^* = \rho^p z^c + \rho^f (z^* - z^f)$ . Since  $z^* - z^f > 0$ , the latter equality implies  $z^* > z^c$ .
- Assume that  $z^* > z^c$ . Again, jointly with algebraic manipulations, (2.31) implies that  $\rho^f z^c = -(\rho^p - \rho^f) z^c + (\rho^p - \rho^f) z^* + \rho^f z^f > -(\rho^p - \rho^f) z^c + (\rho^p - \rho^f) z^c + \rho^f z^f > \rho^f z^f$ , which entails that  $z^c > z^f$ .
- Assume that  $z^c > z^f$ . Algebraic manipulations and (2.31) imply that  $(\rho^p - \rho^f) z^* = \rho^p (z^c - z^f) + (\rho^p - \rho^f) z^f > (\rho^p - \rho^f) z^f$ , which implies  $z^* > z^f$ .  $\square$

**Proposition 2** Referring to (2.33),

- If permanent workers are the only ones hired, then  $\max[z^f, z^*] \leq z^f$ , implying that  $z^* \leq z^f$ . Referring to proposition 2, the latter inequality entails  $z^f \leq z^c$ . As a result,  $z^* \leq z^f \leq z^c$ .
- If job creation is dual, then

$$\begin{cases} 0 < \max[z^f, z^*] \\ z^f < z^* \end{cases}$$

Using proposition 2, the latter system of inequalities boils down to  $\max [0, z^f] < z^*$ .

For each case, the converse propositions are straightforward using (2.33).  $\square$

**Proposition 3**

- Considering the case where job creation is dual and there is endogenous destruction of permanent jobs, We differentiate the different equations.

$$\begin{aligned}\frac{r + \xi}{r + s + \lambda} dz^p &= \frac{\eta\gamma}{1 - \eta} d\theta - (r + s) dF \\ \frac{r + \delta}{r + \delta + \lambda} dz^f &= \frac{\eta\gamma}{1 - \eta} d\theta \\ dz^c &= dz^p + (r + s + \lambda) dF \\ \left( \frac{1}{r + s + \lambda} - \frac{1}{r + \delta + \lambda} \right) dz^* &= \frac{dz^c}{r + s + \lambda} - \frac{dz^f}{r + \delta + \lambda} \\ - \frac{\gamma q'}{(1 - \eta)q^2(\theta)} d\theta &= - (1 - G(z^*)) \left( \frac{1}{r + s + \lambda} - \frac{1}{r + \delta + \lambda} \right) dz^* - \left( 1 - G(z^f) \right) \frac{dz^f}{r + \delta + \lambda}\end{aligned}$$

Substituting the expression of  $dz^p$  into the definition of  $dz^c$ , we get

$$\frac{dz^c}{r + s + \lambda} = \frac{1}{r + \xi} \left( \frac{\eta\gamma}{1 - \eta} d\theta + \lambda G(z^p) dF \right)$$

In turn, this expression for  $dz^c$  can be substituted into the definition of  $dz^*$ .

$$\left( \frac{1}{r + s + \lambda} - \frac{1}{r + \delta + \lambda} \right) dz^* = \frac{1}{r + \xi} \left( \frac{\eta\gamma}{1 - \eta} d\theta + \lambda G(z^p) dF \right) - \frac{1}{r + \delta + \lambda} dz^f$$

Reintroducing the expression of  $dz^f$ , the differentiated job creation condition becomes,

$$- \frac{\gamma q'}{(1 - \eta)q^2(\theta)} d\theta = - \frac{1 - G(z^*)}{r + \xi} \left[ \frac{\eta\gamma}{1 - \eta} d\theta + \lambda G(z^p) dF \right] - \frac{G(z^*) - G(z^f)}{r + \delta} \frac{\eta\gamma}{1 - \eta} d\theta$$

As a result,

$$\frac{\partial \theta}{\partial F} = - \frac{\frac{1 - G(z^*)}{r + \xi} \lambda G(z^p)}{- \frac{\gamma q'}{(1 - \eta)q^2(\theta)} + \frac{\eta\gamma}{1 - \eta} \left[ \frac{1 - G(z^*)}{r + \xi} + \frac{(G(z^*) - G(z^f))}{r + \delta} \right]} < 0$$

This entails  $\frac{\partial z^f}{\partial F} < 0$  and  $\frac{\partial z^p}{\partial F} < 0$ .

In addition,

$$\frac{\partial z^c}{\partial F} \propto \frac{\eta\gamma}{1 - \eta} \frac{\partial \theta}{\partial F} + \lambda G(z^p)$$

$$\frac{\partial z^c}{\partial F} \propto -\frac{\gamma q'}{(1-\eta)q^2(\theta)} + \frac{G(z^*) - G(z^f)}{r+\delta} \frac{\eta\gamma}{1-\eta} > 0$$

Jointly with the fact that  $\frac{\partial z^f}{\partial F} < 0$ , We get that  $\frac{\partial z^*}{\partial F} > 0$ .

