The Network Origin of Slow Labor Reallocation*

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Abstract

How fast do labor markets adjust to technology shocks? This paper introduces a novel network-based framework to model skill frictions between occupations. Using expert data on skills, I construct a network of occupations and find it is sparse, divided in clusters of similar occupations with 'bridge occupations' linking distinct clusters. Leveraging French administrative data, I show that workers transitioning through these 'bridges' move to occupations with higher wages and lower unemployment. Next, I build a tractable model of job search with networked labor markets, and demonstrate that bridge occupations significantly affect reallocation speed, with slow reallocation creating large adjustment costs. I then augment the model with quantitative extensions, leveraging hat-algebra methods to solve counterfactuals without having to estimate large numbers of parameters. Calibrated to French data, the model predicts that robot adoption induces slow reallocation, around 40 quarters, and that this sluggish reallocation reduces welfare gains by approximately 40%— an order of magnitude higher than previous estimates. However, policies targeting bridge occupations can speed-up reallocation, and much more so than policies targeting tight occupations directly. These findings highlight the crucial role of the occupation network in shaping reallocation dynamics and provide new insights for the design of labor market policies.

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1 Introduction

Trade and technology create winners and losers in the labor market. Some occupations experience significant job losses (e.g., supermarket cashiers), while others benefit from job creation (e.g., robot maintenance). Labor reallocation – the process of moving workers from declining to growing occupations – can mitigate the negative impact on affected workers. However, evidence suggests that displaced workers reallocate slowly, leading to prolonged periods of unemployment (Autor et al., 2013, 2014; Dix-Carneiro, 2014; Dix-Carneiro and Kovak, 2019; Adão et al., 2024).

A key barrier to reallocation is the limited transferability of skills across occupations (Gathmann and Schönberg, 2010; Traiberman, 2019; Lise and Postel-Vinay, 2020). Workers in declining jobs often lack the skills needed to transition into growing sectors, e.g. from cashier to robot maintenance. However, analyzing these skill frictions is difficult because skill transferability varies widely between occupation pairs, forming a high-dimensional matrix. With 200 occupations, the skill frictions matrix contains 40,000 elements, making it difficult to analyze the structure of these frictions and their impact on reallocation speed.

In this paper, I introduce a novel network-based approach to model skill frictions between occupations. Specifically, I construct an occupation network, where nodes are occupations and edges represent potential transitions based on skill transferability. For instance, engineers and computer scientists are linked, while cashiers and engineers are not. This network framework provides new tools to analyze the structure of skill frictions, allowing me to "open the black box" of skill frictions.

Leveraging this framework, I examine how the topology of the occupation network influences worker mobility dynamics in response to technology shocks. My main contribution is to uncover the crucial role of *central* occupations in shaping the reallocation process. Intuitively, these central occupations act as hubs, bridging different clusters of occupations, thereby facilitating transitions. I support this argument using a combination of empirical, theoretical and structural methods. I will now outline each part of the paper in more details.

In the first part of the paper, I provide empirical evidence on the structure of the occupation network. To construct this network, I leverage information from a French administrative database on skills, called ROME, which lists possible occupation transitions according to HR experts. I highlight three key features of the occupation network. First, it is sparse, with occupations having relatively few connections. Second, it exhibits a "community structure", with clusters of tightly-connected occupations with similar skills and tasks.[1] Intuitively, occupations within the same 1-digit group share many skills and thus have many links. Third, it features 'bridge occupations' that are located between clusters and connect them. These bridge occupations, such as logistics technician, sales assistant, or consultant, typically involve diverse skill sets, allowing workers to move across groups of occupations.

Then, I examine how the architecture of the occupation network affects worker mobility patterns. Using French employer-employee data,I find that workers are 95% less likely to switch occupations when the transition is not deemed feasible by experts, controlling for time-varying origin and destination occupations fixed-effects. I also explore the impact of transiting through bridge occupations on long-term outcomes. Workers who pass through bridge occupations reallocate to better-paying, lower unemployment risk occupations, have a higher probability of switching 1-digit occupation groups, and travel farther in the network. To address potential self-selection, I control for age, origin occupation, and mobility type. Taken together, these results highlight the important role of bridge occupations in facilitating reallocation to better employment opportunities.

In the second part of the paper, I develop a model of job search with networked labor markets to build intuition about the key theoretical mechanisms at play. The model combines both search frictions and skill frictions, captured in reduced form by the network structure of occupations. I begin with a tractable version of the model, which allows for a full analytical solution but makes three key simplifying assumptions – relaxed thereafter. First, the occupation network is modelled as a a simple line graph: two periphery occupations are connected to a central bridge occupation in-between. Second, workers

[1] The concept of community structure originates from network theory, and describes network in which elements are organized into "communities", i.e. clusters of tightly-knit nodes (e.g. see Newman (2006).

search for jobs randomly, both in their own and adjacent occupations within the network, and only when unemployed.[2] Third, wages and firm-worker separation probabilities are exogenous, keeping the firm side of the model deliberately simple. This setup enables a clear analysis of worker reallocation dynamics following permanent productivity asymmetric shocks, such as those arising from trade or technological changes.

I derive a closed-form analytical solution for aggregate reallocation time, clarifying how different factors shape the aggregate speed of reallocation. First, aggregate reallocation is slow, with transition times on the order of s^{-1}, where the separation rate s is typically 2.5% per quarter, predicting reallocation times of around 40 quarters. Intuitively, this is because workers must wait to become unemployed before switching occupations. Second, the centrality of shocks matters: reallocation is slower following shocks to periphery occupations because they have fewer reallocation opportunities. Third, the centrality of occupations matters too: the job finding rate in bridge occupations has a very significant impact on aggregate reallocation speed, because it can create bottlenecks for workers reallocating between periphery occupations. Later, I show that the results from this simple setting generalize to more complex environments.

In addition, I provide a closed-form analytical characterization of adjustment costs following asymmetric productivity shocks, showing that slow reallocation can significantly reduce long-term welfare gains. Specifically, I show that adjustment costs are substantial when reallocation is slow, with welfare gains reduced by up to 100% in some cases. Intuitively, as reallocation time increases, the present value of future welfare gains diminishes. This is particularly pronounced for shocks affecting periphery occupations, where misallocation is particularly severe along the transition.

In the third part of the paper, I build a quantitative version of the model to test whether the predictions of the tractable model hold in a more realistic environment. While the toy model provides analytical insights, it relies on a simplified network structure and mechanical agent behavior. Here, both assumptions are relaxed. First, I allow for arbitrary network structures, capturing the complexity of real-world occupation net-

[2] In the quantitative model, I relax these assumptions and allow for both endogenous search effort and on-the-job search.

works. Second, I incorporate more realistic worker behavior, including endogenous search effort, on-the-job search, and Nash bargaining on wages. Workers strategically allocate their search efforts to occupations with higher expected returns, both when employed and unemployed.

Despite its complexity, the quantitative model can be estimated and solved leveraging dynamic hat-algebra techniques from the trade literature. The idea is straightforward: following small shocks, the new equilibrium can be expressed as the product of the pre-shock equilibrium and a first-order approximation of the percentage point change (the so-called "hat changes"). Solving for these hat changes requires only a few key elasticities, reducing the parameter space significantly. A crucial elasticity is the search elasticity, which captures how workers adjust their search efforts in response to changes in expected returns. By exploiting the gravity structure of worker flows, I show that this elasticity can be directly estimated from the data. Importantly, this approach extends beyond the specific context of this paper and can be applied to solve counterfactuals in a broader set of models that combine discrete choice and search frictions.

Finally, I use the estimated model to simulate how labor markets adjust following the introduction of robots. To calibrate the robot shock, I match estimates of the response of relative wages and aggregate TFP gains from Webb (2019), Acemoglu and Restrepo (2022). I find that the most affected occupations – expanding and declining as the result of shock – are located at opposite ends of the occupation network, mirroring the periphery shock from the tractable network search model.

There are three main takeaways from this quantitative analysis. First, robot adoption induces slow worker reallocation, of around 40 quarters. This aligns closely with the stylized model's prediction, suggesting that the effects of a larger network and more strategic agents offset each other. Second, this moderate aggregate reallocation time masks important heterogeneity across occupations: a minority of occupations require more than 80 quarters to adjust. Third, and most importantly, the slow dynamics of reallocation generate significant adjustment costs, reducing welfare gains by approximately 40%. This is an order of magnitude higher than previous estimates, such as the 3.5% reduction found by Caliendo et al. (2019) for the US China shock. The key difference lies in the inclusion of search frictions in my model, which increases the cost of unemployment for both workers and the broader economy.

Lastly, I explore the potential of targeted policy interventions to accelerate reallocation. I simulate the effects of government employment subsidies aimed at declining occupations (e.g., cashiers), expanding occupations (e.g., robot maintenance technicians), and bridge occupations (e.g., logistics technicians). The results show that targeting bridge occupations consistently leads to the largest reduction in reallocation times, cutting them by three times more than subsidies for expanding occupations. While subsidies for occupations facing labor shortages increase workers' incentives to switch by raising wages and vacancy postings, they fail to address skill frictions. In contrast, targeting bridge occupations helps workers reskill, facilitating transitions to expanding jobs. Overall, this suggests that exploiting the network structure of skill frictions can help policymakers design more effective labor market policies.

Relation to the literature. My paper connects to three strands of literature. First, it examines labor market adjustments after trade or technology shocks. Motivated by evidence of slow worker reallocation, a series of papers build structural models of dynamic occupational choice with switching costs to estimate welfare effects. (e.g., Artuç et al. (2010); Dix-Carneiro (2014); Caliendo et al. (2019); Traiberman (2019); Humlum (2019)). Recent papers add search frictions to account for the rise in involuntary unemployment following these shocks (Pilossoph (2012), Dix-Carneiro et al. (2021); Chodorow-Reich and Wieland (2020); Carrillo-Tudela and Visschers (2023)).[3] I make two contributions. First, I use network theory to analyze reallocation frictions, highlighting the role of central occupations on reallocation speed. Second, I extend the hat-algebra method from Caliendo et al. (2019) to include search frictions.

[3]In a closely related paper, Restrepo (2016) models labor market adjustments to technological shocks using a task-based framework with search frictions, emphasizing the role of "stepping-stone" jobs in unemployment dynamics. My paper complements this framework by incorporating greater heterogeneity across occupations and linking the existence of stepping-stone occupations to the structure of the occupation network.

Second, my paper connects to the literature on job search in non-segmented labor markets. A growing body of work emphasizes the substantial overlap in labor market boundaries, whether in geographical space (Schmutz and Sidibé (2019); Manning and Petrongolo (2017); Marinescu and Rathelot (2018)) or skill space (Guvenen et al. (2020); Lise and Postel-Vinay (2020); Lise and Postel-Vinay (2020)). Schubert et al. (2021) and Jarosch et al. (2024) show this matters for the measure of labor market power. I contribute by representing the overlap between labor markets as a network, highlighting the role of central occupations. Unlike this literature, which focuses on mismatch or concentration, my focus is on the speed of worker reallocation.

Third, this paper contributes to the literature on heterogeneity in macroeconomics. An important strand of the literature uses spectral methods to study transition dynamics in economies with heterogeneous agents (e.g., Moll (2014); Gabaix et al. (2016); Alvarez and Lippi (2022); Baley and Blanco (2021); Beraja and Wolf (2021); Kleinman et al. (2023); Liu and Tsyvinski (2024)). I show that these tools can be applied in a labor market context, providing a full analytical characterization in a simplified setting. Additionally, the production network literature highlights that central sectors significantly impact aggregate outcomes (e.g., Gabaix et al. (2016); Acemoglu et al. (2012),Carvalho and Tahbaz-Salehi (2019); Baqaee and Farhi (2019)). I extend this insight to labor markets, showing that central occupations disproportionately affect aggregate worker reallocation speed, paving the way for new research avenues.[4]

2 Empirical Evidence on the Occupation Network

In this section, I construct the occupation network using the ROME database and analyze its structure, highlighting three stylized facts.

[4] A literature in complexity economics also emphasizes the network structure of worker flows (see Neffke et al. (2017) or del Rio-Chanona et al. (2021)). I push further their analytical characterization, highlighting the role of bridge occupations for reallocation dynamics.

2.1 Data

To construct an empirical measure of the skill network, I use the French administrative database 'ROME'.[5] This database provides detailed information on the skills and tasks of occupations and is the French counterpart to the US O*NET database.

The ROME database was developed in 1989 by experts in human resources from the French employment agency Pole Emploi, in collaboration with firms and labor unions. Its original purpose was to improve the matching of job seekers with vacancies based on skill similarity, during a period of substantial structural transformation in labor markets. To find new opportunities for displaced workers, experts relied as little as possible on past worker flows. It has been revised many times since its creation, and here I use the fourth version, which was released in 2023.

2.2 Construction of the Main Variables

Definition. To begin, I will introduce a few concepts from network theory. A directed weighted network $\mathcal{G}$ consists of nodes $\mathcal{N}$, edges $\mathcal{E}$ and weights $\mathcal{W}$: $\mathcal{G} = \{\mathcal{N}, \mathcal{E}, \mathcal{W}\}$. This network can be represented by an adjacency matrix $\boldsymbol{G} = (g_{ij})_{(i,j)\in\mathcal{N}^2}$, where $g_{ij} \in \mathcal{W}$ is the weight of the edge $i \to j \in \mathcal{E}$. A network is undirected if all nodes are connected in both directions: $i \to j \in \mathcal{E}$ and $j \to i \in \mathcal{E}$. A network is unweighted if all edges weights are equal.

Nodes. I construct the nodes of the occupation network using the ROME nomenclature of occupations, defined at the 5-digit level of aggregation, totaling around 500 different occupations. Importantly, occupations are grouped by skill similarity rather than hierarchical rank. For example, the 5-digit category "higher education" includes both "lecturers" and "full professors."

Edges. I construct the edges of the skill network using a unique information from ROME: namely, the list of occupation transitions deemed feasible by experts. The edges

[5] The acronym ROME stands for *Répertoire Opérationnel des Métiers et Emplois*, which translates to "Operational Nomenclature of Occupations and Jobs".

of the occupation network are directed, meaning that reverse occupational transitions may not be feasible. For example, a medical doctor can become a nurse, but the reverse may not be true. This directionality partially captures the hierarchical structure of occupations.

Weights. To construct the edge weights, I use additional information from the ROME database. Experts distinguish between two types of feasible occupational transitions: "close" transitions can be made immediately, and "distant" transitions require additional training. I assign different numerical weights to each type of transition: α_{close}, α_{distant}, and $\alpha_{\text{self-loops}}$.

The weights measure the degree of skill transferability across occupations, normalized between zero and one. A weight of one indicates perfect accessibility (identical skills), while zero indicates inaccessibility (no skill similarity). Thus, they can be interpreted as the probability that a worker can acquire the skills to move to a new occupation. Formally, $0 \leq \alpha_{\text{distant}} \leq \alpha_{\text{close}} \leq \alpha_{\text{self}} = 1$. The weights are estimated in next section.[6]

2.3 Plot of the Occupation Network

Figure 2.1 plots the occupation network, using a standard network layout.[7] Manufacturing occupations are clustered together at the bottom of the network, while high-skilled service occupations, e.g. banking and insurance, are clustered at the top. Occupations with general skills, such as sales, transportation and logistics, or management are found at the center of the plot.[8]

[6] Regressing worker flows on the different types of network connections, I find $\alpha_{\text{close}} = 0.1$ and $\alpha_{\text{distant}} = 0.05$.

[7] The spring layout treats edges as springs keeping adjacent nodes close, which otherwise want to repel each other.

[8] Many groups of occupations seem to overlap importantly with sectors, e.g. industry. In fact, a lot of occupations are specific to a certain industry but others, like accounting or sales, exist across all sectors exist across all sectors.

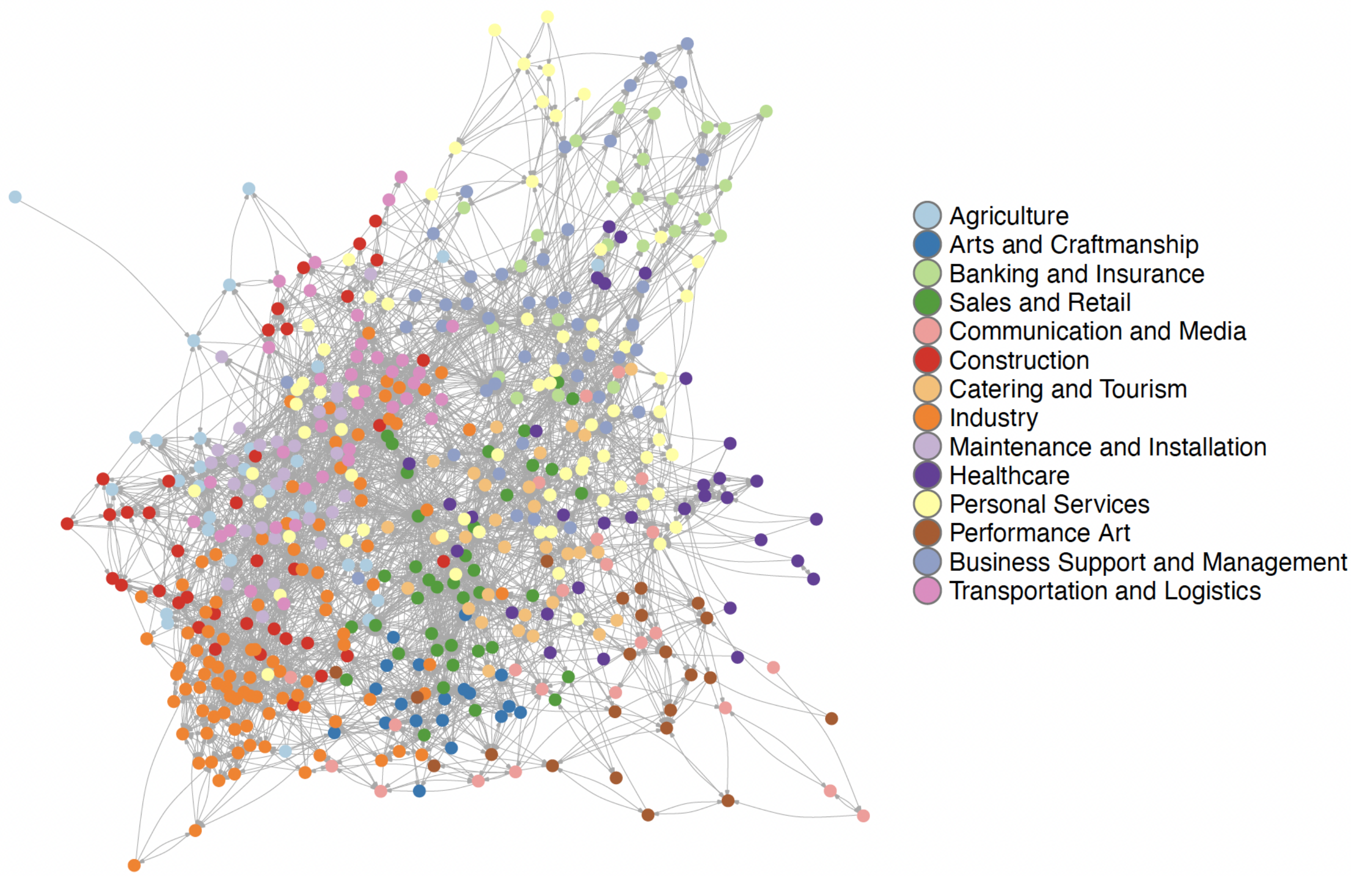


Figure 2.1: **The ROME skill network - with spring layout and close transitions only**

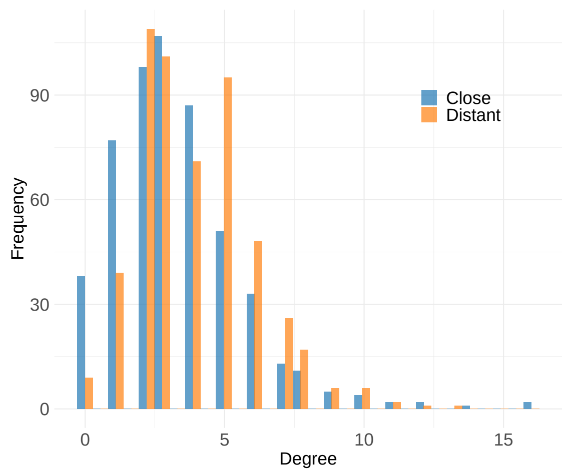
2.4 Stylized Facts

This subsection documents three stylized facts on the architecture of the occupation network.

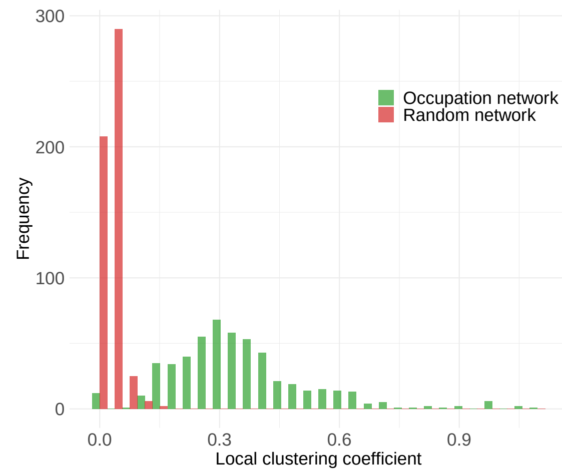
Fact 1. *The occupation network is very sparse* A network is sparse if its nodes have few connections relative to the size of the network. This is captured by out-degree centrality, which counts the number of edges originating from a node.

