

## Optimal Contracts and Competitive Markets with Costly State Verification

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### 1. INTRODUCTION

PAR 13 The insight of Arrow [4] and Debreu [7] that uncertainty is easily incorporated into general equilibrium models is double-edged. It is true that one need only index commodities by the state of nature, and classical results on the existence and optimality of competitive equilibria can be made to apply. Yet it seems there are few contingent dealings among agents relative to those suggested by the theory. For example, closely held firms issue bonds which pay off a fixed constant, independent of investment project returns, at least if bankruptcy does not occur. More generally, common forms of debt are simple rather than contingent. Similarly, individuals carry insurance policies with deductible portions—small losses are uninsured.

PAR 14 What is needed then are models that explain such phenomena. Arrow [1] has argued that the observed absence of contingent dealings is closely related to moral hazard and imperfect information. If a contract is contingent on an event, then it must be known whether or not the event occurred. Though this information is likely to be available to only one party of the contract, the range of possible contingent contracts is limited to those which are easily verified by both. Radner [16] has formalized this notion by exogenously limiting contracts between agents to those which are contingent on the events in the information partitions of both agents. Radner also suggests that the information structure of an economy may be costly and endogenous. This

\* This paper began as a joint effort with Neil Wallace and reflects that collaboration as well as subsequent comments in many ways. I would also like to thank the participants of the *NSF-NBER* Conference on Theoretical Industrial Organization at Carnegie-Mellon University, March 1976; my colleagues at Carnegie-Mellon, especially Arthur Raviv and Edward C. Prescott; Edward J. Green; and the referees for helpful comments. Partial support for this research from the Federal Reserve Bank of Minneapolis is gratefully acknowledged. I assume full responsibility for any errors as well as the views expressed here.

paper elaborates on the themes suggested by Arrow and Radner. A model in which agents are asymmetrically informed on the actual state of nature and in which this information may be transmitted to other agents only at some cost is presented. As will be noted, the model is successful in explaining the above-mentioned observations, at least subject to some qualifications.

This paper begins in Section 2 with a simple, two-agent, pure exchange economy in which the endowment of the consumption good of one of the agents, say agent 2, is random. Preferences and endowments are such that there are gains to trading claims contingent on the realization of the random endowment. But any realization is known only by agent 2 unless a verification (auditing) cost is borne. A contract in such a setting is a prestate agreement as to when there is to be verification and the amount to be exchanged, and a contract is said to be consistent (incentive compatible) if agent 2 submits to verification and honors claims in accordance with the contract. Pareto optimal, consistent contracts are shown in Section 3 to have familiar characteristics. In particular, there exists a set of realizations over which there is no verification. In the case of insurance this corresponds to the region over which no claims are filed. For closely held firms this corresponds to the region over which bonds pay the stated yield. A verification set is a set of low realizations; insurance claims are settled and firms default.

The next two sections examine the robustness of these results by extending the model in several directions. Section 4 introduces a random verification procedure and establishes that the random procedure can dominate, in a Pareto sense, the optimal contract under the assumed deterministic procedure. Though consistent with observations on random audits and the like, this finding represents a major criticism of the deterministic scheme. Section 5 introduces more agents and random variables. Here extensions of the earlier results are established, subject to some *exogenous* restrictions on contingent exchange agreements. One restriction is that the  $m$ -agent model be essentially bilateral in nature.

Section 6 proposes a competitive equilibrium concept for the  $m$ -agent model. It is established that, under specified assumptions, an equilibrium exists and yields optimal allocations. This section represents an attempt to improve our understanding of general equilibrium competitive models with moral hazard and costly information (cf., Helpman and Laffont [11]). It also represents a rigorous analysis of incomplete competitive insurance markets.

Section 7 presents some concluding remarks. The proofs of all lemmas and propositions are contained in an appendix.

The remainder of this introduction deals with the relationship of this paper to other literature. The characterization of optimal contracts may be viewed in part as an extension of the literature on optimal insurance policies. Arrow [2, 3] and subsequently Raviv [17] have shown that under certain nonnega-

tivity constraints and in the presence of loading, an optimal insurance contract can have deductible. It is shown in this paper that consistency conditions yield the requisite nonnegativity constraints and that it is costly state verification which can make complete risk-sharing suboptimal.

This paper is also closely related to the literature on imperfect information and principal-agent relationships. In Spence and Zeckhauser [21], Shavell [20], and Harris and Raviv [10] the random output of the consumption good is not exogenous, but rather depends on an action taken by the agent. Spence and Zeckhauser established in this context that the form of an optimal contract depends on the principal's ability to monitor the state of nature, the action taken by the agent, and the output of the consumption good. Subsequently, Shavell and Harris and Raviv focused on the case in which the output of the consumption good is known to both the principal and the agent, in contrast to the model of this paper, but in which the action of the agent may or may not be observed. Various assumptions can be made on the monitoring technology and the timing of observations. Unlike the model of this paper, in which verification is perfect when it occurs, the authors allow for observation of the agent's action with error. Shavell further allows observations on care to be costly and to be taken either before or after the realization of output. The model of this paper also emphasizes the costly nature of observation, but, in contrast, does not deal at all with the timing question.

Retaining the perfect observation assumption, it might have been supposed here that the decision to verify could be made *ex ante* at some fixed cost of the consumption good and that subsequently all realizations would be observed. This then would be the model suggested by Kihlstrom and Pauly [14], and it has the implication that one agent provides either complete insurance coverage to the other or no coverage at all. Similarly, in Shavell's model, if care is observed perfectly, then an optimal insurance policy offers full coverage. Thus, such an alternative model might explain the complete absence of some dealings, but it could not explain the observations on noncontingent dealings noted at the outset.

If in the model of this paper verification were imperfect, and if the verification cost were a function of the actual realization of the endowment, then the decision to verify might act as a signal of the realization, and aspects of the signaling-incentive literature might be brought to bear. In this regard, one might also weaken the assumption that the probability distribution of the consumption good is known to both agents. In Ross [18] the financial decision of the manager acts as a signal to uninformed investors of the return stream of the firm. An approach that combines the model of this paper and that of Ross might suppose the choice of financial structure signals information that reduces *ex post* auditing costs. In any event, there are many aspects of the present model that could be modified in subsequent work.

## 2. AN ECONOMY WITH TWO AGENTS AND ONE RANDOM VARIABLE

It is supposed that each of two agents has an endowment of the single consumption good of the model. The endowment of agent 2, denoted  $y_2$ , is a random variable with cumulative probability distribution  $F(y_2)$ . It is further assumed that  $y_2$  takes on values in the interval  $[\alpha, \beta]$ ,  $\alpha > 0$ , and is either simple, in which case it has a finite number of realizations, or continuous, in which case it is assumed to have a continuous, strictly positive density function  $f(y_2)$ .<sup>1</sup> The endowment of agent 1, denoted  $y_1$ , is not random and  $y_1 > 0$ .

Each agent  $j$  has a utility function  $U_j$  over riskless consumption which is continuously differentiable, concave, and strictly increasing. It is assumed moreover that  $U_2$  is strictly concave with  $U_2'(0) = \infty$  and  $U_2'(\infty) = 0$ . Letting  $c_j(y_2)$  denote the consumption of agent  $j$  as a function of  $y_2$ , feasibility then requires that  $c_1(y_2) + c_2(y_2) \leq y_1 + y_2$ . Consistent with von Neumann–Morgenstern axioms, each agent  $j$  has as objective the maximization of expected utility,  $\int U_j[c_j(y_2)] dF(y_2)$ .

The model described thus far can be given various interpretations. For example, agent 2 can be viewed as a firm engaged in an investment project with random return  $y_2$ . Agent 2 may issue an asset to agent 1 where the asset is some claim on the returns of the project. The problem is to determine the type of asset that is mutually agreeable to both parties. Alternatively, agent 2 can be viewed as an individual who is to suffer some random loss  $\beta - y_2$ , and would like to purchase insurance from agent 1. Under either interpretation, exchange is motivated by risk-sharing considerations.

If both agents were always fully informed ex post as to the realization (state) of  $y_2$ , then they could agree to an exchange contingent on the realization. In general any such *exchange* which results in a (full information) Pareto optimal allocation will be a nontrivial function of  $y_2$ . But the purpose of this paper is to explain the *absence* of such contingent dealings: firms issue bonds which pay out a fixed constant, independent of investment project returns, and individuals hold insurance contracts with deductible portions. Consequently the full information assumption must be weakened.

Here then it is supposed that the realization of  $y_2$  is known only by agent 2 unless there is verification. If there is verification,  $y_2$  is made known *without error* to agent 1. Verification is costly in that some specified amount of the consumption good is forfeited by agent 2 and disappears from the model. The idea here is that it is costly for a firm to make known its project return to outside investors. Perhaps independent auditors

<sup>1</sup> In what follows I disregard sets of probability zero and properties of functions on sets of probability zero, at least where no ambiguity results.

must be hired, and costly state verification can be interpreted as costly auditing. Similarly, it is costly for individuals to establish claimed losses; the extent of damages must be verified.<sup>2</sup>

Resources are allocated in this model in accordance with specified rules on the execution of a contract. First a contract must be defined. Prior to the realization of  $y_2$ , agents agree to a contingent exchange. Let  $g(y_2)$  denote the actual poststate net transfer of the consumption good from agent 2 to agent 1 as a function of  $y_2$ . Then let  $\bar{g}$  denote the *prestate contractual choice* of the function  $g$ . Similarly, prior to the realization of  $y_2$ , agents agree as to when there is or is not to be verification, contingent on  $y_2$ . A verification region  $S$  (with complement  $S'$ ) is a set of realizations of  $y_2$  such that there is verification. Then let  $\bar{S}$  and  $\bar{S}'$  denote the *prestate contractual choices* of  $S$  and  $S'$ , respectively. Thus, a contract  $[\bar{g}, \bar{S}]$  is a prestate contingent specification of when there is to be verification and the amount to be transferred.

Subsequent to the realization of  $y_2$ , agent 2 announces whether there is or is not to be verification. If there is verification, specified amounts of the consumption good are forfeited by agent 2,  $y_2$  is made known to agent 1, and agent 2 transfers what was agreed upon. (In terms of the notation, if  $y_2 \in S$ ,  $g(y_2) = \bar{g}(y_2)$ .) If there is not verification, then agent 2 may transfer any amount consistent with the prior specification of the amount to be transferred when there was not to be verification. That is, agent 2 may transfer  $\bar{g}(x)$  for any  $x$  in  $\bar{S}'$ . Of course, agent 2 will transfer the least amount possible, so in fact  $g(y_2) = \min_{x \in \bar{S}'} \bar{g}(x)$ .<sup>3</sup> Finally, to resolve any indeterminacy, it is assumed that if agent 2 is indifferent between asking for verification or not, then he does not ask for verification.