- When  $z^p \leq 0$ , we have that  $\frac{\partial \theta}{\partial F} = 0$  and, consequently,  $\frac{\partial z^f}{\partial F} = 0$  and  $\frac{\partial z^c}{\partial F} = 0$ . This leads to  $\frac{\partial z^*}{\partial F} = 0$ .
- Similar computations can be carried out for the other case, where job creation occurs through permanent contracts only. The only change lies in the job creation condition.

$$-\frac{\gamma q'}{(1-\eta)q^2(\theta)} d\theta = -\frac{1-G(z^c)}{r+s+\lambda} dz^c$$

Introducing the expression of  $dz^c$ , the differentiated job creation condition becomes,

$$-\frac{\gamma q'(\theta)}{(1-\eta)q^2(\theta)} d\theta = -\frac{1-G(z^c)}{r+\xi} \left( \frac{\eta\gamma}{1-\eta} d\theta + \lambda G(z^p) dF \right)$$

As a result,

$$\frac{\partial \theta}{\partial F} = -\frac{\frac{1-G(z^c)}{r+\xi} \lambda G(z^p)}{-\frac{\gamma q'}{(1-\eta)q^2(\theta)} + \frac{\eta\gamma}{1-\eta} \frac{1-G(z^c)}{r+\xi}} < 0$$

and

$$\begin{aligned} \frac{\partial z^c}{\partial F} &\propto \frac{\eta\gamma}{1-\eta} \frac{\partial \theta}{\partial F} + \lambda G(z^p) \\ \frac{\partial z^c}{\partial F} &\propto -\frac{\gamma q'}{(1-\eta)q^2(\theta)} > 0 \end{aligned}$$

$\frac{\partial z^p}{\partial F} < 0$  is still true.  $\square$

**Proposition 4** To prove this result, we will rely on the two corner equilibria that are implicitly present in proposition 2.

- In the first case, consider the case where  $z^f = z^c = z^* = \hat{z}$ , which is the knife-edge case associated with proposition 1. In this case, the equilibrium is summed up by equations (2.33) and (2.25), where  $\theta$  and  $z^f$  are respectively replaced by  $\hat{\theta}$  and  $\hat{z}$ . Taking into account the fact that  $z^f = z^c = z^*$  in the job creation equation, the two latter equations boil down to

$$\begin{aligned} \hat{z} &= \left( 1 + \frac{\lambda}{r+\delta} \right) \left( b + \frac{\eta\gamma\hat{\theta}}{1-\eta} \right) - \frac{\lambda}{r+\delta} E_z \\ \frac{\gamma}{(1-\eta)q(\hat{\theta})} &= \int_{\hat{z}}^{+\infty} (1-G(z)) dz \end{aligned}$$

Given the equilibrium values  $(\hat{\theta}, \hat{z})$ ,  $z^c = \hat{z}$  along with (2.23) and (2.28) entail that  $F_1$  verifies

$$\hat{z} - b - \lambda \hat{F} + \frac{\lambda}{r + s + \lambda} \int_{\hat{z} - (r+s+\lambda)F_1}^{+\infty} (1 - G(z)) dz = \frac{\eta \gamma \hat{\theta}}{1 - \eta}$$

Proposition 2 ensures that job creation only occurs through permanent contracts. Replacing  $\mu^f$  by zero in (2.37) demonstrates that  $n^f = 0$  at the equilibrium.

A marginal increase in firing costs immediately entails an increase in  $z^c$  while  $z^f$  diminishes according to proposition 3. The initial situation being  $z^f = z^c = \hat{z}$ , we now face  $z^f < \hat{z} < z^c$ , which is equivalent to  $z^f < z^*$  as Proposition 1 states. Job creation is now dual, as asserts Proposition 2. Conversely, a marginal decrease in firing costs implies a cut in  $z^*$  and  $z^c$  as well as an increase in  $z^f$ , which results in  $z^c < z^f$ . The latter is equivalent to  $z^* < z^c$  under proposition 2. The resulting equilibrium only involves permanent contracts.

- In the second case, consider the case where  $z^p = 0$ . There is no endogenous destruction of permanent matches. The equilibrium can be summed up by  $(\tilde{F}, \theta, z^p, z^c, z^f, z^*)$  verifying equations (2.23), (2.25), (2.28), (2.31) and (2.33) under the additional constraint that  $z^p = 0$ .

