Repurchase Agreements, Collateral Re-Use and Intermediation

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Motivation: Importance of Repos

- **Repo:** sale of an asset combined with a forward contract (to repurchase of the asset)

  difference from collateralized loans: repo lender obtains right to pledged collateral (re-use)

- Repos received considerable attention since the crisis:
  extensively used by market makers, dealer banks, ... to get funds, acquire securities, get safe return on cash.

- Growing body of works on repo, but still some important questions to answer ...

Repos: Important issues

1. Repos vs. Sales : you could sell the asset spot to obtain funds.
   $\rightarrow$ Obtain less cash than with a spot sale if haircut$>$0...

   $$\text{Haircut} = \text{spot price - repo price}$$

   $\rightarrow$ Commit to a future repurchase price.

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2. What determines haircuts?

   $\rightarrow$ Some empirical evidence that haircuts increase with counterparty or asset risk...

   $\rightarrow$ but *"haircuts are a puzzle"* (GM, JFE 2012).

Repos: Haircuts index

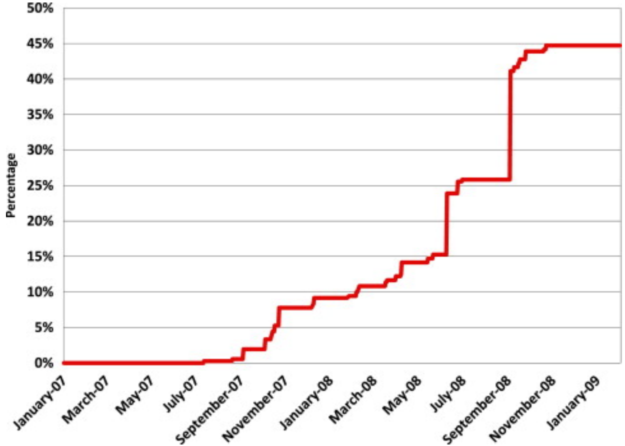

Figure: Haircut index (bilateral repo). Source : GM, JFE 2012

Repo: Some Issues

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   - Acquiring ownership means you can re-use the asset!
     $\rightarrow$ Allows collateral to circulate (Singh 2010, 2011,..).
     $\rightarrow$ Allows agents to intermediate repo:
     Dealers lend to a Hedge Fund and borrows from a Mutual Fund.
     Bilateral vs. Tri-party segments of the market.

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- Duffie & Skeel (2012) and Infante (2013) analyze sale of collateral but focus on the consequence upon default.

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- We show that collateral re-use:
  1. Help agents reach first best through collateral multiplier effect.
  2. Generates endogenous intermediation.

Outline

Environment

- 1 perishable consumption good and 2 agents $i \in \{1, 2\}$.
- 3 periods : $t = 1, 2, 3$
- Supply a of an asset with payoff $s \sim G$ on $[\underline{s}, \bar{s}]$ in period 3.
  $\rightarrow$ Payoff s known in period 2.
- Endowments:

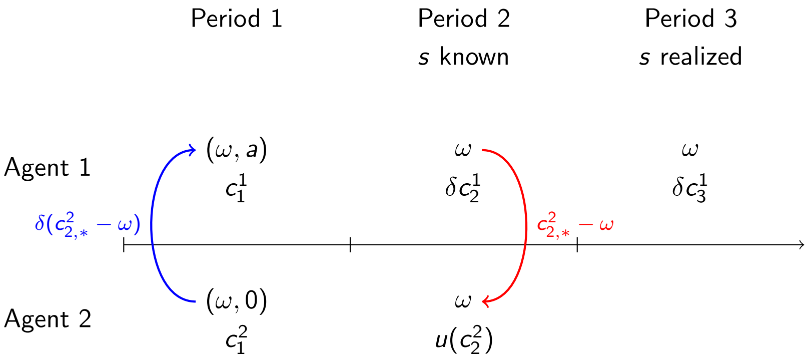
$$\text{Agent 1}: \ (\omega, \omega, \omega), \ a_0^1 = a, \qquad \text{Agent 2}: \ (\omega, \omega, 0), \ a_0^2 = 0$$

- Preferences over consumption (c_1, c_2, c_3):

$$v^1(c_1, c_2, c_3) = c_1 + \delta(c_2 + c_3), \qquad \delta < 1$$
$$v^2(c_1, c_2, c_3) = c_1 + u(c_2)$$

  with u concave satisfying Inada condition.
- Assumption : $u'(\omega) > \delta$. Gains from trade.

