Strategic loyalty reward in dynamic price discrimination*

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Abstract

In a dynamic model with overlapping generations of consumers, we study duopolistic competition when firms can price-discriminate, at each period, between their previous customers and the consumers that they have never served. Long-term contracts are not enforceable. In (Markov-perfect) equilibrium, one firm charges higher prices to its past customers prices than to its new customers, as past customers have revealed their strong preferences for the firm; the other firm, however, rewards its previous customers by charging lower prices to them than to its new customers. This loyalty reward strategy comes from the interplay between the firms' usual incentive to extract surplus from consumers with revealed strong preferences and their incentives to acquire information and to recognize their young loyal customers. The result also relies on the firms' inability a priori to tell different generations apart. It is the outcome of the unique equilibrium of a simplified two-period (or T-period) version of the game and holds with forward-looking impatient enough consumers.

Keywords: Price discrimination, Dynamic pricing, Behavior-based pricing, Loyalty reward, Customer relationship management.

JEL: L11, L40, M31

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1 Introduction

Behavior-based price discrimination (BBPD) is a very simple form of price discrimination that consists in offering different prices to different customers according to their past purchase history. In practice, firms charge their own previous customers a different price than their new ones. This pricing strategy is already widely established in many important industries (e.g. banks, telecommunications, softwares, hotels, airlines and e-retailers[1]) and is likely to become even more prevalent with the development of new information technologies (See OFT (2010)).

When BBPD is possible, one of the basic questions is: should firms charge higher prices to their previous customers when they renew their purchase or to their new customers at their first purchase? The academic literature on BBPD often predicts that firms should charge a lower price to their new customers. The reason is that previous customers of a firm have revealed their relative higher preference for the good it provides, thus inducing the firm to charge them a higher price in subsequent periods. The empirical evidence, however, is rather mixed: Shaffer and Zhang (2000) for example provides many instances in which firms charge a lower price to their previous customers. Shin and Sudhir (2010) also notes that practitioners' intuition leads them to think that previous customers should be offered in general better deals than new ones.

There are many examples of introductory offers to new customers. Newspapers usually offer a discount to their new subscribers. For instance, a new subscriber for 3 months to the French newspaper "*Le Monde*" pays 50 euros whereas a previous customer is charged 131.30 euros. Another example is the online retailer *AuchanDirect* who offers a free delivery to its new customers. A third example is the newly opened online betting industry in France wherein operators offer free bets to their new customers. For instance, *BetClic* and the *PMU* offer respectively 80 euros and 50 euros to their new customers. A last case is the antivirus software developer *McAfee* that tried in 2010 to make its previous customers renew their subscriptions for 79.99 dollars, whereas it offered the same software to its new customers at 69.9 dollars.

Examples of better deals to previous consumers also exist. It is observed in the sport industry. For instance, the Parisian rugby club the "*Stade Francais*" offers a discount to its previous customers when they renew their season ticket. In 2008, for the basic season ticket

[1] See Feinberg *et al.* (2002) for more extensive examples.

a new customer paid 400 euros while the renewal price was only 360 euros. The same is sometimes true in fitness clubs. The *Club Vitam* for instance offers 15% discount on the yearly subscription for those who renew their membership. The *Parisian Club Med Gym* has also launched a discount campaign towards its previous customers, offering one extra month on the renewal of yearly subscription for customers that have patronized the *Club* for at least four years. This new strategy emerges while several aggressive entrants have recently appeared in the Paris area. In the antivirus software industry, *Bitdefender* offers 25% to 35% price reduction to its previous customers when they renew their subscription to its antivirus software.

This last example, in combination with the *McAfee* case, shows that the marketing strategies that consist in offering better deals to new or to previous customers may both arise within the same industry. Another example of market wherein one can observe firms that use the two types of pricing strategies is the market for advertising spaces in magazines. In the French market in 2012, the magazine "*Détente Jardin*" offers larger discounts to new advertisers than to past advertisers that do not more than double their purchase (3% versus 2 %), whereas the magazines "*Vivre Ensemble*" and "*Connaissance des Arts*" offer larger discounts to past advertisers than to new advertisers (5% versus 3%). Shaffer and Zhang (2000) also discuss the competition between *Coca-Cola* and *Pepsi* on their so-called "diet" products markets, in which *Coca-Cola* uses a pay-to-switch strategy while *Pepsi* responds by using a pay-to-stay strategy.

There is also evidence that a firm could change its pricing strategy across time, although it is rather difficult to gather unambiguous data about these pricing policy changes. A suggestive example is given in the French market for satellite television, by looking at the pricing strategy of the historical incumbent *Canal+ / Canalsat*. For the first 20 years of its existence, *Canal+ / Canalsat* has hold a monopoly position on paid-TV in France and has systematically offered better deals to new subscribers. The satellite television network *TPS*, launched in december 1996, emerged progressively as a serious competitor in particular in the early 2000s, with the invalidation of *Canal+*'s exclusivity on the broadcasting rights for the French football league L1 in 2002, the acquisition by *TPS* of the rights for the French basketball league in 2003 and of the rights for the prestigious English Premier League in 2004. In 2004, after a contracting of its earnings, *Canal+ / Canalsat* decided to launch a loyalty program [2], until it merged with

[2] See http://www.strategies.fr/actualites/medias/r33955W/canal-experimente-la-fidelisation.html

TPS in 2007. Since then, the firm does not communicate about any loyalty programm anymore and in contrast heavily advertises on its deals for new subscriptions; looking at various users' forums, it seems that the loyalty rewards have indeed disappeared.

In this paper, we present a new theoretical explanation for why a firm may reward its previous customers with better deals even though long term contracts are not enforceable. This loyalty reward strategy comes from the interplay between the firms' usual incentive to extract surplus from consumers with revealed strong preferences and their incentives to acquire information and to recognize their young loyal customers.

More precisely, we consider and analyze an infinite competition two-firm model with overlapping generations of consumers who live two periods; each new generation of consumers is made of symmetric proportions of price insensitive (hereafter loyal to one firm) consumers and price sensitive consumers (hereafter shoppers), as in Varian (1980) and Iyer *et al.* (2005). In addition, through habit formation, an old consumer may become loyal to the firm he patronized when young. Firms are able to recognize their own previous customers, but cannot distinguish between the consumers of the young generation and the previous, hence old, customers of its competitor. Firms can then price discriminate between their previous customers and their new customers. We characterize a symmetric Markov-perfect equilibrium of this model when the degree of habit formation is small enough; this equilibrium is under mixed strategies with continuous supports, as in the elementary models of Varian (1980) and Narasimhan (1988). This equilibrium outcome is the limit of the sequence of unique equilibrium outcomes of finite horizon versions of our game. The equilibrium implies higher profits for firms at the expense of consumers than under uniform competition. More importantly, it exhibits the property that the firm that has recognized its old loyal customers offers a lower price to its new customers (i.e uses a "pay-to-switch" or a "poaching" strategy) than to its previous customers, while its rival, that cannot tell its old loyal and the old shoppers apart, charges a lower price to its previous customers (i.e uses a "pay-to-stay" or a "loyalty reward" strategy) than to its new customers.

The basic intuition runs as follows. The firm that has recognized its old loyal customers, say firm 1, can extract the whole surplus from this category of consumers as they have revealed their strong preferences for the firm's product. It is more aggressive on its more elastic segment of new customers that consists in the young shoppers, the old shoppers who have not been converted into loyal consumers of its rival and its young loyal consumers, and that exhibits consequently a small proportion of loyal consumers. The other firm, say firm 2, has served both its old loyal consumers and the old shoppers in the previous period. As a consequence, it has turned part of the old shoppers into loyal consumers to its products. This means that firm 2 faces a higher proportion of loyal consumers in its market of past customers than in its market of new customers. Consequently, firm 2 has a first (and well-understood) incentive to charge a higher price on its segment of past customers who have relatively stronger preferences for its product. Nevertheless, the segment of new customers contains firm 2's young loyal customers who are much more "valuable" than its old loyal consumers, since being able to perfectly recognize them enables firm 2 to extract their surplus in the subsequent period. Recognition of these young loyal customers requires a price high enough so that they are the only ones from this generation who buy from firm 2. As a consequence, firm 2 has a countervailing incentive to charge a higher price on its segment of new customers than on its segment of previous customers, so as to increase its chance to acquire information on future demand by recognizing its young loyal consumers. When the degree of habit formation is small enough, the loyal customers recognition effect dominates the standard surplus extraction effect based on revealed preferences. This phenomenon, combined with the inability of firm 1 to distinguish generations, gives rise to a systematic pay-to-stay strategy on the part of firm 2 that offers a lower price to its past customers than to its new customers. Although we are unable to prove the uniqueness of such an equilibrium in the infinite horizon overlapping generations game, we show that the "natural" T-period simplified version of the game has a similar unique equilibrium with one firm that uses a pay-to-stay strategy and the other one a pay-to-switch strategy.

Our main analysis is carried out with myopic consumers who only care about the current price they pay. They do not foresee the strategic use of their purchase behavior by firms for subsequent price discrimination and hence do not attempt to manipulate the revelation of their preferences[3]. However we prove that our main result on previous customers reward is robust to

[3]This assumption is likely to be relevant for new markets, where consumers have not yet learned the firms' pricing strategies (Armstrong (2006)). This makes the myopic assumption fair enough for instance in the context of e-retailing which is still a nascent sector. Turow *et.al* (2005) in a study about online markets reports that two-thirds of adult Internet users surveyed believed incorrectly that it was illegal for online retailers to charge different people different prices. Consequently, consumers are unlikely to act strategically to avoid being recognized. In established industries, the myopic assumption can also be seen as a form of bounded rationality.

the consideration of fully rational consumers (without habit formation) as long as their discount factor for the present is low enough.

Relevant literature. In the terminology of Fudenberg and Villas Boas (2006) our model mixes information price discrimination, as past purchases convey information on consumers' tastes, and the switching cost approach where past purchases do have a direct effect on consumers' utilities. The pure BBPD branch of the literature in competitive environments[4] has been pioneered by Villas-Boas (1999) and Fudenberg and Tirole (2000). The other branch of the literature, with payoff-relevant history has been pioneered by Chen (1997) and Taylor (2003) who consider environments with ex ante homogenous goods and switching costs that cause ex post differentiation [5]. The BBPD literature has then been extended in several directions: asymmetry among firms (Chen (2008)), changes in consumers' preferences (Chen and Pearcy (2010) and Shin and Sudhir (2010)), link with firms' advertising strategies (Esteves (2009a), and De Nijs (2013)), discrete distribution of consumers' preferences (Chen and Zhang (2009) and Esteves (2010)), enhanced services (Aquisiti and Varian (2005) and Pazgal and Soberman (2008)), complement goods (Kim and Choi (2010)) and endogenous product design (Zhang (2011)). [6]

Depending on the underlying consumers' preferences and degree of patience, BBPD has been found to be either profitable or unprofitable. Moreover, a common prediction of these models is that firms should offer lower prices to their rivals' customers to entice them to switch and higher prices to their own previous customers to capture their captive surplus. In our model, the incentives to recognize one's own captive customers interact with these forces, thereby generating a high price on new customers and rewards for previous customers on the part of the firm that has not recognized its old loyal consumers. In a similar infinite competition model with overlapping generations of consumers, Villas-Boas (1999) finds opposite conclusions namely, BBPD always generates poaching strategies. The forces at play in our article are similar

[4]For models with a monopolist and customer recognition, see e.g. Fudenberg and Tirole (1988), Villas-Boas (2004), and Conitzer *et.al* (2012)

[5]See also Gehrig and Stenbacka (2004) and Gehrig *et.al* (2011) for models with both switching costs and horizontal product differentiation.