**Proposition 5** Let me denote  $y(z) = z - r\alpha - (r + \lambda) \left( S_0^p(z)^+ - \int S_0^p(z')^+ dG(z') \right)$ . Notice that  $y$  is increasing in  $z$ . Algebraic manipulations deliver another expression of  $S^f$  as follows

$$S^f(z, \delta) = \frac{1}{r + \delta + \lambda} \left( y(z) - \frac{\lambda}{r + \delta} (r\alpha - Ez) \right) + S_0^p(z)^+ \quad (\text{B.1})$$

Differentiating this expression with respect to  $\delta$  yields

$$\frac{\partial S^f}{\partial \delta} = -\frac{1}{(r + \delta + \lambda)^2} \left( y(z) - \frac{\lambda}{r + \delta} (r\alpha - Ez) \right) + \frac{1}{r + \delta + \lambda} \frac{\lambda}{(r + \delta)^2} (r\alpha - Ez)$$

Consequently, provided that  $r + \delta > 0$ ,

$$\frac{\partial S^f}{\partial \delta} \geq 0 \Leftrightarrow y(z)(r + \delta)^2 - 2\lambda(r\alpha - Ez)(r + \delta) - \lambda^2(r\alpha - Ez) \leq 0$$

Studying the variations of  $S^f$  boils down to assessing the sign of a second-degree polynomial in  $(r + \delta)$ .

- If  $r\alpha = Ez$ ,  $\text{sign} \left( \frac{\partial S^f}{\partial \delta} \right) = \text{sign}(-y(z))$ . In this case,  $\delta^*(z) \in \{0, +\infty\}$  if  $y(z) \neq 0$ . Otherwise, any non-negative  $\delta$  maximizes  $S^f$ .
- If  $r\alpha > Ez$ ,  $\text{sign} \left( \frac{\partial S^f}{\partial \delta} \right) = \text{sign}(P_z(r + \delta))$ , where  $P_z(X) = -x(z)X^2 + 2\lambda X + \lambda^2$ . Several subcases arise.
  - If  $x(z) \leq -1$ ,  $\partial S^f / \partial \delta \geq 0$  and  $\delta^*(z) = +\infty$
  - If  $-1 < x(z) < 0$ ,  $P_z$  has two negative roots and, thus, is positive on  $(0, +\infty)$ . Consequently,  $S^f$  is increasing in  $\delta$  over the latter interval and  $\delta^*(z) = +\infty$

- If  $x(z) = 0$ ,  $P_z$  is linear and positive over  $(0, +\infty)$  and  $\delta^*(z) = +\infty$ .
- If  $x(z) > 0$ ,  $P_z$  has one negative and one positive root, the latter verifying  $r + \delta_0 = \lambda \frac{1 + \sqrt{1 + x(z)}}{x(z)}$ . Either  $\delta_0 < 0$ , in which case  $S^f$  decreases over  $(0, \infty)$  and  $\delta^*(z) = 0$ , or  $\delta_0 \geq 0$ , in which case  $S^f$  attains a maximum at  $\delta^*(z) = \delta_0 \in (0, \infty)$ .

When do we have  $\delta_0 \geq 0$ ? The latter condition is equivalent to  $r(1 + x(z)) - \lambda\sqrt{1 + x(z)} - (r + \lambda) \leq 0$  which is - again - a second-degree polynomial in  $\sqrt{1 + x(z)}$  with roots  $-1$  and  $(\lambda + r)/r$ . Consequently,  $\delta_0 \geq 0$  if and only if  $-1 \leq \sqrt{1 + x(z)} \leq (\lambda + r)/r$ . Since  $x(z) > 0$ , this is true whenever  $x(z) \leq \frac{\lambda}{r} (2 + \frac{\lambda}{r})$ .

- If  $r\alpha < Ez$ ,  $\text{sign} \left( \frac{\partial S^f}{\partial \delta} \right) = -\text{sign} (P_z(r + \delta))$ . One may revisit the cases tackled above.
    - If  $x(z) \leq 0$ ,  $S^f$  is decreasing in  $\delta$  over  $(0, +\infty)$  and  $\delta^*(z) = 0$
    - If  $x(z) > 0$ ,  $P_z$  has one negative and one positive root, the latter verifying  $r + \delta_0 = \lambda \frac{1 + \sqrt{1 + x(z)}}{x(z)}$ . Either  $\delta_0 < 0$ , in which case  $S^f$  increases over  $(0, \infty)$  and  $\delta^*(z) = +\infty$ , or  $\delta_0 \geq 0$ , in which case  $S^f$  attains a minimum at  $\delta_0 \in (0, \infty)$  and  $\delta^*(z) \in \{0, +\infty\}$ .
- 

**Proposition 6** The choice between a temporary and a permanent contract at the hiring stage can be summed up in the sign of the function  $\Delta$  defined as  $\Delta(z) = S_0^f(z) - S_0^p(z)^+$ . The latter can be rewritten as

$$\Delta(z) = S^f(z, \delta^*(z)) - c - S_0^p(z)^+$$

Using (B.1), the previous equation becomes

$$\Delta(z) = \frac{r\alpha - Ez}{r + \delta^*(z) + \lambda} \left( x(z) - \frac{\lambda}{r + \delta^*(z)} \right) - c$$

