Designing large procurement auctions:

India's utility-scale renewable electricity market *

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Abstract

Auctions to procure capacity for large-scale renewable electricity production have become important policy tools in climate change mitigation and improving electricity access. These auctions have procurement targets that often exceed any supplier's capacity, resulting in nuanced allocation rules. In this paper, I use theoretical and structural methods to analyse the auctions in India for their allocation inefficiency and the government's expenditure, which is financed by costly public funds. I theoretically prove that the design of these auctions incentivises lower-capacity firms to be more competitive. Such firms may get larger contracts despite having higher costs, leading to inefficient allocation. To measure the inefficiency, I first estimate firms' cost distributions structurally using a recent dataset. Then, I simulate the theoretical equilibrium with empirical cost distributions of winners to quantify the inefficiency. I further suggest counterfactual designs which lower the inefficiency and government expenditure, without reducing capacity allocation.

JEL classification: D44; Q42; C57; H57; Q48
Keywords: renewable energy; auctions; econometrics of auctions; large procurement; applied market design

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1 Introduction

Large-scale renewable power production is crucial for addressing climate change and expanding electricity access in developing countries.[1] Over 60 countries now use auctions to procure projects increasing their renewable electricity production capacity (IRENA, 2015). These projects usually have very high initial investment and a lifespan of 25 years to make a return on it. Without government intervention, volatility of prices and policies, and contract enforcement challenges over such a large period could lead to high risk premiums being charged by investors which can hinder the development of renewable power market. To mitigate this, some governments offer guaranteed tariffs to power producers using public funds, but they face problems of asymmetric information and procurement targets larger than suppliers' capacities.

Perfectly informed governments could minimize public spending by procuring from the most cost-efficient producers. As cost information is private, procurement auctions are used. Simpler mechanisms require bidders to make flat offers of capacity and price. Unlike standard settings though, procurement targets often exceed any individual firm's supply, and the cumulative supply of the lowest price bidders may not match the target.[2] The allocation rules used to solve these problems may generate incentives for asymmetric bidders to compete differently, which leads to inefficient allocation.[3] Given such incentives and the policy importance of these auctions, studying their design is crucial to improve their efficiency, reduce their cost to public funds, and attain climate goals (Fabra, 2024, EARIE presidential address).

With this broader objective, I theoretically and structurally analyse the auction designed by Solar Energy Corporation of India (SECI), which is responsible for procuring half of India's 109GW capacity.[4] Successful bidders receive power purchase agreements (PPAs) specifying project size (in MW) and a fixed tariff per kilowatt-hour (INR/kWh) for selling their production to SECI for 25 years, both determined in the auction. I theoretically show that when bidders are only privately informed of their cost, the allocation rule of SECI's auction incentivises larger ca-

[1] In this paper, renewable refers to solar and wind-based electricity.

[2] See IRENA (2015) Chapter 5 for examples

[3] Consider sealed-bid first-price auction with asymmetric information. A bidder who believes they have a higher cost may bid more aggressively to avoid losing, potentially winning despite having a lower valuation or higher cost

[4] At 20% utilisation rate, 109 GW could provide electricity to around 152 million Indian households consuming an average of 1255 kilowatt-hour annually in 2022 (PIB, 2022).

pacity bidders to compete less aggressively and lose the auction even if their projects have lower costs. This results in a cost-inefficient allocation. To quantify allocation inefficiency resulting from such strategic behavior, I proceed in two steps. First, I structurally estimate bidders' cost distributions using a recent dataset. Second, I use estimated cost distributions and observed bids to infer the range of cost of the winners. I then simulate the theoretical equilibrium with empirical distribution, truncated by the range of winners' costs, in the current and counterfactual auction designs. My key finding is that we can achieve fully efficient allocation at a lower government expenditure (measured by the auctioneer's payoff), without lowering the capacity allocation. This contrasts with usual results on the rent-efficiency tradeoff, indicating that SECI auctions might be leaving higher information rents to bidders than needed for incentive compatibility.[5] In addition to its technical contributions, my paper is the first, in my knowledge, to formally analyse and estimate the impact of auction design on efficiency and public expenditure in developing countries.

To understand the results, let's look at SECI's auction design, which is also replicated by other important renewable capacity procurement agencies in India. Before the auction, the auctioneer announces her demand (or a procurement target). The auctions have a qualification and a final stage. I model the final stage and make suitable assumptions on the qualification when needed. This is enough to show that the auction design results in inefficient allocation, without explicitly engaging with the complexities of a nuanced two-stage auction. Furthermore, the empirically estimated inefficiency from the final stage equilibrium can be seen as a lower bound because it will only add to any inefficiency arising from qualification stage.

Bidders provide a single bid of their capacity and tariff offer in the qualification stage and are selected for the final stage according to a pre-specified rule. The final stage is an open descending bid auction, where project size offers (or capacity) from the qualification stage are frozen and revealed publicly, but bidders can reduce their tariff offers.[6] A bidder's award is the minimum of her capacity offer and the residual of the target and cumulative capacity of the bidders with lower tariff offers. This way, the market is cleared by awarding a project of a positive, but lower than offered capacity, to one of the bidders.

[5]Efficient mechanisms usually provide higher incentives to more efficient bidders to deter them from bidding like less efficient ones, thereby revealing their true type. The incentives, called information rents, arise due to information asymmetry between the auctioneer and bidders.

[6]Open descending bid in procurement setting can be thought of as a reverse English auction with many winners. Similarly, the Dutch auction in this setting would have ascending bids, where the auctioneer would start with a low bid and keep increasing it until someone agrees to sell.

Like in any open descending price auction, bidders in SECI's auctions final stage (or simply, SECI auctions) are faced with the choice of either undercutting their competitors' last tariff offer, or exiting the auction at their current tariff offer. In the latter case, they get the residual award and their exiting tariff offer is their bid. Any bidder who can get a strictly positive residual on exiting at some tariff is said to be pivotal at that tariff. Her exit ends the auction, and her bid is the tariff discovered in that auction, paid to all the winners. A non-pivotal bidder's exit starts a subgame with remaining bidders. Any non-pivotal bidder should bid her costs to maximize the chance of a positive award. Pivotal bidders balance the trade-off between high bids with residual awards and low bids with own capacity as the award. This trade-off arises due to the residual award rule.

Following Milgrom and Weber (1982), I model the final stage as a "clock auction" with 2 bidders having the same reserve.[7] Such cases appear in auctions accounting for nearly 20% of the total allotted capacity. In this auction, a digital clock shows project size offers of both bidders, and the reserve tariff bid at the start of the auction. The bid continuously reduces as the auction proceeds, unless one bidder exits. On exiting, she gets the residual award and the other bidder is awarded her capacity. The price shown on the clock when she exits is the price discovered in the auction, which would be given to both the bidders. Each bidder has a cutoff strategy, i.e., she ex-ante decides on the bid at she would exit if opponent is still active. Assume that bidders draw their cost independently and identically. Under this assumption, I show that for any given cost, the bidder with the higher capacity offer has the higher bid. In other words, she is more likely to exits at a higher bid than her opponent, making her less aggressive.

Intuitively, the higher capacity bidder is less competitive because she receives a higher residual award when exiting the auction. If she competes and loses, she gets a lower price for a higher residual award. The additional quantity gained on winning equals excess supply in the auction for both the bidders. Thus, aggressively competing is not advantageous to the higher capacity bidder regardless of her cost type. The residual award in the market clearing rule and public information about opponents' capacities create strategic incentives, wherein a high-capacity bidder would

[7] Usually, there are more than 2 bidders in any given auction. At the start, they are all non pivotal and have different reserves. As the auction proceeds, many non pivotal bidders exit with a zero award. The bid of the last exiting bidder becomes the reserve bid for all the bidders who are still active. This reduces the number of bidders, only one or two of whom would be pivotal in most cases, and all of whom would have the same reserve bid.

be less competitive despite having a lower cost, resulting in inefficient allocation. Given the similarities between the auction design and Bertrand-Edgeworth duopoly, the theoretical results are more applicable to more general settings.

Holmberg and Wolak (2018) and Fabra, Fehr, and Harbord (2006) have studied similar auctions with simplifying assumption on bidders' information, which are violated in SECI's design. Fabra, Fehr, and Harbord (2006) assumes that bidders know each-others' capacities and costs. Holmberg and Wolak (2018) assumes that bidders have symmetric capacities and cost information, and have private information of their own costs. This provides them with a closed form equilibrium bid functions. Given the design of SECI's auctions, bidders see each-others' asymmetric capacities and know their costs privately. This game's equilibrium is characterised by coupled ordinary differential equations and boundary conditions, whose implicit solution yields bidding strategies. While proving the existence and uniqueness of this equilibrium through constructive methods, I deal with a singularity at one of the boundaries in the boundary value problem defining it. This proof contributes to the techniques used in auction theory, notably in Lizzeri and Persico (2000) and Lebrun (2006).

The theoretical inefficiency gives rise to an important policy question. Since the auctioneers may not want to complicate the allocation rule and aim to fulfill all of their procurement demand, can the auctions be tweaked to make the allocation more cost-efficient while reducing government expenditure? I address this question econometrically in two steps: identifying and estimating the cost distributions of bidders; and using the estimates to simulate the theoretical equilibrium of SECI auctions and counterfactual designs to compute inefficiency and government expenditure.

Identification involves mapping SECI's data of bidders' identities, their final stage tariff offers, their capacities and awards to their cost distributions. As discussed earlier, the bidders who get an award of zero bid their own cost, thereby revealing some of the order statistics of the costs. The data is similar to Dutch auctions, where the winner identity, the winning bid (lowest order statistic), and set of all the bidders can identify the underlying bid and cost distribution if the costs are drawn independently (Brendstrup and H. Paarsch, 2003). However, the observations in SECI's final stage pertain to the sample of bidders whose qualification bid is below a certain endogenously determined threshold. As we will see, this implies that observed bids in the final round pertain to the costs which are drawn independently of each other, conditional on one of the cost draws during the qualification stage.

I can exploit this conditional independence to establish the identification of cost distributions, using the technique similar to Song (2006). In doing so, I also account for the possibility that bidders may get new information affecting their costs between the 2 stages. The identification result adds to the literature on auction econometrics. Note that we don't need to use the theoretical equilibrium for identification of cost distributions, but it is important for estimating the inefficiency and government expenditure in SECI auctions and counterfactual designs.

I estimate the cost distribution parametrically. To capture bidder heterogeneity, I enhance SECI's auction data by identifying the majority shareholder/owner and managing firm of each bidder from various sources. Based on their parent companies' sectors, firms may differ in capital access and business models. Financial institutions, owning some new firms, can profit by selling their portfolios to energy companies. These new firms often have high liquidity from their institutional parent, which reduces their financing cost volatility. In contrast, energy firms focus on long-term returns from electricity sales and rely on external debt financing. I classify firms into three categories: Indian energy firms, financial firms, and miscellaneous.

Using estimated cost distributions and the theoretical equilibrium, I can infer the cost of bidders with positive residual award in data using their bids. For other winners, I can infer the range of costs. Using this additional information, I simulate the theoretical equilibrium for various draws of opponent's cost. Simulations are conducted for SECI auctions and two counterfactual designs- a discriminatory price auction, and a Vickrey Clarke Groves (VCG) mechanism. In discriminatory price auctions, bidders submit sealed tariff bids and receive their bid, while their capacities are public knowledge. They are popular in various settings, including India's 2G spectrum allocation and UK's electricity auctions. Comparison with VCG is motivated by its efficiency properties. This mechanism incentivises all bidders to bid their costs, thereby providing a good theoretical benchmark. I estimate welfare and SECI's expenditure using simulated bids of the participants under different mechanisms. I find that VCG performs much better than the current design of SECI, as it reduces the probability of inefficient allocation by 40% and government spending by 5%.

The theoretical and structural study of design elements of India's renewable capacity auctions provides us with insights which can help improve their performance. While the methods of the paper contribute to the domains of auction theory and econometrics, and the paper contributes to electricity market design, and renewable energy market design in developing countries. Section 2 highlights these contributions and situates the paper in the literature. Section 3 provides institutional background. Section 4 explains the dataset and establishes the stylized fact of negative correlation between bidders' project size offer with their competitiveness. Sections 5 provides a theoretical explanation of the stylised fact. In section 6, I provide the identification result to obtain cost distribution structurally from SECI's data. Section 7 gives the structural estimates of cost distribution. Section 8 provides the counterfactual analysis of possible improvements in auction design. Section 9 concludes the paper.

2 Contributions to the literature

The paper contributes to four strands of literature. The research question and context adds to the literature on renewable energy market design in developing countries. More broadly, it adds to the papers on energy and electricty market design. The methods of the paper contribute to the domains of auction theory, theory of wars of attrition, and auction econometrics.

My paper contributes to the nascent literature on renewable energy auctions in developing countries. Most of this literature has focused on the effect of risk premiums on investment incentives for power producers in such countries. Regarding India, Ryan (2021) shows that the participation and competition was higher in the auctions conducted by SECI in comparison to auctions conducted by other agencies. This is because other agencies are more likely to default on their payments to the bidders, which makes them risky. As a result bidders charge risk premium through their bids in the auction, which holds up investments in their auctions. The paper, however, abstracts from certain strategically important nuances of the auction procedure used by SECI. Besides this, Probst et al. (2020) provide reduced form results on the impact of local content requirement on the price discovered in SECI auctions. Such a requirement was discontinued in 2017, but has been reinstated since 2021 in a different form. Outside the Indian context, Hara (2023) studies the importance of risk premiums for bidding in Brazilian renewable energy auctions. To the best of my knowledge, my paper is the first one to formally analyse the design of auctions in India, and its affect on allocation efficiency and cost to public funds.

The analysis of SECI auctions adds to the literature on multi-unit procurement. Many papers on multi-unit procurement focus on supply function equilibria where bidders submit a price quantity schedule to the auctioneer, who then decides allocation (Holmberg, 2007; Holmberg, 2009; Schwenen, 2015). However, my paper has flat tariff offers, i.e., the producers have only one price for all of their production. It is closely related to Fabra, Fehr, and Harbord (2006) and Fabra, Fehr, and Frutos (2011), who analyse the implications of uniform versus discriminatory pricing auctions for 2 capacity constrained pivotal bidders with complete information. The setting is similar to Bertrand-Edgeworth (BE) duopoly with asymmetric firms (See Allen and Hellwig, 1993, for example).

Fabra, Fehr, and Harbord (2006) showed that the impact of this design choice on allocation efficiency in electricity markets is ambiguous. The mixed strategy equilibrium in this setting is similar to, but not the same as, the semi-separating Bayes Nash equilibrium (BNE) in SECI auctions. Holmberg and Wolak (2018) further extend this framework to incorporate symmetric private information on cost, and publicly known symmetric capacity constraints on suppliers. They show that discriminatory pricing is better for the auctioneer's payoff when bidders have affiliated distributions, while contending that the comparison between the two designs is ultimately an econometric question. My study extends these papers by finding equilibrium with bidders with asymmetric, but known, capacities (as in SECI auctions); and private information about marginal cost. This extension is non-trivial, as I do not have closed form bidding functions as in Holmberg and Wolak (2018), and the equilibrium of SECI auctions comes from implicit solution of a system of coupled non-linear ODEs.

In the presence of 2 pivotal bidders, the SECI auction is similar to a war of attrition (WoA) or asymmetric second price all-pay auction, except that the loser also gets some award. Levin (2004)'s lecture notes provide a good summary of the literature on WoA in simple settings with complete information. Nalebuff and Riley (1985) analyse the WoA where players have private information on their type and asymmetric beliefs. He shows that there is a continuum of equilibria when the losing bidder gets zero. The same result was found in Amann and Leininger (1996), which could show uniqueness of BNE for a general class of asymmetric all pay auctions with 2 players, except for the second price all pay auction, which is strategically same as WoA. This is in contrast to the result for SECI auction, where we have a unique semi-separating equilibrium in pure strategies. This difference is due to the absence of sunk cost in SECI's case, unlike traditional WoA where the time spent waiting for the opponent to leave is a sunk cost, which changes the boundary conditions of the differential equation system which give equilibrium.

The proof of equilibrium existence and uniqueness is inspired by the exposition of theoretical results in Lebrun (2006) and Lizzeri and Persico (2000). The former provides the conditions for existence and uniqueness of pure strategy monotonic equilibrium in asymmetric sealed bid first price auction. The differential equation system in these auctions have a problem of singularity at the infimum of types. Same problem arises in the SECI auctions, but can't be resolved using same assumptions. The intuition behind uniqueness of equilibrium is same as that of relative toughness in Lizzeri and Persico (2000) for second price all-pay auctions. Furthermore, the uniqueness result of my paper adds to that of Lizzeri and Persico (2000), whose assumption of non positive award to low ranking bidder is invalid in SECI's auctions, and Bertrand Edgeworth duopoly in general. Given these differences, the techniques used in theoretical results of this paper add to the literature on equilibrium existence and uniqueness in auctions.

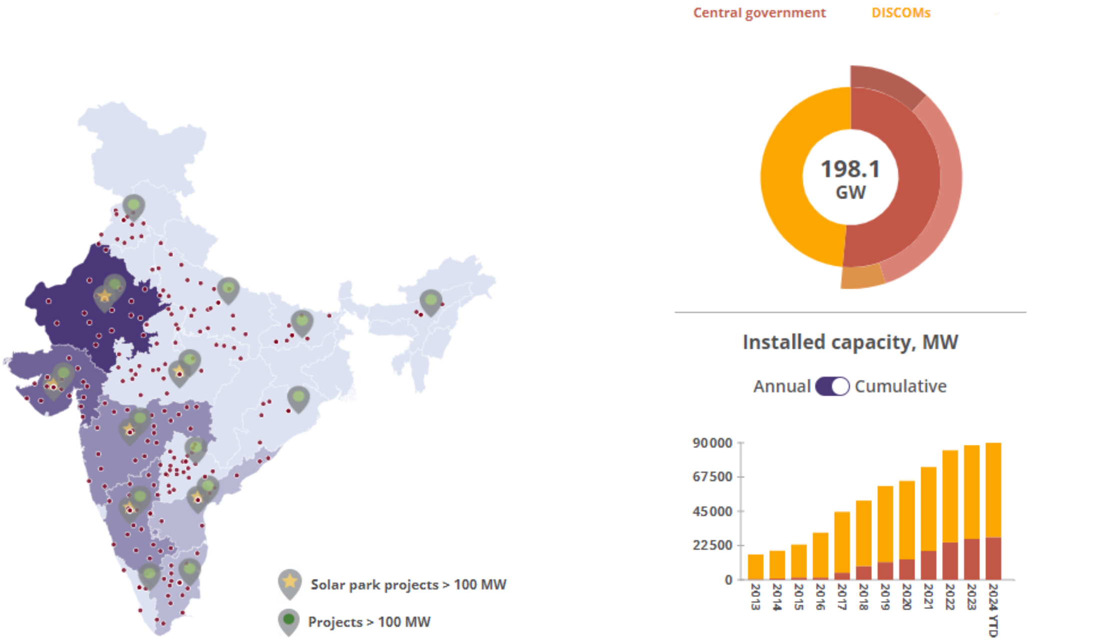
In the empirical part of the paper, I solve for a selection problem arising in the context of SECI auctions due to the presence of a qualifier round. I identify the cost distribution from observed order statistics of the final stage bid of losing non-pivotal bidders, which are also their costs. Furthermore, we observe their respective identities, and identities of all other bidders. Identification is similar to that of bid distribution in Dutch auctions where we observe winning bid, winner and other bidders' identities (Brendstrup and H. Paarsch, 2003).[8] The likelihood function employed in my paper is inspired by Song (2006), who provides conditions for identifying cost distribution from order statistics in a different setting which has unknown an number of anonymous bidders.

To conclude this section, this paper belongs to the literature on auction design for renewable energy, and contributes to empirical and theoretical literature on auctions.

3 Institution and allocation mechanism

At 180 GW, India has 4^{th} largest installed capacity of electricity production from renewable sources. Of this 109 GW is based on solar and wind. The target is

[8]In the Dutch auctions, there is an additional step where the bid distribution is used to estimate cost distribution, using the first order conditions of equilibrium. This is not needed in SECI auctions.



(a) Statewise capacity distribution

(b) Capacity by off-taker and annual trends

Figure 1: Distribution of commissioned and planned Solar and Wind capacity in India

Source of graphics- Bridge to India

to increase it to 500 GW by 2030, in order to have at least 50% of electricity production without fossil fuels. These targets are driven by environmental and pollution concern, increasing demand of electricity, and desire to reduce extreme dependence on oil and gas imports (BEE, 2009). Renewables accounted for around 90% of the additional electricity capacity for India in 2022, which indicates their ever increasing significance (report by Ember-Climate, 2023). The sector attracted foreign direct investment worth USD 9 billion, which exhibits its global importance.

Figure 1 shows the trends of utility scale solar and wind power capacity in India since 2011, and state-wise distribution of the same. The western state of Rajasthan has the highest installed and planned capacity, followed by Gujarat. The reason may be that the western Rajasthan is desert with 300-330 days of clear skies, and has easy availability of land. Nearly half of this planned and installed capacity comes from central government agencies like SECI as shown in Figure 1b. This figure also shows that the overall installed capacity has increased over the years.

Utility scale solar and wind farms are built through auctions of power purchase agreements (PPA), which are signed between the auctioneer and bidders. PPAs mention the size of projects which a single bidder has to construct, and the tariff at which they sell their electricity to the auctioneer for 25 years. Many agencies at state and central level conduct these auctions. Around 50% of the solar and wind capacity is created by SECI and National Thermal Power Corporation (NTPC) (joint report by JMK and IEEFA, 2023). Some other important agencies are the state level energy development corporations. Most of these agencies have similar final stage for allocation.

3.1 Allocation procedure

In this section, I describe SECI's allocation procedure in detail. I focus on SECI because it has almost half of the capacity and is considered relatively risk-free counterparty (Ryan, 2021). Therefore, the analysis of bids can abstract from risk-premium considerations. Moreover, state level agencies like those of Rajasthan and Gujarat, states with largest solar capacity, also share the allocation rules used by SECI in the final stage. While the focus is on inefficiencies in the final stage, I provide econometrically relevant details of qualification stage as well.

Before the 2 stages, the auctioneer releases a Request for Submission (RfS) document, which specifies auction specific details. It mentions if the project has to be solar or wind or hybrid, if it has be located in a particular place in India or if it's location neutral. RfSs state that it's bidders responsibility to find the land (unless the auction is for solar park) and connect their project to the grid. It provides the incentives and penalties, respectively, for good and bad post-auction performance (delay in commissioning and production). RfS mentions procurement target (M) and reserve tariff for the qualifier stage ($\bar{p}$).

In the qualifier stage, each bidder submits two envelops. The first envelop shows the financial and technical competence of the bidder. The second envelope contains the project size and flat tariff offer. SECI doesn't open second envelope until it has ascertained the veracity of the first envelope. If allowed, the tariff could be replaced by Viability Gap Funding (VGF), which is the minimum amount/MW required by the bidder to make her project financially feasible, while selling electricity it would produce at $\bar{p}$. VGF bids were discontinued after 2017, except for very specific cases, as more and more winners were bidding only on tariffs.

The qualification and allocation rules are exhibited through the example in Table 1.

The total number of bidders in the qualifier round can be denoted by N_1, and the mechanism is low price sealed bid. SECI sorts the N_1 bidders in an increasing order of their tariff offers, with the lowest (best) rank for the lowest tariff. If VGF is allowed, the bidders asking for VGF are ranked worse than the ones bidding tariff. SECI selects top l bidders such that their cumulative capacity just exceeds M for the final round. Additionally, top half of the remaining bidders also qualify. To formalise, assume that the project capacity offered by i^{th} ranked bidder is $q(i)$. SECI would select top m ranking bidders for the final stage, such that $m = l + \lceil \frac{N_1 - l}{2} \rceil$ where $l = \min_k \sum_{j=1}^{k} q(j) \geq M$. In the example table 1a, $N_1 = 7, m = 6, l = 4$ and selected bidders are $B_1 - B_6$. If the total of bidders' capacities is less than M, the auctioneer reduces the value of M in final stage in a pre-defined manner and all the bidders qualify. Denote the number of bidders in final round by N.

The N qualified bidders compete online in an open descending bid auction in the final round.[9] Each bidder is able to see opponents' pseudo identities, tariff, desired project size, and tentative award throughout the auction.[10] Bidders can't change their project size in this stage, which can, thus, be treated as exogenous. The starting tariff bid of each bidder is their qualifier tariff offer, and they can only reduce it. In a way, each bidder has a personalised reserve bid. The minimum reduction allowed is 0.01 INR (≈0.00012 USD). Auction lasts for at least one hour and it ends when there has been no change in bids for 8 minutes. At the end of the auction, top W bidders by tariff, whose cumulative capacity just falls below M are awarded a contract are awarded contract to build their desired capacity. The market clearing rule is to award the lowest tariff bidder among the remaining (or, the marginal winner), the residual amount for capacity creation. In the table 1b, $W = 3$, $B_1 - B_3$ are awarded their capacity, while B_4 is marginal and is awarded residual of 500 and 350. If two bidders have same the tariff and both are marginal, then the residual is given to one of them with 50% probability.

This paper focuses on the final stage, and inefficiencies which may arise due to the market clearing rule. To understand bidding behaviour in the auction, lets revisit the example in Table 1. Initially, there is excess supply and all the bidders are non-pivotal in the sense that their decision to not compete doesn't lead to excess demand. At the start, B_5 and B_6 need to bid less than 3.0 in order to get any award. Suppose they jump to this tariff. At this point, B_4's bid is 3.0, B_5 and B_6

[9]It is strategically equivalent to uniform price auction. See Krishna (2009)

[10]Tentative award at some instant is the project size a bidder would be awarded if the auction were to end at that instant.