[6]See Fudenberg and Villas-Boas (2006), Esteves (2009b), and Zhang (2010) for more complete literature reviews.

to those in Villas-Boas (1999). In both papers, lowering the price offered to new customers hurts future profits. This is the reason why firms have incentives to charge a high price to new customers to better recognize their new loyal consumers. In both papers, firms have also an incentive to charge higher prices on past customers, as they have revealed their relatively higher preference for the firm's product. In Villas-Boas (1999), the incentive to extract surplus from consumers with revealed high preferences dominates for both firms, leading them to follow a pay-to-switch strategy, while in our model, the incentive for loyal customers recognition dominates for the firm that has not perfectly identified its old loyal customers. The connections and differences of the current article with Villas-Boas (1999) are further discussed in sub-section ??.

The only two papers on BBPD with short term contracts that generate previous customers reward we are aware of are Shin and Sudhir (2010) and De Nijs and Rhodes (2013). Using a model à la Fudenberg and Tirole (2000), Shin and Sudhir (2010) allow consumers' preferences to vary across periods and they introduce some heterogeneity among customers with respect to the number of units they wish to buy each period. In this context, past purchase history conveys information on both consumers tastes and the quantity they wish to buy. They find that in markets with sufficient heterogeneity in quantities demanded and large enough changes of consumers preferences, it is optimal to reward one's own previous, high-demand customers, since the marginal gain in profit from cutting prices to retain them is greater than the marginal benefit of poaching a mix of low and high demand competitors' customers. Only one category of previous customers is rewarded and both firms use pay-to-stay strategies. Our model does not rely on consumers' mobility neither do we need an additional dimension of heterogeneity to generate previous consumers reward. Besides we predict that only one of the two firms offers a lower price to its previous customers. De Nijs and Rhodes (2013) show that when duopolists that engage in BBPD sell experience goods, the skewness of consumer valuations fully determines whether firms use pay-to-stay or pay-to-switch strategies. Contrary to the current model they find that in equilibrium both firms should use the same type of pricing strategies.

Another closely related article is Chen and Zhang (2009). They use the same underlying consumers' preferences as ours in a two-period model where one single generation leaves through the two periods. Their main finding is that firms can be better off with BBPD than without it, even when consumers behave strategically. The intuition is that, in order to pursue customer recognition,[7] competing firms need to price high to screen out price-sensitive consumers and hence price competition is moderated. The pursuit of loyal consumers recognition plays a similar role in our analysis as it contributes to the profitability of BBPD, but our repeated setting implies in addition that pricing for new customers aims at increasing the chances to win the race for customers recognition, which results in the strategic loyalty reward phenomenon. Note also that our result holds for myopic or strategic and relatively impatient consumers.

Introducing long term contracts is another possibility to derive loyalty rewards in the literature on BBPD. This strand of the literature has been pioneered by Caminal and Matutes (1990) and has been the object of recent advances (See Chen and Pearcy (2010)). The rationale for previous customers reward is the creation of endogenous switching cost through the design of the loyalty program as a way to create an opportunity cost from switching brands.

There are other rationales for pay-to-stay strategies that could be inferred from the literature. The first one is related to variety seeking. If consumers exhibit a variety seeking behavior, loyalty reward emerges naturally as a means for firms to retain their previous customers who now value more their rival's product. Formally, this idea can be formalized by introducing negative correlation in consumers' preferences between the two subsequent periods in the models of Chen and Pearcy (2010) or Shin and Sudhir (2010). Another way to formalize the idea would be to consider negative switching costs in the model of Chen (1997). Another rationale for pay-to-stay strategies could be based on cost considerations. Marginal costs to serve existing consumers might be lower than those for new ones, because for instance their personal information have already been collected. This difference in marginal costs would introduce a force towards pay-to-stay strategies (See Shin et al. (2012)).

Last, our article is also related to the literature on static models of preference-based pricing and especially to Shaffer and Zhang (2000). Shaffer and Zhang (2000) consider a setting with asymmetric inherited market shares and asymmetric levels of brand loyalty. They show that when the average loyalty of the two groups of consumers is sufficiently dissimilar, the firm whose previous customers are the less loyal finds this segment to be the more elastic one and hence

[7] See also Esteves (2009) about the competition softening effect of the pursuit of customers recognition.

offers it a lower price. In this case the other firm charges a lower price to its new customers. So, their result comes from differences in price elasticities while ours is a consequence of dynamic consideration of customer recognition.

The rest of this article is organized as follows. Section **??** describes the model. Section **??** investigates simplified versions of the model. Section **??** provides the main analysis with myopic consumers. Section **??** extends the analysis to forward-looking consumers. Section **??** concludes. Technical proofs are relegated in the appendix.

2 The model

We consider a market for an horizontally differentiated good with overlapping generations of consumers living two periods and two symmetric infinitely-lived firms $i = 1, 2$.

Each period, a unit mass of consumers enters the market and stays until the end of the next period. Consumers have a unit demand per period, with constant per-period valuation equal to $v > 0$. Within a new generation, a proportion $l \in (0, 1/2)$ is only interested in buying from firm 1 and the same proportion l for firm 2; these are "loyal" consumers. The remaining proportion $s = 1 - 2l$ may buy from either firm and are price sensitive; they are called "shoppers"[8]. We also assume that a consumer who was a shopper when he was young becomes with probability α a loyal consumer to the firm he previously bought from when he gets old[9]. This assumption introduces state dependency in consumer preferences. It accounts for habit formation (See for instance Dubé et al. (2010) for empirical substantiation). All consumers discount the future at the same rate $\beta \in [0, 1)$ and choose whether to buy and from which firm at each period of their life. The size of consumer segments is common knowledge to all agents.

Firms are infinitely lived and their production marginal cost are normalized to 0. They maximize their respective intertemporal profit streams, with common discount factor $\delta \in [0, 1)$. At each date, firms choose prices simultaneously.

The information structure is critical. First, firms observe all prices once they have been set. Second, we assume that firms are unable to distinguish loyal consumers from shoppers

[8]Basically this is a duopoly version of Varian (1980).
[9]See Anderson and Kumar (2007) who make a similar modeling assumption.

within a young generation. Third, firms are able to identify their "previous customers", i.e. the customers they have previously served. However, they cannot distinguish between the consumers of the young generation and the previous customers of its competitor: these are just "new" customers for them.

Based on the information they collect, firms can price discriminate between their own previous and their new customers. So, at each period, they simultaneously choose a pair of prices $P_i^t \equiv (p_{o,i}^t, p_{n,i}^t)$ for firm $i = 1, 2$, $p_{o,i}^t$ charged to i's own previous customers and $p_{n,i}^t$ to i's new customers. That is, we only allow for short term contracts.

We will restrict attention to symmetric Markov-perfect equilibria. To be more precise about the equilibrium concept, we focus on the case of myopic consumers ($\beta = 0$) in the remaining of this section and in sections 3 and 4, relegating the more general case with forward-looking consumers in section **??**.

When $\beta = 0$, there is no intertemporal strategic considerations on the consumer side. So, at any period t, loyal consumers buy if and only if the price of their preferred firm is not larger than v. Shoppers, at any period t, buy from the firm offering the lowest price available to them (as a previous customer or a new customer for the firms), provided this price does not exceed v. In case of a tie, we assume that with probability 1/2 all shoppers patronize one of the firms and with probability 1/2 they patronize the other one. This rule has the same impact on current expected profits as the standard rule in which consumers split equally among the firms, but it simplifies the description of the payoff-relevant history as it implies that, next period, all shoppers are previous customers of the same firm.

The game thus basically reduces to a game between the firms. Restricting attention to prices within $[0, v]$,[10] suppose that (P_1^{t-1}, P_2^{t-1}) prevailed at period $t-1$. When, at period t, firms choose prices $(P_1^t, P_2^t) \in [0, v]^4$, they use their information to implement price discrimination that is, to allow a previous (resp. new) customer to be charged a price $p_{o,i}^t$ (resp. $p_{n,i}^t$); then, firm i's total demand and profit simply depend on (P_1^t, P_2^t) and on whether it served the shoppers born at $t-1$ or not, since that determines the composition of their respective segments of potential customers. So, there exists a public sufficient statistics for the whole payoff-relevant

[10] As is intuitive, prices cannot be larger than v in equilibrium. The proof of proposition **??** in an online Appendix characterizes, in the general case of strategic consumers, equilibrium strategies following any price deviation.

history that corresponds to the identity of the firm who served the shoppers born at $t-1$: either firm 1 served all shoppers born at $t-1$, i.e. (P_1^{t-1}, P_2^{t-1}) is such that $p_{n,1}^{t-1} < p_{n,2}^{t-1}$ (or $p_{n,1}^{t-1} = p_{n,2}^{t-1}$ and all shoppers patronized firm 1), or firm 2 served them all, i.e. when $p_{n,2}^{t-1} < p_{n,1}^{t-1}$ or ($p_{n,1}^{t-1} = p_{n,2}^{t-1}$ and all shoppers patronized firm 2).

To ease notation, we will thereafter change the labeling of firms: we let $P_L = (p_{o,L}, p_{n,L})$ denote the prices of the L-firm, that is the firm that served all the shoppers born at the previous period, $P_H = (p_{o,H}, p_{n,H})$ the pricing rule for the H-firm, that is the firm that had the highest price and served no shoppers. Symmetric Markov-perfect equilibria thus are subgame perfect equilibria in which firms choose their prices based solely on whether they are the (current) H-firm or L-firm, and consumers make their purchase decisions based solely on the current prices available to them, given which firm, if any, they patronized previously.

3 Static version and finite dynamics

3.1 One-period version of the game and intuition

Let us first consider a one-period version of the game with asymmetric bases of past customers for the firms and an exogenous bonus for being able to identify one's own young loyal consumers. This static game can be seen as the constituent game of the infinite horizon game when one endogenizes the bonus for loyal consumers recognition, i.e. when it is taken to be equal to the difference between the intertemporal equilibrium valuations of the H-firm and of the L-firm in the infinite horizon game. The analysis of the static game is also useful to build intuition as it enables us to capture the strategic interaction due to the imperfect overlap of the populations of potential customers for the firms, and the incentive to recognize loyal customers of the young generation.