Several cases arise when  $c > 0$

- If  $x(z) \leq 0$ ,  $\delta^*(z) = +\infty$  and  $\Delta(z) = -c < 0$ . Hiring only takes place through open-ended contracts under the constraint that  $z \geq z^c$ .
- If  $0 < x(z) < \frac{\lambda}{r} (2 + \frac{\lambda}{r})$ , using the definition of  $\delta^*$  spelled in proposition 5, one may rewrite  $\Delta(z)$  as

$$\Delta(z) = \frac{r\alpha - Ez}{\lambda} \left( \frac{x(z)}{1 + \sqrt{1 + x(z)}} \right)^2 - c$$

Algebraic manipulations entail that

$$\text{sign}(\Delta(z)) = \text{sign} \left( x(z) + 1 - \beta \sqrt{1 + x(z)} - (1 + \beta) \right)$$

where  $\beta = \sqrt{\frac{c\lambda}{r\alpha - Ez}}$

The right-hand side of the equation above is a second-degree polynomial in  $\sqrt{1 + x(z)}$ . Therefore, since  $x(z) > 0$ ,  $\Delta(z) \geq 0$  if and only if  $x(z) \geq (2 + \beta) \beta$ .

- If  $x(z) \geq \frac{\lambda}{r} (2 + \frac{\lambda}{r})$ ,  $\delta^*(z) = 0$  and  $\Delta(z) = \frac{r\alpha - Ez}{r + \lambda} (x(z) - \frac{\lambda}{r}) - c$ . Thus,  $\Delta(z) \geq 0$  if and only if  $x(z) \geq \frac{\lambda}{r} + \frac{c(r + \lambda)}{r\alpha - Ez}$ .

Notice that the latter condition is always fulfilled in this specific case if and only if  $\beta \leq \frac{\lambda}{r}$ .  
If  $\beta \leq \frac{\lambda}{r}$ ,

$$\begin{aligned} \frac{\lambda}{r} + \frac{c(r + \lambda)}{r\alpha - Ez} &= \frac{\lambda}{r} + \beta^2 \left(1 + \frac{r}{\lambda}\right) \\ &\leq \frac{\lambda}{r} + \beta^2 + \beta \\ &\leq \frac{\lambda}{r} + \left(\frac{\lambda}{r}\right)^2 + \frac{\lambda}{r} \\ &\leq \frac{\lambda}{r} \left(2 + \frac{\lambda}{r}\right) \end{aligned}$$

Conversely, if  $\frac{\lambda}{r} + \frac{c(r + \lambda)}{r\alpha - Ez} = \frac{\lambda}{r} + \beta^2 \left(1 + \frac{r}{\lambda}\right) \leq \frac{\lambda}{r} \left(2 + \frac{\lambda}{r}\right)$ , algebraic manipulations directly entail that  $\beta^2 \leq \left(\frac{\lambda}{r}\right)^2$ , which proves that  $\beta < \frac{\lambda}{r}$ .

We now have all the necessary information to circumscribe the equilibria with dual job creation.

- If  $\beta < \frac{\lambda}{r}$ ,  $\Delta(z) \geq 0$  if and only if  $x(z) \geq (2 + \beta) \beta$ . In other words, temporary contracts are hired whenever  $x(z) \geq (2 + \beta) \beta$ .  $x$  being increasing in  $z$ , there is room for job creation through permanent contracts if and only if  $x(z^c) < (2 + \beta) \beta$ .
- If  $\beta \geq \frac{\lambda}{r}$ ,  $\Delta(z) \geq 0$  if and only if  $x(z) \geq \frac{\lambda}{r} + \beta^2 \left(1 + \frac{r}{\lambda}\right)$ . Job creation is dual if and only if  $x(z^c) < \frac{\lambda}{r} + \beta^2 \left(1 + \frac{r}{\lambda}\right)$ . In this case, all temporary contracts have a zero probability of job destruction.

To end the proof, notice that  $\beta < \frac{\lambda}{r}$  is equivalent to  $c < \frac{\lambda}{r} \left(\alpha - \frac{Ez}{r}\right)$ . Moreover, when  $c = 0$ , revisiting each point above ensures that hiring through a temporary contract is weakly preferable to hiring through a permanent contract.  $\square$

## C Dynamics

In this section, I describe the dynamics of the equilibrium after a shock to firing costs.

### C.1 Dual-to-dual and classic-to-dual transitions

The comments made here also apply to the model that includes policy uncertainty.

