Université d'Orléans

Master International Economics

Competition policy and game theory

Exercise Set 5 : Cournot and Bertrand, advanced

Fall 2019

You should know all about the basic Cournot and Bertrand oligopoly models. The first exercise is long and tedious, but you should come back every day to do a question more, and check for the first question. Like that, it is a very, very good training for that class.

Bertrand competition usually designs an oligopoly where firms compete in price. There is a demand addressed to each firm, that depends on the price posted by each firm ; let denote it $q_1^d(p_1, p_2)$ and $q_2^d(p_1, p_2)$. The basic model have been developed by Bertrand, in the case of an homogeneous good, and similar firms without capacity constraints, and marginal constant cost equal to c. The unique Nash equilibrium is such that $p_1^* = p_2^* = c$. Usually, people think that Bertrand competition is the toughest possible degree of product market competition.	Cournot competition usually designs an oligopoly where each firms posts simultaneouly quantity. It is supposed that there is implicitly an auction market clearing price, and that the selling price is the one that equilibrates demand and the produced goods, q_1^* and q_2^* ; we denote it p^*. In the case of an homogeneous good, and similar firms without capacity constraints, and marginal constant cost equal to c, the unique equilibrium is symmetric, such that the corresponding price $p^* > c$, and the price cost margin is decreasing with the number of competing firms. When $n \to +\infty$, the price tends to be c.

1 Bertrand Competition with capacity constraint

A capacity constraint for firm i, denoted k_i, is the maximum amount of good that she can produce. We study in this exercise TWO firms, i and j, which marginal cost is constant, equal to 1 and that differ in their capacity constraint. In the first part of the exercise, we suppose that capacity constraints are exogeneous, that the aggregate (inverse) demand is $p = a - q$, with $a > 1$. We consider the Bertrand competition game between those two firms.

1) Make a synthetic table in which you put the price and quantity for that economy in the pure competition case and in the monopoly (or cartel) case. After a general presentation, make a second table in the case $a = 2$ and $c = 1$.

For the case being, we suppose that both firms have a capacity constraint, respectively k_1 and k_2 and that $k_1 + k_2 = \frac{2}{3}(a - c)$. In other word, the firms could not serve the whole market in the pure competition setting, but they could act as a cartel.

2) Compute the price such that both firms work on the basis of their entire capacity (and sell all their stock). Show that this price (that we denote p^k) is between p^* and p^M.

3) Show that $k_1 < a - c$. Interpret that condition and then show that $(p_i^*, p_i^*) = (c, c)$ is not a Bertrand equilibrium.

We want to study thereafter under which condition (p^k, p^k) is an equilibrium of the usual price competition between firm 1 and 2.

4) Show that when firm 2 anticipates that firm 1 post $p_1 = p^k$, then firm 2 has no interest to undercut firm 1 (that is to lower the prices).

To study the price competition, we have to make some assumption on the demand, when there is some rationing. In particular we suppose that in situations in which there are two prices $p_i < p_j$, then consumers first buys at the lowest price, and then, the rationed consumers could accept to pay p_j if their reservation price is above p_j : they buy if their reservation price is above p_j and if they were rationed at the price p_i.

5) Analyze when firm 2 anticipates that firm 1 post $p_1 = p^k$, how firm 2 could have interest to propose to increase its prices and propose $p_2 > p_1$. Analyze with details that situation. Show in particular that the residual demand to firm 2 is $k_2 - \varepsilon$, write the deviation profit of firm 2 and think about it : under which condition a deviation could be profitable ?

6) Analyze when firm 1 anticipates that firm 1 post $p_1 = p^k$, how firm 1 could have interest to propose to increase its prices and propose $p_1 > p_2$. Analyze with details that situation. Show in particular that the residual demand to firm 1 is $k_1 - \varepsilon$, write the deviation profit of firm 1 and think about it : under which condition a deviation could be profitable ?

7) From the two preceding questions, deduce that whenever $k_1 = k_2$, then, (p^k, p^k) is an equilibrium of the price competition game

8) Suppose that $k_1 < \frac{1}{3}(a-c) < k_2$ and prove that under that condition, (p^k, p^k) is not an equilibrium.

> *the remaining part is to show that this is not possible to sustain, under the demand behavior that we stated above, an asymmetric equilibrium. We stick in particular to the case $p_1 < p_2$.*

9) In a graphic q, p draw the whole demand addressed to the market representing in particular the point $(k_1 + k_2, p^k)$ and as a dashed line, the residual demand to firm 2 , representing in particular the point $(k_2, p_2 = p^k)$

10) Show that if there is an asymmetric equilibrium $p_1 < p_2$, then a necessary condition for equilibrium is that consumers are rationed at p_1

11) Show then that if there is an asymmetric equilibrium $p_1 < p_2$, the best proposal by firm 2 would be $p_2^{**} = p^M - \frac{1}{2}k_1$

> *Remark that the preceding computation is true whenever $p_1 \leq p_2^{**}$. We suppose in what follows that this condition could is verified, and it is under this assumption that we will now investigate firm 1 's behavior.*

12) check that the value of the quantity sold by firm 2 when she sticks to the preceding condition $p_2 = p_2^{**}$, at the conditions that $q_1 = k_1$ is equal to $q_2^{**} = \dfrac{a - k_1 - c}{2}$, and that thee quantity sold by the market is $a - p_2^{**}$. Check in particular what could happen if firm 1 increase slightly its price when firm 2 does not modify $p_2 = p_2^{**}$.

13) Do you think that if $p_2 = p_2^{**}$, firm 1 will accept to price $p_1 = p^k$? Said differently, is (p^k, p_2^{**}) a Nash equilibrium ?

2 Cournot with prior investment

> *Let consider two firms which technology is $y = \sqrt{L}\sqrt{K}$, competing in a Cournot setting, in which prior, the decision of capital is a long term one, while the decision on the level of work is a short term 1. Their decision of L is done in a Cournot framework. The price of Capital and of work are normalized to 1. We suppose that the inverse demand is $p = a - q$.*

1) Show (it is an accountant equation) that the cost of producing x_i when the level of capital is K_i is $C(x_i, K_i) = K_i + \dfrac{x_i^2}{K_i}$.

2) Let suppose that firm i has chosen K_i and that firm j has chosen K_j as their long term investment.

3) Compute the Cournot equilibrium in the short term, and then write for each firm write their optimal production ; Check that the equilibrium price is $p^* = \frac{a}{2} + a\dfrac{2 - \frac{1}{2}K_1K_2}{4 + 4K_1 + 4K_2 + 3K_1K_2}$.

4) From the previous computations show that $k_i \leq k_j$ is equivalent to $x_i \leq x_j$. Comment.

5) Compare the x_1^* and x_2^* that you obtained with the situation that would have been in pure competition : In particular, what should you conclude, given that the firm are constant return to scale ?

6) From the previous computation, show that the equilibrium price is increasing with the level of capital invested [No need to compute the precise derivative] Comment

> *We should consider now, in the first period, the game of the two firms, choosing in the long term their investment in capital. We model that situation as a game in which in the first stage, each firm anticipates what will be done in the next (Cournot) stage (analyzed precedingly). So after writting the profit as a function of K_1 and K_2 we would compute the derivative $\dfrac{\partial \pi_i}{\partial K_i}$ and then solve for the optimal level of production and then solve for the equilibrium. That exercise is tedious, and we will not do it entirely here*

7) Is there an indirect path to Verify that the optimum corresponds to overinvestment ? [Write $\pi_1 = (a - x_1 - x_2)x_1 - c$]

************************* END OF THE EXERCISE SET 5*************************