First-Best Allocation



- First best allocation: $u'(c_{2,*}^2) = \delta$.
- **Implementation**: Agent 1 borrows $\delta(c_{2,*}^2 - \omega)$ at rate $r = 1/\delta - 1$.

Limited Commitment

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- Agent 2 would purchase the asset in $t = 1$ and resell in $t = 2$.
- Agent 1 is the only one to hold the asset after $t = 2$

$$\text{Spot price} \quad p_2(s) = s = \text{ MWP of Agent 1}$$

  $\rightarrow$ Agent 2 is risk-averse. Dislikes the price risk.

- Alternatively, you can use repos.

Repurchase contract

- **Definition**: A repo $F = \{\bar{p}(s)\}_{s \in [\underline{s}, \bar{s}]}$ is an agreement to repurchase the asset at price $\bar{p}(s)$ in state s of period 2.
- Repo F traded competitively at price p_F in $t = 1$.

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- Default in state s: collateral loss+ penalty $\theta \bar{p}(s) a_F$ with $\theta \in [0, 1]$.

$$\text{No default} \qquad -\theta \bar{p}(s) a_F \leq (s - \bar{p}(s)) a_F$$

$$\bar{p}(s) \leq \frac{s}{1 - \theta} \tag{1}$$

- Set of feasible repos: $\mathcal{F} = \{F \mid (1) \text{ holds}\}$
  $\rightarrow$ No loss of generality in focusing on default free contracts.

Equilibrium Selection of Repos

- We solve for a competitive equilibrium where agents may trade spot and any repo $F \in \mathcal{F}$.
- Equilibrium selects the repo contract(s) that agents actually trade. All contracts (even non-traded ones) in $\mathcal{F}$ are priced in equilibrium.
- **Guess and Verify** : In equilibrium, agents only trade one contract.

Consumer Problem

- Let $F = \{\bar{p}(s)\}_s$ be that repo contract.
- a_t^i: spot position of agent i in period t.
  a_F^i: repo position (period 1). $a_F^i \leq 0$: seller.
- Agent i problem

$$
\begin{aligned}
&\max_{c_t^i, a_t^i, a_F^i} v^i(c_1^i, c_2^i, c_3^i) \\
&\text{subject to} \quad & c_1^i &= \omega - p_1(a_1^i - a_0^i) - p_F a_F^i \geq 0 \\
& & c_2^i(s) &= \omega + s(a_2^i - a_1^i) + \bar{p}(s) a_F^i \geq 0 \\
& & c_3^i(s) &= \mathbf{1}_{\{i=1\}}\omega + a_2^i s \\
& & a_1^i &\geq 0 \qquad \xi_1^i \text{ (No short sale)} \\
& & a_1^i + a_F^i &\geq 0 \qquad \eta_1^i \text{ (Collateral Constraint)}
\end{aligned}
$$

Equilibrium Repo

Proposition

In equilibrium, agents only trade repo:

