Information, contracts and competition

Chapter III
Principal Agent Models with many agents and 1 principal

This part of the course presents a unified generalized framework for contracts, the Principal - Agent model.

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Introducing a Principal in a non cooperative game

A Principal-agent model can be understood as an extension of a noncooperative game.

Indeed, when a non cooperative equilibrium allocation is inefficient, it seems natural to ask if cooperation could improve the game output.

- Players could refrain playing a “bad” strategy
- However, that is only possible if they can credibly <u>commit</u> on the subset of strategies in which they will restrict their action. *possibly by a commitment on a penalty they would inflict on themselves if their y would deviate*

However such *commitments* are not always possible, in particular, when a player has not the ability of modifying the game rules. This leads to consider situations in which, one player, “the Principal” can commit on some modifications of the rules of the game.

Communication, rules between the players

One other ingredient that is often taken into account, when a non cooperative equilibrium allocation is inefficient is that players could speak and communicate among themself, the idea being that speaking could enhance cooperation. A general questions is then : Does Communication helps cooperation ? However what is really intended by communication ?

- should be more that a simple (even symmetric) talk about the strategies to be played ...
- which would stay *cheap talk* until the games of the rules to be modified
- There is cooperation only when players send credible messages on the actions on which they will commit, together with the modified rule of the game.

Still, *commitment* seems necessary, and particularly, the idea of a communication center, a Principal that controls the rules of com-

Communication, the case of information revelation

Communication is sometimes about revealing information, whenever one (decentralized) party reveals to the other party her type that is not *common knowledge*

Is there any commitment attached to such revelation ? Notice that, in that case,

- First, there is an essentiel credibility problem when a private information should be revealed ;
- second, that an information given by one player modifies the perceptions and the anticipations of the other players

- More deeply, when revealing a private information, a players commits himself toward the other players, modifying the game. He also lost some degree of freedom.

Revealing private information could be assimilated to introducing cooperative ingredients in a game.

Communication analysis

How to introduce communication process in non cooperative games ?

- how to synthesize the process by which agents reach an agreement ?
- Is that possible to anticipate the result from those process ?

Communication and equilibrium

As a poker player, an agent could try to cheat when communicating, but its strategy depends upon the strategies of the other players. Game theory is the tool for identifying optimal communication strategies.

- Hurwicz requires that each players has a dominant strategy
- E. Maskin et R. Myerson ask only for a Nash equilibrium

Phases of the game

Informations	distribution of information, private, public	Individual
Signals	cheap talk, signals ; ➜ Sending a cost, a price, a belief, ... ➜ Sending a hidden characteristics, "without lying", etc..	Partial/total information revelation,
Decisions	individual actions, contingent to the received informatione	Game Equilibrium

Exemple 1 Group Coordination about one decision

The generalized principal-agent problem can be interpreted as a social choice problem. whenever there is a principal, he is a social planner and his utility function is a social welfare function

Let consider a group of individuals, which objectives are differents, with potential conflicts, that have to take a common decision. Each individual have private information, i.e. her belief on the world, her preferences. The decision is taken after some talks, where the individual reveal partially their information, with eventually the help of a coordinator.

- This is more than a Bayesian equilibrium
- Communication is part of the strategy of a player
- Equilibrium output can be interpreted as the group decision.

Exemple 2 Planned economy : Evaluation and Coordination

Let consider a *social planer* which is looking for social welfare and aims to allow individuals to reach this ideal. He has to overcome two problems : he knows partially agent's type and he doesn't want (or cannot) control the agents' actions.

- First, the social planner looks for the optimal ressource allocations, given the characteristics of the economy
- Second, as the mechanism designer, he defines a mechanism that allow to decentralize the social welfare.
- Those two goals form a unique program, where the planner decides of the social game rules.

Communicating players - Game master - Principals

When analyzing interactions, it is not so unrealistic to think that for some players communication is easier, either because their information is larger or for any other reason. We could also think that the power of changing the rules of the game is not shared by every body. At the end, we can think that there is ONE player having those two capacities.

We analyze in this chapter games in which it is supposed that there is a which is able to modify the rules of the game (with some instruments) and which role is to facilitate the communication between agents.

Different players ’ status

When there are some communication process, the players ’ roles should be distinguished.

Definition

We define as an *Agent* any player involved in a non cooperative game that accepts in a passive way the rules of the game.

- The agent cannot enforce the end of the game
- The agent does not influence the game while communicating with the other players.