The cost of verification can be modeled formally in several ways. One natural specification is that the cost of verifying  $y_2$  is some constant, say  $\mu > 0$ , independent of the actual realization; this specification is pursued further below. The cost also may be supposed to depend on  $y_2$ , either directly or, alternatively, through the agreed-upon transfer. This latter specification is also pursued below. That is, let  $\xi[\bar{g}(y_2)]$  be the cost of verifying the realization  $y_2$ .<sup>4</sup> One may argue, for example, that the cost of auditing a firm in bankruptcy proceedings depends on outstanding claims.<sup>5</sup> Finally, note that setting

<sup>2</sup> Of course there are no independent third parties such as auditors in the model. Also, it may be natural to view insurance companies as bearing the costs of verifying claimed losses. In this regard the assumption that the cost is borne by the insured is not restrictive as these costs may be passed along to the insurer in an optimal exchange.

<sup>3</sup> It may be assumed without loss of generality that  $\bar{S}'$  is closed.

<sup>4</sup> Analytically this will be equivalent to letting the verification cost depend on the actual transfer,  $g(y_2)$ .

<sup>5</sup> At this level of abstraction, however, this latter specification of the verification cost may seem somewhat unnatural and is motivated, as will be seen below, by analytic convenience.

$\xi[\bar{g}(y_2)] \equiv \mu$ , one obtains the first specification, a constant cost of verification.

With this notation, we may now examine the nature of contracts in this model. A contract  $[\bar{g}, \bar{S}]$  is said to be *consistent* if

- (i)  $S = \bar{S}$ ;
- (ii)  $g(y_2) = \bar{g}(y_2) \quad y_2 \in [\alpha, \beta]$ .

Thus, under a consistent contract, agent 2 has no incentive to misrepresent, relative to the prior agreement, whether there is or is not to be verification or to not pay off what was agreed upon. It is perhaps obvious that under a consistent contract the agreed-upon transfer from agent 2 to agent 1 cannot depend on information which is known only to agent 2. That is, the function  $\bar{g}$  must be identically equal to some constant  $\bar{C}$  whenever there is not to be verification,  $y_2 \in \bar{S}'$ . Similarly, as agent 2 determines whether there is to be verification, he must have an incentive to ask for verification when he is supposed to do so. That is, the transfer plus verification cost must be less than  $\bar{C}$  on  $\bar{S}$ . These conditions are stated formally in

PAR 17 LEMMA 2.1. *A contract  $[\bar{g}, \bar{S}]$  is consistent if and only if  $\bar{g}(y_2)$  equals some constant  $\bar{C}$  on  $\bar{S}'$  and  $\bar{g}(y_2) + \xi[\bar{g}(y_2)] < \bar{C}$  on  $\bar{S}$ .*

In what follows attention is limited to consistent contracts. But intuitively, at least, this restriction should be without loss of generality; given a contract  $[\bar{g}, \bar{S}]$  each agent knows the allocation rules and can determine the actual transfer  $g(\bar{g}, \bar{S})$  and verification region  $S(\bar{g}, \bar{S})$  implied. Both know that in essence they have agreed to a contract  $[\bar{h}, \bar{T}]$  where  $\bar{h} = g(\bar{g}, \bar{S})$  and  $\bar{T} = S(\bar{g}, \bar{S})$ . The implication is summarized in

LEMMA 2.2. *Given any contract  $[\bar{g}, \bar{S}]$ , there exists a consistent contract  $[\bar{h}, \bar{T}]$  which achieves the same allocation of resources.*

Thus the restriction to consistent contracts is without loss of generality. It is in this sense that the problem of "moral hazard" is internalized in this model. It should be noted that this notion of consistency is closely related to the notion of incentive compatibility as discussed by Hurwicz [13]. A contract that is not consistent would require that agent 2 act in a way that is inconsistent with his own (maximizing) inclinations under the rules of the allocation process.

Returning to the interpretations of the model, recall that agent 2 may be viewed as a firm with investment project return  $y_2$ . Then a consistent contract  $[\bar{g}, \bar{S}]$  may be viewed as a bond which promises to pay some fixed constant  $\bar{C}$  unless bankruptcy is declared by agent 2. In that event verification (bankruptcy) costs are incurred, and something less than the fixed yield is paid.

(The payment may be negative.) This interpretation offers a simple theory of closely held corporations. In the model a share would be a claim on some proportion of the profits (project return) of the firm. Individuals such as agent 1, who are not "insiders" but who hold shares, must verify claimed profit levels. Publicly held shares thus require more verification than other forms of debt. (Of course, the model of this paper does not purport to explain the financial structure and bankruptcy decisions of all corporations.)

Alternatively, agent 2 can be viewed as an individual who is to suffer some random loss  $\beta - y_2$  and purchases an insurance contract  $[\bar{g}, \bar{S}]$  from agent 1. (See Arrow [2, 3] and Raviv [17].) Here  $\bar{C}$  is the premium, paid to agent 1 independent of the loss, and  $\bar{I}(y_2) = \bar{C} - \bar{g}(y_2)$  is the insurance payment to agent 2 for loss  $\beta - y_2$  if a claim is filed, in which case verification costs are incurred. Thus if  $y_2 \in \bar{S}'$ , then  $\bar{I}(y_2) = 0$ . Alternatively, if  $y_2 \in \bar{S}$ , then consistency requires that  $\bar{g}(y_2) + \xi[\bar{g}(y_2)] < \bar{C}$  so that  $\bar{I}(y_2) - \xi[\bar{g}(y_2)] > 0$ . This interpretation will motivate some further restrictions on  $\xi[\bar{g}(y_2)]$  in the analysis that follows.

### 3. A CHARACTERIZATION OF OPTIMAL CONTRACTS

The objective in what follows is to characterize the set of optimal contracts. An allocation of the consumption good is said to be optimal if it is Pareto optimal among the set of allocations which can be achieved by consistent contracts, and any contract which achieves an optimal allocation is itself said to be optimal. It should be noted that the consistency conditions and verification costs require that optimal contracts be defined relative to the initial endowments.<sup>6</sup> It should also be noted that optimal contracts are defined relative to the deterministic verification procedure described above. (Stochastic procedures are discussed in Section 4.)

By definition, optimal allocations constitute the contract curve of the two-agent economy. Consistent with the positive intent of this paper, it is assumed here that agents will enter into an optimal contract and thus end up on the contract curve, though the precise allocation will depend on the bargaining power of the two agents. A competitive equilibrium concept that is Pareto satisfactory relative to optimal allocations is the subject of Section 6.

In summary, the objective in what follows is to solve<sup>7</sup>

<sup>6</sup> Here optimal allocations are defined relative to constraints (consistency conditions) derived under the *particular* game described in the text. It is conjectured, however, that these constraints will characterize the outcomes of a large class of alternative games.

<sup>7</sup> As consistency conditions are imposed, the  $\bar{\quad}$  may be dropped from the notation.

**PROBLEM 3.1.** Find a function  $g(y_2)$ , a constant  $C$ , and a region  $S$  that maximize

$$\int_S U_2\{y_2 - g(y_2) - \xi[g(y_2)]\} dF(y_2) + \int_{S'} U_2\{y_2 - C\} dF(y_2)$$

subject to

$$\int_S U_1[y_1 + g(y_2)] dF(y_2) + \int_{S'} U_1[y_1 + C] dF(y_2) \geq K \tag{3.1}$$

$$g(y_2) + \xi[g(y_2)] < C \quad y_2 \in S \tag{3.2}$$

$$y_1 + g(y_2) \geq 0 \quad \text{for } y_2 \in S \quad \text{and} \quad y_1 + C \geq 0 \quad \text{for } y_2 \in S'. \tag{3.3}$$

Here constraint (3.1) specifies that the expected utility of agent 1 be no less than some constant  $K$ . It is further required that  $K \geq U_1(y_1)$  so that agent 1 is at least as well off as in autarky. Constraint (3.2) is the consistency requirement; that  $g(y_2) \equiv C$  on  $S'$  has already been imposed by substitution. Constraint (3.3) is the nonnegativity constraint on the consumption of agent 1; by virtue of the assumption  $U_2'(0) = \infty$ , the analog for agent 2 need not be imposed.

In what follows solutions to Problem 3.1 are characterized under classical and nonclassical assumptions on the verification cost function  $\xi$ . For the classical approach,  $\xi$  is expressed as a continuously differentiable, convex function of the transfer function, and necessary Euler conditions for a maximum are utilized. In contrast, with a fixed cost of verification, the analysis is more tedious; a condition shown by Rothschild and Stiglitz [19] to be equivalent to risk aversion is utilized. Under either approach the important result is that the verification region is a lower interval,  $[\alpha, \gamma]$ ,  $\gamma \leq \beta$ .

The first approach is motivated by the insurance interpretation discussed above.<sup>8</sup> Let  $I(y_2) = C - g(y_2)$  where, again,  $C$  is viewed as the premium and  $I(y_2)$  is the insurance payment. Recall that  $I \equiv 0$  on  $S'$  and  $I > 0$  on  $S$ . Then on  $S$  let  $\xi[g(y_2)] = \Psi[I(y_2)]$  where  $\Psi(I) > 0$ . Hence, in this approach the verification cost is assumed to depend only on the size of the insurance payment. It is further assumed that  $\Psi(I)$  is convex and continuously differentiable. Moreover, defining  $\Psi(0)$  and  $\Psi'(0)$  by taking limits as  $I \rightarrow 0$ , it is assumed that  $\Psi(0) = 0$  and  $\Psi'(0) < 1$ . This last condition states that the marginal cost of verification at  $I = 0$  is less than the marginal payoff to agent 2 from  $I$ . Note that if  $\Psi'(0) \geq 1$  and  $\Psi(I)$  were convex, then  $I - \Psi(I)$

<sup>8</sup> I am much indebted to Artur Raviv, who pointed out to me the mathematical similarity of a preliminary version of Problem 3.1 to one of the insurance literature. The method of proof of Proposition 3.1 emanated from the method employed by Raviv [17].

would be everywhere nonpositive, and no insurance would be trivially optimal.