Figure 2.2.a displays the distribution of out-degree centrality indices across occupations. On average, occupations are connected to 7 other occupations in the skill network (out of 500), with a standard deviation of approximately 3. This indicates that occupations have a limited number of reallocation opportunities, suggesting large frictions to occupational mobility.

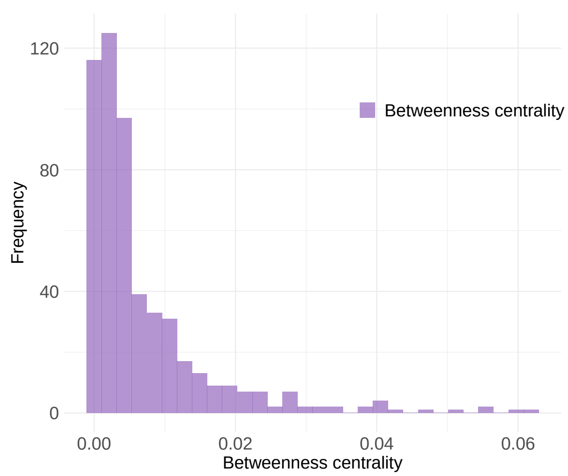
Figure 2.1: **Distribution of network centrality measures in the occupation network**



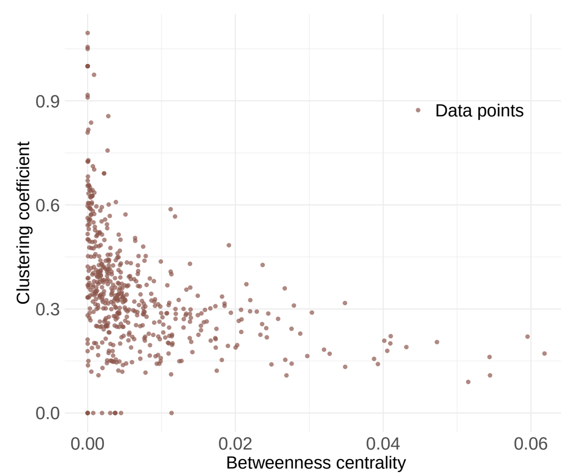
a) Distribution of out-degree centrality



b) Distribution of local clustering



c) Distribution of betweenness centrality



d) Betweenness vs local clustering

Fact 2. *The occupation network exhibits a community structure*

A network exhibits a community structure when nodes are grouped in dense clusters that are sparsely connected to each other (Newman (2006)). This is captured by the local clustering coefficient, which measures the share of a node's neighbors that are also connected to each other.

Figure 2.2.b compares the distribution of local clustering coefficients in the occupation network to a random network with similar density. The occupation network exhibits higher clustering than the random network, indicating that occupations tend to be grouped together. Intuitively, occupations within the same 1-digit family often share

similar skills, and hence are tightly-connected. [9]

Fact 3. *The occupation network features bridge occupations*

In network analysis, bridges are crucial nodes that connect otherwise loosely linked clusters. These bridges are identified by (1) low local clustering and (2) high betweenness centrality, which captures how much a node is 'in-between' others.[10][11]

Figure 2.2.c plots the distribution of betweenness centrality indices. While most occupations have low betweenness centrality, a small subset has significantly higher values. These bridge occupations act as hubs through which a large proportion of reallocation paths between clusters must pass. Examples include logistics technicians, secretaries and business consultants, occupations which typically require a mix of different skill sets. Appendix A provides a list of bridges and map their location in the occupation network.

Intuitively, bridge occupations play a crucial role for reallocation dynamics. If workers find it difficult to transit through bridges, mobility between clusters is severely restricted, impacting reallocation speed. I formalize this intuition in the theoretical model.

3 Empirical Evidence on Worker Flows

This subsection studies workers trajectories within the occupation network. First, I present the data and the main variables. Then, I study how skill frictions affect patterns of worker transitions. Finally, I show that bridge occupations facilitate workers reallocation.

[9] In the appendix, I use community detection algorithms to directly identify clusters within the occupation network. The clusters exhibit a high degree of modularity, a standard measure of community structure in network.

[10] Betweenness centrality is the share of shortest paths in the network that transit through a node.

[11] Low local clustering ensures bridges link different clusters, not just nodes within the same cluster.

3.1 Data

To measure worker transitions across occupations and employment states, I use the DADS panel, a French employer-employee dataset collected by the Social Security agency. The dataset covers about one-twelfth of the French labor force (all workers born in October) from 2002 to 2010, with data drawn from job contracts. Unique individual identifiers allow tracking individuals across jobs. The dataset includes employment spell dates, occupation codes, wages, hours worked, and demographics such as gender, age, and location.

I restrict the sample to male workers aged 25-55 who held at least two job contracts during the period, representing a group with high labor force attachment. The analysis is further limited to main job contracts in mainland France and the private sector. Unemployment spells are defined as periods of more than 30 days between consecutive employment spells, with occupations based on the last job. I focus on the 2009-2019 period, when firms were required to report 4-digit occupation codes, resulting in a panel of approximately 550,000 workers over ten years. More details are provided in Appendix B.

There are two key variables of interest. First, the worker transition rates $\mu_{ijt}^{xy}(t)$, which measure the number of workers in occupation i and employment state x at quarter t who transition to occupation j and employment state y the next quarter $t+1$. Second, worker payoffs for model estimation, where wages w_{it} are the average full-time equivalent wage in occupation i at quarter t, residualized on age. Unemployment benefits $b_i(t)$ are calculated as $bw_i(t)$, where b is the wage replacement rate, calibrated later.[12]

3.2 Gravity Regression

Do workers really transition along the edges of the occupation network built with ROME data? If experts misidentify skill frictions, the previously constructed occupation net-

[12]This is a simplification, as actual benefits depend on the last job's wage and duration. The model can incorporate occupation-specific replacement rates, but using a single parameter is more tractable.

work becomes irrelevant for understanding the constraints on worker reallocation.

Specification. To assess how much worker transitions are constrained by the occupation network, I draw on the analogy with the gravity model in trade. Just as trade economists analyze how geographical distance influences trade flows between countries, I substitute trade flows with worker flows and geographical distance with skill distance.

My baseline specification uses a two-way fixed effects Poisson regression model:[13]

$$\underbrace{\mu_{ijt}^{ue}}_{\substack{\text{transitions}\\ u_i \to e_j}} = \exp\Big\{ \underbrace{\gamma_{it} + \zeta_{jt}}_{\substack{\text{time-varying}\\ \text{origin/destination FE}}} + \underbrace{\beta^{\text{distant}} 1(\text{distant})_{ij} + \beta^{\text{unlinked}} 1(\text{unlinked})_{ij}}_{\text{type of network connection}} \Big\} \eta_{ijt}$$

where μ_{ijt}^{x} represents the number of transitions to occupation j from unemployed workers in occupation $i \neq i$, γ_{it} and ζ_{jt} are time-varying origin and destination occupation fixed effects, $1(\text{distant})ij$ and $1(\text{unlinked})ij$ are dummy variables for each type of network connection, and η_{ijt} is a multiplicative error term.[14]

The coefficients of interest, β^{distant} and β^{unlinked}, capture the change in the likelihood of switching occupations when a pair is classified as distant or unlinked by experts, relative to close occupations.

Including time-varying origin and destination fixed effects is crucial for identification, as they control for local shocks at both the origin and destination occupations. Without these controls, low transitions might be mistakenly attributed to low skill frictions instead of negative shocks.

Regression results. Table 3.1 presents the estimated coefficients.

Notes: Each observation is a quarterly occupation-destination cell. The dependent variable is the number of occupation transitions during a quarter. The variables of interest are dummies capturing whether the transitions are labeled as close, distant, or unconnected by experts. Standard errors are clustered at the origin. * p<0.10, ** p<0.05, *** p<0.01.

The coefficients are all negative, statistically significant at the 1% level, and economically large. Workers switching occupations are about 95% less likely to transition to an

[13]The Pseudo-Poisson regression is appropriate for non-negative dependent variables. Excluding the 90% of pairs with no transitions would bias the estimation (see Santos Silva and Tenreyro (2006)).

[14]Transitions not listed as possible by experts are classified as "unlinked."

'unlinked' occupation than to a 'close' one, all else equal.[15]

This confirms the importance of the occupation network in shaping job transitions. The next subsection examines which network features shape long-term worker reallocation dynamics.

3.3 Bridge Regression

Is worker reallocation facilitated by transiting through bridge occupations?

Due to the sparsity of the occupation network, workers often cannot move *directly* into occupations outside their skill cluster. However, they could *indirectly* access these occupations by transitioning first through bridge occupations, which connect different clusters. This would help displaced workers acquire new skills, escape declining clusters, and enter expanding ones.

Specification. I study how transiting through a bridge occupation affects the long-term outcomes of workers. I estimate the following model:

$$\Delta y_{id} = \alpha_o(id) + \beta 1(\text{transited through bridge})_{id} + \gamma \boldsymbol{Z}_{id} + u_{id} \tag{1}$$

where *id* represents worker identifiers, $o(id)$ is the worker's occupation at the start of the period, Δy_{id} is the long-run change in worker outcomes, and $1(\text{transited through bridge})id$ is a dummy variable equal to one if the worker transitioned through a bridge occupation. $\boldsymbol{Z}_{id}$ includes worker-level controls, such as age, and u_{id} is the error term. Appendix B provides a full list of bridge occupations. The coefficient of interest, β, measures the change in outcomes for workers who transited through bridge occupations relative to those who did not.

Including controls and origin-occupation fixed effects is key for identification. Without these, an endogeneity bias could occur, as transitioning through a bridge may correlate with the worker's initial occupation or age. As a robustness check, I run an

[15]Marginal effects are calculated using the formula $e^{\beta} - 1$.

alternative regression limiting the control group to high-mobility workers who made at least two transitions during the period.

Regression results. Table 3.2 reports the estimated coefficients for the period 2013-2019. Appendix B provides the corresponding estimates for 2008-2012 and for the alternative control group with high-mobility workers.[16]

Each column of the regression table corresponds to a different outcome variable: likelihood of changing 1- or 2-digit occupation groups, distance traveled in the occupation network, wage growth, and change in unemployment risk. All coefficients are statistically significant at the 1% percentage threshold and economically large.

Table 3.2: **Change in worker outcomes vs. transition through bridge, 2013-2019**

	Dependent Variables				
	Change 1-digit Occupation	Change 2-digit Occupation	Distance (Standardized)	Wage Growth	Unemployment Rate Change
Through Bridge	0.296***	0.316***	0.710***	0.083***	-0.785***
	(0.003)	(0.004)	(0.008)	(0.007)	(0.038)
Observations	196,439	196,439	196,439	196,402	196,439
Origin FE	Yes	Yes	Yes	Yes	Yes

Notes: Each observation is a worker over the period 2013-2019. The dependent variables are the probability of changing 1-digit or 2-digit occupation, distance traveled within the network, wage growth, and change in unemployment risk. The variable of interest is a dummy indicating whether workers transited through a bridge during the period. * p<0.10, ** p<0.05, *** p<0.01.

First, workers who transit through bridges are about 30 percentage points more likely to switch 1- or 2-digit occupation groups, consistent with the fact that bridge occupations connect clusters of these groups.

Second, workers who transit through bridges travel farther in the occupation network, about 0.7 standard deviations of the distance distribution. This suggests that bridge occupations are not just 'rest employment' jobs where displaced workers temporarily stay. Instead, these workers continue progressing in their career path without

[16] The sample is split in two to minimize the number of observations lost due to the balanced panel requirement.

reverting to their previous occupations.

Third, workers who transit through bridges experience significantly higher wage growth. Specifically, these workers saw an 8 percentage point cumulative increase in wages, compared to just 3.5% average wage growth over the same period. This indicates that workers who use bridge occupations move to better-paying jobs.

Fourth, workers who transit through bridge occupations end up in jobs with lower unemployment rates, about 0.75 percentage points lower. For comparison, the average unemployment rate in France during this period was around 10%. This suggests that bridge occupations help workers move to jobs with lower unemployment risk.

These findings suggest that transiting through bridge occupations significantly benefits workers by enabling them to move to distant, better-paying jobs with lower unemployment risks. However, these results are correlational, not causal. Additionally, the impact on other workers is unobservable and part of the intercept. To quantify the general equilibrium effects of bridge occupations on workers, a model is required.

4 A Tractable Model of Network Search

This section develops a tractable model of job search across occupations connected by a network structure. The goal is to provide a clear analytical characterization of worker reallocation dynamics before transitioning to a fully-fledged quantitative model in the next section.

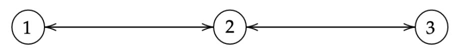
The model builds on the canonical random search and matching frameworks of Diamond (1982) and Mortensen (1982). The key new ingredient is that workers search for jobs not only within their own occupation but also in adjacent occupations within the network, naturally leading to worker mobility across occupations.

4.1 Setting

Time. Time is continuous, starts at zero, runs until infinity, and is discounted at the rate r.

Occupational structure. There are three occupations, indexed by $i \in \mathcal{N} = \{1, 2, 3\}$, connected by an unweighted and undirected occupation network. Edges indicate that workers can switch occupations.

Figure 4.1 : **A stylized occuption network**



The occupation network is assumed to be a line network, as represented Figure 4.1. Occupation 2 is central in the network, while occupations 1 and 3 are at the periphery. The line network is the simplest structure that allows for variations in network centrality. In the quantitative model, I lift this restriction and allow for arbitrary network structures.

Workers. There is a unit mass of infinitely-lived workers distributed across occupations. Workers can be either employed or unemployed. Unemployed workers in occupation i are those whose *last job* was in occupation i.[17] Employed workers in occupation i earn a wage $w_i(t)$, while unemployed workers receive benefits b_i.

The key innovation is that workers search for jobs both within their current occupation and in adjacent ones in the occupation network. For simplicity, only unemployed workers search for jobs, and their search is random. These assumptions are relaxed in the quantitative model.

The distribution of workers across occupations and employment states, referred to as

[17] I abstract from worker birth-death dynamics, as the entry and exit of generations occur on a much slower time scale than the one relevant for this model.

the worker distribution, is represented as:

$$f(t) = ((e_i(t))_{i \in \mathcal{N}}, (u_i(t))_{i \in \mathcal{N}})' = (e_1(t), e_2(t), e_3(t), u_1(t), u_2(t), u_3(t))' \tag{2}$$

where $e_i(t)$ and $u_i(t)$ are the number of employed and unemployed workers in occupation i at t.

Firms. There are infinitely many potential firms in each occupation, each incurring a flow cost c_i to post a job vacancy. When a firm matches with a worker, the vacancy is filled, and the firm-worker pair produces homogeneous output with productivity y_i.[18]

This specification abstracts from worker learning dynamics. In the model, reallocated workers instantly acquire the skills of their new occupation and produce at the same level as experienced workers. Relaxing this assumption would reduce tractability.[19] However, this assumption likely overestimates the speed of worker reallocation, leading to more frequent occupational switches than observed in reality.

The firm pays the worker a wage w_i and collects profits of $y_i - w_i$. Wages are set exogenously as a fixed fraction of productivity, $w_i = \phi y_i$, where ϕ reflects worker bargaining power. Finally, firm-worker matches are subject to exogenous separation shocks at rate s_i.

Search frictions. Each occupation is a distinct labor market where search frictions prevent unemployed workers and firms from matching immediately.[20]

The number of hires in occupation i at date t is governed by a network-augmented matching function:

$$h_i(t) = \zeta_i \quad \cdot \left(u_i(t) + \sum_{j \neq i} g_{ij} u_j(t) \right)^{\alpha} \cdot \quad v_i(t)^{1-\alpha_i}$$

[18]Alternatively, each occupation could produce a specific output sold at an exogenous price y_i. Price exogeneity simplifies the model by avoiding complex general equilibrium feedback effects between prices and worker distributions.

[19]If productivity depends on the worker's origin occupation, the model is no longer block-recursive, as the worker distribution would affect the expected value of a match for firms.

[20]Search frictions do not arise from underlying heterogeneity, but rather from imperfect information between workers and firms about suitable matches.

where ζ_i represents the matching efficiency of labor market i, $v_i(t)$ is the number of vacancies posted by firms in occupation i at time t, $u_i(t) + \sum_{j \neq i} g_{ij} u_j(t)$ is the number of unemployed workers searching *in* occupation i from both occupation i and neighboring occupations, and α is the matching elasticity.

The key novelty is that the pool of job seekers includes unemployed workers from neighboring occupations, which has important implications for the measurement of labor market tightness.[21]

The labor market tightness ratio in occupation i at date t writes

$$\theta_i(t) \equiv \frac{v_i(t)}{u_i(t) + \sum_{j \neq i} g_{ij} u_j(t)}$$

Firms and workers meet randomly, meaning each unemployed worker searching in occupation i has an equal chance of meeting a firm in this occupation. The flow probability of matching with a firm in occupation i at t is given by: $p_i(t) = h_i(t) / \big(u_i(t) + \sum_{j \neq i} g_{ij} u_j(t)\big) = \zeta \theta_i(t)^{1-\alpha}$. Since workers simultaneously search for jobs in all adjacent occupations, their unemployment outflow rate from occupation i is $p_i(t) + \sum_{j \neq i} g_{ij} p_j(t)$.[22] Finally, vacancies in occupation i are filled with flow probability $q_i(t) = h_i(t)/v_i(t) = \zeta \theta_i(t)^{-\alpha}$.

Productivity shock. Consider the following scenario.

At $t = 0^-$, the economy is at steady-state with the productivity distribution denoted $\tilde{\boldsymbol{y}}$ and the worker distribution by $\boldsymbol{f}(0) = \tilde{\boldsymbol{f}}_\infty$, where $\tilde{\boldsymbol{f}}_\infty$ represents the pre-shock steady-state worker distribution. Hereafter, $\tilde{x}$ denotes pre-shock variables.

At $t = 0$, the economy experiences an unanticipated permanent productivity shock, changing the productivity distribution from $\tilde{\boldsymbol{y}}$ to $\boldsymbol{y}$. The productivity change for each occupation is represented by the vector $\boldsymbol{dy} = (dy_i)_{i \in \mathcal{N}}$, where dy_i is the productivity

[21] A new paper by Costa-Dias et al. (2021) explores the implications of labor market overlap for tightness measures.

[22] Workers do not match with more than one firm simultaneously due to the assumptions of continuous time and Poisson processes of job search.

shock to occupation i.[23]

4.2 Values

How does firm vacancy posting react to the productivity shock? This is a function of their value functions, detailed below.

Firm values. The value of a filled job for firms in occupation i at date t is denoted $J_i(t)$, and the value of an unfilled vacancy in occupation i at date t is denoted $V_i(t)$.

The value of a job filled $J_i(t)$ solves the Bellman equation

$$rJ_i(t) = y_i - w_i + s_i(V_i(t) - J_i(t)) + \dot{J}_i(t) \tag{3}$$

The firm collects the flow profits $y_i - w_i$. At rate s_i, the firm-worker match gets separated and the firm suffers the loss $V_i(t) - J_i(t)$. The value can vary over time during the transition dynamics.

The value of a vacant job $V_i(t)$ solves the Bellman equation

$$rV_i(t) = -c_i + q(\theta_i)(J_i(t) - V_i(t)) + \dot{V}_i(t) \tag{4}$$

The firm pays a flow cost c_i to post its vacancy. At rate q_i, the vacancy gets filled and the firm earns $J_i(t) - V_i(t)$. The value function can vary over time during the transition dynamics.

Free entry. Potential entrants can freely post vacancies in each occupation, driving the value of posting a vacancy to zero at all times:

$$V_i(t) = 0 \quad \implies \quad \frac{c}{q(\theta_i(t))} = J_i(t) \tag{5}$$

This condition determines the equilibrium tightness ratio in each occupation. Equilib-

[23] I focus on productivity shocks as Cortes et al. (2020) show that most of the decline in routine employment was driven by reduced job finding rates, attributed to productivity shocks in the model, rather than separation shocks.

rium tightness ratio equalizes the expected benefit of posting a vacancy – the value of a filled vacancy – and the expected cost of posting a vacancy – the flow cost over the expected waiting time.

Importantly, the model assumes the free entry condition holds *at all times*. This assumption is key for tractability but arguably quite restrictive, implying that adjustment in vacancy posting happen infinitely fast relative to other variables, which is counterfactual (see Elsby et al. (2015)).[24] However, allowing for slow vacancy adjustments would only result in slower responses to shocks.

Transition dynamics. In theory, the firm values and tightness ratios can vary during the transition dynamics. However, Proposition 1 shows they remain constant during the transition.

Proposition 1. (Job posting along the transition)

Firm values $J_i(t)$, tightness ratios $\theta_i(t)$ and job finding rates $p_i(t)$ are constant along the transition dynamics, and given by

$$J_i(t) = \frac{\phi y_i}{r + s_i} \qquad \theta_i(t) = \left(\frac{\zeta_i}{c_i}\left(\frac{(1-\phi)y_i}{r+s_i}\right)\right)^{\frac{1}{\alpha}} \qquad p_i(t) = \zeta^{\frac{1}{\alpha}}\left(\frac{(1-\phi)y_i}{c_i(r+s_i)}\right)^{\frac{1-\alpha}{\alpha}}$$

Firm values, tightness ratios, and job finding rates are "jump variables," adjusting instantly to new steady-state levels after productivity shocks. This extends a result from canonical search and matching models to the network case (see Pissarides (2000)).