More precisely, in the static game, both firms face two segments of customers each and can price discriminate between them. The H-firm, faces a first monopolistic segment (of past customers) consisting in l old loyal customers, who are therefore charged the monopoly price v. Its second segment, on which it charges a price $p_{n,H}$, consists of l other loyal consumers,

$(1-\alpha)s$ old shoppers[11] and s young shoppers. The L-firm faces two different segments: its first segment of past customers consists of $l+\alpha s$ old and (now) loyal consumers and $(1-\alpha)s$ old shoppers; its second segment of new customers consists of l young loyal consumers and s young shoppers. The L-firm charges a price $p_{o,L}$ on the first segment and a price $p_{n,L}$ on the second one. In addition, we assume that the firm that serves only its population of loyal consumers from the new generation earns an exogenous bonus $\delta\Delta$, with $0 \leq \delta\Delta < sv$.[12] As previously specified, in case of a tie on the new generation of consumers, we assume that with probability 1/2 all shoppers patronize one of the firm and with probability 1/2 they patronize the other one. Figure 1 depicts the different segments of own previous consumers and new customers identified by each firm.

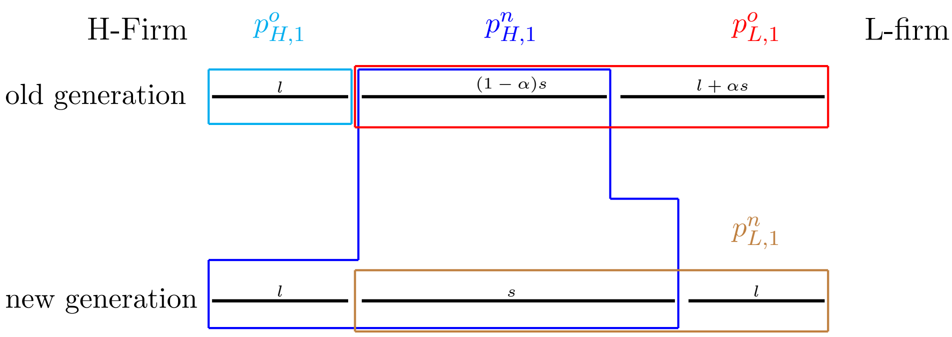

Figure 1

Our setting differs from models such as Varian (1980) or Narasimhan (1988) because of the bonus Δ. The existence of the bonus introduces an incentive for one firm to overprice its rival, which conflicts with the standard Bertrand logic. Yet, the approach followed in these papers can be extended, leading to two sets of results. First, unsurprisingly, one can show that there exists no pure-strategy equilibrium in our setting, whatever the value of the bonus Δ (See Appendix **??**, Claim **??**). Dasgupta and Maskin (1986) ensures that an equilibrium exists in our setting, so we look for equilibria in mixed strategies. Second, any mixed strategy equilibrium can be shown to be such that the support of $p_{n,H}$ and the union of the supports

[11] A fraction αs has been converted into loyal consumers of the L-firm.

[12] If $\delta\Delta > sv$, the tradeoff between capturing the young shoppers with a low enough price and earning the bonus with a high enough price would disappear: the incentive to charge high prices would always dominate.

of $(p_{o,L}, p_{n,L})$ are identical, with no hole and no mass point except perhaps for one firm at the upper bound v of these supports (See Appendix **??**, Claim **??**). Firms' program can then be written as follows:

$$U_L = \max_{p_o \leq v} p_o(l + \alpha s + (1-\alpha)sH(p_o)) + \max_{p_n \leq v} p_n(l + sH(p_n)) + \delta\Delta(1 - H(p_n)), \tag{1}$$

where $H(.)$ is the ddf used by the H-firm for its price $p_{n,H}$ to its new customers, and:

$$U_H = lv + \max_{q \leq v} q(1-\alpha)sL_o(q) + q(l + sL_n(q)) + \delta\Delta(1 - L_n(q)), \tag{2}$$

where $L_o(.)$ and $L_n(.)$ denote the marginal ddf of the joint price distribution used by the L-firm to randomize its prices $(p_{o,L}, p_{n,L})$ for past customers and new customers respectively. The following proposition is proved in Appendix A.

Proposition 1. *For any (Δ, α) such that $\alpha\frac{(l+s)}{l}sv < \delta\Delta < sv$, all mixed-strategy equilibria of the static game are characterized by the same marginal price distributions and are therefore payoff-equivalent; they are such that the firm that has not recognized its old loyal customers, i.e. the L-firm, uses a systematic (i.e with probability one) pay-to-stay strategy, while its rivals, the H-firm, systematically uses a pay-to-switch strategy.*

While Appendix **??** fully characterizes the unique equilibrium (marginal) price distributions (See Claim **??** and Claim **??**), the central result lies in two specific claims: first, the supports of $L_o(.)$ and $L_n(.)$ must necessarily be disjoint and adjacent (See Appendix **??**, Claim **??**) and, second, the support of $L_n(.)$ always lies above the support of $L_o(.)$ when habit formation is not too strong, i.e. provided that $\alpha\frac{(l+s)}{l}sv < \delta\Delta$ (See Appendix **??**, Claim **??**); that is, there is loyalty reward by the L-firm with probability one.

Let us provide some intuition for these results. Given that all prices are bounded from above by v, the monopolistic price, and given that the H-firm has complete monopoly power on its past (loyal) customers, it is immediate that the H-firm follows a pay-to-switch strategy: it offers a better deal to new customers to capture new shoppers and to entice some old shoppers to switch from its rival. The situation of the L-firm is more ambiguous. On the one hand, the L-firm has a higher proportion of loyal consumers in its market of past customers than in its market of new customers, because of habit formation. This creates a well-known incentive for the L-firm to charge a (stochastically) higher price to its more inelastic segment of past customers. On the other hand, the segment of new customers contains the L-firm's young loyal customers who are more "valuable" than its old loyal consumers, since being able to perfectly identify them enables the L-firm to earn the bonus Δ on top of the sales. Recognition of these young loyal customers requires a price high enough so that they are the only ones from this generation who buy from the L-firm. As a consequence, the L-firm has an incentive to charge a higher price to its segment of new customers than to its segment of previous customers. When the degree of habit formation is small enough, the loyal customers recognition effect is strong compared to the elasticity effect. How the two forces combine to deliver our key result depends critically on the H-firm's response.

In our setting with unidentifiable generations, the H-firm uses the same price to challenge the L-firm on the old shoppers segment and on the young consumers segment, due to its inability to tell apart different generations of consumers. Suppose, instead, that generations were identifiable *per se*. The H-firm could therefore charge a specific price to old shoppers (old consumers that it has not served) and another specific price to young consumers (as well as the monopoly price on its old loyal customers). The situation boils down to two separated static games on two sub-markets, one for old shoppers and one for the young generation; one can show easily that in equilibrium both firms randomize their prices to the old shoppers on the interval $[\frac{l+\alpha s}{l+s}v, v]$ with a mass $\frac{l+\alpha s}{l+s}$ at v for the L-firm, and they randomize their prices (symmetrically) to the young generation on the interval $[\frac{lv+\delta\Delta}{l+s}, v]$. The elasticity effect induces the L-firm to charge the old generation a higher price than if there was no habit formation at all ($\alpha = 0$) and, by strategic complementarity it induces the H-firm to charge a higher price as well on the segment of old shoppers. Similarly, the customers recognition effect induces both firms to charge higher prices than if there was no bonus Δ on the segment of young consumers, both through a direct effect due to the bonus and through an indirect effect due to strategic complementarity. When habit formation is limited compared to the bonus for loyal customers recognition, prices charged to the old generation can take smaller value, i.e. be more competitive, than the prices to the young generation: the lower bound of the support of prices is smaller for old consumers than for young consumers ($\frac{l+\alpha s}{l+s}v < \frac{lv+\delta\Delta}{l+s}$). But the upper bound of the supports is the monopolistic price v for both sub-markets, so that both supports overlap. This means that one can observe ex post that the L-firm has used a pay-to-stay or a pay-to-switch strategy: there is only *some* loyalty reward in equilibrium.

In our setting, there is loyalty-reward with probability one by the L-firm in equilibrium. The H-firm cannot randomize its (unique) price so that the two forces towards high prices for the L-firm be exactly matched and the L-firm be indifferent with respect to its two price instruments on a common interval of prices. To accommodate the strong incentives of the L-firm to charge a high price p_n to the young generation, when Δ is large, the H-firm charges a (stochastically) high price (it also has a direct incentive to do that). This shift towards high prices also applies for old shoppers so that the L-firm now strictly prefers to undercut the H-firm and to capture old shoppers with high probability by charging relatively low prices to its past customers.

3.2 Endogenous bonus for loyal consumers recognition in finite dynamics

In order to endogenize the bonus for loyal consumers recognition Δ, let us consider the twofold repetition of the static game presented in the previous sub-section, with the following features. In period 2, there is no exogenous bonus. The H-firm in period 2 is the firm that has recognized its loyal customers in period 1, i.e. the firm that has charged the highest price to the generation born at period 1; its installed base corresponds to its l loyal customers born at period 1. The L-firm in period 2 is the firm that charged the lowest price to the generation born at period 1 and its installed base consists in $l + \alpha s$ loyal customers born at period 1. So, there are $(1-\alpha)s$ old shoppers in period 2. The bonus from charging the highest price to new consumers in period 1 corresponds to the difference between the profit from holding the H-firm position in period 2 and the profit from holding the L-firm position in period 2.

The analysis consists in characterizing the unique (mixed-strategy) equilibrium outcome in terms of profits in the period 2 subgame, and to use the (discounted) difference between the equilibrium profits of the period-2 H-firm and the period-2 L-firm as the bonus for loyal customer recognition in period 1.[13] The subgame in period 2 can be analyzed exactly as the

[13]The analysis of the finitely repeated game and the sketch of proof of the following proposition are provided

previous static game except that the bonus is the null: the consequence is that the period-2 L-firm still randomizes its two prices, one charged to its previous customers and one to its new customers, on two disjoint and adjacent intervals but now, only the elasticity effect is active so that the support of $L_{o,2}(.)$, the price distribution for the previous customers used by the period-2 L-firm, lies above the support of $L_{n,2}(.)$. More precisely, and for α not too large, the firm that has recognized its loyal consumers in period 1, the so-called period-2 H-firm, charges v to its past (loyal) customers, thus earning lv on this segment, and randomizes the price to its new customers on the interval $[\frac{(l+(1-\alpha)s)(l+\alpha s)v}{(l+s)^2}, v]$, thereby earning an additional profit equal to $\frac{(l+(1-\alpha)s)(l+\alpha s)v}{(l+s)^2}(l+(2-\alpha)s)$. The rival firm, the so-called period-2 L-firm, charges a price to its previous customers randomized on the interval $[\frac{(l+(2-\alpha)s)(l+\alpha s)v}{(l+s)^2}, v]$, with a mass point at v, thus earning $(l+\alpha s)v$, and a price to its new customers randomized on the interval $[\frac{(l+(1-\alpha)s)(l+\alpha s)v}{(l+s)^2}, \frac{(l+(2-\alpha)s)(l+\alpha s)v}{(l+s)^2}]$, thereby generating an additional profit equal to $\frac{(l+(1-\alpha)s)(l+\alpha s)v}{(l+s)}$. Consequently the additional profit earned in the last period by the firm that has recognized its own loyal consumers at the end of period 1 can be viewed, from the perspective of period 1, as a bonus for loyal consumers recognition and it is given by:

$$\Delta_1(\alpha) = \frac{s}{(l+s)^2}[(1-2\alpha)l^2 + (1-3\alpha)sl - \alpha^2(2-\alpha)s^2].$$

As long as the degree of habit formation is low enough, this bonus is such that Proposition **??** applies: then, the subgame perfect equilibrium is basically unique and it exhibits loyalty reward by the L-firm in period 1. The argument is valid for any finite repetition of the static game (See Appendix B, section B.1 and B.2).