Definition

We define as an *Principal* any player that can modify the social economy

- by establishing communication rules ;
- and/or incentives to which the player will respond

Agents and Principals : a generalized model

A generalized Principal - Agent model is an hybrid model between a cooperative game and a non cooperative game.

- Agents act in a non cooperative way, by maximizing their objectives, and accept in a passive way the Nash Equilibrium that has been "chosen" by the Principal
- Communication and cooperation possibilities are controled by Principals

Roadmap

1. The Hasanyi model,

Incertainty and coordination

Uncertainty comes sometime from *coordination uncertainty*. For example, in game theory, when a player chooses an action, there is no commitment of the other players on the action they will play. The aim of Nash Equilibrium is to provide situations in which the players beliefs can be understood as rational.

Economic theory approach exploits this idea that every economic agent has beliefs that supports her actions, which allow her making rational and stable decisions.

Beliefs and actions are part of equilibrium strategy.

Jeu, Information et incertitude

The agents ' piece of information is a key element for understanding coordination. Indeed, Game theory lies on the right knowledge of the beliefs of the other players. This leads to the definition of symmetric and asymmetric information.

Complete information is a key assumption. For example, one firm could not know perfectly the cost function of her competitors. A firm dealing with unions does not know very well its members desutility for a long strike.

Definition

in a game with incomplete information, players do not possess full information about their opponents, about their possible actions and the resulting payoffs.

One question : how does asymmetric information affects the coordination between the agents, making it more or less difficult ?

Incomplete Information and beliefs

Two main directions for tackling incomplete information

- How the information will be *de facto* revealed when players chooses their actions
- Belief formation that will rub out the asymetries of information.

Remark : Information asymetries are not total. There is some commun knowledge, for instance about the distribution of information. That distribution will evolve during the game. Such revisions are called bayesian.

Structure of a Bayesien game

In a bayesian game, Nature plays first. It chooses the type of each player. Each player knows her own type, but doesn't know the other players type. Type of player i is denoted t_i, $t_i \in T_i$ where T_i is the set of players i 's types (which is common knowledge).
All types are draw from a distribution $p(t_1, \dots, t_i, \dots, t_n)$ that is common knowledge.
We denote S_i the (whole) set of choices for player i, independent of the contingencies. Player i 's utility function depends on her actions, on the other players actions and also on their characteristics. It is defined on

$$\underbrace{S_1 \times \dots S_n}_{actions} \times \underbrace{T_1 \times \cdots \times T_n}_{types}.$$

bayesian Equilibre

Definition

A Bayesian Nash equilibrium is defined as a strategy profile and beliefs specified for each player about the types of the other players that maximizes the expected payoff for each player given their beliefs about the other players' types and given the strategies played by the other players. Then, given any strategy of the other players, player i 's payoff is,

$$U_i(s,t) = \sum_{t_{-i}} p_i(t_i|t_{-i}) u_i(s_1(t_1), \ldots, s_i, \ldots, s_n(t_n))$$

- this is as if player i was playing with all the types of all the players
- Equilibrium strategies are $(s_1^*(t_1), \ldots, s_i^*(t_i^*), \ldots, s_n^*(t_n))$

Exemple d'un jeu bayesien

Wife and Husband should go to the theater (T) or to see a movie at a cinema (C).

- The wife 's payoff depends on the spectacle and also on being or not with her husband. We suppose that she prefers to go to the theater and that shed dislikes to go alone.

	Husband going to T	Husband going to C
Wife going to T	2	0
Wife going to C	0	1

- The husband payoff depends also on the spectacle and also being or not with her wife. However, at the begining of the story the husband 's type, and particularly its preferences is not known. He is either ***A****social*, prefering going alone to the spectacle or ***B****onhomme*, disliking going alone to the movie.

mari Asocial	AT	AC
FT	0	2
FC	1	0

Bon mari	BT	BC
FT	1	0
FC	0	2

- How many equilibria if the wife knows her husband being Asocial ?
- How many equilibria if the wife knows her husband being B ?
- We suppose some ignorance of the wife, believing that her husband is Asocial with proba $\frac{1}{2}$ and B with proba $\frac{1}{2}$. What is the Bayesian equilibrium of this incomplete information game ?

Analyse du jeu bayesien

- a set of strategies is a set of strategy for each type, that is for the wife *FT* or *FC*, for the Asocial *AT* or *AC* and for the B, *BT* or *BC*.
- In any part of the table, there will be the certain payoffs of the Asocial and of the B, and the *wife 's Expected payoff* depending on the anticipations of the two husband types.

