Competition Policy & Game Theory

Chapitre II
Introduction to Imperfect Competition Models

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Roadmap

0. Introduction
1. Monopoly
2. Oligopoly in static games, with homogeneous goods, Bertrand and Cournot
3. Oligopoly in static games, , with differentiated goods
4. Oligopoly in Dynamic games

Introduction

The Pure competition Reference model

When firms participate to a pure competition market, they cannot choose the selling price, and it is optimal for each firm that

$$p = C_m$$

- This is the reason why public policies promote pure competition market
- The idea : firms produce “as much as they can”

1. Monopoly : single and multi-product

Simple-product Monopoly

The market demand is characterized by $D(p)$, negatively sloped, and concave, there is only one firms, which cost function is $C(q)$, increasing and convex.

The unique firm can decide price and quantity. It is constrained by the demand. When she post one price, the maximum quantity she sells is $D(p)$. So the optimal tarif of the monopole will depend upon the demand.

Let's write the revenue and the cost as a function of the sold quantity : $R(q) = q * p(q)$ where $p(q)$ is the inverse demand function. The monopole behavior is represented by the rule *Marginal revenue=Marginal Cost*, with $R_m(q) = p(q) + qp'(q) = p(1 + p'(q)\frac{q}{p}) = p\left(1 + \frac{1}{\varepsilon}\right)$, with $\varepsilon \leq 0$ being demand elasticity. Optimal behavior :

$$p\left(1 + \frac{1}{\varepsilon}\right) = C_m \Longleftrightarrow \frac{p - C_m}{p} = \frac{-1}{\varepsilon} \tag{1}$$

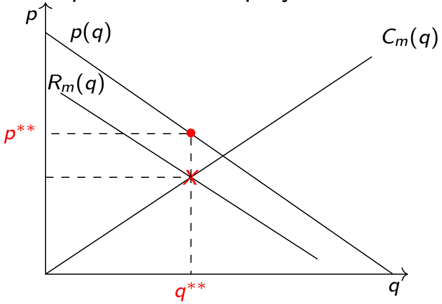
The left-hand side is the Lerner index, a measure of market power, that is the ability of a firm to set prices above marginal costs.

an exemple of monopolistic tarif

Let consider the case such that $c_m = 2q$, $D(p) = 120 - p$, which we rewrite $p(q) = 120 - q$

1) Compute q^* et p^* in a pure competition market

2) Compute q^{**} and p^{**} in a monopoly market if the firm is alone



Simple-product Monopoly example

In case of pure competition (p, q) is on the supply curve when $p = c_m$, i.e. when $p = 2q$ and (p, q) is on the demand curve when $p = 120 - q$. Both curves cross whenever $2q = 120 - q$, that is when $q = 40$, implying $p = 80$.

$$p^* = 80 \qquad q^* = 40$$

The monopolist tarification is such that $R_m = c_m$ **;** In our example, $R(q) = q(120 - q)$, $R_m = 120 - 2q$, $R_m = c_m$ is equivalent to $120 - 2q = 2q$, and $q = 30$. For that quantity, the maximal price that the firm can charge is $p = 120 - 30 = 90$.

$$p^{**} = 90 \qquad q^{**} = 30$$

Multi-product Monopoly

In the real world firms could produce and sell many products. However, whenever demand and cost of one product does not affect demande and costs for other products, then, the problem of the monopolist reduces to the problem studied in the one-product case. Indeed, for instance in the 2 product case, the monopoly profit is the sum of two terms,

$$\pi = \left(p1D_1(p_1) - C_1(D_1(p_1))\right) + \left(p2D_2(p_2) - C_2(D_2(p_2))\right),$$

that can be maximized independently, each maximization problem going to the condition

$$\frac{p_i - C'(q_i)}{p_i} = \frac{1}{\varepsilon_i}$$

- Solving this problem independently is true, even if the demand and the costs structure are different. This is as if there where two divisions in the considered firm
- That will not be true if interdependences exists on the demand side or on the costs side

Multi-product Monopoly with interdependent demands

Often, a firm sells a range of products that are to some extent substitutable with each other. For substitute products, the increase on the price of one product will increase the demand for the others that become more convenient. That is, in the two good case :

$$q_i = q_i(p_i p_j) \qquad \frac{\partial q_i}{\partial p_j} > 0$$

For *complementary goods*, an increase on the price of one product will decrease the demand for the others (by decreasing the demand for the first product, demand for complements is also discouraged). In the two good case :

$$q_i = q_i(p_i p_j) \qquad \frac{\partial q_i}{\partial p_j} < 0$$

Multi-product, interdependent demands, an example

Let assume the two products with the following demand functions

$$q_i = a - bp_i + gp_j,$$

g being positive in the substitute case, and negative in the complement case. Let suppose that the marginal cost to produce the good is constant, the same c for the two products.