Table 1: Qualification and allocation rules

(a) qualification

Bidder	Capacity	Tariff	Qualify
B_1	100	2.4	✓
B_2	50	2.6	✓
B_3	200	2.8	✓
B_4	450	3.0	✓
B_5	150	3.2	✓
B_6	100	3.4	✓
B_7	300	3.5	×

(b) Allocation

Bidder	Capacity	Final Tariff	Award
B_1	100	2.09	100
B_2	50	2.09	50
B_3	200	2.09	200
B_4	450	2.1	150
B_6	100	2.5	0
B_5	150	2.9	0

are at 2.99. If the auction were to stop here, B_4 would get nothing and B_5 or B_6 would get the residual. However, B_4 can now bid 2.98 to get the residual if her cost is lower than that. In fact, she should not bid below 2.98 at this instant unless she wants to be awarded a project of same size as her offered capacity. B_5 and B_6 would then respond by reducing their bid to 2.97. The game continues this way with each bidder trying to just outcompete the others. Proceeding this way, suppose that B_5 has bid 2.9 and B_4 and B_6 have bid 2.89. Suppose B_5's cost is 2.9. In this case, she would prefer to exit the auction as competing further risks getting a negative reward as the tariff would be lower than her cost. The game, although, proceeds without B_5; B_4 and B_6 try to outcompete each other, until B_6 drops out at her cost of 2.5. At this tariff, B_4 becomes pivotal. She can either try to outcompete others and get more than 150MW, or drop out at 2.5 to get 150MW. She decides to compete and eventually drops out at 2.1, even if her cost may have been than that of others, and gets 150 MW.

In this stage, one can think of strategy of each bidder as a cutoff tariff upto which they would compete, given the set of bidders who are yet to drop out. Their cutoff tariff can be seen as their bid. In this scenario, assuming that the bid space is continuous, any non-pivotal bidder would have a bid equal to their cost. Bidding higher reduces their chances of getting positive award by becoming pivotal (like B_4 for bids below 2.5) or getting own capacity award while being non-pivotal (like B_2, B_3) at a price above their cost. Bidding lower would yield negative reward. A pivotal bidder, on the other hand, has a bid higher than her cost as that gives her a positive award. Her bid depends on her cost, the set of pivotal and non-pivotal bidders who are still active in the auction, and her beliefs about their bid.

4 Data and stylized fact

Data for this paper is compiled from various sources. Bidding data is obtained from 2 sets of documents the SECI's website. The first set of documents are requests for submission (RfSs), which are issued by the auctioneer to invited bidders. This document provides auction specific characteristics like technology specifications, location restrictions, procurement target etc. They also provide the details of auction mechanism, allocation, and market clearing rule among other things. Separately in a result sheet, SECI provides bidding data for both stages. For qualifier stage, we observe bidder identity, tariff and project size offers. For the final, we observe the project size awarded to each bidder, bids of bidders with zero award, and bid of the marginal winner. The bidding data is for auctions from 2016 through 2023. In all, there are 536 bids for the qualifier and 421 bids for the final stage from 62 auctions.

Usually, the firms create different entities for each auction. I map these entities to their ultimate corporate parent using Orbis, public data on Zaubacorp, and firms' own websites. For some entities, like Mira Zavas, Halvad, I found the ultimate parent company using the domain of the email ID for the registered contact on Zaubacorp and further confirmed the same on LinkedIn. Since the market is yet to consolidate, there are many mergers and acquisitions during the period of data collection. I map them to the parent in the year in which the auction was conducted. Overall, there are 102 companies.

4.1 Data overview

In total, there is data from 64 auctions conducted by SECI. Two of these auctions restrict participation to Public Sector Enterprises, two have very small procurement target (5MW). One of them is for Round-the-Clock supply with escalating prices and another one has two part tariff. Data is not complete for 2 of the auctions. I remove these eight auctions from the analysis because they are not directly comparable to other auctions. This leaves us with 56 auctions, having a total of 517 bids for the qualifier and 410 for the final stage for 46.8GW allocation. Table 2 shows the average number of participants to the qualifier round across large SECI auctions. In total, 102 firms have participated in the large auctions. High participation in 2017 is driven by auctions for projects in Bhadla Solar Park in the desert state of Rajasthan, which is now one of the largest Solar Parks in Asia. The auctions have

Table 2: Average number of participants in auctions with target ≥ 25

Year	Ind	N	S	SP	Aggregate
2015	-	-	8.5	-	8.50
2016	-	3	8.33	6	4.73
2017	12	-	-	22	17.80
2018	9.5	-	-	7	9.14
2019	4.25	9	-	-	5.20
2020	9.6	13	-	-	10.57
2021	15	22	-	-	17.33
2022	12.75	-	10	-	11.83
2023	9.2	-	-	-	9.2
Aggregate	9.06	7	8.83	16	9.23

been successful in fostering competition and scale, which has reduced the average market clearing price (as explained in section 3) from Rs. 4.5/unit in 2015 auctions to Rs. 2.4/unit in 2021 auctions (Figure 2).

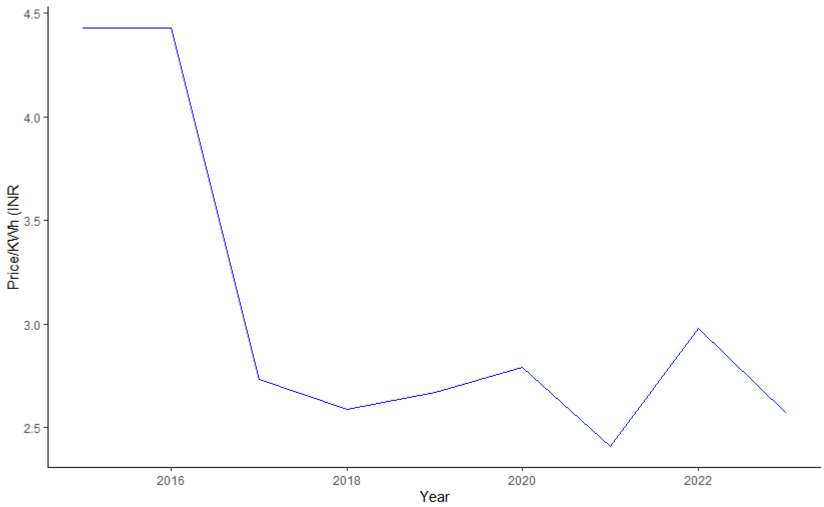


Figure 2: Average market clearing price in large SECI auctions

Of the 102 firms, 34 have never won an award. Table 3 provides total award for each category of firm, in the 56 auctions for each technology- solar, wind, and hybrid. 54% (25.5GW) of the capacity is created by the 27 Indian energy firms. The largest among them includes Adani Green, Renew power, NTPC. Nearly half of the remaining capacity (9.88GW) is awarded to the 13 companies incorporated by financial institutions, specifically for the purpose of gaining returns on investment in India's renewable market. Half of the capacity awarded in this category is won

Table 3: Gigawatts awarded and number of successful bidders for each firm category in SECI auctions with target $\geq$25 MW

Firm Category	Hybrid	Solar	Wind	Total
Non financial	5.91	17.49	13.52	36.92
Indian Energy	5.46	10.99	9.02	25.48
Others	0.45	6.49	4.50	11.44
International		4.04	4.16	8.20
Indian NonEnergy		2.16	0.34	2.49
Financial	1.2	6.58	2.10	9.88
Total	7.11	24.07	15.62	46.8

by Softbank's energy platform and Ayana, energy platform incorporated in 2016 by British government's British International Investment.[11] Among other firms, 17 international energy firms like Total Energy (France), Sembcorp (Singapore) have won most of the PPAs in terms of capacity.

Thus, we can see that a diverse set of firms have participated and won in SECI auctions. Even within Indian energy firms, data shows that awards are being won equally by public sector firms, family firms, and other firms backed by financial institutions. As such, the market has diverse set of producers, and concentrations are still low. Given such high market participation, collusive behaviour may not be stable.

4.2 Stylized fact about bidding behavior

The key element of strategy in the final stage of auctions is the decision of a pivotal bidder to agree to a residual award at a certain price, or to compete further in order to get the desired capacity.

In the tariff auctions with large procurement targets (above 200MW), I observe that there are 37 auctions with a pivotal bidder who doesn't exercise her right to reject the award.[12] In 30 out of these 37 auctions, the bidder who gets residual award has bid within INR 0.02 of the lowest of bids of all the bidders with zero award; or within

[11] Softbank sold all of its portfolio to Adani Green in 2022, despite the allegations of accounting fraud and corruption against the latter.

[12] This right was given to the bidders who were awarded half of their desired capacity, in some auctions since 2019.

0.02 of her qualifying bid, if there is no bidder with zero award. In these cases, the pivotal bidder is said to have exited immediately or not competed. In 26 of these 30 auctions, the bidder with highest quantity bid exits immediately. In 11 auctions, the pivotal bidder competes before exiting at a slightly lower bid. Besides these 37 auctions, two have bidders with same capacity and in three others, no bidder gets rationed in the outcome.

Among the 14 VGF auctions analysed (with target $\geq$ 25MW), 6 auctions have no residual award as each bidder's quantity bid equals M. Among the remaining, the pivotal doesn't compete in 3 auctions. This bidder is also the one with highest capacity in these auctions. In 2 auctions, competition is observed. In 3 auctions, the winner had a very low first round bid and capacity bid equal to M, which led to absence of competition in second round.

Thus, we observe both competitive and non-competitive behaviour in 42 auctions with a pivotal bidder. This can rule out a scheme where the bidders collude to get a high tariff. However, in 78% of all these auctions, we observe that the pivotal bidder agreed to residual award without any competition, and in most of the cases, it was the bidder with highest capacity. There seems to a relationship between capacity offer and competitive behavior. To further explore this relation, I estimate a simple linear probability model where decision to exit immediately is the dependent variable.

In the econometric model, the decision to not compete is captured by indicator variable $concede_{ia}$. For tariff auctions, $concede_{ia} = 1$ if in auction a, B_i gets residual award and bids within INR 0.02 of the lowest bid among all the bidders who got zero award. If no bidder gets award of zero, I compare the final bids to qualifier bids and set $concede_{ia} = 1$ if B_i gets a positive residual award with the bid within INR 0.02 of her qualifier bid. For VGF auctions, same procedure is followed with a threshold of INR 100,000.

I use data from the 42 auctions with pivotal bidders to estimate the linear probability model, where right to reject the residual award was not exercised. As explained in the section 3, whenever a non-pivotal bidder concedes, the auction doesn't end, as the allocation is not yet decided. It continues with lesser number of bidders, any subset of whom (including empty subset) might get a positive award if they decide to not compete further. Thus, a subgame is created among remaining bidders. If I observe such a situation in a particular auction, I consider the subgame generated by exit of a non-pivotal bidder as a separate auction. In each such subgame a where the bidder i decides to compete and not agree to a residual capacity, $concede_{ia} = 0$. In the observed terminal subgame, $concede_{ia} = 1$ if the bidder who gets positive residual concedes immediately. I treat these subgames as independent of each other, which imposes further limitation on the interpretation of the linear probability model. As such, the model just measures a controlled correlation, and not the causal effect of various factors on decision to exit immediately. The bidders who got zero award are not considered for this analysis because their decision to concede is not based on strategic choice regarding agreeing to residual at a higher bid, but on their individual rationality.

Furthermore, it is possible that pivotal bidder exits only when the size of residual award is high enough. To check this, I calculate a potential residual award for all the bidders who were awarded their capacity. This is the capacity award they would have obtained hypothetically if they had chosen to concede at the tariff discovered in the auction, instead of outcompeting it. To this end, I subtract from the procurement target M, the capacity offers of winners whose bids are same for both the stages (if any). This gives us "adjusted M". I also remove such bidders from regression analysis because their behaviour is not observed. The potential residual award is then difference between adjusted M and capacity of all other bidders. The potential residual is then floored at 0. The non-pivotal bidders at the market clearing price are those with zero potential residual.

In three different specifications, I run a regression of $concede_{ia}$ on $capacity_{ia}$ and $residual_{ia}$ scaled by procurement target in a, and the ratio of the two. I remove the non-pivotal bidders from the data fed into regression. Effect of competition on exit decision is captured by number of non pivotal and pivotal competitors. The regression results are provided in Table 4. This econometric model doesn't use any measure for cost of bidders, which is important for exit decisions. However, this should not be a problem because the aim here is not to claim any causality, but find some correlation between a bidder's decision to concede and her capacity. This is indeed what we find. The 3 different measures, bidder capacity, bidder's potential residual award, and the ratio of the 2 are positively related to the probability of them exiting immediately.

The key stylized fact regarding final stage bidding is, thus, the positive correlation between the capacity offer of a pivotal bidder and her decision to exit immediately. The regression results indicate that this correlation may be driven by the residual

Table 4: Relation between residual award size and immediate exit decision

	Dependent variable:					
	Immediate exit decision					
	(1)	(2)	(3)	(4)	(5)	(6)
Constant	−0.060	0.253***	0.155***	−0.209	0.175	0.081
	(0.044)	(0.055)	(0.045)	(0.167)	(0.167)	(0.169)
Capacity	0.654***			0.665***		
	(0.108)			(0.132)		
Residual		0.209***			0.286***	
		(0.058)			(0.076)	
Residual/Capacity			0.008*			0.009*
			(0.004)			(0.005)
Number NonPivotal	−0.015	0.011	0.053	−0.011	−0.016	0.023
	(0.039)	(0.042)	(0.040)	(0.041)	(0.043)	(0.043)
Number Pivotal	0.020	−0.027	0.002	0.025*	−0.026	0.013
	(0.013)	(0.018)	(0.015)	(0.014)	(0.020)	(0.016)
Location				✓	✓	✓
Year				✓	✓	✓
EnergySource				✓	✓	✓
Observations	192	192	192	192	192	192
R^2	0.181	0.084	0.040	0.193	0.146	0.097
Adjusted R^2	0.168	0.070	0.024	0.148	0.099	0.047
Note:				*p<0.1; **p<0.05; ***p<0.01		

award.

5 Theory of bidding behaviour

This section models the final stage as a descending clock auction with residual award. The aim of this section is to provide a theoretical explanation of the stylized facts presented in Section 4. I make assumptions on game timing, and bidder and auctioneer information which incorporate relevant information from qualifier round. Although seemingly strong, such assumptions help this paper remain focused on incentives for bidders to compete or not, when facing the market clearing rule.

Before the auction, government announces the procurement target M for that auction. Each bidder, i announces her capacity $q_i \leq M$, which is the capacity they can create and provide to the government. I assume that this quantity is reported truthfully. Set of all the bidders is denoted by $\mathcal{N}$. The bidders are assumed to be risk-neutral. In the procedure described in section 3, the reserve bid is individualised as it depends on their qualification bid. However, I abstract from this and assume that the announces the reserve price ($=b^R$) which is same for all the bidders.

The abstraction on reserve price doesn't lead to much loss of generality as can be explained in the following example. Let's suppose $M = 100, \mathcal{N} = \{B_1, B_2, B_3, B_4, B_5\}$. Bidder's respective capacities are $\{30, 40, 50, 35, 10\}$ and corresponding qualification bids are $\{3, 4, 5, 6, 7\}$. Then, the highest possible bid for any bidder is 7. However, the 4^{th} and 5^{th} bidder get 0 if they bid 7. Thus, they would gradually reduce their bid from their starting bid, with the hope of out-competing some other bidder at a price above own cost. Suppose they reduce their bid to 5 and then B_5 exits the auction. If the auction were to end at this bid, B_1 and B_2 would get 70 in total as their award. This means that third and fourth bidder have to compete for remaining 30 if the auction continues. The situation is similar to an auction where bidders bid for an award of 30, and have the same reserve bid of 5. Moreover, the game can continue in such a way that the common reserve becomes 4 and the total award size is 70. Thus, assuming a common reserve bid, instead of individualised reserve (as in reality) doesn't affect the theoretical understanding of the bidding strategies in this auction, and this is essentially due to open nature of bidding.

Each bidder is assumed to have a constant marginal cost of supplying the product, denoted by c_i. For each bidder i, c_i is private information, revealed to her before the auction. $c_i \sim F_i(.)$ independently and $c_i \in [0, \bar{c}]$. For the baseline model, $F_i(c) = F(c)$, $\forall i$. I denote the reversed hazard rate of this distribution, $f(c)/F(c)$

by $\sigma(c)$ and assume that $\sigma'(c) < 0, \forall c > 0$. It is possible that there might be some learning among bidders from the qualification bids of their opponents. Any such learning can be captured by assuming heterogeneous priors over opponents' costs. As I show through a extensions in appendix B, heterogenous priors can be easily accommodated in the baseline model.

M is allotted via an open descending price auction, modelled as descending clock auctions as in Milgrom and Weber (1982). Bidders bid the per unit price they would ask the government for providing their capacity.[13] At the start of the auction, auctioneer displays bid b^R, M, and $q_i \forall B_i \in \mathcal{N}$ on a screen and all the bidders enter an arena. As auction proceeds, the displayed bid reduces in a continuous manner. If a bidder wishes to exit at a bid b, she leaves the arena when screen displays $b \leq b^R$. When she leaves, she gets a residual quantity award of $Max\{0, M - \sum_i q_i \mathbb{1}_{B_i \in \mathcal{I}(b)}\}$, where $\mathcal{I}(b)$ is the set of bidders in the arena at bid b. The auction stops when a bidder gets a positive award when she exits, or if $M - \sum_i q_i \mathbb{1}_{B_i \in \mathcal{I}(b)} = 0$. The bidders who are still in the arena at the end of auction are awarded their quantity at the bid displayed on the screen at that time. Thus at any point, the bidders who would get a positive residual on exiting the arena decide to either accept the residual at the current bid, or to wait for the bid to drop so that another opponent exits. If they decide the former, they get higher price but lower quantity, and vice-versa if they decide the latter.

In such a game, any bidder who would get a zero award on exiting, would not exit until the displayed price is same as her cost. If they exit at a higher bid, they still get a payoff of zero. However, if the don't exit, there is a chance that some other players will exit and this bidder may get a positive award. Thus, it's beneficial for her to not exit at a bid above cost. This characteristic of equilibrium bids of zero award bidders plays crucial role in identification of the cost distribution from SECI data.

The descending clock auction is essentially a dynamic game, where the bidders have 2 options (exit and continue) at any given instant. However, one can also think of this as a stage game. At the start of the game, each bidder chooses a cutoff bid at which she would exit, if none of her opponents would have exited by that bid.

[13]This is an abstraction from the idea of price bids being the tariff on produced electricity and not the price of constructed capacity. The price bids in this model can be thought to be the sum of per unit markup these bidders desire and the Lifetime average Cost of Electricity they expect to produce. Any adjustments made for this equivalence don't harm the equilibrium results as long as capacity utilisation factors and future discounting rates are assumed same across all the bidders.

If a bidder exits and gets an award of zero, a subgame starts, and each bidder in this subgame finds a new cutoff bid. If in any subgame, the exiting bidder gets a positive residual award, the game ends. Thus, bidders have cutoff strategies in this stage game, where the cutoff bid depends on the set of quantities of all the players in the subgame. Bidder i's strategy is to choose her cutoff bid (or simply, bid) b_i in each subgame. The analysis amounts to finding Bayes Nash Equilibria (BNE) in pure strategies of this game. To keep the results simple and tractable, I focus on games with just 2 bidders.

5.1 Pure strategy equilibrium with 2 players

This section provides the results on characteristics and existence of pure strategy equilibria for auctions with 2 players and 3 players. In general, opponent of B_i is denoted by B_{-i}, her bid by b_i, and her equilibrium bid function by $\beta_i(c)$. A bidder is said to be large if their capacity is larger than the procurement target. The simplest case with 2 bidders would be when $M < q_i$ for both i, i.e., both are large. This case reduces the auction to a simple english auction, where $\beta_i(c) = c$ for both i. The other cases are a bit more involved.

5.1.1 A large bidder and a small bidder

Assume $M = q_1 > q_2$ without loss of generality. In this case B_2 gets 0 if her bid is higher. On the other hand, B_1 gets her capacity in all the cases. B_i's ex-post payoff, conditional on winning and losing respectively, are:

$$\pi_i^W(b_i; c_i, \mathbf{q}, b_{-i}) = q_i(p - c_i)$$
$$\pi_i^L(b_i; c_i, \mathbf{q}, b_{-i}) = Max\{0, M - q_{-i}\}(p - c_i)$$

where $p = Max\{b_1, b_2\}$

B_2 would find it weakly dominant to bid her cost. If she bids above and loses, she gets 0. If she wins with this bid, she pays price equal to opponent's bid, which would higher than her cost. Thus, she isn't really better off by bidding above her cost. Bidding lower than cost is dominated as that gives negative payoff. Thus, her equilibrium bid function, $\beta_2(c) = c$.

$\beta_1(c)$ is obtained as B_1's best response to $\beta_2(c) = c$. This is obtained by maximisation of her expected payoff, which is given by:

$$\pi_1(b_1; c_1, \beta_2(c)) = (M - q_2)(b_1 - c_1)F(b_1) + q_1 \int_{b_1}^{b^R} (x - c_1)dF(x)$$

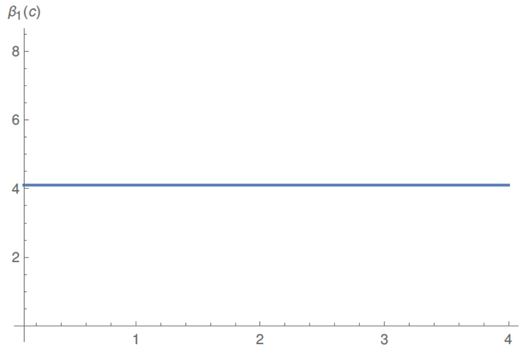
For B_1, this situation reduces, analytically, to a decision problem, rather than a game. $\beta_1(c_1)$ is attained by finding $b_1 \in \underset{b \leq b^R}{ArgMax} \pi_1(b; c_1, \beta_2(c))$ for each c_1. If $\beta_1(c_1) < b^R$, then $\sigma(\beta_1(c_1))(\beta_1(c_1) - c_1) = \frac{M-q_2}{q_2}$ which is the first order condition of optimisation at an interior point. If for some c_1 this equality doesn't hold $\forall b < b^R$, $\beta_1(c_1) = b^R$, i.e., B_1 exits immediately at b^R. Strategies $\beta_1(c), \beta_2(c)$ constitute the equilibrium of this case.

To have an illustration of equilibrium, suppose $c_i \overset{iid}{\sim} U(0, 1)$ without an atom. This implies that if there is an internal optimum for some cost type, she bids according to function $\beta_1(c) = \frac{q_2}{2q_2 - M}c$. Note that if $q_2 < M/2$, this yields negative bids, which are dominated. Thus, if $q_2 < M/2$, there is no internal optimum and B_1 bids b^R regardless of her cost ($\beta_1(c) = b^R$), which implies complete pooling. Otherwise, she would be pooling partially. For example, when $M = q_1 = 3, q_2 = 2$, she would bid b^R for $c > 0.2\sqrt{31} - 0.8 \approx 0.313$. For other values of c, $\beta_1(c) = 2c$. Notice that the bidding function is discontinuous. This discontinuity is further illustrated in Figure 3b where a truncated lognormal distribution is assumed. Since it is dominant for B_2 to bid her type c_2, and the computed $\beta_1(c)$ is the unique maximiser of B_1's payoff, the equilibrium described here is unique BNE.

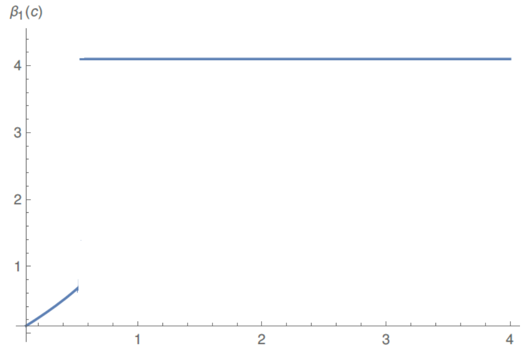
B_1 bids b^R for a type c_1 if $\sigma(b)(b - c_1) < \frac{M-q_2}{q_2}$ $,\forall b < b^R$. If M or q_1 rise, and/or q_2 declines, this inequality is likely to be satisfied for a wider range of c_1. Thus, the extent of bunching would increase. Intuitively, rise in M and decline in q_2 reduces the extent of rationing faced by B_1. This makes her reluctant to compete when her cost isn't low enough to defeat B_2 who bids truthfully.

5.1.2 2 small bidders

In this case, $M > q_1 > q_2$, and $q_1 + q_2 > M$. In this case, both bidders would get a positive reward in case their bids are higher. B_i's ex-post win and loss payoffs can



(a) Complete pooling: $q_1 = 100$, $q_2 = 40$



(b) Partial pooling: $q_1 = 100$, $q_2 = 80$

Figure 3: Equilibrium bid function of B_1

Equilibrium bid function for B_1 when $M = 100$, $b^R = 4.1$, and $F : [0, 4] \to [0, 1]$ is constrained Log-Normal with $\mu = 1$, $\sigma = 1$. Note that the scales on x-axis and y-axis are different.

be written as:

$$\pi_i^W(b_i; c_i, \mathbf{q}, b_{-i}) = q_i(p - c_i)$$
$$\pi_i^L(b_i; c_i, \mathbf{q}, b_{-i}) = (M - q_{-i})(p - c_i)$$

where $p = \max\{b_1, b_2\}$.

