Competition Policy & Game Theory

Chapitre II
Introduction to Imperfect Competition Models
Roadmap

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1. Monopoly
2. Oligopoly in static games, with homogeneous goods, Bertrand and Cournot
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Introduction
1. Monopoly: single and multi-product
The market demand is characterized by $D(p)$, negatively sloped, and concave, there is only one firm, which cost function is $C(q)$, increasing and convex.

The unique firm can decide price and quantity. It is constrained by the demand. When she post one price, the maximum quantity she sells is $D(p)$. So the optimal tariff of the monopole will depend upon the demand.

Let’s write the revenue and the cost as a function of the sold quantity: $R(q) = q \times p(q)$ where $p(q)$ is the inverse demand function. The monopole behavior is represented by the rule $\text{Marginal revenue} = \text{Marginal Cost}$, with $R_m(q) = p(q) + qp'(q) = p(1 + p'(q)\frac{q}{p}) = p \left(1 + \frac{1}{\varepsilon}\right)$, with $\varepsilon \leq 0$ being demand elasticity. Optimal behavior:

$$p \left(1 + \frac{1}{\varepsilon}\right) = C_m \iff \frac{p - C_m}{p} = -\frac{1}{\varepsilon} \quad (1)$$

The left-hand side is the Lerner index, a measure of market power, that is the ability of a firm to set prices above marginal costs.
Let consider the case such that \( c_m = 2q, \ D(p) = 120 - p \), which we rewrite \( p(q) = 120 - q \)

1) Compute \( q^* \) et \( p^* \) in a pure competition market

2) Compute \( q^{**} \) and \( p^{**} \) in a monopoly market if the firm is alone
In case of pure competition \((p, q)\) is on the supply curve when \(p = c_m\), i.e. when \(p = 2q\) and \((p, q)\) is on the demand curve when \(p = 120 - q\). Both curves cross whenever \(2q = 120 - q\), that is when \(q = 40\), implying \(p = 80\).

\[
p^* = 80 \quad q^* = 40
\]

The monopolist tarification is such that \(R_m = c_m\); In our example, \(R(q) = q(120 - q)\), \(R_m = 120 - 2q\), \(R_m = c_m\) is equivalent to \(120 - 2q = 2q\), and \(q = 30\). For that quantity, the maximal price that the firm can charge is \(p = 120 - 30 = 90\).

\[
p^{**} = 90 \quad q^{**} = 30
\]
Multi-product Monopoly

In the real world firms could produce and sell many products. However, whenever demand and cost of one product does not affect demande and costs for other products, then, the problem of the monopolist reduces to the problem studied in the one-product case. Indeed, for instance in the 2 product case, the monopoly profit is the sum of two terms,

\[ \pi = (p_1D_1(p_1) - C_1(D_1(p_1))) + (p_2D_2(p_2) - C_2(D_2(p_2))), \]

that can be maximized independently, each maximization problem going to the condition

\[ \frac{p_i - C'(q_i)}{p_i} = \frac{1}{\varepsilon_i} \]

- Solving this problem independently is true, even if the demand and the costs structure are different. This is as if there where two divisions in the considered firm
- That will not be true if interdependences exists on the demand side or on the costs side
Often, a firm sells a range of products that are to some extent substitutable with each other. For substitute products, the increase on the price of one product will increase the demand for the others that become more convenient. That is, in the two good case:

\[ q_i = q_i(p_i, p_j) \quad \frac{\partial q_i}{\partial p_j} > 0 \]

For complementary goods, an increase on the price of one product will decrease the demand for the others (by decreasing the demand for the first product, demand for complements is also discouraged). In the two good case:

\[ q_i = q_i(p_i, p_j) \quad \frac{\partial q_i}{\partial p_j} < 0 \]
Let assume the two products with the following demand functions

\[ q_i = a - bp_i + gp_j, \]

\( g \) being positive in the substitute case, and negative in the complement case. Let suppose that the marginal cost to produce the good is constant, the same \( c \) for the two products.

\[ p_1 = p_2 = \frac{a + bc}{2b - g} \]

Write each profit function as a function of \( p_i, p_j \), and derive it relative to \( p_i \).