- Prove in the two good case that $p_1 = p_2 = \dfrac{a + bc}{2b - g}$

  Write each profit function as as function of p_i, p_j, and derive it relative to p_i.

- Interpret why we only care when $|g| < |b|$; Interpret the role of g

- Interpret with comparing to the case $g = 0$. (monopoly pricing)

When the products are complements, they exercise a positive externalityon each other and the monopolist internalises it by decreasing its prices. A lower price for good one stimulates sales of good 2 and vice versa.

If products 1 and 2 were sold by two distincts monopolists, consumers would pay more for them than when they are sold by the same firm.

When the products are substitutes, teh externality they ewxwercise on each other is negative, and the monopolist controls for it by raising prices (a lower price of good 1 crowds out sales of good 2 and vice versa).

If products 1 and 2 were sold by two distinc firms, consumers would pay less than when they are sold by the same firm.

A product on multi period with interdependent demands

The preceding model apply to the case the same product is sold in sequential markets. For instance, with q_1, q_2, p_1, p_2 quantity and prices of period one and two, when the demand is

$$q_1 = a - bp_1 \qquad q_2 = a - bp_2 + \lambda q_1$$

- Prove in the case $c = 0, a = 1, b = \frac{1}{2}$ that $p_1 = 2\frac{1-\lambda}{2-\lambda}$ and $p_2 = \frac{2}{2-\lambda}$

  Write *ONLY ONE* profit function as as function of p_1, p_2, and derive it relative to the two prices.

- Check that when λ rises, then, the first period price goes down and the second period price, up. Interpret

  Notice that the solution in the general case is $p_1 = \dfrac{a(1-\lambda)+cb}{b(2-\lambda)} \qquad p_2 = \dfrac{a+cb(1-\lambda)}{b(2-\lambda)}$

Intertemporal complementarity : Introductory price offers When $\lambda > 0$, the monopolist realises that there is a positive intertemporal demand externality, and it internalises it by decreasing thee price relative to the price it would set if there was no future market.

Intertemporal substituability : Durable goods In the case where $\lambda < 0$, higher sales in the first period decrease the demand in the second period. As a result, the monopolist keeps the fist period price higher than in the hypothetical cas where the product is sold only once, to internalise the negative demand externalityarising across periods. This a a reduced form of the case of a durable good monopolist : the more consumers buy in the first period the fewer will buy in the second. In that class of models, the equilibrium prices tend to decrease over time.

We say that there are cost externalities, in a two product production model, when the global cost cannot be written as the sum of the cost of producing q_1 good 1 and the cost of producing q_2 good 2 as, in the following :

$$C(q_1, q_2) = c(q_1 + q_2) + \mu q_1 q_2$$

- When $\mu > 0$, there exist diseconomies of scope between the two products, as the higher the outpt of one product, the higher the marginal cost of the other product. This is the case, when both products make use of limited natural resources or inputs.
- When $\mu < 0$, there exists economies of scope. There are many instances inthe real world where producing two foods jointly gives rise to cost savings relative to the case where each product is produced separately.

Interdependant costs (II)

One can check that under symmetry, when the demand are $q_i = a - bp_i$, and not trivial (in particular, $a - bc > 0$, then when the prices are identical, $p_1 = p_2 = p_m$ one can check that

$$p_m = \frac{a(1 + b\mu) + cb}{b(2 + b\mu)}$$

It happens then that the derivative of p_m is increasing with μ : the stronger the cost externality between the two products, the higher the equilibrium price set by the monopolist.

- When $\mu < 0$, the prices are lower than in the benchmark case where $\mu = 0$: the firm wants to stimulate its output. a multiproduct monopolist charges lower prices than two distinct monopolists
- When $\mu > 0$, the prices are higher than in the benchmark case. The ponopolists wants to reduce the output of each good.

The cost of production of many goods and services decreases with the experience accumulated in producing those goods and services

$$\mu < 0$$

In such a situation a monopolist will want to decrease prices in the early stages of life of a product, in order to increase output and “go down the learning curve”.

2. Oligopoly I : Market competition in static games, with homogeneous goods, Bertrand and Cournot

Cournot and Bertrand

Two static theories developed in the XIX century

Very different results in terms of :

- the degree of competition,
- the nature of the first-mover advantage,
- and the relationship between market structure (concentration) and the price-cost margin.