$$a_1^2 = 0 = a - a_1^1$$

Define $s^* := s^*(a, \theta, \delta)$ *as the solution to*

$$u'\left(\omega + \frac{s^*}{1-\theta} a\right) = \delta = u'(c_{2,*}^2)$$

The (essentially) unique equilibrium repo is $F = \{\bar{p}(s)\}_s$ *where:*

i) *If* $s^* > \bar{s}$ *(a low)*

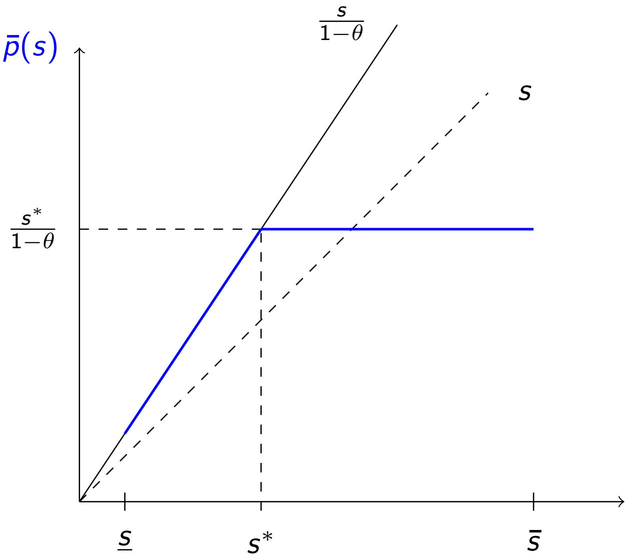
$$\bar{p}(s) = \frac{s}{1-\theta}$$

ii) *If* $s^* \in [\underline{s}, \bar{s}]$ *(a intermediate):*

$$\bar{p}(s) = \begin{cases} \frac{s}{1-\theta} & \text{if} \quad s \leq s^* \\ \frac{s^*}{1-\theta} & \text{if} \quad s > s^* \end{cases}$$

iii) *If* $s^* \leq \underline{s}$, $\bar{p}(s) = \bar{p}$ *is constant and* $\bar{p} \in [s^*/(1-\theta), \underline{s}/(1-\theta)]$

Equilibrium Repo: Borrowing vs. Hedging Motive



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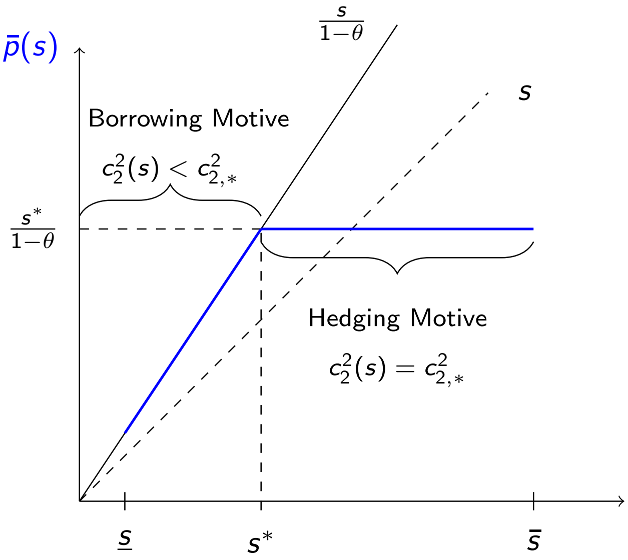


Intuition

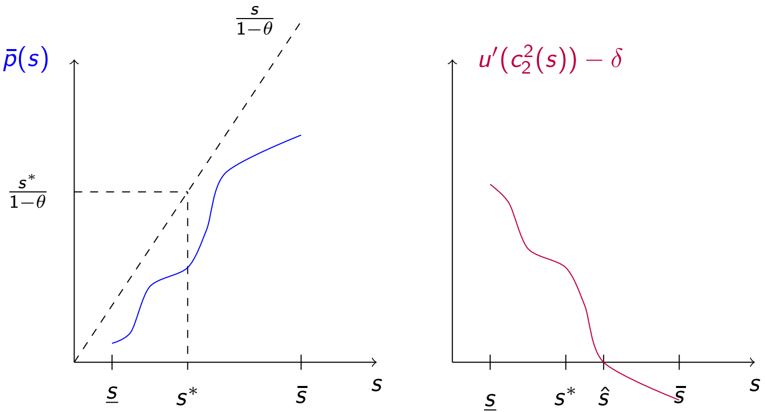
- Intermediate Region :

$$\bar{p}(s) = \begin{cases} \frac{s}{1-\theta} & \text{if} \quad s \leq s^* \\ \frac{s^*}{1-\theta} & \text{if} \quad s > s^* \end{cases}$$

- FB allocation $c_2^2(s) = c_{2,*}^2$ such that $u'(c_{2,*}^2) = \delta$
- Feasible repo $\bar{p} \Rightarrow c_2^2(s) \leq \omega + as/(1-\theta)$.
- When $s \leq s^*$, hit the no default constraint $\bar{p}(s) = s/(1-\theta)$.
  $\rightarrow$ Borrowing Motive.
- When $s > s^*$, you can have $c_2^2(s) = c_{2,*}^2$.
  $\rightarrow$ Hedging Motive: Flat part in the repo contract.

