Information, contracts and competition - Applications to financial markets

Chapter II
Principal Agent Models with 1 agent and 1 Principal

This part of the course introduce information asymmetries and presents a review of different classical frameworks for contracts, first insights in the Principal - Agent model.

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From the market to game theory

Game theory is a modeling approach which drops perfect competitions's assumption that individuals are price-takers and instead requires them to behave strategically, taking into account that their actions will alter the behaviour of the rest of the market.

It is crucial to delineate carrefully the order of actions and the information available to each player : precommitment and information transmissions are the two pilars of modern game theory.

Game Theory in Finance, *The new Palgrave Dictionary of Money and Finance*

The Principal-Agent model

A Principal delegates a task to one or many agents. Principal and agents have conflicting objectives. Agents and Principal interact in the sense that their payoffs depends on all players actions.

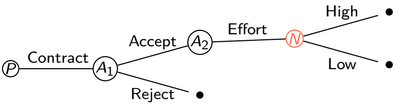
- Moreover agents decisions could not be entirely controled by the Principal, or, they hold some private information.

- Principal can ask about information, and, after the analysis of the different informations that he gathered, he could send messages to the agents, depending on which they will choose their actions.

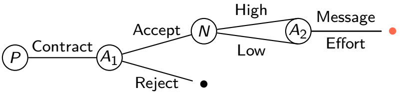
Principal-Agent models : different sequences

The uninformed player proposes a contract to the informed player and natures plays.

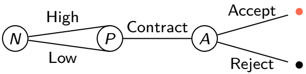
(a) moral hazard with hidden action



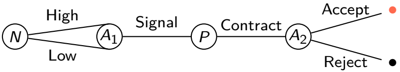
(b) post-contractual hidden knowledge



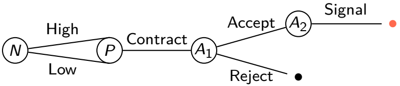
(c) adverse selection



(d) signalling



(e) screening



Typical example

A problem we will consider in detail arises when an employer (the principal) hires a worker (the agent). If the employer knows the worker's ability but not his effort level, the problem is moral hazard with hidden actions. If neither player knows the worker's ability at first, but the worker discovers it once he starts working, the problem is moral hazard with hidden knowledge. If the worker knows his ability from the start, but the employer does not, the problem is adverse selection. If, in addition to the worker knowing his ability from the start he can acquire credentials before he makes a contract with the employer, the problem is signalling. If the worker acquires his credentials in response to a wage offer made by the employer, the problem is screening.

Many Applications of the Principal agent Model

	Principal	Agent	Effort, or type and signal
Moral hazard with hidden actions	Insurance company	Policyholder	Care to avoid theft
	Insurance company	Policyholder	Drinking and smoking
	Plantation owner	Sharecropper	Farming effort
	Bondholders	Stockholders	Riskiness of corporate projects
	Tenant	Landlord	Upkeep of the building
	Landlord	Tenant	Upkeep of the building
	Society	Criminal	Number of robberies
Moral hazard with hidden knowledge	Shareholders	Company president	Investment decision
	FDIC	Bank	Safety of loans
Adverse selection	Insurance company	Policyholder	Infection with HIV virus
	Employer	Worker	Skill
Signalling and screening	Employer	Worker	Skill and education
	Buyer	Seller	Durability and warranty
	Investor	Stock issuer	Stock value and % retained

Roadmap

0) Introduction

1) An particular exemple of Moral Hazard in Finance

2) Categories of Asymmetric information models

   1. Moral Hazard
   2. Adverse selection
   3. Post-contractual hidden knowledge
   4. Signalling and Screening

1. A model and a question in Finance

(i) Moral Hazard ex post (costly state verification)

(ii) Incentive design : why use Financial intermediaries

Costly State Verification

Townsend, R.M., 1979. Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory* 21(2), 265–293.

Moral hazard in the borrowing-lending relationship

We are interested in a lending relationship, typically, an entrepreneurs borrows I for a project, which payoff $\tilde{Y}$ is a distribution between $\underline{Y}$ and $\overline{Y}$. In that case, it sounds natural that the reimbursement scheme could depend upon the states of nature. A contract is then a function : $Y \mapsto R(Y)$, when Y is *verifiable* :

$$I \left\langle \begin{array}{lcl} \overline{Y} & \rightarrow & R(\overline{Y}) \\ \vdots & & \\ \underline{Y} & \rightarrow & R(\underline{Y}) \end{array} \right.$$

However, in the case Y is not verifiable, or costly verifiable, then, the contract should be redefined, and particularly the optimal contract. This is what we will study.