Corporate strategy is, of course, very broad embracing all the activities of the firm – price, output, investment, advertising, R & D and so on.

Cournot (1838) takes the view that the firm's strategic variable is its *output*

Bertrand (1883) takes the view that the firm's strategic variable is *price*

Cournot Nash with homogeneous goods

n firms, producing $q_1, q_2, ..., q_n$, the agregate output being $q = \sum q_i$.

Costs : marginal cost constant, identical for all firms, being equal to c

Demand : $p = a - bq$, with $a > c$

Definition The Cournot equilibrium is the Nash equilibrium of the simultaneous game in which firms choose quantity

$$\pi_i = q_i(a - bq - c)\,;\ \frac{\partial p_i}{\partial q} = a - c - bq - bq_i\,;\ \text{max if } q + q_i = \frac{a-c}{b}$$

Adding all equations : $(n+1)q = n\frac{a-c}{b}$, $q = \frac{n}{n+1}\frac{a-c}{b}$, $q_i = \frac{1}{n+1}\frac{a-c}{b}$

Symmetric equilibrium, price $p^c = \frac{a}{n+1} + c\frac{n}{n+1}$, decreasing in n.

Negative relationship between nb of firms and price-cost margin

As in the monopoly we define the price-cost margin $\dfrac{p-c}{p}$

$\dfrac{p^c - c}{p^c} = \dfrac{a-c}{n+1}\dfrac{n+1}{a+cn} = \dfrac{a-c}{a+nc}$, decreasing with n, limit equal to zero.

Intuition : **with more firms, each firm's own demand becomes more elastic**

The industry price elasticity is $\varepsilon = \frac{p}{q}\frac{dq}{dp} = \dfrac{a-bq}{q}\frac{1}{b} = \dfrac{a}{bq} - 1$

The representative firm's elasticity is $\varepsilon_i = \frac{p}{q_i}\frac{dq_i}{dp}$. However, under the Nash assumption firms treat the other firms' outputs as given and the change in industry output q equals the change in firm i's output. Hence $dqi/dp = dq/dp$. In equilibrium, each firm elasticity is larger than the industry elasticity

$$\varepsilon_i = n\varepsilon$$

when n gets large, so does ε_i, leading to approximately "price-taking" behavior.

A Benchmark : joint profit maximization

Let consider the case of n identical firms, but, instead of playing a non cooperative game, they maximize the joint profit of the firms (and after, they share equally).

They will produce the same amount, that we write Q/n where Q is the aggregate production. One producer profit is $\pi_i = (p(Q) - c)Q/n$ and the global profit :

$$\Pi = (p(Q) - c)Q = (a - bQ - c)Q$$

a value that is maximum when $a - 2bQ - c = 0$, or $Q = (a - c)/2b$. Collusion is then

$$q_{col} = \frac{1}{2n}\frac{a-c}{b} \qquad p^{col} = a - \frac{a-c}{2} = \frac{a+c}{2}$$

Notice that clearly $p^{col} = p^m$, the cartel acts as a monopoly

Criticism of Cournot's model by Bertrand

Firms set prices not quantities : the output sold by the firm is determined by the demand it faces at the price it sets.

A more complex world : If firms set prices the model is rather more complicated than in the Cournot framework since there can be as many prices in the market as there are firms. In the Cournot framework the inverse industry demand curve implies a single "market" price. In the Bertrand framework each firm directly controls the price at which it sells its output and, in general, the demand for its output will depend on the price set by each firm and the amount that they wish to sell at that price (see Dixon 1987b).

Price competition with two identical firms

Consider two firms, selling an homogeneous good, playing a one shot game, choosing independently the selling price, having no capacity constraint, i.e., they are able to serve all the demand that is addressed to them (in particular we suppose that the marginal cost is constant).

We analyze at the first glance the case of identical firms, with a same marginal cost.

A firm wins the market whenever she posts a strictly lower price. Il is necessary in that kind of game to specify the tie-break rule, that is, how is divided the market when the two firms posts the same price.

We will suppose that the demand will be equally split when the two firms proposes the same price

▶**Proposition** One unique equilibrium when $p_1 = p_2 = c$, $\frac{p_i - c}{p_i} = 0$

Difference between Cournot and Bertrand

The Bertrand Competition equilibrium corresponds to the toughest possible degree of product market competition : with one firm the monopoly outcome occurs ; with two or more firms the competitive outcome occurs.

The Cournot-Nash equilibrium is such that a large numbers of firms are necessary to obtain the competitive outcome.