Any ties are broken in favour B_2.[14] Unlike, the previous case and second price auction, none of the players would bid truthfully in this case. B_i's expected payoff from the auction when she bids b_i, conditional on opponent's bid, b_{-i} and capacities q_1, q_2, M is:

$$\pi_i(b_i; b_{-i}, c_i, \mathbf{q}, M) = (M - q_{-i})(b_i - c_i)Pr(b_i > b_{-i}) + q_i \mathbb{E}_F(b_i - c_i | b_i < b_{-i})Pr(b_i < b_{-i})$$

There are 2 complete pooling Bayes Nash Equilibria, where either B_1 or B_2 never exit the arena, and their opponent exits immediately. In other words, one of the bidder commits to bid lower than the other bidder, who in turn bids b^R. Such BNE are sustained by some crazy type, and lead to completely inefficient screening.[15] Thus, it is natural to look for any other possible BNE, where screening is better. Following lemma characterises such a BNE:

[14]This tie-breaking rule is not without loss of generality. In fact, it is set in this way in order to have equilibrium existence. This is similar to the idea in Simon and Zame (1990) on endogenising the tie-breaking rule. They prove that in the game where indeterminacy can arise due to unspecified tie-breaking rule, one can always find a tie-breaking rule consistent with equilibrium existence.

[15]If we look at the descending auction in dynamic version explained earlier, such an equilibrium will not be a perfect bayesian equilibrium.

Lemma 1. *For each B_i, $\beta_i(c)$ constitute a semi-separating Bayes Nash Equilibrium of the 2-player clock auction with rationing if and only if it satisfies following properties:*

(i) *$\beta_i(c)$ is non-decreasing in c*

(ii) *$\beta_i(c)$ is continuous and atomless for $b < b^R$ for both i*

(iii) *$\beta_i(0) = 0, \forall i$*

(iv) *For each i, define $\phi_i(b) = \beta_i^{-1}(c)$ for $b \in (0, b^R)$. $\phi_i(b)$ solves:*

$$\frac{f(\phi_{-i}(b))}{F(\phi_{-i}(b))}\phi'_{-i}(b)(b - \phi_i(b))(q_1 + q_2 - M) = (M - q_{-i}), \forall i \tag{1}$$

(v) *$\beta_2(b^R) = \bar{c}$, and $\beta_1(c) = b^R$, $\forall c \in [c^*, \bar{c}]$, where $c^* = \phi_1(b)$.*

Proof. See Appendix A.1 ■

Characteristic (i) can be shown by exhibiting that payoff function satisfies increasing differences property. (ii) can be shown through standard arguments for continuity and atomlessness. If there is an atom at some bid b, the opponent's type which bids b will deviate to a bid slightly lower than b, if latter's strategy is continuous. If there is a discontinuity in strategies, such that the type $\beta(c) = b$ and type $\beta(c^-) = b' < b$, than the opponent types bidding between b' and b would prefer to bid b. These deviations are shown in Figure 4. Characteristic (iii) can be shown through arguments similar to Bertrand competition.Characteristic (iv) can be obtained through first order conditions for optimum at an interior point. It requires invertibility of bid function, which is ensured by conditions (i) and (ii).

Property (v) is the key characteristic of interest. It implies that a positive mass of high cost types of B_1 bid b^R, i.e., B_1 bunches at b^R. It relies on the relative marginal payoffs of two players at any point of intersection of the solution curves, which are such that $\frac{\beta_2'(c)}{\beta_1'(c)} = \frac{M-q_1}{M-q_2} < 1$ if $\beta_i(c)$s intersect at the cost c. The marginal payoffs are such that their solution curves intersect at most once. Then, by continuity, strict monotonicity at $b < b^R$, and property (iii) and (iv), I show that even in the immediate neighbourhood of 0, $\beta_1(c) > \beta_2(c)$. Thus, there is no point of intersection for strictly positive costs and bids and $\beta_1(c) > \beta_2(c) \forall c > 0$. Combined with the

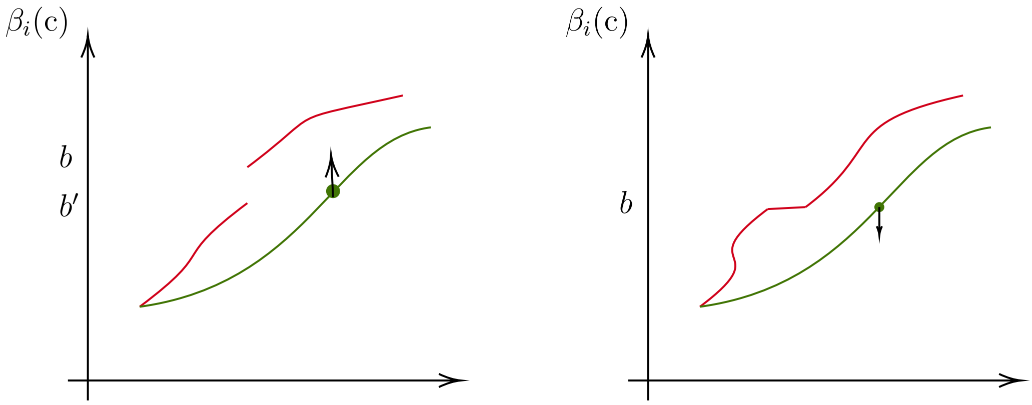


Figure 4: Possible deviations in case of discontinuity and presence of atom

property that highest types of both players should bid b^R, it implies that $\beta_1(c) = b^R, \forall c \in [c^*, \bar{c}]$, while $\beta_2(\bar{c}) = b^R$. This property also shows the importance of tie-breaking rule in favor of B_2. In absence of this rule, whenever the two players bid b^R, B_2 has an incentive to reduce the bid slightly below b^R and avoid rationing with positive probability because B_1 is bunching at b^R. This tie-breaking rule makes B_2 indifferent between bidding b^R or slightly below b^R. Such an incentive doesn't exist for B_1 as possibility of tie for her is 0 because B_2 doesn't bunch.

Intuitively, B_1 is less aggressive and bunches because she has a higher marginal cost of competing (or reducing her bid) for any given cost type. She has a higher residual award, and if she loses after competing, she gets a lower price. The gain in quantity conditional on winning is same for both the bidders ($=q_1 + q_2 - M$). Being less aggressive gives B_1 a higher markup ($= \beta_1(c) - c$), which balances her higher marginal cost of competing. B_1's bid function is above B_2's for all positive costs. This also implies that for high cost types, B_1 has no incentive to compete at all, which leads to bunching. An important implication of the property (v) of the Lemma is that we can rule out existence of any completely separating equilibrium in this auction as long as the capacities of the two bidders are different. Figure 5 shows the equilibrium as characterised in Lemma 1.

This figure also exhibits the selection inefficiency in these auctions. If B_2 has cost c_2 and B_1 has cost $c_1 < c_2$ as in the figure, B_2 will be bidding lower. As such, she will be awarded q_2 and B_1 gets $M - q_2$. Total cost of production in this scenario is $c_2q_2 + c_1(M - q_2) = c_1M + (c_2 - c_1)q_2$. On the other hand, if B_2 was rationed, the cost would have been $c_1q_1 + c_2(M - q_1) = c_2M - (c_2 - c_1)q_1 < c_1M + (c_2 - c_1)q_2$. Thus, the allocation is not cost efficient.

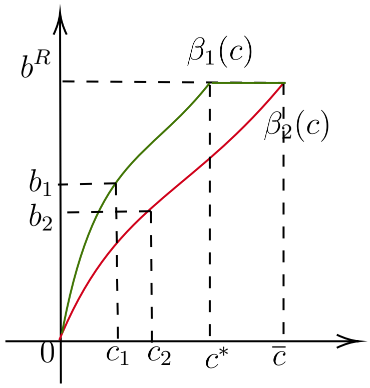


Figure 5: Asymmetric equilibrium with 2 players $q_1 > q_2$

So far, I haven't analysed the existence and uniqueness of equilibrium described in the lemma. This is important because in absence of such an equilibrium, the game only has the complete pooling equilibria. The functions, $\phi_1(b), \phi_2(b)$ are hereafter called solution curves. Since Lemma 1 implies that $\beta_1(c) > \beta_2(c), \forall c \in (0, \bar{c})$, it also implies that $\phi_1(b) < \phi_2(b), \forall b > 0$.

Any equilibrium is attained from the solution to Boundary value problem (BVP) given by FOCs (equations 1) and boundary conditions given by $\phi_2(b^R) = \bar{c}$, $\phi_1(b^R) = c^* < \bar{c}$ such that $\phi_1(0) = \phi_2(0) = 0$. The differential equations of this BVP have a division by 0 at the left boundary and hence, Picard Lidelof theorem is not applicable at $(0, 0)$. Thus, right boundary has to be used to establish existence, which is endogenously determined for $\phi_1(b)$. Similar problem of existence and uniqueness is encountered in asymmetric sealed bid first price auction, which is usually resolved by assumptions like a small atom at lower bound of support.[16] While this assumption doesn't help in SECI auctions, I can nevertheless show existence, by proving $\exists c^*$ such that when $\phi_1(b^R) = c^*$, $\phi_1(0) = \phi_2(0) = 0$. Theorem 1 is formal statement of existence and uniqueness of equilibrium in Lemma 1, which I prove in the appendix.

Theorem 1. *The BNE as described in Lemma 1, exists and is unique.*

Proof. See Appendix A.2 ■

Uniqueness can be understood through the argument similar to relative toughness in Lizzeri and Persico (2000). Consider two sets of solution curves $\phi_i(b)$ and $\hat{\phi}_i(b)$ such that $\phi_2(b^R) = \hat{\phi}_2(b^R) = \bar{c}$ and $\phi_1(b^R) = c^* < \hat{\phi}_1(b^R) = \hat{c}^*$, pertaining to "$\phi$" and "$\hat{\phi}$" situations respectively. As I show formally in appendix, this would imply

[16] Formal treatment and other useful assumptions can be found in Lebrun (2006)

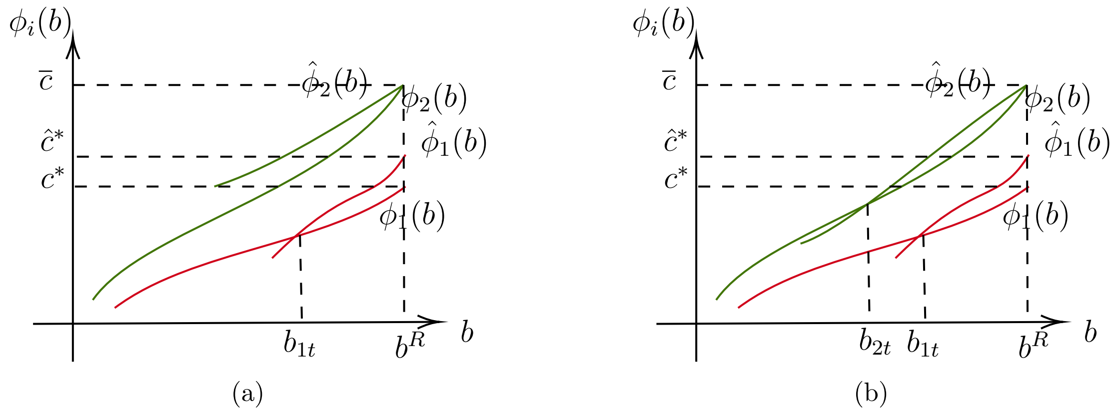


Figure 6: Intersecting solution curves

that $\hat{\phi}_1(b) > \phi_1(b)$ and $\hat{\phi}_2(b) < \phi_2(b)$ for all $b > 0$, as exhibited in Figure 7. If this doesn't hold, we have intersections as shown in Figure 6. If $\hat{\phi}_1(b)$ and $\phi_1(b)$ intersect as in figure 6a, then we have 2 solutions to the Initial value problem with $\phi_2(b^R) = \bar{c}$ and $\phi_1(b_{1t}) = c_{1t}$ and equations (1), violating Picard Lindelof theorem. Other possibility is that $\hat{\phi}_2(b)$ and $\phi_2(b)$ intersect as in figure 6b, which necessitates an intersection between $\hat{\phi}_1(b)$ and $\phi_1(b)$, as proven in the appendix. This too violates Picard Lindelof theorem. Thus, $\hat{\phi}_1(b) > \phi_1(b)$ and $\hat{\phi}_2(b) < \phi_2(b)$ for all $b > 0$.

To understand the intuition, let's look at Figure 7. At b^R, B_2 is bidding the same in both equilibria but is "marginally" less aggressive at b^R in $\hat{\phi}$ equilibrium (i.e., $\hat{\phi}_2'(b^R) > \phi_2'(b^R)$). As such, the probability of B_2's exit when B_1 bids in the immediate neighbourhood of b^R is lower in ϕ. Thus, B_1 of type $\hat{c}^*$ should be less aggressive in scenario ϕ, which compensates for this lower probability through a higher markup. $\hat{\phi}_1(b^R)$ needs to be higher to equalise marginal benefit and marginal cost of competition. The same logic applies at $b < b^R$. As $\hat{\phi}_2(b) < \phi_2(b)$, B_2 is more aggressive in scenario ϕ, and hence has a lower probability of exit at any bid. B_1 needs to compensate for that by charging a higher markup, i.e., $\phi_1(b) < \hat{\phi}_1(b)$.

While the result on existence and uniqueness is in line with the results on all-pay auctions without any residual reward for the losing bidder, there are some subtle differences. For example, results in Lizzeri and Persico (2000) required loss payoff to be nonpositive. The result I have is attained even when the "loss" payoff is positive. Moreover, my result is in contrast with the result on 2 player asymmetric war of attrition in Nalebuff and Riley (1985), which had a continuum of equilibria in asymmetric war of attrition. In their case, many possible solutions to the FOCs satisfy the condition that player with highest type can wait infinitely.

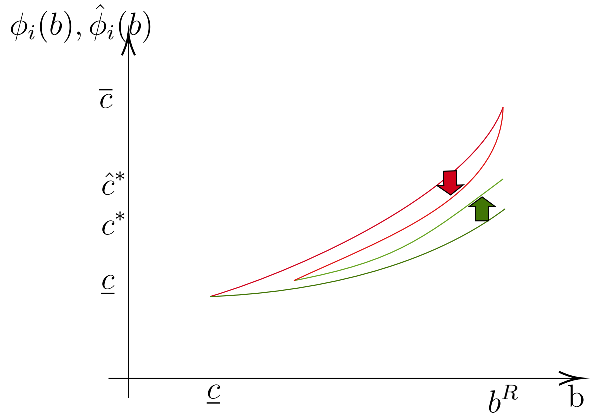


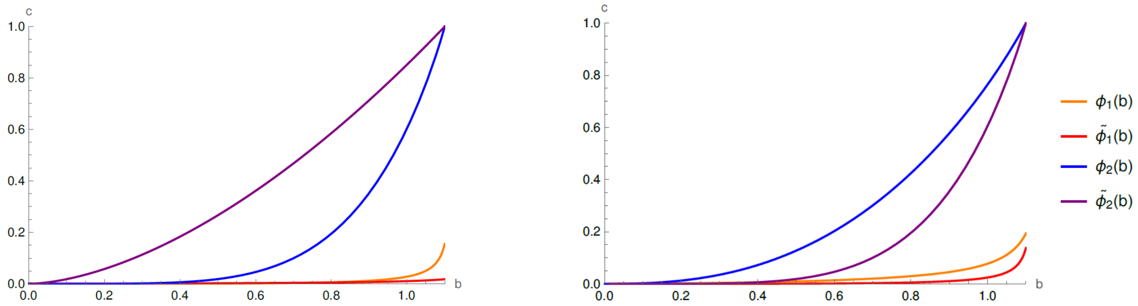
Figure 7: Co-movement of $\phi_1(b)$ and $\phi_2(b)$ in response to change in c^*

The equilibrium characteristic that B_1 bunches depends crucially on the finite reserve bid and assumption that ex-post payoff are the only source of ex-ante asymmetry. So far in the paper, this asymmetry has been imposed by capacity differences and the cost distribution is same for both bidders. However, ex-ante asymmetry can arise from differences in cost distributions too. Till now, I have focused only on the former in order to clearly understand the effect of such an asymmetry. The insights developed here on the effect of quantity award heterogeneity also carry on to the situations where both sources of asymmetry are considered. However, the identity of bunching bidder depends on the net effect of dominance of cost distribution and ex-post award. I show this in Appendix B, where I provide a formal characterisation of the equilibrium and proofs for following 2 cases of heterogeneity in cost distribution of the two players:

1. $c_i \in [0, \bar{c}_i]$, such that $\bar{c}_1 < \bar{c}_2$ and $c_i \overset{i.i.d}{\sim} F_i(c)$ such that $\sigma_1(c) = \sigma_2(c), \forall c \in [0, min\{\bar{c}_1, \bar{c}_2\}]$. Intuitively speaking, B_2 is likely to have larger costs than B_1.

2. $c_i \overset{i.i.d}{\sim} F_i(.)$ where each F_i has same support, $[0, \bar{c}]$. Denote by $\sigma_i(c)$ the reversed hazard rate (RHR) of $F_i(c)$; $\sigma_i'(c) < 0$. Suppose that the distribution F_1 RHR dominates F_2, i.e., $\sigma_1(c) \geq \sigma_2(c) \forall c \in [0, \bar{c}]$. Dominance can imply having higher probability of higher costs.

Through these cases, I can show that the intuition regarding the effect of differences in ex-post quantity award in the case of same cost distributions for each bidder case is robust to differences in cost distributions, even though the net effect is differ-



(a) (b)

Figure 8: Change in $\phi_i(b)$s in response to quantity changes

In the left figure, $\phi_i(b)$s are defined for $q_1 = 60, q_2 = 50, m = 100$ and $\tilde{\phi}_i(b)$ are defined for $q_1 = 80, q_2 = 50, m = 100$. In the right figure, $\phi_i(b)$s are defined for $q_1 = 60, q_2 = 50, m = 100$ and $\tilde{\phi}_i(b)$ are defined for $q_1 = 70, q_2 = 50, m = 100$. The costs are drawn in i.i.d manner from $U[0, 1]$ with a $b^R = 1.1$.

ent. What matters for the equilibrium structure, and specially for the identity of bunching bidder is the net effect of cost distribution dominance and quantity bids.

To conclude the analysis, I provide the comparative statics with respect to M and q_i. The simulations show that any effect of increase in $q_1/(M - q_2)$, depends on its value, and the extent of change in it. This is shown in Figure 8. In Figure 8a, q_1 rises from 60 to 90, and that leads to B_1 being very less aggressive ($\tilde{\phi}_1(b) < \phi_1(b)$), while B_2 becomes more aggressive ($\tilde{\phi}_2(b) > \phi_2(b)$). In Figure 8b, q_1 rises from 60 to 70, which makes both the players less aggressive. Thus, changes in bidding behavior in response to change in q_1 and extent of rationing are not obvious and not monotonic.

The theoretical exercises of this section and the appendix C on 3 players, show that the descending clock auction with rationing allocates inefficiently. While such an auction design is attractive because of the simplicity of allocation rules and transparency, the market of renewable electricity created by it is not cost-efficient. Thus a question arises regarding possibility of making this market more cost-efficient without using more complicated methods. I take an empirical approach to answer this question. This not only helps me quantify the cost-inefficiency in the auctions, but also tells the extent to which auctions can be made more efficient by slightly different mechanisms. The first step of this approach is to identify the cost distribution of the bidders from the observables in the data. The second step is to estimate the cost distribution, and final step is to conduct a simulation based study of various counterfactual mechanisms.

6 Identification of cost distributions

Identification requires mapping the observables in SECI's data to the auction model's primitives of interest. The model primitives of interest are cost distribution of bidders in final stage. Relevant observables are the identities of bidders, their tariff offers, project sizes of the bidders and their awards in the final stage, and the ranking of bidders in the qualification stage.

Model primitives:

Denote the set of all auctions in SECI data by $\mathcal{T}$ and bidders in qualification stage of auction $t \in \mathcal{T}$ by $\mathcal{N}_t^I$. Any bidder $B_i \in \mathcal{N}_t^I$ has capacity $q_{it} \leq M_t$. The vector of the capacities in qualification is denoted by $\mathbf{q}_t^I$.[17] An arbitrary bidder $B_i \in \mathcal{N}_t^I$ has cost $c_{it}^I \sim F_{it}^I(.)$ at the time of qualification bid submission. c_{it}^I is assumed to be drawn independently of costs of other bidders and the vector of capacities. Both c_{it}^I and q_{it} are B_i's private information during qualification. Given the time difference between the stages, bidders may receive new information which can affect their final stage costs. B_i's cost during the final stage is $c_{it} = c_{it}^I + \varepsilon_{it}$, where $\varepsilon_{it} \sim H_{it}(.)$, independent of $\mathbf{c}_t^I$ and $\mathbf{q}_t^I$. Bidder B_i's final stage cost distribution $F_{it}(.)$ is a convolution of $F_{it}^I(.)$ and $H_{it}(.)$, $F_{it}(c) = \int_{-\infty}^{\infty} h_{it}(x) F_{it}^I(c-x)dx$. The model primitives of interest are bidders' final stage cost distributions $F_{it}(.)$.

[17] Boldface character denote vectors throughout this section.

Observables:

For the final stage, SECI provides us the bidders' identities, capacities, final tariff offer r_{it}, and award a_{it}. In the clock model, a bidder's tariff offer is not necessarily same as her bid, which is the lowest price at which she bidder would supply any capacity. For all the bidders with $a_{it} = q_{it}$, the tariff offer is usually same as the bid of the rationed bidder, and is larger than their own bid. However, each bidder with $a_{it} = 0$ bids her own cost, which is higher than the cost of bidders with a positive award, i.e., B_i with $a_{it} = 0$, $r_{it} = c_{it}$.[18] This reveals some of the order statistics of the cost $c_t^{(k:N)}$ and the identity of the bidder bidding them. Besides this, SECI's data also tells us the identity of the marginal qualifier from the qualification stage.[19]

Identification:

If $c_{it} \sim F_{it}(.)$ independently, then some k^{th} cost order statistic ($c_t^{(k:N_t)}$), the identity of bidder bidding it ($B_t^{(k:N_t)}$), and set of bidders $\mathcal{N}_t$ can identify the underlying cost distribution. H. J. Paarsch, Hong, et al. (2006) and Athey and Haile (2007) provide relevant identification result adapted from Berman (1963) for single-unit Dutch auctions where winner, winning bid, and set of bidders can identify bid and cost distributions. This result relies critically on the independence of cost draws, which is not unconditional in the final stage of SECI auctions.

In the qualification stage, bidder $B_i \in \mathcal{N}_t^I$ has tariff bid $b_{it}^I = \beta_{it}^I(c_{it}^I, q_{it})$. Denote the k^{th} order statistic of bids in this stage by $b_t^{I,(k:N_t^I)}$. The qualification rule from section 3 states that bidder B_i qualifies for the final stage iff $\beta_{it}^I(c_{it}^I, q_{it}) \leq b_t^{I,(N(\mathbf{q}_t^I, \mathbf{b}_t^I, M_t):N_t^I)}$, where $N(\mathbf{q}_t^I, \mathbf{b}_t^I, M_t) = N_t$ is the number of qualifying bidders in auction t. Suppose, without loss of generality, $b_t^{I,(N(\mathbf{q}_t^I, \mathbf{b}_t^I, M_t):N_t^I} = b_{ut}^I$, bid of some arbitrary bidder B_u.

The costs of bidders in final stage are drawn from convolution of $H_{it}(.)$ and truncated distribution $F_{it}^I(. \mid \beta_{it}^I(c_{it}^I, q_{it}) \leq \beta_{ut}^I(c_{ut}^I, q_{ut}))$. Denote this convolution by $F_{it}(. \mid \beta_{it}^I(c_{it}^I, q_{it}) \leq \beta_{ut}^I(c_{ut}^I, q_{ut}))$. Assuming first stage bids are monotonic in c_{it}^I, the final stage cost c_{it} is drawn from the distribution $F_{it}(. \mid c_{it}^I \leq \bar{c}_{it}^I(c_{ut}^I, q_u, q_i))$ for all bidders. An increase in c_{ut}^I increases b_{ut}^I, which then increases $\bar{c}_{it}^I(c_{ut}, q_u, q_i)$. This affects the probability of drawing $c_{it}, \forall B_i \in \mathcal{N}_t$. Figure 9 illustrates this for a special case when all bidders draw from same distribution. The observed costs are drawn independently, conditional on c_{ut}^I (or b_{ut}^I). The conditional independence of draws

[18] This is specific to clock model of open descending bid auctions. Haile and Tamer (2003) provide results on identification of bounds of distribution without clock model assumption, which allows bidders to engage in jump bidding.

[19] Recall: marginal qualifier is the bidder who succeeds qualification stage with the highest bid.

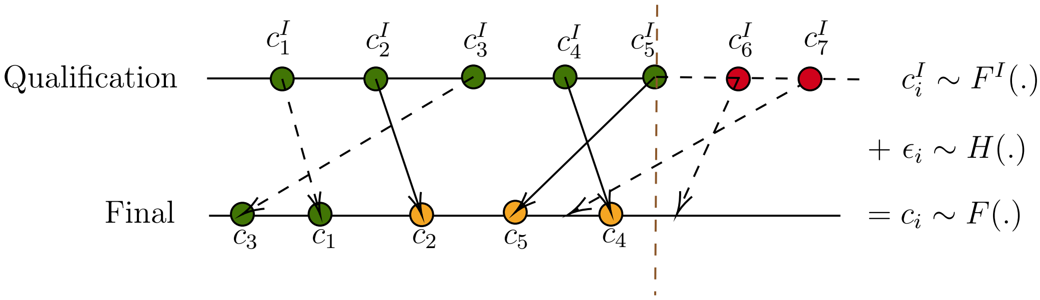


Figure 9: Selection problem

Solid segments represent the observed support of bidder cost distributions, with dots representing realizations. The figure is drawn assuming symmetric distributions across bidders. Solid arrows pertain to the bidders whose final stage costs are observed. Green dots are successful bidders at each stage, yellow dots are qualified bidders with 0 award, and red dots are disqualified bidders. Data provides orange bidders' costs and all bidders' identities. Qualification rules implies that we do not observe final costs of red bidders which may be below c_4. As c_5^I changes, length of solid segment for qualification stage changes, affecting the probability of observed cost draws.

enables us to combine Song (2006) use the identification results for Dutch auctions. As long as at least 2 order statistics of costs are observed, higher one of which pertains to the marginal qualifier, identification is achieved. As I illustrate in Appendix D, the distribution of c_{it} conditional on $c_{it} < c_{ut}$ and $c_{it}^I < c_{ut}^I$ is independent of c_{ut}^I.