Graphic Argument

- $\bar{p}$ optimal contract among $\tilde{p} \in \mathcal{F}$: $E[(\bar{p}(s) - \tilde{p}(s)(u'(c_2^2(s)) - \delta)] \geq 0$



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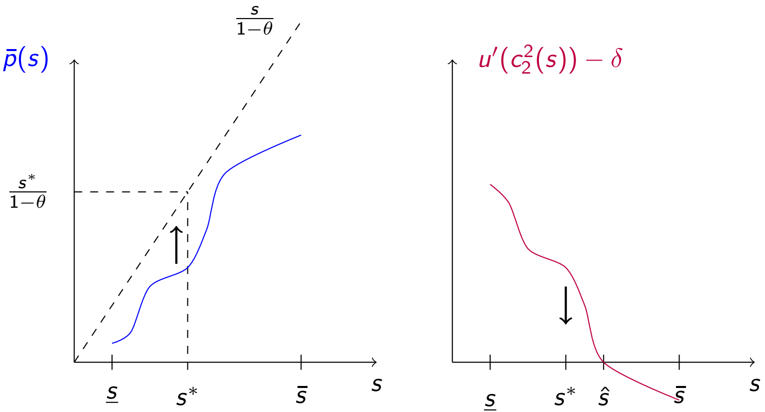

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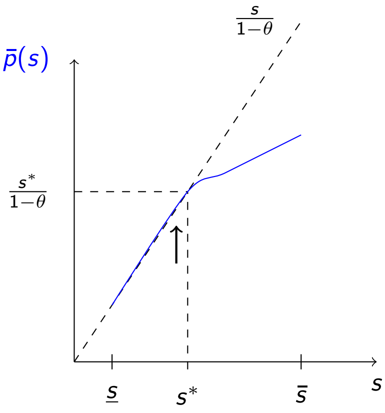

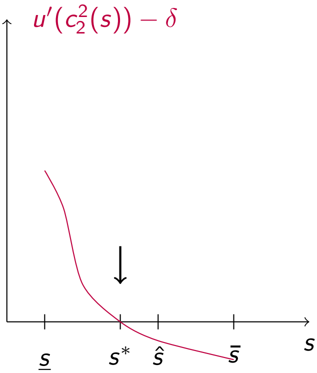

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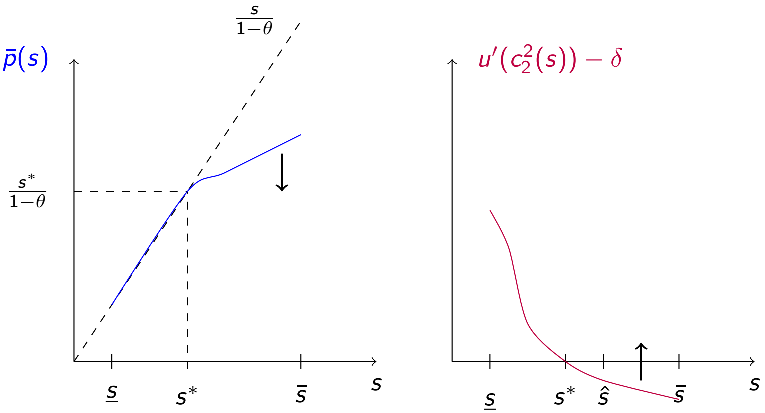

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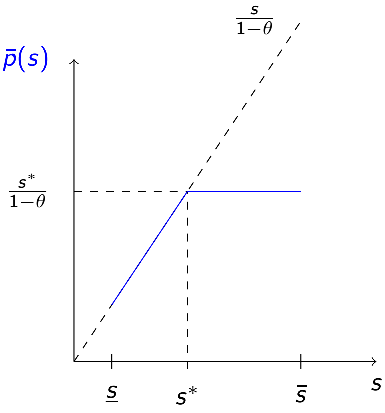
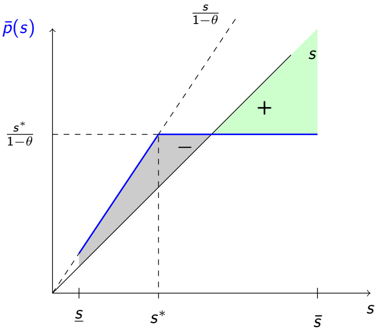


