

# Public goods

## Microeconomics 2

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# I. The nature of goods – I.1. Examples and definitions

Suppose I grow rare flowers....

- ... I can sell them to you: rivalry and exclusion
- ... I can open a flower exhibition and charge you an entry fee for the delightful view: non-rivalry but exclusion
- ... I can keep them but it improves the chances that these rare seeds continue to exist, i.e. I contribute to biodiversity: non-rivalry and non-exclusion
- In the first case, there is a market that "works" probably well enough
- In the second case, there is some sort of a market that works differently
- In the third case, there is (yet) no market: if I stop incurring the cost, we (on earth) will all become "poorer" !

# I.1. Examples and definitions

## Definitions

- A good is **rival** (in consumption) if the same unit of the good cannot be consumed by more than one person at the same time.
- A good is **excludable** if it is technologically or/and institutionally feasible to prevent some people to consume the good.

- **Rival and excludable goods:** private consumption goods... we know that !
- **Rival and non-excludable goods:** common resources, e.g. red tuna in the sea
- **Non-rival and excludable goods:** pay-TV, computer software, patented knowledge - ideas
- **Non-rival and non-excludable goods:** **pure public goods**, e.g. national defense, scientific knowledge - ideas, public TV

## I.1. Examples and definitions

- In fact, a matter of degree of rivalry and exclusion
- A public good makes collective consumption possible
- But the satisfaction from consuming it may depend on others consuming it (e.g. network effects, congestion,...)
- Subtle difference: reduction of value vs destruction by consumption!
- Strong link between public goods and externalities
  
- "Public" goods are not necessarily supplied by the government: e.g. TF1, research in private universities
- "Private" goods may be supplied by public firms / organizations: health services, mail delivery

## I.1. Examples and definitions

### The intriguing example of roads

- A non-toll road with fluid traffic is a public good: I can drive without bothering others and I cannot be prevented from driving on this road
- Toll highways are not pure public goods, they are excludable. Roads may also be forbidden for heavy trucks.
- Paris' circular highway (Boulevard Périphérique) is packed almost always: one additional driver prevents the others from using this facility: the good become (almost) rival due to extreme congestion externalities.

## I.2. Road map for today

Objectives: analysis of economy with a *pure public good*

- Markets tend not to provide public goods efficiently
- Foundations for public / market intervention
- Introduction to public economics and environmental economics

Precise roadmap:

- The basic market failure in a simple example: BLS conditions for efficiency, inefficient private provision
- Remedies: quotas, taxes, Lindhal equilibria, voting on public good provision
- Link between externalities and public goods
- Why asymmetric information is a major concern

## II. The basic market failure – II.1. A simple economy with public good

We will investigate the case of a *pure public good*, that is of a non-rival, non-excludable good, in a simple environment

- An economy with  $I$  consumers, and  $H + 1$  goods
- Goods  $h = 1, \dots, H$  are standard private goods; good  $h = 1$  is normalized as the numéraire
- Good  $h = 0$  is a public good: when  $x_0$  is available in the economy, all consumers benefit from  $x_0$
- Consumers' preferences are represented by:  $u^i(x_0, x^i)$  in which  $x^i$  is the bundle of private good consumption  $x^i = (x_1^i, x_2^i, \dots, x_H^i)$
- $u^i(\cdot)$  is assumed differentiable, increasing and concave

## II.1. A simple economy with public good

- Production of the public good through a firm (or equivalently a sector of identical firms) with technology:

$$y_0 \leq f(y)$$

with  $y = (y_1, y_2, \dots, y_H)$  the vector of input (counted positively)

- $f(\cdot)$  is assumed differentiable increasing and concave
- Available total initial endowments in private goods  $\omega = (\omega_1, \omega_2, \dots, \omega_H)$

## II.2. Optimal provision of public good

We look for Pareto optima in this economy

$$\begin{aligned} \max u^1(x_0, x^1) & \quad [\mu_1 = 1] \\ \forall i \neq 1, u^i(x_0, x^i) \geq v_i & \quad [\mu_i] \\ y_0 \leq f(y) & \quad [\nu] \\ y_0 = x_0 \text{ and } \forall h > 0, \sum_i x_h^i + y_h = \omega_h & \quad [\lambda_h] \end{aligned}$$

