

You should know the definition of the competitive equilibrium market, understand the behavior of a monopoly, it's strategy that can be formulated in terms of price or in terms of quantity, and the basic Cournot and Bertrand first basic and historical oligopoly models.

<p>Price competition usually designs an oligopoly where each firms posts simultaneously prices. Then, there is a demand addressed to each firm, that depends on the price posted by each firm ; let denote it $q_1^d(p_1, p_2)$ and $q_2^d(p_1, p_2)$. The basic model have been developed by Bertrand, in the case of an homogeneous good, and similar firms without capacity constraints, and marginal constant cost equal to c. The unique Nash equilibrium is such that $p_1^* = p_2^* = c$.</p>	<p>Quantity competition usually designs an oligopoly where each firms posts simultaneously quantity. Then, we suppose that the price will be the one that equilibrates demand and the produced goods, q_1^* and q_2^* ; we denote it p^*. The basic model have been developed by Cournot, in the case of an homogeneous good, and similar firms without capacity constraints, and marginal constant cost equal to c. The equilibrium is such that the corresponding price $p^* > c$, and the price cost margin is decreasing with the number of competing firms. In the case $n = 1$, the unique firm behave as a monopoly and p^* is the monopoly price.</p>
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1 Back to the monopoly, regular and discriminant

Let consider an homogeneous non divisible good, 28 consumers and one producer. Let suppose that in that market the consumers are interested in consuming 1 or 0, depending on the comparison of the selling price with their reservation price. The firm capacity is equal to maximum 28 goods produced, with a marginal cost increasing. The characteristics of the consumers and of the producers are the following :

$r^1 = 979 \quad r^2 = 934 \quad r^3 = 868 \quad r^4 = 836 \quad r^5 = 806 \quad r^6 = 799 \quad r^7 = 792 \quad r^8 = 775 \quad r^9 = 685 \quad r^{10} = 654 \quad r^{11} = 604 \quad r^{12} = 591 \quad r^{13} = 532 \quad r^{14} = 512$
 $r^{15} = 512 \quad r^{16} = 490 \quad r^{17} = 469 \quad r^{18} = 460 \quad r^{19} = 436 \quad r^{20} = 330 \quad r^{21} = 311 \quad r^{22} = 240 \quad r^{23} = 238 \quad r^{24} = 219 \quad r^{25} = 190 \quad r^{26} = 92 \quad r^{27} = 91 \quad r^{28} = 22$
 $c^1 = 5 \quad c^2 = 12 \quad c^3 = 29 \quad c^4 = 94 \quad c^5 = 94 \quad c^6 = 134 \quad c^7 = 134 \quad c^8 = 139 \quad c^9 = 156 \quad c^{10} = 325 \quad c^{11} = 370 \quad c^{12} = 395 \quad c^{13} = 435 \quad c^{14} = 470$
 $c^{15} = 501 \quad c^{16} = 511 \quad c^{17} = 521 \quad c^{18} = 549 \quad c^{19} = 581 \quad c^{20} = 652 \quad c^{21} = 729 \quad c^{22} = 738 \quad c^{23} = 748 \quad c^{24} = 778 \quad c^{25} = 808 \quad c^{26} = 850 \quad c^{27} = 886 \quad c^{28} = 903$

1) Describe the efficient allocation of this market et denote by q^* the corresponding quantity produced. Compute the global surplus.

The allocation is that one that maximises the total surplus of the economy. Then, to built this allocation, we can sequentially associate the best consumer (i.e. the consumer whose reservation price is the highest) with the best firm (i.e. the firm which cost is the smallest)

Put on excel the ordered increasing reservation price, and the ordered costs. Production is efficient and can be increased when the reservation price of next consumer is greater than the cost of next firm. In the spread sheet, it is clear than the efficient production should not exceed $q = 15$.

label i	1	2	14	15	16	17
Consumer r_i	979	934	512	512	490	469
Firm c_i	5	12	470	501	511	521

[Here above I mask some column of the spread sheet, in order to show the label (i=16) after which the reservation price is lower than the cost.]