Intuitively, the identification is made possible by restricting ourselves only to those auctions where marginal qualifier reveals her cost and considering the subsample of bidders cost samller than marginal qualifier's. Identification from an order statistic in this restricted subsample is same as in dutch auctions where costs are drawn from distribution truncated at the cost of marginal qualifier. Variation across auctions in the value of an order statistic and the bidder bidding that order statistic then helps with identification of distribution.

7 Estimating cost distribution

In this section, I provide parametric estimates for the cost distribution. While this imposes additional non-testable structure, the limited amount of data prevents using non-parametric estimation. Unlike non parametric estimation which is truncated at some highest observed cost, parametric estimation enables us to find a non-truncated distribution.[20] Using these estimates, one can also see the effect of auction and

[20] An analysis of tradeoff between the two methods can be found in H. J. Paarsch (1997) section 3.4.

Table 5: Average observed cost and number of non-pivotal bids

Auction Type	Pre 2018	2019 to 2021	Post 2021	**Aggregate**
Hybrid	-	2.526 (13)	2.593 (4)	2.542 (17)
Solar	3.094 (37)	2.393 (19)	2.597 (15)	2.801 (71)
North	2.882 (27)	2.328 (13)	-	2.702 (40)
South	4.558 (5)	-	2.520 (4)	3.652 (9)
Unspecified	2.774 (5)	2.535 (6)	2.625 (11)	2.634 (22)
Wind	2.985 (17)	2.925 (12)	3.103 (3)	2.974 (32)
Aggregate	3.060 (54)	2.578 (44)	2.665 (22)	**2.810 (120)**

bidder characteristics on the cost distribution.

Given the limitation imposed by identification requirement, I use data from 27 auctions. These 27 auctions have 242 bids, of which 120 pertain to cost order statistics. The auctions are different in three aspects:

1. Temporal: There is a trend of decline in costs of renewable technology, which affects the cost distribution. Furthermore, there have been many policy interventions, notably on import duties on solar panels and modules.
2. Technological: Some auctions are specifically for solar power, some for wind, and some are technologically neutral (hybrid).
3. Geographical: Some auctions specify the state where the project must be constructed, some are specific to certain solar parks, and some others are location-neutral. This affects the average cost of production because different parts of India have different solar irradiance. Moreover, a part of cost is purchasing the land and connecting the project to the grid, which can be affected by state-specific laws.

I provide a naive average of costs across all these different dimensions in Table 5. These costs are the bids of the bidders with zero award, and hence, the averages in the table are positively biased. However, these numbers can still provide some insights. For the temporal part, I divide the sample into pre-2018 (including 2018), 2019-21, and post 2021. While the data mentions the exact state, I club the states or solar parks as south Indian if they are to the south of tropic of cancer, which are also more industrialised (coincidentally). We can observe that the costs are lower in the later years. Specifically, the cost of solar power are much lower in 2019-21 period, which maybe because of reduced duties on Solar panels imported from China. We can also notice that hybrid auctions usually have slightly lower costs than solar, much lower cost than wind. Such auctions provide bidders flexibility, which enables them to create the optimal mix of solar and wind. Furthermore, these auctions don't restrict projects to specific locations, which gives additional flexibility to the bidders. This is also true for solar auctions with unspecified location, which have lower costs.

A major part of bidder heterogeneity in such large auctions emerges from financing costs. As such, an entity incorporated by financial institutions would have a different cost structure than the others. More precisely, given their assured access to finance from parent fund, the variance of their cost distribution maybe lower than others. Besides the heterogeneity on ownership, I also capture the effect of being an Indian energy firm. The idea is that such a firm would have better knowledge of the India market and policies, which can enable them to have a lower cost.

Table 6 provides average of the observed cost of each firm type. It can be noticed that Indian energy firms usually have lower costs in hybrid and solar power. For wind power, other types of firms tend to have lower cost. Similarly, firms incorporated as platforms by financial institutions tend to have lower cost in each category of auction. However, one must read these average costs wutg some caution, as they pertain only to the bidders who have lost in the auction. Thus, these numbers have a systematic upward bias. For example, it may be the case that many Indian energy firms are also winning the PPAs in wind auctions, but their costs are not used in calculating the averages in this table. The sole purpose of these descriptive statistics is to indicate that some types of firms may have systematically lower cost than the others.

Table 6 also shows that variance of costs is lower for financial firms vis-a-vis others, regardless of the auction type. Based on these indicative results, I estimate the following parametric distribution for the costs of firms.

$$
\begin{aligned}
c_{it} &\sim \log \mathcal{N}(\mu_{it}, var_i) \\
where\ \mu_{it} &= \alpha_0 + \alpha_1 Ind_i + \alpha_2 X_t, \\
var_i &= v_0 + v_1 Fin_i
\end{aligned}
$$

where X_t is the vector of characteristics, and $Ind_i = 1$ for Indian energy firms,

Table 6: Average and variance (in parenthesis) of costs of firms of each type

Auction Type	**Indian Energy**	**Others**	**Financial**	**Others**
Hybrid	2.492	2.597	2.494	2.562
	(0.014)	(0.049)	(0.005)	(0.042)
Solar	2.724	2.872	2.709	2.839
	(0.291)	(0.344)	(0.261)	(0.345)
Wind	3.117	2.848	2.729	3.055
	(0.203)	(0.095)	(0.016)	(0.182)
Aggregate	2.830	2.789	2.682	2.861
	(0.265	(0.242)	(0.168)	(0.277)

$Fin_i = 1$ for firms incorporated by financial institutions. For the formal likelihood function, I introduce some more notations:

- For each $t \in \mathcal{T}$, $\mathcal{B}_t^L$ denotes the set of bidders with zero award, and $B_t^{I,(N(\mathbf{q}_t^I,\mathbf{b}_t^I,M_t):N_t^I)}$ denotes identity of marginal qualifier,.
- $\mathfrak{S} = \{\{(B_i, q_{it}, r_{it}, a_{it})\}_{B_i \in \mathcal{N}_t} \mid t \in \mathcal{T} \ \& \ |\mathcal{B}_t^L| \geq 2 \ \& \ B_t^{I,(N(\mathbf{q}_t^I,\mathbf{b}_t^I,M_t):N_t^I)} \in \mathcal{B}_t^L\}$, set of observations from auctions with at least 2 bidder with zero award, one of which belongs to marginal qualifier
- For each $t \in \mathcal{T}$, r_{ut}, the final stage bid of marginal qualifier
- For each $t \in \mathcal{T}$, r_{lt}, the lowest observed cost order statistic in t
- For each $t \in \mathcal{T}$, $\mathcal{B}_t^{Lsub}$, set of bidders with zero award, and cost below r_{ut}

The parameter vectors $\alpha_0, \alpha_1, \alpha_2, v_0$ and v_1 are estimated by maximising following likelihood function:

$$\mathcal{L}(\alpha_0, \alpha_1, \alpha_2, v_0, v_1 | \mathfrak{S}, \cup_{t \in \mathcal{T}} X_t, \mathbf{Ind}, \mathbf{Fin})$$
$$= \prod_{t \in \mathcal{T}} \left(\frac{\prod_{B_i \in \mathcal{N}_t \setminus \mathcal{B}_t^L} F\left(\frac{ln(r_{lt}) - (\alpha_0 + \alpha_1 Ind_i + \alpha_2 X_t)}{\sqrt{v_0 + v_1 Fin_i}}\right) \prod_{B_i \in \mathcal{B}_t^{Lsub}} f\left(\frac{ln(r_{it}) - (\alpha_0 + \alpha_1 Ind_i + \alpha_2 X_t)}{\sqrt{v_0 + v_1 Fin_i}}\right)}{\prod_{B_i \in (\mathcal{N}_t \setminus \mathcal{B}_t^L) \cup B_t^{Lsub}} F\left(\frac{ln(r_{ut}) - (\alpha_0 + \alpha_1 Ind_i + \alpha_2 X_t)}{\sqrt{v_0 + v_1 Fin_i}}\right)} \right)$$

where $f(.)$ and $F(.)$ denote PDF and CDF respectively of standard normal distribution.

Table 7 provides bootstrapped estimates of parameters of cost distribution. Most results are in line with descriptive statistics. Solar auctions attract lower tariffs, and tariffs have been lower after 2018. In the years 2019-21, the tariffs are the lowest. This may be due to reduced duties on solar panels, or even due to overcompetition in the period. Among the firms, the Indian energy firms seem to have lower cost, which can be explained by their better understanding of the market and institutions. Financial firms usually have lower variance.

Table 7: Cost distribution parameters

Parameter	Estimate	Standard Error
Constant	1.094	0.074
Solar	-0.106	0.075
Y2019-21	-0.150	0.068
Y21post	-0.045	0.184
Indian Energy	-0.071	0.102
Variance	0.153	0.057
Financial	-0.068	0.064

8 Alternatives designs

In this section, I compare SECI's existing mechanism to following alternatives:

1. Sealed bid discriminatory pricing: Bidders' capacity offers are publicly known, but the tariff bids are sealed whenever some player becomes pivotal. The bidders than make a tariff bid at which they are willing to supply electricity from the project, and this is the bid SECI will pay to them. The rest of the allocation mechanism remains the same. Sealed bid auctions are also regularly used in other sectors of Indian economy, an important one of them being the spectrum allocation.[21] In the literature on comparing different auction designs with multiple winners, there is no consensus on whether uniform pricing is better or discriminatory for the auctioneer's payoff and/or the allocation efficiency. As per Holmberg and Wolak (2018), the suitability of low/high price sealed bid vs

[21] These auctions were marred by corruption scandal accusations (most notably 2G scam of 2008), which may have been a motivation behind having open auctions in renewable energy.

open ascending/descending bid auction depends on the application at hand, and hence warrants a separate investigation for each setting. Fabra, Fehr, and Harbord (2006) compared the two auctions theoretically when bidders have complete information about each other, and concluded that the effect this design choice on allocation efficiency is ambiguous.

2. Vickrey Clarke Groves (VCG): Consider the case of 2 pivotal players. VCG mechanism is a pair (A_i, t_i) where A_i is allocation to B_i and t_i is the transfer from SECI to the bidders, such that

$$A_i = \begin{cases} M - q_{-i}, & b_i > b_{-i} \\ q_i, & b_i < b_{-i} \end{cases}$$

$$t_i = \begin{cases} (M - q_{-i})\bar{c}, & b_i > b_{-i} \\ (M - q_{-i})\bar{c} + (q_1 + q_2 - M)b_{-i}, & b_i < b_{-i} \end{cases}$$

where b_i is B_i's sealed bid. The transfer has a constant part, which is set in a way which makes the mechanism individually rational. It can be seen that this allocation and transfer rule yields $b_i = c_i$ for each i as a weakly dominant strategy, under same information assumptions as before. Any other bid either leads to a negative payoff or reduces the probability of getting project of desired size. This leads to perfect revelation and fully efficient allocation. However, these auctions maybe costly to SECI's budget.

I compare these designs for their allocation efficiency and the ex-post payoffs for the auctioneer. I estimate the following metrics:

- The **probability** that the lower cost bidder was given a residual award (i.e., got smaller capacity award).
- The **production cost** incurred in creation of capacity when award is made as per a particular auction mechanism.
- **Productive inefficiency**: Difference of production cost in different mechanisms vis-a-vis VCG mechanism (efficiency benchmark)
- **Ex-post payment** made by SECI to the auctioneers.

I estimate these metrics for the auctions which have 1 or 2 pivotal bidders, or where project size is same for each pivotal bidder. These are the cases where we have theoretical resolution of the bidding behavior. Such auctions account for around 10 GW of total capacity allocations.

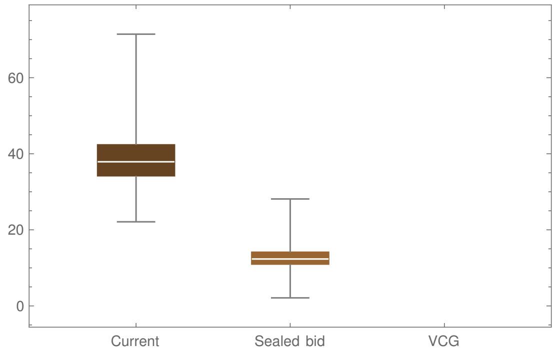
While estimating for the auctions with 2 pivotal players, I assume that they can't account for information heterogeneity among them because they can't see each other's identities. However they know past auction results. Thus, I assume that they respond to the following belief regarding their opponent's cost:

$$c_{ia} \sim Log\mathcal{N}(\hat{\mu}_{ia}, \hat{var}_i)$$
$$where\ \hat{\mu}_{ia} = \hat{\alpha}_0 + \hat{\alpha}_1 \bar{X}_i + \hat{\alpha}_2 X_a, \hat{var}_i = \hat{var}_0 + \hat{var}_1 \bar{X}_i$$

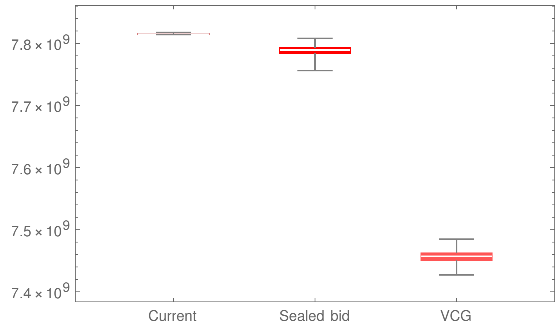
Here, $\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2$ are estimated parameters from Section 6. $\bar{X}_i$ is the weighted average of characteristics of all the bidders. Each bidder's weight is the fraction of auctions where she has won a positive award. Using this distribution, I compute bidders' bids under alternative mechanisms and calculate the metrics for each draw in each auction. Figure 10 shows the bootstrapped estimates for inefficiency and payment metrics.

The estimates show that the probability of inefficient allocation is much higher in open uniform price in comparison to sealed bid discriminatory price auction and VCG, and the production costs are slightly higher. To understand the gains in efficiencies when switching to discriminatory pricing, we can compare equilibrium bidding functions in each, which are shown in figure 11. We notice that bidder's bidding functions (and hence, bid distributions) are closer to each other in sealed bid discriminatory price auction, vis-a-vis open uniform price auction. The reason being that in the latter, bidders receive what they bid when they win, not what their opponent bids. As such, they are inclined to make bids with higher markup. Given that the highest cost type bids reserve, higher markup implies that the bidding functions are at higher levels for all costs in sealed bid compared to open bidding. This would imply that their bidding functions and the bid distributions are close, which reduces the probability of inefficient allocation. Such difference in bidding behavior across auctions should be seen as long as bidders are asymmetric. If the bidders are symmetric, we don't have inefficient allocation.

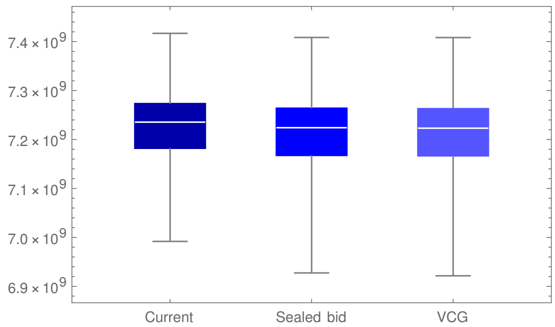
Finally, it is possible that bidders may be under-reporting in mechanisms other than VCG, which adds to inefficiency levels presented in this paper. Thus, the efficiency



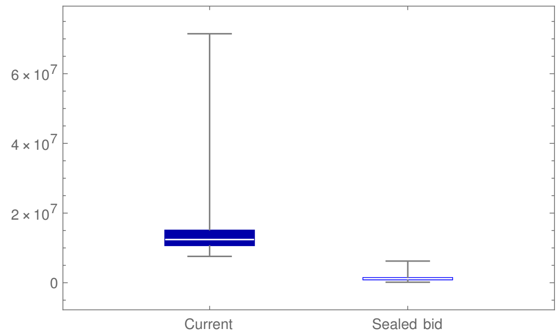
(a) Probability of Inefficient allocation



(b) SECI's ex-post payment



(c) Production Cost



(d) Productive Inefficiency

Figure 10: Comparing different mechanisms

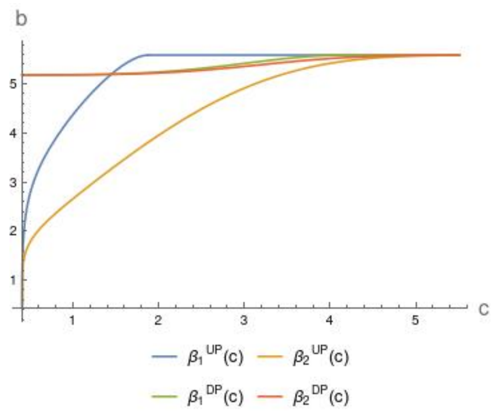


Figure 11: Bidding behavior: uniform versus discriminatory price

estimates here are a lower bound. Moreover, VCG also performs better when it comes to payments by SECI to the bidders. There is roughly 5% savings which can be achieved if SECI switches her design, which could be a policy recommendations

9 Conclusion

In this paper, I attempt to provide insights into designing large scale renewable energy auctions. As more and more countries adopt these auctions, and as these auctions enable very large scale capacity creation, little elements of their design can have significant effect. This paper analyses such effects in the final stage of auction mechanism used by Solar Energy Corporation of India (SECI), theoretically as well as empirically. India already has 5^{th} largest capacity of solar and wind power, and the aim is to scale it up further in order to generate 50% of electricity from renewable sources by 2030. In this capacity creation, SECI has an important role to play.

SECI auctions' winners receive 25 year long PPAs. The terms of the PPA are set in the final stage, whose participants are chosen on the basis of their bids in a qualification stage. This stage is an open descending price auction (uniform pricing) with multiple winners, and a market clearing rule which gives residual award to the winner with highest bid. The stylized fact and semi-separating equilibrium of a descending clock version of this auction show that bidders with higher project size bid less aggressively. I show that public knowledge of opponent's project sizes, finite reserve bid, and the market clearing rule are drivers of this result. These results clearly show that these auctions are allocating inefficiently.

The paper then attempts to suggest improvements in this design. This is done empirically, in order to measure the possible reductions in inefficiency and SECI's payment. For this purpose, the paper first deals with an identification problem. Here the identification of parameters is enabled by observation of bids of bidders with zero award, who bid their cost. These costs provide us order statistics, albeit only of the bidders whose costs is below a threshold in the qualification stage, which poses a sample selection problem. This problem can be resolved by exploiting the density of probability of observing a certain order statistic, conditional on the observation of cost of the worst ranked qualifying bidder. In addition to the order statistics, we can also observe identities all the bidders mapped to their bids. The identification then follows from the literature on dutch auctions with observation of winning bid, winner, and set of all the bidders.

Using the aforementioned conditional density, I create a likelihood function, and estimate the parameters using MLE. The estimates show that Indian energy firms are likely to have lower expected cost. The firms which are incorporated by financial institutions, with the sole aim of making some financial returns, are likely to have lower variance. With these parameters, I conduct a counterfactual exercise and show that switching from uniform pricing of open auction to a sealed bid discriminatory pricing, or VCG mechanism, can reduce SECI's payoffs as well as inefficient allocation probability. The results show that VCG performs the best in terms of efficiency as well as SECI's payoff. The paper ends with a policy suggestion on moving to a VCG mechanism.

To conclude, in this paper, I study auction design in a very important market. In doing so, I also contribute to literature in auction theory and empirics. The paper opens a lot of different avenues for research on India's energy markets. For example, research questions can focus on effect of auction design choices on investment incentives. While VCG does away with underinvestment, we can't say if the currently used SECI mechanism has such incentives or not. Fabra, Fehr, and Frutos (2011) has answered such a question in complete information setting. Extending this work to IPV setting would be theoretically interesting and can help with empirically measuring the extent of underinvestment. More empirical questions try to understand why Indian energy firms have lower cost than others, or on the differences in costs realised in North India vis-a-vis South India in locations with similar solar and wind conditions. Another question can be on the impact of changes in the duties on solar panel and module imports. The Indian renewable energy market is still growing, and researching these and other related questions can help shape it's final form; and also provide insights for other countries.

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A Proofs for section 5

Throughout the proof, assume that $q_1 > q_2$ denote $\lim_{x \to x^-} u(x)$ by $u(x^-)$ and $\lim_{x \to x^+} u(x)$ by $u(x^+)$ for any function $u(x)$.

A.1 Proof of Lemma 1

Proof. First I prove that the equilibrium should satisfy the specified conditions. Then, I show that there is no unilateral deviation from a bid suggested by these properties (only if direction), for any type of any bidder (if direction).

Only if direction:

Condition (i)**, Monotonicity of bidding functions**: It is sufficient to show that payoff of a player satisfies Single crossing property of incremental returns (SCP IR). Consider any 2 arbitrary cost types of B_i, c_i and c_i' such that $c_i < c_i'$ and 2 bids b_i, b_i' such that $b_i < b_i'$. Then the property is satisfied if $\pi_i(b_i', c_i) - \pi_i(b_i, c_i) > 0$ implies $\pi_i(b_i', c_i') - \pi_i(b_i, c_i') > 0$ when the opponent B_{-i} bids with a non-decreasing strategy. Without loss of generality, assume $i = 1$.

$$\begin{aligned} \pi_1(b_1', c_1; b_2) &= (M - q_2)(b_1' - c_1)Pr(b_2 < b_1') + q_1\mathbb{E}(b_2 - c_1|b_2 > b_1')Pr(b_2 > b_1') \\ \pi_1(b_1, c_1; b_2) &= (M - q_2)(b_1 - c_1)Pr(b_2 < b_1) + q_1\mathbb{E}(b_2 - c_1|b_2 > b_1)Pr(b_2 > b_1) \end{aligned} \tag{2}$$

where b_2 is the random variable denoting B_2's bid.

$$\begin{aligned} \therefore & A(b_1', b_1, c_1, b_2) \equiv \pi_1(b_1', c_1; b_2) - \pi_1(b_1, c_1; b_2) \\ =& (M - q_2)[(b_1' - c_1)Pr(b_2 < b_1') - (b_1 - c_1)Pr(b_2 < b_1)] \\ & + q_1[\mathbb{E}(b_2 - c_1|b_2 > b_1')Pr(b_2 > b_1') - \mathbb{E}(b_2 - c_1|b_2 > b_1)Pr(b_2 > b_1)] \end{aligned} \tag{3}$$

Suppose $A(b_1', b_1, c_1, b_2) > 0.$,

$$\begin{aligned}
&\pi_1(b_1', c_1'; b_2) - \pi_1(b_1, c_1'; b_2) \\
=&(M - q_2)[(b_1' - c_1')Pr(b_2 < b_1') - (b_1 - c_1')Pr(b_2 < b_1)] \\
&+ q_1[\mathbb{E}(b_2 - c_1'|b_2 > b_1')Pr(b_2 > b_1') - \mathbb{E}(b_2 - c_1'|b_2 > b_1)Pr(b_2 > b_1)] \\
=&(M - q_2)[(b_1' - c_1 + c_1 - c_1')Pr(b_2 < b_1') - (b_1 - c_1 + c_1 - c_1')Pr(b_2 < b_1)] \\
&+ q_1[\mathbb{E}(b_2 - c_1 + c_1 - c_1'|b_2 > b_1')Pr(b_2 > b_1') - \mathbb{E}(b_2 - c_1 + c_1 - c_1'|b_2 > b_1)Pr(b_2 > b_1)] \\
=&A(b_1', b_1, c_1, b_2) + (M - q_2)(c_1 - c_1')[Pr(b_2 < b_1') - Pr(b_2 < b_1)] + q_1(c_1 - c_1')[Pr(b_2 > b_1') - Pr(b_2 > b_1)] \\
=&A(b_1', b_1, c_1, b_2) + (M - q_2)(c_1 - c_1')[Pr(b_2 < b_1') - Pr(b_2 < b_1)] + q_1(c_1 - c_1')[-Pr(b_2 < b_1') + Pr(b_2 < b_1)] \\
=&\underbrace{A(b_1', b_1, c_1, b_2)}_{>0} + \underbrace{(M - q_2 - q_1)}_{<0}\underbrace{(c_1 - c_1')}_{<0}\underbrace{[Pr(b_2 < b_1') - Pr(b_2 < b_1)]}_{>0 \text{ as } b_1' > b_1}
\end{aligned} \tag{4}$$

$Pr(b_1' = max\{b_1', b_2\}) - Pr(b_1 = max\{b_1, b_2\}) > 0$. This, along with $A(b', b, c_1, b_2) > 0$, $c_1 < c_1'$, $M < q_1 + q_2$, ensures that above expression above is positive. Thus, $\pi_1(b_1', c_1'; b_2) - \pi_1(b_1, c_1'; b_2) > 0$, which proves the SCP-IR. Thus, equilibrium is monotonic.

Condition (ii), continuity and atomlessness

Continuity: For this, I proceed in two steps. First I show that their bidding functions have same range, and then I show that the range is a convex subset of real line. Given the monotonicity of equilibrium, the only type of discontinuity is the one where for some type c_1 of B_1, $\beta_1(c_1^-) = b' < \beta_1(c_1) = b$. Suppose first, that bidders bidding function ranges are different. Then, as shown in the figure 12a, $\exists \tilde{c}_2$ s.t. $\beta_2(\tilde{c}_2) \in [b', b]$. The expected payoff of this type of B_2 is $\pi_2(\beta_2(\tilde{c}_2), \tilde{c}_2) = (\beta_2(\tilde{c}_2) - \tilde{c}_2)(M - q_1)Pr(b_1 < \beta_2(\tilde{c}_2)) + q_2\mathbb{E}(b_1 - \tilde{c}_2|b_1 > \beta_2(\tilde{c}_2))Pr(b_1 > \beta_2(\tilde{c}_2))$.