- Usual FOC across private goods:  $\forall(h, k)$  non-null and  $\forall i$

$$\frac{\lambda_h}{\lambda_k} = MRS_{h,k}^i = \frac{\partial_h u^i}{\partial_k u^i} = \frac{\partial_h f}{\partial_k f} = MRT_{h,k}$$

- **Bowen-Lindhal-Samuelson conditions** (BLS):  $\forall h, i$

$$\frac{\nu}{\lambda_h} = \sum_{i=1}^I MRS_{0,h}^i = \sum_{i=1}^I \frac{\partial_0 u^i}{\partial_h u^i} = \frac{1}{\partial_h f} = MRT_{0,h}$$

## II.2. Optimal provision of public good

$dx_0$  requires  $\frac{dx_0}{\partial_h f}$  of input  $h$  and increases by  $\partial_0 u^i \cdot dx_0$  any agent  $i$ 's utility; maintaining  $i$ 's utility ( $i \neq 1$ ) constant by reducing  $i$ 's consumption of  $h$  by  $dx_h^j = -MRS_{0,h}^i \cdot dx_0$ ; overall, for agent 1:

$$\partial_0 u^1 \cdot dx_0 + \partial_h u^1 \cdot \left[ -\frac{dx_0}{\partial_h f} + \sum_{i \neq 1} MRS_{0,h}^i \cdot dx_0 \right] = 0 \Leftrightarrow \text{BLS}$$

**Remark:** Given FOC wrt private goods, BLS for  $h = 1$  is sufficient

**Particular case:** for separable utilities without revenue effect,

$$\sum_{i=1}^I \partial_0 u^i(x^{Opt}) = \frac{1}{\partial_1 f(x^{Opt})} = mc(x_0^{Opt})$$

sum of marginal benefits = marginal cost of public good

## II.2. Optimal provision of public good

### Linear quadratic example

- Only two goods, the public good and the numéraire
- $u^i(x_0, x_i) = x_i - \frac{\gamma_i}{2}(1 - x_0)^2$ , with  $\gamma_1 \geq \gamma_2 \geq \dots \geq \gamma_I$
- $f(y) = (2y)^{\frac{1}{2}}$ , so that  $c(y_0) = \frac{1}{2}y_0^2$  and  $mc(y_0) = y_0$
- BLS conditions:

$$\sum_{i=1}^I \partial_0 u^i(x) = \sum_{i=1}^I mb^i(x_0) = \left( \sum_{i=1}^I \gamma_i \right) (1 - x_0) = x_0 = mc(x_0)$$

- the **social marginal benefit** of increasing public good is the sum of private marginal benefits (one unit benefits all!), it must be equal to the (social) marginal cost
- Pareto optimum:  $x_0^{Opt} = \frac{\sum_{i=1}^I \gamma_i}{1 + \sum_{i=1}^I \gamma_i}$
- Symmetric case:  $x_0^{Opt} = \frac{I\gamma}{1 + I\gamma}$  which goes to 1 when  $I \rightarrow \infty$

## II.3. Private provision for the public good

Do market allocation mechanisms attain Pareto optimality ?

- Suppose total quantity of public good = sum of all quantities purchased individually by consumers
- Each consumer  $i$  chooses how much of the public good  $x_0^i$  to buy, taking as given the price system AND the amount of public good purchased by other consumers

Subscription equilibrium, i.e. private provision of public good

$(x_0^{i*}, x^{i*})_{i=1}^I, (y_0^*, y^*), (p_0^*, p^*)$  such that:

- $(x_0^{i*}, x^{i*}) = \arg \max_{x_0^i, x^i} u^i(x_0^i + \sum_{j \neq i} x_0^{j*}, x^i)$  under budget constraint  $p_0^* x_0^i + p^* \cdot x^i \leq B(p_0^*, p^*)$  and  $x_0^i \geq 0$ ; non-negativity not trivial if others make purchases!
- $(y_0^*, y^*) = \arg \max_{y_0, y} (p_0^* y_0 - p^* \cdot y)$  with  $y_0 \leq f(y)$
- Markets clear:  $\sum_i x_0^{i*} = y_0^*$  and  $\forall h > 0, \sum_i x_h^{i*} + y_h^* = \sum_i \omega_h^i$