Proposition 2. *In the T-period model with installed bases, when the degree of habit formation is sufficiently small, all equilibria involve mixed strategies in all periods of the game and are all equivalent in terms of marginal ddf per period, with the property that, at each period except the last one, the firm that has not recognized its old loyal customers at the previous period uses a systematic (i.e with probability one) pay-to-stay strategy, while its rival systematically uses a pay-to-switch strategy.*

When the degree of habit formation is limited, identifying its own young loyal consumers

in Appendix **??**.

is profitable for a firm as it allows it to extract higher profits in the following period than if it had not identify them and this dynamic loyal consumers recognition effect is powerful enough to dominate the standard incentive to charge higher prices on the segment of consumers with less elastic demand. The basic intuition provided in the static game carries on in the finite dynamic version of the game.

3.3 Comparison with Villas-Boas (1999) and Fudenberg-Tirole (2000)

The forces at play in our model are the same than in Villas-Boas (1999), but Villas-Boas (1999) finds that firms always use pay-to-switch strategies while we find that a pay-to-stay strategy may emerge in equilibrium for one firm. The two-period model provides a tractable framework in which one can analyze this major difference.

We now consider a simplified two-period version of Villas-Boas (1999) with features similar to our two-period model developped in the previous subsection, but with consumers uniformly distributed on a Hotelling line of length 1 and firms located at each extremity. Consumers incur a linear transportation cost and the transportation parameter is normalized to one. We assume that in the first period, firm L (on the left extremity) has an installed base constituted by all consumers on the line while its rival (firm H) has no installed base. This situation mimics our model with asymmetric installed bases. We also assume that a new generation of consumers enters the market in the second period and that consumers are myopic.[14] As in Villas-Boas (1999) we also assume that firms choose first their prices for new customers and then for past customers.[15]

Solving for the (unique) subgame perfect equilibrium in this two-period version of Villas-Boas (1999), following the steps spelled out in the original paper, is a simple matter of calculus and is only sketched in Appendix **??** section B.3. Two results are significant. First, when the L-firm's (resp. the H-firm's) market share among young consumers in period 1 is $\hat{x}$ (resp. $1-\hat{x}$) with $\hat{x} \in [3/7, 4/7]$, the second period profits of each firm can be shown to be decreasing in this

[14] As shown by Villas-Boas(1999), in a Hotelling-type model with new generations at each period, when the discount factor of consumers increase, the demand from new customers becomes more elastic thus reducing the scope for the emergence of a pay-to-stay strategy.

[15] This timing is different from ours but it is very unlikely to be the key factor that tilts the balance between the use of a pay-to-stay or a pay-to-switch strategy.

firm's market share among young consumers in period 1. That is, charging the highest price to young consumers in period 1 induces one firm to better identify the customers who prefer it and to earn higher profits in the ensuing competition game in period 2. As in our setting, viewed from period 1, there is a bonus for close-by customers recognition. Second, in equilibrium and in period 1, the L-firm can be shown to charge a higher price to its new customers than to its previous customers. That is, firm L uses a pay-to-stay strategy, just as in our setting. The result relies on a similar effect as in our model: the bonus for close-by customers recognition leads firm-L to charge a higher prices to the young generation in period 1. Moreover, when firm L's initial installed base is large, it faces elastic consumers on average and cannot afford charging too high a price to the old generation. The combination of the two effects leads to the pay-to-stay strategy for the L-firm.

The use of a pay-to-stay strategy cannot however be part of equilibrium strategies at the steady state of the Markov-perfect equilibrium of the infinite competition game with overlapping generations of consumers, as shown by Villas-Boas (1999). A pay-to-stay strategy is not a stable property of the market described by Villas-Boas (1999) in the long run. To the contrary, in our setting, we show in the next section that the use of a pay-to-stay strategy is observed at the steady-state of the Markov-perfect equilibrium of the infinite dynamic game. The key ingredient that drives this difference is the structure of preferences. In Villas-Boas (1999), the indifferent consumer of a new generation between the two products invariably becomes closer to the center of the line thus making the incentive to poach the rivals' past customers dominate the incentive to screen new consumers; both firms end up using a pay-to-switch strategy in equilibrium in the long run. Such a phenomenon is not possible in our framework because the split of consumers of a new generation between the two firms always occurs in the same way with one firm that serves only its loyal consumers and the other firm that serves all the other consumers. The L-firm is therefore unable to poach any past consumers from its rival and its pricing strategy for its new consumers is only driven by the learning/recognition effect that tends to push up prices. In addition the L-firm has a weak incentive to charge a high price to its past customers who on average exhibit a low level of loyalty. This explains why the L-firm uses a pay-to-stay strategy both in our two-period and infinite period models.

When describing preferences among differentiated products, a smooth model à la Villas-

Boas (1999) or Fudenberg-Tirole (2000) is probably better suited than our setting. If, instead, the heterogeneity among consumers comes from different information structures, our setting seems adequate to formalize a situation in which some consumers are only informed about the existence of one firm (the so-called loyal consumers) and some others know about both firms (the shoppers) (and possibly forget with probability α about the other firm once they have patronized one firm). Then, the emergence of pay-to-stay strategies corresponds to the possibility of screening out consumers who are uninformed about the existing competition and who can later be squeezed, once perfectly identified.

4 Infinite horizon dynamic model

We now turn to the dynamic overlapping-generation version of the game. As explained in section **??**, we look for symmetric Markov-perfect equilibria in which the behavior of one firm at period t depends only on whether it is the current H-firm or the current L-firm. The analysis of the equilibrium behavior at any period is similar to the analysis of the equilibrium behavior in the static game except that the exogenous bonus for recognition of one's own loyal consumers is now endogenized as being equal to the difference between V_H, the intertemporal equilibrium valuation of the H-firm, and V_L, the intertemporal equilibrium valuation of the L-firm.

For similar reasons as in the two-period game, there exists no symmetric pure-strategy Markov-perfect equilibria and all symmetric Markov-perfect equilibria involve mixing (except for $p_{o,H} = v$). We exhibit one equilibrium characterized by two thresholds, $\underline{p}$ and $\hat{p}$, such that the H-firm randomizes $p_{n,H}$ on the interval $[\underline{p}, v]$ according to the absolutely continuous marginal d.d.f. $H(.)$, the L-firm randomizes $(p_{o,L}, p_{n,L})$ according to the absolutely continuous marginal d.d.f. $L_o(.)$ for $p_{o,L}$ on the interval $[\underline{p}, \hat{p}]$ and $L_n(.)$ for $p_{n,L}$ on the interval $[\hat{p}, v]$, with a mass point at v. Equilibrium conditions can be stated as follows:

$$\begin{aligned} V_L &= \max_{p_o \leq v} p_o[l + \alpha s + (1-\alpha)sH(p_o)] \\ &\quad + \max_{p_n \leq v}\{p_n[l + sH(p_n)] + \delta V_L H(p_n) + \delta V_H(1 - H(p_n))\} \\ &= \underline{p}(l+s) + vl + \delta V_H \\ &= \hat{p}(l + \alpha s + (1-\alpha)sH(\hat{p})) + vl + \delta V_H \\ &= \underline{p}(l+s) + \hat{p}(l + sH(\hat{p})) + \delta V_L H(\hat{p}) + \delta V_H(1 - H(\hat{p})), \end{aligned}$$

$$\begin{aligned} V_H &= vl + max_{q<v}\{q[l + (1-\alpha)sL_o(q) + sL_n(q)] + \delta V_L L_n(q) + \delta V_H(1 - L_n(q))\} \\ &= vl + \underline{p}(l + (2-\alpha)s) + \delta V_L \\ &= vl + \hat{p}(l+s) + \delta V_L, \end{aligned}$$

The analysis in Appendix **??** proves the next proposition, which generalizes Propositions **??** and **??**.

Proposition 3. *In the dynamic model with myopic consumers, when the degree of habit formation is sufficiently small, there exists Markov-perfect equilibria that exhibit the property that the firm that cannot currently identify its old loyal consumers follows a pay-to-stay strategy. These equilibria are equivalent in terms of outcome and marginal ddf and they correspond to the class of equilibria that are the limits of all equilibria obtained in the sequence of T-period versions of the game, when T tends to infinity.*

The equilibrium characterized in the previous proposition exhibits the same remarkable feature than the one of the two-period model: the L-firm charges uniformly lower prices to its own previous customers than to its new customers, i.e. its previous customers are rewarded in equilibrium with probability 1. This feature is, again, in striking contrast with the usually described pricing strategies in behavior-based price discrimination models in which, firms usually extract more surplus from their previous customers than from the consumers they have never served.

In our model, the firm that has perfectly identified its previous loyal customers actually extracts all their surplus ($p_{o,H} = v$). The intuition to understand the L-firm's behavior is the same than in the two-period model. It relies on the profitability of identifying one's young loyal consumers and on the impossibility to discriminate among old and young shoppers when the firm has never served any of them previously. On the one hand, the L-firm has a higher proportion of loyal consumers in its market of past customers than in its market of new customers, hence the usual incentive to follow a pay-to-switch strategy. However, the segment of new customers contains the L-firm's young loyal customers whose recognition is profitable as it enables this firm to secure the next H-firm position and to fully extract these consumers' surplus at the next period. In order to screen out these young loyal customers, the L-firm has to charge a price high enough so that only these young loyal consumers buy from it. When the degree of habit formation is small enough, this force dominates and the L-firm ends up charging its new customers higher prices than its previous customers.

The characterization of the equilibrium reveals that the payoff-relevant state, i.e. the distribution of the H-firm position and the L-firm position, follows a Markov process. The current L-firm remains the L-firm with some probability at the next period and becomes the H-firm with the complement probability. More precisely, it is easy to show that when α is sufficiently small: $\Pr\{p_{n,H} < \hat{p}\} = 1 - H(\hat{p}) = \frac{1+s-4\alpha s}{1+(3-2\alpha)s} > \frac{1}{2}$. Therefore, $\Pr\{p_{n,H} < p_{n,L}\} > \Pr\{p_{n,H} < \hat{p}\} > 1/2$: the current H-firm (resp. L-firm) has a probability higher than 1/2 of becoming the next L-firm (resp. H-firm). In other words, firms change roles more often than not in equilibrium. The prediction clearly singles out our model compared to other explanations of strategic loyalty reward and it could easily be confronted to data.