For the surplus, I add the line $r_i - c_i$ in the spreadsheet, and then, I compute the sum of the surplus till $q = 15$. We find

$$S^* = 7586$$

Then, the following spread sheet to compute the profit of each unit produced at different prices

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
34		Firm participation															
35	979		1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	934		1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
37	868		1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
38	836		1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
39	806		1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
40	799		1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
41	792		1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
42	775		1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
43	685		1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
44	654		1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
45	604		1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
46	591		1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
47	532		1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
48	512		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
49	512		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
50		Firm profit															
51	979		979	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	934		929	922	0	0	0	0	0	0	0	0	0	0	0	0	0
53	868		863	856	839	0	0	0	0	0	0	0	0	0	0	0	0
54	836		831	824	807	742	0	0	0	0	0	0	0	0	0	0	0
55	806		801	794	777	712	712	0	0	0	0	0	0	0	0	0	0
56	799		794	787	770	705	705	665	0	0	0	0	0	0	0	0	0
57	792		787	780	763	698	698	658	658	0	0	0	0	0	0	0	0
58	775		770	763	746	681	681	641	641	636	0	0	0	0	0	0	0
59	685		680	673	656	591	591	551	551	546	529	0	0	0	0	0	0
60	654		649	642	625	560	560	520	520	515	498	329	0	0	0	0	0
61	604		599	592	575	510	510	470	470	465	448	279	234	0	0	0	0
62	591		586	579	562	497	497	457	457	452	435	266	221	196	0	0	0
63	532		527	520	503	438	438	398	398	393	376	207	162	137	97	0	0
64	512		507	500	483	418	418	378	378	373	356	187	142	117	77	42	11
65	512		507	500	483	418	418	378	378	373	356	187	142	117	77	42	11
66																	

Then, we finish our spread sheet by compute the agregate profit at different prices

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
34		Firm participation															
35	979		1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	934		1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
37	868		1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
38	836		1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
39	806		1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
40	799		1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
41	792		1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
42	775		1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
43	685		1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
44	654		1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
45	604		1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
46	591		1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
47	532		1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
48	512		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
49	512		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
50		Firm profit															
51	979	Q51	974	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	934	1851	929	922	0	0	0	0	0	0	0	0	0	0	0	0	0
53	868	2558	863	856	839	0	0	0	0	0	0	0	0	0	0	0	0
54	836	3204	831	824	807	742	0	0	0	0	0	0	0	0	0	0	0
55	806	3796	801	794	777	712	712	0	0	0	0	0	0	0	0	0	0
56	799	4426	794	787	770	705	705	665	0	0	0	0	0	0	0	0	0
57	792	5042	787	780	763	698	698	658	658	0	0	0	0	0	0	0	0
58	775	5559	770	763	746	681	681	641	641	636	0	0	0	0	0	0	0
59	685	5368	680	673	656	591	591	551	551	546	529	0	0	0	0	0	0
60	654	5418	649	642	625	560	560	520	520	515	498	329	0	0	0	0	0
61	604	5152	599	592	575	510	510	470	470	465	448	279	234	0	0	0	0
62	591	5205	586	579	562	497	497	457	457	452	435	266	221	196	0	0	0
63	532	4594	527	520	503	438	438	398	398	393	376	207	162	137	97	0	0
64	512	4387	507	500	483	418	418	378	378	373	356	187	142	117	77	42	11
65	512	4387	507	500	483	418	418	378	378	373	356	187	142	117	77	42	11
66																	

It comes that that maximum profit comes when $p^{**} = 654$, that is, when $q^{**} = 10$. The maximum profit is then :

$$\pi^{**} = 5418.$$

4) Analyze the behavior of the monopoly if he is able to price discriminate, and if he knows (still) the characteristics of the consumers. Compute in particular, π^d the profit of the discriminant monopoly. [Hint, back to the Spreadsheet].

When one allows for price discrimination, then, each unit will be sold at the reservation price of the corresponding consumer.

Then, the same number of goods is sold, as in the competitive market, so, here, $q^d = 15$.