## II.3. Private provision for the public good

FOC:  $\forall(h, k)$  non-null and  $\forall i$

$$MRS_{h,k}^i = \frac{\partial_h u^i}{\partial_k u^i} = \frac{p_h^*}{p_k^*} = \frac{\partial_h f}{\partial_k f} = MRT_{h,k}$$

$$MRS_{0,h}^i = \frac{\partial_0 u^i + \xi^i}{\partial_h u^i} = \frac{p_0^*}{p_h^*} = \frac{1}{\partial_h f} = MRT_{0,h}$$

with  $\xi^i \geq 0$  multiplier associated to non-negativity constraint

- If at the equilibrium,  $y_0^* > 0$ , then  $\exists \hat{i}$  with a positive demand for public good, hence  $\xi^{\hat{i}} = 0$  and  $MRS_{0,h}^{\hat{i}} = \frac{p_0^*}{p_h^*} = MRT_{0,h}$
- If the public good is always a "good", positive  $MRS_{0,h}^i$  for all  $i$  and therefore:  $\sum_{i=1}^I MRS_{0,h}^i > MRT_{0,h}$

## II.3. Private provision for the public good

The sum of MRS across agents not equal to MRT ! **Inefficiency.**

**The free-rider problem:** agent  $i$  only takes into account his own private marginal benefit of purchasing the public good, and not positive effects on others. With concavity: demand for the public good too low, compared to optimum

**Particular separable linear case:**

- $\partial_0 u^i(x_0^*, x^{i*}) = p_0^*$ , for  $i$  with positive demand, so:

$$\sum_{j=1}^I \partial_0 u^j(x_0^*, x^{j*}) > p_0^*$$

- Under-provision of public "good", over-provision if public "bad"

No obvious conclusion in general equilibrium (prices move)