This follows from the free entry assumption, which imposes that expected benefits and costs equalize at all times. When a productivity shock occurs, expected benefits from posting vacancies increase instantly. To maintain equilibrium, expected costs must rise simultaneously, achieved by an immediate increase in the tightness ratio, which increases the expected filling time.

[24] A nascent literature studies departures from free-entry in search and matching models (Den Haan et al. (2021)).

4.3 Laws of Motion

How do workers reallocate across occupations? First, I describe the laws of motion governing the evolution of the worker distribution. Then, I introduce a measure of aggregate speed of reallocation.

Laws of motion. The employment in occupation i at date t evolves as

$$\dot{e}_i(t) = p_i \left(u_i(t) + \sum_{j \neq i} g_{ij} u_j(t) \right) - s_i e_i(t) \tag{6}$$

Workers searching for jobs in occupation i are hired at the rate p_i from a pool of job seekers including unemployed workers from both their own and neighboring occupations $u_i(t) + \sum_{j \neq i} g_{ij} u_j(t)$. Workers in occupation i get separated at rate s_i.

The unemployment in occupation i at date t evolves as

$$\dot{u}_i(t) = s_i e_i(t) \quad - \left(p_i + \sum_{j \neq i} g_{ij} p_j \right) u_i(t) \tag{7}$$

Newly separated workers from occupation i enter unemployment at rate s_i, while unemployed workers find a job in their own and neighboring occupations at the rate $p_i + \sum_{j \neq i} g_{ij} p_j$.

Transition matrix. The laws of motion can be succinctly written as $\dot{\boldsymbol{f}}(t) = \boldsymbol{Q}\boldsymbol{f}(t)$, where $\boldsymbol{f}(t)$ is the worker distribution at date t and $\boldsymbol{Q}(t)$ is the transition matrix.[25]

$$\boldsymbol{Q} \equiv \begin{pmatrix} -s & 0 & 0 & p_1 & p_1 & 0 \\ 0 & -s & 0 & p_2 & p_2 & p_2 \\ 0 & 0 & -s & 0 & p_3 & p_3 \\ s & 0 & 0 & -(p_1 + p_2) & 0 & 0 \\ 0 & s & 0 & 0 & -(p_1 + p_2 + p_3) & 0 \\ 0 & 0 & s & 0 & 0 & -(p_2 + p_3) \end{pmatrix} \tag{8}$$

[25] The matrix $Q(t)$ is the continuous time equivalent of the discrete time transition matrix.

The upper-left and lower-left blocks represent job destructions, while the upper-right and lower-right blocks represent job creations. Worker mobility across occupations is reflected by positive off-diagonal terms in the upper-right block, while the absence of on-the-job search is shown by the null off-diagonal terms in the upper-left block.

The transition matrix contains all the necessary information to predict the evolution of the worker distribution.

Measure of transition speed. Measuring the speed of worker reallocation is challenging due to the high dimensionality of distributions. However, the heterogeneous agents literature in macroeconomics has developed valuable metrics for quantifying transition times of distributions (Gabaix et al., 2016; Alvarez and Lippi, 2022; Baley and Blanco, 2021; Beraja and Wolf, 2021).

Building on this work, I introduce a new definition of aggregate reallocation time, defined as the normalized cumulative distance to the steady state.

Definition 1. (Aggregate worker reallocation time)

The aggregate worker reallocation time T is defined as

$$T \equiv \frac{1}{\|f(0) - f_\infty\|_2} \int_0^{+\infty} \|f(t) - f_\infty\|_2 dt \tag{9}$$

where $\|f(t) - f_\infty\|_2 = \sqrt{\sum_k (f_k(t) - f_{k,\infty})^2}$.

The intuition is as follows. The integrand is the distance of the worker distribution to its steady-state. By integrating this distance over time, we obtain the cumulative distance to the steady-state distribution. A larger cumulative distance indicates that the distribution stayed far from the steady-state longer, signifying slower convergence. The normalization constant ensures the measure is not inflated merely by starting farther from the steady-state.[26]

As an illustration, consider the dynamics of unemployment in the canonical search

[26] This measure of transition time allows for closed-form solutions in higher-dimensions, unlike the half-life. Moreover, it can be biased, as it only focuses on the initial half of the transition dynamics and ignores slower transition thereafter.

and matching model. The unemployment rate evolves as $\dot{u}(t) = s(1 - u(t)) - pu(t)$. The general solution is $u(t) = u_\infty + (u(0) - u_\infty)e^{-(p+s)t}$, implying that a transition time of $T = \int_0^{+\infty} e^{-(p+s)t}\, dt = \frac{1}{p+s}$. This is the inverse of the exponential decay rate, a popular measure of transition time, corresponding to the time for 63% of the transition to occur.[27]

4.4 Characterizing Reallocation Speed

What are the determinants of worker reallocation speed? Determining worker reallocation time is challenging in the general case, as it involves computing eigenvalues and eigenvectors of large matrices, which typically lack closed-form solutions. However, the tractability of the simple model allows me to derive closed-form expressions.

Additionnal simplifying assumptions. Assume equal productivity y_P in periphery occupations and productivity y_C in the central occupation. Let p_C and p_P denote job finding rate in central and non-central occupations. Also, assume homogeneous separation rates, vacancy costs, and matching efficiencies s, c, ζ, with s small compared to job finding rates.

Asymmetric shocks. I focus on asymmetric shocks, such as trade or technology shocks, which impact occupations unevenly, with the employment-weighted sum of occupation-specific productivity shocks equaling zero.[28] Some occupations are negatively affected, while others benefit, leading to worker reallocation.

Two types of asymmetric shocks are examined: periphery and central shocks, defined as

$$\mathbf{dy}_P = (-\Delta, 0, \Delta) \quad \text{and} \quad \boldsymbol{dy}^C = (\frac{2p_C + p_P}{2p_C + 2p_P}\Delta, -\Delta, \frac{2p_C + p_P}{2p_C + 2p_P}\Delta)$$

where $\Delta > 0$ controls the magnitude of the shock. The periphery shock affects periphery occupations with opposite sign, while the central shock negatively impacts the central

[27] In one-dimension, we have $T = \log(2) \times t_{1/2}$, with $t_{1/2}$ the half-life, but this relationship is lost in higher dimensions.

[28] Aggregate shocks affect all occupations equally, whereas asymmetric shocks, like trade or technology shocks, are orthogonal to these and result in an employment-weighted sum of productivity shocks that equals zero.

occupation but benefits the periphery occupations. [29]

Worker reallocation time. Lemma 1 provides an approximation of worker reallocation time after periphery and central shocks.

Lemma 1. (Reallocation time after periphery and central shocks)

For small shock, reallocation times after periphery and central shocks are approximately

$$T_P \approx \left(1 + \frac{p_P}{p_C}\right) s^{-1} \quad \text{and} \quad T_C \approx s^{-1}$$

Equipped with this result, we can determine the reallocation time for any asymmetric shock. An asymmetric shock is defined as a shock such that the employment weighted occupation productivity shocks sum to zero. Assuming additionnally that productivity differentials are not too great, such that $p_C \approx p_P$, then any asymmetric shock can be written as a combination of these eigen-shocks: $\boldsymbol{dy} = \gamma^C \boldsymbol{dy}^C + \gamma^P \boldsymbol{dy}^P$.

Proposition 2. (Reallocation time after any asymmetric shock)

For small shocks, transition time after any asymmetric productivity shock is approximately

$$T \approx \alpha_P \cdot \left(1 + \frac{p_P}{p_C}\right) s^{-1} + (1 - \alpha_P) \cdot s^{-1}$$

where the weight on transition time after periphery shock α_P is equal to $\alpha_P \equiv \frac{|(p_P+p_C)\gamma_P|}{|(p_P+p_C)\gamma_P|+|\frac{(2p_P+p_C)^2}{p_P+p_C}\gamma_C|}$ *and where γ_P and γ_C are the loadings on periphery and central shocks.*

The transition time after asymmetric shocks is a weighted average of the transition times after periphery $\left(1 + \frac{p_P}{p_C}\right) s^{-1}$ and central shocks s^{-1}. The weights depend on the relative importance of productivity shocks to periphery versus central occupations.

There are three main take-aways. First, transition is very slow following asymmetric shocks. Indeed, transition time is of order s^{-1}, where the separation rate is typically 2.5% per quarter, predicting reallocation times of around 40 quarters. Intuitively, this is

[29] These productivity shocks are the *eigen-shocks* of the economy, as defined by Kleinman et al. (2023). The transition speed following an eigen-shock is determined by the associated eigenvalue.

because workers must wait to become unemployed before switching occupations.[30]

Notably, this transition time is an order of magnitude larger than in the canonical search and matching model, which predicts transition times of order p^{-1} (about 3 months for a 30% monthly job-finding rate in the US). This suggests that adding labor market heterogeneity can help match the persistence of shocks.

Second, the centrality of productivity shocks matters for reallocation speed. Transition times are longer after shocks to periphery occupations than after shocks to central occupations. This is because peripheral occupations provide fewer opportunities for displaced workers to transition to new jobs, resulting in slower reallocation processes.

Third, bridge occupations matter for worker reallocation speed. The central occupation's job finding rate has a *granular* effect on reallocation time: if it approaches zero, the transition time becomes arbitrarily large, while it remains finite if the job finding rate of periphery occupations approaches zero. Intuitively, bridge occupations can act as bottlenecks, especially when workers need to transit between periphery occupations.

4.5 Characterizing Welfare Effects

Does the speed of worker reallocation matter for welfare? This subsection examines the welfare effects of permanent asymmetric productivity shocks. First, I define a measure of welfare and adjustment costs. Second, I provide closed-form analytical characterization of adjustment costs after any asymmetric shock.

Defining welfare. Let define social welfare over the transition as the present discounted value of aggregate output net of vacancy costs

$$SW^{\text{slow}} \equiv \int_0^{+\infty} e^{-rt} \sum_{i \in \mathcal{N}} \left[e_i(t) y_i + (b_i - c_i \theta_i) u_i(t) \right] dt$$

The term $e_i(t)y_i + (b_i - c_i\theta_i)u_i(t)$ is the sum of worker and firm payoffs net of hiring costs

[30] On-the-job search would not significantly alter the result, as job-to-job transitions across occupations are infrequent.

in occupation i at date t.[31] Summing over occupation yields the *aggregate* flow welfare at date t, and multiplying by the discount factor and integrating over time provides the present value of the economy's welfare over the transition. The 'slow' upperscript denotes the fact that the worker distribution evolve slowly over time.

By contrast, the social welfare if the economy remains at the old steady-state writes $SW^{\text{old}} \equiv \frac{1}{r}\sum_{i\in\mathcal{N}}\left[\widetilde{e_{i\infty}}\widetilde{y}_i + (\widetilde{b}_i - c_i\widetilde{\theta}_i)\widetilde{u_{i\infty}}\right]$, where variables denoted by $\widetilde{x}$ represent their old steady-state level. Moreover, the social welfare if the economy immediatly jumps to the next steady-state writes $SW^{\text{new}} \equiv \frac{1}{r}\sum_{i\in\mathcal{N}}\left[e_{i\infty}y_i + (b_i - c_i\theta_i)u_{i\infty}\right]$.

Adjustment costs. The adjustment costs AC measure the percentage point reduction in welfare gains due to slow reallocation, relative to a counterfactual scenario where transition would be instantaneous.

$$AC \equiv \frac{SW^{\text{slow}} - SW^{\text{old}}}{SW^{\text{new}} - SW^{\text{old}}} - 1$$

The numerator measures the change in social welfare if transition is slow, while the denominator measures the change in social welfare if transition is instantaneous.[32] For instance, if the welfare change with slow transition is only a half of the welfare change with instantaneous-transition scenario, then $\frac{SW^{\text{slow}} - SW^{\text{old}}}{SW^{\text{new}} - SW^{\text{old}}} = 0.5$ and adjustment costs are equal to -50%.

Characterizing adjustment costs. First, I begin by providing closed-form analytical characterizations of adjustment costs after periphery and central shocks.

Proposition 3. (Adjustment cost after periphery and central shocks)

The adjustment cost after the periphery shock and central shocks can be approximated as

$$AC^P \approx -\frac{T^P}{r^{-1} + T^P} \quad \text{and} \quad AC^P \approx -\frac{T^C}{T^C + r^{-1}} \cdot \frac{p_C + p_P + p_P^2/(p_C + p_P)}{2p_C + 3p_P + p_P^2/(p_C + p_P)} \tag{10}$$

[31] Note that wages cancel out since they are a transfer, and hiring costs can be written $c_i v_i(t) = c_i\theta_i u_i(t)$.

[32] This definition of adjustment costs aligns closely with that in Caliendo et al. (2019), who define adjustment costs as $AC = \log\frac{SW^{\text{slow}} - SW^{\text{old}}}{SW^{\text{new}} - SW^{\text{old}}}$. Both definitions are approximately equivalent when $\frac{SW^{\text{slow}} - SW^{\text{old}}}{SW^{\text{new}} - SW^{\text{old}}} \approx 1$. An advantage of this approach is that it allows for closed-form solutions.

where $T^P \equiv \frac{p_P+p_C}{sp_C}$ and $T^C \equiv \frac{1}{s}$ are the transition times after periphery and central shock, resp.

There are two main take-aways. First, these expressions establish a direct and positive connection between the magnitude of adjustment costs and the speed of reallocation after periphery and central shocks. Longer reallocation times lead to larger adjustment costs.

Second, adjustment costs can be very large after periphery shocks. Intuitively, this is because misallocation is particularly severe along the transition, with labor shortages and productivity shocks being perfectly negatively correlated. As a result, delays in adjustment are very costly, leading to large adjustment costs. For example, if reallocation time is very long, adjustment costs approach -1, indicating a complete reduction in welfare gains compared to an instantaneous transition.

In contrast, adjustment costs are lower after central shocks because transitory misallocation is less severe, with labor shortages and productivity shocks not perfectly anticorrelated. Adjustment costs are bounded from below by a misallocation term and cannot drop to -100%.

Equipped with this, I am now ready to derive an analytical characterization of adjustment costs after *any* asymmetric shock.

Proposition 4. (Adjustment cost after any asymmetric shock)

The adjustment cost after any asymmetric productivity shock is approximatively

$$AC \approx \Lambda_P \underbrace{\frac{-T^P}{r^{-1}+T^P}}_{=AC^P} + (1-\Lambda_P)\underbrace{\frac{-T^C}{r^{-1}+T^C}\frac{p_C+p_P+p_P^2/(p_C+p_P)}{2p_C+3p_P+p_P^2/(p_C+p_P)}}_{=AC^C} \tag{11}$$

where T^P and T^C are the transition times after the periphery and central shock, respectively, and where the weight on periphery adjustment costs Υ_P writes $\Lambda_P \equiv \frac{\gamma^P 2(2p_C+p_C)}{\gamma^P 2(2p_C+p_C)+\gamma^C(2p_C+3p_P+{p_P}^2/(p_C+p_P))}$.

This means that the adjustment costs after any asymmetric shock is a weighted average of adjustment costs after central and periphery shock. The weight depends on the relative importance of the periphery and central shocks, γ^P and γ^C respectively, in the shock decomposition.

An important implication is that shock centrality matters for the size of adjustment costs. Adjustment costs are larger after shocks to periphery occupations, than shocks to central occupation. This is for two reasons: after periphery shocks, transition is slower and misallocation is more severe along the transition.

5 A Quantitative Model of Network Search

This section develops a quantitative version of the network job search model.

While the tractability of the stylized model allowed for an analytical characterization of reallocation time, it came at the cost of assuming a simplistic occupation network structure and rather mechanical worker behavior. The quantitative model lifts these restrictions, allowing for an arbitrary occupation network and strategic decision-making in response to shocks.

5.1 Extensions

Occupation structure. There are N distinct occupations, indexed by $i \in \mathcal{N}$, connected by the edges of the occupation network. The network structure is captured by the adjacency matrix $\boldsymbol{G} = (g_{ij})_{(i,j)\in\mathcal{N}^2}$, where $g_{ij} \in (0,1)$ represents the probability that workers in occupation i can learn the skills of occupation j. The matrix $\boldsymbol{G}$ is asymmetric ($g_{ij} \neq g_{ji}$), with each occupation perfectly accessible to itself ($g_{ii} = 1$ for all i).[33]

Workers. Employed workers can search *on-the-job*, but have less time for search compared to unemployed workers ($\eta_i < 1$). As a result, firm-worker matches can end due to exogenous separation shocks or because the worker found a job in another occupation.

Workers *choose their search effort*, allowing them to partially direct their search toward certain occupations. Unemployed workers have 1 unit of time, while employed workers

[33] The occupation network is strongly connected, ensuring a positive worker population in all occupations over time.

have η_i. The search time in occupation j by workers in i and state $x \in \{e, u\}$ is denoted τ_{ij}^x. Search time translates into effective search effort, σ_{ij}^x, according to:

$$\sigma_{ij}^x = \tau_{ij}^{x\,\frac{\psi}{\psi+1}}$$

where $\psi \geq 0$ measures the degree of concavity of the search effort technology. The assumption of decreasing returns to search captures the idea that workers initially contact a few salient firms, but subsequent contacts require more time, resulting in fewer applications per hour invested.

Wage determination. Workers and firms negotiate wages using a generalized Nash bargaining rule, where the worker bargaining strength ϕ_i determines the surplus split.

Wages are continuously renegotiated during the transition. The model abstracts from wage-setting frictions, such as downward wage rigidity or infrequent negotiations. However, adding these frictions would slow down reallocation dynamics even further.

Search frictions. The details of the search frictions are provided in Appendix D. Importantly, the transition rate of workers in occupation i and state x to employment in occupation j date t writes

$$\mu_{ij}^x(t) \equiv g_{ij}\sigma_{ij}^x(t)p_j(t)$$

where $\sigma_{ij}^x(t)$ is the search effort, and $p_j(t)$ the the job finding rate per unit of search effort.

The transition rate is the product of three terms: (1) the skill friction g_{ij} reflects whether the transition is *feasible*, (2) the search effort σ_{ij}^x reflects whether the transition is *desirable* for workers, while (3) the job finding rate per unit of search $p_j(t)$ reflects whether the transition is *desirable* for firms (i.e., their hiring rate).

5.2 Values

How do workers allocate their search efforts? How are wages negotiated, and how does it affect vacancy posting? Workers' and firms' decisions depend on their values, which I define now.

Workers values. The value of being employed in occupation i at date t is denoted $E_i(t)$, and the value of being unemployed in occupation i at date t is denoted $U_i(t)$. The value of being unemployed from occupation i at date t $U_i(t)$ solves

$$rU_i(t) = \max_{\{\tau^u_{ij}(t)\}_j \geq 0} b_i + \sum_{j \in \mathcal{N}} \mu^u_{ij}(t)(E_j(t) - U_i(t)) + \dot{U}_i(t) \tag{12}$$
$$\text{s.t.} \quad \mu^u_{ij}(t) = g_{ij} p_j(t) \tau^u_{ij}(t)^{\frac{\psi}{\psi+1}} \text{ and } \sum_j \tau^u_{ij}(t) = 1$$

Unemployed workers from occupation i receive flow unemployment benefits b_i and allocate their search time τ^u_{ij} to maximize the expected value of being hired in their own or adjacent occupations. They match with firms in j at the rate μ^u_{ij}, in which case they earn $E_i(t) - U_i(t)$, hereafter the switching gain. The value function can vary over time during the transition dynamics.

The value of being employed in occupation i at date t $E_i(t)$ solves the Bellman equation

$$rE_i(t) = \max_{\{\tau^e_{ij}(t)\}_j \geq 0} w_i(t) + \sum_{j \in \mathcal{N}} \mu^e_{ij}(t)(E_j(t) - E_i(t)) + s_i(U_i(t) - E_i(t)) + \dot{E}_i(t) \tag{13}$$
$$\text{s.t.} \quad \mu^e_{ij}(t) = g_{ij} p_j(t) \tau^e_{ij}(t)^{\frac{\psi}{\psi+1}} \text{ and } \sum_j \tau^e_{ij}(t) = \eta_i$$

Employed workers earn a flow wage $w_i(t)$ and allocate their search time to maximize the expected value of being hired, but have less time to search than unemployed workers ($\eta_i < 1$). They match with firms in j at rate $\mu^e_{ij}(t)$ and gain $E_j(t) - E_i(t)$. At the exogenous rate s_i, the firm-worker match is separated, and the worker becomes unemployed. The value function can vary over time during the transition dynamics.