As δ increases, it is easy to prove that $\underline{p}$ and $\hat{p}$ increase and that the price distributions shift in the sense of first order stochastic dominance so that higher prices become more likely: price competition becomes less intense as δ increases, i.e. as the incentives to enter a race for the H-firm position become stronger. Also, when δ increases, the per-period profit of the L-firm improves while the per-period profit of the H-firm position diminishes: $(1-\delta)V_L$ increases and $(1-\delta)V_H$ decreases. The L-firm engages in a race to ensure the next H-firm position and therefore enjoys the benefits associated to the H-position eventually; similarly, the H-firm does not secure the H-position for ever.

Corollary 4. *When the degree of habit formation is small enough, behavior-based price discrimination increases the profits of both firms at the expense of consumers compared to the situation in which price discrimination is banned (or impossible)..*

It is immediate to check that $V_J > V_J^{nd}$ with $J = H, L$ and V_J^{nd} being the intertemporal equilibrium valuations when price discrimination is not feasible,[16] which means that behavior-based price discrimination boosts the industry profits in comparison with uniform price competition. This result is driven by the surplus appropriation effect of price discrimination against recognized old captive customers and the related pursuit of young loyal customers recognition. It is in line with Chen and Zhang (2009). But, here a novelty arises in the sense that even the firm that did not recognize its old loyal customers derives a higher profit on its segment of previous customers. This effect is a direct consequence of the loyalty reward: it is due to the infinite nature of competition and to the structure of information available to firms that make the L-firm benefit, on its segment of previous customers, from the price softening effect induced by the race for young consumers recognition. In our model, welfare does not depend on the type of competition and is fixed to $2v$ by period. Consequently the profit boosting effect from price discrimination comes at the expense of the consumers' surplus.

5 Price discrimination with forward-looking consumers

We finally turn to the case of non-myopic consumers: $\beta > 0$. To simplify the analysis we assume in this section that $\alpha = 0$. Consumers' behavior can exhibit two types of patterns that were absent in the previous section: a young loyal consumer may decide not to buy so as to avoid being identified and being charged an excessive price by his favorite firm when old; and a young shopper may decide either not to buy or even to buy from the highest-price firm so as to benefit from a more advantageous array of prices when old.

[16] The characterization of these is a trivial application of standard models in our setting, hence omitted. One finds:

$$\begin{aligned} V_L^{nd} &= \frac{(1+s)(1-(1-\alpha)s)v}{(1-\delta)(1+(1+\alpha\delta)s)} \\ V_H^{nd} &= \frac{(1+(1-(1-\delta)\alpha)s)(1-(1-\alpha)s)v}{(1-\delta)(1+(1+\alpha\delta)s)}. \end{aligned}$$

The equilibrium exhibited in the previous section (with $\alpha = 0$) is disrupted by such strategic manipulation. Suppose $v > p^t_{n,1} > p^t_{n,2}$, so that at $t + 1$ firm 1 will become the H-firm. If he buys from firm 1, a young consumer loyal to firm 1 is identified and receives no surplus when old (since $p^{t+1}_{o,H} = v$). If instead he refrains from buying, he will face a price distribution $H(.)$ when old, the expectation of which is bounded away from v: he will enjoy a positive expected surplus, which makes this deviation profitable even for small β when $p^t_{n,1}$ is close enough to v. This suggests that firms cannot charge prices too close to v in equilibrium.

With forward-looking consumers, the definition of an equilibrium is quite cumbersome because the state variable that describes the payoff-relevant history should record the proportions of loyal consumers and of shoppers served by each firm.[17] In the following, we adopt a more modest approach. We present a mild modification of the Markovian strategies of the equilibrium with loyalty reward and myopic consumers and show that, for a range of small discount factors β, they support a (Markov-perfect) equilibrium in the following sense: no firm and no (individual) consumer has any profitable deviation after any history of prices either on the equilibrium path or off the equilibrium path, in any subgame subsequent to a price deviation by one firm.[18]

More precisely, let the L-firm at period t be the firm who had the lowest price for its segment of new customers at period $t - 1$; the other firm is the H-firm. Strategies on the equilibrium path are as follows: the H-firm charges $p_{o,H} = v$ and chooses $p_{n,H}$ according to a d.d.f. $H^*(.)$ with support $[\underline{a}, \bar{a}]$, for some $\bar{a} \leq v$; the L-firm chooses $(p_{o,L}, p_{n,L})$ according to a joint distribution with marginal d.d.f. $L^*_o(.)$ and $L^*_n(.)$ respectively, $L^*_o(.)$ has support $[\underline{a}, \hat{a}]$ and $L^*_n(.)$ has support $[\hat{a}, \bar{a}]$ with a mass at $\bar{a}$; young consumers purchase at the lowest acceptable price to them if and only if this price is not larger than $\bar{a}$, and they refrain from consuming when the lowest acceptable price is larger than $\bar{a}$; old consumers follow their static dominant strategy, as previously. These behaviors are similar to the ones generated by the equilibrium

[17] On the general theory of Markov-perfect equilibria, see Maskin - Tirole (2001). In our setting in which only prices are observed, tools developed by Fershtman and Pakes (2009) should be used.

[18] Note first that, since strategies are mixed, the issue is about deviating on prices above the maximal observable price (low prices can be easily handled). We omit the description of strategies in subgames following price deviations by both firms above the maximal observable price. This enables us to reduce the possible values of the state variables to the H-firm / L-firm statistics and the proportion of loyal consumers served when a price exceeds the maximal observable price. A full description of strategies, and of the associated Markov-perfect equilibrium, is possible but it would require an extremely heavy presentation without any economic insights.

with reward of previous customers when consumers are myopic, except for the maximal price $\bar{a}$ that firms can charge in equilibrium. The maximal price $\bar{a}$ is determined so that no individual consumer has an incentive to deviate from the straightforward behavior he would follow if he were myopic, provided firms charge prices below $\bar{a}$, but not all young loyal consumers of a firm purchase from this firm when it charges a price above $\bar{a}$. The possibility that strategic non-consumption implies that the maximal price $\bar{a}$ will be strictly smaller than v. The next proposition shows in what sense Proposition **??** is robust to forward-looking consumers.

Proposition 5. *For small enough values of the consumers' discount factor β and no habit formation, there exists an equilibrium with reward of previous customers by the L-firm of each period, characterized by a maximal price $\bar{a}$ strictly smaller than v.*

Proposition **??** whose proofs are relegated in a separate appendix shows that strategic loyalty rewards can survive to the intertemporal considerations of forward-looking consumers. When consumers become very patient, the equilibrium is likely to be qualitatively different with consumers who forego their purchase when young to have a better price when old. The characterization of such equilibria is beyond the scope of this paper.

6 Discussion

Summary. In this article, we have analyzed an infinite competition model with overlapping generations and firms that are able to recognize their own previous customers when charging prices. The game admits a Markov-perfect equilibrium which exhibits interesting properties regarding which segments of customers (i.e previous or new customers) a firm should offer a better price. We find that the firm that has recognized its old loyal customers charges a lower price to its new customers than to its own previous customers. But we showed that the firm that did not recognize its old loyal consumers charges its previous customers a lower price, in contrast with much of the literature on behavior-based price discrimination. This equilibrium is unique in a simplified two-period model that constitutes the key building block of the full dynamic game. It survives when we consider forward-looking consumers provided they are impatient enough.

Testable implications. In order to test the implications of our model, one should focus on markets wherein preferences à la Varian (1980) with loyal and price-sensitive consumers is a good representation of consumers heterogeneity. This is likely to be the case for markets in which there is little intrinsic horizontal product differentiation but an important heterogeneity in consumers' awareness about product existence or prices. This environment is a standard framework to model e-retail competition (See Baye *et al.* (2006)). Among other things, it generates price dispersion for homogenous products across firms and across time, which is in line with many empirical investigations of online competition (See Baye *et al.* (2006)) and which is compatible with mixed strategy equilibria. Indeed, a report on consumers' view on switching service providers for the European Commission (Flash Eurobarometer 243, The Gallup Organization 2009) provides evidence that consumers' search online is quite limited in markets for rather homogenous goods such as electricity retailing, Internet and telephony providers. Note, though, that a model with consumers heterogeneity à la Varian (1980) can also be suitable for offline retailing as argued in Iyer *et al.* (2005). In all these types of markets, the main testable implications of our analysis of loyalty reward are the following ones.

The first implication of our model is that one should observe in a given geographical market intra-firm variations of pricing strategies across time. In other words, a firm should switch from pay-to-stay types of strategy (existence of loyalty programs, cash return policies, and other non-price loyalty rewards) to pay-to-switch types (promotional offers, discount on new subscriptions,...) and vice versa as time passes. The relevant period of time to test this implication would be the typical length of contracts in the industry. In addition, because the L-firm has more chances to recognize its young loyal customers than the H-firm in our model, one should observe that across time the probability that a firm changes the nature of its pricing strategy is greater than the probability that it maintains the same type of strategies. In more technical terms, the transition matrix for the types of pricing strategies used by firms from one period to the other should exhibit higher off-diagonal coefficients than diagonal ones.

Second, according to our model, one should observe in a given geographical market cross-sectional variations of pricing strategies among firms. Of course, this is quite a common observation and one that is also simply compatible with mixed strategies. A more discriminatory prediction of the model is that markets with dynamic price discrimination should never exhibit a uniform use of pay-to-stay strategies, i.e. all firms offering loyalty rewards; indeed, when loyalty rewards are observed, the model predicts that other firms should use standard pay-to-switch strategies because of the different composition of their respective segments of past / new customers. Our model also offers a testable prediction when comparing industries with different identification of customers namely that the probability of observing no pay-to-stay strategy should be higher when information about consumers is available that helps identify consumers with a promising future in the market. This corresponds to situations in which firms can identify "generations of consumers" per se, because they use age-related or experience-related tariffs (student rates, senior rates) or because they need personal information to serve consumers. A way to test this prediction would be to gather data on prices offered by firms engaging in BBPD in different geographical markets (at the MSA level for instance) for two different industries (within the class of industries previously described): an industry that can a priori price discrimination between generations of consumers, and another industry that cannot. The share of geographical markets without firms engaging in a pay-to-stay strategy should be higher in the first industry.

Third, our theory of loyalty reward is plausible only in markets with relatively low degree of habit formation. This means that one should observe more pay-to-stay pricing strategies in markets experiencing a decrease in the degree of habit formation, because of some standardization in the products or because of a shock in switching costs. The introduction of number portability in the telephony industry would provide such a suitable "quasi-experiment" to test this prediction (See Viard (2007) and Park (2011)). This type of policy gives consumers the option of keeping their current number when they change providers thus reducing their switching cost.

Managerial implications. Our analysis of the competitive forces at play in dynamic price discrimination lead to some suggestions for managerial decisions. These suggestions apply in markets in which there is a high rate of consumers' renewal, in which past purchase information about past customers can be collected and used by firms in order to price discriminate, and in which switching costs and habit formation in consumption are limited. In such a market, decisions about launching or improving a loyalty program for loyal (hence past) customers and decisions about promotional offers or discounts to new consumers may be viewed as the two sides of the same coin, depending on whether the firm decides to face competition more aggressively on its segment of new or of past customers.