Optimal contract when $c = 0$ and risk neutral investor

As a starting point we study optimal contracting when Y is verifiable, and the investor risk neutral

- A Pareto optimal allocation is a solution of :

$$\begin{array}{rl} \max_{R(Y)} & E(U(Y - R(Y)) \\ \text{s.c.} & -I + E(R(Y)) \geq \underline{\Pi} \end{array}$$

- We look for the optimal R for each value of Y
- Lagrangean is $L = E(U(Y - R(Y)) + \lambda(E(R(Y)) - I)$
  derivative relative to $R \; \forall Y$: $U'(Y - R(Y)) - 1 = \lambda$
  $Y - R(Y)$ is constant : Risk goes to investors
  This contract could be interpreted as a stock sale.

- You can show that, at the contrary, we would have a debt contract when the entrepreneur would have been risk neutral and the investor risk averse.

Optimal contract with *Costly state verification*

Let suppose that Y is not verifiable, unless by paying the verification cost c. To share the risk among lender and borrower we define a mechanism that precises when the borrower will be audited, and a penalty function.

A contract has three components :

- A reimbursement function $\hat{Y} \mapsto R(\hat{Y})$ where $\hat{Y}$ is the *declared* income
- An audit rule that can be formalized by S, a subset of $\mathcal{Y}$, the set of all possible messages of the borrower, for which there will be an audit.
- A penalty function $P(Y, \hat{Y})$ which defines the additional transfer depending on the result Y of the audit. Clearly $P(Y, \hat{Y}) = 0$ when there is no false declaration.

Mécanisme direct révélateur

- *Revelation Principle* tells that there is no loss of generality if we consider direct mechanism, when looking for Pareto optimal allocations.

Définition

A mechanism $(R(\cdot), S, P(\cdot, \cdot))$ e direct when it induces agents to make truthful declarations

three properties of the mechanism

- There exists a truthful mechanism
- $R(\cdot)$ should be constant outside of S ;
- the constant R could not be less the the maximum reimbursement on the audit zone S.

three properties of the mechanism

- There exists a truthful mechanism. [The penalty could be such important that the borrower could say the truth, at least in the audit zone.]
- $R(\cdot)$ should be constant outside of S ; we denote best R this constant [by contradiction : if not, the borrower would never declare the income level corresponding to a high level of reimbursement]
- the constant R could not be less the the maximum reimbursement on the audit zone S. [by contradiction : if not, the borrower will ask for R in case an audition would imply a big reimbursement.]

The the second best revelating contract $(R(\cdot), S, +\infty)$ satisfies :

$$\begin{array}{lcll} \text{if } Y \in S & : & R(Y) \leq R & (\hat{Y} = Y \in S) \\ \text{if} Y \notin S & : & R(Y) = R & (\hat{Y} = Y \notin S) \end{array}$$

Optimal Contrat in the risk neutral case

Program

$$\begin{aligned} \max_{R(Y)} \quad & E(Y - R(Y)) \\ \text{s.c.} \quad & -I + E(R(Y)) - C(S) \geq \underline{\Pi} \\ & R(Y) \leq Y \end{aligned}$$

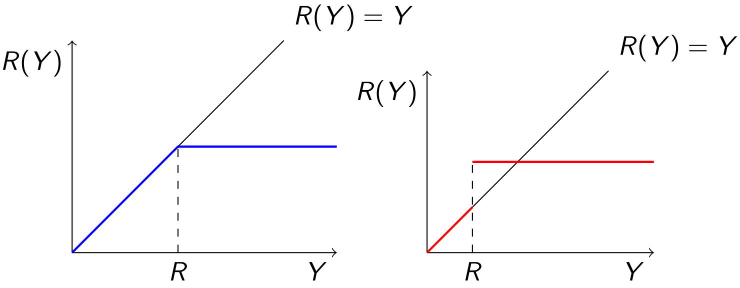
To make it easy, let think that Y is uniformly distributed of $[0, \overline{Y}]$. Then,

- Binding constraint 1 : *If not it could be possible to reduce the reimbursement.*
- The program is then to minimize the audit cost, i.e., $\min_S \; m(S)$ with the constraint $E(R(Y)) \geq C(S) + I + \underline{\Pi}$
  Then, an efficient way to get $E(R(Y))$ as big as possible given S is to choose $R(Y) = Y$ in the audit zone.
  Then, given $m(S)$ and $R(Y) = Y$ when $y \in S$, choose the lowest Y possible, that is : $S = [0, m(S)]$.

  in other words, Efficient contracts are then obtained by minimizing the probability of an audit for a fixed expected repayment, or equivalently, by maximizing the expected repayment for a fixed probability of an audit.