If she bids b, her expected payoff is $\pi_2(b, \tilde{c}_2) = (b - \tilde{c}_2)(M - q_1)Pr(b_1 < b) + q_2\mathbb{E}(b_1 - \tilde{c}_2|b_1 > b)Pr(b_1 > b)$. The monotonicity of B_1's strategy and a hole in her bid distribution on (b', b), and atomlessness of cost distribution, $Pr(b_1 > b) = Pr(b_1 > \beta_2(\tilde{c}_2))$ and $Pr(b_1 < b) = Pr(b_1 < \beta_2(\tilde{c}_2))$. Thus, $\pi_2(b, \tilde{c}_2) - \pi_2(\beta_2(\tilde{c}_2), \tilde{c}_2) = (b - \beta_2(\tilde{c}_2))(M - q_1)Pr(b_1 < b) + q_2\mathbb{E}(b_1 - \tilde{c}_2|b_1 > b)Pr(b_1 > b) > 0$.

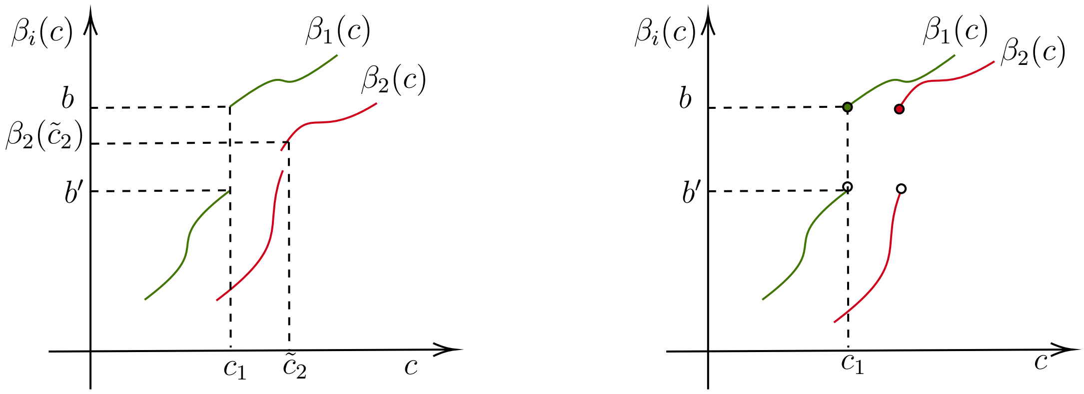
Now suppose that there is a range of bids $[b', b)$, $b' < b$ which are bid by none of the bidder and all bids below b' are being bid by some type of each bidder. This is shown in Figure 12b. Consider a type $c_1 - \epsilon$, $\epsilon \to 0$ of B_1 such that her type c_1 bids b. Given the monotonicity, this type would bid $b' - \delta(\epsilon)$, $\delta(\epsilon) \to 0$ when $\epsilon \to 0$. Her payoff is:

$$\begin{aligned}
&\pi_1(b' - \delta(\epsilon); c_1 - \epsilon, b_2)\\
=&(M - q_2)(b' - \delta(\epsilon) - c_1 + \epsilon)Pr(b_2 < b' - \delta(\epsilon)) + q_1\mathbb{E}(b_2 - c_1 - \epsilon(\epsilon)|b_2 > b' - \delta(\epsilon))Pr(b_2 > b' - \delta(\epsilon))
\end{aligned}$$

If she instead bids b, her payoff is:

$$\begin{aligned}
&\pi_1(b; c_1 - \epsilon, b_2)\\
=&(M - q_2)(b - c_1 + \epsilon)Pr(b_2 < b) + q_1\mathbb{E}(b_2 - c_1 + \epsilon|b_2 > b)Pr(b_2 > b)\\
=&(M - q_2)(b - c_1 + \epsilon)Pr(b_2 < b') + q_1\mathbb{E}(b_2 - c_1 + \epsilon|b_2 > b')Pr(b_2 > b')\\
=&(M - q_2)(b - c_1 + \epsilon)(Pr(b_2 < b' - \delta(\epsilon)) + Pr(b' - \delta(\epsilon) < b_2 < b'))\\
&+ q_1(\mathbb{E}(b_2 - c_1 - \epsilon|b_2 > b' - \delta(\epsilon))Pr(b_2 > b' - \delta(\epsilon)) - \mathbb{E}(b_2 - c_1 - \epsilon|b' - \delta < b_2 < b')Pr(b' - \delta(\epsilon) < b_2 < b'))
\end{aligned}$$

where the last expression follows from law of total expectations.



(a) (b)

Figure 12: Discontinuity of bidding functions

$$\begin{aligned}
&\pi_1(b; c_1 - \epsilon, b_2) - \pi_1(b' - \delta(\epsilon); c_1 - \epsilon, b_2)\\
=&(b - b' + \delta(\epsilon))(M - q_2)Pr(b_2 < b' - \delta(\epsilon)) + ((M - q_2)(b - c_1 + \epsilon)\\
&- q_1\mathbb{E}(b_2 - c_1 - \epsilon|b' - \delta(\epsilon) < b_2 < b'))Pr(b' - \delta(\epsilon) < b_2 < b')\\
\therefore &\lim_{\epsilon\to 0} \pi_1(b; c_1 - \epsilon, b_2) - \pi_1(b' - \delta(\epsilon); c_1 - \epsilon, b_2) = (b - b')(M - q_2)Pr(b_2 < b') > 0
\end{aligned}$$

Thus, there is a strictly positive deviation for B_1 when the bids do not have full support. Similar deviation can be shown for B_2 too. Thus, the result on common and full support for bids of both players tells us that their strategies are continuous.

No atom at bids below b^R: In any equilibrium, a cost type of a bidder has to be locally indifferent between the bid suggested by equilibrium and a bid slightly lower or higher. Suppose that in equilibrium, B_1 has an atom of probability mass $d > 0$ at some bid $b_1 < b^R$. If opponent bids continuously. Then B_2 has a type $c_2 + \delta$, where $\delta \to 0$ and type c_2 bids b_1. This is exhibited in Figure 13. If this type decides to reduce her bid to b_1^-, then her marginal cost is almost zero, but marginal benefit is $(q_1 + q_2 - M)d(b_1 - c_2)$. Thus, B_2 of this type (c_2^+) can profit by bidding slightly lower than b_1. Thus, there is no equilibrium where there is an atom for $b < b^R$.

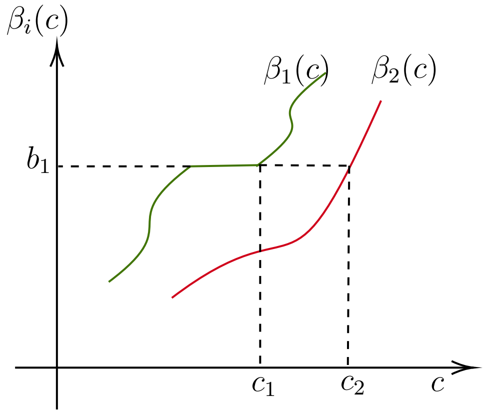


Figure 13: Deviation if there is an atom in bids

From (i) and (ii), we know that $\beta_i(c)$ is invertible for all c as long as $\beta_i(c) \neq b^R$. Thus, for each i, I can define the functions $\phi_i(b), \forall i$ as follows:

$$\phi_i(b) := \begin{cases} \beta_i^{-1}(b) & \text{for } b < b^R \\ Inf\{c : \beta_i(c) = b^R\} & \text{for } b = b^R \end{cases}$$

$\phi_i(b)$ gives the cost type of B_i who would some bid $b < b^R$ in equilibrium. If the bidder bids b^R, then $\phi_i(b)$ gives the smallest cost type of B_i who would bid b^R. Since the equilibrium bids are continuous monotonic, $\phi_i(b)$s are also continuous and monotonic.

Condition (iii), bid of $c = 0$: can be argued as follows. Suppose without loss of generality that in equilibrium $\beta_1(0) = \underline{b}$ but $\beta_2(c_*) = \underline{b}$ for some $c_* > 0$ and $\underline{b} > 0$. Given the strict monotonicity of $\phi_i(b)$, the type $c_* + \epsilon$, of B_2 would bid some $\underline{b} + \delta(\epsilon)$.

As $\epsilon \to 0, \delta(\epsilon) \to 0$ by continuity. It's payoff is:

$$
\begin{aligned}
&\pi_2(\underline{b} + \delta(\epsilon), c_* + \epsilon) \\
=&(M - q_1)F(\phi_1(\underline{b} + \delta(\epsilon)))(\underline{b} + \delta(\epsilon) - c_* - \epsilon) + q_2 \int_{\underline{b}+\delta(\epsilon)}^{b^R} (x - c_* - \epsilon) dF(\phi_1(x)) \\
=&(M - q_1)(F(\phi_1(0)) + \delta(\epsilon) f(\phi_1(\underline{b}) + (\delta(\epsilon))^2 \frac{\partial^2}{\partial b^2} F(\phi_1(\underline{b})) + \dots)(\underline{b} + \delta(\epsilon) - c_* - \epsilon) \\
&+ q_2 \int_{\underline{b}+\delta(\epsilon)}^{b^R} (x - c_* - \epsilon) dF(\phi_1(x)) \\
=&q_2 \int_{\underline{b}}^{b^R} (x - c_* - \epsilon) dF(\phi_1(x)) \\
&- (q_1 + q_2 - M)(\underline{b} - c_* - \epsilon)\big(\underbrace{\delta(\epsilon) f(\phi_1(\underline{b}))\phi_1'(\underline{b})}_{>0} + \underbrace{(\delta(\epsilon))^2 \frac{\partial^2}{\partial b^2} F(\phi_1(\underline{b})) + \dots}_{\to 0 \text{ as } \epsilon \to 0}\big) \\
&+ (M - q_1) \underbrace{F(\phi_1(\underline{b}))}_{=0}(\underline{b} + \delta(\epsilon) - c_* - \epsilon) + (M - q_1) \underbrace{\delta(\epsilon)\big(\delta(\epsilon) f(\phi_1(\underline{b}) + (\delta(\epsilon))^2 \frac{\partial^2}{\partial b^2} F(\phi_1(\underline{b})) + \dots\big)}_{\to 0 \text{ as } \epsilon \to 0} \\
<&q_2 \int_{\underline{b}}^{b^R} (x - c_* - \epsilon) dF(\phi_1(x)) \\
<&q_2 \int_{\underline{b}}^{b^R} (x - c_* - \epsilon) dF(\phi_1(x)) \\
=&\pi_2(\underline{b} - \gamma, c_* + \epsilon) \ , \forall \gamma \geq 0
\end{aligned}
$$

Thus, there is a strictly profitable deviation for the type $c_* + \epsilon$. This deviation doesn't exist if $\underline{b} = 0$. Similar deviation can be shown if $\beta_2(0) = \underline{b} > \beta_1(0) = 0$. Therefore, in equilibrium $\beta_i(0) = 0$ for both i.

Condition (iv)**, First order condition:** Suppose that B_{-i} is playing as per solution curve $\phi_{-i}(b)$, which satisfies equation (1) (when replacing $\beta_i^{-1}(c)$ with $\phi_i(b)$. Then, the payoff of B_i of type c_i when she bids b_i is:

$$\pi_i(b_i; c_i, \phi_{-i}(b)) = F(\phi_{-i}(b_i))(b_i - c_i)(M - \sum_{j \neq i} q_j) + q_i \int_{b_i}^{b^R} (x - c_i) dF(\phi_{-i}(x)) \quad (5)$$

Any interior optimum of this payoff will satisfy the first order condition of optimisation, which is:

$$f(\phi_{-i}(b_i))\phi_{-i}'(b_i)(b_i - c_i)(M - q_{-i} - q_i) + F(\phi_{-i}(b_i))(M - q_{-i}) = 0$$

Replacing c_i by $\phi_i(b)$, we can obtain (1) for B_i.

Condition (v)**, bid of** $\bar{c}$ which states that B_1 partially pools at b^R in equilibrium. For this, I first prove that there can be at most one intersection between $\phi_2(b)$ and $\phi_1(b)$ and that intersection should be as in Figure 14. Then I show that even in the immediate right neighbourhood of 0, $\phi_2(b) > \phi_1(b)$, which shows that any intersection as shown in the figure is not possible. These two together imply that $\phi_2(b) > \phi_1(b)$ for $b > 0$.

For first step, note that at any point of intersection of $\phi_1(b)$ and $\phi_2(b)$, one can see from (1) that $\dfrac{\phi_2'(b)}{\phi_1'(b)} = \dfrac{M - q_2}{M - q_1} > 1$. This implies that $\phi_2(b)$ should intersect that $\phi_1(b)$ just once and from below and left of it, as show in figure 14. The inequality $\phi_2'(b) > \phi_1'(b)$ will not be satisfied at the second point of intersection. If $\phi_1(b) < \phi_2(b)$ for some $b < b^R$, there will be no intersection between the two functions for bids above this b. Suppose that $\exists b_t \leq b^R$, such that $\phi_1(b) \geq \phi_2(b), \forall b \leq b_t$ with equality only at $b = b_t$ (as shown in Figure 14). Since $\phi_2(b)$ can intersect $\phi_1(b)$ only from left and below, all other cases are ruled out.

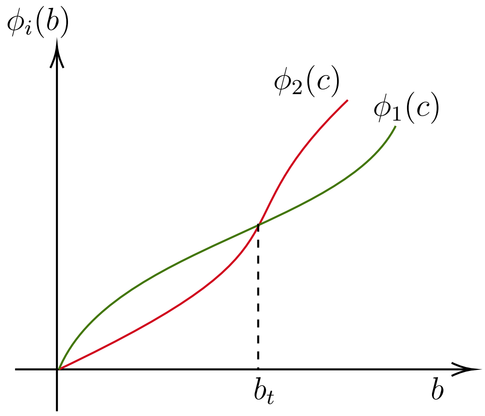


Figure 14: Possible intersection between $\phi_1(b)$ and $\phi_2(b)$

From (iii), we know that as $c \to 0^+$, $\beta_1(c) \to 0^+, \beta_2(c) \to 0^+$. This implies that $\beta_1(c) \to \beta_2(c)$ as $c \to 0^+$. From (i) and (ii), $\beta_i(c)$ is continuous and strictly monotonic when $c \to 0^+$, which implies that $\phi_i(b)$ is defined for all $b > 0$, and that $\lim\limits_{b \to 0^+} \phi_i(b) = 0$.

For the second step, consider some $\delta \to 0^+$ and suppose $\phi_i(\delta/n) = 0 + \epsilon_i(\delta/n)$ for some natural number $n \geq 1$. Then, for each i, $\phi_i(\delta) - \phi_i(\delta/n) = \frac{n-1}{n}\delta\phi_i'(\delta) +$

$\left(\frac{n-1}{n}\delta\right)^2\phi_i''(\delta)+\dots$ Therefore,

$$\begin{aligned}\frac{\phi_2'(\delta)}{\phi_1'(\delta)} &= \frac{\phi_2(\delta)-\phi_2(\delta/n)-\frac{n-1}{n}\delta\phi_2''(\delta)-\frac{n-1}{n}^2\delta^2\phi_2'''(\delta)-\dots}{\phi_1(\delta)-\phi_1(\delta/n)-\frac{n-1}{n}\delta\phi_1''(\delta)-\frac{n-1}{n}^2\delta^2\phi_1'''(\delta)-\dots}\\ &= \frac{\epsilon_2(\delta)-\epsilon_2(\delta/n)-\frac{n-1}{n}\delta\phi_2''(\delta)-\frac{n-1}{n}^2\delta^2\phi_2'''(\delta)-\dots}{\epsilon_1(\delta)-\epsilon_1(\delta/n)-\frac{n-1}{n}\delta\phi_1''(\delta)-\frac{n-1}{n}^2\delta^2\phi_1'''(\delta)-\dots}\end{aligned} \tag{6}$$

From FOCs (equations 1), $\dfrac{\phi_2'(\delta)}{\phi_1'(\delta)}=\dfrac{M-q_2}{M-q_1}\dfrac{\sigma(0)+\epsilon_1(\delta)\sigma'(0)+\epsilon_1^2(\delta)\sigma''(0)+\dots}{\sigma(0)+\epsilon_2(\delta)\sigma'(0)+\epsilon_2^2(\delta)\sigma''(0)+\dots}\dfrac{\delta-\epsilon_2(\delta)}{\delta-\epsilon_1(\delta)}$.

Thus,

$$\begin{aligned} &\frac{M-q_2}{M-q_1}\frac{\sigma(0)+\epsilon_1(\delta)\sigma'(0)+\epsilon_1^2(\delta)\sigma''(0)+\dots}{\sigma(0)+\epsilon_2(\delta)\sigma'(0)+\epsilon_2^2(\delta)\sigma''(0)+\dots}\frac{\delta-\epsilon_2(\delta)}{\delta-\epsilon_1(\delta)}\\ &=\frac{\epsilon_2(\delta)-\epsilon_2(\delta/n)-\frac{n-1}{n}\delta\phi_2''(\delta)-\frac{n-1}{n}^2\delta^2\phi_2'''(\delta)-\dots}{\epsilon_1(\delta)-\epsilon_1(\delta/n)-\frac{n-1}{n}\delta\phi_1''(\delta)-\frac{n-1}{n}^2\delta^2\phi_1'''(\delta)-\dots}\\ \implies \frac{M-q_2}{M-q_1} &= \frac{(\delta-\epsilon_1(\delta))(\epsilon_2(\delta)-\epsilon_2(\delta/n)-\frac{n-1}{n}\delta\phi_2''(\delta)-\frac{n-1}{n}^2\delta^2\phi_2'''(\delta)-\dots)}{(\delta-\epsilon_2(\delta))(\epsilon_1(\delta)-\epsilon_1(\delta/n)-\frac{n-1}{n}\delta\phi_1''(\delta)-\frac{n-1}{n}^2\delta^2\phi_1'''(\delta)-\dots)}\\ &\quad\frac{\sigma(0)+\epsilon_2(\delta)\sigma'(0)+\epsilon_2^2(\delta)\sigma''(0)+\dots}{\sigma(0)+\epsilon_1(\delta)\sigma'(0)+\epsilon_1^2(\delta)\sigma''(0)+\dots}\end{aligned}$$

As $n\to\infty$, $\frac{n-1}{n}\to 1$ and $\epsilon_i(\delta/n)\to 0, \forall i$. Furthermore, $\frac{\sigma^{(k)}(c)}{\sigma^{(n)}(c)}=0$ for $k<n$, where $\sigma^{(k)}(c)$ is the k^{th} differential of reversed hazard rate function.

$$\therefore \underbrace{\frac{M-q_2}{M-q_1}}_{>1}=\underbrace{\frac{(\delta-\epsilon_1(\delta))(\epsilon_2(\delta)-\delta\phi_2''(\delta)-\delta^2\phi_2'''(\delta)-\dots)}{(\delta-\epsilon_2(\delta))(\epsilon_1(\delta)-\delta\phi_1''(\delta)-\delta^2\phi_1'''(\delta)-\dots)}}_{>1,\text{ iff }\epsilon_2(\delta)>\epsilon_1(\delta)\text{ as }\delta\to 0}\underbrace{\frac{\sigma(0)+\epsilon_2(\delta)\sigma'(0)+\epsilon_2^2(\delta)\sigma''(0)+\dots}{\sigma(0)+\epsilon_1(\delta)\sigma'(0)+\epsilon_1^2(\delta)\sigma''(0)+\dots}}_{>1,\text{ iff }\epsilon_2(\delta)>\epsilon_1(\delta)}$$

The relation above would hold if and only if $\epsilon_2(\delta)>\epsilon_1(\delta)$. This implies that $\phi_2(b)>\phi_1(b)$ in the immediate right neighbourhood of 0. Hence, there is no point of intersection between $\phi_2(b)$ and $\phi_1(b)$ for $b>0$. Thus, for any $b\in(0,b^R], \phi_2(b)>\phi_1(b)$ and, in particular, $\exists c^*<\bar{c},\ s.t.\ \phi_1(b^R)=c^*<\phi_1(b^R)=\bar{c}$. The bidding function is then:

$$\beta_1(c)=\begin{cases}\phi_1^{-1}(c), & \text{for } c\le c^*\\ b^R, & \text{for } c^*<c<\bar{c}\end{cases}$$

$$\beta_2(c)=\phi_2^{-1}(c), \forall c\in(0,\bar{c}]$$

If direction:

The conditions give equilibrium, if there is no deviation for any type c_i of any player B_i, from the bid recommended by any function $\beta_i(c_i)$ which satisfies the properties in Lemma 1. While the calculations here are for $i = 1$, the proof for $i = 2$ is the same. Suppose $\phi_1(b_1) = c_1$, where $0 < b_1 < b^R$.

Define $\Pi_1(b_1', b_1, c_1; \phi_2(b)) := \pi_1(b_1', c_1; \phi_2(b)) - \pi_1(b_1, c_1; \phi_2(b))$ as the change in payoff of B_1 if she bids $b_1' \in [0, b^R]$ instead of b_1.

Given the continuity, monotonicity and full support of bids, $\exists$ a type c_1' such that $\phi_1(b_1') = c_1'$. Since $\phi_1(b)$ satisfies 1,

$$\begin{aligned}
&\frac{\partial}{\partial b_1'}\Pi_1(b_1', b_1, c_1; \phi_2(b))\\
=&\frac{\partial}{\partial b_1'}\pi_1(b_1', c_1; \phi_2(b))\\
=&(M - q_2 - q_1)(b_1' - c_1)f(\phi_2(b_1'))\phi_2'(b_1') + (M - q_2)F(\phi_2(b_1'))\\
=&(M - q_2 - q_1)(b_1' - c_1' + c_1' - c_1)f(\phi_2(b_1'))\phi_2'(b_1') + (M - q_2)F(\phi_2(b_1'))\\
=&(c_1' - c_1)(M - q_2 - q_1)f(\phi_2(b_1'))\phi_2'(b_1')\\
=&\underbrace{(\phi_1(b_1') - \phi_1(b_1))}_{>(<)0 \text{ if } b_1' > (<) b_1}\underbrace{(M - q_2 - q_1)}_{<0}\underbrace{f(\phi_2(b_1'))\phi_2'(b_1')}_{>0}
\end{aligned}$$

$\Pi_1(b_1, b_1, c_1; \phi_2(b)) = 0$. If $b_1' > b_1$, then $\frac{\partial}{\partial b_1'}\Pi_1(b_1', b_1, c_1; \phi_2(b)) < 0$. This implies that any deviation from b_1 to a higher bid would lead to reduction in expected payoff. Similarly, when $b_1' < b_1$, $\frac{\partial}{\partial b_1'}\Pi_1(b_1', b_1, c_1; \phi_2(b)) > 0$, which would ultimately imply that any deviation from b_1 to a lower bid will lead to reduction in expected payoff. Thus, there is no strictly positive deviation for type c_1 of B_1 from the strategy recommended by conditions of Lemma 1. Since c_1 was chosen arbitrarily, I can infer that no such deviation can be found for any other type. Similar calculations can be done for B_2. The absence of any unilateral deviation implies that any function $\beta_i(c)$ satisfying the conditions of the Lemma gives a bayes nash equilibrium. ■

A.2 Proof of Theorem 1

To show that equilibrium exists and is unique amounts to showing that there is exactly one pair of two functions $\beta_1(c)$ and $\beta_2(c)$ such that the conditions of Lemma 1 are satisfied. To do so, I proceed in following steps:

1. Consider an Initial Value Problem $\mathcal{P}$ as follows:

$$\begin{aligned}
\phi_2'(b) &= \frac{M-q_2}{q_1+q_2-M}\frac{1}{\sigma(\phi_2(b))(b-\phi_1(b))} \\
\phi_1'(b) &= \frac{M-q_1}{q_1+q_2-M}\frac{1}{\sigma(\phi_1(b))(b-\phi_2(b))}
\end{aligned} \tag{7}$$

   $\phi_2(b^R) = \bar{c}$, and $\phi_1(b^R) = c^* \leq \bar{c}$. Cauchy Lipschitz theorem implies that $\exists a$ such that a unique solution to $\mathcal{P}$ exists for interval $[b^R - a, b^R + a]$ because $b^R > \bar{c}$.

2. Show that this solution is positive, and monotonic and extend the local solution to the interval $(0, b^R]$.

3. Consider another IVP $\hat{\mathcal{P}}$, with which is same as $\mathcal{P}$ in step 1, except that $\hat{\phi}_1(b^R) = \hat{c}^*$. Show that it's solution $(\hat{\phi}_1(b), \hat{\phi}_2(b))$ is such that $\hat{\phi}_2(b) < \phi_2(b), \forall b \in (0, b^R), \hat{\phi}_1(b) < \phi_1(b), \forall b \in (0, b^R]$.

4. Using 2 and 3, show that there is exactly one value of c^* (and hence one IVP) such that $\lim_{b \to 0^+} \phi_1(b) = \lim_{b \to 0^+} \phi_2(b) = 0$, where $\phi_i(b)$s solve $\mathcal{P}$.

5. Extend $\phi_i(b)$s to include 0 in the domain by setting $\phi_1(0) = \phi_2(0) = 0$.

6. Invert $\phi_i(b)$s. Note that the domain of $\phi_1^{-1}(c)$ is $[0, c^*]$. Thus, $\beta_i(c)$ are defined as:

$$\begin{aligned}
\beta_1(c) &= \begin{cases} \phi_1^{-1}(c) & 0 \leq c \leq c^* \\ b^R & c^* < c \leq \bar{c}. \end{cases} \\
\beta_2(c) &= \phi_2^{-1}(c)
\end{aligned}$$

Step 1 is obvious from Cauchy Lipschitz theorem. I now prove steps 2,3, and 4. Steps 5 and 6 do not require any proof.