	AT BT	AC BT	AT BC	AC BC
FT	2, 0, 1	1, 2, 1	1, 0, 0	0, 2, 0
FC	0, 1, 0	$\frac{1}{2}$, 0, 0	$\frac{1}{2}$, 1, 1 ;	1, 0, 1

- Then follows the analysis of the rationality of each type, considering what they will not do, given the assumption on the belief of the types of the others agents doing.
  - Cross four non rational choices of the wife (in red)
  - Cross four non rational choices of the Asocial (in blue)
  - Cross four non rational choices of the B (in green)
    - *There is a unique pure strategies Nash Equilibrium, in $(FC, AC, BT,)$: under this equilibrium, the wife goes to the theater, the Asocial, to the movie, and the B, to the theater.*

A simple firm selling model with private information

Let consider the following story :
A risk neutral investor buys project which payoffs follow q normal distribution $\tilde{R}(\theta), \mathcal{N}(\theta, \sigma)$, with σ known (the macro environment) and θ unknown (this is the seller 's type).
p is the price of a project. We suppose a uniform distribution of $\theta \in [0, 1]$.
We suppose also the seller risk averse, the VNM being $u(x) = -e^{-\rho x}$.

- Describe the game and compute the Bayesian equilibrium.
- We suppose that, given the entrepreneur preferences, then, $Eu(W_0 + \tilde{R}(\theta)) = u(W_0 + \theta - \frac{1}{2}\rho\sigma^2)$.

2. Generalized mechanism,

From - Roger B. MYERSON, P. (1982). Optimal Coordination Mechanisms In Generalized Principal-Agent Problems. *Journal of Mathematical Economics*, 10 (1982) 67-81.

Myerson words

This is a paper about game design. The central question to be considered is how an individual should structure a social situation which he controls, so as to maximize his expected utility.

The Principal-Agent model

A Principal delegates a task to one or many agents. Principal and agents have conflicting objectives. Agents and Principal interact in the sense that their payoffs depends on all players actions.

- Moreover agents decisions could not be entirely controled by the Principal, or, they hold some private information.

- Principal can ask about information, and, after the analysis of the different informations that he gathered, he could send messages to the agents, depending on which they will choose their actions.

Origin of the Principal-Agent model

Any situation where an actor (the Principal) wants to delegate a task to someone else (the Agent), a task which affect his payoff. In that context, there are two types of information problems that he could face :

- The result depends upon the Agent 's characteristics, that are private information
- The result depends upon the Agent 's actions, what we call *effort* that are unoservable by the Principal (or by the legal system). In that situation, given some alea, the Principal could observe some variables that are correlated to the agent 's effort.

In such a situation, there should be an agreement between the Principal and the Agent, concerning the mechanism of retribution of the agent. This is the aim of the model.

Motivating the role of the principal

Even with differents objective,

- The Principal acts as a *monopole* : its benefit will be maximum when he gather the maximum possible information an maintain a full control on agents.

- The Principal is *benevolent* : He needs information to find the best social welfare. Controling agent's communication is only an implementation matter.

Form of the optimal mechanism

We address two questions about the optimal mechanism

- either it reveals or not, partialy or totally, the agents types
- either it induces or not, agents to choose particular actions

The Principal constraints

As in the Harsanyi model, the principal takes into account two types of constraints

- He cannot observe some Agent's type : $t \in T = T_1 \times T_2 \times \cdots \times T_n$
- Agents can choose some action that cannot be controlled : $d \in D_1 \times \cdots \times D_n$

We suppose that types and actions could modify the payoffs of all the agents.

Rules of the principal

The principal can take some decision d_0 that will affect the agents. Those decisions are usually transfers, eventually contingent on some message.

- All the strategies of the Principal and of the agents are summarized by d :

$$d \in D = D_0 \times D_1 \times \cdots \times D_n$$

- Principal and agents ' payoffs are defined on $D \times T$

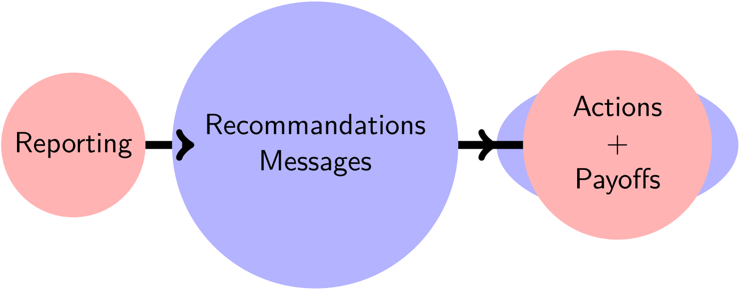
$$U_0 : D \times T \to \mathbb{R} \qquad \forall\, i\ U_i : D \times T \to \mathbb{R}$$

What differs with Bayesian games

First, agents do not have uncertainty on d_0 (as the principal can commit). However, it could be possible that the payoff be contingent on the messages of the others agents. (which is different from the precedent elementary examples).