Haircuts

- The haircut is the difference between the spot and the repo price:

$$\mathcal{H} = p_1 - p_F = \delta(E[s] - E[\bar{p}(s)])$$



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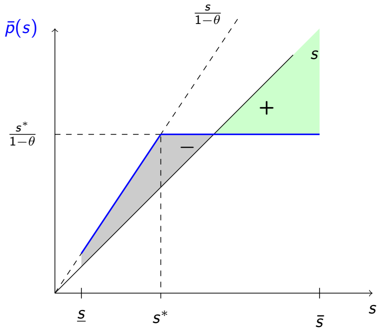
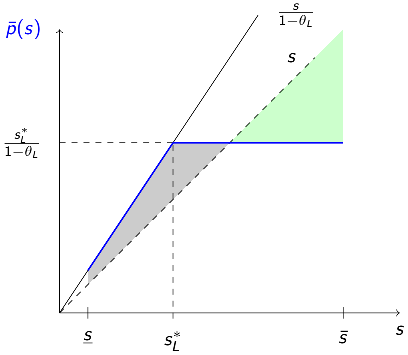


- Asset is scarce = s^* high: can have negative haircut.

  → Documented for some securities: on the run treasuries, collateral "on special" (cf. Vayanos & Weill 2008).

Haircuts - Pledgeability : θ

- Haircut $\mathcal{H} = \delta E[s - \bar{p}(s)]$
- Comparative Statics: change from θ_L to $\theta_H > \theta_L$: better borrower.



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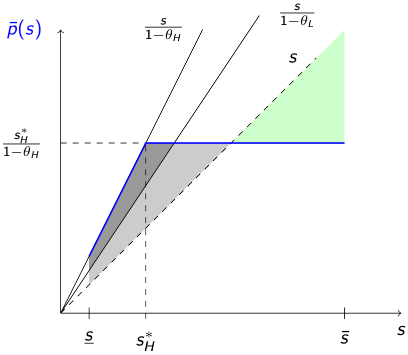

- Agent with θ_H can borrow more: $\mathcal{H} \searrow$
  $\rightarrow$ High quality counterparties face lower haircut.

Asset Risk

- Introduce two assets with perfectly correlated payoffs.
  $\rightarrow$ Safe asset pays s.
  $\rightarrow$ Risky asset is mean-preserving spread of safe asset.

$$s_{risky} = \alpha(s_{safe} - E[s]) + E[s], \quad \alpha > 1$$

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- We can apply the previous analysis with these two assets.
- **Claim** : risky asset has a higher haircut and a lower liquidity premium (LP) where

$$LP = p_1 - E[s]$$

- Riskier asset is worse collateral.
  - $\rightarrow$ Can borrow less per unit of asset : higher haircut.
  - $\rightarrow$ Less desirable than safe asset : lower liquidity premium.

Outline

Collateral re-use

- So far, repo = collateralized loan.

  $\rightarrow$ Main difference: collateral is sold in a repo: lender can re-use it.

  $\rightarrow$ Proposition 1 (a low): Agent 1 wants to buy to pledge more!

- Agent 2 becomes owner of fraction $\nu_2 \in [0, 1]$ of collateral pledged.

Legal Definition

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  Legal Definition

- When pledging a_F, Agent 1 lends $\nu_2 a_F$ units of asset Agent 2.

  → Is lender implicit promise to return $\nu_2 a_F$ credible?

- We treat "lender default" default in a symmetric way and show the lender no default constraint does not bind in equilibrium. Lender Default

Re-use: Sequential process

- How much can you pledge with collateral re-use ?
  → Illustration as a sequential process.

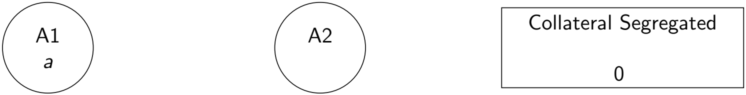

Figure: Re-use and Pledgeability: Step 0

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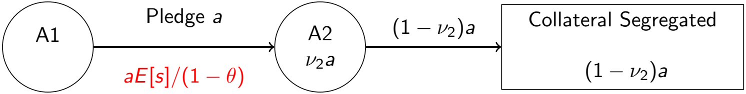

Figure: Re-use and Pledgeability: Step 1

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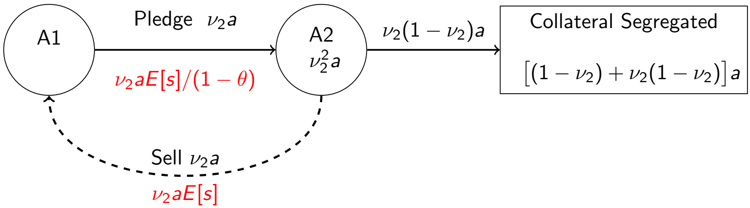