Optimal search times. The optimal search times, derived from the first-order conditions of the Bellman equation, are

$$\tau^u_{ij}(t) = \frac{\left(g_{ij} p_j(t) \max\{E_j(t) - U_i(t), 0\}\right)^{\psi}}{\sum_k \left(g_{ik} p_k(t) \max\{E_k(t) - U_i(t), 0\}\right)^{\psi}} \quad \tau^e_{ij}(t) = \frac{\left(g_{ij} p_j(t) \max\{E_j(t) - U_i(t), 0\}\right)^{\psi}}{\sum_k \left(g_{ik} p_k(t) \max\{E_k(t) - U_i(t), 0\}\right)^{\psi}} \tag{14}$$

Workers exert higher search time to occupations with (1) high skill similarity g_{ij}, (2) high job finding rate $p_j(t)$ or (3) high switching gains, $E_j(t) - U_i(t)$ or $E_j(t) - E_i(t)$. Conversely, they do not exert effort in unconnected occupations, or where switching would result in a loss.

The parameter ψ represents the elasticity of search effort to expected gains from switching occupations. This formulation encompasses random and directed search as special cases: when ψ approaches zero, workers search randomly across occupations, whereas when ψ approaches infinity, workers fully direct their search effort towards the highest-return occupations. This parameter governs how workers adjust their search in response to economic shocks.

This specification mirrors choice probabilities in discrete choice models. However, there are two main differences. First, effective search efforts σ_{ij}^x do *not* sum to one, allowing workers in high-return occupations to search more. Second, the Bellman equation is linear in payoffs, significantly simplifying the estimation of values. Further discussion on this comparison can be found in Appendix D.

Firm values. The Bellman expressions for firm values are similar to those in the stylized model and are therefore relegated to Appendix D. The main difference is that the value of a filled job now accounts for the possibility of endogenous job separation due to worker on-the-job search.

Nash bargaining. When a firm and a worker meet, they bargain over the wage. At any date t, the negotiated wage in occupation i, denoted $w_i(t)$, maximizes the generalized Nash product

$$\max_{w_i(t)} \left(E_i(t) - U_i(t)\right)^{\phi_i} \left(J_i(t) - V_i(t)\right)^{1-\phi_i}$$

Importantly, the worker's outside option is the value of being unemployed from occupation i onwards, regardless of previous occupations or employment status. This reflects that wages are negotiated only after the worker has transitioned to a new occupation, consistent with the idea that only *ex post* negotiations are credible in a world of incomplete contracts. This assumption ensures analytical tractability by making all workers applying in occupation i have the same outside options and receive the same wage.[34]

The negotiated wage can be expressed as

$$w_i(t) = \phi_i y_i + (1-\phi_i)\bar{w}_i \quad \text{with } \bar{w}_i \equiv \underbrace{b_i + \sum_j \mu_{ij}^u(t)(E_j(t) - U_i(t))}_{\text{outside option}} - \underbrace{\sum_j \mu_{ij}^e(t)(E_j(t) - U_i(t))}_{\text{option value of on-the-job}} \tag{15}$$

The proof is detailed in Appendix D.

The wage $w_i(t)$ is a weighted average of the worker-firm productivity y_i and the worker's reservation wage $\bar{w}_i(t)$, with the weight determined by worker's bargaining power ϕ_i. In other words, Nash bargaining strikes a balance between the maximum wage acceptable to the firm (productivity y_i) and the minimum wage acceptable to the worker (reservation wage $\bar{w}_i$).

The worker's reservation wage $\bar{w}_i$ has two components: the outside option, which raises the wage, and the option value of future on-the-job search, which leads workers to accept a lower current wage in anticipation of better future prospects. However, the bargaining channel always dominates, raising wages, since $\mu_{ij}^u(t) \geq \mu_{ij}^e(t)$.

A key new implication is that the reservation wage now depends on job prospects in adjacent occupations within the network, creating spillover effects across occupations. A wage increase in one occupation raises the reservation wages in neighboring occupations, enhancing their negotiated wages. In the appendix, I show that wages are proportional to occupations' Katz-Bonacich centrality in a certain network, implying that centrality increases bargaining power and wages.

Free entry. As in the stylized model, potential entrants can freely post vacancies in each occupation, driving the value of posting a vacancy to zero at all times: $V_i(t) = 0$.

A new implication is that tightness ratios are interconnected across occupations through the reservation wage channel, leading to negative spillovers in vacancy postings.

[34] Otherwise, wages would depend on both the current and previous occupations, w_{ij}, leading firm and worker values to depend on both—breaking the block-recursive structure of the model.

Higher vacancy postings in neighboring occupations improve workers' search prospects, raising their reservation wages and potentially reducing the firm's incentive to post vacancies.

5.3 Law of Motion

The laws of motion of worker employment and unemployment closely resemble those in the stylized model and are therefore relegated to Appendix D.

Importantly, the worker distribution's law of motion still writes $\dot{\boldsymbol{f}}(t) = \boldsymbol{Q}(t)\boldsymbol{f}(t)$, but where the quantitative transition matrix $\boldsymbol{Q}(t)$ has a slightly more complex expression:

$$\boldsymbol{Q}(t) = \begin{pmatrix} \underbrace{\text{diag}\big(\boldsymbol{p}\big)\big(\boldsymbol{G} \odot \boldsymbol{\sigma}^{\boldsymbol{e}}\big)'\text{diag} - \text{diag}\big(\big(\boldsymbol{G} \odot \boldsymbol{\sigma}^{\boldsymbol{e}}\big)\boldsymbol{p}\big) - \text{diag}\big(\boldsymbol{s}\big)}_{\text{e} \to \text{e}} & \underbrace{\text{diag}\big(\boldsymbol{p}\big)\big(\boldsymbol{G} \odot \boldsymbol{\sigma}^{\boldsymbol{u}}\big)'}_{\text{u} \to \text{e}} \\ \underbrace{\text{diag}\big(\boldsymbol{s}\big)}_{\text{e} \to \text{u}} & \underbrace{-\text{diag}\big(\big(\boldsymbol{G} \odot \boldsymbol{\sigma}^{\boldsymbol{u}}\big)\boldsymbol{p}\big)}_{\text{u} \to \text{u}} \end{pmatrix} \tag{16}$$

where $\odot$ denotes matrix element-wise multiplication.

First, the upper-left block now has non-zero diagonal elements, reflecting on-the-job search by employed workers. Second, the upper-right and lower-right blocks incorporate worker search efforts, accounting for the semi-directed nature of job search

5.4 Definition of the Equilibrium

Definition of the equilibrium. *The equilibrium is a collection of time-varying value functions $\{E_i(t)\}_{i\in\mathcal{N}}$, $\{U_i(t)\}_{i\in\mathcal{N}}$, $\{J_i(t)\}_{i\in\mathcal{N}}$, $\{V_i(t)\}_{i\in\mathcal{N}}$, time-varying search times $\{\tau^e_{ij}(t)\}_{(i,j)\in\mathcal{N}^2}$, $\{\tau^u_{ij}(t)\}_{(i,j)\in\mathcal{N}^2}$, time-varying wages $\{w_i(t)\}_{i\in\mathcal{N}}$, time-varying labor market tightness ratios $\{\theta_i(t)\}_{i\in\mathcal{N}}$ and a time-varying distribution of workers $\boldsymbol{f}(t)$, such that:*

i) The value functions satisfy the worker and firm Bellman equations

ii) The search times satisfy the first order conditions of the Bellman equations

iii) The wages satisfy the Nash bargaining condition at all dates *iv) The tightness ratios are such that the free entry condition holds at all dates*

v) The distribution of workers evolves according to the matrix law of motion

Block recursive equilibrium. The equilibrium in this model is block recursive, as defined by Menzio and Shi (2010) and Menzio and Shi (2011). Block recursivity means that the agents' value and policy functions do not depend on the worker distribution. Block recursivity is crucial for maintaining the tractability of models with heterogeneous agents.

This gives me my solution strategy. First, I solve for the agents' optimal actions and values. This determines the equilibrium transition rates. Second, I solve for the dynamics of worker reallocation, given the previously determined equilibrium matrix of transition rates.

Stability, existence and unicity of equilibrium. Although, in theory, worker and firm value and policy functions might vary during the transition, Appendix D provides a sufficient condition under which these functions must remain constant throughout the transition.

Furthermore, I show that a steady-state equilibrium always exists. Under additional assumptions, detailed in appendix D, which can be verified numerically, this equilibrium is also unique.

5.5 Dynamic Hat-Algebra

This subsection solves for counterfactuals following small permanent productivity shocks.

Solving for counterfactuals in this quantitative model is challenging due to the number of variables and their complex interactions. Moreover, it requires estimating a large number of parameters. To address this, I build on the 'hat algebra' method from trade, which drastically reduces the number of parameters to estimate (Jones (1965), Dekle et al. (2008), Caliendo et al. (2019)). Here, I extend dynamic hat algebra to incorporate search frictions. In what follows, $\hat{x} \equiv dx/x$ denotes log-changes.

Proposition 5. (Dynamic Hat-Algebra)

Following small productivity shock $\boldsymbol{dy}$*, the approximate hat-changes in values and actions solve*

$$\hat{E}_i = \frac{1}{rE_i}\left(w_i\hat{w}_i + \sum_j \mu^e_{ij}\Delta^e_{ij}\left(\hat{\mu}^u_{ij} + \hat{\Delta}^e_{ij}\right)\right) \qquad \hat{U}_i = \frac{1}{rU_i}\left(\sum_j \mu^u_{ij}\Delta_{ij}\left(\hat{\mu}^u_{ij} + \hat{\Delta}^u_{ij}\right)\right)$$

$$\hat{\mu}^u_{ij} = (\psi+1)\hat{p}_j + \psi\hat{\Delta}^u_{ij} - \psi\sum_j \sigma^u_{ij}(\hat{p}_k + \hat{\Delta}^u_{ik}) \qquad \hat{\mu}^e_{ij} = (\psi+1)\hat{p}_j + \psi\hat{\Delta}^e_{ij} - \psi\sum_j \sigma^e_{ij}(\hat{p}_k + \hat{\Delta}^e_{ik})$$

$$\hat{w}_i = \frac{1}{w_i}\left(\phi dy_i + (1-\phi)\sum_j \mu^u_{ij}\Delta^u_{ij}\left(\hat{\mu}^u_{ij} + \hat{\Delta}^u_{ij}\right) - (1-\phi)\sum_j \mu^e_{ij}\Delta^u_{ij}\left(\hat{\mu}^e_{ij} + \hat{\Delta}^u_{ij}\right)\right) \qquad \hat{p}_i = \frac{1-\alpha}{\alpha}\hat{\Delta}^u_{ii}$$

where $\Delta^u_{ij} \equiv (E_j - U_i)$ and $\Delta^e_{ij} \equiv (E_j - E_i)$ denote switching gains. Proof in Appendix D.

The first two equations describe how worker values react to changes in agents' actions. The other four describe how agents' actions react to changes in values and baseline actions.

Several key elasticities determine the magnitude of equilibrium changes. The search elasticity ψ controls how transition rates respond to changes in worker values, while the matching elasticity α influences how job-finding rates adjust to variations in worker values. Worker bargaining power ϕ affects how wages respond to changes in both productivity and worker values.

An important implication is that, given baseline transition rates $(\mu^u_{ij})_{i,j}, (\mu^e_{ij})_{i,j}, (s_i)_i$, baseline payoffs $(w_i)_i, (b_i)_i$ and a vector of productivity shocks (dy_i), only the discount rate r and three elasticities ψ, α, ϕ are needed to compute counterfactuals.

6 Estimation

This section presents the estimation and calibration strategy for the key model parameters.

Few parameters are needed to solve counterfactuals. The search elasticity, which determines how workers reallocate their search in response to shocks, can be estimated using a reduced-form regression with structural interpretation. This method leverages the gravity structure of worker transitions across occupations, making the estimation both efficient and transparent. The other parameters are calibrated.

Discount rate and replacement rate. I begin by setting the discount rate r at a quarterly value of 3%, corresponds to a time horizon of 8 years as in Le Barbanchon et al. (2021). The replacement rate b is calibrated at 40%, which falls within the mid-range of values reported in the literature, spanning from 0.1 to 0.9 (e.g. Hagedorn and Manovskii (2008), Conlon et al. (2018)).[35]

Given worker payoffs, transition rates and the discount rate, the worker value functions can be computed for each quarter t. The vector of worker values at quarter t, denoted $\boldsymbol{V}(t) = (E_1(t), \ldots, E_N(t), U_1(t), \ldots U_N(t))'$, is

$$\boldsymbol{V}(t) = (r\boldsymbol{I} - \boldsymbol{Q}'(t))^{-1} \begin{pmatrix} \boldsymbol{w}(t) \\ \boldsymbol{b}(t) \end{pmatrix}$$

The intuition is that worker values are the discounted sum of future income flows, accounting for transitions to other occupations and employment states. Assuming workers do not anticipate further shocks, expected future incomes and expected future transition probabilities are equal to today's, and therefore worker values can be computed.[36]

From this, the switching gains for employed and unemployed workers can be computed as $\boldsymbol{\Delta}^e(t) = (E_j(t) - E_i(t))_{,ij \in \mathcal{N}^2}$ and $\boldsymbol{\Delta}^u(t) = (E_j(t) - U_i(t))_{i,j \in \mathcal{N}^2}$.

Search elasticity. To estimate search elasticity ψ, I study how worker occupation transitions respond to changes in switching gains, controlling for confounding factors.

The model predicts that worker flows have a gravity structure:

[35] There is no consensus on the calibration of flow unemployment values. Shimer (2005) sets b at 40% for the US based on monetary benefits (in France, the replacement rate was approximately 70% of net wages at that time). However, this approach ignores non-monetary factors like leisure (utility) and social stigma (disutility). A low replacement rate increases job search incentives, speeding up reallocation, providing a lower bound on reallocation speed.

[36] Each quarter a new productivity shock hits, resulting in a new equilibrium with a distinct vector of worker values.

$$\log \mu^u_{ijt} = (\psi + 1) \log g_{ij} + (\psi + 1) \log p_{jt} - \log N^u_{it} + \psi \log \Delta^u_{ijt}$$

where μ^u_{ijt} is the transition rate, g_{ij} is the skill friction, $p_j(t)$ is the job finding rate, N^u_{it} is a normalization constant, and Δ^x_{ijt} is the switching gain.[37]

The search elasticity ψ could ideally be estimated directly by regressing switching gains on observed variables and extracting the relevant coefficient. However, many elements such as skill frictions and job finding rates are not directly observable. Omitting these factors introduces a missing variable bias, as worker values are correlated with skill frictions and job finding rates.

Building on the similarity with the gravity framework, I control for these unobserved terms with fixed effects (FE): time-invariant origin-destination FE control for skill frictions, while time-varying destination FE control for job finding rates. However, the number of fixed effects can become too large, leading to non-robust estimates, in particular because of the large number of time-invariant origin-destination fixed. To mitigate this, I follow Head and Mayer (2014) recommendation to first demean along the time dimension, before running a two-way fixed effect Poisson regression. Further details are provided in Appendix E.

I find a value of 0.09, much lower than Caliendo et al. (2019)'s quarterly estimate of 0.2. This means a 1% increase in switching gains results in a 0.09% increase in worker transitions, all else equal. This lower estimate might be due to the fact that I do not model explicitly the geographical frictions, leading to an attenuation bias.

Matching elasticity and worker bargaining power. I calibrate the matching elasticity and worker bargaining power α at $\phi = 0.5$, a standard calibration value in macro-labor models.[38] [39]

[37] In seminal papers, Artuç and McLaren (2015) and Cortes et al. (2020) also exploit the gravity structure of worker flows to estimate occupation switching costs. I extend their approach by incorporating search frictions.

[38] This corresponds to the Hosios condition, which ensures efficiency in search & matching models. Interestingly, the model predicts new sources of inefficiency as firms and workers do not internalize congestion effects on *neighboring* occupations. But the Hosios condition still holds approximately.

[39] Lacking job vacancy data, I cannot estimate matching elasticity directly. Ideally, this would involve

7 Application: Robots

This section uses the estimated quantitative model as a 'laboratory' to explore the effects of the robots on the French labor market. First, I calibrate the robot shock. Then, I study the labor market adjustments after the shock. Finally, I simulate policy interventions and assess their effectiveness in accelerating worker reallocation.

7.1 Calibration of the Robot Shock

The robot shock is modelled as a permanent unexpected shock to the productivity vector. It is decomposed into two components: an aggregate shock, representing economy-wide productivity gains, and an asymmetric shock, reflecting the relative effects of technology across occupations.

Calibration of productivity shocks. To calibrate the asymmetric productivity shocks, I use Webb (2019) measure of occupation exposure to robots. I first apply Webb's estimated effect of robot exposure on wages to predict wage changes.[40] I then infer productivity changes to match the predicted wage response. Details are in Appendix E.

The aggregate part of the productivity shock is calibrated to match the 3.5% aggregate productivity gain reported by Acemoglu and Restrepo (2022) for robots and software in the US from 1980-2016.[41] Since I focus on robots and my data starts in 2009, I calibrate the aggregate productivity gains at approximately 1%.

Most affected occupations. Which occupations are most affected by robots? The "declining" occupations, negatively impacted by robots, are those where tasks involve moving objects, such as construction operators, rail operators and industrial unskilled workers. In contrast, the "expanding" occupations, positively affected by robots, involve

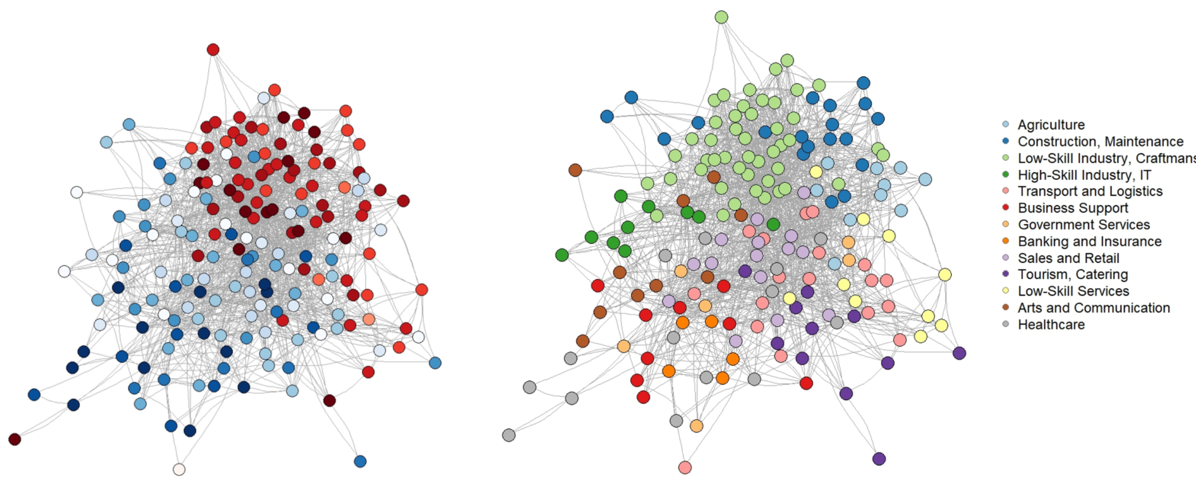
regressing hires on vacancies and job seekers within an occupation. Note that the model includes job seekers from adjacent occupations, and not controlling for this may introduce measurement bias.

[40] Ideally, I would estimate these effects directly, but the short time span of my data precludes such analysis.

[41] Recovering the GE effects of robot would require a more complex model, beyond the scope of this study.

socio-linguistic skills, including interpreters, telemarketers, journalists and social workers.

Figure 7.1: **Expanding (blue) vs. declining (red) occupations in the occupation network**



a) Expanding vs. Declining b) Labels

Notes: The occupation network is aggregated at the level of 3-digit FAP occupations.

Figure 7.1 plots the location of declining and expanding occupations within the network of occupation after the robot shock. It show that expanding and declining occupations are very clustered, forming two clearly divided groups of occupations with little overlap between them. This suggests that the robot shock strongly resembles the periphery shock from the toy model.

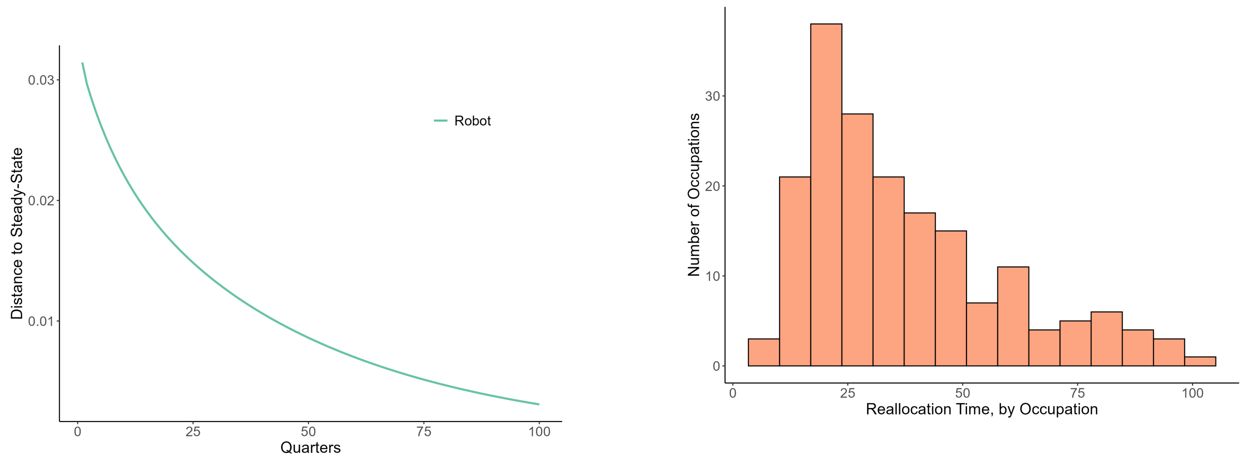
7.2 Transition Dynamics

How do French labor markets adjust to the introduction of robots? In this subsection I simulate the economy's transition dynamics after the robot shock. The baseline is calibrated at the time-average of the French economy over 2009-2012.