Optimal Contrat in the risk neutral case

we obtain the following mechanism, depending on its continuity :



- that is, the efficient incentive-compatible mechanism is a *debt contract*. The agent will be audited when he cannot repay R, and the bank will then take it all.

- A central result of CSV approach is that it is generally optimal to commit to a partial, state-contingent disclosure rule.

Incentive design : why use Financial intermediaries
Financial intermediaries as Delegated monitoring

Diamond, D. W. 1984. Financial intermediation and delegated monitoring. Review of Economic Studies 51 (3) : 393–414.

Financial intermediaries

The modern theory of financial intermediation analyzes, mainly, the functions of financial intermediation, the way in which the financial intermediation influences the economy on the whole and the effects of government policies on the financial intermediaries.

Main explanation starting from imperfections of the financial market : high cost of transaction, lack of complete information in useful time generating deviations from the theory of perfect markets in an Arrow -Debreu sense.

Following Diamond, the purpose of a financial intermediary is to eliminate redundancy by replacing decentralised monitoring by a single central monitor. Monitoring typically involves increasing returns to scale, which implies that it is more efficiently performed by specialized firms.

Financial intermediation as delegated monitoring

M risk-neutral investors wish to finance N risk-neutral entrepreneurs. Capital $I = 1$ and Y is not observed by the investors.

Investors must rely either on monitoring or on an incentive contract :

- Under monitoring each investor pays K to observe Y, which makes it a contractible variable, on which the repayment can be made contingent.
- Under the incentive contract, the entrepreneur suffers a dissipative punishment $\phi(z)$ if he repays z.

In the absence of an intermediary the incentive contract is prefered if $E\phi < MK$

The use of this model is to show that (a) an intermediary helps on ly if there are both many investors and many entrepreneurs, and (b) incentive contracts have economies of scale compared to monitoring. The institutions takes its particular form to avoid information problems by contracting while information is still symmetric.

2. Categories of Asymmetric information models

Moral Hazard - Adverse selection - Post-contractual hidden knowledge - Signalling and Screening

The production Game

Eric Rasmusen, *Games and Information*, 4th edition (2006)

The production game

The principal is a manager and the agent a worker. Denote the monetary value of output by $q(e)$, which is increasing in effort, e. The agent's utility function $U(e, w)$ is decreasing in effort and increasing in the wage, w, while the principal's utility $V(q - w)$ is increasing in the difference between output and the wage.

Timing

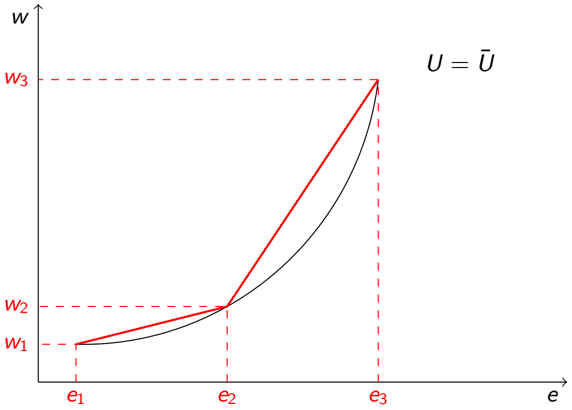
1. The principal offers the agent a wage w.
2. The agent decides whether to accept or reject the contract.
3. If the agent accepts, he exerts effort e.
4. Output equals $q(e)$, where $q' > 0$.

Payoffs

- If the agent rejects the contract, then $\pi_{\text{agent}} = \bar{U}$ and $\pi_{\text{principal}} = 0$.
- If the agent accepts the contract, then $\pi_{\text{agent}} = U(e, w)$ and $\pi_{\text{principal}} = V(q - w)$.

Production Game I : Some details

$U(e, w)$ is a convex Utility function, in particular the set $\{(e, w)/U(e, w) \geq \bar{U}\}$ is convex. The more you increase the effort, the more you pay to compensate the agent : the mean variation of w increases with e as in the figure :



Production Game I : Full information

Assumption : Every move is common knowledge, **w** and **e** are contractible.