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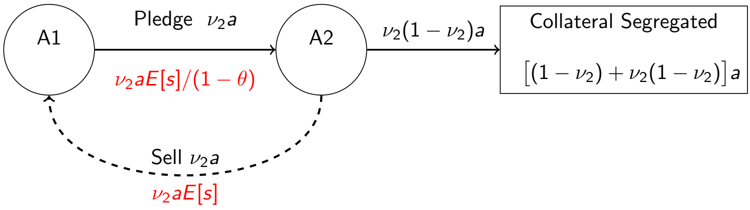

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- In the model, simultaneous trades with a consolidated collateral constraint. Details

Equilibrium Repo: Re-use

Proposition

Define $s^*(\nu) := s^*(a, \theta, \nu)$ *as the solution to*

$$u'\left(\omega + \frac{as^*(\nu)}{1-\nu}\left[\frac{1}{1-\theta} - \nu\right]\right) = \delta$$

The equilibrium repo contract with re-use is

$$\bar{p}(s, \nu) = \begin{cases} \frac{s}{1-\theta} & \text{if } s < s^*(\nu) \\ \frac{s^*(\nu)}{1-\theta} + \nu(s - s^*(\nu)) & \text{if } s \geq s^*(\nu) \end{cases}$$

If $s^*(\nu) > \underline{s}$

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$$a_1^1 = -a_F^1 = \frac{1}{1-\nu}a, \qquad a_2^1 = -\frac{\nu}{1-\nu}a,\ a_F^2 = -a_F^1$$

Corollary:

$$s^*(\nu) = \frac{1-\nu(1-\theta)}{1-\nu}s^*(0)$$

There exists $\bar{\nu}$ such that if $\nu_2 \geq \bar{\nu}$, equilibrium allocation is FB.

Re-use: Multiplier Effect

- From Proposition 2:

$$u'\left(\omega + \frac{as^*(\nu)}{1-\nu}\left[\frac{1}{1-\theta} - \nu\right]\right) = \delta$$

- Ultimate quantity of collateral available is

$$a' = \underbrace{\frac{1-\nu(1-\theta)}{1-\nu}}_{\text{Collateral Multiplier } k(\theta,\nu)} a$$

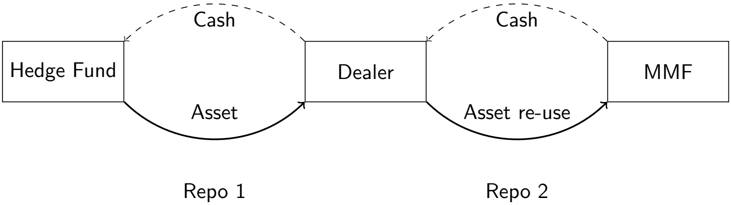
$\rightarrow$ Observe that $k(0,\nu) = 1$ for all ν

$\rightarrow$ No multiplier effect without some commitment (Maurin, 2015).

- Period 2 allocation $(c_2^1(s), c_2^2(s))$ with asset a and re-use ν is the same than with asset a' and re-use 0

Re-use and Intermediation

- Re-use also helpful to intermediate:



- Dealers lend on bilateral repo market and borrow on the tri-party repo market. Why does the Hedge Fund not borrow directly from the MMF. Is is just regulation ?

- We provide an explanation with differences in counterparty quality θ.
  $\rightarrow$ Endogenous intermediation through better borrower.

Extending the model: 3 agent types

- Let us have 2 type 1 agents:

  $\rightarrow$ Agent $1L$: $a_0^L = a$, θ_L, δ_L

$$v^L(c_1, c_2, c_3) = c_1 + \delta_L(c_2 + c_3)$$

  $\rightarrow$ Agent $1H$; $a_0^H = 0$, $\theta_H \geq \theta_L, \delta_H \geq \delta_H$:

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$$v^H(c_1, c_2, c_3) = c_1 + \delta_H(c_2 + c_3)$$

- For this presentation, only agent $1H$ can re-use collateral:

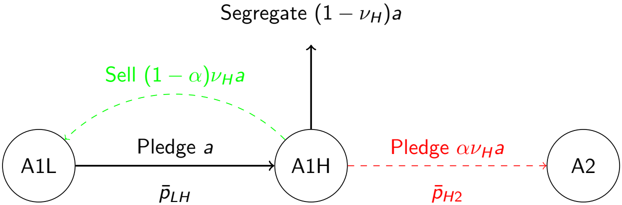
$$\nu_L = \nu_2 = 0$$

- $1H$ is still a “borrower” with respect to 2

$$\delta_L \leq \delta_H < u'(\omega)$$

Intermediation Equilibrium: Conjecture

- Intermediation + Asset circulation in background.