Let us consider the model starting from the equilibrium defined by  $(\theta_0, z_0^p, z_0^c, z_0^f, z_0^*)$  with firing costs  $F_0$  at time  $t = 0^-$  and the associated permanent and temporary employment values  $n_0^p$  and  $n_0^f$ . The former tuple solely consists in forward looking variables. Therefore, assuming that firing costs jump from  $F_0$  to  $F$  on time  $t = 0$ , the equilibrium tuple defined in definition 1 immediately jumps to its new value  $(\theta, z^p, z^c, z^f, z^*)$ . As for employment, notice that if  $z^p > z_0^p$  - which occurs if  $F < F_0$  - permanent matches such that  $z_0^p < z < z^p$  split right away. Denoting  $\delta_0$  Dirac function in 0, permanent and temporary contracts, equations (2.34)-(2.35) are amended as follows

$$\begin{cases} \dot{n}_t^p &= -\xi n_t^p + \mu^p (1 - n_t^p - n_t^f) - \frac{G(\max\{z_0^p, z^p\}) - G(z_0^p)}{1 - G(z_0^p)} \delta_0 n_t^p \\ \dot{n}_t^f &= -\delta n_t^f + \mu^f (1 - n_t^p - n_t^f) \end{cases}$$

where  $\xi$ ,  $\mu^p$  and  $\mu^f$  are computed as described in section 2.4 using the equilibrium tuple  $(\theta, z^p, z^c, z^f, z^*)$ .

Laplace transforms are particularly useful when it comes to solving systems of differential equations embedding Dirac functions. Denoting  $N^p$  and  $N^f$  the Laplace transforms of  $n^p$  and  $n^f$ , the latter system becomes

$$\begin{cases} sN^p(s) - n_0^p &= \frac{\mu^p}{s} - \alpha_0 n_0^p - (\xi + \mu^p) N^p(s) - \mu^p N^f(s) \\ sN^f(s) - n_0^f &= \frac{\mu^f}{s} - (\delta + \mu^f) N^f(s) - \mu^f N^p(s) \end{cases}$$

where  $\alpha_0 = \frac{G(\max\{z_0^p, z^p\}) - G(z_0^p)}{1 - G(z_0^p)}$ .

The system above being linear in  $N^p(s)$  and  $N^f(s)$ , one may isolate the latter to obtain

$$\begin{aligned} N^p(s) &= \frac{(1 - \alpha_0)n_0^p s^2 + (\mu^p + (\delta + \mu^f)(1 - \alpha_0)n_0^p - \mu^p n_0^f)s + \delta\mu^p}{s(s^2 + (\xi + \delta + \mu^p + \mu^f)s + \xi\delta + \xi\mu^f + \mu^p\delta)} \\ N^f(s) &= \frac{n_0^f s^2 + (\mu^f + (\xi + \mu^p)n_0^f - \mu^f(1 - \alpha_0)n_0^p)s + \xi\mu^f}{s(s^2 + (\xi + \delta + \mu^p + \mu^f)s + \xi\delta + \xi\mu^f + \mu^p\delta)} \end{aligned}$$

The common denominator of these two expressions is a third-degree polynomial with three real roots: 0,  $\alpha_1$  and  $\alpha_2$  defined as

$$\begin{aligned} \alpha_1 &= \frac{1}{2} \left( -(\xi + \delta + \mu^p + \mu^f) + \sqrt{(\xi + \delta + \mu^p + \mu^f)^2 - 4(\xi\delta + \xi\mu^f + \mu^p\delta)} \right) \\ \alpha_2 &= \frac{1}{2} \left( -(\xi + \delta + \mu^p + \mu^f) - \sqrt{(\xi + \delta + \mu^p + \mu^f)^2 - 4(\xi\delta + \xi\mu^f + \mu^p\delta)} \right) \end{aligned}$$

Thus,  $N^p(s)$  and  $N^f(s)$  verify

$$\begin{aligned} N^p(s) &= \frac{(1 - \alpha_0)n_0^p s^2 + (\mu^p + (\delta + \mu^f)(1 - \alpha_0)n_0^p - \mu^p n_0^f)s + \delta\mu^p}{s(s - \alpha_1)(s - \alpha_2)} \\ N^f(s) &= \frac{n_0^f s^2 + (\mu^f + (\xi + \mu^p)n_0^f - \mu^f(1 - \alpha_0)n_0^p)s + \xi\mu^f}{s(s - \alpha_1)(s - \alpha_2)} \end{aligned}$$

For  $i \in \{p, f\}$ , a partial fraction decomposition yields

$$N^i(s) = \frac{n_\infty^i}{s} + \frac{\rho_1^i}{s - \alpha_1} + \frac{\rho_2^i}{s - \alpha_2}$$

where  $n_\infty^i$ ,  $\rho_1^i$  and  $\rho_2^i$  verify

$$\begin{aligned} n_\infty^p &= \frac{\mu^p \delta}{\mu^p \delta + \xi \delta + \mu^f \xi} \\ n_\infty^f &= \frac{\mu^f \xi}{\mu^p \delta + \xi \delta + \mu^f \xi} \\ \rho_1^i &= -\frac{\alpha_1 A_i + B_i}{\alpha_2 - \alpha_1} \quad \forall i \in \{p, f\} \\ \rho_2^i &= \frac{\alpha_2 A_i + B_i}{\alpha_2 - \alpha_1} \quad \forall i \in \{p, f\} \end{aligned}$$

where

$$\begin{aligned} A_p &= (1 - \alpha_0)n_0^p - n_\infty^p \\ A_f &= n_0^f - n_\infty^f \\ B_p &= \mu^p + (\delta + \mu^f)(1 - \alpha_0)n_0^p - \mu^p n_0^f + (\alpha_1 + \alpha_2)n_\infty^p \\ B_f &= \mu^f + (\xi + \mu^p)n_0^f - \mu^f(1 - \alpha_0)n_0^p + (\alpha_1 + \alpha_2)n_\infty^f \end{aligned}$$