What grounds do we have for choosing between those two models ? First, and perhaps most importantly, there is the question of the type of market. In some markets (for primary products, stocks and shares) the people who set prices (brokers) are different to the producers. There exists what is essentially an auction market : producers/suppliers release a certain quantity into the market and then brokers will sell this for the highest price possible (the market clearing price). *The Cournot framework would thus seem natural where there are auction markets*. While there are auction markets, there are also many industrial markets without "brokers" where the producers directly set the price at which they sell their produce. Clearly, the "typical" sort of market which concerns industrial economists is not an auction market but a market with price- setting firms. *How can the use of the Cournot framework be justified in markets with price-setting firms ?*

Difference between Cournot and Bertrand : flexibility

In the Bertrand framework firms set prices and then produce to order. Thus, once set, prices are fixed while output is perfectly flexible.

In the Cournot framework once chosen, outputs are fixed while the price is flexible in the sense that it clears the market.

Does prices are more flexible than quantities (e.g. Hart 1985) and hence the Cournot equilibrium is more appropriate ? To study that question, we develop the *Bertrand competition with capacity constraints*.

Bertrand under (fixed) capacity constraints

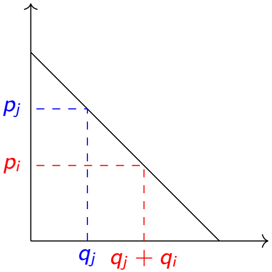
2 firms, same marginal constant cost c, selling an homogeneous good, playing a one shot game, choosing the selling price, with capacity constraint $k_i < D(p_i = c)$.

Proposition Under capacity constraint, $(p_i^*, p_i^*) = (c, c)$ is not an equilibrium of the Bertrand competition

Indeed, if the two firms was to price c, they would not be able to serve all the market, which cannot be an equilibrium (easy deviation for at least one of the firms).

- *solve the game with the demand $p = a - q$ when $c = 1$ and each firm capacity is k_i, k_j.*

Let consider the possibility of an asymmetric equilibrium $p_i < p_j$. We suppose that the demand will be satisfied under that condition : the agent which pay p_j are the one such that their reservation's price is the higher.



$$q_j = a - p_j$$
$$q_j + q_i = a - p_i$$

which implies that $q_i = p_j - p_i$ Corresponding profit functions are

$$\pi_i = (p_i - c)(p_j - p_i)$$
$$\pi_j = (p_j - c)(a - p_j)$$

Best response of firm i is such that $p_i = \dfrac{p_j + c}{2}$

Best response of firm j is such that $p_j = p^m = \dfrac{a + c}{2}$

Here, the capacities does not reduce the quantities, The game would be different if one of the right constraint would not be satisfied.

We suppose that $q_j = a - \left(\dfrac{a+c}{2}\right) \leq k_j$ and that $q_i = \dfrac{p_j - c}{2} = \dfrac{a - c}{4} \leq k_i$

Bertrand under (endogeneous) capacity constraints

If we introduce two stages, the first one in which firms commit to their capacity constraints, and the second one in which firms compete in price, then, the price competition nature is deeply modified.

Proposition (Kreps Scheinkman (1983) In a two stage game, where firms first choose (simultaneously) capacities, and then prices, the final equilibrium outcome is the same as in a one-shot game where firms choose quantities

Flexilibity of production

When flexibility of production is endogeneous, Bertrand and Cournot equilibria come out as limiting cases corresponding to when production is perfectly flexible (a horizontal marginal cost curve yields the Bertrand outcome) or totally inflexible (a vertical marginal cost curve at capacity yields Cournot).

3. Oligopoly II : Market competition in static games, with differentiated goods

Competition with differentiated products

We explore and contrast the Bertrand and Cournot approaches within a common framework of differentiated products with symmetric linear demands.

"As we shall see, there are again significant contrasts between markets where firms compete with prices and quantities. Firstly, we will compare the equilibrium prices and show that the Cournot equilibrium yields a higher price than the Bertrand equilibrium. Thus, as in the case of homogeneous products, Cournot competition is less competitive than Bertrand competition although the contrast is less."

In the case of two goods, the considered demand is :

$$x_1 = 1 - p_1 + ap_2 \tag{2}$$

$$x_2 = 1 - p_2 + ap_1, \tag{3}$$

the goods are substitute whenever $a > 0$ and complement when $a < 0$.