Principal 's program : $\max_{e,w} V(q(e) - w) \quad s.c. \quad U(e,w) \geq \bar{U}$.

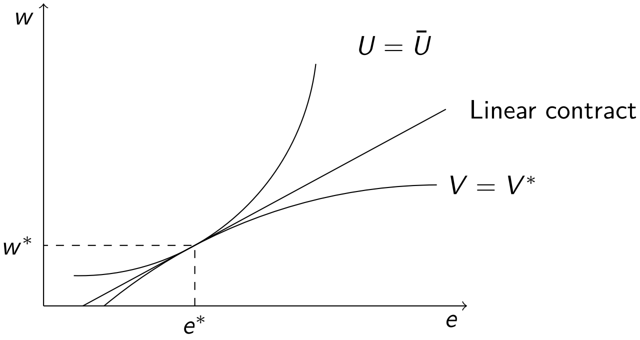
FOC : $U_w q_e = -U_e$

At the optimal effort, the marginal utility to the agent which would result if he kept all the marginal output from extra effort equals the marginal disutility to him of that effort.

or, the productivity of effort in terms of income equal the MRS of effort in terms of income, $q_e = -U_e/U_w$

Production Game I : Full information

At least two contracts, are equally effective under full information.
1 The **forcing contract** sets $w(e^*) = w^*$ and $w(e \neq e^*) = 0$. This is certainly a strong incentive for the agent to choose exactly $e = e^*$.
2 The **linear contract**, shown in the Figure sets $w(e) = \alpha + \beta e$, where α and β are chosen so that $w^* = \alpha + \beta e^*$ and the contract line is tangent to the indifference curve $U = \bar{U}$ at e^*.



Production Game III with constrained flat wages and certainty

In this version of the game, the principal can condition the wage neither on effort nor on output. This is modelled as a principal who observes neither effort nor output, so information is asymmetric. More precisely, he cannot base wages on output, however, because a contract must be enforceable by some third party such as a court.

Assumption : q and e is not contractible, Contract is only w constant

Principal 's program : $\max_{w} V(q(e) - w) \quad s.c. \quad e = argmax_e U(e, w).$

The solution is trivial $e = 0$, $w = 0$. More precisely, the Principal anticipates that the agent will choose $e = 0$ whatever be the value of w so that w 's choice has no consequence on the agent effort 's choice, and then $w = 0$ is the optimal choice of the principal.

▶ If there is nothing on which to condition the wage, however, the agency problem cannot be solved by designing the contract carefully.

Production Game IV : An Output-Based Wage under Certainty

In this version, the principal cannot observe effort but he can observe output and specify the contract to be $w(q)$. Notice that such a contract $w(q)$ renders it possible to contract on a variable correlated to e, here $q = q(e)$.

Assumption : e is not contractible, q is contractible, $q = q(e)$; Contract is $w(q)$

Principal 's program : $\max_{w(\cdot)} V(q - w(q))\ s.c.\ \ q = q(e)$ with $e = argmax_e U(e, w(q(e))$.

▶ Let q^* and e^* denote the first best (program I) ; then, the following mechanism, such that $w(q^*)$, as in the model I, is defined by solving $U(e(q^*, w(q^*)) = \bar{U}$ and also $w(q)$, for $q \neq q^*$, such that $U(e(q), w(q)) < \bar{U}$ gives the agent the proper incentives.

Production Game IV shows that the unobservability of effort is not a problem in itself, if the contract can be conditioned on something which is observable and perfectly correlated with effort

That is what we call certainty.

- The general assumption of Moral Hazard is that one cannot distinguish the effort level from the output. There are in fact two relevant models, that have been deeply studied in the litterature

$$\text{Output} = \text{Effort} + \text{Alea}$$

$$\text{Output} = \text{Effort} + \text{Hidden Variable}$$

Two reference models :

- Grossman, Sanford and Hart, Oliver, (1983), An Analysis of the Principal-Agent Problem, *Econometrica*, 51, issue 1, p. 7-45.

- Laffont, Jean-Jacques, and Jean Tirole. "Using Cost Observation to Regulate Firms." Journal of Political Economy, vol. 94, no. 3, 1986, pp. 614–641.