**Prove in the two good case that**

- Interpret why we only care when \(|g| < |b|\); Interpret the role of \( g \)
- Interpret with comparing to the case \( g = 0 \). (monopoly pricing)
When the products are complements, they exercise a positive externality on each other and the monopolist internalises it by decreasing its prices. A lower price for good one stimulates sales of good 2 and vice versa.

If products 1 and 2 were sold by two distincts monopolists, consumers would pay more for them than when they are sold by the same firm.

When the products are substitutes, the externality they exercise on each other is negative, and the monopolist controls for it by raising prices (a lower price of good 1 crowds out sales of good 2 and vice versa).

If products 1 and 2 were sold by two distinct firms, consumers would pay less than when they are sold by the same firm.
The preceding model apply to the case the same product is sold in sequential markets. For instance, with $q_1, q_2, p_1, p_2$ quantity and prices of period one and two, when the demand is

$$q_1 = a - bp_1 \quad \quad q_2 = a - bp_2 + \lambda q_1$$

Prove in the case $c = 0, a = 1, b = \frac{1}{2}$ that $p_1 = 2 \frac{1 - \lambda}{2 - \lambda}$ and $p_2 = \frac{2}{2 - \lambda}$

Write ONLY ONE profit function as a function of $p_1, p_2$, and derive it relative to the two prices.

Check that when $\lambda$ rises, then, the first period price goes down and the second period price, up. Interpret

Notice that the solution in the general case is $p_1 = \frac{a(1 - \lambda) + cb}{b(2 - \lambda)} \quad \quad p_2 = \frac{a + cb(1 - \lambda)}{b(2 - \lambda)}$
**Intertemporal complementarity : Introductory price offers** When \( \lambda > 0 \), the monopolist realises that there is a positive intertemporal demand externality, and it internalises it by decreasing the price relative to the price it would set if there was no future market.

**Intertemporal substituability : Durable goods** In the case where \( \lambda < 0 \), higher sales in the first period decrease the demand in the second period. As a result, the monopolist keeps the first period price higher than in the hypothetical case where the product is sold only once, to internalise the negative demand externality arising across periods. This is a reduced form of the case of a durable good monopolist: the more consumers buy in the first period the fewer will buy in the second. In that class of models, the equilibrium prices tend to decrease over time.
Interdependant costs

We say that there are cost externalities, in a two product production model, when the global cost cannot be written as the sum of the cost of producing $q_1$ good 1 and the cost of producing $q_2$ good 2 as, in the following:

$$C(q_1, q_2) = c(q_1 + q_2) + \mu q_1 q_2$$

- When $\mu > 0$, there exist diseconomies of scope between the two products, as the higher the output of one product, the higher the marginal cost of the other product. This is the case, when both products make use of limited natural resources or inputs.

- When $\mu < 0$, there exists economies of scope. There are many instances in the real world where producing two foods jointly gives rise to cost savings relative to the case where each product is produced separately.
Interdependant costs (II)

One can check that under symmetry, when the demand are $q_i = a - bp_i$, and not trivial (in particular, $a - bc > 0$, then when the prices are identical, $p_1 = p_2 = p_m$ one can check that

$$p_m = \frac{a(1 + b\mu) + cb}{b(2 + b\mu)}$$

It happens then that the derivative of $p_m$ is increasing with $\mu$: the stronger the cost externality between the two products, the higher the equilibrium price set by the monopolist.

- When $\mu < 0$, the prices are lower than in the benchmark case where $\mu = 0$: the firm wants to stimulate its output. A multiproduct monopolist charges lower prices than two distinct monopolists.
- When $\mu > 0$, the prices are higher than in the benchmark case. The monopolists wants to reduce the output of each good.
Learning by doing

The cost of production of many goods and services decreases with the experience accumulated in producing those goods and services

\[ \mu < 0 \]

In such a situation a monopolist will want to decrease prices in the early stages of life of a product, in order to increase output and “go down the learning curve”.
2. Oligopoly I : Market competition in static games, with homogeneous goods, Bertrand and Cournot
To be continued