Loyalty programs may rely on long term commitments (club membership, customers' cards), and they may then play a role in attracting new consumers based on long term promises. But other loyalty reward policies do not rely on long run commitment, either because they are not strongly advertised or because they are not contractual obligations. These reward policies are often presented as a rebate on a base price (paid by new customers) and constitute a way to retain consumers with high preferences for the firm's products for future profitable business relationships. We suggest that this type of strategy can alternatively be viewed as a surcharge to new consumers compared to a base price (paid by past customers), which is a way to select and screen high willingness-to-pay consumers among the newcomers through high prices, so as to constitute a niche of highly profitable loyal consumers in the future. This strategy should be considered in markets in which the degree of consumers' heterogeneity makes a niche position potentially highly profitable (the H-position is more valuable). It should be considered by a firm that has a dominant position in the sense that it has won a large market share among the price sensitive or well-informed consumers already active on the market (it is the L-firm, in our model). In such a situation, the firm uses different prices to keep these shoppers that have already participated in the market and to compete for the newcomers who just enter the market; so, a strategy of high prices to new consumers will not induce a desertion of past customers, especially if the price structure is presented as a loyalty reward for past customers, and it may build a valuable niche of consumers with large future earning potential for the firm. Conversely, a firm that has constituted a niche (a H-firm) can separate the price instruments to exploit its niche consumers and the price instruments to compete for new customers, either among the newcomers or through poaching in the rivals' existing market share, through promotions or discounts for the first purchase.

A related interpretation is possible in terms of incumbent and new entrant in the industry. Our analysis would then recommend the leader to take care of its past customers by charging them a lower price that to its new customers. As for the challenger, our analysis suggests an aggressive pricing on the segment of its new customers -in particular to target the old shoppers not turned into loyal consumers to its rival. Although we did not solve the model with increasing market size over time, we can reasonably conjecture that the discount for loyalty reward would be smaller when the market size is increasing (this is similar to an increase in the discount rate). In this case, the incentive to recognize loyal young consumers is very strong so that the challenger would not be so aggressive on its segment of new customers. Therefore, by strategic complementarity the leader would not need to offer important discounts to retain its past customers. This interpretation is compatible with the anecdotical evidence on the Canalsat / TPS example sketched in the introduction.

Another managerial implication is that the decision to reward its new or past customers cannot be a long-lasting pricing practice, and hence requires sufficient flexibility in terms of organization and management of distribution channels to implement this behavioral pricing. Indeed, in some cases, a distributional channel may require from a firm the guarantee that the firm will never offer a lower price for the same product elsewhere. This is the case for instance for *Booking.com*[19]. This could be problematic for a firm when it has to switch from a pay-to-switch to a pay-to-stay strategy and the channel primarily used to target new consumers requires the lowest available price. The bottom line is that firms should closely pay attention to the pricing flexibility afforded by their different channels of distribution when they engage in highly time-varying behavior based pricing strategies.

[19] $http://www.booking.com/general.fr.html?aid=356982;label=gog235jc-partnerreg-fr-XX-XX-unspec-fr-com;sid=24448b615e8aa13baaea0a40f6f7700f;dcid=2;tmpl=docs/partner_print$ §4.8.4

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A Analysis of the static model.

The proof consists in several claims. Claims 1 shows that there exists no pure strategy equilibrium and Claim 2 characterizes standard necessary properties of support of the distribution of $p_{n,H}$, hereafter denoted q, and of the union of the supports of the marginal distributions of $p_{o,L}$ and $p_{n,L}$, hereafter simply denoted by p_o and p_n, and the possibility of mass points. In both claims, the presence of the bonus Δ prevents us from simply applying Narasimhan (1988). In the proofs, we only provide the specific elements due to the presence of the bonus. Claims 3 to 6 consist in disentangling the marginal distributions of $p_{o,L}$ and $p_{n,L}$; they are more original and require more complete proofs. Finally Claim 7 is a technical result that is used in the other appendices.

Fix from now on parameters $\alpha \in (0,1)$ and $\Delta \in [0, \frac{sv}{\delta})$.

Claim 1. *There exists no pure strategy equilibrium.*

Proof. Suppose there exists a pure strategy equilibrium (q, p_0, p_n). The equilibrium payoffs are given by:

$$\begin{aligned}
U_H &= vl + ql + q(1-\alpha)s\mathbf{1}_{\{q<p_o\}} + qs\mathbf{1}_{\{q<p_n\}} + \delta\Delta\mathbf{1}_{\{q>p_n\}} \\
&\quad + \frac{qs+\delta\Delta}{2}\mathbf{1}_{\{q=p_n\}} + \frac{q(1-\alpha)s}{2}\mathbf{1}_{\{q=p_o\}}; \\
U_L &= p_o(l+\alpha s) + p_o(1-\alpha)s\mathbf{1}_{\{p_o<q\}} + p_n l + p_n s\mathbf{1}_{\{p_n<q\}} + \delta\Delta\mathbf{1}_{\{p_n>q\}} \\
&\quad + \frac{p_n s+\delta\Delta}{2}\mathbf{1}_{\{p_n=q\}} + \frac{p_o(1-\alpha)s}{2}\mathbf{1}_{\{p_o=q\}}.
\end{aligned}$$

It is standard to prove that the only possible pure strategy configuration is: $q = p_n < p_o = v$.

Suppose now that $0 < q = p_n < p_o = v$. Then a deviation to $q-\varepsilon$ yields an additional profit to the H-firm approximately equal to $\frac{qs-\delta\Delta}{2}$ when ε vanishes; this deviation is not profitable only if $qs \leq \delta\Delta$. A deviation to $q = v - \varepsilon$ yields an additional profit to the H-firm equal $(v-q)(l+(1-\alpha)s) + \frac{\delta\Delta-qs}{2}$ when ε vanishes; given the previous condition that $\delta\Delta \geq qs$, this deviation is strictly profitable. So, the configuration of prices $0 < q = p_n < p_o = v$ cannot constitute an equilibrium configuration.

Finally, suppose that $q = p_n = 0 < p_o = v$. A deviation for the H-firm to $q = v - \varepsilon$, ε vanishing, yields profit equal to $[l + (1-\alpha)s]v + \frac{\delta\Delta}{2} > 0$. So, this cannot be an equilibrium either. $\square$

So, any equilibrium is under mixed strategies. The proof that the union of the supports of the L-firm's prices, p_o and p_n, has no hole and the support of q has no hole either follows from Narasimhan's Proposition 2.

Claim 2. *Neither firm can have a mass point in the interior or at the lower boundary of the other firm's supports (union of supports), and only one firm can have a mass point at the upper boundary of the other firm's support (union of supports).*

Proof. Suppose in equilibrium there is a mass m at price $q = \mu$ for the distribution of the H-firm's price. If the L-firm charges $p_o \in [\mu, \mu + \varepsilon)$ with some positive probability in equilibrium, a standard argument (Narasimhan (1988), Proposition 3) shows that moving some mass from $p_o \in [\mu, \mu + \varepsilon)$ to $p_o \in (\mu - \varepsilon', \mu)$ for ε and ε' small enough would be a profitable deviation for the L-firm. The same argument applies if the L-firm charges $p_n \in [\mu, \mu + \varepsilon)$ with some positive probability in equilibrium, provided $\mu > \frac{\delta\Delta}{s}$, since the gain is then proportional to $\mu s - \delta\Delta > 0$. The argument also holds if μ is the upper bound of the union of the supports of p_o and p_n and the L-firm charges it with positive probability.

What if $\mu \leq \frac{\delta\Delta}{s}$? If the H-firm were to charge a price q with positive probability, it would obtain a profit on its new consumers equal to:

$$q\left[l + (1-\alpha)s\left(\Pr\{p_o > q\} + \frac{\Pr\{p_o = q\}}{2}\right)\right] + \delta\Delta$$
$$+ \left(\Pr\{p_n > q\} + \frac{\Pr\{p_n = q\}}{2}\right)[qs - \delta\Delta].$$

If, the H-firm charges $q = \mu + \varepsilon$ instead of $q = \mu$ for ε small enough ($\mu + \varepsilon$ is still smaller than v as $\delta\Delta < sv$ by assumption), note first that $\left(\Pr\{p_n > q\} + \frac{\Pr\{p_n = q\}}{2}\right)$ decreases in q so that given that $\mu s - \delta\Delta \leq 0$, the second line of the expression cannot decline. Moreover, as seen in the first paragraph of the proof, in equilibrium the L-firm cannot charge $p_o \in [\mu, \mu + \varepsilon)$ with positive probability, so that the probability terms in the first line are the same for $q = \mu$ and $q = \mu + \varepsilon$, and the first line therefore strictly increases: the H-firm could therefore profitably deviate from the mass on $q = \mu$, a contradiction. So, in equilibrium there cannot exist a mass point for $q = \mu \leq \frac{\delta\Delta}{s}$.

The proof for the L-firm is similar and omitted. $\square$

The next two steps in Narasimhan's Appendix (his Propositions 4 and 5) extend without difficulty to our analysis: the support of q and the union of the supports of p_o and p_n are identical, and the common upper bound is v. Any equilibrium is therefore in mixed strategies characterized by $H(.)$, the ddf of q and $L_o(.)$ and $L_n(.)$ the marginal ddf of p_o and p_n, all these are absolutely continuous within $[\underline{a}, v)$, except perhaps for one firm at v. The equilibrium payoffs are given by:

$$\begin{aligned} U_H &= vl + q[l + (1-\alpha)sL_o(q) + sL_n(q)] + \delta\Delta(1 - L_n(q)); \\ U_L &= p_o[l + \alpha s + (1-\alpha)sH(p_o)] + p_n[l + sH(p_n)] + \delta\Delta(1 - H(p_n)). \end{aligned}$$

The next result shows that there cannot be any non-degenerate overlap between the supports of p_o and p_n.

Claim 3. *The intersection of the supports of $L_o(.)$ and $L_n(.)$ is reduced to isolated points.*

Proof. Suppose that there exists an open interval Ω that is included in both supports. By the equilibrium condition of mixing, there exists two positive constants, K_o and K_n, such that for any $p \in \Omega$:

$$\begin{aligned} p[l + \alpha s + (1-\alpha)sH(p)] &= K_o \\ p[l + sH(p)] + \delta\Delta(1 - H(p)) &= K_n. \end{aligned}$$

Note that K_o and K_n have to be larger or equal to lv, since this is a lower bound on the payoffs that the L-firm can earn on one segment by charging the price v on this segment.

Eliminating $H(p)$ between these two equations, one obtains that for any $p \in \Omega$:

$$\alpha s(l+s)p^2 + [(1-\alpha)sK_n - sK_o - \delta\Delta(l+s)]p + \delta\Delta K_o = 0.$$

This can never be satisfied for $\alpha \in (0, 1)$. $\square$

The only thing that remains to be characterized is how the supports of L_o and L_n are related in equilibrium. The next result is key in pinning down the only possible equilibrium configuration, namely that, when habit formation is not too strong, the support of p_o lies entirely below that of p_n.