Production Game V : An Output-Based Wage under Uncertainty

Assumption : e, not contractible, q, contractible ; $q = q(e, s)$, $s \in S$; Contract $w(q)$

Principal 's program :
$$\begin{array}{rl} \max_{w(\cdot)} & EV(q - w(q)) \\ s.c. & q = Eq(e, s) \\ & e = argmax_e EU(e, w(q(e, s))) \geq \bar{U} \end{array}.$$

Two main ingredients :

- the Principal cannot control the agent 's effort
- it should satisfies the agent 's reservation constraint

$\Rightarrow$ The combination of unobservable effort and lack of invertibility in Production Game V means that no contract can induce the agent to put forth the efficient effort level without incurring extra costs, which usually take the form of extra risk imposed on the agent.

Compared with the situation without this informational constraint, the contract is inefiicient. However, we will still try to find a contract that is efficient in the sense of maximizing welfare given the informational constraints.

Definition

A *first-best* contract achieves the same allocation as the contract that is optimal when the principal and the agent have the same information set and all variables are contractible.
A *second-best* contract is Pareto optimal given information asymmetry and constraints on writing contracts.

- We define the cost of the agency problem to be the difference in welfare between the first best and the second best

Participation constraint :$EU(\tilde{e}, w(q(e,s))) \geq \bar{U}$
This constraint is classical, and should be computed, for any e candidate as optimal second best effort.

IC constraint :$\tilde{e} = argmax_e EU(e, w(q(e,s)))$
This is the specific constraint or moral hazard, more complex to deal with.

- We usually say that the second contraint introduces non convexities.

More precisely, we cannot apply the first- order condition approach to solve that problem, which would be, to write the FOC conditions resulting from IC constraint and to plug it in the Principal program. Indeed, as the constraint are usually non convex, usual standard programming sufficient conditions do not apply, since they suppose convexity.

Solving the MH Game in three steps

$$\begin{array}{rl} \max_{w(\cdot)} & V(q - w(q)) \\ s.c. & q = Eq(e,s) \\ & e = argmax_e EU(e, w(q(e,s))) \geq \bar{U} \end{array}$$

The classical way to analyze such a principal problem is in 3 steps :

1. find for each possible effort level the set of wage contracts that induce the agent to choose that effort level ;
2. find the contract which supports that effort level at the lowest cost to the principal ;
3. choose then the effort level that maximizes profits,

Production Game VI : Adverse Selection

Under adverse selection, the agent has private information about his type or the state of the world before he agrees to a contract, which means that the emphasis is on which contract he will accept

Timing

0. Nature chooses the agent's ability θ, observed by the agent but not by the principal, according to distribution F(θ).
1. The principal offers the agent one or more wage contracts w1(q), w2(q),. . .
2. The agent accepts one contract or rejects them all.
3. Nature chooses a value for the state of the world, s, according to distribution G(s). Output is then q = q(θ, s).

Payoffs

- If the agent rejects the contract, then $\pi_{\text{agent}} = \bar{U}(\theta)$ which might or might not vary with his type, θ ; and $\pi_{\text{principal}} = 0$.
- Otherwise If the agent accepts the contract, then $\pi_{\text{agent}} = U(w, \theta)$ and $\pi_{\text{principal}} = EV(q(\theta, s) - w)$.

Assumption : θ is not contractible, as it is unknown from the Principal. Non linear wage $w(q, \theta)$

Principal 's program :

$$\begin{array}{rl} \max_{w1(\cdot),w2(\cdot)} & EV\left(q(s,\theta) - w(q,\theta)\right) \\ s.c. & \text{Participation of } \textit{each} \text{ type} \\ & \text{Revelation constraints of } \textit{each} \text{ type} \end{array}.$$

Two main ingredients :

- the Principal should induce the agent to participate
- the Principal should induce the agent 's to choose the right contract

Exemple : Public monopoly regulation

Baron and Myerson [Baron, D., Myerson, R.B., 1982. Regulating a monopolist with unknown costs. *Econometrica* 50, 911–930]

Exemple : Public monopoly regulation

- A principal wants the agent to produce q. The principal gross benefit is q. The cost incured by a type θ agent is $\frac{1}{2\theta}q^2$.

Symmetric information case

- the monopolist gives the *transfer* $t = \frac{1}{2\theta}q^2$ to the agent and receives $\pi = q - \frac{1}{2\theta}q^2$
- the principal ask to produce $q^* = \theta$ in exchange of $t^* = \dfrac{\theta}{2}$ profit : $\pi^* = \dfrac{\theta}{2}$
- In this model, the principal designs the agent' s game .