For all  $i \in \{p, f\}$ , using the inverse Laplace transform, we get

$$n_t^i = n_\infty^i + \rho_1^i \exp\{-\alpha_1 t\} + \rho_2^i \exp\{-\alpha_2 t\}$$

The classic-to-dual transition obeys the same equations with the only additional condition  $n_0^f = 0$ .

## C.2 Dual-to-classic transition

Keeping the same notations as above, with the exception that the possibility to hire through temporary contracts is shut down after the reform. The resulting equilibrium is defined by the tuple  $(z^p, z^c, \theta)$  which verifies equations (2.23), (2.28) and the modified job creation condition

$$\frac{\gamma}{(1 - \eta)q(\theta)} = \frac{1}{r + s + \lambda} \int_{z^c}^{+\infty} (1 - G(z)) dz \quad (\text{C.1})$$

The only remaining transition probabilities on the labor market are the job-finding probability  $\mu = p(\theta)(1 - G(z^c))$  and the job-destruction probability  $\xi$ , which is left unchanged. Thus, the system of equation describing dynamics after the reform on time  $t = 0$  is

$$\dot{n}_t^p = -\xi n_t^p + \mu(1 - n_t^p - n_t^f) - \alpha_0 \delta_0(t) n_t^p \quad (\text{C.2})$$

$$\dot{n}_t^f = -\delta n_t^f \quad (\text{C.3})$$

where  $\alpha_0$  is defined as in the dual-to-dual-reform paragraph.



(C.3) can be solved right away

$$n_t^f = n_0^f \exp\{-\delta t\}$$

As for (C.3), using Laplace transforms yield

$$sN^p(s) - n_0^p = \frac{\mu}{s} - (\xi + \mu)N^p(s) - \frac{\mu}{s + \delta}n_0^f - \alpha_0 n_0^p$$

id est

$$N^p(s) = \frac{(1 - \alpha_0)n_0^p s^2 + \left(\mu + (1 - \alpha_0)\delta n_0^p - \mu n_0^f\right)s + \mu\delta}{s(s + \delta)(s + \xi + \mu)}$$

A partial fraction decomposition yields

$$N(s) = \frac{n_\infty^p}{s} + \frac{\rho_1^p}{s + \delta} + \frac{\rho_2^p}{s + \xi + \mu}$$

where

$$\begin{aligned} n_\infty^p &= \frac{\mu}{\xi + \mu} \\ \rho_1^p &= \frac{\mu}{\delta - \xi - \mu} n_0^f \\ \rho_2^p &= ((1 - \alpha_0)n_0^p - n_\infty^p) - \frac{\mu}{\delta - \xi - \mu} n_0^f \end{aligned}$$

Using the inverse Laplace transform,  $n_t^p$  verifies

$$n_t^p = n_\infty^p + \rho_1^p \exp\{-\delta t\} + \rho_2^p \exp\{-(\xi + \mu)t\}$$

## D The model with regulatory uncertainty

The Bellman equations defining the firms' and workers' programs now include the shock in firing costs. Firing costs become a new state variable. When a shock hits the latter, which occurs with probability  $\epsilon$ , firing costs jumps to either  $F_1$  or  $F_2$  with an equal probability  $1/2$  and the present discounted values adjust accordingly. Whenever possible, I simplify notations and substitute dependence in  $F_i$  with the corresponding subscripts  $i \in \{1, 2\}$ .

$$\begin{aligned} rV(F) &= -\gamma + q(\theta(F)) \int \max \left[ J_0^p(z, F) - V(F), J^f(F) - V(F), 0 \right] dG(z) \\ &\quad + \epsilon \left( \frac{V_1 + V_2}{2} - V(F) \right) \\ rU(F) &= b + p(\theta(F)) \int \max \left[ W_0^p(z', F) - U(F), W^f(z', F) - U(F), 0 \right] dG(z') \\ &\quad + \epsilon \left( \frac{U_1 + U_2}{2} - U(F) \right) \end{aligned}$$