Aggregate reallocation times. How fast do workers reallocate following the robot shock? I find that the worker distribution converges quite slowly to its new steady-state distribution.

Figure 7.2.a shows the dynamics of the worker distribution's distance to its steady

Figure 7.2: **Characterizing the worker reallocation transition dynamics**



(a) Aggregate distance to steady-state

(b) Reallocation time, by occupation

state after the robot shock. The results indicate slow reallocation, with an aggregate reallocation time of approximately 39 quarters, or 10 years. Interestingly, this reallocation time aligns closely with predictions from the simple model, suggesting that the quantitative extensions balance each other. A larger occupation network may slow reallocation by requiring workers to move to more distant occupations, while richer search behaviors—such as on-the-job and semi-directed search—allow workers to switch occupations without waiting for unemployment, thus speeding up reallocation.

Although the predicted reallocation time might seem short, it should be considered a lower bound. Simplifying assumptions—such as continuously renegotiated wages, free entry in vacancy creation, and immediate skill acquisition—likely overestimate the speed of adjustment.

Heterogeneous reallocation times. Do all occupations adjust at the same speed? The moderate aggregate reallocation time actually masks substantial heterogeneity across occupations.

To capture this, let define reallocation time at the *occupation level* as

$$T_i = \frac{1}{\max_h |m_i(h) - m_i|} \int_0^{+\infty} |m_i(t) - m_i| dt$$

where $m_i(t) \equiv e_i(t) + u_i(t)$ is the mass of workers in occupation i at t, and m_i its steady-state level. This metric measures the cumulative distance of an occupation's population from its steady state.

Figure 7.2.b plots the distribution of occupation-specific worker reallocation times, measured in quarters. The results reveal substantial heterogeneity in transition times across occupations. While most occupations adjust relatively quickly to their new steady-state worker levels—with a median reallocation time of 25 to 40 quarters—a small minority experience very long transition times, exceeding 80 quarters. These occupations include retail assistants, security guards, skilled warehouse worker. Intuitively, this is because occupations at the center of the occupation network experience persistent flows until the entire reallocation process is complete, leading to prolonged adjustment periods.[42]

Adjustment costs. How does the robot shock affect social welfare? Does the slow reallocation dynamics lead to adjustment costs? I find that flow social welfare adjusts slowly after the robot shock, leading to large adjustment costs of around -40%.

Figure 7.3: **Flow social welfare over time**

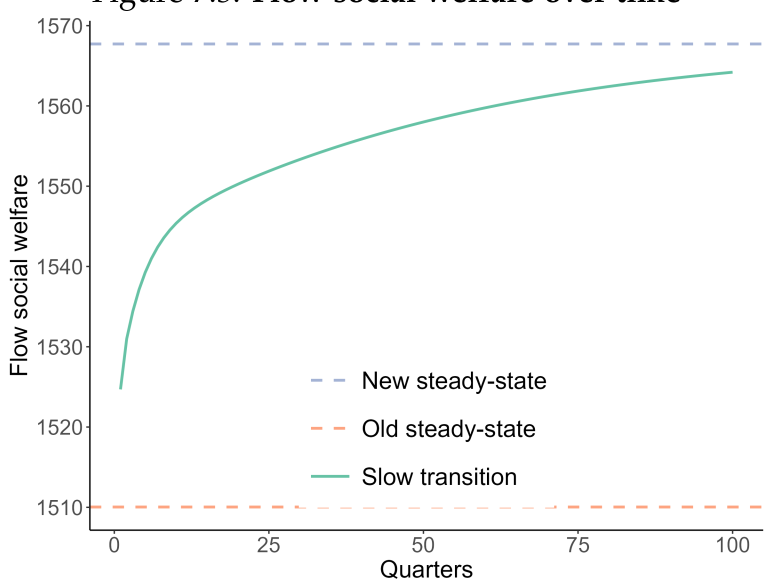


Figure 7.3 shows the evolution of flow social welfare over time under three scenarios.

[42]Results on unemployment are presented in Appendix F. Interestingly, I find that aggregate unemployment decreases significantly after the shock – even though the aggregate productivity shock is small –, and the adjustment is fast. However, this masks substantial heterogeneity across occupations in terms of speed of adjustment, with a minority of occupations adjusting very slowly.

The upward-sloping curve represents social welfare in the slow transition case, where workers reallocate gradually. The top horizontal line represents social welfare in the case where the worker distribution jumps immediately to its new steady state, while the bottom line shows the case where the robot shock does not occur. The figure illustrates that social welfare adjusts slowly, converging toward its new steady-state level.

This slow reallocation results in large adjustment costs, measured graphically as the area between the top line and the upward-sloping curve. I find that adjustment costs amount to approximately -40%, much higher than previous estimates of comparable shocks. For instance, Caliendo et al. (2019) estimate adjustment costs from the China shock at around -3.5%. One reason for this discrepancy is the inclusion of search frictions. Since unemployed workers are not immediately matched with firms, staying unemployed while waiting to transition to new jobs is costly and leads to welfare losses. This highlights the importance of accounting for search frictions when measuring welfare gains from shocks, as ignoring these frictions could overestimate the benefits of technological advances by nearly a factor of two.

7.3 Policy Experiment

How can policymakers speed up worker reallocation? In this subsection, I simulate the effect of different policy interventions on reallocation speed.

Employment subsidies. I examine the effect of targeted employment subsidies on worker reallocation speed. These subsidies, commonly used to support specific groups like young or low-education workers, involve the government paying part of workers' wages when they find a job. Since economic incidence often differs from legal incidence, I model subsidies as an exogenous productivity increase, shared between workers and firms through Nash bargaining.

Subsidies can be targeted to specific occupations under three schemes: (1) declining occupations, negatively impacted by shocks (e.g., unskilled manufacturing workers); (2) expanding occupations, positively impacted (e.g., telemarketers); and (3) bridge occupations, those linking declining and expanding occupations (e.g., maintenance technicians, sales representatives, and sports instructors).

The subsidy is set at 5% of pre-shock productivity for targeted occupations, normalized to ensure comparability across policy schemes. More details are in Appendix F.

Effect on reallocation speed. Which targeted policy interventions are the most efficient at speeding up worker reallocation? To answer this, I simulate a scenario where policymakers permanently distribute targeted employment subsidies after a technology shock, calculate the counterfactual reallocation time, and compare the change to the baseline.

Table 7.1: **Change in reallocation time after different targetings, in p.p.**

	Declining	**Expanding**	**Bridges**
$\Delta \log T$	0.82 %	-0.73 %	-2.28%

Table 7.1 shows the percentage-point changes in reallocation times for different targeting strategies. There are two main takeaways. First, targeting declining occupations makes worker reallocation *longer*, as displaced workers stay inefficiently long in their declining occupations. Second, targeting both expanding and bridge occupations speeds up reallocation, with bridge-targeting being three times more effective than targeting expanding occupations.

It is not immediately obvious why targeting bridge occupations is more effective than targeting expanding occupations. Conventional thinking suggests that targeting jobs with labor shortages (e.g., web developers) would boost wages and job postings, encouraging workers to switch. However, this overlooks the role of skill frictions: workers may lack the skills needed for expanding occupations, limiting their ability to switch, even with higher wages. By targeting bridge occupations, policymakers enable workers to reskill into jobs with skills similar to those in expanding occupations, broadening the pool of workers who can transition effectively.

This shows that leveraging the occupation network structure allows policymakers to design more effective strategies. Identifying and targeting bridge occupations can significantly accelerate reallocation following large structural shocks.

8 Conclusion

This paper introduces a novel network-based framework for understanding labor market reallocation in response to technology shocks. By modeling skill frictions as connections within a network of occupations, I provide new insights into the slow dynamics of worker reallocation and its welfare implications. The analysis highlights the critical role of bridge occupations in shaping reallocation speed, and the significant adjustment costs due to slow transitions.

These findings have important implications for labor market dynamics in the context of technological change, trade disruptions, and the green transition. Targeting bridge occupations with policy interventions can significantly accelerate reallocation and reduce adjustment costs. This framework offers a valuable tool for policymakers aiming to mitigate the effects of labor market shocks.

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9 Appendix A: The Occupation Network

9.1 Alternative Algorithms to Detect Community Structure

Several algorithms have been developed to detect community structures in large networks (e.g. Louvain, Walktrap, Label Propagation, or Edge Betweenness). These methods, either divisive or constructive, aim to maximize metrics like modularity, which compares the actual number of edges within clusters to the expected number in a random network.[43] For example, the Louvain algorithm starts by assigning each node to its own community, then iteratively reassigns nodes to maximize modularity until a local maximum is reached, defining community boundaries. Below is a plot of the occupation network, with nodes colored by their community as identified by the Louvain algorithm.

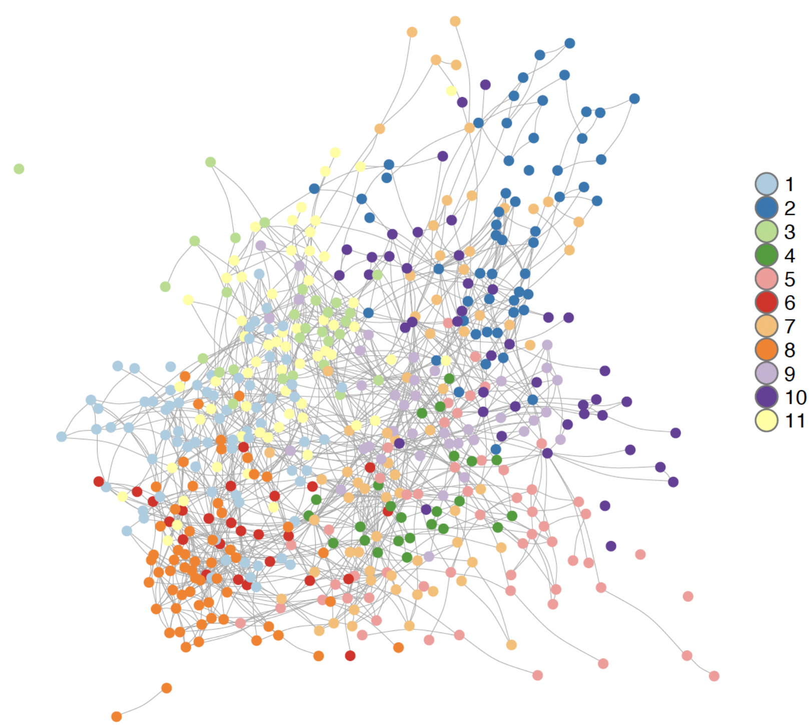


Figure 9.1: The occupation network with communities found with Louvain algorithm

[43] Values above 0.3 indicate significant community structure.

9.2 Bridge Occupations

An occupation is classified as a bridge if its clustering coefficient is in the lowest decile and its betweenness centrality is in the highest decile of their respective distributions. Below is a list of the top 10 bridge occupations.

Rank	Betweenness	Clustering	Job Title
1	0.062	0.171	Emergency Management
2	0.054	0.109	Recreation Workers
3	0.054	0.162	Scientific Research
4	0.052	0.090	Technical Sales Representatives
5	0.043	0.190	Fine Artists
6	0.041	0.179	Logistics Worker
7	0.039	0.141	Management Consultant
8	0.039	0.156	Sales Secretary
9	0.035	0.133	General and Operations Managers
10	0.033	0.171	Maintenance Workers

Figure 1.2 plots the occupation network with node sizes proportional to betweenness centrality, revealing that bridges are concentrated in social services, sales, retail, transport, and logistics.

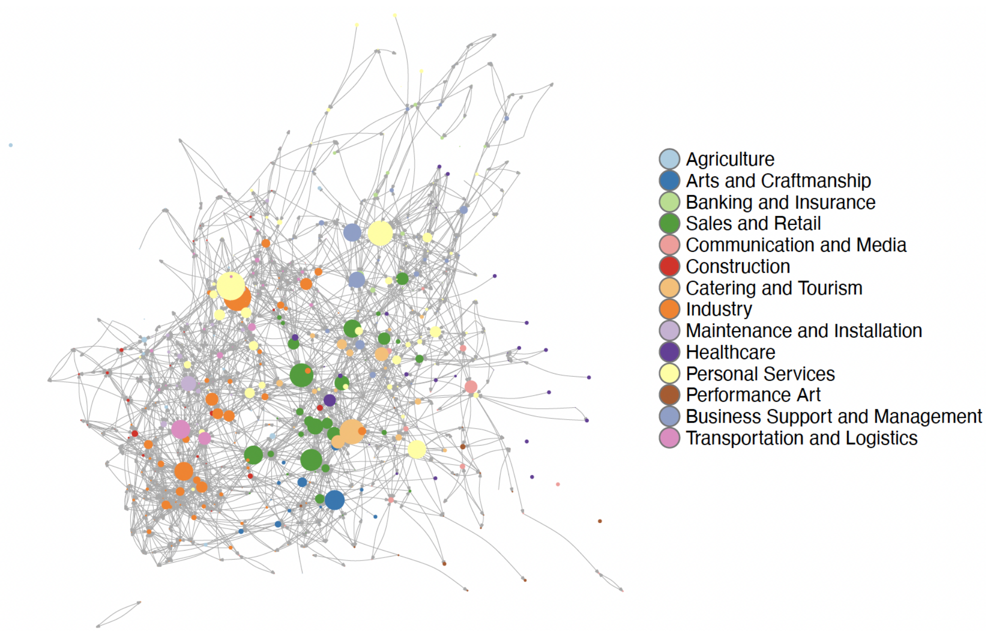


Figure 9.2: The bridge occupations in the occupation network

10 Appendix B: Worker Flows

10.1 Data

Unemployment definition. I assign occupations to unemployed workers based on their last job, rather than the occupations they are currently searching in. This approach has several advantages: it only requires matched employer-employee data, accounts for workers searching in multiple occupations, and aligns better with theories of skill frictions by reflecting workers' current skills and the difficulty of acquiring new ones.

Multiple job holding. Some workers hold multiple jobs despite focusing on main jobs. I interpret this as transitioning between main jobs. To address this, I assign the worker to the job with the most recent start date. If the new job ends before the old one, I drop the new contract, assuming it to be a side job.

Occupation codes. The DADS panel provides imperfect 4-digit occupation codes, especially before 2008 when reporting was not mandatory. Missing values are not uniformly distributed, particularly in low-skilled and public sector jobs, so I restrict the analysis to post-2008 data when reporting became mandatory.

Additionally, DADS and ROME use different occupational nomenclatures: DADS uses PCS, which groups by social rank, while ROME groups by skills. For consistency, I map both onto a third nomenclature, FAP, at the 3-digit level, resulting in around 200 distinct occupations.

10.2 Construction of the main variables

Discrete time transition matrix. Let $\boldsymbol{Q}(t)$ denote the transition matrix between occupations and employment states at quarter t. It can be decomposed into four block matrices $\boldsymbol{Q}^{x \to y}(t)$ representing the occupation transition from employment state x to employment state y, with $x, y \in \{u, e\}$. For example, $\boldsymbol{Q}^{u \to e}(t)$ represents occupation transitions from unemployment to employment.

The general coefficient of $\boldsymbol{Q}^{x\to y}(t) = (\mu_{ij}^{x\to y}(t))_{i,j\in\mathcal{N}^2}$ is computed as

$$\mu_{ij}^{x\to y}(t) \equiv \frac{\text{\# workers in occupation } i \text{ and state } x \text{ at } t \text{ who are in occupation } j \text{ and state } y \text{ at } t+1}{\text{\# workers in occupation } i \text{ and state } x \text{ at } t}$$

Intuitively, this is akin to conducting a large-scale survey of the entire labor force at a fixed quarterly frequency, recording each worker's occupation and employment state at t and $t+1$. Transitions are as the number of workers who change occupation or state between these dates.

Time Frequency. I use a quarterly sampling frequency to balance two risks in measuring transition probabilities. First, *time-aggregation bias* occurs because transitions happen continuously, but are measured at discrete intervals, potentially underestimating actual transitions. While Shimer (2007) offers corrections for two or three states, no general corrections exist for larger state spaces. Shorter intervals (e.g., monthly) would reduce this bias.

Second, *small sample bias* arises when there are too few observations to estimate transition rates accurately. With around 40,000 rates to estimate and limited transitions recorded, small sample bias can significantly affect estimates, especially when no transitions are observed in a given quarter. A shorter sampling interval would worsen this bias.

A quarterly frequency offers a good compromise, reducing time-aggregation bias while limiting small sample bias, ensuring more accurate mobility rate estimates.

Controlling for Seasonality. Estimating transition rates is complicated by strong seasonality, with more transitions occurring at certain times, such as January or September.[44] To control for seasonality, I apply a simple strategy: using a moving average of the last four quarterly transition matrices. This assumes that seasonality effects cancel out over time, focusing on relative rather than absolute effects.

[44] This is especially true for employment-to-employment transitions, where many within-firm transitions happen on January 1st, as firms often report these changes annually due to the administrative burden of continuous reporting.

Estimation of the continuous-time transition matrix. The transition matrix constructed above is a *discrete-time* transition matrix. However, our theoretical model operates in continuous time, so we need to construct a continuous-time transition matrix. To convert a discrete-time transition matrix to a continuous-time transition matrix, I use the fact that $\boldsymbol{f}(t+h) = e^{\boldsymbol{Q}h}\boldsymbol{f}(t)$. For small time intervals, the matrix exponential can be approximated as $e^{\boldsymbol{Q}(t)h} \approx \boldsymbol{I} + \boldsymbol{Q}(t)h$. Setting $h = 1$, we find $\boldsymbol{Q}(t) \approx e^{\boldsymbol{Q}(t)} - \boldsymbol{I}$.[45]

Wages. Let me explain each step in more detail, following the methodology of Le Barbanchon et al. (2021) on the same dataset. First, I measure net wages (in constant euros) rather than gross wages, as the former are more relevant for workers considering job changes. Second, I construct full-time equivalent quarterly net wages. For full-time workers, I calculate daily earnings and multiply by 120 (30 days × 4 months) for quarterly earnings. For part-time workers, I compute hourly earnings and multiply by 35 × 4 × 4. Third, I control for age by regressing wages on a second-order polynomial of age and quarter-specific occupation fixed effects, isolating wage variation by occupation and quarter. Fourth, I adjust for seasonality using a four-quarter moving average.

Recoding of Skill Frictions. An important methodological challenge arises from the recoding of expert-based measures of skill frictions into a new occupation nomenclature. Worker occupation transitions use a different code (PCS) than the expert measure of skill frictions (ROME). Both nomenclatures must be converted into a third one, labelled FAP, at the 3-digit level. This is a many-to-one matching: each FAP code is associated with one or many ROME codes.

First, I assign numerical values to ROME skill frictions: 1 for self-connections, 2 for close transitions, 3 for distant transitions, and 4 for unlinked transitions. Second, I compute FAP-level connection values by averaging ROME connection weights across all ROME occupations associated with each FAP occupation:

$$g_{ij}^{FAP} = \frac{1}{|N_i \times N_j|} \sum_{(k,l) \in N_i \times N_j} g_{kl}^{ROME}$$

[45]This approximation makes sense because the diagonal terms of the discrete-time transition matrix are $1 - \sum_{j \neq i} q_{ij}$, so subtracting one gives $-\sum_{j \neq i} q_{ij}$, ensuring rows sum to zero.

where N_i denotes the set of ROME occupations associated with FAP i, and $N_i \times N_j$ is the set of all ROME connections between two FAP occupations i and j. Third, I discretize the continuous FAP connection values into four categories (self, close, distant, unlinked) by partitioning them into intervals: $g_{ij}^{FAP} = 1$ (self), $g_{ij}^{FAP} \in (1, 2.5]$ (close), $g_{ij}^{FAP} \in (2.5, 3.5]$ (distant), and if $g_{ij}^{FAP} \in (3.5, 4]$(unlinked).

10.3 Bridge Regression

Outcome variables. The long-run difference in worker outcome, Δy_{id}, is measured as the difference between the outcome at the end of the sub-period (4th quarter of 2012 or 2019) and the outcome at the beginning of the sub-period (1st quarter of 2008 or 2013). I divide the sample into two periods to avoid dropping too many observations due to the balanced panel assumption (the worker must be present at both the beginning and the end of the period).

I consider several outcome variables: probability of changing 1-digit or 2-digit occupation group, distance traveled in the occupation network (centered and standardized), wage growth, and change in unemployment risk (measured as the difference in the unemployment rate between the origin and destination occupations in percentage points).

The dummy $1(\text{bridge})$ equals one if the worker transited through a bridge occupation, defined as occupations in the top tenth decile of betweenness centrality and bottom tenth decile of clustering coefficient. It excludes workers who start or end in bridge occupations, requiring at least two transitions: from a non-bridge occupation to a bridge occupation, and then to another non-bridge occupation.