Asymmetric information case

- Suppose that θ is unknown to the principal but not the distribution $\theta \in \{\overline{\theta} > \underline{\theta}\}$, with equiprobability
- Mechanism : $\{(\overline{q}, \overline{t}), (\underline{q}, \underline{t})\}$
  - Impossibilité de donner le first best : calculer le mécanisme optimal, tel que l'agent choisit le contrat lui étant destiné qui maximise le profit.
  - vérifier qu'il donne plus à l'agent le plus productif.

Mecanism design (1) de l'exemple : les contraintes

- *les contraintes de révélation sur les contrats* $(\overline{q}, \overline{t})$ *et* $(\underline{q}, \underline{t})$

chaque type choisit le contrat lui étant destiné si

$$\overline{t} - \frac{1}{2\overline{\theta}}\overline{q}^2 \geq \underline{t} - \frac{1}{2\overline{\theta}}\underline{q}^2 \qquad (R\overline{\theta})$$

$$\overline{t} - \frac{1}{2\underline{\theta}}\overline{q}^2 \leq \underline{t} - \frac{1}{2\underline{\theta}}\underline{q}^2 \qquad (R\underline{\theta})$$

- *les contraintes de participation sur les contrats* $(\overline{q}, \overline{t})$ *et* $(\underline{q}, \underline{t})$

les agents acceptent les contrats s'ils leur assurent un bénéfice minima

$$\overline{t} - \frac{1}{2\overline{\theta}}\overline{q}^2 \geq 0 \qquad (P\overline{\theta})$$

$$0 \leq \underline{t} - \frac{1}{2\underline{\theta}}\underline{q}^2 \qquad (P\underline{\theta})$$

Mecanism design (2) : les propriétés des contrats

Première propriété : production croissante avec la productivité

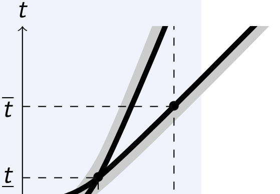
- R$\overline{\theta}$ et R$\underline{\theta}$ s'écrit : $\frac{1}{2\overline{\theta}}\left[\overline{q}^2-\underline{q}^2\right] \leq \overline{t}-\underline{t} \leq \frac{1}{2\underline{\theta}}\left[\overline{q}^2-\underline{q}^2\right]$. Or, puisque $\frac{1}{2\overline{\theta}} < \frac{1}{2\overline{\theta}}$
- nécessairement les trois membres sont positifs, cad $\overline{t} \geq \underline{t}$ et $\overline{q} \geq \underline{q}$

Seconde propriété : P$\underline{\theta}$ saturée

- On peut ordonner les deux membres de droite de R$\overline{\theta}$ et R$\underline{\theta}$: $\underline{t} - \frac{1}{2\overline{\theta}}\underline{q}^2 > \underline{t} - \frac{1}{2\underline{\theta}}\underline{q}^2$. Il en résulte que P$\overline{\theta}$ n'est jamais saturée. Or à l'évidence, à l'optimum,
- une des deux contraintes est saturée ($\Rightarrow \underline{\theta}$), sinon, on pourrait réduire chacun des deux transferts, de ε, sans violer les contraintes de révélation : $\underline{t} = \frac{1}{2\underline{\theta}}\underline{q}^2$

Troisième propriété : R$\overline{\theta}$ saturée

- On représente les objectifs des deux types dans un espace q, t : Le type $\underline{\theta}$ a une utilité $\underline{U} = t - \frac{1}{2\underline{\theta}}q^2$, de pente q / $\underline{\theta}$, et le type $\overline{\theta}$, $\overline{U} = t - \frac{1}{2\overline{\theta}}q^2$, de pente plus faible q / $\overline{\theta}$.
- On représente $(\underline{q}, \underline{t})$ sur $t - \frac{1}{2\underline{\theta}}q^2 = 0$, les parties grisées représentent les contraintes de révélation,
- On représente alors un $\overline{q}$ "compatible". Si le principal



Mecanism design (3) : Arbitrage du principal

D'après le graphique précédent, le contrat obtenu par le type $\underline{\theta}$ dépend immédiatement du niveau de la rente informationnelle de $\overline{\theta}$. On donnera donc plus de rente à $\overline{\theta}$ si on espère pouvoir faire plus de profit sur $\underline{\theta}$.