Now consider

PROBLEM 3.2. Find a function  $I(y_2)$  and a constant  $C$  that maximize

$$\int_{\alpha}^{\beta} U_2\{y_2 + I(y_2) - C - \Psi[I(y_2)]\} dF(y_2)$$

subject to

$$\int_{\alpha}^{\beta} U_1[y_1 - I(y_2) + C] dF(y_2) \geq K \tag{3.4}$$

$$I(y_2) \geq 0 \tag{3.5}$$

$$y_1 - I(y_2) + C \geq 0. \tag{3.6}$$

Under the specified assumptions, if  $I^*, C^*$  is a solution to Problem 3.2, then  $g^*, C^*, S^*$  is a solution to Problem 3.1 where  $g^*(y_2) = C^* - I^*(y_2)$  and  $S^* = \{y_2 : I^*(y_2) > 0\}$ .<sup>9</sup>

This yields

PROPOSITION 3.1. Any solution  $I^*, C^*$  to Problem 3.2 with either  $y_2$  simple or  $y_2$  and  $I(y_2)$  continuous has the property that  $S^* = \{y_2 : y_2 < \gamma\}$  for some parameter  $\gamma$ .<sup>10</sup>

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Proposition 3.1 would of course be vacuous if the verification region  $S^*$  were always either empty or the entire interval. It is shown here by way of an example that  $S^*$  can depend on the verification cost in a nontrivial way. For the example, suppose that  $U_1$  is linear,  $\Psi(I) = \lambda I$  with  $0 \leq \lambda < 1$ , and  $y_2$  is uniformly distributed on  $[\alpha, \beta]$ . Agent 1 is constrained to have the same utility as in autarky. A solution to Problem 3.2 can be characterized on adjacent intervals. On  $[\alpha, \rho]$ , constraint (3.6) is binding, so  $I(y_2) = C + y_1$ ; on  $[\rho, \gamma]$ ,  $I(y_2) = (\gamma - y_2)/(1 - \lambda)$ ; and on  $[\gamma, \beta]$ , constraint (3.5) is binding,

<sup>9</sup> To see this, transform Problem 3.1 to an equivalent problem by making the substitutions  $g(y_2) = C - I(y_2)$  and  $\xi[g(y_2)] = \Psi[I(y_2)]$ . Next, in lieu of constraint (3.2), impose the apparently weaker restriction that  $I(y_2) > 0$  on  $S$ . From the nature of this modified problem and the monotonicity of  $U_i$  if  $I^* > 0$ , then  $I^* - \Psi(I^*) > 0$  so that a solution to the modified problem will satisfy constraint (3.2). Next, recalling that  $I \equiv 0$  on  $S'$  and  $\Psi(0) = 0$ , enter the expression  $I(y_2) - \Psi[I(y_2)]$  in the second branch of the objective function of the modified problem and enter  $I(y_2)$  in the second branch of constraint (3.1). This yields problem (3.2) with  $S = \{y_2 : I(y_2) > 0\}$ .

<sup>10</sup> Existence and uniqueness of a solution is ensured by the continuity and strict concavity of the objective function and compactness and convexity of the set of feasible solutions. (If  $y_2$  is continuous, the class of functions  $I(y_2)$  is restricted.)

so  $I(y_2) = 0$ . Hence, for this example, Problem 3.2 is equivalent to finding constants  $\gamma$  and  $C$  which maximize

$$\frac{1}{\beta - \alpha} \left\{ \int_{\alpha}^{\rho} U_2[y_2 + y_1 - \lambda(C + y_1)] dy_2 + \int_{\rho}^{\gamma} U_2(\gamma - C) dy_2 + \int_{\gamma}^{\beta} U_2(y_2 - C) dy_2 \right\}$$

subject to

$$\int_{\alpha}^{\rho} (C + y_1) dy_2 + \int_{\rho}^{\gamma} [(\gamma - y_2)/(1 - \lambda)] dy_2 = C(\beta - \alpha)$$

where  $\alpha \leq \gamma \leq \beta$ ,  $0 \leq C \leq \beta$ , and  $\rho = \gamma - (1 - \lambda)(C + y_1)$ . Let  $\gamma_{\lambda}$  denote a maximizing  $\gamma$  given the cost parameter  $\lambda$ . If verification is costless, i.e.,  $\lambda = 0$ , full insurance is optimal and verification always occurs, i.e.,  $\gamma_{\lambda} = \beta$ . It can also be shown that we approach atuarkey as  $\lambda \rightarrow 1$ , i.e.,  $\gamma_{\lambda} \rightarrow \alpha$ . (If  $\lambda = 1$ , then there is no role for insurance and the verification region is empty.) In fact,  $\gamma_{\lambda}$  can take on any value between  $\alpha$  and  $\beta$  by appropriate choice of  $\lambda$  between zero and one.<sup>11</sup>

Under the specified assumptions, the function  $\Psi$  is inconsistent with a fixed cost of verification. Yet it has been argued by some that a fixed cost of acquiring information is typical. It is now established, at least under some further assumptions, that the verification region  $S$  will still have the same property.

The analysis is facilitated by the assumption that agent 1 is risk neutral, so that the consumption of agent 2 will equal some constant on  $S$ . This is stated formally in

**LEMMA 3.1.** *Any solution  $g^*$ ,  $C^*$ ,  $S^*$  to Problem 3.1 with the cost of verification equal to some constant  $\mu$ , with agent 1 risk neutral, and with nonbinding nonnegativity constraints has the property that the consumption of agent 2 equals some positive constant on  $S^*$ .*

This lemma enables one to prove

**PROPOSITION 3.2.** *Any solution  $g^*$ ,  $C^*$ ,  $S^*$  to Problem 3.1 with  $y_2$  continuous; with a fixed verification cost  $\mu$ ; with agent 1 risk neutral; and with a nonnegativity condition on the consumption  $c_2^*(y_2)$  of agent 2,  $\sup c_2^*(y_2) \leq y_1 + \mu - \alpha$ , has the property that  $S^* = \{y_2: y_2 < \gamma\}$  for some parameter  $\gamma$ .*

<sup>11</sup> It should be mentioned here that risk neutrality on the part of agent 1 is not necessary for a nontrivial verification region. Also, the example suggests that there might be a more general monotone dependence between the verification region and the cost.

The contracts characterized in Propositions 3.1 and 3.2 have familiar characteristics. Viewing agent 2 as the insider of a firm financing an investment project, the propositions assert that the firm will default on a bond promising to pay  $C^*$  and suffer a costly audit only when the firm does poorly, i.e., when  $y_2 < \gamma$ . Alternatively, viewing agent 2 as a purchaser of insurance, the propositions can be viewed as an extension of some results in the insurance literature. Here the insured files a claim only if the loss  $(\beta - y_2)$  exceeds  $(\beta - \gamma)$ . Hence  $(\beta - \gamma)$  may be viewed as a deductible. In the insurance literature, nontrivial deductibles ( $\gamma \neq \alpha, \beta$ ) are generated by the assumption that  $I(y_2) \geq 0$  and by the assumption of loading,  $(1 + \lambda) \int_{\alpha}^{\beta} I(y_2) dF(y_2) \leq C$  for some positive constant  $\lambda$ —that is, the actuarial value of the policy must be less than the premium. In this paper the first constraint is motivated by consistency considerations, and the loading assumption is replaced by an explicit treatment of costly state verification.

#### 4. STOCHASTIC VERIFICATION

Thus far attention has been limited to a deterministic verification procedure. That is, verification occurs with probability one or zero, depending on whether or not agent 2 asks for verification. This may be contrasted with schemes in which the decision to verify is determined in a random way. One might conjecture that random procedures can lessen the resource cost of verification while the threat of verification induces honesty. Indeed this turns out to be so; this section describes a stochastic verification scheme that can dominate the deterministic procedure. It goes without saying that this result limits the force of the results presented in this paper for deterministic verification.

For the purpose of establishing that stochastic verification schemes can dominate the deterministic procedure, it is enough to provide a simple, but hopefully generic, example. Consequently, it is assumed throughout this section that  $y_2$  is simple with only two realizations,  $y_2(s)$  and  $y_2(t)$ ,  $0 < y_2(s) < y_2(t)$ , with probabilities  $p(s)$  and  $p(t)$ , respectively.

The stochastic scheme is as follows. Prior to the realization of  $y_2$ , agents 1 and 2 agree to exchange specified amounts contingent on the realization. The amount to be transferred depends on whether there is or is not verification, and the latter is determined in a random way. Agent 2 begins by claiming a realization of  $y_2$ , either  $y_2(s)$  or  $y_2(t)$ . Let  $\pi(w)$  denote the agreed-upon probability that there is verification given that  $y_2(w)$  is claimed,  $w = s, t$ . (Presumably there is some machine (urn) that is known by both agents to generate outcomes with the specified probabilities.) Let  $h(w)$  denote the number of units of the consumption good to be transferred from agent 2 to agent 1 given that  $y_2(w)$  is claimed by agent 2 and there is not verification.

Let  $d(w, w')$  denote the amount to be transferred if  $y_2(w)$  is realized,  $y_2(w')$  is claimed, and there is verification. Let  $\mu$  denote the fixed cost of verification as incurred by agent 2 if there is verification.

It should be noted that the scheme just described differs in various ways from the allocation procedure of Section 2. There agent 2 merely announced whether or not there was to be verification, and then, if there was any discretion, determined the transfer. Here agent 2 announces a particular realization of  $y_2$ , and, subsequent to his announcement, the transfer is completely determined, albeit in a random way. Yet these schemes are not dissimilar; it is established below that any allocation of resources achievable by the deterministic procedure is achievable here without randomization.

It remains to show that the present scheme can generate a (random) allocation of resources which both agents can count on. That is, that there is some known relationship between actual realizations of  $y_2$  and announced realizations. A condition on the probabilities  $\pi(w)$  and transfers  $h(w)$ ,  $d(w, w')$  which ensures such a relationship is

$$\begin{aligned}
 & [1 - \pi(w)] U_2[y_2(w) - h(w)] + \pi(w) U_2[y_2(w) - d(w, w) - \mu] \\
 & \geq [1 - \pi(w')] U_2[y_2(w) - h(w')] + \pi(w') U_2[y_2(w) - d(w, w') - \mu]
 \end{aligned}
 \tag{4.1}$$

for  $w, w' = s, t$ . Inequality (4.1) states that, given the realization  $y_2(w)$ , the expected utility of agent 2 if he claims  $y_2(w)$  as a realization is no less than his expected utility if he claims  $y_2(w')$ . With an indifference convention, then, (4.1) ensures that agent 2 would claim  $y_2(w)$  whenever  $y_2(w)$  is realized,  $w = s, t$ .

For the purpose of establishing that the above-described stochastic scheme can dominate the deterministic procedure, one may consider

**PROBLEM 4.1.** Find the  $\pi(w)$ ,  $h(w)$ , and  $d(w, w')$  that maximize

$$\sum_{w=s,t} p(w) \{ [1 - \pi(w)] U_2[y_2(w) - h(w)] + \pi(w) U_2[y_2(w) - d(w, w) - \mu] \}
 \tag{4.2}$$

subject to (4.1) and

$$\sum_{w=s,t} p(w) \{ [1 - \pi(w)] U_1[y_1 + h(w)] + \pi(w) U_1[y_1 + d(w, w)] \} \geq K
 \tag{4.3}$$

$$c_1[y_2(w)] \geq 0
 \tag{4.4}$$

$$0 \leq \pi(w) \leq 1.
 \tag{4.5}$$

Here constraint (4.3) bounds the expected utility of agent 1, (4.4) is the non-negativity constraint on the consumption of agent 1, and (4.5) restates that the  $\pi(w)$  are probabilities.