Proof of step 2: Solution to $\mathcal{P}$ is positive and monotonic

Proof. To see the positivity, rewrite (7) as:

$$\begin{aligned}
\frac{\partial}{\partial b}\ln(F(\phi_2(b)) &= \frac{M-q_2}{q_1+q_2-M}\frac{1}{(b-\phi_1(b))} \\
\frac{\partial}{\partial b}\ln(F(\phi_2(b)) &= \frac{M-q_1}{q_1+q_2-M}\frac{1}{(b-\phi_2(b))}
\end{aligned}$$

The solution of $\mathcal{P}$ exists, and it should satisfy:

$$F(\phi_2(b)) = \exp\left(\frac{M-q_2}{q_1+q_2-M}\frac{1}{(b^R-\bar{c})} - \int_b^{b^R} \frac{M-q_2}{q_1+q_2-M}\frac{1}{(b-\phi_1(b))}\right) > 0$$
$$F(\phi_1(b)) = \exp\left(\frac{M-q_1}{q_1+q_2-M}\frac{1}{(b^R-c^*)} - \int_b^{b^R} \frac{M-q_1}{q_1+q_2-M}\frac{1}{(b-\phi_2(b))}\right) > 0$$

$F(\phi_i(b)) > 0$, only if $\phi_i(b) > 0$.

Monotonicity can be shown through a contradiction argument. Since $\phi_1(b)$ and $\phi_2(b)$ are solutions to ODEs on some interval containing b^R, they are continuous and differentiable in that interval.

Thus, if the solution was not monotonic, then $\exists b_i \in (0, b^R]$ such that $\phi_i'(b_i) = 0$. Under the assumption that there is a small atom at 0, $F(c) > 0 \forall c \in (0, \bar{c}]$. Thus, $\phi_i'(b_i) = 0$ only if $|\phi_{-i}(b_i)| = \infty$. This violates the boundedness theorem. Thus, the solutions $\phi_1(b)$ and $\phi_2(b)$ are monotonic. Moreover, this monotonicity is positive. To see this, note that $b^R > \bar{c} \implies b^R - \phi_{2n}(b^R) > 0 \implies \phi_{1n}'(b^R) > 0$. A negative monotonicity would contradict this. ■

The monotonicity result from step 2 implies that $\phi_i'(b) > 0$, which requires that $b > \phi_{-i}(b)$ for each i. Thus, as $b \to 0$, positivity implies that $\phi_i(b) \to 0$ in order to satisfy the monotonicity.

Proof of step 3: At most 1 IVP where $\lim_{b\to 0}\phi_1(b) = \lim_{b\to 0}\phi_2(b) = 0$

Proof. Consider 2 IVPs, $\mathcal{P}$ and $\hat{\mathcal{P}}$, with following ODEs:

$$\phi_2'(b) = \frac{M-q_2}{q_1+q_2-M}\frac{1}{\sigma(\phi_2(b))(b-\phi_1(b))}$$
$$\phi_1'(b) = \frac{M-q_1}{q_1+q_2-M}\frac{1}{\sigma(\phi_1(b))(b-\phi_2(b))}$$

with initial values
$\phi_2(b^R) = \bar{c}$, and $\phi_1(b^R) = c^* \leq \bar{c}$ for $\mathcal{P}$;
$\hat{\phi}_2(b^R) = \bar{c}$, and $\hat{\phi}_1(b^R) = \hat{c}^*$ for some $\hat{c}^* \in (c^*, \bar{c})$ for $\hat{\mathcal{P}}$.

Since $\phi_2(b^R) = \bar{c} = \hat{\phi}_2(b^R)$, $\sigma(\hat{\phi}_2(b^R)) = \sigma(\phi_2(b^R))$. Using FOCs, it can be inferred that $\hat{\phi}_2'(b^R)(b^R - \hat{\phi}_1(b^R)) = \phi_2'(b^R)(b^R - \phi_1(b^R)) = \dfrac{M-q_2}{(q_1+q_2-M)\sigma(\phi_2(b^R))}$ which

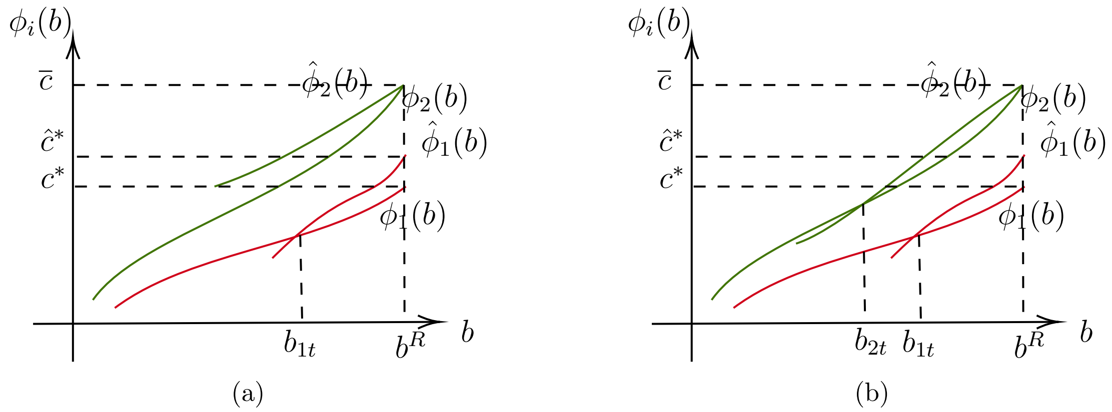


Figure 15: Intersecting solution curves

implies $\hat{\phi}_2'(b^R)(b^R - \hat{c}^*) = \phi_2'(b^R)(b^R - c^*)$. Since $\hat{c}^* > c^*$, $b^R - \hat{c}^* < b^R - c^*$. Therefore, $\hat{\phi}_2'(b^R) > \phi_2'(b^R)$. This implies that for any b in the immediate left-neighbourhood of b^R, $\hat{\phi}_2(b) < \phi_2(b)$.

Intersection between $\phi_1(b)$ and $\hat{\phi}_1(b)$, as in figure 15a, is prohibited by uniqueness of solution to IVPs having same initial value. Suppose for any $b_{2t} \in (0, b^R)$, $\hat{\phi}_2(b)$ and $\phi_2(b)$ intersect as shown in the figure ??. Then, $\hat{\phi}_2(b_{2t}) = \phi_2(b_{2t})$ and $\hat{\phi}_2'(b_{2t}) < \phi_2'(b_{2t})$, which imply that $\sigma(\hat{\phi}_2(b_{2t}))\hat{\phi}_2'(b_{2t}) < \sigma(\phi_2(b_{2t}))\phi_2'(b_{2t})$. From the FOCs, it can then be inferred that $b_{2t} - \hat{\phi}_1(b_{2t}) > b_{2t} - \phi_1(b_{2t})$, which implies that $\hat{\phi}_1(b_{2t}) < \phi_1(b_{2t})$. This requires an intersection between $\hat{\phi}_1(b)$ and $\phi_1(b)$ at some point $b_{1t} \in (b_{2t}, b^R)$. Thus, there are two solutions to the IVP defined by ODEs 7, and boundary at points b_{1t} and b_{2t}, which violates the cauchy-lipschitz theorem of uniquenss of IVP solution. Thus, $\forall b \in (0, b^R)$, $\hat{\phi}_2(b) < \phi_2(b)$.

■

Proof of Step 4:

Proof. I need to show that there is exactly one value of c^* such that the solution $\phi_i(b)$ to resultant IVP $\mathcal{P}$ satisfies $\lim_{b \to 0} \phi_i(b) = 0$. To see this, note that the condition $\hat{\phi}_1(b) > \phi_1(b) \forall b \in (0, b^R]$, and $\hat{\phi}_2(b) < \phi_2(b) \forall b \in (0, b^R)$ and $\hat{\phi}_2(b^R) = \phi_2(b^R)$, and implies that $\hat{\phi}_2(b) - \hat{\phi}_1(b) < \phi_2(b) - \phi_1(b) \forall b > 0$. Alongwith result of step 3, this further implies that if $\phi_2(b_t) = \phi_1(b_t)$ for some $b_t > 0$, then $\hat{\phi}_2(\hat{b}_t) = \hat{\phi}_1(\hat{b}_t)$ for some $\hat{b}_t > b_t$, where $\hat{\phi}_i(b)$ are solutions to $\hat{\mathcal{P}}$. Since the choice of $c^* = \phi_1(b^R)$ and $\hat{c}^* = \hat{\phi}_1(b^R)$, where $c^* < \hat{c}^*$ was arbitrary, this amounts to saying that the x-coordinate of point of intersection of $\phi_1(b)$ and $\phi_2(b)$ is strictly increasing in c^*.

Now, consider a function $H(c) : [\iota, \bar{c}] \to [\iota, \bar{c}]$ which maps c^* to b_t where $\phi_2(b_t) = \phi_1(b_t)$ for IVP with $\phi_1(b^R) = c^*$. This mapping is strictly positively monotonic. Since the RHS of the differential equations (7) is continuous, the solution to these equations is also continuous in the initial value c^*.[22] The continuity of solutions with respect to initial value further implies that the slopes, $\phi_1'(b)$ and $\phi_2'(b)$ also change continuously.

If $H(c_t^*) = b_t$ for some c_t^*, i.e., $\phi_1(b_t) = \phi_2(b_t)$ when $\phi_1(b^R) = c_t^*$, then $\frac{\phi_2'(b_t)}{\phi_1'(b_t)} = \frac{M-q_2}{M-q_1}$ and $\phi_2(b) - \phi_1(b) < 0$ for $b < b_t$ since solution curves intersect just once. Now consider IVP system $\tilde{\mathcal{P}}$ the with same ODEs as (7), and slightly perturbed initial value $\tilde{\phi}_1(b^R) = c_t^* - \omega, \omega \to 0^+$. Given the continuity of IVP solution to initial values, $\tilde{\phi}_1(b_t) - \tilde{\phi}_2(b_t) = \epsilon(\omega) \to 0$, and $\frac{\tilde{\phi}_2'(b_t)}{\tilde{\phi}_1'(b_t)} \to \frac{M-q_2}{M-q_1}$. Continuity and monotonicity of $\tilde{\phi}_1(b_t), \tilde{\phi}_2(b_t)$, then imply that $\exists \delta(\epsilon(\omega)) \to 0$, such that $\tilde{\phi}_1(b_t - \delta(\epsilon(\omega))) = \tilde{\phi}_2(b_t - \delta(\epsilon(\omega)))$. Thus, $H(c_t - \omega) = b_t - \delta(\epsilon(\omega))$ for some $\delta(\epsilon(\omega)) \to 0$, thereby establishing continuity of $H(c)$.

So far, we have established continuity and strictly positive monotonicity of $H(c)$. Notice further that $H(\bar{c}) = \bar{c}$ because we can always set $\phi_1(b^R) = \phi_2(b^R) = \bar{c}$ for the IVP. Therefore, using Extreme Value Theorem we can say that $H(c)$ will attain it's minimum, which is equal to ι, for exactly one value of c. This result holds $\forall \iota > 0$, and in particular for $\iota \to 0$. Thus, the solution to IVP given by equations (7), $\phi_1(b^R) = c^*$, $\phi_2(b^R) = \bar{c}$, is such that $\lim_{b\to 0}\phi_2(b) = \lim_{b\to 0}\phi_1(b) = 0$. ■

The proof of theorem is then completed by Steps 5 and 6, which themselves don't require any proof.

B 2 player extensions

In this section, I present two extensions with asymmetric cost information. In the first extension the 2 bidders have cost distributions which can ordered as per their Reversed Hazard Rates. In the second extension, I assume that the distribution of one of the bidders is truncated version of that of another bidder. While both cases enable me to extend the equilibrium result for the case with same cost distribution, the second is important for the formalisation of 2P1F equilibrium characterisation.

[22] See Hirsch, Smale, and Devaney, 2012 chapters 7 and 17 for results on sensitivity analysis of IVP.

B.1 Different reversed hazard rates

Suppose $c_i \overset{i.i.d}{\sim} F_i(c)$, and $c_i \in [0, \bar{c}]$ for each i. Denote reversed hazard of $F_i(c)$ by $\sigma_i(c)$. Suppose that they can be ordered in terms of their reversed hazard rate, i.e $\sigma_i(c) < \sigma_{-i}(c)$. Furthermore assume that $\lim\limits_{c \to 0^+} \sigma_i'(c) = \lim\limits_{c \to 0^+} \sigma_{-i}'(c)$. Then, as before, I can characterise the equilibrium in following lemma:

Lemma 2. *For each B_i, $\beta_i(c)$ constitutes a non-trivial BNE of the asymmetric 2 player button auction with rationing if and only if it satisfies following properties:*

(i) $\beta_i(c)$ is non-decreasing in c.

(ii) $\beta_i(c)$ is continuous and atomless for $b < b^R$ for both i.

(iii) $\beta_i(0) = 0$, $\forall i$.

(iv) For each player B_i, $\beta_i(c)$ solves:

$$\sigma_{-i}(\beta_{-i}^{-1}(\beta_i(c)))\beta_{-i}^{-1'}(\beta_i(c))(\beta_i(c) - c)(q_1 + q_2 - M) = (M - q_{-i}) \tag{8}$$

(v) If $\frac{\sigma_1(c)}{\sigma_2(c)} > \frac{M-q_1}{M-q_2}, \forall c$, $\exists c_1^*$ such that $\beta_1(c_1^*) = b^R$, $\forall c \in [c_1^*, \bar{c}]$, and $\beta_2(\bar{c}) = b^R$. If $\frac{\sigma_1(c)}{\sigma_2(c)} < \frac{M-q_1}{M-q_2}, \forall c$, $\exists c_2^*$ such that $\beta_2(c_2^*) = b^R$, $\forall c \in [c_2^*, \bar{c}]$, and $\beta_1(\bar{c}) = b^R$.

Proof. Proof of $(i), (ii), (iii), (iv)$, are same as in case with same cost distributions for each bidder. For (v), I can proceed in the same way as before. Define ϕ_i as

$$\phi_i(b) := \begin{cases} \beta_i^{-1}(b) & for\ b < b^R \\ Inf\{c : \beta_i(c) = b^R\} & for\ b = b^R \end{cases}$$

At any point of intersection of $\phi_1(b)$ and $\phi_2(b)$, I can write $\dfrac{\phi_2'(b)}{\phi_1'(b)} = \dfrac{(M - q_2)\sigma_1(\phi_1(b))}{(M - q_1)\sigma_2(\phi_2(b))}$. If $\frac{\sigma_1(c)}{\sigma_2(c)} > \frac{M-q_1}{M-q_2}, \forall c$, $\phi_2'(b) > \phi_1'(b)$ at point of intersection. Given the assumption $\lim\limits_{c \to 0^+} \sigma_i'(c) = \lim\limits_{c \to 0^+} \sigma_{-i}'(c)$, I can use same arguments as in proof of Lemma 1 to show that B_1 will bunch.

However, if $\frac{\sigma_1(c)}{\sigma_2(c)} < \frac{M-q_1}{M-q_2}, \forall c$, then B_2 bunches at b^R. ∎

The result here implies that B_2 will bunch only if the likelihood that she has higher cost than B_1 is large. This provides a larger marginal benefit of reducing the bid, as there is now a higher probability of B_2's exit. If it is large enough, B_1 would be more aggressive as it offsets the effect of having a larger residual, which leads to higher cost of competition.

Existence and uniqueness can be proved with steps similar to the case of same distribution for both bidders.

B.2 Asymmetric support, same RHR

For each B_i, $c_i \in [0, \bar{c}_i]$. $\sigma(c)$ is same for both i for $c \in [0, min_i\{\bar{c}_i\}]$. If other words, cost distribution of one of the bidders is truncation of that of the other. Equilibrium is characterised by the lemma below:

Lemma 3. *For each B_i, $\beta_i(c)$ constitutes a non-trivial BNE of the 2 player asymmetric button auction with rationing if only if it satisfies following properties:*

(i) $\beta_i(c)$ *is non-decreasing in* c.

(ii) $\beta_i(c)$ *is continuous and atomless for* $b < b^R$ *for both* i.

(iii) $\beta_i(0) = 0$ $, \forall i$.

(iv) For each player B_i, $\beta_i(c)$ *solves:*

$$\sigma_{-i}(\beta_{-i}^{-1}(\beta_i(c)))\beta_{-i}^{-1'}(\beta_i(c))(\beta_i(c) - c)(q_1 + q_2 - M) = (M - q_{-i}) \tag{9}$$

(v) $\exists \Delta$ *such that if* $\bar{c}_2 - \bar{c}_1 < \Delta$, $\exists c_1^*$ *such that* $\beta_1(c) = b^R$ $, \forall c \in [c_1^*, \bar{c}_1]$ *and* $\beta_2(\bar{c}_2) = b^R$, *else,* $\exists c_2^*$ *such that* $\beta_2(c) = b^R$ $, \forall c \in [c_2^*, \bar{c}_2]$ *and* $\beta_1(\bar{c}_1) = b^R$

Proof. Proof of $(i), (ii), (iii), (iv)$ are same as in case with same cost distributions for each bidder. As before, define $\phi_i(b)$ as inverse of $\beta_i(c)$. For (v), it can be seen in the same way as in proof of Lemma 1 that $\phi_2(b) > \phi_1(b), \forall b > 0$ for a given set of least upper bounds (LUBs) of support of cost distribution, $\{\bar{c}_1, \bar{c}_2\}$. Consider a bid δ/n, where $\delta \to 0^+$ and $n \geq 1$ is some natural number. Then $\phi_i(\delta/n) = \epsilon_i(\delta/n)$ such that $\epsilon_i(\delta/n) \to 0$. Therefore, as in 2P0F, I can write

$$\frac{\phi_2'(\delta)}{\phi_1'(\delta)} = \frac{\phi_2(\delta) - \phi_2(\delta/n) - \delta^2\kappa_2(\delta, \delta/n)}{\phi_1(\delta) - \phi_1(\delta/n) - \delta^2\kappa_1(\delta, \delta/n)} = \frac{\epsilon_2(\delta) - \epsilon_2(\delta/n) - \delta^2\kappa_2(\delta, \delta/n)}{\epsilon_1(\delta) - \epsilon_1(\delta/n) - \delta^2\kappa_1(\delta, \delta/n)}$$

where $\kappa_i(.)$ is a bounded function. From the FOCs, I can further infer that:

$$\begin{aligned}
&\frac{\phi_2'(\delta)}{\phi_1'(\delta)} = \frac{M-q_2}{M-q_1}\frac{\sigma(\phi_1(\delta)}{\sigma(\phi_2(\delta)}\frac{\delta-\epsilon_2(\delta)}{\delta-\epsilon_1(\delta)} \\
\implies &\frac{\epsilon_2(\delta)-\epsilon_2(\delta/n)-\delta^2\kappa_2(\delta,\delta/n)}{\epsilon_1(\delta)-\epsilon_1(\delta/n)-\delta^2\kappa_1(\delta,\delta/n)} = \frac{M-q_2}{M-q_1}\frac{\sigma(\phi_1(\delta)}{\sigma(\phi_2(\delta)}\frac{\delta-\epsilon_2(\delta)}{\delta-\epsilon_1(\delta)} \\
\implies &\frac{M-q_2}{M-q_1} = \frac{\epsilon_2(\delta)}{\epsilon_1(\delta)}\frac{\delta-\epsilon_1(\delta)}{\delta-\epsilon_2(\delta)}\frac{\epsilon_2(\delta)-\epsilon_2(\delta/n)-\delta^2\kappa_2(\delta,\delta/n)}{\epsilon_1(\delta)-\epsilon_1(\delta/n)-\delta^2\kappa_1(\delta,\delta/n)}
\end{aligned} \tag{10}$$

Using the same reasoning as in Appendix A.1, I can conclude that $\epsilon_2(\delta) > \epsilon_1(\delta)$

If $\bar{c}_1 > \bar{c}_2$, B_1 would bunch because $\phi_2(b^R) = \bar{c}_2$ which needs to be higher than $\phi_1(b^R)$. This would imply that $\phi_1(b^R) < \bar{c}_2 < \bar{c}_1$.

Consider the case where $\bar{c}_1 \leq \bar{c}_2$. Consider two pairs of supremum of support of (c_1, c_2), $(\bar{c}_1, \bar{c}_1)$ and $(\bar{c}_1, \hat{\bar{c}}_2)$ such that $\hat{\bar{c}}_2 > \bar{c}_1$. Denote the corresponding equilibrium inverse bid functions generated from these suprema as $\phi_i(b)$ and $\hat{\phi}_i(b)$ respectively. From Lemma 1, we know that $\phi_1(b^R) = c^* < \bar{c}_1$ and $\phi_2(b^R) = \bar{c}_1$ and that $\lim_{b\to 0^+} \phi_i(c) = 0$ for both i.

With regards to $\hat{\phi}_i(b)$, there are 2 possibilities- either $\hat{\phi}_2(b^R) > \phi_2(b^R) = \bar{c}_1$ or $\hat{\phi}_2(b^R) = \hat{c}_2^* < \phi_2(b^R) = \bar{c}_1$.

Let's consider the first case. Suppose $\exists b_t$ $s.t.$ $\hat{\phi}_2(b_t) = \phi_2(b_t)$, then $\hat{\phi}_2'(b_t) > \phi_2'(b_t)$. This implies that $\sigma(\hat{\phi}_2(b_t))\hat{\phi}_2'(b_t) > \sigma(\phi_2(b_t))\phi_2'(b_t)$, which implies that $\hat{\phi}_1(b_t) > \phi_1(b_t)$. This, further implies that $\hat{\phi}_1(b) > \phi_1(b)$, $\forall b > 0$. Otherwise there are two solutions to IVP characterised by ODEs given by 9, and boundary values given by point of intersection of $\phi_i(b), \hat{\phi}_i(b)$ for each i, defined over any compact interval in $(0, b^R]$ containing the point of intersection. This violates the Cauchy-Lipschitz theorem.

Next, let's look at $\phi_i(b)$ and $\hat{\phi}_i(b)$ in the immediate neighbourhood of 0. For this, I can write following, as in (11),

$$\frac{M-q_2}{M-q_1} = \frac{\hat{\epsilon}_2(\delta)}{\hat{\epsilon}_1(\delta)}\frac{\delta-\hat{\epsilon}_1(\delta)}{\delta-\hat{\epsilon}_2(\delta)}\frac{\hat{\epsilon}_2(\delta)-\hat{\epsilon}_2(\delta/n)-\delta^2\hat{\kappa}_2(\delta,\delta/n)}{\hat{\epsilon}_1(\delta)-\hat{\epsilon}_1(\delta/n)-\delta^2\hat{\kappa}_1(\delta,\delta/n)},$$

where $\hat{\kappa}_i(.)$ is a bounded function. Above implies that $\hat{\phi}_2(b) > \hat{\phi}_1(b)$. I can further infer that:

$$\frac{\epsilon_2(\delta)}{\epsilon_1(\delta)}\frac{\delta-\epsilon_1(\delta)}{\delta-\epsilon_2(\delta)}\frac{\epsilon_2(\delta)-\epsilon_2(\delta/n)-\delta^2\kappa_2(\delta,\delta/n)}{\epsilon_1(\delta)-\epsilon_1(\delta/n)-\delta^2\kappa_1(\delta,\delta/n)} = \frac{\hat\epsilon_2(\delta)}{\hat\epsilon_1(\delta)}\frac{\delta-\hat\epsilon_1(\delta)}{\delta-\hat\epsilon_2(\delta)}\frac{\hat\epsilon_2(\delta)-\hat\epsilon_2(\delta/n)-\delta^2\hat\kappa_2(\delta,\delta/n)}{\hat\epsilon_1(\delta)-\hat\epsilon_1(\delta/n)-\delta^2\hat\kappa_1(\delta,\delta/n)}$$
$$\implies \frac{\epsilon_2(\delta)}{\hat\epsilon_2(\delta)}\frac{\delta-\hat\epsilon_2(\delta)}{\delta-\epsilon_2(\delta)}\frac{\epsilon_2(\delta)-\epsilon_2(\delta/n)-\delta^2\kappa_2(\delta,\delta/n)}{\hat\epsilon_2(\delta)-\hat\epsilon_2(\delta/n)-\delta^2\hat\kappa_2(\delta,\delta/n)} = \frac{\epsilon_1(\delta)}{\hat\epsilon_1(\delta)}\frac{\delta-\hat\epsilon_1(\delta)}{\delta-\epsilon_1(\delta)}\frac{\epsilon_1(\delta)-\epsilon_1(\delta/n)-\delta^2\kappa_1(\delta,\delta/n)}{\hat\epsilon_1(\delta)-\hat\epsilon_1(\delta/n)-\delta^2\hat\kappa_1(\delta,\delta/n)}$$

Above relation should hold for all n. As $\delta \to 0$ and $\kappa_i(.)$ and $\hat\kappa_i(.)$ are bounded functions $\delta^2\kappa_i(\delta,\delta/n) \approx 0$ and $\delta^2\hat\kappa_i(\delta,\delta/n) \approx 0$ for both i. Since both $\phi_i(b)$ and $\hat\phi_i(b)$ converge to 0^+ as $b \to 0^+$, I can further say that $\epsilon_i(\delta/n) \approx \hat\epsilon_i(\delta/n)$ as $n \to \infty$. If $\hat\epsilon_2(\delta) > (<)\epsilon_2(\delta)$, then LHS is above (below) 1. Thus, RHS will be above (below) 1 only if $\hat\epsilon_1(\delta) > (<)\epsilon_1(\delta)$.