- Information Revelation at interim stage could be partial.
- The strategy of the Principal, i.e. the description of the mechanism, is known *ex ante*. However, the realizations of the mechanism could be contingent on the informations transmitted by the agents, via their messages, or the Principal could also choose to give back to the agents some of the information that he receives.

Timing of the mechanism



- The principal chooses the format of the reports, i.e. $R_1 \times R_2 \times \cdots \times R_n$.
- Principal 's messages contains some recomandations to the agents, and also, the information that the center share with the agents, particularly when d_0 is contingent on that info. The set $M_1 \times M_2 \times \cdots \times M_n$ reflects the communication flavor.
- The agents' game played after the reporting and the recommandation stage is a bayesian game. We look for the equilibria of this game.

The Principal 's strategy

- The Principal 's strategy is the definition of the mechanism, that is, the reporting space, $R_1 \times R_2 \times \cdots \times R_n$, the message space, $M_1 \times M_2 \times \cdots \times M_n$, and all the commitments of the principal :
  - Principal 's recommandations and actions after the reporting time
  - i.e. a joint distribution contingent on the reporting :
    $\pi(d_0, m_1, \ldots, m_n | r_1, \ldots, r_n)$.

definition

A *coordination mechanism* $\left((R_i, M_i)_{i=1}^n, \pi\right)$ the elements of the mechanism, including the communication rules, from the exterior to the center, and from the center to the exterior, and the Principal 's recommandations and decisions.

Agents ' strategies

- Agents take only into account the mechanism to choose their strategy
- Agents strategies are not only reporting but also decisions after receiving the principal messages. Principal commits on the whole game .
- The reporting is formalized as a function $\rho_i : T_i \rightarrow R_i$ associating to any type the reporting.
- The decision time can be formalized as a function $\delta_i : M_i \times T_i \rightarrow D_i$ associating to any type $\times$ recommandation an action.

The pair $\left(\rho_i, \delta_i\right)$ is called agent i's *Participation Strategy*.

Payoffs of the agents given all the strategies

Given the coordination mechanism, $((R_i, M_i)_{i=1}^n, \pi)$, given the participation strategies of *all* the agents, (ρ_i, δ_i), the *ex ante* value for agent i is :

Ex Ante value

$$\begin{aligned} V_i &= V_i((\rho_1, \delta_1), \ldots, (\rho_n, \delta_n)) \\ &= \sum_{t \in T} \sum_{d_0 \in D_o} \sum_{m \in M} P(t)\ \pi(d_0, m | \rho(t))\ U_i(d_0, \delta(m, t), t) \end{aligned}$$

- One agent's payoff depends on the other players strategies
- The computation is done *ex ante* at the moment of choosing the strategy. Be carreful, one agent computes what obtains all his possible types
- ... which is different from the *interim* payoff :
- $\sum_{t_{-i} \in T_{-i}} \sum_{d_0 \in D_o} \sum_{m \in M} P(t)\ \pi\big(d_0, m | \rho(t_i, t_{-i})\big)\ U_i\big(d_0, \delta(m, t), t_i, t_{-i}\big)$

One example of coordination mechanism

▶ **Exemple** One Principal, one agent, two types $t \in \{t_1, t_2\}$, with equal probability, three possibles allocations A, B, C. The payoffs depend on those type (first the principal one, second, the agent).

if agent 's type is t_1		**if agent 's type is t_2**	
A	$(1, 1)$	A	$(0, 0)$
B	$(0, 0)$	B	$(1, 1)$
C	$(0, 0)$	C	$(0, 1)$

Consider the following mechanism, $R = \{r_1, r_2, r_3\}$, and d_0 the action contingent on the receive message :

- if the agent declares r_1 he will be gifted by allocation A
- if the agent declares r_2 he will be gifted by allocation B
- if the agent declares r_3 he will be gifted by allocation C

Could you compute the best participation strategy for the agent ? Is that the best mechanism for the Principal ?