Now suppose a solution  $g^*$ ,  $C^*$ ,  $S^*$  to Problem 3.1 has the property that there is verification at  $y_2(s)$ , but not at  $y_2(t)$ . Then there is a feasible solution to Problem 4.1 which achieves the same allocation of resources.<sup>12</sup> For let  $\pi(s) = 1$ ,  $\pi(t) = 0$ ; that is, verify with probability one or zero at  $s$  and  $t$ , respectively. Also, let  $h(t) = C^*$ ,  $d(s, s) = g^*[y_2(s)]$ , and  $d(t, s) = y_2(t) - \mu$ . Then by constraint (3.2),  $d(s, s) + \mu < h(t)$ . It follows that

$$U_2[y_2(s) - d(s, s) - \mu] > U_2[y_2(s) - h(t)] \quad (4.6)$$

$$U_2[y_2(t) - h(t)] > U_2[y_2(t) - d(t, s) - \mu] \quad (4.7)$$

where  $y_2(t) - h(t) > 0$ . With  $\pi(s) = 1$  and  $\pi(t) = 0$ , inequalities (4.6) and (4.7) are consistent with constraint (4.1), and hence, the desired allocation can be achieved.

It is now established that this feasible solution to Problem 4.1 is not maximizing. In addition to the above specification let  $h(s) = g^*[y_2(s)]$ . Then keeping  $\pi(s) = 1$  and  $\pi(t) = 0$ , (4.6) and (4.7) can be rewritten as

$$[1 - \pi(s)] U_2[y_2(s) - h(s)] + \pi(s) U_2[y_2(s) - d(s, s) - \mu] > U_2[y_2(s) - h(t)] \quad (4.8)$$

$$U_2[y_2(t) - h(t)] > [1 - \pi(s)] U_2[y_2(t) - h(s)] + \pi(s) U_2[y_2(t) - d(t, s) - \mu]. \quad (4.9)$$

Note that  $y_2(t) - h(s) > 0$  and  $y_2(t) - d(t, s) - \mu = 0$ . It follows that, *ceteris paribus*,  $\pi(s)$  can be diminished *somewhat* without changing the direction of the inequality in (4.9). As for constraint (4.8), note that with  $h(s) = d(s, s) = g^*[y_2(s)]$ , agent 2 is clearly better off without verification at  $y_2(s)$  by virtue of the resource savings  $\mu$ . Hence a diminution of  $\pi(s)$  will not cause constraint (4.8) to be violated. With the transfer to agent 1 independent of verification at  $y_2(s)$ , constraints (4.3) and (4.4) will still be satisfied. Hence there exists a feasible solution to Problem 4.1 with  $\pi(t) = 0$  and  $0 < \pi(s) < 1$  which dominates the (deterministic) solution to Problem 3.1.

Given the dominance of stochastic verification, some further comment on Problem 4.1 and its solutions would seem to be in order. First, one may question whether the constraints (4.1) may be imposed without loss of generality, as were the consistency conditions in Section 2. That is, suppose the  $\pi(w)$ ,  $h(w)$ , and  $d(w, w')$  were such that both of the constraints (4.1) were violated. Then there is a specification of transfers (essentially a relabeling) which achieves the same allocation and satisfies constraints (4.1). If only one constraint is violated, say for example, agent 2 would always announce that  $y_2(s)$  is realized, then there is a *modified* game in which agent 2

<sup>12</sup> A similar argument establishes that *whatever* the relationship between  $\pi(s)$  and  $\pi(t)$ , the allocation achieved in a solution to Problem 3.1 is also attainable under the stochastic scheme with nonrandom verification.

must always announce  $y_2(s)$ , effecting either  $h(s)$ ,  $d(s, s)$ , or  $d(t, s)$ . Hence there is a modified, albeit more complicated, version of Problem 4.1 which may be imposed without loss of generality.

As to the nature of solutions to problems similar to 4.1, little has been determined.<sup>13</sup> One might *conjecture*, based on the results for deterministic verification, that the probability of verification should be a nonincreasing function of  $y_2$  and perhaps should be zero in states with high realizations. It may be noted in this regard that, in the example discussed above, resource savings are limited by the extent to which  $\pi(s)$  can be diminished without creating an incentive for agent 2 to cheat at  $y_2(t)$ . If  $U_2$  were unbounded from below, then it seems that the value of the objective function could be made arbitrarily close to the corresponding value with optimal contracts and costless verification by making the  $\pi(w)$  arbitrarily close to zero,  $w = s, t$ . For let  $g^*(w)$  denote a maximizing transfer as a function of  $w$  with costless verification. Then, ignoring nonnegativity constraints, let  $h(w) = d(w, w) = g^*(w)$ . Since  $U_2(c) \rightarrow -\infty$  as  $c \rightarrow 0$ ,  $\pi(w)$  can be made arbitrarily close to zero by appropriate choice of  $d(w, w')$  without violating the constraints (4.1).<sup>14</sup>

In summary, stochastic verification procedures can dominate deterministic procedures. In fact, stochastic procedures are not uncommon. The timing of bank audits by government agencies is somewhat random. Similarly, corporations use stochastic procedures in monitoring internal divisions. It is also said that tax audits by the IRS are determined in part at random.

## 5. CONSTRAINED OPTIMAL CONTRACTS IN AN $m$ -AGENT ECONOMY

This section returns to deterministic verification procedures in an attempt to generalize the earlier results on other dimensions—the number of agents and unobserved random variables. It will be seen that this attempt raises some new and interesting problems with regard to the characterization of optimal contracts.

We begin with a symmetric two-agent economy. That is, the realization of the endowment  $y_j$  of each agent  $j$  ( $j = 1, 2$ ) is known only by agent  $j$  unless a verification cost is borne. Each random variable  $y_j$  is associated with a cumulative distribution function  $F(y_j)$  and takes on values in the interval  $[\alpha_j, \beta_j]$ ,  $\alpha_j > 0$ . The  $y_j$  are all either simple or continuous. In the latter

<sup>13</sup> The difficulty is that constraints (4.1) seem quite messy analytically; examination of the necessary conditions for a maximum, as in the proof of Proposition 3.1, has not yet provided much insight. In order to avoid putting measures on measures, a restriction to simple rather than continuous random variables has been imposed. Yet this seems to make the characterization more difficult.

<sup>14</sup> Note that in such cases, Problem 4.1 cannot attain its supremum; at  $\pi(w) = 0$  for all  $w$  there are no disincentives to cheating.

case each  $y_j$  possesses a continuous, strictly positive density function  $f_j$ . It is assumed moreover that the  $y_j$  are independent so that the realization of  $y_j$  conveys no information about  $y_i$ ,  $i \neq j$ . Each agent  $j$  has a utility function  $U_j$  over riskless consumption which is continuously differentiable, strictly concave, and strictly increasing with  $U_j'(0) = \infty$  and  $U_j'(\infty) = 0$ .

Prior to the realizations of  $y_1$  and  $y_2$ , both agents make exchange and verification plans which are contingent on the realizations. That is, let  $g(y_1, y_2)$  denote the actual poststate net transfer of the consumption good from agent 2 to agent 1 as a function of the realizations of  $y_1$  and  $y_2$ , and let  $\bar{g}(y_1, y_2)$  denote the poststate contractual choice of this transfer function. Also, let  $S_j$  denote the set of realizations of  $y_j$  under which there actually is verification of  $y_j$ , and let  $\bar{S}_j$  denote the prestate contractual choice of this set. Thus a contract in this economy is a specification of  $\bar{g}$ ,  $\bar{S}_1$ , and  $\bar{S}_2$ .

Subsequent to the realization of  $y_j$ , each agent  $j$  announces whether there is or is not to be verification. If there is verification,  $y_j$  is made known to agent  $i$  ( $i \neq j$ ) and  $\phi_j(y_j)$  units of the consumption good are forfeited by agent  $j$ .<sup>15</sup> It is agreed that if both agents are verified, then they transfer what was agreed upon, i.e.,  $g(y_1, y_2) = \bar{g}(y_1, y_2)$ . If agent 1 is verified but agent 2 is not, then it is agreed that agent 2 can effect any transfer consistent with the known value of  $y_1$  and any  $y_2$  in the agreed-upon non-verification region of  $y_2$ , i.e.,  $g(y_1, y_2) = \min \bar{g}(y_1, x)$  where the minimum is over  $x \in \bar{S}'_2$ . Similarly, if agent 2 is verified but agent 1 is not, then agent 1 determines the transfer, i.e.,  $g(y_1, y_2) = \max \bar{g}(x, y_2)$  where the maximum is over  $x \in \bar{S}'_1$ . If neither agent is verified, it may be supposed without loss of generality that agent 2 determines the transfer, i.e.,  $g(y_1, y_2) = \min \bar{g}(x_1, x_2)$  where the minimum is over  $(x_1, x_2) \in \bar{S}'_1 \times \bar{S}'_2$ .<sup>16</sup> Finally, if some agent  $j$  asks for verification, but  $y_j$  is not in the agreed-upon verification region  $\bar{S}_j$ , then agent  $j$  incurs the verification cost, and the transfer is determined as if agent  $j$  had not asked for verification. Note that this effectively precludes such an event, and hereafter we disregard this possibility. (The scheme is easily modified to allow for binding nonnegativity constraints.) Any remaining indeterminacy is resolved by an indifference convention as in Section 2.

<sup>15</sup> Here it should be understood that the verification cost  $\phi_j(y_j)$  can depend in an exogenous way on the agreed-upon transfer  $\bar{g}$ , which in turn has  $y_j$  as an argument. Thus  $\phi_j(y_j)$  should be viewed as a composite function and is not meant to imply that the costs depend in an exogenous way on the realization  $y_j$ .

<sup>16</sup> Symmetry might suggest that both should determine the transfer, but this leads to an obvious inconsistency. The implication of the present specification will be that the agreed-upon transfer must be some constant on  $\bar{S}'_1 \times \bar{S}'_2$ , an implication that would also follow if agent 1 determined the transfer. The constant is determined in a solution to a Pareto problem, and thus the process does not favor agent 2 a priori.

The strategy of *telling the truth* for agent  $j$  means the poststate announcement of whether he is or is not to be verified in accord with  $\bar{S}_j$  and  $\bar{S}'_j$ . Now one may define a contract  $[\bar{g}, \bar{S}_1, \bar{S}_2]$  to be *consistent* if (i) telling the truth is a dominant strategy for each agent  $j$ , and (ii)  $g = \bar{g}$ . Note that condition (i) implies  $S_j = \bar{S}_j, j = 1, 2$ , so in this sense the definition of consistency of Section 2 has been generalized.