$$\begin{aligned}
rJ^p(z, F) &= z - w^p(z, F) + s(V - J^p(z, F)) + \lambda \int (\max [J^p(z', F), V(F) - F] - J^p(z, F)) dG(z') \\
&\quad + \epsilon \left( \frac{1}{2} \max [J^p(z, F_1), V_1 - F_1] + \frac{1}{2} \max [J^p(z, F_2), V_2 - F_2] - J^p(z, F) \right) \\
rW^p(z, F) &= w^p(z, F) + \lambda \int (\max [W^p(z', F), U(F)] - W^p(z, F)) dG(z') + s(U(F) - W^p(z, F)) \\
&\quad + \epsilon \left( \frac{1}{2} \max [W^p(z, F_1), U_1] + \frac{1}{2} \max [W^p(z, F_2), U_2] - W^p(z, F) \right) \\
rJ_0^p(z, F) &= z - w_0^p(z, F) + s(V(F) - J_0^p(z)) + \lambda \int (\max [J^p(z', F), V(F) - F] - J_0^p(z, F)) dG(z') \\
&\quad + \epsilon \left( \frac{1}{2} \max [J^p(z, F_1), V_1 - F_1] + \frac{1}{2} \max [J^p(z, F_2), V_2 - F_2] - J_0^p(z, F) \right) \\
rW_0^p(z, F) &= w_0^p(z, F) + \lambda \int (\max [W^p(z', F), U(F)] - W_0^p(z, F)) dG(z') + s(U(F) - W_0^p(z, F)) \\
&\quad + \epsilon \left( \frac{1}{2} \max [W^p(z, F_1), U_1] + \frac{1}{2} \max [W^p(z, F_2), U_2] - W_0^p(z, F) \right) \\
rJ^f(z, F) &= z - w^f(z, F) + \lambda \int (J^f(z', F) - J^f(z, F)) dG(z') + \delta (V(F) - J^f(z, F)) \\
&\quad + \epsilon \left( \frac{1}{2} J^f(z, F_1) + \frac{1}{2} J^f(z, F_2) - J^f(z, F) \right) \\
rW^f(z, F) &= w^f(z, F) + \lambda \int (W^f(z', F) - W^f(z, F)) dG(z') + \delta (U(F) - W^f(z, F)) \\
&\quad + \epsilon \left( \frac{1}{2} W^f(z, F_1) + \frac{1}{2} W^f(z, F_2) - W^f(z, F) \right)
\end{aligned}$$

Free entry implies  $V_1 = V_2 = 0$ . Taking into account the Nash-bargaining rules (2.1)–(2.3), the job creation condition is

$$\frac{\gamma}{(1-\eta)q(\theta(F))} = \int \max [S_0^p(z', F), S^f(z', F), 0] dG(z')$$

Considering the unemployed's value function, the previous equation and Nash-bargaining rules (2.1)–(2.3) imply

$$(r + \epsilon)U_i = \left(1 + \frac{\epsilon}{r}\right)b + \frac{\eta\gamma}{1-\eta} \left(\theta_i + \frac{\epsilon}{r} \frac{\theta_1 + \theta_2}{2}\right)$$

Algebraic manipulations along with the Nash-bargaining assumption deliver the following expressions for surpluses

$$\begin{aligned}
(r + s + \lambda + \epsilon)S^p(z, F) &= z - (r + \epsilon)U(F) + (r + s + \epsilon)F + \lambda \int \max [S^p(z', F), 0] dG(z') \\
&\quad + \frac{\epsilon}{2} (\max [S^p(z, F_1), 0] + \max [S^p(z, F_2), 0]) \\
&\quad + \epsilon \left( \frac{U_1 + U_2}{2} - \frac{F_1 + F_2}{2} \right)
\end{aligned}$$

$$S_0^p(z, F) = S^p(z, F) - F$$

$$(r + \delta + \lambda + \epsilon)S^f(z, F) = z - (r + \epsilon)U(F) + \lambda \int S^f(z', F) dG(z') \\ + \epsilon \left( \frac{S^f(z, F_1) + S^f(z, F_2)}{2} + \frac{U_1 + U_2}{2} \right)$$

$S^p$  is increasing in both  $z$  and  $F$  and has its values in  $\mathbb{R}$ . Thus, there exists  $z_1^p$  and  $z_2^p$  such that  $S^p(z_1^p, F_1) = 0$  and  $S^p(z_2^p, F_2) = 0$ . Notice that  $\partial S^p / \partial z$  does not depend on  $F$  and verifies

$$\frac{\partial S^p}{\partial z} = \begin{cases} \frac{1}{r+s+\lambda+\epsilon} & \text{if } z \leq z_2^p \\ \frac{1}{r+s+\lambda+\frac{\epsilon}{2}} & \text{if } z_2^p \leq z \leq z_1^p \\ \frac{1}{r+s+\lambda} & \text{if } z \geq z_1^p \end{cases}$$