Back to the spreadsheet, we compute the profit for each unit, as the difference between the reservation price and the cost.

The discriminant monopoly profit is then the whole surplus of the competitive market, that is

$$\pi^d = 7586$$

5) Would you say that the discriminant monopoly is efficient.

The answer is yes. From a normative point of view, the global surplus is the same as in the competitive market, the production also, the consumers that are served are the same as in the competitive market. The only difference being that here, all the surplus comes to the firm. But firms belong to humans...

2 A Cournot Game with asymmetric costs

Let consider an oligopolistic market such that the inverse demande is $p = p(q)$. Two and only two firms, firm 1 and firm 2 produce the good, respectively at the unit cost c_1 and c_2 and sell the produced units of good to the consumers. They choose individually a quantity to produce (denoted q_1, q_2), thereafter the selling price is set to $p(q_1 + q_2)$ (all produced goods are sold).

1) Compute the firm's payoff and explain why the two firms are in strategic interaction

In the general formulation

$$\pi_1 = (p(q_1 + q_2) - c_1)q_1 \quad (4)$$

$$\pi_2 = (p(q_1 + q_2) - c_2)q_2 \quad (5)$$

The firms are in strategic interaction, as for example, firm 1's profit depend not only on q_1 but also on q_2 .

2) Consider that $p(q) = 1 - q$ and write the profit functions

In the particular example, when the demand is $p(q) = 1 - q$, then, the payoffs are written :

$$\pi_1 = (1 - q_1 - q_2 - c_1)q_1 \quad (6)$$

$$\pi_2 = (1 - q_1 - q_2 - c_2)q_2 \quad (7)$$

3) Compute reaction functions for each firm and the Nash equilibrium

We compute the derivative of π_1 relative to q_1 and the derivative of π_2 relative to q_2 .

$$\frac{\partial \pi_1}{\partial q_1} = (1 - 2q_1 - q_2 - c_1) \quad (8)$$

$$\frac{\partial \pi_2}{\partial q_2} = (1 - q_1 - 2q_2 - c_2) \quad (9)$$

and then, we find the optimal choice of firm i when the derivative of π_i relative to q_i is equal to zero.

We find then

$$q_1 = \frac{1}{2}(1 - q_2 - c_1) \quad (10)$$

$$q_2 = \frac{1}{2}(1 - q_1 - c_2) \quad (11)$$

To solve that, we add the two equations, and it comes $q = \frac{1}{2}(1 - q - c_1 - c_2)$, which is equivalent to $3q = 1 - c_1 - c_2$ or $q = \frac{1}{3}(1 - c_1 - c_2)$. Then (??) and (??) can be rewritten

$$\frac{1}{2}q_1 + \frac{1}{2}q = \frac{1}{2}(1 - c_1) \quad \frac{1}{2}q_2 + \frac{1}{2}q = \frac{1}{2}(1 - c_2)$$

which ends up to :

$$q_1 = \frac{2}{3}(1 - c_1) + \frac{1}{3}c_2 \quad (12)$$

$$q_2 = \frac{2}{3}(1 - c_2) + \frac{1}{3}c_1 \quad (13)$$

4) Check that $q_1^* > q_2^*$ and interpret.

We only make the computation $q_1^* - q_2^*$:

$$\begin{aligned} q_1^* - q_2^* &= \left(\frac{2}{3} - \frac{2}{3}c_1 + \frac{1}{3}c_2\right) - \left(\frac{2}{3} - \frac{2}{3}c_2 + \frac{1}{3}c_1\right) \\ &= c_2 - c_1, \end{aligned}$$

the conclusion is immediate, as by assumption $c_2 > c_1$.

$c_2 > c_1$ means that firm 1 is more efficient than firm 2, so it is not surprising that $q_1^* > q_2^*$: the Cournot market use more efficient firms.

5) Check the conditions under which $q_2^* = 0$.