One particular mechanism

Analyzing the strategic form game

There is a unique principal strategy, six strategies of the types t_1 and t_2, with the following payoff $\left(0 \;\; \begin{matrix} 1 \\ 2 \end{matrix} \right.$ is 0 for the principal, 1 for type t_1 and 2 for type $\left. t_2 \right)$:

$\begin{matrix} A \\ A \end{matrix}$	$\begin{matrix} A \\ B \end{matrix}$	$\begin{matrix} A \\ C \end{matrix}$	$\begin{matrix} B \\ B \end{matrix}$	$\begin{matrix} B \\ C \end{matrix}$	$\begin{matrix} C \\ C \end{matrix}$
$\frac{1}{2} \; \begin{matrix} 1 \\ 0 \end{matrix}$	$1 \; \begin{matrix} 1 \\ 1 \end{matrix}$	$\frac{1}{2} \; \begin{matrix} 1 \\ 1 \end{matrix}$	$\frac{1}{2} \; \begin{matrix} 0 \\ 1 \end{matrix}$	$0 \; \begin{matrix} 0 \\ 1 \end{matrix}$	$0 \; \begin{matrix} 0 \\ 1 \end{matrix}$

At the equilibrium, type 1 dominant strategy is to report r_1, while type 2 is indifferent between reporting r_2 or r_3. One can consider the following equilibrium :

- type 1 always declares r_1
- type 2 declares r_2 with proba $1/2$ and r_3 avec proba $1/2$

- Les payoffs sont alors $(3/4, 1, 1)$

Additionnal comments

Does the type of the agent be revealed with this mechanism ?
Does the Principal can do the same, with $R = \{t_1, t_2\}$ and no stochastic mechanism ?
The answer is no.

The Principal cannot develop the same incentives with a direct mechanism that would not be stochastic ?
Le principal est incapable d'inciter un tel outcome via un mécanisme direct révélateur qui ne soit pas stochastique. En effet, un tel mécanisme direct impliquerait que l'agent ne pourrait choisir qu'entre deux messages seulement et on ne pourrait pas voir l'émergence des trois allocations.

3. Equilibrium and the revelation principle

Equilibrium

Bayesien Equilibrium

Given the coordination mechanism $\left((R_i, M_i)_{i=1}^n, \pi\right)$, a set of participation strategies $\left(\rho_i, \delta_i\right)$ form a bayesian equilibrium, if there is no profitable deviation at the ex ante stage. Formally, for any other set of participation strategies $(\tilde{\rho}_i, \tilde{\delta}_i)$, it is always true that :

$$V_i\big((\rho_1, \delta_1), \ldots, (\rho_i, \delta_i), \ldots, (\rho_n, \delta_n)\big) \geq V_i\big((\rho_1, \delta_1), \ldots, (\tilde{\rho}_i, \tilde{\delta}_i), \ldots, (\rho_n, \delta_n)\big)$$

At this stage, we should notice that agent i 's choices are done after the Principal has chosen d_0. Why then we focus on the *ex ante* payoff and not on the interim payoff ? The fact is that the choice criterium is identical, given the anticipations of all the other types of the agents over agent i 's types.

Optimal mechanism

The Principal problem is to find a coordination mechanism maximizing its objective.

Three distributions resulting from the mechanism

- What results specifically from the mechanism
Given a coordination mechanism $\big(\pi(d_0, m|r_i)\big)$, given the participation strategies of all agents, $r_i = \rho_i(t_i)$ et $d_i = \delta(t_i, m)$, one can compute the distribution reports, messages, actions of every one. One obtains *ex ante*

$$\big(P(r, m, d)\big). \tag{1}$$

- Ex ante Distribution of the economy We have a better information, that the one described above in equation (**??**). We can cross the preceding joint distribution with the distribution on the types. We obtain a more exhaustive distribution

$$\big(P(t, r, m, d)\big). \tag{2}$$

- Distribution between types and actions Starting from the exhaustive distribution of the economy, as formalized by equation (**??**), we extract a joint distribution on types and actions (equation (**??**)), in which one made "disppear" the mechanism

$$\big(P(t, d)\big) \tag{3}$$

Representation of a Mechanism (Myerson, p.74)

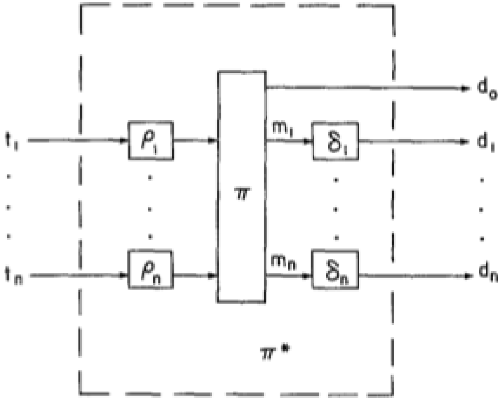


Fig. 1

A Simplified analysis of a Mechanism

Fondamentally, a mechanism can be represented by (equation (**??**))

$$\big(P(t, d)\big) \tag{3}$$

and its interpretation is that the mechanism links some actions with some types.