## II.3. Private provision for the public good

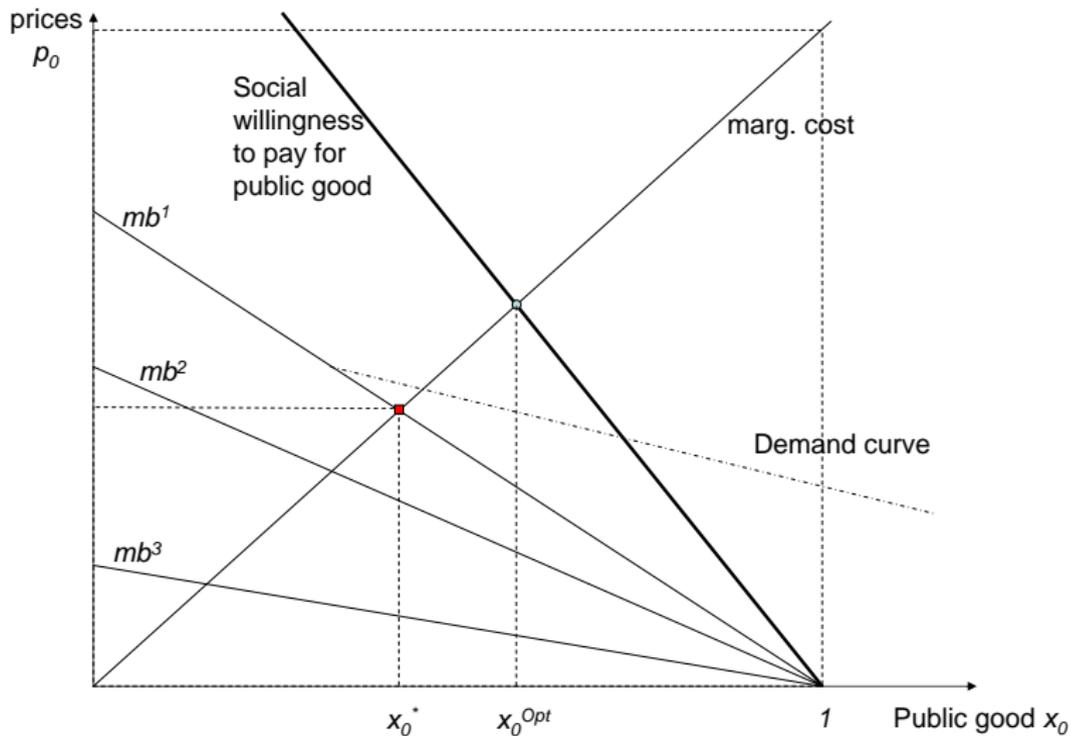
### Linear quadratic example

- For any  $i$ ,

$$\xi^i + \gamma_i(1 - x_0^*) =^{cons.} p_0^* =^{prod.} x_0^*$$

- Only one agent can have a positive demand at equilibrium, agent 1 who has the largest private marginal benefit: hence,  $x_0^1 = \frac{\gamma_1}{1+\gamma_1} = x_0^*$  while for  $i > 1$ ,  $x_0^i = 0$
- Under-provision:  $x_0^* = \frac{\gamma_1}{1+\gamma_1} < \frac{\sum_{i=1}^I \gamma_i}{1+\sum_{i=1}^I \gamma_i} = x_0^{Opt}$
- In the symmetric case, equilibrium is  $x_0^* = \frac{\gamma}{1+\gamma}$ , shared in any way among agents: insensitive to  $I$ , while Pareto optimum  $x_0^{Opt} \rightarrow 1$  when  $I$  increases: large inefficiency in large economies, the free-rider problems become more serious

## II.3. Private provision for the public good



## II.3. Private provision for the public good

Graphical analysis in the separable linear case with  $L = 1$ :

To determine the aggregate demand for a private good, we sum marginal benefit curves **horizontally**

To determine the social marginal benefit for the public good, which then dictates Pareto optimality characterization, we have to sum marginal benefit curves **vertically** !

Yet, in the equilibrium of private provision of the public good, only the private marginal benefit curve for the highest marginal benefit agent matters.

As for externalities, the problem of public good provision opens the door for government intervention !

### **First solution: regulation** (quotas)

- Government (central planner) directly manages the provision of public good, imposing a level  $x_0^0$
- Through government-owned public firm / service (Defense, Meteo)
- Through government regulation of private firm / service (Water treatment, Bus in local community)
- Concessions, Delegation of Public Service: mandatory service explicit in contract between public authority and firm
- Informationally demanding and benevolent government

## III.1. Government regulation and taxes

**Second solution: a tax (or subsidy) on private purchases of public good**

- Consumer  $i$  subject to (personalized) tax on public good purchase  $s_i = -\sum_{j \neq i} \frac{\partial_0 u^j(x^{Opt})}{\partial_1 u^j(x^{Opt})}$  (subsidy to encourage demand)
- New budget constraint:  $(p_0^* + s_i)x_0^i + p^*x^i \leq B(p_0^*, p^*)$
- FOC involving the public good and the numéraire:  $\forall i$

$$MRS_{0,1}^i = p_0^* + s_i \Leftrightarrow \frac{\partial_0 u^i(x_0^i + \sum_{j \neq i} x_0^j, x^i)}{\partial_1 u^i(x_0^i + \sum_{j \neq i} x_0^j, x^i)} + \sum_{j \neq i} \frac{\partial_0 u^j(x^{Opt})}{\partial_1 u^j(x^{Opt})} = p_0^*$$

- There exists an efficient equilibrium
- Informationally demanding and high cost of implementation

## III.1. Government regulation and taxes

### Linear quadratic example

- For any  $i$ , the public good purchase is taxed (in fact subsidized, as is intuitive given the under-provision without intervention) at rate  $s_i = -\frac{\sum_{j \neq i} \gamma_j}{1 + \sum_j \gamma_j} < 0$
- FOC for equilibrium with taxes are: for all  $i$ ,

$$\gamma_i(1 - x_0^i - \sum_{j \neq i} x_0^j) + \frac{\sum_{j \neq i} \gamma_j}{1 + \sum_j \gamma_j} = p_0^* = x_0$$

- $x_0 = x_0^0 = \frac{\sum_j \gamma_j}{1 + \sum_j \gamma_j}$  solves this system of equations: i.e. the Pareto optimal allocation
- Individual purchases are undetermined: there are multiple equilibria that yield same global public good level !