The implications of consistency should not be too surprising. Under a consistent contract the transfer function  $\bar{g}$  cannot depend on information that is known only to one agent. That is, the transfer cannot depend on  $y_j$  if agent  $j$  is not verified. Also, certain incentive inequalities must be satisfied. More formally, we have

LEMMA 5.1. *A contract  $[\bar{g}, \bar{S}_1, \bar{S}_2]$  is consistent if and only if  $\bar{g}(y_1, y_2)$  equals some constant  $\bar{C}$  on  $\bar{S}'_1 \times \bar{S}'_2$ , equals some function  $\bar{g}^1(y_1)$  on  $\bar{S}_1 \times \bar{S}'_2$ , equals some function  $\bar{g}^2(y_2)$  on  $\bar{S}'_1 \times \bar{S}_2$ , and the inequalities below obtain:*

$$\begin{aligned} \bar{g}(y_1, y_2) - \phi_1(y_1) &> \bar{g}^2(y_2) && (y_1, y_2) \in \bar{S}_1 \times \bar{S}_2 \\ \bar{g}^1(y_1) - \phi_1(y_1) &> \bar{C} && y_1 \in \bar{S}_1 \\ \bar{g}(y_1, y_2) + \phi_2(y_2) &< \bar{g}^1(y_1) && (y_1, y_2) \in \bar{S}'_1 \times \bar{S}_2 \\ \bar{g}^2(y_2) + \phi_2(y_2) &< \bar{C} && y_2 \in \bar{S}_2. \end{aligned}$$

It may also be noted that under the dominant strategy equilibrium concept for determining the outcome of a contract  $[\bar{g}, \bar{S}_1, \bar{S}_2]$ , consistency requirements may be imposed without loss of generality, as in Section 2.

One may now proceed in an attempt to characterize optimal contracts. Motivated by the classical approach of Section 3, one might hope to formulate an analog to Problem 3.2 in which inequality constraints define the space of feasible functions and in which there is no explicit reference to regions. First, define functions  $\bar{I}_{21}(y_1)$  and  $\bar{I}_{12}(y_2)$  as follows. Let  $\bar{I}_{21}(y_1) = \bar{g}^1(y_1) - \bar{C}$  on  $\bar{S}_1, \bar{I}_{21}(y_1) \equiv 0$  on  $\bar{S}'_1, \bar{I}_{12}(y_2) = \bar{C} - \bar{g}^2(y_2)$  on  $\bar{S}_2$ , and  $\bar{I}_{12}(y_2) \equiv 0$  on  $\bar{S}'_2$ . Also, define a function  $\bar{K}(y_1, y_2) = \bar{g}(y_1, y_2) - \bar{C} - \bar{I}_{21}(y_1) + \bar{I}_{12}(y_2)$  on  $\bar{S}_1 \times \bar{S}_2$  and zero otherwise. Then by substitution into the inequality constraints of Lemma (5.1) one obtains the restrictions

$$\bar{I}_{21}(y_1) - \phi_1(y_1) > 0 \quad y_1 \in \bar{S}_1 \tag{5.1}$$

$$\bar{I}_{12}(y_2) - \phi_2(y_2) > 0 \quad y_2 \in \bar{S}_2 \tag{5.2}$$

$$\begin{aligned} -\bar{I}_{21}(y_1) + \phi_1(y_1) &< \bar{K}(y_1, y_2) \\ &< \bar{I}_{12}(y_2) - \phi_2(y_2) \quad (y_1, y_2) \in \bar{S}_1 \times \bar{S}_2. \end{aligned} \tag{5.3}$$

The difficulty with this approach is constraint (5.3) and the appearance of the function  $\bar{K}(y_1, y_2)$  on  $\bar{S}_1 \times \bar{S}_2$ . If, however,  $\bar{K}(y_1, y_2)$  were restricted exogenously to be identically zero, then  $\bar{g}(y_1, y_2) = \bar{I}_{21}(y_1) - \bar{I}_{12}(y_2) + \bar{C}$

everywhere. One could then postulate that the cost of verification of  $y_j$  depends only on the agreed-upon insurance payment  $\bar{I}_{ij}$ . That is,  $\phi_j(y_j) = \Psi_j[\bar{I}_{ij}(y_j)]$  with  $\Psi_j(0) = 0$ . Then, as in Section 3, one could formulate an optimization problem with the verification region  $\bar{S}_j$  defined by  $\bar{S}_j = \{y_j: \bar{I}_{ij}(y_j) > 0\}$ . This is done below in greater generality. The maximizing contract is said to be a constrained optimum.

One should consider the implication of the constraint  $\bar{K}(y_1, y_2) \equiv 0$ .<sup>17</sup> Roughly speaking, this restriction precludes certain risk-sharing arrangements. To get some feel for this suppose  $U_1(c) = c^{\gamma+1}/(\gamma + 1)$ ,  $U_2(c) = c^{\rho+1}/(\rho + 1)$  with  $\gamma = -\frac{1}{4}$  and  $\rho = -\frac{1}{2}$ . Then the optimal (full information) transfer function  $g^*$  is of the form

$$2g^*(y_1, y_2) = (\lambda^{-4} + 2y_2) \pm \{(\lambda^{-4} + 2y_2)^2 - 4(y_2^2 - \lambda^{-4}y_1)\}^{1/2},$$

where  $\lambda$  is some positive constant. To be noted is that  $g^*$  is not separable with respect to  $y_1$  and  $y_2$  as is required by the exogenous restriction. Thus it seems that, among other things, the transfer function is constrained in the region in which both agents are verified.

For the remainder of this section we consider the  $m$ -agent generalization of the symmetric two-agent economy. Much of the notation introduced at the outset of this section applied in an obvious way. For example,  $y_j$  denotes the endowment of each agent  $j$ , where now  $j = 1, 2, \dots, m$ . Let  $F(y_1, y_2, \dots, y_m)$  denote the joint distribution of the endowments. Again, independence is assumed. Any realization of  $y_j$  is assumed to be known only to agent  $j$  unless a verification cost is borne; in that event  $y_j$  is made known to *all* agents. Let  $g_{ij}(y_1, y_2, \dots, y_m)$  denote the net transfer of the consumption good from agent  $i$  to agent  $j$  as a function of the realization of each of the endowments.<sup>18</sup> Then a social contract  $\{\bar{g}_{ij}\}_{i,j=1}^m, \{\bar{S}_j\}_{j=1}^m$  is a prestate agreement as to the amounts to be transferred and when there is to be verification. Such a contract is said to be consistent if: (i) telling the truth is a dominant strategy for each agent  $j$ , and (ii)  $g_{ij} = \bar{g}_{ij}$ ,  $i, j = 1, 2, \dots, m$ .

Again, one would like to find an analytically tractable maximization problem whose solutions characterize an optimal social contract. Unfortunately this is done here only after imposing several exogenous restrictions

<sup>17</sup> The intent here and below is to impose enough exogenous restrictions that Proposition 3.1 can be generalized. It is hoped that the reader finds these restrictions, motivated as they are by technical considerations, as unpleasant as the author. It may be noted, however, that under these restrictions feasible contracts seem to mimic what we actually observe in some insurance markets; each agent pays a premium independent of the state and receives compensation only as a function of his *own* loss. Additional work should be devoted to finding an environment under which these restrictions are endogenous so that Proposition 5.1 and the results of Section 6 below have more force.

<sup>18</sup> Thus  $g_{ii} = -g_{ij}$ . Also, it is convenient in what follows to define  $g_{ii} = 0$  and similarly (except in Section 6) for all variables with an identical double subscript.

on the exchanges, including that they be bilateral in nature. That is, the agreement  $\bar{g}_{ij}$  is restricted to depend at most on  $y_i$  and  $y_j$ . It bears repeating that this restriction is imposed for analytical convenience and is not motivated by economic considerations.<sup>19</sup> Given this restriction it may be presumed that each pair of agents  $i$  and  $j$  adopts a resource allocation procedure virtually identical to the two-agent procedure described above, and, in similar fashion, restrictions on the transfer function  $\bar{g}_{ij}$  analogous to those of the first part of Lemma 5.1 may be derived, with subscripts  $i$  and  $j$  where appropriate. Imposing restrictions analogous to  $K(\cdot, \cdot) \equiv 0$ , the  $\bar{g}_{ij}$  can be shown to be of the form  $\bar{g}_{ij}(y_i, y_j) = \bar{I}_{ij}(y_j) - \bar{I}_{ji}(y_i) + \bar{C}_{ij}$  where  $\bar{C}_{ij}$  is some constant and  $\bar{I}_{ij}(y_j) \equiv 0$  on  $\bar{S}'_j$ . Also impose the restrictions that  $I_{ij}(y_j) \geq 0$ .

Now suppose the cost of verifying  $y_j$  depends only on the sum of the "insurance payments" from other agents. That is, let the verification cost be  $\Psi_j[\sum_i \bar{I}_{ij}(y_j)]$  where  $\Psi_j$  is a continuously differentiable, convex function with  $\Psi_j(0) = 0$  and  $\Psi'_j(0) < 1$ .

Motivated by this discussion, then, a prestate social contract  $\{\bar{g}_{ij}\}_{i,j=1}^m$ ,  $\{\bar{S}_j\}_{j=1}^m$  is restricted to be of the form

$$\begin{aligned} \bar{g}_{ij}(y_i, y_j) &= \bar{I}_{ij}(y_j) - \bar{I}_{ji}(y_i) + \bar{C}_{ij} \\ \bar{I}_{ij}(y_j) &\geq 0 \\ \bar{S}_j &= \left\{ y_j: \sum_i \bar{I}_{ij}(y_j) - \Psi_j \left[ \sum_i \bar{I}_{ij}(y_j) \right] > 0 \right\} \\ \bar{S}'_j &= \{ y_j: \bar{I}_{ij}(y_j) = 0 \text{ for all } i \}. \end{aligned}$$

By construction, such a social contract is consistent, and subsequently the - may be dropped from the notation.

One may now characterize a *constrained optimal social contract* by consideration of

**PROBLEM 5.1.** Find functions  $I_{ij}$  and constants  $C_j$ ,  $i, j = 1, 2, \dots, m$  that maximize

$$\int U_1 \left\{ y_1 + \sum_i I_{i1}(y_1) + C_1 - \sum_i I_{1i}(y_i) - \Psi_1 \left[ \sum_i I_{i1}(y_1) \right] \right\} dF(y_1, y_2, \dots, y_m)$$

<sup>19</sup> Jerry Green [8], among others, has stressed the need for bilateral models of exchange, but their study here (making the restriction endogenous, perhaps by an explicit treatment of the technology of communication) would constitute a separate paper. In contrast Wilson [23] has stressed the collective nature of decisions under risk.

subject to

$$\int U_j \left\{ y_j + \sum_i I_{ij}(y_j) + C_j - \sum_i I_{ji}(y_i) - \Psi_j \left[ \sum_i I_{ij}(y_j) \right] \right\} \times dF(y_1, y_2, \dots, y_m) \geq K_j \quad j = 2, 3, \dots, m \tag{5.4}$$

$$I_{ij}(y_j) \geq 0 \quad i, j = 1, 2, \dots, m \tag{5.5}$$

$$\sum_{j=1}^m C_j = 0. \tag{5.6}$$

Here  $S_j$  is defined by  $S_j = \{y_j: \sum_i I_{ij}(y_j) > 0\}$ . Here also the constants  $C_j$  may be interpreted as a premium received by agent  $j$  independent of the realization of the  $y_i$ . Note that these completely determine the desired constants  $C_{ij}$ .<sup>20</sup> Also impose the better-than-autarky condition,  $K_j \geq \int U_j(y_j) dF(y_j)$ .

