Université d'Orléans

Master International Economics

Competition policy and game theory

Exercise Set 1 : game theory 's basic elements

Fall 2019

You should know some definitions about an objet we call a game : simultaneous and dynamic games, normal form and extensive form game, and the main solutions concepts.

In a game, a finite (sometimes infinite) number of rational agents have to take decisions, Which affects the welfare of every body. A game is described by the list of players, the rules of the games, i.e., the allowed actions of the players and the interactions the payoff resulting in any realizable history. The actions choices are either simultaneous or asynchronous.	A main feature of game theory is payoff's interdependency. That is for each player, her payoff could depend not only on the action she takes but also on the actions taken by the other players. Then, when choosing its action, the player should anticipate what the other players would choose. More precisely, we define for each player what we call her strategies, i.e., her unilateral decision of what she will do in any *node* she has to take an action. Analyzing a game is then to make the list of consistent a set of strategies.	The solution concept you should know is Nash Equilibrium : a set of strategies is a Nash Equilibrium is there is no unilateral deviation of any player. That is, when considering the equilibrium set of strategies, none of the player could increase her payoff by changing unilaterally her deviation.

1 Analyzing the equilibrium in a simultaneous game

This is about a simultaneous game in which two players A and B, called *firms* have to choose a price to sell at least one unit of good. For each firm the choice set is continuous, equal to $\mathbb{R}_+$

Let consider the following competition game between two firms, A and B. Both of them share a market in which there is a continuum of agents. Each buyer reservation price is equal to 1. Each firm 's marginal cost is equal to $c > 0$. The game is simultaneous : whenever $1 \geq p_A$ and $p_A < p_B$, firm A wins all the market, $q_A = 1$ whenever $1 \geq p_A = p_B$, there is a tie break rule : the market is divided among the competitors and $q_A = 1/2$. Firm i 's payoff is :

$$\pi_i = q_i(p_i - c)$$

1) Prove that (c, c) is one equilibrium of the game
2) Prove that there is only one equilibrium of the game, that induces zero profit.

2 Three finite Games

In seaching for the Nash equilibria of a game, you have to analyze the rationality of each player by eliminating the strategy they would never choose, because they are dominated, contingent on the strategies of the other players. Consider the three following games (player A 's action $\in \{a_1, a_2, a_3, a_4\}$, player B 's action $\in \{b_1, b_2, b_3, b_4\}$) :

	b_1	b_2	b_3	b_4
a_1	1,2	3,4	5,6	7,8
a_2	9,10	11,12	13,14	15,16
a_3	17,18	19,20	21,22	23,24
a_4	25,26	27,28	29,30	31,32

Left

	b_1	b_2	b_3	b_4
a_1	19,2	15,10	13,16	1,20
a_2	17,28	11,4	3,12	29,18
a_3	9,24	5,30	31,6	27,14
a_4	7,22	33,26	23,32	21,8

Center

	b_1	b_2	b_3	b_4
a_1	1,32	2,31	3,30	4,29
a_2	5,28	6,27	7,26	8,25
a_3	9,24	10,23	11,22	12,21
a_4	13,20	14,19	15,18	16,17

Right

1) Compute the Nash equilibrium of the left game. Be very precise on the followed methodology.
2) Compute the Nash equilibrium of the right game. Be very precise on the followed methodology.
3) Compute if there is some Nash equilibrium in the center game. Be very precise on the followed methodology.