Claim 4. *For any $(\Delta, \alpha) \in \mathbb{R}^2_+$ such that $\alpha \frac{(l+s)}{l} sv < \delta\Delta < sv$, there exists $\hat{a} \in (\underline{a}, v)$ such that the equilibrium support of p_o is $[\underline{a}, \hat{a}]$ and the equilibrium support of p_n is $[\hat{a}, v]$ (both possibly with mass points at v).*

Proof. We know that the supports of p_o and p_n do not overlap, that they are adjacent and that they are no mass point below v. Suppose that there exists a price $\tilde{p} \in (\underline{a}, v)$ and $\varepsilon > 0$ such that $(\tilde{p} - \varepsilon, \tilde{p})$ is included in the support of p_n (hence not of p_o) and $(\tilde{p}, \tilde{p} + \varepsilon)$ is included in the support of p_o (hence not of p_n).

The mixing condition of the L-firm implies that $\forall p \in (\tilde{p}, \tilde{p} + \varepsilon)$,

$$p(l + \alpha s + (1 - \alpha)sH(p)) = K \geq vl.$$

Locally then, within $(\tilde{p}, \tilde{p} + \varepsilon)$, $H(.)$ is differentiable and its derivative $h(.)$ within $(\tilde{p}, \tilde{p} + \varepsilon)$ satisfies:

$$h(p) = -\frac{K}{(1 - \alpha)sp^2} \leq -\frac{vl}{(1 - \alpha)sv^2} = -\frac{l}{(1 - \alpha)sv}. \tag{3}$$

And the (right-)derivative at $\tilde{p}$ of the profit function on the old customers has to be equal to 0:

$$l + \alpha s + (1 - \alpha)sH(\tilde{p}) + (1 - \alpha)s\tilde{p}h(\tilde{p}^+) = 0. \tag{4}$$

In equilibrium, charging a price $p_n \in (\tilde{p}, \tilde{p}+\varepsilon)$ must yield weakly smaller profit for the L-firm on the segment of its new consumers; therefore, since $H(.)$ is continuous at $\tilde{p}$ and differentiable on $(\tilde{p}, \tilde{p} + \varepsilon)$, it is necessary that the (right-)derivative at $\tilde{p}$ of the L-firm's profit function on the new consumers be non-positive:

$$l + sH(\tilde{p}) + (\tilde{p}s - \delta\Delta)h(\tilde{p}^+) \leq 0. \tag{5}$$

Using (**??**)-(**??**) to eliminate $H(\tilde{p})$, one has necessarily:

$$-\alpha(l + s) - \delta\Delta(1 - \alpha)h(\tilde{p}^+) \leq 0.$$

If $\alpha < \frac{l\delta\Delta}{(l+s)sv}$, however, one gets $-\alpha(l+s) > -\frac{l\delta\Delta}{sv}$ and, given (**??**), this implies:

$$-\alpha(l+s) - \delta\Delta(1-\alpha)h(\tilde{p}^{+}) > 0,$$

a contradiction. One concludes that there cannot exist such a price $\tilde{p}$ below which p_n is chosen and above which p_o is chosen. Since there are no mass points below the upper bound of the union of the supports, the only possibility in equilibrium is that the support of p_o lies below and is adjacent to the support of p_n. □

So, for any (Δ, α) such that $\alpha\frac{(l+s)}{l}sv < \delta\Delta < sv$, any equilibrium has the following form:

- the H-firm draws q according to the ddf $H(.)$ with support $[\underline{a}, v]$ and possibly with mass point at v;
- the L-firm draws p_o and p_n according to $L_o(.)$ on $[\underline{a}, \hat{a}]$ and $L_n(.)$ on $[\hat{a}, v]$ with possible mass points m_o and m_n at v;
- only one firm can have a mass point at v.

Claim 5. *For any (Δ, α) such that $\alpha\frac{(l+s)}{l}sv < \delta\Delta < sv$, there cannot exist an equilibrium such that there is a mass point for the H-firm at $q = v$.*

The presence of the bonus Δ does not change the strategy of the proof of Claim 5, compared to Narasimhan (1988); hence we omit it. The next claim proves that the equilibrium is therefore such that the L-firm has a mass point for $p_n = v$, but no mass for p_o (that is, $m_o = 0$ and $m_n > 0$).

Claim 6. *For any (Δ, α) such that $\alpha\frac{(l+s)}{l}sv < \delta\Delta < sv$, an equilibrium of the static game exists, all equilibria are such that $m_n > 0$ and $m_o = 0$, and they are all equivalent in terms of intervals of randomization and of marginal distributions of prices.*

Proof. Consider an equilibrium configuration with possible mass points $m_o > 0$ and m_n at v. The following system, with unknown variables $(\underline{a}, \hat{a}, \hat{H})$,

$$\begin{aligned} \underline{a}(l+s) &= \hat{a}(l+\alpha s+(1-\alpha)s\hat{H}) = v(l+\alpha s) \\ \hat{a}(l+s\hat{H}) &= vl+\delta\Delta\hat{H} \end{aligned}$$

implies that either $\hat{H} = \frac{\alpha sv(s+l)-\delta\Delta(l+\alpha s)}{(1-\alpha)s\delta\Delta}$ or $\hat{H} = 0$. The assumption that $\alpha\frac{(l+s)}{l}sv < \delta\Delta$ immediately rules out the first case, as it would lead to a negative value for $\hat{H}$. The second case corresponds to an equilibrium configuration in which $p_n = v$ with probability 1 while p_o is distributed on $[\frac{(l+\alpha s)}{l+s}v, v]$, with possible mass m_o at v. Then, the indifference condition for the H-firm is: for any $q \in (\frac{(l+\alpha s)}{l+s}v, v)$

$$q(l+s+(1-\alpha)sL_o(q)) = (l+(2-\alpha)s)\frac{(l+\alpha s)}{l+s}v.$$

Letting q tend to v from below, the LHS goes to: $v(l+s+(1-\alpha)sm_o)$ which has to be larger than $v(l+s)$ for $m_o > 0$. The RHS, however, is equal to $v(l+s) - v\frac{(1-\alpha)^2s^2}{l+s}$, hence smaller than $v(l+s)$. This contradiction rules out this equilibrium configuration.

We are left with one unique possible equilibrium configuration: there is no mass point for p_o and, possibly, a mass point m_n for p_n at v . The following system, with unknown variables $(\underline{a}, \hat{a}, \hat{H})$,

$$\begin{aligned}
\underline{a}(l+s) &= \hat{a}(l+\alpha s+(1-\alpha)s\hat{H}) \\
\hat{a}(l+s\hat{H}) &= vl+\delta\Delta\hat{H} \\
\underline{a}(l+(2-\alpha)s) &= \hat{a}(l+s)
\end{aligned}$$

has a unique solution given by:

$$\begin{aligned}
\hat{H} &= \frac{(1-\alpha)s}{l+(2-\alpha)s} \\
\hat{a}(\Delta) &= \frac{vl+\delta\Delta\hat{H}}{l+s\hat{H}} \\
\underline{a}(\Delta) &= \frac{l+s}{l+(2-\alpha)s}\cdot\frac{vl+\delta\Delta\hat{H}}{l+s\hat{H}}.
\end{aligned}$$

One can easily check that $\hat{H} \in (0,1)$ and $\hat{a}(\Delta) > \underline{a}(\Delta) > 0$. Moreover, $\hat{a}(\Delta) < v \Leftrightarrow \delta\Delta <$

sv. The overall price distributions are given by:

$$\begin{aligned} p(l+\alpha s+(1-\alpha)sH(p)) &= \underline{a}(l+s) \text{ for } p\in(\underline{a},\hat{a}) \\ p(l+sH(p))+\delta\Delta(1-H(p)) &= vl+\delta\Delta \text{ for } p\in(\hat{a},v) \\ p(l+s+(1-\alpha)sL_o(p)) &= \underline{a}(l+(2-\alpha)s) \text{ for } p\in(\underline{a},\hat{a}) \\ p(l+sL_n(p))+\delta\Delta(1-L_n(p)) &= \hat{a}(l+s) \text{ for } p\in(\hat{a},v). \end{aligned}$$

It is immediate to check that these equalities define ddf, that $\lim_{p\to x} H(p)=0$, $\lim_{p\to\hat{a}} L_o(p)=0$ and $\lim_{p\to v} L_n(p)=\frac{l}{l+(1-\alpha)s}=m_n>0$, which corresponds to the mass at v.

We finally prove that any pair of mixed strategies that satisfy the above constitutes an equilibrium. For that, remark first that any price deviation by any firm below $\underline{a}$ is obviously dominated.[20] Second, given $H(p)=\frac{\underline{a}(l+s)-(l+\alpha s)p}{(1-\alpha)sp}$ on $(\underline{a},\hat{a})$, a deviation by the L-firm to a price $p_n\in(\underline{a},\hat{a})$ yields the following profit on its new customers:

$$p_n(l+sH(p_n))+\delta\Delta(1-H(p_n))=\frac{(l+s)}{(1-\alpha)}(\frac{\delta\Delta}{s}+\underline{a})-\frac{\alpha(l+s)}{(1-\alpha)}p_n-\delta\Delta\underline{a}\frac{(l+s)}{(1-\alpha)sp_n}.$$

Over $(\underline{a},\hat{a})$, the derivative of this profit in p_n is equal to $\frac{(l+s)}{(1-\alpha)s}[\delta\Delta\frac{\underline{a}}{p_n^2}-\alpha s]$. The bracketed term is larger than its value for $p_n=\hat{a}$ and using $\delta\Delta>\frac{(l+s)}{l}\alpha sv$, it is a simple matter of tedious calculation to prove that the bracketed term is positive. Therefore, charging $p_n=\hat{a}$ weakly dominates a price deviation to $p_n\in(\underline{a},\hat{a})$ and such a deviation is not profitable.

Third, given $H(p)=\frac{(v-p)l}{sp-\delta\Delta}$ on $(\hat{a},v)$, a deviation by the L-firm to a price $p_o\in(\hat{a},v)$ yields the following profit on its old customers:

$$p_o(l+\alpha s+(1-\alpha)sH(p_o))=p_o(l+\alpha s)+(1-\alpha)sp_o\frac{l(v-p_o)}{sp_o-\delta\Delta}.$$

Over $(\hat{a},v)$, the derivative of this profit in p_o is equal to $\alpha(l+s)-(1-\alpha)l\frac{\delta\Delta(sv-\delta\Delta)}{(sp_o-\delta\Delta)^2}$. It is smaller than its value for $p_o=v$ and it is again a simple matter of calculation to show that it

[20] We have a priori restricted prices to be not larger than v. It is possible to relax this restriction and prove that firms will not charge above v. This requires to specify the strategies in continuation subgames after a deviation above v: this can be done as a special case of the proof of Proposition **??**. The construction, however, is rather involved and in the current proof, we have chosen the a priori restriction in order to facilitate the reading.

is actually negative for $p_o = v$. Therefore, charging $p_o = \hat{a}$ weakly dominates a price deviation to $p_o \in (\hat{a}, v)$ and such a deviation is not profitable.