Finally, we obtain the sought-after analog of Proposition 3.1 in

**PROPOSITION 5.1.** *Any solution  $I_{ij}^*, C_j^*$  to Problem 5.1 with either the  $y_j$  simple or the  $y_j$  and the  $I_{ij}^*$  continuous has the property that each  $S_j^* = \{y_j: y_j < \gamma_j\}$  for some parameter  $\gamma_j$ .*

### 6. A PARETO SATISFACTORY COMPETITIVE EQUILIBRIUM

The purpose of this section is to analyze the properties of a competitive equilibrium concept for the  $m$ -agent economy. In particular it is established that a competitive equilibrium exists and that any equilibrium allocation is a constrained optimum, i.e., can be achieved with a constrained optimal social contract. This result is important in establishing the way in which agents end up on the contract curve and thereby supports the contention that optimal trades will be observed.

For the purpose of this section each  $y_j$  will be taken to be simple with  $n$  possible realizations.<sup>21</sup> The realization  $y_j(s_j)$  occurs with probability  $p_j(s_j)$ ,  $s_j = 1, 2, \dots, n$ . The commodities which are traded in competitive markets prior to the realization of the endowments are claims contingent on the realization of each endowment and unconditional claims.<sup>22</sup> Let  $J_{ij}(s_j)$  denote the number of claims contingent on the  $s_j$ th realization of  $y_j$  purchased by agent  $i$ , where one such claim entitles the holder to one unit of the consumption good

<sup>20</sup> The relationship is  $\sum_i C_{ij} = C_j$ , where as usual  $C_{jj} = 0$  and  $C_{ji} = -C_{ij}$ .

<sup>21</sup> One could easily permit a different number of realizations for each agent.

<sup>22</sup> For an earlier discussion of the relationship between insurance contracts and contingent commodity markets in the standard competitive model see Kihlstrom and Pauly [14].

if  $y_j(s_j)$  is realized and zero otherwise. The direction of trade in such contingent claims is restricted: Agent  $i$  can purchase claims contingent on his own endowment and issue claims contingent on the endowments of others. That is,  $J_{ii}(s_i) \geq 0$ , and  $J_{ij}(s_j) \leq 0$ ,  $i \neq j$ . Let  $q_j(s_j)$  denote the price of a unit claim contingent on  $y_j(s_j)$ . Let  $D_i$  denote the number of unconditional claims on the consumption good purchased by agent  $i$  in the market for claims, where one such claim entitles the holder to unit of the consumption good regardless of the realization of the endowments. There is no direct restriction on the direction of trade in such unconditional claims. Let  $r$  denote the price of one such unconditional claim. After the realization  $y_i(s_i)$ , each agent  $i$  must decide whether (or not) to collect the "insurance payment"  $J_{ii}(s_i)$ , incurring the verification cost  $\Psi_i[J_{ii}(s_i)]$ .

All agents take the prices  $q_j(s_j)$  and  $r$  as parameters and maximize expected utility subject to the budget constraint. That is, each agent  $i$  chooses the  $J_{ij}(s_j)$  and  $D_i$  to maximize

$$\sum_{s_1=1}^n \sum_{s_2=1}^n \cdots \sum_{s_m=1}^n [p_1(s_1) p_2(s_2) \cdots p_m(s_m)] \times U_i \left\{ y_i(s_i) + \sum_{j=1}^m J_{ij}(s_j) + D_i - \Psi_i[J_{ii}(s_i)] \right\} \quad (6.1)$$

subject to

$$\sum_{j=1}^m \sum_{s_j=1}^n q_j(s_j) J_{ij}(s_j) + rD_i \leq 0 \quad (6.2)$$

$$J_{ii}(s_i) \geq 0 \quad J_{ij}(s_j) \leq 0, \quad j \neq i \quad (6.3)$$

$$J_{ii}(s_i) - \Psi_i[J_{ii}(s_i)] \geq 0 \quad (6.4)$$

$$y_i(s_i) + \sum_{j=1}^m J_{ij}(s_j) + D_i - \Psi_i[J_{ii}(s_i)] \geq 0. \quad (6.5)$$

Here, (6.2) is the budget constraint, (6.3) restricts the direction of trade, as noted, (6.4) ensures that the proceeds of insurance cover verification costs, and (6.5) is the nonnegativity constraint on consumption. Note that these last two constraints could be suppressed.

A *competitive equilibrium* is a set of nonnegative prices  $q_j^*(s_j)$  and  $r^*$  (not all zero) and commodity demands  $J_{ij}^*(s_j)$  and  $D_i^*$  for each agent  $i$  such that

- (i)  $J_{ij}^*(s_j)$  and  $D_i^*$  maximize (6.1) subject to constraints (6.2)–(6.5), and
- (ii)  $\sum_{i=1}^m J_{ij}^*(s_j) \leq 0$ ,  $\sum_{i=1}^m D_i^* \leq 0$  (market clearing).

The existence and constrained optimality of such an equilibrium is established.

**PROPOSITION 6.1.** *Under the assumptions of the model there exists a competitive equilibrium.*

**PROPOSITION 6.2.** *The allocation of any competitive equilibrium is a constrained optimum.*

An equilibrium concept may be said to be Pareto satisfactory if any equilibrium allocation is optimal and if any optimal allocation can be supported as an equilibrium. (See Hurwicz [13].) Proposition 6.2 establishes the first property. As for the second, it is clear that if agents were endowed with the unconditional and contingent claims associated with a constrained optimal allocation, then there would exist an autarkic competitive equilibrium. Hence the equilibrium described in this section is Pareto satisfactory relative to the constrained optimal allocations described in the previous section.

Finally, some unusual characteristics of this equilibrium concept should be noted. The contingent claims which are traded in this model are not anonymous. A contingent claim on  $y_j$  is associated with agent  $j$ . Though for large  $m$  there are many possible sellers of such commodities, there is only one possible buyer, agent  $j$ . Hence the assumption that agent  $j$  is a price taker may be troublesome. Ideally, the way to proceed in this context is to formulate a game with endogenous price setters, and with restrictions on trade tied closely to incentive compatibility conditions, and to establish that the equilibrium allocations of such a game approach in the limit those of the competitive equilibrium (as defined above) as the economy is replicated. It would seem crucial in establishing such a result that the "bargaining power" of any agent become negligible in the limit, despite the fact that for any finite economy traders do not have identical initial endowments. Caspi [6] provides evidence to this effect in a simpler (full information) context: in a pure exchange economy in which traders have identical preferences and independent but identically distributed random endowments, a *vanishing* function of traders receive in the core a claim which differs from the mean of their common endowment as the economy is replicated. The point is that in the context of Caspi's model the monopsony power of each buyer is limited because of the presence of near ex ante substitutes. One strongly suspects this result will carry over to the limited information context of this model, despite the need for idiosyncratic verification.<sup>23</sup>

<sup>23</sup> Again it may be noted that competitive insurance markets seem consistent with the imposed restrictions.

## 7. CONCLUDING REMARKS

Perhaps one of the more interesting aspects of this paper is the attempt to explain the financial organization of firms by way of information asymmetries. As Ross [18] indicates in taking a similar approach, attempts to reconcile observations on financial structure with the Miller–Modigliani theorem have been less than satisfactory. Yet on this account, at least, this paper cannot be termed a success. The model as it stands may contribute to our understanding of closely held firms, but it cannot explain the coexistence of publicly held shares and debt. And one would like to model bankruptcy at a deeper level. Thus this paper can only be regarded as a first step.

The extent to which a model may be said to *explain* economic phenomena depends on the nature of exogenous restrictions on the behavior of agents of the model, that is, restrictions which are not implied by the environment. Perhaps the most troublesome is the restriction to deterministic verification. There are also exogenous restrictions on feasible transfer functions. Risk-sharing arrangements when each of two agents is verified are restricted in a way which is motivated by technical considerations, and mutually advantageous trades contingent on the realized endowment of a third party are also excluded. Clearly here as in much of the contract literature more work is needed in multiagent environments. In this regard we may note again that the existence and optimality of the competitive equilibrium concept of Section 6 are established subject to these exogenous restrictions. As is well known, the presence in some settings of exogenous restrictions can affect the existence of equilibrium. It is hoped that the analysis of this paper will prove useful in subsequent work in characterizing optimal contracts and in establishing the existence of equilibrium when fewer exogenous restrictions are imposed. Of course, the propositions of this paper will have more force to the extent that the restrictions which have been imposed here can be derived endogenously in environments with more structure, with limitations on multilateral communication, for example.

For the most part, the model deals with information in an entirely classical way. There has been some discussion in the literature to the effect that there are increasing returns to scale in the production of information; see for example Wilson [22] and Radner [16]. Grossman and Stiglitz [9] have shown that costly information can be revealed completely by the equilibrium prices of competitive markets. In contrast Hirshleifer [12] has argued that competitive markets induce the acquisition of too much information. The results of this paper would seem to illustrate that the nature of information varies with the phenomena of interest to economists and that one should be wary of generalizations.

The model provides an example of the suggestion by Radner [16] that convexity in the technology of information production is reasonable in

situations in which information depends on actions which can be scaled down to any desired size; it is postulated that resources used in state verification vary directly with the size of insurance claims. However, convexity is lost under the apparently reasonable specification that there is a fixed cost of verification. It may be argued by way of Proposition 3.2 that the characterization of optimal contracts will remain valid even under such a specification. But nonconvexities can be the source of considerable difficulty in establishing the existence and optimality of a competitive equilibrium. Ongoing joint research with Edward C. Prescott [15] indicates that, in some contexts, these difficulties may be overcome by stochastic schemes. This leads us back again to Section 4 and the very real possibility of obtaining existence and welfare results with stochastic verification. But this must be the subject of another paper.

APPENDIX

*Proof of Lemma 2.1.* First, note that given any contract  $[\bar{g}, \bar{S}]$ , if  $y_2$  is such that there is not to be verification (i.e.,  $y_2 \in \bar{S}'$ ), then agent 2 has no incentive to ask for verification. For if agent 2 were not to ask for verification, he would transfer  $\min_{x \in S'} \bar{g}(x)$  to agent 1. Alternatively, if agent 2 were to ask for verification, then  $\xi[\bar{g}(y_2)]$  would be used in verification and  $\bar{g}(y_2)$  would be transferred. Clearly agent 2 can only be made worse off by asking for verification.

Necessity is now established. If a contract  $[\bar{g}, \bar{S}]$  is consistent, then  $\bar{g}(y_2)$  is identically equal to some constant  $\bar{C}$  on  $\bar{S}'$ . This may be established by contradiction. Let  $K = \min_{x \in S'} \bar{g}(x)$ , and suppose for some  $y_2 \in \bar{S}'$ ,  $\bar{g}(y_2) > K$ . If this  $y_2$  were realized, there would not be verification. Consequently, the actual transfer  $g(y_2)$  would be  $K$ , which is less than  $\bar{g}(y_2)$ , contradicting condition (ii).