## III.2. Lindhal equilibria

Pure market solution: one market per consumer  $i$  for the **public good benefits experienced by consumer  $i$  !**

- $i$ 's consumption of public good is a distinct commodity with own market and price  $p_0^i$
- $i$  chooses his **total consumption** of public good (unlike in subscription eqbl!) and private consumptions, given prices

### Lindhal equilibrium

$(x_0^{i**}, x^{i**}), (y_0^{i**}, y^{**}), (p_0^{i**}, p^{**})$  for  $i = 1, 2, \dots, I$  such that:

- $(x_0^{i**}, x^{i**})$  maximizes  $u^i(x_0^i, x^i)$  under budget constraint  $p_0^{i**} x_0^i + p^{**} x^i \leq B(p_0^{i**}, p^{**})$
- $(y_0^{i**}, y^{**})$  maximizes the firm's profit  $\sum_{i=1}^I p_0^{i**} y_0^i - p^{**} y$  under the **joint production** technological constraint  $y_0^1 = y_0^2 = \dots = y_0^I = f(y)$
- Markets clear:  $\forall i, x_0^{i**} = y_0^{i**}$  and  $\forall h, \sum_i x_h^{i**} + y_h^{**} = \sum_i \omega_h^i$

## III.2. Lindhal equilibria

The firm in fact maximizes  $(\sum_{i=1}^I p_0^{i**})f(y) - p^{**}y$ , hence:

$$\frac{\partial_h f}{\partial_k f} = \frac{p_h^{**}}{p_k^{**}} \text{ and } \frac{1}{\partial_h f} = \frac{\sum_{i=1}^I p_0^{i**}}{p_h^{**}}$$

Consumers' immediate FOC:

$$\frac{\partial_h u^i}{\partial_k u^i} = \frac{p_h^{**}}{p_k^{**}} \text{ and } \frac{\partial_0 u^i}{\partial_h u^i} = \frac{p_0^{i**}}{p_h^{**}}$$

Hence the BLS conditions:

$$\sum_{i=1}^I \frac{\partial_0 u^i}{\partial_h u^i} = \frac{\sum_{i=1}^I p_0^{i**}}{p_h^{**}} = \frac{1}{\partial_h f}$$

In a Lindhal-version of the economy, the competitive equilibrium yields the Pareto optimal allocation

## III.2. Lindhal equilibria

No surprise ! Lindhal economy has only private goods and complete markets under perfect competition: so, equilibria are efficient. Moreover Pareto optima in the Lindhal economy correspond to Pareto optima in the original economy. The joint production aspect is inconsequential

- **Critical assumption 1:** public good must be **excludable**, otherwise consumer would not buy the public good for his own experience, he would free-ride on others' purchases
- **Critical assumption 2:** only **one** agent on demand side in individualized markets for public good! **The perfect competition assumption is not tenable**
- Lindhal equilibria: fine theoretical but unrealistic solution
- Market solutions not convincing for public goods (contrast with localized externalities)

### III.3. Political economy equilibria

From the Lindhal equilibrium theory, individualized lump sum taxes  $t_i = p_0^{i**} x_0^0$ , with consumers choosing their private consumption on private markets, enable the government to produce and finance the optimal quantity of public good (but information...!)

We now go further in the direction of formalizing public finance by looking at government budget constraint for financing the public good

- Same economy, but for simplicity, public good produced from the numéraire only
- A **budget** is an quantity of public good and its financing by lump sum transfers from consumers:  $(x_0, \{t^i\}_{i=1}^I)$  such that  $\sum_{i=1}^I t^i = f^{-1}(x_0)$ , where  $t^i$  is paid (input supplied) by consumer  $i$

### III.3. Political economy equilibria

#### A political economy equilibrium

$(p^*, (x_0^*, \{t^{i*}\}_{i=1}^I), \{x^{i*}\}_{i=1}^I)$  with  $x_0^* = f(\sum_{i=1}^I t^{i*})$  such that:

- letting  $X^i(p, t^i, x_0)$  denote  $i$ 's demand functions under constraint  $p \cdot x^i + t^i \leq p \cdot \omega^i$
- Consumers maximize utility and pay taxes:  $x^{i*} = X^i(p^*, t^{i*}, x_0^*)$
- Markets clear:  $\forall h > 1, \sum_{i=1}^I x_h^{i*} = \sum_{i=1}^I \omega_h^{i*}$  and  $\sum_{i=1}^I x_1^{i*} = \sum_{i=1}^I \omega_1^{i*} - \sum_{i=1}^I t^{i*}$  (taxes in the numéraire)
- **and there exists no budget  $(x_0, \{t^i\}_{i=1}^I)$  with  $x_0 = f(\sum_i t^i)$ , that improves all agents' welfare, i.e. such that  $u^i(x_0, X^i(p^*, t^i, x_0)) \geq u^i(x_0^*, x^{i*})$  (one at least strict)**

Idea is to study mode of financing (government budgets) that cannot be unanimously defeated by agents

### III.3. Political economy equilibria

#### Optimality of political economy equilibrium

A political economy equilibrium is a Pareto optimum

- If not,  $\exists(\bar{x}_0, \bar{x}^i)$ ,  $\bar{x}_0 = f(\sum_i(\omega_1^i - \bar{x}_1^i))$ , that dominates i.e.:  
 $u^i(\bar{x}_0, \bar{x}^i) \geq u^i(\bar{x}_0^*, \bar{x}^{i*})$
- Consider budget  $(\bar{x}_0, \bar{t}^i)$  with  $\bar{t}^i = p^*(\omega^i - \bar{x}^i)$
- It finances the public good, i.e.:  $\sum_i \bar{t}^i = f^{-1}(\bar{x}_0)$  since  $\sum_i \omega_h^i - \bar{x}_h^i = 0$  and  $\sum_i \omega_1^i - \bar{x}_1^i = f^{-1}(\bar{x}_0)$  (by feasibility)
- It is preferred by all, since  $\max u^i(\bar{x}_0, x^i)$  under constraint  $p^*x^i \leq p^*\omega^i - \bar{t}^i = p^*\bar{x}^i$  necessarily yields (weakly) larger maximum than  $u^i(\bar{x}_0, \bar{x}^i)$ , hence than  $u^i(x_0^*, x^{i*})$
- Contradiction

### III.3. Political economy equilibria

Very heavy procedure to attain a non-unanimously-dominated budget; in particular, assume that agents reveal their preferences to block budget proposal, although they may reduce their taxes by misrepresenting their preferences

Describe a more realistic mode of political decision process: voting procedures to determine a budget

#### Linear quadratic example

- With initial endowment  $\omega^i = 1$  in the numéraire and  $I$  odd
- Constitution: egalitarian financing of the public good, at level  $t$  per capita, simple majority vote on  $t$
- Using budget constraint  $x^i = 1 - t$  and  $x_0 = f(\sum_i t^i) = f(It) = (2It)^{1/2}$ , agents' indirect utility is:

$$v^i(t, \gamma_i) = 1 - t - \frac{\gamma_i}{2}(1 - (2It)^{1/2})^2$$

### III.3. Political economy equilibria

#### Linear quadratic example, cont'd

- Note:  $\partial_t v^i = -1 + \gamma_i(\sqrt{\frac{I}{2t}} - I)$
- $v^i(\cdot)$  concave,  $\partial_t v^i$  positive for  $t \rightarrow 0$ , negative for  $t \rightarrow 1$ , maximum at  $t_i = \frac{I\gamma_i^2}{2(1+I\gamma_i)^2}$ : unimodal
- **Median voter theorem:** with unimodal preferences, the median voter preferred policy is a Condorcet winner, i.e. wins in a simple majority vote
- Let  $\bar{\gamma}$ , the median coefficient, the level  $t^* = \frac{I\bar{\gamma}^2}{2(1+I\bar{\gamma})^2}$  is adopted, which yields  $x_0 = \frac{I\bar{\gamma}}{1+I\bar{\gamma}} \neq x_0^{Opt} = \frac{\sum_{i=1}^I \gamma_i}{1+\sum_{i=1}^I \gamma_i}$
- In general inefficient, except if the median equals the mean coefficient (e.g. under symmetry): inefficiency in very non-equalitarian economies

### III.3. Political economy equilibria

- Political economy equilibria with very sophisticated public decision procedures lead to efficiency, but are also quite unrealistic
- More realistic procedures miss efficiency
- Moreover, there are no well-behaved model of voting for more complicated situations of public good decision

Going further in this direction requires full courses in public finance and in the theory of collective choices

## IV. Public goods and multilateral externalities

Consider air pollution / foul air:

- Non-source specific externality from factories on people
- Foul air as a public "bad", non-rival and non-excludable !