$q_2^* = 0$ is equivalent to $2(1 - c_2) + c_1 = 0$ or to $c_2 = 1 + \frac{1}{2}c_1$. So when the difference in the costs is quite large, firm 2 is so much less efficient than firm 1, that it will not be used.

6) Explain why in your solution the condition $c_2 < 1 + \frac{1}{2}c_1$ is needed in the computation of question 3. Find the formal reason, and then make the interpretation.

As we already understood from the preceding question, $c_2 = 1 + \frac{1}{2}c_1$. In the same line, when $c_2 > 1 + \frac{1}{2}c_1$ we would have had $q_2^* \leq 0$, which does not make sense. Then, we have to conclude that Firm 2 does not enter in the Cournot market when its costs are too large compared to the firm 1 cost.

7) Explain how the market works under the condition $c_2 \geq 1 + \frac{1}{2}c_1$. Find and interpret the behavior of firm 1.

As we understood, under the condition $c_2 \geq 1 + \frac{1}{2}c_1$ firm 2 does not enter the market. Then, (??) is no more verified, and (??) should be rewritten with the additional condition $q_2 = 0$:

$$q_1^* = \frac{1}{2}(1 - c_1) \quad (14)$$

As we suspect, this should correspond to a monopoly behavior. To make sure of that, we compute the corresponding price-cost margin. Notice that the price is $p^* = 1 - \frac{1}{2}(1 - c_1) = \frac{1}{2}(1 + c_1)$ and firm 1 price cost margin equals to

$$\frac{p^* - c_1}{p^*} = \frac{\frac{1}{2}(1 - c_1)}{\frac{1}{2}(1 + c_1)} = \frac{1 - c_1}{1 + c_1}$$

One checks that this is exactly the opposite of the inverse of the elasticity of demand of the market computed when the quantity is q_1 . Indeed, $q = 1 - p$, $\varepsilon_p = \frac{p}{q} * -1$, which value is at p^* and q_1^* :

$$\varepsilon_p = \frac{\frac{1}{2}(1 + c_1)}{\frac{1}{2}(1 - c_1)}.$$

In conclusion, under the condition $c_2 \geq 1 + \frac{1}{2}c_1$, firm 1 behaves as a monopolist.

Let consider the introduction of a VAT. When any firms receive p , the consumers pay $p(1 + t)$.

8) Explain how the VAT will affect the equilibrium. What are the variables that change significantly?

The price depending on the quantity will change : When any firms receive p , the consumers pay $p(1 + t)$. Say it differently, when any firms post the price $p/(1 + t)$, the consumers pay p . Then when the firms produce q , the firm will sell everything when they post the price $p(q)/(1 + t)$

Then, the payoff functions will change :

$$\pi_1 = \left(\frac{1 - q_1 - q_2}{1 + t} - c_1 \right) q_1 \quad (15)$$

$$\pi_2 = \left(\frac{1 - q_1 - q_2}{1 + t} - c_2 \right) q_2 \quad (16)$$

Then, the derivative of the payoff functions will change :

$$\frac{\partial \pi_1}{\partial q_1} = \left(\frac{1 - 2q_1 - q_2}{1 + t} - c_1 \right) \quad (17)$$

$$\frac{\partial \pi_2}{\partial q_2} = \left(\frac{1 - q_1 - 2q_2}{1 + t} - c_2 \right) \quad (18)$$

and Then, the first order conditions will change :

$$1 - 2q_1 - q_2 = (1 + t)c_1 \quad (19)$$

$$1 - q_1 - 2q_2 = (1 + t)c_2 \quad (20)$$

$$(21)$$

In this particular framework, all is *as if* costs had been multiplied by $(1 + t)$. There is no need to make more computations. The formulas that we found in the preceding question are enough to compute the equilibrium : quantity will be smaller !

3 Bertrand and Cournot Models

- 1) In two three lines, describe a basic model of duopoly, the differences between the Cournot and the Bertrand mechanisms,
- 2) In two three lines, describe a basic model of duopoly, the differences between the Cournot and the Bertrand equilibria.