- $1H$ must be indifferent between red and green:

$$\frac{1}{1-\theta_H}\int_s \bar{p}_{H2}(s)\left[u'\left(c_2^2(s)\right)-\delta_H\right]\mathrm{d}F(s)=\frac{(\delta_H-\delta_L)\theta_L}{(1-\nu_H)(1-\theta_L)}$$

- $1H$ lends b_{LH} to $1L$ and borrows b_{H2} from 2.

$$b_{LH}(1-\nu_H)+b_{H2}=a$$

Intermediation Equilibrium

- $1H$ more patient, better counterparty: $\theta_L < \theta_H$, $\delta_L < \delta_H$.

Proposition

If $\theta_H - \theta_L \geq \Delta\bar{\theta}$ and $a \leq \bar{a}$, the only contracts traded in equilibrium are :

1. *A repo $\bar{p}_{LH}$ where $1L$ borrows from $1H$*

$$\bar{p}_{LH}(s) = \frac{s}{1 - \theta_L}$$

2. *A repo $\bar{p}_{H2}$ where $1H$ borrows from 2*

- Importantly, $1L$ and 2 do not trade directly but $1H$ intermediates.

Benchmark cases

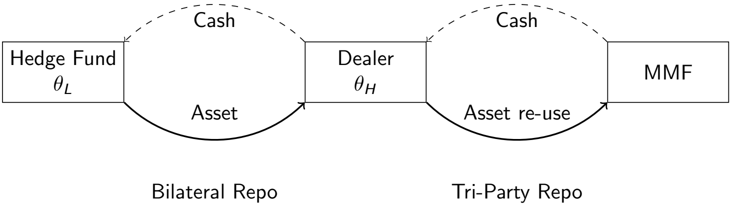
Intermediation Equilibrium: Intuition

- Why does $1L$ not trade with 2?
  $\rightarrow$ He has the asset and higher gains from trade with 2: $u'(\omega) - \delta_L$.
  $\rightarrow$ But θ_L is low : cannot pledge much period 1 income.
- $1H$ intermediates for $1L$ with his higher borrowing capacity $\theta_H > \theta_L$.
- Condition for intermediation:
  Let γ_1^H be the shadow price of the collateral constraint for $1H$

$$\underbrace{(1-\nu_H)\gamma_1^H}_{\text{Intermediation Cost}} \leq \underbrace{E\left[\left(\bar{p}_{H2}(s) - \bar{p}_{LH}(s)\right)\left(u'(c_2^2(s)) - \delta_H\right)\right]}_{\text{Intermediation Benefit}}$$

Intermediation Equilibrium: Interpretation

- Intermediation valuable for non-trustworthy borrowers:



- With partial commitment, counterparty and asset quality mixed.
  → Intermediation makes the collateral more acceptable to MMF..

Conclusion

- We presented a simple model of repurchase agreements :
  - $\rightarrow$ Equilibrium repo among contracts with limited commitment.
  - $\rightarrow$ Haircut increase with asset risk and counterparty risk.
- Repos are an efficient way of using asset for borrowing/lending:
  - $\rightarrow$ Equilibrium contract trades off borrowing/hedging motive.
  - $\rightarrow$ Selling collateral allows for asset circulation/intermediation.
- What's next?
  - $\rightarrow$ Collateral Transformation.
  - $\rightarrow$ Central Bank Policy : Effect of an asset purchase on repos..