**Multilateral externalities** are non source-specific homogenous externalities. They have a lot in common with public goods

Market-based solutions are less convincing and quotas / taxes more appropriate for public goods than for bilateral externalities: so what in the case of multilateral externalities ?

## IV. Public goods and multilateral externalities

Multilateral **depletable** (private, rivalrous) externality: experience of the externality by one agent reduces the amount felt by another agent

- E.g. dumping of garbage on people's property
- Characteristics of a private good (garbage on  $i$ 's land does not affect  $i'$ )
- Market solutions appropriate: property rights + trade

Multilateral **non-depletable** (public, non-rivalrous) externality

- Air pollution, smog through automobile use, congestion
- Close to public goods
- Quotas / taxes more appropriate

## IV. Public goods and multilateral externalities

Very simple partial equilibrium model in a much reduced form:

- Through producing,  $J$  firms generate a negative externality on  $I$  consumers
- Emitting an externality  $z_j$  corresponds to a profit for firm  $j$  equal to  $\pi_j(z_j)$ ,  $\pi_j(\cdot)$  concave: profit maximization yields  $\pi'_j(z_j^*) = 0$
- Externality is homogenous: total externality is  $z = \sum_j z_j$
- When experiencing externality  $y_i$  and consuming the amount of numéraire  $x_i$ , consumer  $i$  gets quasi-linear utility  $x_i + u_i(y_i)$ ,  $u_i(\cdot)$  decreasing (negative externality) concave

## IV. Public goods and multilateral externalities

Pareto optimum with **depletable multilateral externality**:

$$\begin{aligned} \max_{(y,z)} \quad & \sum_j \pi_j(z_j) + \sum_i u_i(y_i) \\ \text{s.t.} \quad & \sum_j z_j = \sum_i y_i \end{aligned}$$

**FOC:** for all  $i, j$ ,  $u'_i(y_i^{Opt}) = -\pi'_j(z_j^{Opt})$

- Conditions similar to optimality in a one-good economy with  $-\pi'_j(\cdot)$  as firm  $j$ 's marginal cost of producing the externality
- With well-defined property rights and large number of participants, market solutions likely to be effective

## IV. Public goods and multilateral externalities

Pareto optimum with **non-depletable multilateral externality**:

$$\max_z \sum_j \pi_j(z_j) + \sum_i u_i(\sum_j z_j)$$

**FOC:**  $\sum_i u'_i(\sum_j z_j^{Opt}) = -\pi'_j(z_j^{Opt}), \forall j$

- Condition is analogous to the BLS conditions with  $-\pi'_j(\cdot)$  as firm  $j$ 's marginal cost of production
- By analogy with public goods, a market for the externality will not restore efficiency: **free rider problem** remains

## IV. Public goods and multilateral externalities

With non-depletable externality, market-based solutions are dubious. Rather rely on quotas (impose  $z_j^{Opt}$ , possibly as a ceiling quota) and taxes (tax externality at  $t = -\sum_{i=1}^I u'_i(\sum_j z_j^{Opt})$ )

- The market can still be used with *global quota and permits*: distribute permits  $\bar{z}_j$  with  $\sum_j \bar{z}_j = \sum_j z_j^{Opt}$  that are tradable on a permit market at equilibrium price  $p_z$
- The equilibrium is such that:  $\pi'_j(z_j) = p_z^*$ ,  $\sum_j z_j = \sum_j \bar{z}_j = \sum_j z_j^{Opt}$  and necessarily  $p_z^* = -\sum_{i=1}^I u'_i(\sum_j z_j^{Opt})$
- There exists an equilibrium that implements the optimum
- Relax the informational burden on the government

## V. Public goods and asymmetric information

In many of the remedies, the government has to know much about the economy and the agents: unrealistic