4 Bertrand under capacity constraint

Consider two firms that have the same marginal cost c , but they differ, holding a different capacity $k_i < D(p_i = c)$: when charging at marginal cost, a firm would have to supply a larger number of units than its capacity would allow it to do. We suppose that those two firms compete in prices.

- 1) Show that $(p_i^*, p_j^*) = (c, c)$ is not an equilibrium of the competition game.

To prove this result, we just need to find a deviation that leaves one of the firms better off. At the candidate $(p_i^*, p_j^*) = (c, c)$, firms make zero profit. If firm i deviate by proposing $p_i' = c + \varepsilon$, with ε small, contrary to the standard Bertrand game, some consumers will address it and it will make positive profits. Indeed, at the deviation stage, all consumers would like to buy from firm j, but j cannot serve them all (as j's capacity $k_j < D(c)$). Some consumers are rationed, and will have to buy instead from firm 1, that will therefore make positif profits.

We suppose that the market demand is $q = 1 - p$ and that the cost is $c = 0$.

- 2) Prove that if firm 2 produces at capacity $q_2 = k_2$, then, firm 1 best response (without considering its capacity) is : $R_1(k_2) = (1 - k_2)/2$. Compute in the same spirit firm 2 's best response (without considering its capacity) whenever firm 1 produces at capacity $q_1 = k_1$.

This is as in the Cournot model, firm 1 chooses q_1 in order to maximize

$$q_1(1 - q_1 - k_2),$$

a value that is maximum when

$$(1 - 2q_1 - k_2) = 0 \iff q_1 = \frac{1}{2}(1 - k_2),$$

which we can denote $R_1(k_2) = \frac{1}{2}(1 - k_2)$.

a symmetric computation gives $R_2(k_1)$.

We say that capacities are small whenever $k_1 \leq R(k_2)$ and when $k_2 \leq R(k_1)$

3) Interpret that condition, and already think about what will happen in that market.

When one of the firms produces at its capacity, then, the other firm is constrained on her production : She cannot produce her best response \Rightarrow she will also produce at her capacity constraint.

4) When both firms compete with prices, show that $p_1^* = p_2^* = 1 - k_1 - k_2$ is an equilibrium.

ROADMAP : to prove that there is no deviation for any of the firms ; as the problem is symmetric, we do the job for firm 1.

Let suppose that firm 2 's price is $p_2^* = 1 - k_1 - k_2$, and that firm 2 produces q_2^* less (or equal) than k_2 . Remember that at that price, the market is asking for the quantity $k_1 + k_2$, which insures that firm 1 could sell any of its production less than k_1 .

So, at the price $p_2^* = 1 - k_1 - k_2$, F1 would sell k_1 , earning $\pi_1^* = k_1(1 - k_1 - q_2^*)$

If F1 decreases the price, F1 would have more demande, but he could not afford it, then, still producing k_1 selling at a lower price. That deviation makes the profit decrease.

Can F1 increase the price? F1 knows that due to the capacity constraint of F2, the demand she will have will not be zero. We have then to compute the marginal revenue of F1 when she increases the price to $p_2^* + \varepsilon$. The revenue is $(D(p_2^* + \varepsilon) - k_2) * (p_2^* + \varepsilon)$, the marginal revenue will then be

$$D(p_2^*) - k_2 + p_2^* D'(p_2^*). \tag{22}$$

If p_2^* is near the monopoly price then, the value $D(p_2^*) + p_2^* D'(p_2^*)$ is nearby 0, and the term $-k_1$ in equation (??) make the expression negative. So increasing the price does not imply increasing profits.

5) When both firms compete with prices, show that $p_1^* = p_2^* = 1 - k_1 - k_2$ is the unique equilibrium.

First, there is no asymmetric equilibrium $p_1^* \neq p_2^*$. This is the classical argument

If both firms post a lower price than $p(k_1 + k_2)$ one of the firm will propose $p(k_1 + k_2)$, making greater profits

If both firms post a greater price than $p(k_1 + k_2)$ this is not a good strategy, as the firms should try to produce more (at a lower pricee)

***** END OF EXERCISE SET 4 *****