Period 2008-2012. The Table 2.1 gives the estimated coefficients over the period 2009-2012. All coefficients are statistically significant and have the same sign as in the 2013-2019 period. Compared to the subsequent period, workers transiting through bridge occupations have slightly higher chances of changing 1- or 2-digit occupation groups and travel farther in the network. However, their wage growth and reduction in unemployment risk are halved. Despite this, the effect remains economically significant, especially

Table 10.1: **Change in worker outcomes vs. transition through bridge, 2009-2012**

	Dependent Variables				
	Change 1-digit Occupation	Change 2-digit Occupation	Distance (Standardized)	Wage Growth	Unemployment Rate Change
Through Bridge	0.317***	0.342***	0.816***	0.044***	-0.345***
	(0.003)	(0.003)	(0.008)	(0.002)	(0.026)
Observations	196,439	196,439	196,439	253,375	253,423
Origin FE	Yes	Yes	Yes	Yes	Yes

Notes: Each observation is a worker over the period 2008-2012. The dependent variables are the probability of changing 1-digit or 2-digit occupation, distance traveled within the network, wage growth, and change in unemployment risk. The variable of interest is a dummy indicating whether workers transited through a bridge during the period. * p<0.10, ** p<0.05, *** p<0.01.

considering that wages declined and unemployment rates rose over the period.

Robustness check. As a robustness check, I restrict the control group to workers who made at least two transitions over the period. This controls for unobserved personality traits, such as high mobility, which could confound the effect of transiting through a bridge occupation.

Table 2.2 presents the estimated coefficients for the 2013-2019 subperiod with the alternative control group. The main take-away is that most results go through. All coefficients are still statistically significant at the 1% threshold. The magnitude of the first three coefficients is reduced but still quite high: workers transiting through bridges still have a higher probability of changing 1-digit and 2-digit occupation groups and travel farther away, but the difference with the control group is reduced. What is more, the magnitude of the fourth and firth coefficients are unchanged, if not slightly increased for unemployment risk.

Table 10.2: **Change in worker outcomes vs. transition through bridge, 2013-2019, robust control**

	Dependent Variables				
	Change 1-digit Occupation	Change 2-digit Occupation	Distance (Standardized)	Wage Growth	Unemployment Rate Change
Through Bridge	0.103***	0.053***	0.168***	0.049***	-0.744***
	(0.004)	(0.004)	(0.008)	(0.010)	(0.048)
Observations	110,368	110,368	110,368	110,343	110,368
Origin FE	Yes	Yes	Yes	Yes	Yes

Notes: Each observation is a worker over the period 2008-2012. The dependent variables are the probability of changing 1-digit or 2-digit occupation, distance traveled within the network, wage growth, and change in unemployment risk. The variable of interest is a dummy indicating whether workers transited through a bridge during the period. * p<0.10, ** p<0.05, *** p<0.01.

11 Appendix C: Stylized Network Search Model

11.1 Firm Values

Proposition 1. (Job posting along the transition)

Firm values $J_i(t)$, tightness ratios $\theta_i(t)$ and job finding rates $p_i(t)$ are constant along the transition dynamics, and given by

$$J_i(t) = \frac{\phi y_i}{r+s} \qquad \theta_i(t) = \left(\frac{\zeta_i}{c_i}\left(\frac{(1-\phi)y_i}{r+s_i}\right)\right)^{\frac{1}{\alpha}} \qquad p_i(t) = \zeta^{\frac{1}{\alpha}}\left(\frac{(1-\phi)y_i}{c_i(r+s_i)}\right)^{\frac{1-\alpha}{\alpha}}$$

Proof: Firm value is unstable, thus the only stable dynamics is the constant-steady-state value. Plugging the free entry condition, the Bellman equation of firm value writes $\dot{J}_i(t) = (r+s_i)J_i(t) - (y_i - w_i)$. Solving for this differential equation gives $J_i(t) = \frac{\phi y_i}{r+s} + \left(\frac{y_i - w_i}{r+s_i} - J_i(0)\right)e^{(r+s_i)t}$. If the initial condition is not equal to the steady-state value of firm value, then firm value diverges over time. Consequently, the only stable trajectory is the constant steady-state value. Using the free entry condition $\frac{c_i}{q(\theta_i)=J_i}$ then gives the equilibrium tightness ratios. Using the functional form the matching function yields the job finding rate.

11.2 Approximating worker reallocation dynamics

For small enough shocks, the baseline pre-shock transition matrix is sufficient to compute the dynamics of the worker distribution, as the next lemma shows.

Lemma (Approximation of worker reallocation dynamics)

The deviation from steady-state, $\boldsymbol{\delta f}(t) \equiv \boldsymbol{f}(t) - \boldsymbol{f}_\infty$ evolves approximately as:

$$\dot{\boldsymbol{\delta f}}(t) \approx \widetilde{\boldsymbol{Q}}\, \boldsymbol{\delta f}(t) \quad \text{with} \quad \boldsymbol{\delta f}_0 \approx \widetilde{\boldsymbol{f}_\infty} - \boldsymbol{f}_\infty$$

where $\widetilde{\boldsymbol{Q}}$ is the pre-shock transition matrix, and $\widetilde{\boldsymbol{f}_\infty} - \boldsymbol{f}_\infty$ is the deviation in steady-states.

Proof: The proof is in three steps, and extensively leverages properties of first-order approximations. First, for small enough shocks, the post-shock job finding rates can be expressed as $\boldsymbol{p}_{\text{post-shock}} \approx \boldsymbol{p}_{\text{pre-shock}} + \kappa \boldsymbol{\delta y}$ with $\kappa \equiv \frac{1-\alpha}{\alpha} \left(\frac{\zeta(1-\phi)}{c(r+s)}\right)^{\frac{1-\alpha}{\alpha}} \bar{y}^{\frac{1-2\alpha}{\alpha}}$ The new job finding rate is the sum of the old job finding rate and a shock term, which is proportional to productivity shock. Because the shock size can be rescaled by $1/\kappa$, in what follows I simply denote the job finding rate shock by Δ.

Second, for small enough shocks, the post-shock steady-state distribution can be expressed as $\boldsymbol{f}_\infty = \tilde{\boldsymbol{f}}_\infty + \Delta \cdot \boldsymbol{x}_0$, where Δ controls the magnitude of the shock, and $\boldsymbol{x}_0$ is a vector independent of shock size (which depends on the shock structure, and will specified later). This implies that the initial deviation from the new steady-state is small, and of order Δ.

Third, a first-order Taylor approximation of the post-shock transition matrix gives $\boldsymbol{Q} \approx \tilde{\boldsymbol{Q}} + \Delta \cdot \boldsymbol{\delta Q}$, where Δ controls the magnitude of the shock, and $\boldsymbol{\delta Q}$ is a matrix independent of shock size (which depends on the shock structure, and will specified later).

Taking together, these three results imply that

$$\dot{\delta f}(t) \approx (\tilde{\boldsymbol{Q}} + \Delta\delta\boldsymbol{Q})\delta\boldsymbol{f}(t) \approx \tilde{\boldsymbol{Q}}\delta\boldsymbol{f}(t) + \Delta\delta\boldsymbol{Q}\delta\boldsymbol{f}(t)$$

Integrating each term yields: $\delta\boldsymbol{f}(t) = e^{\tilde{\boldsymbol{Q}}t}\delta\boldsymbol{f}_0 + \Delta^2 e^{\delta\boldsymbol{Q}t}\boldsymbol{x}_0$ The second term is of order Δ^2, and hence negligible, yielding the desired result.

To sum up, this means the pre-shock transition matrix is sufficient to simulate worker reallocation dynamics after small shocks.

11.3 Spectral decomposition

Lemma. (Spectral decomposition)

The worker distribution at date t can be expressed as a function of the post-shock transition matrix's eigenvalues and eigenvectors

$$\boldsymbol{f}(t) = \boldsymbol{f}_\infty + \sum_{k=2}^{6} \gamma_k e^{\lambda_k t}\boldsymbol{v}_k \tag{17}$$

where $\lambda_k < 0$ are the eigenvalues of the baseline pre-shock transition matrix $\boldsymbol{Q}$, $\boldsymbol{v}_k$ are the associated right-eigenvectors, and γ_k are the associated loadings. The loadings depend on the initial distribution: $\gamma_k = \boldsymbol{w}_k'\boldsymbol{f}_\mathbf{0}$, where $\boldsymbol{w}_k$ are the left-eigenvectors of the transition matrix.

Intuitively, the eigenvalue-eigenvector decomposition expresses the complex transition dynamics of the worker distribution as a sum of simpler convergence modes. Generally, the worker distribution converges at varying speeds, but along these eigenvectors, each segment converges at the same speed determined by the eigenvalue.

11.4 Spectral properties of transition matrix

Here, I turn to characterizing the eigenvalues and left- and right-eigenvectors of the *baseline* pre-shock transition matrix. The details of the computation have limited economic interest, and hence are not shown - but are available upon request.

Lemma. *The eigenvalues of the pre-shock transition matrix are approximately*

$$\lambda_1 = 0, \quad \lambda_2 \approx \frac{p_C s}{p_P + p_C + s}, \quad \lambda_3 \approx s + \frac{s p_P p_C}{(p_P + p_C)(2p_P + p_C)}$$
$$\lambda_4 \approx p_P + p_C + s - \frac{2sp_C}{p_P + p_C + s}, \quad \lambda_5 \approx p_P + p_C + s - \frac{sp_C}{p_P + p_C + s}, \quad \lambda_6 \approx 2p_P + p_C + s - \frac{2sp_P}{(2p_P + p_C)}$$

Lemma. *The second and third right-eigenvectors of the pre-shock transition rate matrix are*

$$v_2 = \begin{pmatrix} p_C + p_P & 0 & -(p_C + p_P) & s & 0 & -s \end{pmatrix}'$$
$$v_3 = \begin{pmatrix} p_C + p_P & --2(p_C + p_P) & p_C + p_P & s & -s & s \end{pmatrix}'$$

I do not provide an analytical characterization of other right-eigenvectors, as it turns out that the loadings on these eigenvectors is negligible for *asymmetric* shocks. It is, however, not the case for aggregate shocks - which is not focus of the current focus.

Lemma. *The left-eigenvectors of the pre-shock transition rate matrix are*

$$w_2 = \mathcal{Z}_2^{-1} \begin{pmatrix} 1 & 0 & -1 & \frac{p_P}{p_P+p_C} & 0 & -\frac{p_P}{p_P+p_C} \end{pmatrix}'$$
$$w_3 = \mathcal{Z}_3^{-1} \begin{pmatrix} 1 & -\frac{2p_P}{p_C} & 1 & -\frac{p_P p_C}{(p_P+p_C)(2p_P+p_C)} & \frac{2p_P}{p_C}\frac{p_P p_C}{(p_P+p_C)(2p_P+p_C)} & -\frac{p_P p_P}{(p_P+p_C)(2p_P+p_C)} \end{pmatrix}'$$
$$w_4 = \mathcal{Z}_4^{-1} \begin{pmatrix} 1 & -\frac{2s}{p_P+p_C} & 1 & \frac{p_P+p_C}{s} & -2 & \frac{p_P+p_C}{s} \end{pmatrix}'$$
$$w_5 = \mathcal{Z}_5^{-1} \begin{pmatrix} s & 0 & -s & -(p_P + p_C) & 0 & -(p_P + p_C) \end{pmatrix}'$$
$$w_6 = \mathcal{Z}_6^{-1} \begin{pmatrix} s & \frac{p_P(2p_P+p_C)}{p_C} & s & -(2p_P + p_C) & -\frac{p_P(2p_P+p_C)^2}{sp_C} & -(2p_P + p_C) \end{pmatrix}'$$

where the normalization constants are set such that $w_k^T v_k = 1$.

11.5 Reallocation times after periphery and central shocks

In this subsection, I derive an approximation of the aggregate reallocation time after both periphery and central shocks. All proofs are relegated to the online appendix.

Periphery Shocks. I begin by analytically characterizing reallocation time following periphery shocks. With the eigenvalues already computed, the next step is to characterize the distribution of loadings in the spectral decomposition, and thus the initial deviation from steady state.

Lemma. *After the periphery productivity shock, the initial deviation from steady-state writes*

$$\delta \boldsymbol{f}(0)_{\mathrm{P}} \equiv \tilde{\boldsymbol{f}}_{\infty} - \boldsymbol{f}_{\infty,\mathrm{P}} \approx \Delta \mathcal{Z}^{-1} \big(-(2p_P + p_C), 0, (2p_P + p_C), -s, 0, s \big)'$$

where $\mathcal{Z}$ is the normalization constant of the steady-state worker distribution before the shock.

Equipped with this result, and the distribution of left eigenvectors, I can now compute the distribution of loadings using the formula: $\gamma_k = \boldsymbol{w}_k'(\tilde{\boldsymbol{f}}_{\infty} - \boldsymbol{f}_{\infty,\mathrm{P}})$, where $\boldsymbol{w}_k$ are the left-eigenvectors of the baseline transition matrix.

Lemma. *The distribution of loadings after the periphery shock is approximately*

$$\gamma_2 \approx -\Delta \mathcal{Z}^{-1} (p_P + p_C) \quad \gamma_3 \approx 0 \quad \gamma_4 \approx 0 \quad \gamma_5 \approx -s \Delta \mathcal{Z}^{-1} \frac{3p_P + 2p_C}{p_P + p_C} \quad \gamma_6 \approx 0$$

Using the spectral decomposition, one can now solve entirely for the worker distribution at any point in time - and thus compute the aggregate reallocation time.

Proposition. *For small enough shock size Δ, the transition time after the central productivity shock can be approximated as*

$$T_{\mathrm{C}} = \frac{p_C + p_P}{s p_C}$$

Central shocks. I begin by analytically characterizing reallocation time following central shocks. The approach is the same as for periphery shocks.

Lemma. *After central shock, the initial deviation writes*

$$\delta \boldsymbol{f}(0)_{\mathrm{C}} \equiv \tilde{\boldsymbol{f}}_{\infty} - \boldsymbol{f}_{\infty,\mathrm{C}} \approx \Delta \mathcal{Z}^{-1} \begin{pmatrix} -\frac{2p_P + p_C}{2} & 2p_P + p_C & -\frac{2p_P + p_C}{2} & -\frac{s}{2} & s & -\frac{s}{2} \end{pmatrix}'$$

where $\mathcal{Z}$ is the normalization constant of the steady-state worker distribution before the shock.

Lemma. *The distribution of loadings after the central productivity shock is approximately*

$$\gamma_2 \approx 0 \quad \gamma_3 \approx \Delta \mathcal{Z}^{-1} \frac{(2p_P + p_C)^2}{p_C} \quad \gamma_4 \approx -s\Delta \mathcal{Z}^{-1} \frac{2p_P + p_C}{2(p_P + p_C)} \quad \gamma_5 \approx 0 \quad \gamma_6 \approx s\Delta \mathcal{Z}^{-1} \frac{2p_P}{p_C}(2p_P + p_C)^2$$

Proposition. *For small enough shock size Δ, the transition time after the central productivity shock can be approximated as*

$$T_{\text{C}} = \frac{1}{s}$$

11.6 Reallocation time after any asymmetric shock

In this subsection, I show that the reallocation time after *any asymmetric* shock can be expressed as a weighted average of reallocation times after periphery and central shocks. To make this result, I first need to assume further that the productivity levels in periphery and central occupations are not too different, such that the gap $p_P - p_C$ is small.

Lemma. (Linear decomposition of asymmetric shocks)

Assume $p_P - p_C$ is small. Then all asymmetric productivity shocks, i.e. productivity shocks such that the employment weighted occupation productivity shocks sum to zero, can be decomposed as a linear combination of the central and periphery productivity shocks, respectively, $\boldsymbol{\delta y}_{\text{P}} = (-\Delta, 0, \Delta)$ and $\boldsymbol{\delta y}_{\text{C}} = (\frac{2p_C + p_P}{2p_C + 2p_P}\Delta, -\Delta, \frac{2p_C + p_P}{2p_C + 2p_P}\Delta)$

$$\boldsymbol{\delta y}_{\text{asymmetric}} = \gamma_{\text{P}} \boldsymbol{\delta y}_{\text{P}} + \gamma_{\text{C}} \boldsymbol{\delta y}_{\text{C}}$$

where γ_P is loading on periphery shocks γ_C is the loading on central shocks.

Leveraging the linarity of the first-order approximations, it can be shown that the change in job finding rate, transition matrix and steady-state deviation can be expressed as a linear combination of the associated periphery and central shocks' changes; characterized above.

For small asymmetric shocks, the job finding rates can be approximated as follows $\boldsymbol{p}_{\text{after-shock}} \approx \boldsymbol{p}_{\text{before-shock}} + \gamma_{\text{P}} \kappa \Delta \boldsymbol{\delta y}_{\text{P}} + \gamma_{\text{C}} \kappa \Delta \boldsymbol{\delta y}_{\text{C}}$ Similarly, the post-shock transition matrix can be approximated as $\boldsymbol{Q}_{\text{after-shock}} = \boldsymbol{Q}_{\text{pre-shock}} + \gamma_{\mathrm{P}}\Delta\delta\boldsymbol{Q}_{\mathrm{P}} + \gamma_{\mathrm{C}}\Delta\delta\boldsymbol{Q}_{\mathrm{C}}$. Finally, the deviation from the new steady-state is a linear combination of the deviations from steady-state after the periphery and central shocks:$\delta\boldsymbol{f}(0) \equiv \tilde{\boldsymbol{f}}_\infty - \boldsymbol{f}_\infty = \gamma_{\mathrm{P}}\Delta\delta\boldsymbol{f}_{\mathrm{P}}(0) + \gamma_{\mathrm{C}}\Delta\delta\boldsymbol{f}_{\mathrm{C}}(0)$ Equipped with these results, I can now compute the reallocation time after any asymmetric shock.

Proposition. *For small shocks, the transition time after an asymmetric productivity shock is*

$$T \approx \alpha_{\mathrm{P}} \cdot \underbrace{\frac{p_C + p_P + s}{p_P} s^{-1}}_{\equiv T_{\mathrm{P}}} + (1-\alpha_{\mathrm{P}}) \underbrace{\cdot s^{-1}}_{\equiv T_{\mathrm{P}}} \quad \text{with} \quad \alpha_{\mathrm{P}} \equiv \frac{|\,(p_P + p_C)\,\gamma_{\mathrm{P}}\,|}{|\,(p_P + p_C)\,\gamma_{\mathrm{P}}\,| + |\,-\frac{(2p_P + p_C)^2}{p_P + p_C}\gamma_{\mathrm{C}}\,|}$$

Proof: The worker distribution at date t can be approximated as

$$\boldsymbol{f}(t) \approx \widetilde{\boldsymbol{f}}_\infty - \Delta\mathcal{Z}^{-1}\gamma_{\mathrm{P}}(p_P + p_C)e^{-\lambda_2 t}\boldsymbol{v}_2 - \Delta\mathcal{Z}^{-1}\gamma_{\mathrm{C}}\frac{(2p_P + p_C)^2}{p_P + p_C}e^{-\lambda_3 t}\boldsymbol{v}_3$$

Therefore, the transition time is approximately

$$T \int_0^{+\infty} \frac{\|-\gamma_{\mathrm{P}}(p_P + p_C)e^{-\lambda_2 t}\boldsymbol{v}_2 - \gamma_{\mathrm{C}}\frac{(2p_P+p_C)^2}{p_P+p_C}e^{-\lambda_3 t}\boldsymbol{v}_3\|_2}{\|-\gamma_{\mathrm{P}}(p_P + p_C)\boldsymbol{v}_2 - \gamma_{\mathrm{C}}\frac{(2p_P+p_C)^2}{p_P+p_C}\boldsymbol{v}_3\|_2} dt$$

where $\Delta\mathcal{Z}^{-1}$ cancel out from the numerator and denominator.

Now, I use the fact that both right eigenvectors $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$ are orthogonal to each other. Hence, by Pythagorean theorem, we have

$$\begin{aligned}\|-\gamma_{\mathrm{P}}(p_P + p_C)e^{-\lambda_2 t}\boldsymbol{v}_2 - \gamma_{\mathrm{C}}\frac{(2p_P + p_C)^2}{p_P + p_C}e^{-\lambda_3 t}\boldsymbol{v}_3\|_2 &= \|-\gamma_{\mathrm{P}}(p_P + p_C)e^{-\lambda_2 t}\boldsymbol{v}_2\|_2 + \|-\gamma_{\mathrm{C}}\frac{(2p_P + p_C)^2}{p_P + p_C}e^{-\lambda_3 t}\boldsymbol{v} \\ &= |\gamma_{\mathrm{P}}(p_P + p_C)|e^{-\lambda_2 t} + |\gamma_{\mathrm{C}}\frac{(2p_P + p_C)^2}{p_P + p_C}e^{-\lambda_3 t}|\end{aligned}$$

where the last line uses the fact that $\|\boldsymbol{v}_2\|_2 = \|\boldsymbol{v}_3\|_2 = 1$.

Finally, integrating over time yields the desired result.

11.7 Adjustment costs

The adjustment costs can be written more compactly in terms of the worker distribution and the vector of agents' payoffs as follows:

$$AC = \frac{SW^{\text{slow}} - SW^{\text{new}}}{SW^{\text{new}} - SW^{\text{old}}} = \frac{\int_0^{+\infty} e^{-rt}(\boldsymbol{f}(t) - \boldsymbol{f}_\infty)^T \boldsymbol{u} dt}{(1/r) \cdot (\boldsymbol{f}_\infty^T \boldsymbol{u} - \widetilde{\boldsymbol{f}_\infty}^T \widetilde{\boldsymbol{u}})}$$

where $\boldsymbol{u} = (y_1, \dots, b_N - c_N\theta_N)$ is the vector of flow utilites.