- Said it differently, as some actions are correlated with some types, mechanims produce *de facto* information.
This is not far from the question whe had at the begining of the model : how make agents reveal enough information to obtain a better coordination in the economy.

- Next, a corrolarry
does there exists simple mechanism producing the same effects (in terms of types and actions of equation (**??**))

Direct and incentive mechanisms

Definition

A mechanism is said *direct* if all the reporting set are identical to the types sets, and if the Principal messages are restricted to be recommendations on the actions that agent i should overtake. Formally,

- $R_i = T_i \qquad M_i = D_i$

- **Honest and obeying Agents**

Given a direct mechanism, agents are said Honest and obeying if their participation strategy is to reveal their type and to follow the Principal recommandations.

- $\rho_i^*(t_i) = t_i \qquad \delta_i^*(d_i, t_i) = d_i$

Definition

A Direct Mechanism is incentive if all honest and obeying strategies form an equilibrium.

Revelation Principle

Theorem

Given a set of equilibrium participation, (ρ_i, δ_i), in response to the Principal coordination mechanism $((R_i, M_i)_{i=1}^n, \pi)$, then, there exists (another) direct incentive mechanism in which the Principal obtains the same expected payoff as in the initial equilibrium.

Corolarry

It follows that that direct mechanism is optimal in the set of all coordination mechanism

Proof of the Revelation Principle (1/2)

- **the starting π mechanism**

Consider *ex ante* the participation strategies $\left(\rho_i, \delta_i\right)_i$ corresponding to the mechanism $\left((R_i, M_i)_{i=1}^n, \pi\right)$.

- **Building a direct mechanism π^***

We look for a direct mechanism that induces the same actions that induced the Principal messages, whatever be the types. However, any choice of action d_i was an immediate consequence of a message taken into the set $\{m_i \ / \ \delta_i(t_i, m_i) = d_i \ \}$.
We then define a direct mechanism

$$\pi^*(d|t) = \sum_{m \ / \ \delta_i(t_i, m_i) = d_i, \forall \ i} \pi(d, m|\rho(t))$$

We proove in next slide that π^* is a revealing mechanism, that is, each agent will reveal her own type and make d_i when this action is recommended by the principal.

Proof of the Revelation Principle (2/2)

- **What would be an *interim* deviation for the agent**

Let suppose that agent i, of type τ_i has interest to announce a type $\hat{\tau}_i$, and that this same agents has interest not to follow the Principal instructions, formally, by choosing $d_i \mapsto \hat{\delta}(d_i)$.

- **Build an alternative participation of the agent** $| \; \pi \ldots$

  - consider the reporting strategy $\tilde{\rho}_i$ identical to ρ_i, except for the image of $\tau_i : \tilde{(}\tau_i) = \hat{\tau}_i$.
  - consider the decision strategy $\tilde{\delta}_i$ identical to $\tilde{\delta}_i$, except for the image of $\tau_i, m_i \; (\forall m_i) : \tilde{\delta}_i(\tau_i, m_i) = \hat{\delta}()$

- **... giving to the agent a greater utility**
  - Consider $\tilde{\rho}_i$ with $\tilde{(}\tau_i) = \hat{\tau}_i$.
  - Consider $\tilde{\delta}_i$ with $\tilde{\delta}_i(\tau_i, m_i) = \hat{\delta}()$
  - Then it is immediate to verify that this strategy gives to the agent a greater utility than its initial participation strategy, a contradiction.

Interpretation of the revelation principle

- To every decision process, we can associate a direct incentive mechanism, which is simpler

- Looking for direct mechanism is enough for maximizing the principal objective.

- In a generalized Principal-Agent model, all the information is to be revealed. It is always in the interest of the principal to extract all the information, even if it is costly

- In a generalized Principal-Agent model, the agent always choose her action following the recommendation of the principal

- This revelation principle is robust, as its proof, quite simple, does not rely on any particular assumption, unless the principal controls the reporting phase and part of the decision of the agents.