Quatrième propriété : sous production de $\underline{\theta}$, production optimale de $\overline{\theta}$

- remarquons que pour chaque niveau de profit $\underline{\pi}$ (pour $\underline{\theta}$), on peut associer au plus deux contrats satisfaisant $q - t = \underline{\pi}$ et $t - \frac{1}{2\underline{\theta}} q^2 = 0$, de part et d'autres de la valeur $q = \underline{\theta}$ optimale. Il n'y a lieu de considérer que la solution induit une moindre rente informationnelle pour $\overline{\theta}$ cad pour $\underline{q} \leq \underline{\theta}$.
- Il est alors immédiat qu'en ce point $(\underline{q}, \underline{t})$, la pente de la courbe d'indifférence de l'agent de type $\overline{\theta}$ est inférieure à 1, et que le principal, contraint de se déplacer sur cette courbe d'indifférence augmente ses profits s'il l'incite à augmenter sa prod. Ceci, jusqu'à $\overline{q} = \overline{\theta}$.

Résolution de l'exemple

l'arbitrage du principal, la solution

- Son profit s'écrit $\pi = \frac{1}{2}(\underline{q} - \underline{t}) + \frac{1}{2}(\overline{q} - \overline{t})$, droites d'iso-profit de pente 1.
- qu'il donne ou non un contrat efficace aux $\underline{\theta}$, il en donne un aux $\overline{\theta}$: $\overline{q} = \overline{\theta}$.
- Il donne une rente à $\overline{\theta}$ pour pouvoir faire produire $\underline{\theta}$: rente max si $\underline{q} = \underline{\theta}$
- S'il réduit inefficacités côté $\overline{\theta}$ (cad la rente), il perd production côté $\underline{\theta}$ ($\underline{q} < \underline{q}^*$)
- le mécanisme est paramétré par $\underline{q}$: $\overline{t} - \frac{\overline{\theta}}{2} = \frac{1}{2\underline{\theta}}\underline{q}^2 - \frac{1}{2\overline{\theta}}\underline{q}^2$; d'où une fct $\pi(\underline{q})$
- la solution optimale, après calcul, est $\underline{q}^{**} = \underline{\theta}\left[\frac{\overline{\theta}}{2\overline{\theta} - 2\underline{\theta}}\right] < \underline{\theta} = \underline{q}^*$.

Description de l'équilibre

Ce que produit l'équilibre

- Un niveau de production optimale, contingent à θ : $\underline{q}, \overline{q}$
- Un mécanisme de transfert, $\underline{t}, \overline{t}$, pour l'*implémenter*
- Note : toute l'information est révélée ($\Leftarrow$ rentes)

Le mécanisme de l'équilibre

- Le principal s'engage sur (l'offre) des contrats $\{(\overline{q}, \overline{t}), (\underline{q}, \underline{t})\}$
- En réponse (au mécanisme) les agents ont une stratégie dominante

Un mécanisme alternatif aux mêmes effets

- Les agents révèlent dans un premier temps leur θ.
- L'offre du principal est contingente à θ :
  $\underline{\theta} \mapsto \{(\underline{q}, \underline{t})\}$ $\overline{\theta} \mapsto \{(\overline{q}, \overline{t})\}$

Mécanisme et engagement

- Cette description d'un échange d'information et de recommandation n'a d'intérêt que s'ils produisent quelque chose. Ils produisent un effet s'ils sont crédibles, c'est-à-dire que la séquence est connue par tous les joueurs et lorsque le principal est capable de se LIER à cette séquence. On appelle Mécanisme la mise en place de telles règles de communication.

- On parlera de contrat pour souligner cet aspect d'engagement du principal

Implémentation et Mécanisme optimal

Mécanisme optimal

- Le niveau de production optimale, contingent à θ : $\underline{q}, \overline{q}$

Implémentation (parfois un problème indépendant)

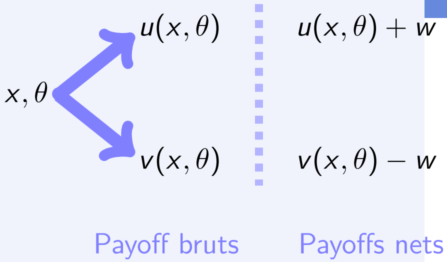
- Quels sont les contrats qui permettent d'implémenter $\underline{q}, \overline{q}$?
- *Ex Ante*, le principal s'engage sur des contrats sur lesquels il ne pourra pas revenir après. C'est

$$\{\underline{\theta} \mapsto \{(\underline{q}, \underline{t})\} \;\; ; \quad \overline{\theta} \mapsto \{(\overline{q}, \overline{t})\}\}$$

- Les agents révèlent dans un premier temps leur θ. Ils le font ayant l'assurance que les principaux se tiendront aux contrats auxquels ils se sont engagés.
- La recommandation du principal est alors contingente à θ :

Le même exemple revisité

Un joueur P décide d'une transaction avec un agent, portant sur $x \in X \subset \mathbb{R}^n$ et sur $w \in \mathbb{R}$. L'agent a une information privée θ qui affecte à la fois ses préférences et le bénéfice du joueur P.