Now, if $\hat\epsilon_2(\delta) < \epsilon_2(\delta)$, then $\hat\epsilon_1(\delta) < \epsilon_1(\delta)$. Since $\hat\phi_1(b_t) > \phi_1(b_t)$, where b_t is the point of intersection of $\hat\phi_i(b)$ and $\phi_i(b)$, this implies that $\hat\phi_i(b)$ intersects $\phi_i(b)$ for both i because $\hat\phi_2(b^R) = \hat{\bar{c}}_2 > \phi_2(b^R) = \bar c_1$. This situation is depicted in Figure 16. As explained in appendix A2, such intersections violate the Cauchy-Lipschitz theorem of unique solution. Thus, if $\hat\phi_2(b^R) = \hat{\bar{c}}_2 > \phi_2(b^R) = \bar c_1$, then $\hat\phi_2(b) > \phi_2(b) \forall b > 0$ which implies $\hat\phi_1(b) > \phi_1(b) \forall b > 0 \implies \hat\phi_1(b^R) = \hat c_1^* > \phi_1(b^R) = c^*$.

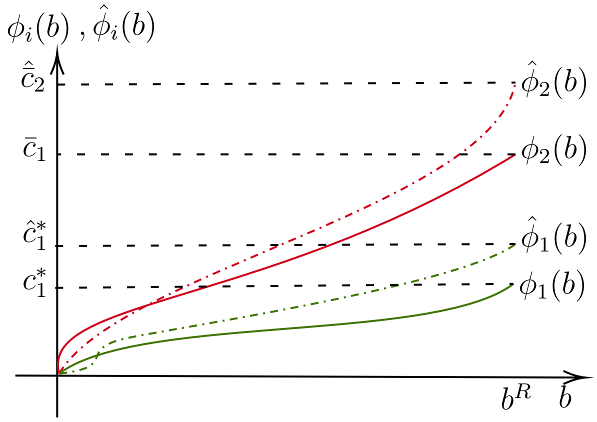


Figure 16: Intersecting solution curves

The second case is where $\hat\phi_2(b^R) = \hat c_2^* < \phi_2(b^R) = \bar c_1$. In this case, $\hat\phi_1(b^R) = \bar c_1$, else both players will have an atom, which is not possible in equilibrium. Thus, here, $\hat\phi_1(b^R) > \phi_1(b^R)$. As before, I can show that any intersection between $\hat\phi_2(b)$ and $\phi_2(b)$ would imply intersection between $\hat\phi_1(b)$ and $\phi_1(b)$. Hence, $\hat\phi_2(b) < \phi_2(b)$, and $\hat\phi_1(b) > \phi_1(b)$, $\forall b > 0$. However, as shown above, this inequality wouldn't hold for the bids close to 0. Thus, this case leads to contradictions and hence, is not possible.

Therefore, when if the supremum of support of c_2 is higher, i.e., $\hat{\bar{c}}_2 > \bar c_2$, $\hat\phi_2(b) > \phi_2(b)$, and $\hat\phi_1(b) > \phi_1(b)$, $\forall b > 0$.

Define a function $M(\bar{c}_2) : [\bar{c}_1, \infty) \to \mathbb{R}^+$ such that $M(\bar{c}_2)$ maps LUB of support of c_2 to $\phi_1(b^R)$, where $\bar{c}_1$ is LUB of an arbitrary support of c_1. Since the choice of $\hat{\bar{c}}_2$ above is arbitrary, we can say that $M(\bar{c}_2) > 0$ is an increasing function. Continuity can be argued in the same way as in proof of Theorem 1 in Appendix A.2. Thus, for a given $\bar{c}_1$, as $\bar{c}_2$ increases from $\bar{c}_1$, c^* increases, and the size of B_1's atom at b^R reduces. The maximum value of c^* can be $\bar{c}_1$, which corresponds to atom size of 0. Due to monotonicity and continuity of $M(\bar{c}_2)$, $\exists \bar{c}_2^T$ such that $M(\bar{c}_2^T) = \bar{c}_1$. Then for $\bar{c}_2 \in [\bar{c}_1, \bar{c}_2^T)$, B_1 bunches at b^R and for $\bar{c}_2 > \bar{c}_2^T$, B_2 would bunch. This holds true regardless of the value of $\underline{c}_1$. I can thus define $\Delta \equiv \underline{c}_2^T - \underline{c}_1$, such that $B_1(B_2)$ bunches if $\underline{c}_2 < (>) \underline{c}_1 + \Delta$. This proves (v). ■

This result here has similar intuition as in previous extension. B_2 would bunch at b^R only if it is likely to have costs much higher than that of B_1. This extension is important not only for robustness checks, but also for formalising equilibrium in case with 2 small and 1 very small player.

Finally, I establish existence and uniqueness of this PBE in order to have characterisation of equilibrium of 2P1F case.

Theorem 2. *Equilibrium defined by Lemma 3 exists and is unique.*

Proof. From Lemma 3, it can be inferred that for some given values of $\bar{c}_1, \bar{c}_2$, only one of the bidders, B_1 or B_2 will be bunching.

The boundary value problem which gives equilibrium bid function is characterised by the differential equation 9, and boundaries given by $\phi_1(0) = \phi_2(0)$, and $\phi_2(b^R) = \bar{c}_2$ when $\bar{c}_2 > \bar{c}_1 + \Delta$, and $\phi_1(b^R) = \bar{c}_1$ otherwise. Comparing to the boundary value problem for 2P0F case, it can be noticed that the differential equation and left boundary are the same, while right boundary can be different.

From the proof of Theorem 1, we already know that equilibrium exists and is unique if the right boundary is $\phi_2(b^R) = \bar{c}_2$. Moreover, same arguments can be applied to the case where the right boundary is $\phi_1(b^R) = \bar{c}_1$. ■

C 3 player extension: 2 small and 1 very small bidder

Suppose 3 bidders B_1, B_2, and B_3 have quantities q_1, q_2, and q_3 respectively, such that, $q_1 > q_2 > q_3$, $q_1 + q_2 > M$ but $q_1 + q_3 < M$ and $q_2 + q_3 < M$. Thus, B_1 and B_2 can together cover the whole demand. For B_3, it is dominant to bid her cost, for the reasons same as in section 5.1.1. In this game, exit of B_1 or B_2 will end the game, but exit of B_3 will start a new subgame between the other two. As before, there are equilibria which require crazy types but the analysis here will focus on the semi-separating equilibrium which don't require such types. This equilibrium is also the perfect bayesian equilibrium of this game.

Denote the set of all players by $\mathcal{N}$, and set $\{B_1, B_2\}$ by $\mathcal{A}2$. In this section B_i refers to the elements of $\mathcal{A}2$ and B_{-i} is the element of $\mathcal{A}2 \setminus B_i$. For $i \in \{1, 2\}$, denote the equilibrium bid function of B_i by $\beta_{i,\mathcal{N}}(c)$ in the subgame with all players, and $\beta_{i,\mathcal{A}2}(c)$ in the subgame started by B_3's exit. $\mathbf{b}$ denotes the vector of bids of all the players. If a bidder in $\mathcal{A}2$ exits at any bid, she gets a strictly positive quantity award. As such, these bidders can be called partially rationed as opposed to fully rationed bidder, B_3. A partially rationed bidder B_i bids b_i, and the other partially rationed bidder bids b_{-i}, and B_3 bids b_3, her payoff when her type is c_i is:

$$
\begin{aligned}
\pi_i(b_i; c_i, \mathbf{b}) = &(M - q_{-i} - q_3)(b_i - c_i)Pr(b_i = max_j\{b_j\}) \\
&+ q_i\mathbb{E}(b_{-i} - c_i | b_{-i} > b_3, b_{-i} > b_i)Pr(b_{-i} = max_j\{b_j\}) \\
&+ \mathbb{E}(\pi^*_{i,\mathcal{A}2}(b_3) | b_i < b_3, b_{-i} < b_3)Pr(b_3 = max_j\{b_j\})
\end{aligned}
$$

where $\pi^*_{i,\mathcal{A}2}(b_3)$ is the payoff for B_i in the subgame started by B_3's exit.

$\beta_{3,\mathcal{N}}(c) = c$. B_1 and B_2 best respond to that and to each other in equilibrium, which is characterised in the following lemma:

Lemma 4. *$\beta_{3,\mathcal{N}}(c) = c$. $\beta_{i,\mathcal{N}}(c)$ and $\beta_{i,\mathcal{A}2}(c)$ for $i \in \{1, 2\}$, give a PBE if and only if:*

(i) $\beta_{i,\mathcal{N}}(c)$ is non-decreasing in c.

(ii) $\beta_{i,\mathcal{N}}(c)$ is continuous and atomless for $b < b^R$ for both i.

(iii) $\beta_{i,\mathcal{N}}(0) = 0$, $\forall i$.

(iv) $\forall i, \beta_{i,\mathcal{A}2}(c_i)$*, solve following differential equations:*

$$\begin{aligned}&(\pi^*_{i,\mathcal{A}2}(b;c_i)-(M-q_{-i}-q_3)(\beta_{i,\mathcal{N}}(c_i)-c_i))\frac{f(\beta_{i,\mathcal{N}}(c_i))}{F(\beta_{i,\mathcal{N}}(c_i))}\mathbb{1}_{b\leq\bar{c}}\\&+(\beta_{i,\mathcal{N}}(c_i)-c_i)(\sum_j q_j-M)\frac{f(\beta^{-1}_{-i,\mathcal{N}}(\beta_{i,\mathcal{N}}(c_i)))\beta^{-1'}_{-i,\mathcal{N}}(\beta_{i,\mathcal{N}}(c_i))}{F(\beta^{-1}_{-i,\mathcal{N}}(\beta_{i,\mathcal{N}}(c_i)))}=M-q_{-i}-q_3\end{aligned} \tag{11}$$

where $\pi^*_{i,\mathcal{A}2}(b;c_i)$ *is the payoff of* B_i *in the subgame started with exit of* B_3*.*

(v) $\exists c^*_1 \leq \bar{c}$ *such that* $\beta_{1,\mathcal{N}}(c) = b^R, \forall c \in [c^*_1, \bar{c}]$*.* $\beta_{2,\mathcal{N}}(\bar{c}) = b^R$ *if* $b^R > \bar{c}$ *and* $\lim_{c\to\bar{c}^-}\beta_{2,\mathcal{N}}(c) = b^R$ *if* $b^R = \bar{c}$*.*

(vi) $\beta_{i,\mathcal{A}2}(c)$ *for* $i \in \{1,2\}$ *are given by semi-seperating equilibrium in the subgame started by* B_3*'s exit at a bid b, which is characterised in Lemma 3 in Appendix B.2.*

Proof. See Appendix C.1. ■

PBE described here looks the same that of section 5.1.2, except that there is a kink at $b = \bar{c}$. The intuition behind a similar equilibrium as in case with 2 small bidders is that B_3's presence affects both B_1 and B_2 in the same way. It reduces their residual capacity by the same amount and the marginal probability of B_3's exit at any bid is same for both the bidders. Thus, B_1 is still less reluctant to compete vis-a-vis B_2.

The proof is also similar, except for some additional steps for (i) and (v). For (v), I show that there will be at most one point of intersection between $\beta_1(c)$ and $\beta_2(c)$. At any point of intersection, $\frac{\beta'_{1,\mathcal{N}}(c)}{\beta'_{2,\mathcal{N}}(c)} = \frac{M-q_2-q_3-(\pi^*_{1,\mathcal{A}2}(b,c)-(M-q_2-q_3)(b-c))\sigma(b)}{M-q_1-q_3-(\pi^*_{2,\mathcal{A}2}(b,c)-(M-q_1-q_3)(b-c))\sigma(b)}$ for $b \leq \bar{c}$. If B_3 were to exit at bid b pertaining to the point of intersection, then a subgame same as 2P0F starts with b as reserve. As we know from Lemma 1(v), B_1 of type c pertaining to this bid, will also exit at b in this subgame. This gives us the values for $\pi^*_{i,\mathcal{A}2}(b,c)$ for each i, which are such that the aforementioned slope ratio is above 1. Thus, there is only one possible point of intersection between $\beta_{1,\mathcal{N}}$ and $\beta_{2,\mathcal{N}}$, and that point is $(0,0)$ for reasons same as in section 5.1.2.

Furthermore, as I show in appendix, the PBE is such that in the subgame, B_2 would be bunching. This result eases the analysis for existence and uniqueness, as it gives explicit expressions of continuation values.

Looking at the equilibrium characteristics, it can be noticed that apart from FOC, every other property is same that of equilibrium in 2 players. FOC here is such that LHS is not continuous, unlike previous case. The key condition leading to uniqueness and existence in that case was that the solution to the boundary value problem for different boundaries is such that $\phi_2(b)$ is lower if $\phi_1(b)$ is higher for a given boundary (as in Figure 7). Although, this condition still holds, the lack of continuity leads to negative result on existence of pure strategy PBE.

If $b^R > \bar{c}$, there is a kink in the bidding function at $\bar{c}$. In this case, B_2 becomes more aggressive on the margin at $\bar{c}$. The best response for B_1 is, then, to be less aggressive in absolute manner, unless the quantities have some very specific values. This creates discontinuity in B_1's bidding function, by a logic similar to 2P0F. This violates property (i) of BNE described in Lemma 4. Thus, in such a case, we can only have trivial BNE. However, such a problem doesn't exist when $b^R = \bar{c}$. Thus, the result on existence and uniqueness of equilibrium doesn't extend to this case when $b^R > \bar{c}$.

Theorem 3. *If $b^R > \bar{c}$, equilibrium described by Lemma 4 may not always exist, but when it exists, it is unique. If $b^R \leq \bar{c}$, the equilibrium exists and is unique.*

Proof. See Appendix C.2. ∎

C.1 Proof of Lemma 4

Proof. For the very small bidder B_3, it is weakly dominant to bid her cost. The reason is same as for 1P1F case. The proof proceeds in the way similar to that in 2P0F (Appendix A2). However, there are some additional nuances involved in proving property (i) and (v).

As in Section A.1, I show (i) condition by proving that a player's expected payoff satisfies SCP-IR property, when opponent is playing as per an increasing strategy. As before, I will show it for B_1. Consider any two types c_1, c_1' of B_1, such that $c_1 < c_1'$, and any two arbitrary bids b_1, b_1', where $b_1 < b_1'$. To show monotonicity, all I need to show is that when B_2 follows a non-decreasing strategy, if $\pi_1(b_1', c_1; b_2, c_3) - \pi_1(b_1, c_1; b_2, c_3) > 0$, then $\pi_1(b_1', c_1'; b_2, c_3) - \pi_1(b_1, c_1'; b_2, c_3) > 0$, where b_2 is random variable (RV) denoting B_2's bid, and c_3 is RV for B_3's cost type (and equivalently, her bid).

$$
\begin{aligned}
\pi_1(b_1', c_1; b_2, c_3) =& (M - q_2 - q_3)(b_1' - c_1)Pr(b_1' = max\{b_1', b_2, c_3\}) \\
&+ q_1\mathbb{E}(b_2 - c_1 | b_2 = max\{b_1', b_2, c_3\})Pr(b_2 = max\{b_1', b_2, c_3\}) \\
&+ \mathbb{E}(\pi^*_{1,\mathcal{A}2}(c_3, c_1) | c_3 = max\{b_1', b_2, c_3\})Pr(c_3 = max\{b_1', b_2, c_3\}) \\
\pi_1(b_1, c_1; b_2, c_3) =& (M - q_2 - q_3)(b_1 - c_1)Pr(b_1 = max\{b_1, b_2, c_3\}) \\
&+ q_1\mathbb{E}(b_2 - c_1 | b_2 = max\{b_1, b_2, c_3\})Pr(b_2 = max\{b_1, b_2, c_3\}) \\
&+ \mathbb{E}(\pi^*_{1,\mathcal{A}2}(c_3, c_1) | c_3 = max\{b_1, b_2, c_3\})Pr(c_3 = max\{b_1, b_2, c_3\})
\end{aligned} \tag{12}
$$

Denote $\pi_1(b_1', c_1; b_2, c_3) - \pi_1(b_1, c_1; b_2, c_3)$ by $A(b_1', b_1, c_1, b_2, c_3)$, or simply, A. Suppose that $A > 0$ always. Furthermore,

$$
\begin{aligned}
\pi_1(b_1', c_1'; b_2, c_3) =& (M - q_2 - q_3)(b_1' - c_1')Pr(b_1' = max\{b_1', b_2, c_3\}) \\
&+ q_1\mathbb{E}(b_2 - c_1' | b_2 = max\{b_1', b_2, c_3\})Pr(b_2 = max\{b_1', b_2, c_3\}) \\
&+ \mathbb{E}(\pi^*_{1,\mathcal{A}2}(c_3, c_1') | c_3 = max\{b_1', b_2, c_3\})Pr(c_3 = max\{b_1', b_2, c_3\}) \\
\pi_1(b_1, c_1'; b_2, c_3) =& (M - q_2 - q_3)(b_1 - c_1')Pr(b_1 = max\{b_1, b_2, c_3\}) \\
&+ q_1\mathbb{E}(b_2 - c_1' | b_2 = max\{b_1, b_2, c_3\})Pr(b_2 = max\{b_1, b_2, c_3\}) \\
&+ \mathbb{E}(\pi^*_{1,\mathcal{A}2}(c_3, c_1') | c_3 = max\{b_1, b_2, c_3\})Pr(c_3 = max\{b_1, b_2, c_3\})
\end{aligned} \tag{13}
$$

which implies,

$$
\begin{aligned}
\pi_1(b_1', c_1'; b_2, c_3) =& (M - q_2 - q_3)(b_1' - c_1' + c_1 - c_1)Pr(b_1' = max\{b_1', b_2, c_3\}) \\
&+ q_1\mathbb{E}(b_2 - c_1' + c_1 - c_1 | b_2 = max\{b_1', b_2, c_3\})Pr(b_2 = max\{b_1', b_2, c_3\}) \\
&+ \mathbb{E}(\pi^*_{1,\mathcal{A}2}(c_3, c_1') | c_3 = max\{b_1', b_2, c_3\})Pr(c_3 = max\{b_1', b_2, c_3\}) \\
&+ \mathbb{E}(\pi^*_{1,\mathcal{A}2}(c_3, c_1) | c_3 = max\{b_1', b_2, c_3\})Pr(c_3 = max\{b_1', b_2, c_3\}) \\
&- \mathbb{E}(\pi^*_{1,\mathcal{A}2}(c_3, c_1) | c_3 = max\{b_1', b_2, c_3\})Pr(c_3 = max\{b_1', b_2, c_3\}) \\
\pi_1(b_1, c_1'; b_2, c_3) =& (M - q_2 - q_3)(b_1 - c_1' + c_1 - c_1)Pr(b_1 = max\{b_1, b_2, c_3\}) \\
&+ q_1\mathbb{E}(b_2 - c_1' + c_1 - c_1 | b_2 = max\{b_1, b_2, c_3\})Pr(b_2 = max\{b_1, b_2, c_3\}) \\
&+ \mathbb{E}(\pi^*_{1,\mathcal{A}2}(c_3, c_1') | c_3 = max\{b_1, b_2, c_3\})Pr(c_3 = max\{b_1, b_2, c_3\}) \\
&+ \mathbb{E}(\pi^*_{1,\mathcal{A}2}(c_3, c_1) | c_3 = max\{b_1, b_2, c_3\})Pr(c_3 = max\{b_1, b_2, c_3\}) \\
&- \mathbb{E}(\pi^*_{1,\mathcal{A}2}(c_3, c_1) | c_3 = max\{b_1, b_2, c_3\})Pr(c_3 = max\{b_1, b_2, c_3\})
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
&\therefore \pi_1(b_1', c_1'; b_2, c_3) - \pi_1(b_1, c_1'; b_2, c_3)\\
&= A + (M - q_2 - q_3)(c_1 - c_1')Pr(b_1' = max\{b_1', b_2, c_3\}) - (M - q_2 - q_3)(c_1 - c_1')Pr(b_1 = max\{b_1, b_2, c_3\})\\
&\quad + q_1\mathbb{E}(c_1 - c_1'|b_2 = max\{b_1', b_2, c_3\})Pr(b_2 = max\{b_1', b_2, c_3\}) - q_1\mathbb{E}(c_1 - c_1'|b_2 = max\{b_1, b_2, c_3\})Pr(b_2 = max\{b_1, b_2, c_3\})\\
&\quad + \mathbb{E}(\pi^*_{1,\mathcal{A}2}(c_3, c_1')|c_3 = max\{b_1', b_2, c_3\} - \pi^*_{1,\mathcal{A}2}(c_3, c_1)|c_3 = max\{b_1', b_2, c_3\})Pr(c_3 = max\{b_1', b_2, c_3\})\\
&\quad - \mathbb{E}(\pi^*_{1,\mathcal{A}2}(c_3, c_1')|c_3 = max\{b_1, b_2, c_3\} - \pi^*_{1,\mathcal{A}2}(c_3, c_1)|c_3 = max\{b_1, b_2, c_3\})Pr(c_3 = max\{b_1, b_2, c_3\})
\end{aligned} \tag{15}
$$

From Lemma 3, I can write continuation value in the subgame following B_3's exit, $\pi^*_{1,\mathcal{A}2}(c_3, c_1)$, as:

$$
\pi^*_{1,\mathcal{A}2}(c_3, c_1) = \underset{b_1'' \le c_3}{Max}\left[(M - q_2)(b_1'' - c_1)\frac{F(\phi_2^{sg}(b_1''))}{a(c_3)} + q_1\int_{b_1''}^{c_3}(x - c_1)\frac{dF(\phi_2^{sg}(x))}{a(c_3)}\right]
$$

where $\phi_2^{sg}(b)$ is given by Lemma 3 in Appendix A.3.2 and $a(c_3)$ denotes the probability that B_2's cost type is from that subset of $[0, \bar{c}]$ which bids less than c_3 in the subgame with preceding B_3's exit. I can further write,

$$
\begin{aligned}
\pi^*_{1,\mathcal{A}2}(c_3, c_1) =& \underset{b_1'' \le c_3}{Max}\left[(M - q_2)(b_1'' - c_1 + c_1' - c_1')\frac{F(\phi_2^{sg}(b_1''))}{a(c_3)} + q_1\int_{b_1''}^{c_3}(x - c_1 + c_1' - c_1')\frac{dF^{sg}(\phi_2(x))}{a(c_3)}\right]\\
\implies \pi^*_{1,\mathcal{A}2}(c_3, c_1) \le& \underset{b_1'' \le c_3}{Max}\left[(M - q_2)(x - c_1')\frac{F(\phi_2^{sg}(b_1''))}{a(c_3)} + q_1\int_{b_1''}^{c_3}(x - c_1')\frac{dF(\phi_2^{sg}(x))}{a(c_3)}\right]\\
&+ \underset{b_1'' \le c_3}{Max}\left[(M - q_2)(c_1' - c_1)\frac{F(\phi_2^{sg}(b_1''))}{a(c_3)} + q_1\int_{b_1''}^{c_3}(c_1' - c_1)\frac{dF(\phi_2^{sg}(x))}{a(c_3)}\right]\\
\implies \pi_1(c_3, c_1') - \pi_1(c_3, c_1) \ge& - \underset{b_1'' \le c_3}{Max}\left[(M - q_2)(c_1' - c_1)\frac{F(\phi_2^{sg}(b_1''))}{a(c_3)} + q_1\int_{b_1''}^{c_3}(c_1' - c_1)\frac{dF(\phi_2^{sg}(x))}{a(c_3)}\right]
\end{aligned} \tag{16}
$$

Since we have supposed that B_2 has non-decreasing strategies in the subgame before B_3's exit, and Lemma 3(i) states that $\phi_2^{sg}(x)$ is an increasing function, (16) implies

$$
\begin{aligned}
&\pi_1(c_3, c_1') - \pi_1(c_3, c_1) \ge -\underset{b_1'' \le c_3}{Max}\left[(M - q_2)(c_1' - c_1)\frac{F(\phi_2^{sg}(b_1''))}{a(c_3)} + q_1(c_1' - c_1)\frac{a(c_3) - \phi_2^{sg}(b_1'')}{a(c_3)}\right]\\
\implies &\pi_1(c_3, c_1') - \pi_1(c_3, c_1) \ge -q_1(c_1' - c_1)
\end{aligned} \tag{17}
$$

where the last line follows from the idea that this objective function will be maximised when $b_1'' = 0$.

$$
\begin{aligned}
&\pi_1(b_1', c_1'; b_2, c_3) - \pi_1(b_1, c_1'; b_2, c_3) \\
&\geq A + (M - q_2 - q_3)(c_1 - c_1')Pr(b_1' = max\{b_1', b_2, c_3\}) - (M - q_2 - q_3)(c_1 - c_1')Pr(b_1 = max\{b_1, b_2, c_3\}) \\
&+ q_1(c_1 - c_1')Pr(b_2 = max\{b_1', b_2, c_3\}) - q_1(c - c_1)Pr(b_2 = max\{b_1, b_2, c_3\}) \\
&+ q_1(c_1 - c_1')Pr(c_3 = max\{b_1', b_2, c_3\}) - q_1(c - c_1)Pr(c_3 = max\{b_1, b_2, c_3\}) \\
&= A + (M - q_2 - q_3)(c_1 - c_1')Pr(b_1' = max\{b_1', b_2, c_3\}) - (M - q_2 - q_3)(c_1 - c_1')Pr(b_1 = max\{b_1, b_2, c_3\}) \\
&+ q_1(c_1 - c_1')Pr(b_1' \neq max\{b_1', b_2, c_3\}) - q_1(c_1 - c_1')Pr(b_1 \neq max\{b_1, b_2, c_3\}) \\
&= A + (M - q_2 - q_3)(c_1 - c_1')Pr(b_1' = max\{b_1', b_2, c_3\}) - (M - q_2 - q_3)(c_1 - c_1')Pr(b_1 = max\{b_1, b_2, c_3\}) \\
&+ q_1(c_1 - c_1')(1 - Pr(b_1' = max\{b_1', b_2, c_3\})) - q_1(c_1 - c_1')(1 - Pr(b_1 = max\{b_1, b_2, c_3\})) \\
&= \underbrace{A}_{>0} + \underbrace{(M - q_2 - q_3 - q_1)}_{<0}\underbrace{(c_1 - c_1')}_{<0}\underbrace{(Pr(b_1' = max\{b_1', b_2, c_3\}) - Pr(b_1 = max\{b_1, b_2, c_3\}))}_{>0}
\end{aligned} \tag{18}
$$

$Pr(b_1' = max\{b_1', b_2, c_3\}) - Pr(b_1 = max\{b_1, b_2, c_3\}) > 0$ because $b_1' > b_1$ and event that b_1 is greater than both b_2 and c_3 is subset of the event that b_1' is greater than both b_2 and c_3. This along with $A > 0$, $c_1 < c_1'$, $M < q_1 + q_2 + q_3$, $b_1' > b_1$, ensures that above expression is positive. This proves condition (i).