By applying spectral decomposition, the adjustment costs can be further decomposed into the transition matrix's eigenvectors and eigenvalues, as clarified in the next lemma.

Lemma. (Spectral decomposition of adjustment costs)

After small productivity shocks, the adjustment costs can be approximated as

$$AC = \sum_{k=1}^{2N} \underbrace{\frac{r}{r + |\lambda_k|}}_{\text{speed}} \cdot \underbrace{\boldsymbol{w}_k^T (\boldsymbol{f}_0 - \boldsymbol{f}_\infty)}_{\text{loading}} \cdot \underbrace{\frac{\boldsymbol{v}_k^T \boldsymbol{u}}{\boldsymbol{f}_\infty^T \boldsymbol{u} - \widetilde{\boldsymbol{f}_\infty}^T \widetilde{\boldsymbol{u}}}}_{\text{misallocation}} \tag{18}$$

where λ_k is the k-th smallest eigenvalue of the pre-shock transition matrix, while $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are the associated left- and right-eigenvectors, respectively.

The interpretation is as follows: the spectral decomposition breaks down the dynamics of worker transitions into a sum of simpler transition patterns, each characterized by a fixed speed of convergence, given by the eigenvalues of the transition matrix. Along each of these simpler transition patterns—represented by the eigenvectors of the transition matrix—the welfare effect can be easily quantified as the product of three terms.

The first term $\frac{r}{r+|\lambda_k|}$ captures the effect of *transition speed* on welfare. Smaller eigenvalues $|\lambda_k|$ indicate slower reallocation and higher adjustment costs. This is because slower reallocation delays the realization of long-term benefits from the shock, which reduces the present value of these benefits and increases adjustment costs.

The second term $\boldsymbol{w}_k^T (\boldsymbol{f}_0 - \boldsymbol{f}_\infty)$ reflects the *magnitude* of the simpler transition pattern in the spectral decomposition. It measures the extent to which the initial deviation of the worker distribution from the steady-state $\boldsymbol{f}_0 - \boldsymbol{f}_\infty$ aligns with the right eigenvector $\boldsymbol{v}_k$. For example, if the initial deviation is proportional to $\boldsymbol{v}_k$, then the loading is one for $\boldsymbol{v}_k$ and zero for all other eigenvectors.[46]

The third term $\frac{\boldsymbol{v}_k^T \boldsymbol{u}}{\boldsymbol{f}_\infty^T \boldsymbol{u} - \widetilde{\boldsymbol{f}_\infty}^T \widetilde{\boldsymbol{u}}}$ captures the impact of *transitory misallocation* on aggregate welfare. For example, if labor shortages occur in certain occupations ($\boldsymbol{v}_k(i) < 0$)) where the flow payoff u_i is high, the numerator turns negative, indicating that workers are misallocated over the transition. The denominator scales these misallocations by the total welfare change that would occur if the transition were instantaneous, thus yielding percentage-point variations in welfare due to the slow transition.

By plugging in the analytical characterizations of the eigenvalues and eigenvectors of the baseline transition matrix and simplifying the terms, we obtain the desired results.

12 Appendix D: Quantitative Network Search Model

12.1 Search frictions

The number of hires in occupation i at date t is now given by

$$h_i(t) = \zeta_i \quad \cdot \left(\sigma_{ii}^u(t) u_i(t) + \sum_{j \neq i} \sigma_{ji}^u(t) u_j(t) + \sum_{j \neq i} \sigma_{ji}^e(t) e_j(t) \right)^\alpha \cdot \quad v_i(t)^{1-\alpha_i}$$

where $v_i(t)$ is the total number of vacancies posted by firms in occupation i at date t, and $\sigma_{ii}^u u_i + \sum_{j \neq i} \sigma_{ji}^u g_{ji} u_j + \sum_{j \neq i} \sigma_{ji}^e g_{ji} \eta_j e_j$ represents total search exerted by workers in occupation i at date t, from unemployed and employed workers.

The tightness ratio in occupation i at date t is now defined as $\theta_i(t) \equiv v_i(t) / \big(\sigma_{ii}^u(t) u_i(t) + \sum_{j \neq i} \sigma_{ji}^u(t) u_j(t) + \sum_{j \neq i} \sigma_{ji}^e(t) e_j(t) \big)$.

[46] Without loss of generality, the left-eigenvectors can be normalized such that $\boldsymbol{W}^T \boldsymbol{V} = \boldsymbol{I}$, where $\boldsymbol{W}$ and $\boldsymbol{V}$ are the matrices of left- and right-eigenvectors, respectively. In other words, $\boldsymbol{w}_l^T \boldsymbol{v}_k = 1$ if $k = l$, and 0 otherwise.

Matching is random conditional on search efforts, so the job finding rate in occupation i at date t per unit of search effort is $p_i(t) = h_i(t) / \big(\sigma^u_{ii}(t) u_i(t) + \sum_{j \neq i} \sigma^u_{ji}(t) u_j(t) + \sum_{j \neq i} \sigma^e_{ji}(t) e_j(t)\big) = \zeta \theta_i(t)^{1-\alpha}$. Similarly, vacancies in occupation i face a flow probability $q_i(t) = h_i(t)/v_i(t) = \zeta \theta_i(t)^{-\alpha}$ of matching with a worker at date t.

12.2 Firm values

The value of a filled job for firms in occupation i at date t is denoted $J_i(t)$, and the value of an unfilled vacancy in occupation i at date t is denoted $V_i(t)$.

The value of a job filled in occupation i at date t $J_i(t)$ solves a modified Bellman equation

$$rJ_i(t) = \underbrace{y_i - w_i}_{\text{flow profits}} + \underbrace{\left(s_i + \sum_{j \in \mathcal{N}} \sigma^e_{ij}(t) p_j(t)\right)}_{\text{on-the-job adjusted separation rate}} [V_i(t) - J_i(t)] + \dot{J}_i(t) \tag{19}$$

The main differences stems from the job separation rate $s_i + \sum_{j \in \mathcal{N}} g_{ij} \sigma^e_{ij}(t) p_j(t)$, which accounts for the possibility that the employed worker leaves because she has matched with another firm.

The value of a vacant job in occupation i at date t solves the Bellman equation

$$rV_i(t) = -c_i + q(\theta_i)[J_i(t) - V_i(t)] + \dot{V}_i(t) \tag{20}$$

It is the same as the Bellman equation in the tractable model.

12.3 Search efforts

In this subsection, I provide an analytical characterization of the equilibrium search time and search effort of both unemployed and employed workers.

Lemma (Equilibrium search time of unemployed workers)

The equilibrium search time of unemployed worker in occupation i searching in occupation j at date t writes

$$\tau_{ij}^{u}(t) = \frac{\left(g_{ij}\max\{E_j(t) - U_i(t), 0\}\right)^{\psi+1}}{\sum_j \left(g_{ij}\max\{E_j(t) - U_i(t), 0\}\right)^{\psi+1}}.$$

Proof: The first order condition of the unemployed worker problem writes

$$g_{ij}(E_j(t) - U_i(t))\tau_{ij}^{u}(t)^{\frac{-1}{\psi+1}} = \Lambda_i^{u}(t),$$

where $\Lambda_i^u(t) \geq 0$ is the Lagrange multiplier associated with the time constraint $\sum_k \tau_{ik}^u = 1$. Intuitively, the optimal search time equalizes the marginal return and the marginal cost of searching.

Rearranging terms, we get

$$\tau_{ij}^{u}(t) = \left(\frac{g_{ij}(E_j(t) - U_i(t))}{\Lambda_i^u(t)}\right)^{\psi+1}.$$

Summing over j and using the time constraint, we obtain

$$\Lambda_i^u(t)^{\psi+1} = \sum_j \left(g_{ij}(E_j(t) - U_i(t))\right)^{\psi+1}.$$

This yields the desired expression. From this, we can derive optimal search effort:

$$\sigma_{ij}^{u}(t) = \tau_{ij}^{u\,\frac{\psi}{\psi+1}} = \frac{\left(g_{ij}p_j(t)\max\{E_j(t) - U_i(t), 0\}\right)^{\psi}}{\left[\sum_k \left(g_{ik}p_k(t)\max\{E_k(t) - U_i(t), 0\}\right)^{\psi+1}\right]^{\frac{\psi}{\psi+1}}}.$$

Let me turn now to the equilibrium search time of employed workers, which has very similar expression from that of unemployed workers.

Lemma. (Equilibrium search time of employed workers)

The equilibrium search time of employed worker in occupation i searching in occupation j at date *t writes*

$$\sigma^{e}_{ij}(t) = \tau^{e}_{ij}{}^{\frac{\psi}{\psi+1}} = \frac{\eta_i^{\frac{\psi}{\psi+1}} \left(g_{ij} p_j(t) \max\{ E_j(t) - E_i(t), 0\}\right)^{\psi}}{\left[\sum_k \left(g_{ik} p_k(t) \max\{ E_k(t) - E_i(t), 0\}\right]^{\psi+1}\right]^{\frac{\psi}{\psi+1}}}$$

Comparison with discrete choice models. The formulation of optimal search efforts mirrors choice probabilities in *discrete choice models* with Fréchet taste shocks. In such models, the probability that a worker in i chooses occupation j at date t is $\mathbb{P}(j \mid i)(t) = \frac{v_{ij}(t)^{\psi}}{\sum_k v_{ik}(t)^{\psi}}$, where $v_{ij}(t)$ is the expected payoff of occupation j for workers in i at date t and ψ is the shape parameter of the Fréchet distribution.

However, the endogenous search setting differs significantly from discrete choice models in several key aspects. Firstly, normalization constants in discrete choice models must sum to one, which is not required in endogenous search. Secondly, the endogenous search model preserves the linearity of the Bellman equation in terms of transition rates, unlike the non-linear aggregation in discrete choice settings:

$$rU_i(t) = b_i + \left(\sum_k (g_{ij} p_j(t) \max\{E_j(t) - U_i(t)\})^{\psi+1} \right)^{\frac{1}{\psi+1}}.$$

The linearity of the Bellman equation enhances tractability and simplifies estimation of values.

12.4 Wages

Lemma (Equilibrium wages)

The equilibrium wage in occupation i at date t, denoted $w_i(t)$, is given by

$$w_i(t) = \phi_i y_i + (1 - \phi_i)\Big(b_i + \sum_j (\mu^{u}_{ij}(t) - \mu^{e}_{ij}(t))(E_j(t) - U_i(t))\Big)$$

Proof: The equilibrium wage in occupation i at date t maximizes the generalized Nash product $\max_{w_i(t)} (E_i(t) - U_i(t))^{\phi} J_i(t)^{1-\phi}$. The FOC of the maximization program above gives the so-called Nash bargaining condition $(1 - \phi)(E_i(t) - U_i(t)) = \phi J_i(t)$. This is implicitly a function of the wage, through the value functions $E_i(t)$ and $J_i(t)$.

We now use the Bellman equation to make explicit the dependance of the value functions on the wage. The value of a filled job to the firm in i at date t, denoted $J_i(t)$, solves $\left(r + s_i + \sum_j \mu^e_{ij}(t)\right) J_i(t) = y_i - w_i(t) + \dot{J}_i(t)$ and the value of being employed in occupation i at date t, denoted $E_i(t)$, solves $\left(r + s_i + \sum_j \mu^e_{ij}(t)\right) E_i(t) = w_i(t) + \sum_j \mu^e_{ij}(t) E_j(t) + s_i U_i(t) + \dot{E}_i(t)$ Therefore, the net value of employment in occupation i at date t is $\left(r + s_i + \sum_j \mu^e_{ij}(t)\right)(E_i(t) - U_i(t)) = w_i(t) - rU_i(t) + \sum_j \mu^e_{ij}(t)(E_j(t) - U_i(t)) + \dot{E}_i(t)$. We can further plug the flow value of unemployment $rU_i(t)$ from the Bellman equation, yielding $\left(r + s_i + \sum_j \mu^e_{ij}(t)\right)(E_i(t) - U_i(t)) = w_i(t) - b_i + \sum_j \mu^e_{ij}(t)(E_j(t) - U_i(t)) - \sum_j \mu^u_{ij}(t)(E_j(t) - U_i(t)) + \dot{E}_i(t) - \dot{U}_i(t)$. Plugging both expressions in the Nash bargaining condition and dividing by the effective discount rate gives

$$(1-\phi_i)\left(w_i(t) - b_i + \sum_j \mu^e_{ij}(t)(E_j(t) - U_i(t)) - \sum_j \mu^u_{ij}(t)(E_j(t) - U_i(t)) + \dot{E}_i(t) - \dot{U}_i(t)\right) = \phi_i(y_i - w_i(t) + \dot{J}_i(t))$$

Note that the time-derivative terms cancel out. Indeed, taking the derivative of the Nash condition with respect to time gives $(1-\phi)(\dot{E}_i(t) - \dot{U}_i(t)) = \phi \dot{J}_i(t)$. Finally, we isolate the wage on the left-hand side and we obtain the desired result.

12.5 Vacancy posting

Lemma (Equilibrium tightness ratios)

The equilibrium tightness ratio in occupation i at date t, denoted $\theta_i(t)$, is given by

$$\theta_i(t) = \left(\frac{\zeta_i}{c_i}\frac{1-\phi}{\phi}(E_i - U_i)\right)^{\frac{1}{\alpha}}$$

Proof: The equilibrium tightness ratio in occupation i at date t satisfies the free entry condition $V_i(t) = 0$. Plugging this condition in the Bellman equation for the value of vacancy, I get $\frac{c_i}{q(\theta_i(t))} = J_i(t)$. Given $q_i(t) = q(\theta_i(t)) = \zeta_i \theta_i(t)^{-\alpha}$, we get $\theta_i(t) = \left(\frac{\zeta_i}{c_i} J_i(t)\right)^{\frac{1}{\alpha}}$. Making use of the Nash bargaining condition, we have $J_i(t) = \frac{1-\phi}{\phi}(E_i(t) - U_i(t)$. Replacing in the expression above gives the desired result.

Corollary (Equilibrium job finding rates)

The equilibrium job finding rates are given by

$$p_i(t) = \zeta_i \theta_i(t)^{1-\alpha} = \xi_i \left((E_i(t) - U_i(t)) \right)^{\frac{1-\alpha}{\alpha}} \quad \text{with} \quad \xi_i \equiv \zeta_i^{\frac{1}{\alpha}} c_i^{\frac{\alpha-1}{\alpha}} \left(\frac{1-\phi}{\phi} \right)^{\frac{1-\alpha}{\alpha}}$$

12.6 Laws of motion

How do workers reallocate across occupations?

Variations in employment. The employment in occupation i at date t evolves as follows

$$\dot{e}_i(t) = p_i(t) \underbrace{\left(\sum_{j \in \mathcal{N}} \sigma_{ji}^u(t) u_j(t) + \sigma_{ji}^e(t) e_j(t) \right)}_{\text{search effort by neighboring u and e workers}} - \underbrace{\left(s_i + \sum_{j \in \mathcal{N}} \sigma_{ij}^e(t) p_j(t) \right)}_{\text{OJS-adjusted separation rate}} e_i(t)$$

The variation in employment for occupation i at time t is determined by the balance of new hires and job destructions. The *flow of new hires* is the product of the job finding rate per unit of search effort and the total search effort from neighboring employed and unemployed workers targeting occupation i. Conversely, the *flow of job destruction* is the product of the separation rate and the number of employed workers in occupation i, adjusted for on-the-job search.

Variations in unemployment. The unemployment in occupation i at date t evolves as follows

$$\dot{u}_i(t) = s_i e_i(t) - \underbrace{\left(\sum_{j \in \mathcal{N}} \sigma_{ij}^u(t) p_j(t) \right)}_{\text{unemployment outflow rate adjusted for search effort}} u_i(t)$$

As above, the variation in unemployment for occupation i at date t reflects the difference between job destructions and hires. The *flow of job destruction* is the product of the separation rate—unadjusted for on-the-job search—with the number of employed workers in occupation i. Conversely, the *flow of hires* in neighboring occupations is the product of the outflow rate from unemployment—adjusted for search effort—by the mass of unemployed workers originally from occupation i.

12.7 The Master equation

In this subsection, I show that the dynamics of the whole economy can by described by a single matrix differential equation.

Lemma 4. (The Master equation)

Let $\boldsymbol{V}(t) = (E_1(t), \ldots, E_N(t), U_1(t), \ldots, U_N(t))'$ denote the vector of worker values at date t. The vector of worker values $\boldsymbol{V}(t)$ is sufficient to characterize the economy over the transition, and evolves as

$$\dot{\boldsymbol{V}(t)} = \boldsymbol{G}(\boldsymbol{V}(t))$$

with $\boldsymbol{G}(\cdot)$ defined as follows

$$\boldsymbol{G}_i(\boldsymbol{V}) = rE_i(t) - w_i(\boldsymbol{V}(t)) - \sum_j \mu^e_{ij}(\boldsymbol{V}(t))(E_j(t) - E_i(t)) - s_i(U_i(t) - E_i(t)) \tag{21}$$

$$\boldsymbol{G}_{i+N}(\boldsymbol{V}) = rU_i(t) - b_i - \sum_j \mu^u_{ij}(\boldsymbol{V}(t))(E_j(t) - U_i(t)) \tag{22}$$

and where

$$w_i(\boldsymbol{V}(t)) = \phi_i y_i + (1-\phi_i)\Big(b_i + \sum_j (\mu^u_{ij}(\boldsymbol{V}(t)) - \mu^e_{ij}(\boldsymbol{V}(t)))(E_j(t) - U_i(t))\Big) \tag{23}$$

$$\mu^u_{ij}(\boldsymbol{V}(t)) = \frac{g_{ij}^{\psi+1} p_j(\boldsymbol{V}(t))^{\psi+1}(E_j(t) - U_i(t))_+^{\psi}}{\sum_k g_{ik}^{\psi+1} p_k(\boldsymbol{V}(t))^{\psi+1}(E_k(t) - U_i(t))_+)^{\psi}} \tag{24}$$

$$\mu^e_{ij}(\boldsymbol{V}(t)) = \frac{\eta_i g_{ij}^{\psi+1} p_j(\boldsymbol{V}(t))^{\psi+1}(E_j(t) - E_i(t))_+^{\psi}}{\sum_k g_{ik}^{\psi+1} p_k(\boldsymbol{V}(t))^{\psi+1}(E_k(t) - E_i(t))_+)^{\psi}} \tag{25}$$

$$p_i(\boldsymbol{V}(t)) = \xi_j (E_i(t) - U_i(t))^{\frac{1-\alpha}{\alpha}} \tag{26}$$

Proof: The lemmas above show that all control variables – eg. transition rates, wages and tightness – can be expressed as a function of worker values only. Collecting all equations from these lemmas, I can to construct $\boldsymbol{G}(.)$.

The first two equations and give the change in equilibrium worker values as a function of equilibrium agents' actions, while the other four equations give the equilibrium agents' actions as a function of worker values. The firm values are omitted because Nash bargaining implies they are proportional to the net value of employment. Finally, note the worker distribution is absent from the Master equation, because the model is block-recursive.

Proposition (Linear approximation of the Master equation)

We compute the total differential of the Master equation with respect to worker values, evaluated at $\boldsymbol{V}_0$. For expositional clarity, we drop the dependence on $\boldsymbol{V}_0$ for all functions.[47] *Moreover, we define: $\hat{x} = \frac{dx}{x}$. The total differential writes*

$$df_i = rdE_i - dw_i - \sum_j \mu^e_{ij}\Gamma^e_{ij}\left(\hat{\mu}^u_{ij} + \hat{\Gamma}^e_{ij}\right)$$

$$df_{N+i} = rdU_i - \sum_j \mu^u_{ij}\Gamma_{ij}\left(\hat{\mu}^u_{ij} + \hat{\Gamma}^u_{ij}\right)$$

with

$$dw_i = (1-\phi)\sum_j \mu^u_{ij}\Gamma^u_{ij}\left(\hat{\mu}^u_{ij} + \hat{\Gamma}^u_{ij}\right) - (1-\phi)\sum_j \mu^e_{ij}\Gamma^u_{ij}\left(\hat{\mu}^e_{ij} + \hat{\Gamma}^u_{ij}\right)$$

$$\hat{\mu}^u_{ij} = d\log\mu^u_{ij} = (\psi+1)\hat{p}_j + \psi\hat{\Gamma}^u_{ij} - \psi\sum_j \sigma^u_{ij}(\hat{p}_k + \hat{\Gamma}^u_{ik})$$

$$\hat{\mu}^e_{ij} = d\log\mu^e_{ij} = (\psi+1)\hat{p}_j + \psi\hat{\Gamma}^e_{ij} - \psi\sum_j \sigma^e_{ij}(\hat{p}_k + \hat{\Gamma}^e_{ik})$$

$$\hat{p}_i = \frac{1-\alpha}{\alpha}\hat{\Gamma}^u_{ii}$$

$$\hat{\Gamma}^u_{ij} = \frac{dE_i - dU_i}{\Gamma^u_{ij}} \quad \text{and} \quad \hat{\Gamma}^e_{ij} = \frac{dE_i - dE_i}{\Gamma^e_{ij}}$$

Proof: We begin by computing the total differential of f_{N+i}. The formula for the total

[47] For example, $w_i = w_i(\boldsymbol{V}_0)$, etc.

differential of f_i and the total differential of the wage follows the same steps. We have

$$\begin{aligned} df_{N+i} &= d\Big(rU_i - \sum_j \mu^u_{ij}\Gamma^u_{ij}\Big) \\ &= rdU_i - \sum_j d\left(\mu^u_{ij}\Gamma^u_{ij}\right) \end{aligned}$$

where the second equality follows from the linearity of the differential operator. Now, we exploit the fact that

$$\begin{aligned} d\left(\mu^u_{ij}\Gamma^u_{ij}\right) &= d\mu^u_{ij} \cdot \Gamma^u_{ij} + \mu^u_{ij} \cdot d\Gamma^u_{ij} \\ &= \frac{d\mu^u_{ij}}{\mu^u_{ij}}\mu^u_{ij}\Gamma^u_{ij} + \frac{d\Gamma^u_{ij}}{\Gamma^u_{ij}}\mu^u_{ij}\Gamma^u_{ij} \\ &= \mu^u_{ij}\Gamma^u_{ij}\left(\hat{\mu}^u_{ij} + \hat{\Gamma}^u_{ij}\right) \end{aligned}$$

Plugging in the expression above gives the desired expression for the Bellman equation of the value of unemployment.