Using an integration by parts, we know that  $z_1^p$  and  $z_2^p$  verify

$$z_2^p - (r + \epsilon)U_2 + (r + s + \epsilon)F_2 + \epsilon \left( \frac{U_1 + U_2}{2} - \frac{F_1 + F_2}{2} \right) \\ + \frac{\lambda}{r + s + \lambda + \frac{\epsilon}{2}} \int_{z_2^p}^{z_1^p} (1 - G(x)) dx + \frac{\lambda}{r + s + \lambda} \int_{z_1^p}^{+\infty} (1 - G(x)) dx = 0 \\ z_1^p - (r + \epsilon)U_1 + (r + s + \epsilon)F_1 + \epsilon \left( \frac{U_1 + U_2}{2} - \frac{F_1 + F_2}{2} \right) \\ + \frac{\lambda}{r + s + \lambda} \int_{z_1^p}^{+\infty} (1 - G(x)) dx + \frac{\epsilon}{2} \frac{z_1^p - z_2^p}{r + s + \lambda + \frac{\epsilon}{2}} = 0$$

The present discounted value of unemployment  $U$  depends on the firing cost through the labor market tightness, which accounts for the  $i$  subscript, with  $i \in \{1, 2\}$ . Their expressions still verify (2.18).

The derivative above yields another expression for  $S^p$ .

$$S^p(z, F_2) = \begin{cases} \frac{z - z_2^p}{r+s+\lambda+\epsilon} & \text{if } z \leq z_2^p \\ \frac{z - z_2^p}{r+s+\lambda+\frac{\epsilon}{2}} & \text{if } z_2^p < z < z_1^p \\ \frac{z - z_1^p}{r+s+\lambda} + \frac{z_1^p - z_2^p}{r+s+\lambda+\frac{\epsilon}{2}} & \text{if } z \geq z_1^p \end{cases} \\ S^p(z, F_1) = \begin{cases} \frac{z - z_2^p}{r+s+\lambda+\epsilon} + \frac{z_2^p - z_1^p}{r+s+\lambda+\frac{\epsilon}{2}} & \text{if } z \leq z_2^p \\ \frac{z - z_1^p}{r+s+\lambda+\frac{\epsilon}{2}} & \text{if } z_2^p < z < z_1^p \\ \frac{z - z_1^p}{r+s+\lambda} & \text{if } z \geq z_1^p \end{cases}$$

Considering the surplus of temporary contracts,  $\partial S^f / \partial z$  equals  $1/(r + \delta + \lambda)$ . One may define  $z_i^f$  such that  $S^f(z_i^f, F_i) = 0$ . Thus, another definition of  $S^f$  is

$$S^f(z, F_i) = \frac{z - z_i^f}{r + \delta + \lambda}$$

where thresholds  $z_i^f$  are such that

$$z_i^f = \frac{r + \delta + \lambda}{r + \delta + \epsilon} \left( (r + \epsilon)U_i - \frac{\delta}{r + \delta} \epsilon \frac{U_1 + U_2}{2} \right) - \frac{\lambda}{r + \delta} E z$$

In the same manner, one may define  $z_i^c$  as  $S_0^p(z_i^c, F_i) = 0$ . Since for all  $z \geq z_2^p$ ,  $\partial S^p / \partial z = 1/(r + s + \lambda) > 1/(r + \delta + \lambda) = \partial S^f / \partial z$ , there still exists  $z_i^*$  such that  $S^f(z_i^*, F_i) = S_0^p(z_i^*, F_i)$ , for  $i \in \{1, 2\}$ . I assume that  $\epsilon$  and the distance between  $F_1$  and  $F_2$  is small enough so that  $z_i^* > z_2^p$  for all  $i \in \{1, 2\}$ . In this case,  $z_i^*$  verify

$$\left( \frac{1}{r + s + \lambda} - \frac{1}{r + \lambda + \delta} \right) z_i^* = \frac{z_1^p}{r + s + \lambda} - \frac{z_1^p - z_2^p}{r + s + \lambda + \frac{\epsilon}{2}} \mathbf{1}\{i = 2\} + F_i - \frac{z_i^f}{r + \lambda + \delta}$$

As previously, assume that  $\epsilon$  and the distance between  $F_1$  and  $F_2$  is small enough so that  $z_i^c > z_2^p$  for all  $i \in \{1, 2\}$ , we can rewrite the job creation conditions (2.14), which pinpoint  $\theta_i$  with  $i \in \{1, 2\}$ .

$$\frac{\gamma}{(1 - \eta)q(\theta_i)} = \frac{1}{r + s + \lambda} \int_{\max[z_i^c, z_i^*]}^{+\infty} (1 - G(z)) dz + \frac{1}{r + \delta + \lambda} \int_{z_i^f}^{\max[z_i^f, z_i^*]} (1 - G(z)) dz$$

Consequently, the equilibrium can be summed up by a continuous time Markov process over space  $\left\{ \left( \theta_i, z_i^p, z_i^c, z_i^f, z_i^* \right) \right\}_{i=1,2}$  with switching probability  $\epsilon$  and 2x2 transition matrix filled with 1/2. See Appendix C for the study of the corresponding dynamic behavior.