Trading Surplus

- Net trading surplus $S^i = v^i_{eq}(c^i_1, c^i_2, c^i_3) - v^i_{aut}$.
- Agent 1 enjoys the liquidity premium on his asset holdings:

$$S^1 = \eta^1_1 a = E[\bar{p}(s)(u'(c^2_2(s) - \delta)]a$$

An increase in the asset supply gives

$$\frac{\partial S^1}{\partial a} = \eta^1_1 + a\frac{\partial \eta^1_1}{\partial a}$$
$$= \int_s^{s^*} \frac{s}{1-\theta}\left[u'\left(\omega + \frac{as}{1-\theta}\right) - \delta + u''\left(\omega + \frac{as}{1-\theta}\right) as\right] \mathrm{d}G(s)$$

Agent 2 gets hedging:

$$\frac{\partial S^2}{\partial a} \geq 0$$

Back to Presentation

Lender Default

- Let now θ_2 be the reliability of agent 2.
- Lender default : lose payment $\bar{p}(s)a_F$ + penalty $\theta_2\nu_2 a_F s$.
  $\rightarrow$ No (lender) default if:

$$\begin{aligned} \bar{p}(s)a_F &\geq \nu_2(1-\theta_2)a_F s \\ \bar{p}(s) &\geq \nu_2(1-\theta_2)s \end{aligned} \tag{2}$$

- For a given $\{\bar{p}(s)\}_s$, (2) gives the maximal feasible ν_2.
- In equilibrium, agents trade $\{\bar{p}(s)\}_s$ such that (2) holds for $\theta_2 = 0$ and $\nu_2 = 1$.

Back to Presentation

Collateral Constraint with Re-use

- Repo F: distinguish long $a^i_{F,+} \geq 0$ vs. short $a^i_{F,-} \geq 0$ positions.
- Agent 1 (Borrower):
$$a^1_1 \geq a^1_{F,-}$$
$\rightarrow$ Collateral constraint unchanged.
- Agent 2 (Lender):
$$a2_1 + \nu_2 a^2_{F,+} \geq 0$$
$\rightarrow$ Agent can re-use (sell) $\nu_2 a^2_{F,+}$ with... agent 1
- Agent 1 has more asset to borrow by market clearing
$$a^2_1 + a^2_2 = a$$

Back to Presentation

3 agents: Benchmark case 1

- **Identical Agents** : $\theta_L = \theta_H$ and $\delta_L = \delta_H$
- **Equilibrium**:

  $\rightarrow$ $1L$ borrows a with contract $\bar{p}_{L2}$ with agent 2

$$\bar{p}_{L2}(s) = \begin{cases} \frac{s}{1-\theta} & \text{if} \quad s \leq s^* \\ \frac{s^*}{1-\theta} & \text{if} \quad s > s^* \end{cases}$$

  $\rightarrow$ $1H$ stays inactive

- $1H$ would like to borrow from 2 as well but $a_1^H = 0$.

3 agents: Benchmark case 2

- **Gains from trade**: $\theta_L = \theta_H$, $\delta_L < \delta_H$.
- Equilibrium:

  $\rightarrow$ $1L$ borrows b_{L2} with contract $\bar{p}_{L2}$ from agent 2

$$\bar{p}_{L2}(s) = \begin{cases} \frac{s}{1-\theta} & \text{if} \quad s \leq s^*(b_{L2}, a, \theta_L) \\ \frac{s^*(b_{L2}, a, \theta_L)}{1-\theta} & \text{if} \quad s > s^*(b_{L2}, a, \theta_L) \end{cases}$$

  $\rightarrow$ $1H$ borrows b_{LH} from $1H$ with contract $\bar{p}_{LH}(s) = s/(1-\theta_L)$

- **Equilibrium conditions**

  Agent $1L$ solves a portfolio allocation problem:

$$a = (1-\nu_H)b_{LH} + b_{L2}$$

  Must be indifferent between borrowing from $1H$ and 2.

- Importantly $1H$ and 2 do not trade !

Back to Presentation

Definitons : Source ICMA

- **Repurchase agreement**: *In a repo, one party sells an asset (usually fixed-income securities) to another party at one price at the start of the transaction and commits to repurchase the fungible assets from the second party at a different price at a future date.*

- **Lender Rights**: *In Europe, repo transfers legal title to collateral from the seller to the buyer by means of an outright sale.* [Under New York law], *collateral is pledged but exempted from certain provisions of the US Bankruptcy Code that normally apply to pledges, in particular, the automatic stay on enforcement of collateral in the event of insolvency. In addition, unlike in traditional pledges, the pledgee/buyer in a US repo is given a general right of use of collateral. Consequently, the resulting rights are deemed to be much the same as those achieved by an outright sale.* Back to Presentation