If a contract  $[\bar{g}, \bar{S}]$  is consistent, then  $\bar{g}(y_2) + \xi[\bar{g}(y_2)] < \bar{C}$  for all  $y_2$  such that there is to be verification (i.e.,  $y_2 \in \bar{S}$ ). Again, arguing by contradiction, suppose this property fails to hold for some  $y_2 \in \bar{S}$ . Then, if this  $y_2$  were realized, agent 2 would not ask for verification, contradicting condition (i).

Sufficiency is now established. If  $y_2 \in \bar{S}'$ , there will not be verification (so  $y_2 \in S'$ ), and  $\bar{C}$  will be transferred (so  $g(y_2) = \bar{g}(y_2)$ ). If  $y_2 \in \bar{S}$ , there will be verification (so  $y_2 \in S$ ), and  $\bar{g}(y_2)$  will be transferred (so  $g(y_2) = \bar{g}(y_2)$ ).

*Proof of Lemma 2.2.* The contract  $[\bar{h}, \bar{T}]$  as defined in the text is consistent.

*Proof of Proposition 3.1.* If  $y_2$  is continuous, among the necessary Euler conditions for a maximum are

$$\{1 - \Psi'[I^*(y_2)]\} U_2'(y_2 + I^*(y_2) - C^* - \Psi[I^*(y_2)])f(y_2) - \theta_1^* U_1'[y_1 - I^*(y_2) + C^*]f(y_2) + \theta_2^*(y_2) - \theta_3^*(y_2) = 0 \quad (A1)$$

$$\theta_1^* > 0$$

$$\theta_2^*(y_2) \geq 0 \quad I^*(y_2) \geq 0 \quad \theta_2^*(y_2) I^*(y_2) = 0$$

$$\theta_3^*(y_2) \geq 0 \quad y_1 - I^*(y_2) + C^* \geq 0 \quad \theta_3^*(y_2)[y_1 - I^*(y_2) + C^*] = 0.$$

Let  $\gamma$  be chosen so that

$$[1 - \Psi'(0)] U_2'(\gamma - C^*) - \theta_1^* U_1'(\gamma_1 + C^*) = 0. \quad (A2)$$

Suppose  $I^*(y_2) = 0$  for some  $y_2 \in [\alpha, \gamma]$ . Then from (A2)

$$[1 - \Psi'(0)] U_2'(y_2 - C^*)f(y_2) - \theta_1^* U_1'(\gamma_1 + C^*)f(y_2) > 0. \quad (A3)$$

With  $I^*(y_2) = 0$ , it follows that  $\theta_2^*(y_2) \geq 0$  and  $\theta_3^*(y_2) = 0$ , and therefore (A3) contradicts (A1).

Similarly, suppose  $I^*(y_2) > 0$  for some  $y_2 \in [\gamma, \beta]$ . Then  $I^*(y_2) - \Psi[I^*(y_2)] > 0$ , and from (A2)

$$\{1 - \Psi'[I^*(y_2)]\} U_2'(y_2 + I^*(y_2) - C^* - \Psi[I^*(y_2)])f(y_2) - \theta_1^* U_1'[y_1 - I^*(y_2) + C^*]f(y_2) < 0. \quad (A4)$$

With  $I^*(y_2) > 0$  it follows that  $\theta_2^*(y_2) = 0$  and  $\theta_3^*(y_2) \geq 0$ , and therefore (A4) contradicts (A1).

If  $y_2$  is simple, the proof proceeds as above with obvious changes in notation.

*Proof of Lemma 3.1.* The proof is rather standard and is not given here for the sake of brevity.

*Proof of Proposition 3.2.* The proof is by contradiction. Suppose  $S^*$  is not a lower interval, i.e.,  $S^* \neq \{y_2: y_2 < \gamma\}$  for any parameter  $\gamma$ . Then, roughly speaking, push the verification region to the left while retaining its mass so that it becomes a lower interval. More precisely, let  $\delta$  be chosen so that  $\text{Prob}([\alpha, \delta]) = \text{Prob}(S^*) > 0$ . Then let the verification set be  $T = \{y_2: \alpha \leq y_2 < \delta\}$  and its complement be  $T' = \{y_2: \delta \leq y_2 \leq \beta\}$ . A new consumption path  $c_2^*(y_2)$  will be constructed on  $T$  and  $T'$  in such a way as to both satisfy constraints (3.1)–(3.3) of Problem 3.1 and to increase the value of the objective functional, the expected utility of agent 2. See Figs. 1A and 1B. This will be the desired contradiction.

By Lemma 3.1 and the nonnegativity condition, the initial consumption path  $c_2^*(y_2)$  equals some constant  $K^*$  on  $S^*$ . Of course,  $c_2^*(y_2) = y_2 - C^*$  on  $S^*$ . For purposes of this proof it will be assumed that  $\alpha - C^* > 0$  and

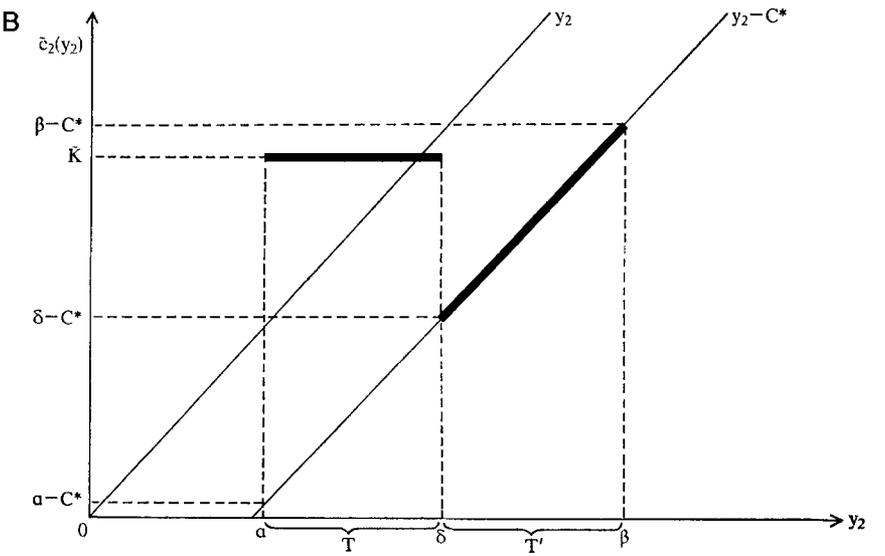
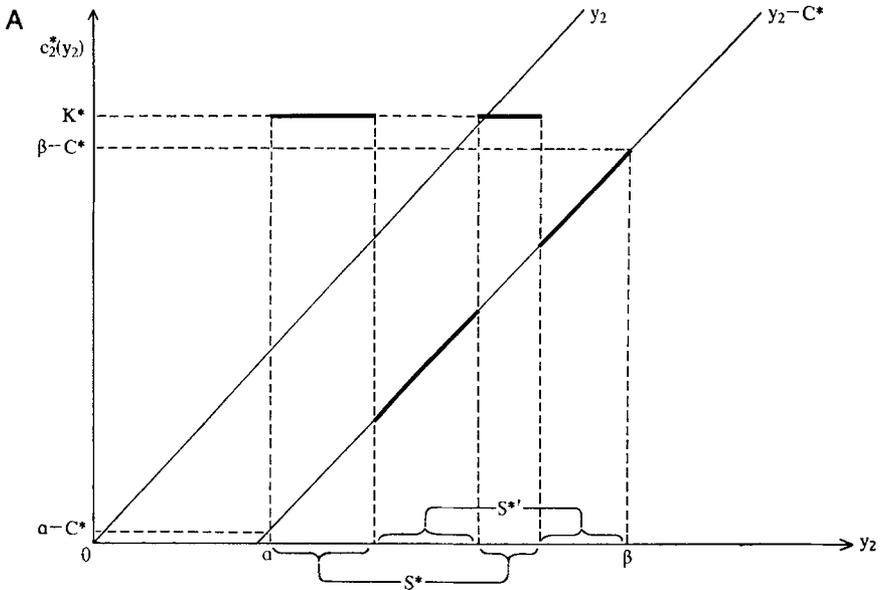


FIGURE 1

$K^* > \beta - C^*$ . The other possible cases can be treated in a similar manner, but this is not done here for the sake of brevity. On  $T'$  let  $\tilde{c}_2(y_2) = y_2 - C^*$ . Now given some constant  $\tilde{K}$  (with a property described momentarily), on  $T$  let  $\tilde{c}_2(y_2) = \tilde{K}$  if  $\tilde{K} > y_2 - C^*$  and let  $\tilde{c}_2(y_2) = y_2 - C^*$  otherwise. The constant  $\tilde{K}$  is chosen so that the expected consumption of agent 2 is the same under the partitions  $\{S^*, S^{*'}\}$  and  $\{T, T'\}$ . It is assumed that  $\delta - C^* < \tilde{K} < \beta - C^*$ ; again, this is a special case, though other cases are similar. With the same expected cost of verification, the expected consumption of agent 1 will remain unchanged, so constraint (3.1) is satisfied. With the non-negativity condition, constraint (3.3) is satisfied. By construction, constraint (3.2) is satisfied.

Let  $F^*(x)$  and  $\tilde{F}(x)$  denote the cumulative distribution functions of  $c_2^*(y_2)$  and  $\tilde{c}_2(y_2)$ , respectively. That is,  $F^*(x) = \text{Prob}\{c_2^*(y_2) \leq x\}$ , and so on. Under the specified assumptions both  $c_2^*$  and  $\tilde{c}_2$  are bounded between  $\alpha - C^*$  and  $K^*$ . Then, following Rothschild and Stiglitz [19], agent 2 with strictly concave  $U_2$  will prefer  $\tilde{c}_2$  to  $c_2^*$  if

$$\int_{\alpha - C^*}^z [\tilde{F}(x) - F^*(x)] dx \leq 0 \quad z \in (\alpha - C^*, K^*) \tag{A5}$$

with a strict inequality for at least one such  $z$  and

$$\int_{\alpha - C^*}^{K^*} [\tilde{F}(x) - F^*(x)] dx = 0. \tag{A6}$$

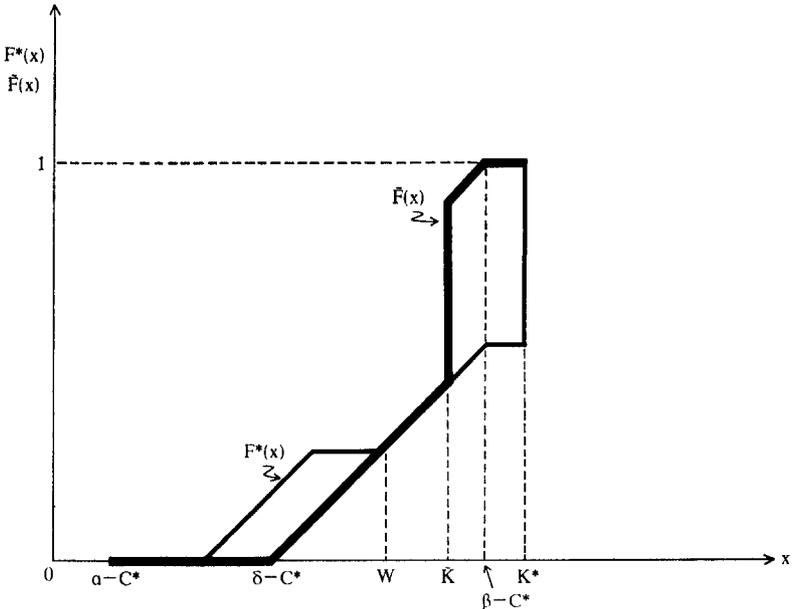


FIGURE 2

Condition (A6) holds since  $\tilde{c}_2$  and  $c_2^*$  are constructed to have the same mean. It remains to verify (A5).