Proof of $(ii), (iii)$ is same as 2P0F. (iv) can be shown from first order conditions of optimisation of B_i's payoff.

For (v), consider a point of intersection (b_t, c_t) of $\phi_{1,\mathcal{N}}$ and $\phi_{2,\mathcal{N}}$ where $b_t < \bar{c}$. At this point,

$$
\frac{\phi_{2,\mathcal{N}}'(b_t)}{\phi_{1,\mathcal{N}}'(b_t)} = \frac{M - q_2 - q_3 - (\pi_{1,\mathcal{A}2}^*(b_t, c_t) - (M - q_2 - q_3)(b_t - c_t))\sigma(b_t)}{M - q_1 - q_3 - (\pi_{2,\mathcal{A}2}^*(b_t, c_t) - (M - q_1 - q_3)(b_t - c_t))\sigma(b_t)} \tag{19}
$$

Note that $\pi_{1,\mathcal{A}2}^*(b_t, c_t)$ is the payoff if B_3 exits at b_t. Since this is also a point of intersection, the subgame started by B_3's exit is same as 2P0F, with $c_i \in [0, c_t]$. Moreover, at this point, both players have type c and the reserve bid for 2P0F is b_t. Thus, from Lemma 1, B_1 of type c_t bids b_t, but is bunching and hence, gets residual. B_2 of type c_t will also bid b_t, but is not bunching. Consequently, their continuation value at this point are $\pi_{1,\mathcal{A}2}^*(b_t, c_t) = (M - q_2)(b_t - c_t)$, $\pi_{2,\mathcal{A}2}^*(b_t, c_t) = q_1(b_t - c_t)$. Thus, we can write

$$
\frac{\phi_{2,\mathcal{N}}'(b_t)}{\phi_{1,\mathcal{N}}'(b_t)} = \frac{(M - q_2 - q_3) - q_3(b_t - c_t)\sigma(b_t)}{(M - q_1 - q_3) - (\sum_{j=1}^{3} q_j - M)(b_t - c_t)\sigma(b_t)} > 1
$$

where inequality arises because $M - q_1 - q_3 < M - q_2 - q_3$ while $\sum_j q_j - M > q_3$. This implies that $\phi_1(b)$ intersects at most once with $\phi_2(b)$ for $b > 0$.

The exit of B_3 starts a subgame which is same as the extension in Appendix A.3.2. In this subgame, either B_1 or B_2 is bunching. This further means that at any given bid b, if B_3 exits, then Lemma 3 tells us that either B_1 or B_2 of the type $\phi_i(b)$ would also exit at b and get a residual.

Consider a bid δ/n, where $\delta \to 0$ and $n \geq 1$ is some natural number. Then, $\phi_{i,\mathcal{N}}(\delta/n) = \epsilon_i(\delta/n)$, where $\epsilon_i(\delta) \to 0$ by continuity. Suppose that B_1 is bunching in the subgame started by B_3's exit at (δ). Then, in the same way as in other cases, I can write the following from the FOCs of case 2P1F:

$$(\delta - \epsilon_1(\delta))(q_3\sigma(\delta) + (q_1 + q_2 + q_3 - M)\sigma(\epsilon_2(\delta))(\epsilon_2(\delta) - \epsilon_2(\delta/n) - \delta^2\kappa_2(\delta, \delta/n))) = M - q_2 - q_3$$
$$(\delta - \epsilon_2(\delta))(q_1 + q_2 + q_3 - M)(\sigma(\delta) + \sigma(\epsilon_1(\delta))(\epsilon_1(\delta) - \epsilon_1(\delta/n) - \delta^2\kappa_1(\delta, \delta/n))) = M - q_1 - q_3$$

Using the fact that $\sigma(0)/\sigma'(0) = 0$ and that $\sigma'(0) = \infty$, I can infer the following from above:

$$\frac{\delta - \epsilon_1(\delta)}{\delta - \epsilon_2(\delta)} \frac{(q_3\delta + \epsilon_2(\delta)(\epsilon_2(\delta) - \epsilon_2(\delta/n) - \delta^2\kappa_2(\delta, \delta/n))(q_1 + q_2 + q_3 - M))}{(q_1 + q_2 + q_3 - M)(\delta + \epsilon_1(\delta)(\epsilon_1(\delta) - \epsilon_1(\delta/n) - \delta^2\kappa_1(\delta, \delta/n)))} = \frac{M - q_2 - q_3}{M - q_1 - q_3} \tag{20}$$

$$\frac{\epsilon_2(\delta)(\epsilon_2(\delta) - \epsilon_2(\delta/n) - \delta^2\kappa_2(\delta, \delta/n))}{\epsilon_1(\delta)(\epsilon_1(\delta) - \epsilon_1(\delta/n) - \delta^2\kappa_1(\delta, \delta/n))} \approx \frac{q_3}{q_1 + q_2 + q_3 - M} < 1 \tag{21}$$

Inputting (21) in (20), I obtain

$$\frac{\delta - \epsilon_1(\delta)}{\delta - \epsilon_2(\delta)} \frac{\epsilon_2(\delta)(\epsilon_2(\delta) - \epsilon_2(\delta/n) - \delta^2\kappa_2(\delta, \delta/n))}{\epsilon_1(\delta)(\epsilon_1(\delta) - \epsilon_1(\delta/n) - \delta^2\kappa_1(\delta, \delta/n))} = \frac{M - q_2 - q_3}{M - q_1 - q_3} > 1$$

As in 2P0F, above implies that $\epsilon_2(\delta) > \epsilon_1(\delta)$. However that is a contradiction because (21) implies otherwise. Thus, B_1 can't be bunching.

Now, consider the case where B_2 is bunching in the subgame started by B_3's exit at the bid δ, $\delta \to 0$. From the FOCs for 2P1F, I can infer following using facts that $\sigma(0)/\sigma'(0) = 0$ and $\sigma'(0) = \infty$:

$$\frac{\delta - \epsilon_1(\delta)}{\delta - \epsilon_2(\delta)} \frac{(q_1 + q_2 + q_3 - M)(\delta + \epsilon_2(\delta)(\epsilon_2(\delta) - \epsilon_2(\delta/n) - \delta^2\kappa_2(\delta, \delta/n)))}{q_3\delta + \epsilon_1(\delta)(\epsilon_1(\delta) - \epsilon_1(\delta/n) - \delta^2\kappa_1(\delta, \delta/n)))(q_1 + q_2 + q_3 - M)} = \frac{M - q_2 - q_3}{M - q_1 - q_3} \tag{22}$$

$$\frac{\epsilon_2(\delta)(\epsilon_2(\delta) - \epsilon_2(\delta/n) - \delta^2\kappa_2(\delta, \delta/n))}{\epsilon_1(\delta)(\epsilon_1(\delta) - \epsilon_1(\delta/n) - \delta^2\kappa_1(\delta, \delta/n))} = \frac{q_1 + q_2 + q_3 - M}{q_3} \tag{23}$$

Inputting (23) in (22) gives:

$$\frac{\delta-\epsilon_1(\delta)}{\delta-\epsilon_2(\delta)}\frac{\epsilon_2(\delta)(\epsilon_2(\delta)-\epsilon_2(\delta/n)-\delta^2\kappa_2(\delta,\delta/n))}{\epsilon_1(\delta)(\epsilon_1(\delta)-\epsilon_1(\delta/n)-\delta^2\kappa_1(\delta,\delta/n))}=\frac{M-q_2-q_3}{M-q_1-q_3} \tag{24}$$

As argued before, above requires $\epsilon_2(\delta)>\epsilon_1(\delta)$ (which, unlike the previous case, is not in contradiction with (23)).

Finally, I need to check if the necessary and sufficient condition for B_2's bunching in the subgame are also satisfied. The FOCs of 2P0F with asymmetric support (Appendix A.3.2) imply that when B_2 bunches $\exists\ \tilde{\epsilon}_2(\delta)<\epsilon_2(\delta)$ such that B_2 pools for costs between $\tilde{\epsilon}_2(\delta)$ and $\epsilon_2(\delta)$. Therefore,

$$\frac{\sigma(\tilde{\epsilon}_2(\delta))}{\sigma(\epsilon_1(\delta))}\frac{\phi'_{2,\mathcal{A}2}(\delta)}{\phi'_{1,\mathcal{A}2}(\delta)}\frac{\delta-\epsilon_1(\delta)}{\delta-\tilde{\epsilon}_2(\delta)}=\frac{M-q_2}{M-q_1}$$

which implies that $\frac{\delta-\epsilon_1(\delta)}{\delta-\tilde{\epsilon}_2(\delta)}\frac{\tilde{\epsilon}_2(\delta)(\tilde{\epsilon}_2(\delta)-\tilde{\epsilon}_2(\delta/n)-\delta^2\tilde{\kappa}_2(\delta,\delta/n))}{\epsilon_1(\delta)(\epsilon_1(\delta)-\epsilon_1(\delta/n)-\delta^2\kappa_1(\delta,\delta/n))}=\frac{M-q_2}{M-q_1}$, where $\tilde{\kappa}_2(.)$ is a bounded function. Since $\tilde{\epsilon}_2(\delta)<\epsilon_2(\delta)$, this further implies

$$\frac{\delta-\epsilon_1(\delta)}{\delta-\epsilon_2(\delta)}\frac{\tilde{\epsilon}_2(\delta)(\epsilon_2(\delta)-\epsilon_2(\delta/n)-\delta^2\tilde{\kappa}_2(\delta,\delta/n))}{\epsilon_1(\delta)(\epsilon_1(\delta)-\epsilon_1(\delta/n)-\delta^2\kappa_1(\delta,\delta/n))}>\frac{M-q_2}{M-q_1}$$

Given the convergence of $\phi_i(b)$ to 0 as $b\to 0$ and its continuity, $\epsilon_i(\delta/n)\to 0$ as $n\to\infty$. Since, $\tilde{\epsilon}_2(\delta/n)<\epsilon_2(\delta/n)$, $\tilde{\epsilon}_2(\delta/n)\to 0$ too as $n\to\infty$. Thus, I can infer that inequality $\frac{\delta-\epsilon_1(\delta)}{\delta-\epsilon_2(\delta)}\frac{\epsilon_2(\delta)}{\epsilon_1(\delta)}\frac{(\epsilon_2(\delta)-\epsilon_2(\delta/n)-\delta^2\tilde{\kappa}_2(\delta,\delta/n)))}{(\epsilon_1(\delta)-\epsilon_1(\delta/n)-\delta^2\kappa_1(\delta,\delta/n)))}>\frac{M-q_2}{M-q_1}$ should hold when B_2 is bunching in the subgame.

As Lemma 3 lists all the necessary and sufficient conditions for the equilibrium, and this inequality is derived from the conditions listed in that lemma, it is a necessary and sufficient condition for B_2 to bunch in the subgame started by exit of B_3. Since $\frac{M-q_2-q_3}{M-q_1-q_3}>\frac{M-q_2}{M-q_1}$ when $q_1>q_2$ and $\delta^2\approx 0$ when $\delta\to 0$, equation (24) implies that the condition is satisfied.

Therefore, $\epsilon_2(\delta)>\epsilon_1(\delta)$ and given that at the point of intersection, solution curve of B_2 needs to have higher slope than that of B_1; the curves will not intersect. Thus, $\phi_{2,\mathcal{N}}(b)>\phi_{1,\mathcal{N}}(b)\ \forall b>0$. This would imply that $\phi_{2,\mathcal{N}}(b^R)=\bar{c}>\phi_{1,\mathcal{N}}(b^R)=c_1^*$.

Finally, notice that if B_2 is bunching in subgame started by B_3's exit at any bid b, she is bunching in such a subgame for all b. Else, there exists a bid b_T such that for $b<b_T$, B_2 bunches and above that, B_1 bunches in the subgame. Thus, B_1's payoff

in the subgame, $\pi^*_{1,\mathcal{A}2}(b;c_i)$ would fall discontinuously at b_T. As such, the FOC is satisfied only if $\phi'_{2,\mathcal{N}}(b_T^-) < \phi'_{2,\mathcal{N}}(b_T^+)$. Similarly, $\phi'_{1,\mathcal{N}}(b_T^-) > \phi'_{1,\mathcal{N}}(b_T^+)$. The distance between $\phi_1(b)$ and $\phi_2(b)$ would increase which, as per Lemma 3, implies that B_2 should bunching in the subgame started by B_3's exit at bids above b_T, which is a contradiction. As such, there is no such b_T. Thus, if B_2 is bunching in subgame started by B_3's exit at any bid b, she is bunching in such a subgame for all b. ■

C.2 Proof of Theorem 3

Proof. The proof is similar to that of Theorem 1 (Appendix A.2). To see this, notice that the proof of Lemma 4 tells us that B_2 is bunching in the subgame started by B_3's exit. Thus, I can rewrite the FOCs as:

$$\begin{aligned}
&(q_1+q_2+q_3-M)(b-\phi_{1,\mathcal{N}}(b)))\sigma(b)\mathbb{1}_{b\leq\bar{c}}\\
&\;+(b-\phi_{1,\mathcal{N}}(b))(q_1+q_2+q_3-M)\sigma(\phi_{2,\mathcal{N}}(b))\phi'_{2,\mathcal{N}}(b)=M-q_2-q_3\\
&q_3(b-\phi_{2,\mathcal{N}}(b))\sigma(b)\mathbb{1}_{b\leq\bar{c}}\\
&\;+(b-\phi_{2,\mathcal{N}}(b))(q_1+q_2+q_3-M)\sigma(\phi_{1,\mathcal{N}}(b))\phi'_{1,\mathcal{N}}(b)=M-q_1-q_3
\end{aligned} \tag{25}$$

Suppose first that $b^R > \bar{c}$. For any $b \in (\bar{c}, b^R]$, the FOCs are similar to that of 2P0F. The solution to any IVP given by those FOCs, and boundary conditions $\phi_{2,\mathcal{N}}(b^R) = \bar{c}$, and $\phi_{1,\mathcal{N}}(b^R) = c^*$ exists for all possible c^* and is unique. Furthermore, a structure similar to that of 2P1F also implies that if $\hat{\phi}_{2,\mathcal{N}}(b) < \phi_{2,\mathcal{N}}(b)$, then $\hat{\phi}_{1,\mathcal{N}}(b) > \phi_{1,\mathcal{N}}(b)$ for solutions to any two IVPs which are same except for the initial value $\phi_{1,\mathcal{N}}(b^R)$.

Thus, for any 2 such IVPs, if $\hat{\phi}_{2,\mathcal{N}}(b) < \phi_{2,\mathcal{N}}(b)$, then $\hat{\phi}_{2,\mathcal{N}}(b) < \phi_{2,\mathcal{N}}(\bar{c})$ and $\hat{\phi}_{1,\mathcal{N}}(b) > \phi_{1,\mathcal{N}}(\bar{c})$.

For any bids less than $\bar{c}$, the equations 25 can be rewritten as:

$$\begin{aligned}
(b-\phi_{1,\mathcal{N}}(b)))(\sigma(b)+\sigma(\phi_{2,\mathcal{N}}(b))\phi'_{2,\mathcal{N}}(b)) &= \frac{M-q_2-q_3}{q_1+q_2+q_3-M}\\
(b-\phi_{2,\mathcal{N}}(b))\left(\frac{q_3}{(q_1+q_2+q_3-M)}\sigma(b)+\sigma(\phi_{1,\mathcal{N}}(b))\phi'_{1,\mathcal{N}}(b)\right) &= \frac{M-q_1-q_3}{q_1+q_2+q_3-M}
\end{aligned} \tag{26}$$

Consider a sequence $\{\frac{\delta}{2^n}\}_{n\in\mathbb{N}}$. For each n, consider two initial value problems $\mathcal{P}_n$ and $\hat{\mathcal{P}}_n$ defined on $[\frac{\delta}{2^n}, \bar{c}]$. The problems have same ODEs as (25) except that I replace function $\phi_{i,\mathcal{N}}$ by $\phi_{in,\mathcal{N}}$. The initial values are $\phi_{2n,\mathcal{N}}(\bar{c}) = c^*_{2n}$, $\phi_{1n,\mathcal{N}}(\bar{c}) = c^*_{1n}$, and $\hat{\phi}_{2n,\mathcal{N}}(\bar{c}) = \hat{c}^*_{2n}$, $\hat{\phi}_{1n,\mathcal{N}}(\bar{c}) = \hat{c}^*_{1n}$, where $c^*_{2n} > \hat{c}^*_{2n}$ and $c^*_{1n} < \hat{c}^*_{1n}$.[23]

Now, I can proceed as in 2P0F to show that for each n there is a unique pair of boundary conditions $\phi_{1n,\mathcal{N}}(\bar{c}) = c^*_{1n}$ and $\phi_{2n,\mathcal{N}}(\bar{c}) = c^*_{2n}$, such that solution to $\mathcal{P}_n$ is such that $\phi_{1n,\mathcal{N}}(0) = \phi_{2n,\mathcal{N}}(0)$. Furthermore, it can be shown from arguments similar to 2P0F that the solution is positively monotonic function. Thus, I can argue that as $n \to \infty$, we will get solution such that $\phi_{in,\mathcal{N}}(0) \to 0$. The rest of the argument is same as before to show that there is a unique pair of initial values (c^*_1, c^*_2) such that $\lim_{c \to 0^+} \phi_{i,\mathcal{N}}(c) = 0$.

Now consider the IVP below, defined on $[\bar{c}, b^R]$:

$$(b - \phi_{1,\mathcal{N}}(b))(q_1 + q_2 + q_3 - M)\sigma(\phi_{2,\mathcal{N}}(b))\phi'_{2,\mathcal{N}}(b) = M - q_2 - q_3$$
$$(b - \phi_{2,\mathcal{N}}(b))(q_1 + q_2 + q_3 - M)\sigma(\phi_{1,\mathcal{N}}(b))\phi'_{1,\mathcal{N}}(b) = M - q_1 - q_3$$

$\phi_{i,\mathcal{N}}(\bar{c}) = c^*_i$

This IVP has a unique solution. However, there is exactly one value of b^R where the solution is such that $\phi_{2,\mathcal{N}}(b^R) = \bar{c}$. Thus, there is no guarantee that the equilibrium exists. However, parameters are such that it does, it is unique.

Note however that when $b^R = \bar{c}$, there is a singularity on the right boundary also. However, I can still proceed as in 2P0F barring some changes.The sequence of BVPs with ODEs as in 25, would be defined on $[\frac{\delta}{2^n}, \bar{c} - \frac{\delta}{2^n}]$ with boundaries $\phi_{2n,\mathcal{N}}(\bar{c} - \frac{\delta}{2^n}) = \bar{c} - \frac{\delta}{2^n} + \frac{\delta^2}{4^n}$ and $\phi_{1n,\mathcal{N}}(\frac{\delta}{2^n}) = \phi_{2n,\mathcal{N}}(\frac{\delta}{2^n})$. The solution to BVPs will generate a sequence of non-decreasing functions $\phi_{in,\mathcal{N}}(b)$. This can be used to generate another sequence of functions $w_{in}(b)_{n \in \mathbb{N}}$ defined as:

$$w_{in}(b) = \begin{cases} \phi_{in,\mathcal{N}}(\bar{c} - \frac{\delta}{2^n}), \ b \in [\bar{c} - \frac{\delta}{2^n}, \bar{c}] \\ \phi_{in,\mathcal{N}}(b), \ b \in [\frac{\delta}{2^n}, \bar{c} - \frac{\delta}{2^n}] \\ \phi_{in,\mathcal{N}}(\frac{\delta}{2^n}), \ b \in [0, \frac{\delta}{2^n}] \end{cases}$$

The rest of the argument leverages the monotone convergence theorem as in 2P0F, to show that the $\lim_{n \to \infty} w_{in}$ converges. Define $\phi_{i,\mathcal{N}}(b)$ as $\lim_{n \to \infty} w_{in}$, which then implies that $\lim_{c \to 0^+} \phi_{i,\mathcal{N}}(c) = 0$ for each i, and $\lim_{c \to \bar{c}^-} \phi_{2,\mathcal{N}}(c) = \bar{c}$.

[23] Case where $c^*_{2n} > \hat{c}^*_{2n}$ and $c^*_{1n} > \hat{c}^*_{1n}$ is of no interest because it violates the condition that if $\hat{\phi}_{2,\mathcal{N}}(b) < \phi_{2,\mathcal{N}}(\bar{c})$, then $\hat{\phi}_{1,\mathcal{N}}(b) > \phi_{1,\mathcal{N}}(\bar{c})$.

■

D Identification illustration

Fix a set $\mathcal{N}$ with $|\mathcal{N}| = 3$ (suppressing t from notation). Suppose we observe $\mathcal{S} = \{(B_1, q_1, r_1, q_1), (B_2, q_2, r_2, 0), (B_3, q_3, r_3, 0)\}$, where $r_1 \leq r_2 < r_3$, B_3 is the marginal qualifier, and bidder identities are arbitrary. $\mathcal{B}^L = \{B_2, B_3\}, r_1 < c_2, \; r_2 = c_2, \; r_3 = c_3$. Here, cost order statistics $c^{(k:3)} = c_k, B^{(k:3)} = B_k$ where $k \in \{2, 3\}$. Denote the CDF of the event that B_i bids order statistic $c^{k:3}$ by $G_i^{k:3}(.)$, and the corresponding PDF by $g_i^{k:3}(.)$. Adapting from Song (2006), the probability density of observing that B_2 bids second order statistic $c^{2:3} = c_2$, conditional on B_3 bidding third order statistic $c^{3:3} = c_3$ can be written as:

$$
\begin{aligned}
p_{2|3}^{2|3:3}(c_2|c_2 \leq c_3; \mathcal{S}) &= \frac{F_1(c_2|c_1^I \leq \bar{c}_1^I(c_3^I, q_3, q_1))}{F_1(c_3|c_1^I \leq \bar{c}_1^I(c_3^I, q_3, q_1))} \frac{f_2(c_2|c_2^I \leq \bar{c}_2^I(c_3^I, q_3, q_2)))}{F_2(c_3|c_2^I \leq \bar{c}_2^I(c_3^I, q_3, q_2)))} \frac{h_3(c_3 - c_3^I)}{h_3(c_3 - c_3^I)} \\
&= \frac{\int_{-\infty}^{\infty} h_1(x) \frac{F_1^I(c_2 - x)}{H_1(\bar{c}_1^I(c_3^I, q_3, q_1))} dx}{\int_{-\infty}^{\infty} h_1(x) \frac{F_1^I(c_3 - x)}{F_1^I(\bar{c}_1^I(c_3^I, q_3, q_1))} dx} \frac{\int_{-\infty}^{\infty} h_2(x) \frac{f_2^I(c_2 - x)}{F_2^I(\bar{c}_2^I(c_3^I, q_3, q_1))} dx}{\int_{-\infty}^{\infty} h_2(x) \frac{F_2^I(c_3 - x)}{F_2^I(\bar{c}_2^I(c_3^I, q_3, q_1))} dx} \\
&= \frac{F_1(c_2)}{F_1(c_3)} \frac{f_2(c_2)}{F_2(c_3)} \\
&= g_2^{2:2}(c_2|c_2 \leq c_3; \mathcal{S})
\end{aligned} \tag{27}
$$

Corresponding CDF is:

$$
\begin{aligned}
G_2^{2:2}(c_2|c_2 \leq c_3) &= \int_{\underline{c}}^{c_2} F_1(t|t \leq c_3) dF_2(t|t \leq c_3) \\
&= \int_{\underline{c}}^{c_2} \frac{F_1(t|t \leq c_3) F_2(t|t \leq c_3)}{F_2(t|t \leq c_3)} dF_2(t|t \leq c_3)) \\
&= \int_{\underline{c}}^{c_2} \frac{Pr(c^{2:2} < t|t \leq c_3, B^{2:2} = B_2) + Pr(c^{2:2} < t|t \leq c_3, B^{2:2} = B_1)}{F_2(t|t \leq c_3)} dF_2(t|t \leq c_3) \\
&= \int_{\underline{c}}^{c_2} (G_2^{2:2}(t|t \leq c_3) + G_1^{2:2}(t|t \leq c_3)) d\ln(F_2(t|t \leq c_3))
\end{aligned}
$$

Differentiating with respect to c_2, we get

$$F_2(c_2|c_2 \leq c_3; \mathcal{S}) = \exp\left(\int_{\underline{c}}^{c_2} \frac{dG_2^{2:2}(y|y \leq c_3; \mathcal{S})}{G_2^{2:2}(y|y \leq c_3; \mathcal{S}) + G_1^{2:2}(y|y \leq c_3; \mathcal{S})}\right)$$

$G_i^{2:2}(y|y \leq c_3; \mathcal{S}), i = 1, 2$ can be observed in the data, and hence we can identify $F_2(c_2|c_2 \leq c_3; \mathcal{S})$. Same argument can be provided for any $F_i(x|x \leq c_{ut}; \mathcal{S})$, where c_{ut} is the cost of marginal qualifier round of auction t. The result can extends to the cases where observed order statistics are not adjacent. The identification here is illustrated for the simplest possible case with 2 adjacent order statistics for the ease of exposition.