- A ce stade de la modelisation, le joueur P a le pouvoir de choisir w et x sans toutefois connaître θ, mais la distribution $F(\theta)$. On pourrait analyser cette interaction comme un jeu bayesien.
- Mais, si P est un principal, il peut tenter d'extraire un peu d'information et décider par la suite de x et de θ.
- Pour ce faire, il peut se lier à un contrat.
- Un contrat (ou un mécanisme) est un jeu caractérisé par un ensemble de stratégie de reporting Θ et de recommandations $(w(\cdot), x(\cdot)) : \Theta \longrightarrow X \times \mathbb{R}$,
- cad le principal impose un ensemble de messages possibles, Θ (le

Under adverse selection, the agent has private information about his type or the state of the world before he agrees to a contract, which means that the emphasis is on which contract he will accept, even in presence of Moral Hazard

Timing

0. Nature chooses the state of the world s, observed by the agent but not by the principal, according to distribution F(s), where the state s is Good with probability 0.5 and Bad with probability 0.5
1. The principal offers the agent a wage contract w(q).
2. The agent accepts or rejects the contract.
3. The agent chooses effort level e.
4. Output is q = q(e, s). where q(e, good) = 3e and q(e, bad) = e.

Payoffs

- If the agent rejects the contract, then $\pi_{\text{agent}} = \bar{U}(\theta) = 0$ and $\pi_{\text{principal}} = 0$.
- Otherwise If the agent accepts the contract, then $\pi_{\text{agent}} = U(e, w, s) = w - e^2$ and $\pi_{\text{principal}} = EV(q - w)$.

Production Game VII : Adverse Selection and Moral Hazard

Assumption : θ is not contractible, as it is unknown from the Principal. Non linear wage $w(q, \theta)$

In the second-best world of information asymmetry, the effort in the good state remains the first-best effort, but second-best effort in the bad state is lower than first- best. This results from the principal's need to keep the bad- state contract from being too attractive in the good state. Bad-state output and compensation must be suppressed. Good-state output, on the other hand, should be left at the first-best level, since the agent will not be tempted by that contract in the bad state.

Also, observe that in the good state the agent earns an informational rent. As explained earlier, this is because the good-state agent could always earn a positive payoff by pretending the state was bad and taking that contract, so any contract that separates out the good-state agent (while leaving some contract acceptable to the bad-state agent) must also have a positive payoff.

Production Game VIII : Mechanism design

A mechanism is a set of rules that one player constructs and another freely accepts in order to convey information from the second player to the first. The mechanism contains an information report by the second player and a mapping from each possible report to some action by the first.

Mechanism design goes beyond simple adverse selection. It can be useful even when players begin a game with symmetric information or when both players have hidden infor- mation that they would like to exchange.

Timing

1. The principal offers the agent a wage contract of the form w(q, m), where q is output and m is a message to be sent by the agent.
2. The agent accepts or rejects the principal's offer.
3. Nature chooses the state of the world s, according to probability distribution F (s), where the state s is good with probability 0.5 and bad with probability 0.5. The agent observes s, but the principal does not.
4. If the agent accepted, he exerts effort e unobserved by the principal, and sends message m 2 good, bad to him.

Assumption : θ is not contractible, as it is unknown from the Principal. Non linear wage $w(q, \theta)$

This game is almost the same as Production Game VII. The big difference is that now the agent does not know his type at the point in time at which he must accept or reject the contract. A smaller difference is that we have added the message m which the agent sends to the principal. This message is cheap talk– it does not affect payffs directly and there is no penalty for lying. It is useful as a modelling convenience, to indicate which output-wage combination the agent chooses.

Exemple : Public monopoly regulation

Baron and Myerson [Baron, D., Myerson, R.B., 1982. Regulating a monopolist with unknown costs. *Econometrica* 50, 911–930]