Now, we compute the hat-change of transition rates from unemployment to employment. The formula for the hat-change of transition rates from employment to employment and the hat-change of job finding rates follows the same steps. We exploit the fact that

$$\hat{\mu^u_{ij}} = \frac{d\mu^u_{ij}}{\mu^u_{ij}} = d\log\mu^u_{ij}$$

This substantially simplifies computation because log transforms products into sums. Plugging the expression of μ^u_{ij} we find

$$\hat{\mu^u_{ij}} = d\left((\psi+1)\log g_{ij} + (\psi+1)\log p_j + \psi\log\Gamma^u_{ij} - \log N_i\right) = (\psi+1)\hat{p}_j + \psi\hat{\Gamma}^u_{ij} - \hat{N}_i$$

where $N_i \equiv \sum_{ik}(g_{ik}p_k\Gamma_{ik})^\psi$ is the normalization constant ensuring that the search probabilities of unemployed workers sum to one: that is, $\sum_{ik}\sigma^u_{ik} = 1$. We recover $\hat{N}_i$ by differentiating the constraint, which gives

$$\begin{aligned}
& d\sum_k \sigma_{ik}^u = 0 \\
\Leftrightarrow & \sum_k \sigma_{ik}^u \hat{\sigma}_{ik}^u = 0 \\
\Leftrightarrow & \sum_k \sigma_{ik}^u \left(\psi \hat{p}_k + \hat{\Gamma}_{ik}^u - \hat{N}_i\right) = 0 \\
\Leftrightarrow & \hat{N}_i = \sum_k \sigma_{ik}^u \left(\psi \hat{p}_k + \hat{\Gamma}_{ik}^u\right)
\end{aligned}$$

where the second equality uses same trick as above, namely: $d\sigma_{ik}^u = \sigma_{ik}^u \frac{d\sigma_{ik}^u}{\sigma_{ik}^u} = \sigma_{ik}^u \hat{\sigma}_{ik}^u$. The third line replaces the hat-change of search efforts σ_{ik}^u by its expression, which can be found using the same steps as above. The fourth line makes use of the fact that $\sum_k \sigma_{ik} = 1$. Plugging $\hat{N}_i$ in the equation above gives the desired expression.

These total differentials can be stored into a single Jacobian matrix denoted $\boldsymbol{D_G})(\boldsymbol{V})$. The exact analytical expression of the Jacobian matrix is delegated to the online appendix.

12.8 Stability, existence and uniqueness

The stability and uniqueness of the equilibrium depend on the properties of this Jacobian matrix.

Theorem. (Stability)

The values are instable over time. Therefore, the only stable solution is that values immediately jump to steady-state and stay constant afterwards.

Proof: The Master equation can be rewritten as: $\frac{d\boldsymbol{V}_t}{dt} = \left[r\boldsymbol{I} - \boldsymbol{Q}(\boldsymbol{V}_t)'\right]\boldsymbol{V}_t$. Note that the dependence of the transition rate matrix on the worker values, through the endogenous search efforts and tightness ratios. This implies that the equation above defines a *non-linear* matrix differential equation, which is very hard to solve in the general case.

My strategy is to discretize the process, and characterize the properties of small changes along the transition. Let Δ denote a small time interval. The Master equation can be approximated as $\frac{\boldsymbol{V}_{t+\Delta}-\boldsymbol{V}_t}{\Delta} = \left[r\boldsymbol{I} - \boldsymbol{Q}(\boldsymbol{V_t})^{'}\right]\boldsymbol{V}_t$. Rearranging terms yields: $\boldsymbol{V}_{t+\Delta} = \left[\boldsymbol{I} + \Delta(r\boldsymbol{I} - \boldsymbol{Q}(\boldsymbol{V}_t)^{'}\right]\boldsymbol{V}_t$.

Assume that the worker values is not at steady-state, such that $\boldsymbol{V}_t \neq \boldsymbol{V}_\infty$.[48]. I show this implies values grow far away from the origin without bound: that is, $\|\boldsymbol{V}_{t+\Delta}\| > \|\boldsymbol{V}_t\|$. Plugging the equation above gives $\|\boldsymbol{V}_{t+\Delta}\| = \|\left[\boldsymbol{I} + \Delta(r\boldsymbol{I} - \boldsymbol{Q}(\boldsymbol{V}_t)^{'}\right]\boldsymbol{V}_t\| \geq \|\left[\boldsymbol{I} + \Delta(r\boldsymbol{I} - \boldsymbol{Q}(\boldsymbol{V}_t)^{'}\right]\| \cdot \|\boldsymbol{V}_t\|$, where the inequality follows from properties of the vector norms. Let λ be the eigenvalues of $\left[\boldsymbol{I} + \Delta(r\boldsymbol{I} - \boldsymbol{Q}(\boldsymbol{V}_t)^{'}\right]$. It holds that: $|\lambda| \leq \|\left[\boldsymbol{I} + \Delta(r\boldsymbol{I} - \boldsymbol{Q}(\boldsymbol{V}_t)^{'}\right]\|$, and therefore: $\|\boldsymbol{V}_{t+\Delta}\| \geq |\lambda| \cdot \|\boldsymbol{V}_t\|$. Note that if $\lambda > 1$, then $\|\boldsymbol{V}_{t+\Delta}\| > \|\boldsymbol{V}_t\|$, which is the desired result.

I turn to characterize λ, the eigenvalues of $\left[\boldsymbol{I} + \Delta(r\boldsymbol{I} - \boldsymbol{Q}(\boldsymbol{V}_t)^{'}\right]$. Because the values are not at the steady-state $\left[r - \boldsymbol{Q}(\boldsymbol{V}_t)^{'}\right]\boldsymbol{V}_t \neq \boldsymbol{0}$.[49] Let ν denote the eigenvalues of the transition matrix $\boldsymbol{Q}(\boldsymbol{V}_t)$. By properties of continuous time transition rate matrix, we always have $\nu \leq 0$. Because a matrix and its transpose share the same spectrum, ν is also an eigenvalue of $\boldsymbol{Q}(\boldsymbol{V}_t)^{'}$. Hence, $r - \nu > 0$ is an eigenvalue of $r\boldsymbol{I} - \boldsymbol{Q}(\boldsymbol{V}_t)^{'}$, and $1 + \Delta(r - \nu) > 1$ is an eigenvalue of $\left[\boldsymbol{I} + \Delta(r\boldsymbol{I} - \boldsymbol{Q}(\boldsymbol{V}_t)^{'}\right]$, which completes the proof.

Existence and uniqueness. Let me turn now to the existence and uniqueness of a vector of steady-state worker values. The steady-state vector of worker values solves $\boldsymbol{0} = \boldsymbol{G}(\boldsymbol{V})$ This can be rewritten as a fixed-point problem

$$\boldsymbol{V} = \boldsymbol{H}(\boldsymbol{V})$$

where $\boldsymbol{H}(\boldsymbol{V}) = \boldsymbol{G}(\boldsymbol{V}) + \boldsymbol{V}$. Let $\boldsymbol{D_H}(\boldsymbol{V}) = \boldsymbol{D_H}(\boldsymbol{V}) + \boldsymbol{I}$ denote the associated Jacobian.

Theorem. (Existence and uniqueness of steady-state worker values)

The vector of steady-state worker values always exists. Furthermore, assume that for any $\boldsymbol{V}$

$$\Re[\lambda(\boldsymbol{D_H})](\boldsymbol{V}) \neq 1$$

[48]Otherwise the solution is trivial: the process simply stays at steady-state thereafter.

[49]By definition, the steady-state values solve $r\boldsymbol{V}_\infty = \boldsymbol{Q}(\boldsymbol{V}_\infty)^{'}\boldsymbol{V}_\infty$.

Then the steady-state vector of worker values is unique. Consequently, steady-state worker search efforts, wages and tightness ratios exist and are unique too.

Proof: We begin by proving the existence of a solution to the fixed-point equation above. The existence proof relies on the Poincaré-Miranda theorem, which is equivalent to the more widely known Brouwer fixed-point theorem, often used in economics.

The Poincaré-Miranda theorem can be stated as follows: Consider N functions $f_1, \ldots, f_N$ of N variables $x_1, \ldots, x_N$, where each x_i varies between a lower bound l_i and an upper bound L_i. If f_i is non-negative when $x_i = l_i$ and non-positive when $x_i = L_i$, then there exists a point where all functions f_i are simultaneously zero.

In our case, we have $2N$ functions $\{f_i, f_{N+i}\}_i$ of $2N$ variables $\{E_i, U_i\}$, where the f_i functions follow the same conditions as above. The lower bounds are $l_i = 0$ for each function, as worker values cannot be negative. The upper bounds are $L_i = \max_j \frac{y_j}{r}$, which is the value a worker would receive if employed indefinitely at the maximum wage a firm is willing to pay. This upper bound also applies to unemployment values, as workers always prefer employment.

The functions f_i are negative at the lower bound. Consider $U_i = 0$, then

$$f_{N+i}(0) = -b_i - \sum_j \mu^u_{ij}(\boldsymbol{V}) E_j < 0$$

The sum is negative since $b_i > 0$, $\mu^u_{ij} > 0$ and $E_j > 0$ for all i, j. The demonstration is exactly the same for $E_i = 0$.

The functions f_i are positive at the upper bound. Consider $U_i = \max_j y_j/r$, then

$$f_{N+i}(\max_j y_j/r) \geq r \max_j y_j/r + \sum_j \mu^u_{ij}(\max_j y_j/r - E_j) > 0$$

where the second term is positive because $E_j \leq \max_j y_j/r$, by definition of the upper bound.

Taken together, this means that the assumptions of the Poincaré-Miranda theorem are verified, and therefore that there exists a vector such that all functions f_i are null at the same time. This implies that a fixed-point exists, and consequently, a steady-state vector of worker values exists.

Finally, uniqueness follows directly from a result in Kellogg (1976).

I am unable to provide an analytical demonstration of this assumption, but numerical computations show that this assumption is always verified.

12.9 Reservation wages and network centrality

Definition 1. (Katz-Bonacich centrality)

Given a graph with adjacency matrix $\mathbf{G}$, the index K_i of Katz-Bonacich centrality of the node i is

$$K_i(\mathbf{G}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}) = \alpha_i + \sum_j \gamma_i \beta_j g_{ij} K_j(\mathbf{G}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma})$$

The Katz-Bonacich centrality index of node i is the sum of two terms. The first component is a node-specific term α_i, which provides an exogenous centrality value to the node. The second component is a linear combination of the Katz-Bonacich centrality indices of node i's neighbors, reflecting how much centrality is conveyed by neighboring nodes. The parameters γ_i and β_j control how much this effect is dampened as it propagates through the network. [50]

Proposition 1. (Distribution of reservation wages)

Assume no on-the-job search. The steady-state reservation wages are equal to their occupation's index of Katz-Bonacich centrality in the appropriate network, and they solve the recursion

$$\bar{w}_i = b_i + \frac{\sum_{k \in \mathcal{N}} \mu^u_{ik}}{r + \sum_{k \in \mathcal{N}} \mu^u_{ik}} \left(\sum_{j \in \mathcal{N}} \frac{\mu^u_{ij}}{\sum_k \mu^u_{ik}} \frac{\phi_j r}{r + s_j} y_j - b_i \right) + \sum_{j \in \mathcal{N}} \frac{1}{r + \sum_k \mu^u_{ik}} \frac{r(1 - \phi_j) + s_j}{r + s_j} \mu^u_{ij} \bar{w}_j \qquad (27)$$

[50] This measure of centrality stems from the sociology literature, where it was interpreted as a proxy for agents' power in networks of social interactions. The intuition for the recursion is the following: agents are central in the network - and therefore powerful - because they are linked to other central powerful agents.

Proof: See online appendix.

The worker reservation wage is closely linked to the Katz-Bonacich centrality index of their occupation, because it follows a similar a recursive relationship. Here, the reservation wage in an occupation is high if it is connected to occupations with high reservation wages too. Indeed, a worker's reservation wage is a function of the wages she could potentially earn in neighboring occupations. Thanks to the Nash sharing rule, the former is a function of the reservation wages in these same neighboring occupations.

A crucial implication is that, everything else being equal, occupations more central within the network have larger outside options. Workers in central occupations enjoy many reallocation possibilities should the bargaining with the firm fail. They can leverage this better outside option in order to secure higher wages. In fact, this prediction is robust to assuming on-the-job search. In the appendix, I derive the closed-form expression of worker outside options in the general case.[51]

13 Appendix E: Estimation

Following Head and Mayer (2014) recommendation, I first implement a demean to reduce the number of fixed effects before running the two-way fixed effects Poisson regression.

First, I drop all observations with no transitions or switching gains over the entire period. Let me briefly explain this sample restriction. Intuitively, occupation pairs with no transitions or gains throughout the time period provide very little information and are likely true "zeros" that can be safely discarded. In contrast, pairs that alternate between zero and positive transitions are informative, and these "zeros" should be retained.

[51] The closed-form expression is however not as clearly interpretable as in the simple case. The relation with Katz-Bonacich centrality still holds, but in a more complicated network. In addition, I show that the value of being employed in an occupation is also given by the occupation's index of Katz-Bonacich centrality, in the appropriate network. Given the option value is a linear combination of the values of being employed and unemployed, this means that that the option value is a function to Katz-Bonacich centrality too, although in a less clear way than for the outside option.

Second, I divide by the average transition over the entire period. This is feasible due to the previous sample restriction. Crucially, I retain the zero transitions for occupation pairs that experienced at least one transition during the time interval. This demeaning procedure allows me to remove time-invariant fixed effects, which are numerous (around 40,000).

Finally, this gives

$$\frac{\mu_{ijt}}{\overline{\mu_{ij}}} = \exp\left\{\log\left(\frac{N_{it}^{-1}}{\overline{N_i^{-1}}}\right) + (\psi+1)\log\left(\frac{p_{jt}}{\overline{p_j}}\right) + \psi\log\left(\frac{\Delta_{ijt}}{\overline{\Delta_{ij}}}\right)\right\} \tag{28}$$

In other words, the gravity equation be rewritten as a Poisson regression with time-varying origin and destination fixed effects. Using standard packages, the search elasticity is then simply estimated as the coefficient associated to switching gains.

14 Appendix F: Application to Robots

14.1 Occupation exposure to robots

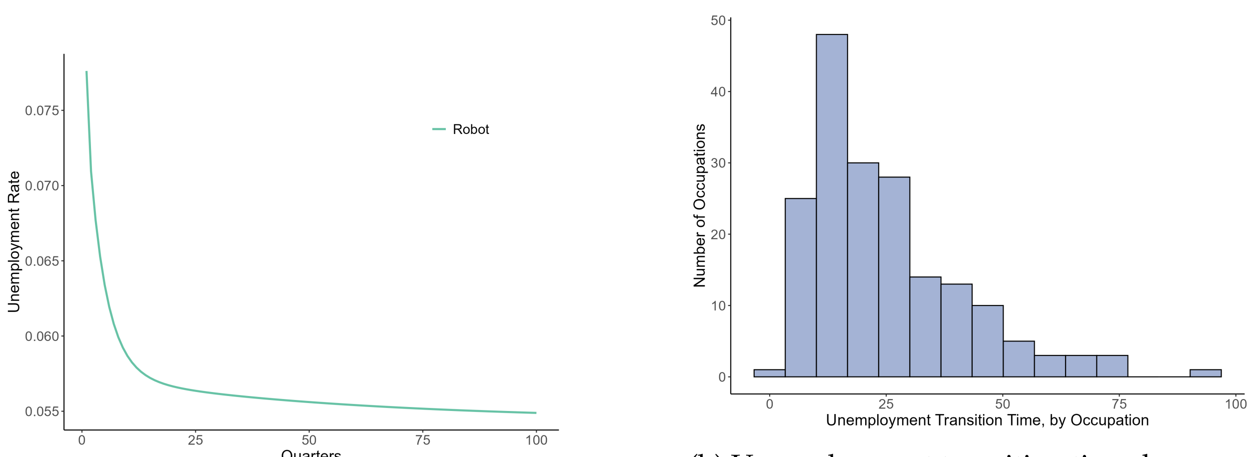
Webb's methodology for measuring occupation exposure involves three steps. First, he groups patents by technology class (e.g., robots or software) and identifies the tasks these patents perform. Second, he examines the tasks carried out by various occupations using US occupation data and compares them to the tasks performed by the patents. Third, he calculates the overlap between the tasks performed by patents and workers, ranking occupations based on the degree of overlap.

To apply his measure to the French labor market, I map the US ONET occupation categories to the French nomenclature of occupations, specifically FAP. This process involves several crosswalks: first, from ONET to the European nomenclature ISCO, then from ISCO to PCS, and finally from PCS to FAP. After mapping, I rank the occupations based on their exposure and normalize these ranks between zero and one, with zero corresponding to the least exposed occupations. This provides the baseline measure of occupation exposure to various new technologies, and in particular robots.

14.2 Calibrating productivity shocks

Solving for productivity shocks presents a complex, high-dimensional problem. Indeed, the wage changes induced by productivity shocks cannot be computed in isolation due to their interconnections within the occupational network: wage adjustments in one occupation influence the reservation wages of adjacent occupations, subsequently affecting their wages, and so forth. This interdependence quickly leads to the curse of dimensionality. To circumvent this issue, and because it is not the heart of the paper, I assume the outside option channel is quantitatively small: that is, $dw_i = \phi dy_i + (1-\phi)d\bar{w}_i \approx \phi dy_i$, where $\bar{w}_i$ is the reservation wage in occupation i. If worker bargaining power is not too large, this approximation is reasonable.

14.3 Unemployment effects



(a) Aggregate unemployment rate

(b) Unemployment transition time, by occupation

Figure 14.1: Unemployment over the transition

Aggregate unemployment. How does aggregate unemployment respond to the robot shock? The aggregate unemployment rate quickly decreases to its new lower steady-state level.

Figure 14.1.a plots the dynamics of the aggregate unemployment rate after the robot shock. There are two main take-aways. First, the figure shows that aggregate unemployment quickly adjusts following the robot shock, with a transition time of only a few quarters. This contrasts with the pattern observed when examining the worker distribution and suggests that the aggregate unemployment rate might be a misleading metric of the underlying movements across disaggregated labor markets.

Second, the robot shock leads to a decrease in the long-term aggregate unemployment rate. Intuitively, this is because robots reduce employment in high-unemployment occupations, such as manufacturing, while increasing employment in low-unemployment occupations, like services. Perhaps surprisingly, the economy does not experience a temporary increase in unemployment due to the worker reallocation dynamics.

Heterogeneity in unemployment adjustment speed. Does unemployment adjust uniformly across occupations? The rapid adjustment of aggregate unemployment actually masks substantial heterogeneity in unemployment adjustment at the occupation level.

I define the unemployment transition time at the occupation level similarly to the occupation-level reallocation times as $T_i^u = \frac{1}{\max_h |u_i(h) - u_i|} \int_0^{+\infty} |u_i(t) - u_i| dt$.

Figure 14.1.b plots the distribution of unemployment transition times for each occupation after the robot shock. There are two main takeaways: First, it shows that, on average, the transition time of unemployment at the occupation level is much longer than at the aggregate level. Intuitively, this is because upward and downward adjustments at the occupation level cancel out in the aggregate, masking the slower adjustments occurring within individual occupations.

Second, there is significant heterogeneity in unemployment transition times across occupations. While the majority of occupations adjust relatively quickly, a small minority adjust very slowly. These occupations typically include journalists, unskilled workers in metallurgy, banking technicians, and architects. These occupations are spread throughout the network, encompassing both expanding and declining occupations, whether central or peripheral.

14.4 Targeted subsidies

In practice, I define declining occupations and expanding occupations as the occupation with in the top or bottom deciles of productivity changes. Bridge occupations are defined as the occupation with the top 5% indices of betweenness centrality in the FAP occupation network, where betweenness centrality is computed using declining occupations as sources and expanding occupations as targets. In other words, it represents the occupations *in-between* the declining and expanding occupations.