### Build or not build under asymmetric information

- $I$  consumers, the numéraire, binary public good  $z \in \{0, 1\}$
- Building the public good costs  $c$
- Consumer  $i$ 's utility:  $x_i + \gamma_i z$ , endowed with  $\omega_i$  in the numéraire, such that  $\sum_{i=1}^I \omega_i > c$
- $c$  is publicly known,  $\gamma_i \in \Gamma \subset \mathbb{R}$  is  $i$ 's private information
- Ex post efficiency: build whenever  $\sum_{i=1}^I \gamma_i \geq c$

Take the asymmetry of information seriously: using quotas and taxes, can the government achieve efficiency for all realizations of  $(\gamma_1, \dots, \gamma_I)$ , i.e. **ex post efficiency**?

## V. Public goods and asymmetric information

The egalitarian, but naive procedure...

- Agents are asked to report simultaneously their valuations  $\gamma_i$ : let  $\gamma_i^a$  denote the announcements
- Then,  $z = 0$  if  $\sum_{i=1}^I \gamma_i^a < c$  and no transfers,  $z = 1$  otherwise with equal financing by agents
- The difference between the two outcomes is that if  $y = 1$ ,  $i$  gets  $\omega_i + \gamma_i - \frac{c}{I}$  while if  $y = 0$ , he simply gets  $\omega_i$
- So, if  $\gamma_i > c/I$ ,  $i$  should maximally over-report, while otherwise he should maximally under-report
- The outcome is (a.a.) inefficient and the government does not extract the relevant information

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A procedure with the same decision rule and where agents pay what they claim the public good is worth for them, when  $y = 1$

- Each agent  $i$  gets  $\gamma_i - \gamma_i^a$  on top of  $\omega_i$ , in all circumstances in which  $y = 1$
- So, by announcing  $\gamma_i^a = \gamma_i$ , they never get more than  $\omega_i$
- Each agent will shade his value and announce  $\gamma_i^a < \gamma_i$ , so as to get a positive rent, even though it may happen less often
- The outcome therefore involves systematic under-evaluation of the public good benefits, hence inefficiency
- The government may possibly infer the relevant information about the  $\gamma_i$ s (by inverting the Bayesian equilibrium strategies,  $\gamma_i^a = m_i^*(\gamma_i)$ ), but the procedure does not allow it to use this information

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### The Groves mechanism

The procedure characterized by the same decision rule and, when  $y = 1$ ,  $i$ 's payment equal to  $c - \sum_{j \neq i} \gamma_j^a$ , induce all agents to report their parameter  $\gamma_i$  truthfully **as a dominant strategy**, and it leads to the efficient decision.

- Suppose  $\gamma_i + \sum_{j \neq i} \gamma_j^a \geq c$ ; then if  $i$  reports so that  $\sum_{j=1}^I \gamma_j^a < c$ , he gets  $\omega_i$ , if he reports so that  $\sum_{j=1}^I \gamma_j^a \geq c$ , he gets  $\omega_i + \gamma_i + \sum_{j \neq i} \gamma_j^a - c \geq \omega_i$ ; among latter optimal choices, truthful revelation !
- Suppose  $\gamma_i + \sum_{j \neq i} \gamma_j^a < c$ ; then if  $i$  reports so that  $\sum_{j=1}^I \gamma_j^a < c$ , he gets  $\omega_i$ , if he reports so that  $\sum_{j=1}^I \gamma_j^a \geq c$ , he gets  $\omega_i + \gamma_i + \sum_{j \neq i} \gamma_j^a - c < \omega_i$ ; among former optimal choices, truthful revelation

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- Suppose that  $\sum_{i=1}^I \gamma_j > c$ , then  $y = 1$  (efficiently) and the public budget is:

$$\sum_{i=1}^I [c - \sum_{j \neq i} \gamma_j] - c = -(I - 1) [\sum_{j=1}^I \gamma_j - c] < 0$$

- The government does not collect enough to finance the public good
- Adding on  $i$ 's transfer a term that only depends on the others' announcements preserves truth-telling and efficiency, and ensures a non-negative budget
- But in general impossible to ensure **exact budget balance** !
- Going further requires advanced methodology in dealing with asymmetric information (later in this course !)

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