Cournot with differentiated products

To explore the Cournot model, we need to invert (2) and (3) :

$$p_1 = a_0 - a_1 x_1 - a_2 x_2 \tag{4}$$

$$p_2 = a_0 - a_1 x_2 - a_2 x_1, \tag{5}$$

with $a_0 = \frac{1+\alpha}{1-\alpha^2}$, $a_1 = \frac{1}{1-\alpha^2}$, $a_0 = \frac{\alpha}{1-\alpha^2}$

We can check that the Cournot competition ends up with symmetry, each firm choosing $x_1 = x_2 = x^c$ such that

$$x^c = \frac{1 + \alpha - c(1 - \alpha^2)}{2 + \alpha}$$

with the same clearing price $p_1 = p_2 = p^c$

$$p^c = \frac{1 + c(1 - \alpha)}{(2 + \alpha)(1 - \alpha)}$$

Bertrand with differentiated products

By differentiating (2) and (3) we find that there is a equilibrium price $p1 = p2 = pb$ with corresponding output, price and price-cost margins :

$$p^b = \frac{1+c}{2-\alpha} \tag{6}$$

$$x^b = \frac{1-c(1-\alpha)}{2-\alpha} \tag{7}$$

$$\mu^b = \frac{1-c(1-\alpha)}{2-\alpha} \tag{8}$$

- Clearly, $p^b < p^c$, $x^c < x^b$ and $\mu^c > \mu^b$
- With product differentiation firms have some monopoly power even with price competition and do not have the same incentives for undercutting their competition as in the homogeneous goods case.

General result with differentiated products

With *multiple equilibria* the comparison between Cournot and Bertrand becomes conceptually more complex. Vives (1985) has established a result that for very general conditions there exists a Bertrand equilibrium which involves a lower price than any Cournot equilibrium.

Of course there are other contrasts to be drawn between Cournot and Bertrand- Nash equilibria. For example, there is the question of welfare analysis employing standard consumer surplus. A simple example employing the linear demand system (2) and (3) is provided by Singh and Vives (1984) which shows that the sum of consumer and producer surplus is larger in Bertrand than in Cournot-Nash equilibrium both when goods are substitutes and complements.

4. Oligopoly III : Market competition in dynamic games

Why should we consider dynamics ?

When studying models where firms have several strategic variables, it is important to recognize that some are typically more long run variables than others.

Particularly, if we analyze games where firms take entry, R&D investment are long term decision while price decisions are short term.

By dynamic games, we think about games in which in the first stage, firms choose wether to enter or not, and in the second stage, they decide how much R&D they want to carry out, and in the third stage, what price they want to sell their product at.

Precommitment

Dynamics allow to consider precommitment strategies, in which firms commit at the very begining to a long term variable that will modify the competition game.

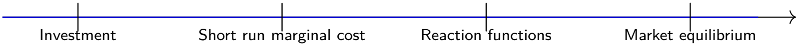
By "precommitment" it is meant that the firm takes some action prior to competing in the product market which commits it to a certain course of action.

Recent literature has investigated two methods of precommitment which have received much recent attention –precommitment through investment and precommitment through delegation.

Firms can take actions such as investment decisions and choice of managers that are irreversible (in the sense of being "fixed" over the market period) and which alter the firm's reaction function thus shifting the Nash equilibrium in the market.

Precommitment to capital stocks

The dynamic sequence of the game is the following



More clearly, there are two stages of the game, a stage in firms choose their capital, and a second stage in which they choose output/price.

We suppose that the production function is

$$x_i = \sqrt{k_i}\sqrt{L_i} \quad \text{implying} \quad c(x_i, k_i) = rk_i + \frac{x_i^2}{k_i}$$

an increase in investment lowers the marginal cost of producing output.

Analyzing the two stage game by backward induction

In stage 2, firm choose quantity. As

$$\pi_i = xi(a0 - a1xi - a2xj) - rki - \frac{x_i^2}{k_i}$$

the optimal x_i, given k_i and x_j is

$$x_i = \frac{a0 - a2xj}{2a1 + (2/ki)}$$

We see that by increasing its investment, firm I shift its reaction function to a lower production.

In stage 1, firm 1 choose k_1, by setting $\frac{\partial \pi_1}{\partial k_1} = 0$. One verifies that this induces :

$$\frac{\partial c}{\partial k_1} = -a_2 \frac{\partial x_2}{\partial k_1} x_1 > 0$$

The optimal strategy is not to minimize the cost, but an overcapitalization.

Bertrand in the Investment game

In the Bertrand case an exactly analogous argument applies for strategic investment. However, there is the opposite result of undercapitalization.

Clearly, the result of strategic investment models depends on the nature of product market competition.