Finally, given $L_o(p) = \frac{\underline{a}(l+(2-\alpha)s)-(l+s)p}{(1-\alpha)sp}$ on $(\underline{a}, \hat{a})$ and $L_n(p) = \frac{\hat{a}(l+s)-\delta\Delta-lp}{ps-\delta\Delta}$ on $(\hat{a}, v)$, the only deviation to consider for the H-firm is to $q = v$. Comparing the gain between $q = v^-$ and $q = v$, the additional profit for the H-firm would be $-[vs - \delta\Delta]\frac{m_n}{2} < 0$, which invalidates this deviation. $\square$

It is now possible to find out that, for any (Δ, α) such that $\alpha\frac{(l+s)}{l}sv < \delta\Delta < sv$, in equilibrium,

$$\begin{aligned} U_H &= vl + \hat{a}(l+s), \\ U_L &= \underline{a}(l+s) + vl + \delta\Delta, \end{aligned}$$

so that:

$$U_H - U_L = (l+s)(\hat{a} - \underline{a}) - \delta\Delta.$$

Plugging in the values of $\underline{a}$ and $\hat{a}$, we obtain:

$$U_H - U_L \equiv T_\alpha(\Delta) = \frac{(1-\alpha)slv}{l+(1-\alpha)s} - \delta\Delta\frac{[(l+(1-\alpha)s)(l+(2-\alpha)s) - (1-\alpha)^2s^2]}{(l+(1-\alpha)s)(l+(2-\alpha)s)}.$$

For later use, we gather in the next claim the properties of the mapping $T_{\alpha(.)}$.

Claim 7. *There exists $\bar{\alpha}$ such that for any $\alpha < \bar{\alpha}$, the following holds:*

$$\alpha\frac{(l+s)}{l}sv < \delta\Delta < \frac{l}{l+s}\delta sv \Rightarrow \alpha\frac{(l+s)}{l}sv < \delta T_\alpha(\Delta) < \frac{l}{l+s}\delta sv,$$

and there exists one unique fixed-point Δ^*_α of $T_\alpha(.)$ within the interval $(\frac{\alpha(l+s)sv}{l}, \frac{\delta lsv}{l+s})$ and it is stable.

Proof. It is a simple matter of computation to prove that the mapping $T_\alpha(.)$ is affine and strictly decreasing with slope $-\frac{(l^2+(3-2\alpha)s+(1-\alpha)s^2)}{(l+(1-\alpha)s)(l+(2-\alpha)s)} \in (-1, 0)$. The intercept is a decreasing function of α and the absolute value of the slope is an increasing function of α, so that $T_\alpha(\Delta)$ decreases in α for a given Δ within the admissible range.

Assume that $\alpha < \frac{l^2\delta}{(l+s)^2}$. Since $T_\alpha(.)$ is decreasing, for any Δ such that $0 < \frac{\alpha(l+s)sv}{l} < \delta\Delta < \frac{l\delta sv}{l+s}$ we have:

$$\delta T_\alpha(\frac{lsv}{l+s}) < \delta T_\alpha(\Delta) < \delta T_\alpha(0).$$

Note that $\delta T_\alpha(0) = \delta\frac{(1-\alpha)slv}{l+(1-\alpha)s} \leq \frac{l}{l+s}\delta sv$ for any α. Also, $\delta T_\alpha(\frac{lsv}{l+s})$ is a decreasing function of α that takes strictly positive value for α small enough by continuity, since $T_0(\frac{lsv}{l+s}) > 0$. As $\alpha\frac{(l+s)sv}{l}$ is strictly increasing in α, equal to 0 when $\alpha = 0$, there exists a critical value of α such that, for any α smaller than this critical value, $\alpha\frac{(l+s)}{l}sv < \delta T_\alpha(\Delta)$. Hence, for any Δ such that $\frac{\alpha(l+s)sv}{l} < \delta\Delta < \frac{l\delta sv}{l+s}$,

$$\frac{\alpha(l+s)sv}{l} < \delta T_\alpha(\Delta) < \frac{l\delta sv}{l+s}.$$

So, when habit formation is not strong, one can define the decreasing mapping $T_\alpha(.)$ from one interval into itself; hence the existence, uniqueness and stability of a fixed point in this interval. □

B Dynamic models with myopic consumers.

B.1 The two-period model

Consider first the two-period model presented in the text. The characterization of the equilibrium strategies in period 2 follows the same approach as in Appendix A, with $\Delta = 0$. Claims 1-3 hold. Claim 4 is modified as p_n takes values smaller than the values taken by p_o. This is where the absence of a bonus kicks in. Claim 5 is still valid and Claim 6 now implies that the distribution of p_o can have a mass point at v. The equilibrium conditions can be written as follows: for any $p_o \in (\hat{a}, v)$, $p_n \in (\underline{a}, \hat{a})$ and $q \in (\underline{a}, v)$

$$p_o(l + \alpha s + (1-\alpha)sH(p_o)) = vl = \hat{a}(l + \alpha s + (1-\alpha)H(\widehat{a}))$$

$$p_n(l + sH(p_n)) = \underline{a}(l+s) = \hat{a}(l + sH(\widehat{a}))$$

$$q(1-\alpha)sL_o(q) + q(l + sL_n(q)) = \underline{a}(l + (2-\alpha)s) = \hat{a}(1 + (1-\alpha)s).$$

From these, one can easily deduce the expression of the endogenous bonus:

$$\Delta_1(\alpha) = \frac{s}{(l+s)^2}[(1-2\alpha)l^2 + (1-3\alpha)sl - \alpha^2(2-\alpha)s^2].$$

It is immediate that $\Delta_1(\alpha)$ is decreasing in α and strictly positive for $\alpha = 0$. So, there exists α_1 such that $\Delta_1(\alpha)$ is positive, decreasing in α from $\frac{l}{l+s}sv$ to 0 when α goes from 0 to α_1. Therefore, there exists $\alpha_2 > 0$ such that for any $\alpha < \alpha_2$,

$$\alpha\frac{(l+s)}{l}sv < \delta\Delta_1(\alpha) < \frac{l}{l+s}\delta sv.$$

Let then $\alpha^* \equiv \inf\{\alpha_2, \bar{\alpha}\}$. For any $\alpha < \alpha^*$, Claim 7 in Appendix A applies: there exists basically a unique equilibrium outcome and it is characterized by equilibrium valuations U_{2H} and U_{2L}, viewed from the beginning of the two-period game, such that: $U_{2H} - U_{2L} \equiv \Delta_2(\alpha) = T_\alpha(\Delta_1(\alpha))$.

B.2 The T-period model

Consider now the T-repeated version of the game. From the analysis in two periods, when $\alpha < \alpha^*$ the series of bonus computed by iterating the mapping $T_\alpha(.)$ remains within the appropriate interval so that one can apply Appendix A recursively from the last period on. Therefore, with T periods, there also exists a basically unique equilibrium outcome and the difference between the discounted intertemporal profits of the initial H-firm and the initial L-firm is given:

$$\Delta_N(\alpha) = T_\alpha(\Delta_{N-1}(\alpha)) = T_\alpha^{(N-1)}(\Delta_1(\alpha)).$$

As N goes to infinity, for $\alpha < \alpha^*$, the relative value of holding the initial H-firm's position tends towards Δ_α^*, the unique fixed point characterized in Claim 7, in equilibrium.

B.3 The infinite horizon OLG model

Finally, in the infinite OLG model, for $\alpha < \alpha^*$, consider the (stationary) strategies consisting in the repetition of the static equilibrium strategies for $\Delta = \Delta_\alpha^*$. Note that the fixed-point equation yields:

$$\Delta^*_\alpha[1+\delta\frac{(1-\alpha)^2s^2}{(l+(1-\alpha)s)(l+(2-\alpha)s)}]=\frac{(1-\alpha)slv}{l+(1-\alpha)s}.$$

By construction, for any $\alpha < \alpha^*$, $(\alpha, \Delta^*_\alpha)$ satisfy the conditions of Appendix A. Therefore, these strategies trivially constitute a Markov-perfect equilibrium of the infinite horizon game. Moreover, this Markov perfect equilibrium is the limit of any sequence of subgame perfect equilibrium in approximating finite horizon games in which the last period has no newborn generation and the intertemporal equilibrium valuations satisfy: $V_H - V_L = \Delta^*_\alpha$.

B.4 The comparison with Villas-Boas (1999)

We here sketch the proof of our statement regarding the two-period version of Villas-Boas (1999). Let denote $\hat{x}$ the cutoff point on the line for the segment of young consumers of generation 1 who bought from the L-firm in period 1. Following Villas-Boas (1999) and Fudenberg and Tirole (2000), it is immediate that when $\hat{x} \in [3/7, 4/7]$, the equilibrium in period 2 is such that:

- Firm L charges its past (resp. new) customers a price $p^o_{L,2} = \frac{3+\hat{x}}{4}$ (resp. $p^n_{L,2} = \frac{2-\hat{x}}{2}$).
- Firm H charges its past (resp. new) customers a price $p^o_{H,2} = \frac{4-\hat{x}}{4}$ (resp. $p^n_{H,2} = \frac{1+\hat{x}}{2}$).

Second-period profits for both firms are given by $\pi_{L,2} = \frac{1}{32}(33+17\hat{x}^2-18\hat{x})$ which is decreasing in $\hat{x}$, and $\pi_{H,2} = \frac{1}{32}(22+17\hat{x}^2+4\hat{x})$ which is increasing in $\hat{x}$. Hence the first statement in the text: the second period profits of a firm are decreasing in its first period market share on the new generation of consumers arriving in period 1.

In period 1, the consumer $\tilde{x}$ from the installed base of firm L who is indifferent between buying to firm L or H is given by $\tilde{x} = \frac{1}{2} + \frac{1}{2}(p^n_{H,1} - p^o_{L,1})$. The indifferent consumer $\hat{x}$ from the new generation is given by: $\hat{x} = \frac{1}{2} + \frac{1}{2}(p^n_{H,1} - p^n_{L,1})$. Consequently intertemporal firms profits are given by:

$$\Pi_L(p^n_{L,1}, p^o_{L,1}, p^n_{H,1}) = p^n_{L,1}\hat{x} + p^o_{L,1}\tilde{x} + \delta\pi_{L,2}$$
$$\Pi_H(p^n_{L,1}, p^o_{L,1}, p^n_{H,1}) = p^n_{H,1}(1-\hat{x}) + p^n_{H,1}(1-\tilde{x}) + \delta\pi_{H,2}$$

Solving the equilibrium yields: $p^n_{L,1} = \frac{2816+(312-77\delta)\delta}{2560-336\delta}$, $p^o_{L,1} = \frac{5632-\delta(200+77\delta)}{5120-672\delta}$, and $p^n_{H,1} = \frac{3072+(136-77\delta)\delta}{2560-336\delta}$. It is easy to check that $\hat{x}$ in indeed in [3/7, 4/7]. More importantly, $p^n_{L,1} > p^o_{L,1}$. The L-firm uses a pay-to-stay strategy, as stated in the text.