Now, roughly speaking,  $F^*(x)$  increases at a rate determined by the density  $f(y_2)$  as  $y_2$  ranges through  $S^{*'}$  and has flats as  $y_2$  ranges through  $S^*$ . The mass  $\text{Prob}(S^*)$  is picked up at  $x = K^*$  and  $F^*(K^*) = 1$ . Also,  $\tilde{F}(x)$  remains at zero until  $x = \delta - C^*$ , and then increases at a rate determined by  $f(y_2)$  as  $y_2$  ranges through  $T'$ , with a jump of  $\text{Prob}(T)$  at  $x = \tilde{K}$ . Note  $\tilde{F}(\beta - C^*) = 1$ . (Fig. 2 is derived from Fig. 1 on the assumption that  $y_2$  is uniformly distributed on  $[\alpha, \beta]$ .) Thus by construction there exists some  $W \in (\delta - C^*, \beta - C^*)$  such that  $\tilde{F}(x) \leq F^*(x)$  for  $\alpha - C^* \leq x < W$  and  $\tilde{F}(x) \geq F^*(x)$  for  $W \leq x \leq \beta - C^*$ . It follows that given (A6), (A5) must hold also.

*Proof of Lemma 5.1.* The proof mimics that of Lemma 2.1 and is not included here for the sake of brevity.

*Proof of Proposition 5.1.* Proceeding as in the proof of Proposition 3.1, let  $\gamma_{j1}$  be chosen so that

$$\begin{aligned} & \int_{\alpha_2}^{\beta_2} \cdots \int_{\alpha_m}^{\beta_m} [1 - \Psi'_1(0)] U'_1 \left\{ \gamma_{j1} + C_1^* - \sum_i I_{1i}^*(y_i) \right\} f(y_1) \cdots f(y_m) dy_2 \cdots dy_m \\ & - \theta_j \int_{\alpha_2}^{\beta_2} \cdots \int_{\alpha_m}^{\beta_m} U'_j \left\{ y_j + \sum_i I_{ji}^*(y_i) + C_j^* - \sum_{i \neq 1} I_{ji}^*(y_i) \right. \\ & \left. - \Psi_j \left[ \sum_i I_{ji}^*(y_i) \right] \right\} f(y_1) \cdots f(y_m) dy_2 \cdots dy_m = 0 \quad j = 2, 3, \dots, m. \quad (A7) \end{aligned}$$

Let  $\gamma_1 = \max\{\gamma_{j1}\}_{j=2}^m$ . Then it can be shown by contradiction that there does not exist any  $y_1 \in [\alpha_1, \gamma_1)$  such that  $I_{j1}^*(y_1) = 0$  for each  $j$ , and there does not exist any  $y_1 \in [\gamma_1, \beta_1]$  such that  $I_{j1}^*(y_1) > 0$  for some  $j$ . The  $\{\gamma_{j1}\}_{j=2}^m$  can be found in a similar manner.

*Proof of Proposition 6.1.* Construct an  $(nm + 1)$ -dimensional commodity space as follows. Let the first  $n$  commodities be associated with the excess demand for claims contingent on the realizations  $\{y_1(s_1); s_1 = 1, 2, \dots, n\}$  of the endowment of the first agent as ordered by  $s_1$ . Let the commodities  $n + 1$  to  $2n$  be those associated with the excess demand for claims contingent on the realizations  $\{y_2(s_2); s_2 = 1, 2, \dots, n\}$  of the second agent with the obvious ordering induced by  $s_2$ . Continue in this way through agent  $m$ , numbering the first  $nm$  elements. Let the  $nm + 1$  commodity be associated with the excess demand for unconditional claims.

For each agent  $i, i = 1, 2, \dots, m$ , there is associated a set  $X_i \subset R^{nm+1}$  of possible consumption vectors (excess demands) defined by (6.3)–(6.5). Thus, for agent 1, for example, given some  $x_1 \in X_1$ , the first  $n$  components of  $x_1$  must be nonnegative, the next  $(m - 1)n$  components must be nonpositive, and the last component is unrestricted in sign. Also, by construction,  $X_i$  is

closed and convex for every  $i$ . The endowment of agent  $i$  in  $R^{nm+1}$  may be taken as the null vector.

For each agent  $i$  there is a preference ordering over  $X_i$  as defined by

$$V_i(x_i) = \sum_{s_1=1}^n \sum_{s_2=1}^n \cdots \sum_{s_m=1}^n [p_1(s_1) p_2(s_2) \cdots p_m(s_m)] \\ \times U_i \left\{ y_i(s_i) + \sum_{j=1}^m J_{ij}(s_j) + D_i - \Psi_i[J_{ii}(s_i)] \right\}$$

for  $x_i \in X_i$ . As  $U_i$  is concave and  $\Psi_i$  is convex, and both are continuous, this ordering is closed and convex.

Now by suitably modifying the argument of Arrow and Hahn [5] it can be established<sup>24</sup> that there exists a price vector  $q^* \in R_+^{nm+1}$ , a utility allocation  $\{V_i^*\}_{i=1}^m$ , and a consumption allocation  $\{x_i^*\}_{i=1}^m$  which constitute a *compensated equilibrium* in that  $q^* \geq 0$ ,  $\sum_{i=1}^m x_i^* \leq 0$ ,  $x_i^*$  minimizes  $q^* \cdot x_i$  subject to  $V_i(x_i) \geq V_i^*$  and (6.3)–(6.5), and  $q^* \cdot x_i^* = 0$ .

Associated with  $q^*$  are the prices  $r^*$ ,  $\{q_j^*(s_j)\}$ . (Recall the labeling convention adopted above.) It is claimed that for every  $j$ ,  $\sum_{s_j=1}^n q_j^*(s_j) \leq r^*$ . For suppose the contrary inequality. Then any agent  $i \neq j$  could issue claims contingent on the realization of the endowment of agent  $j$  and purchase unconditional claims in such a way as to leave relationships (6.4) and (6.5) unaltered and reduce expenditures without limit. This is an obvious contradiction.

Thus  $q^* \neq 0$  implies  $r^* > 0$ . Let  $b_i \equiv \min_{s_i} y_i(s_i)$ . (Recall  $y_i(s_i) > 0$  for every  $s_i$ .) Then the vector  $\hat{x}_i = (0, 0, \dots, -b_i) \in X_i$  is such that  $q^* \cdot \hat{x}_i < 0$ . Hence, by [7, (1) of Sect. (4.9)],  $x_i^*$  is a maximal element in  $X_i$  subject to  $q^* \cdot x_i \leq 0$ . Hence the compensated equilibrium is a competitive equilibrium.

*Proof of Proposition 6.2.* It is first established that the allocation of a competitive equilibrium is Pareto optimal relative to the commodities  $\{D_i\}$  and  $\{J_{ij}(s_j)\}$  which are traded. Retaining the notation of the proof of Proposition 6.1, note first that as the  $U_i$  are strictly increasing,  $q^* > 0$ , and hence  $\sum_{i=1}^m x_i^* = 0$ . Therefore the competitive equilibrium is an equilibrium relative to the price system  $q^*$  as defined in [7, Sect. 6.2], and hence by [7, (1) of Sect. 6.3] is also an optimum.

Finally, note that any allocation such that  $\sum_{i=1}^m x_i = 0$  and  $x_i \in X_i$  for each  $i$  defines a social contract of the restricted form and conversely. For suppose the commodities  $J_{ij}(s_j)$  and  $D_i$  are such that  $\sum_{i=1}^m x_i = 0$  and  $x_i \in X_i$  for all  $i$ . Then  $-J_{ii}(s_i) = \sum_{j \neq i} J_{ji}(s_j)$ . Let  $\bar{I}_{ji}[y_i(s_i)] = -J_{ji}(s_i)$ ,  $j \neq i$ . Let  $\bar{g}_{ij}(y_i, y_j) =$

<sup>24</sup> The assumption that  $X_i \subseteq R_+^k$  in Arrow and Hahn is not crucial to their analysis. Here also the set of feasible allocations is convex and compact. Also, the  $\hat{x}_i = 1, 2, \dots, m$  as defined below serve as the feasible allocation associated with the null utility which can be Pareto dominated.

$\bar{I}_{ij}(y_j) - \bar{I}_{ji}(y_i) + \bar{C}_{ij}$  where the  $\bar{C}_{ij}$  are determined in the obvious way from the  $D_i$ . Let  $\bar{S}_i = \{y_i(s_i): J_{ii}(s_i) > 0\}$ . Then from (6.3) and (6.4)  $\sum_j \bar{I}_{ji}[y_i(s_i)] - \Psi_i\{\sum_j \bar{I}_{ji}[y_i(s_i)]\} > 0$  on  $\bar{S}_i$  and  $\bar{I}_{ji}[y_i(s_i)] = 0$  on  $\bar{S}_i$  for all  $j$ . The converse is similarly established.

## REFERENCES

1. K. J. ARROW, Limited information and economic analysis, *Amer. Econ. Rev.* **64** (1974), 1-10.
2. K. J. ARROW, On optimal insurance policies, in "Essays in the Theory of Risk Bearing," appendix to essay 8, Markham, Chicago, 1971.
3. K. J. ARROW, "Optimal Insurance and Generalized Deductibles," Rand Report, 1973.
4. K. J. ARROW, The role of securities in the optimal allocations if risk bearing, *Rev. Econ. Studies* **31** (1964), 91-96.
5. K. J. ARROW AND F. HAHN, "General Competitive Analysis," Holden Day, San Francisco, Calif., 1971.
6. Y. M. CASPI, A limit theorem on the core of an economy with individual risks, *Rev. Econ. Studies* (1978), 267-271.
7. G. DEBREU, "Theory of Value," New York, Wiley, 1959.
8. J. GREEN, Pre-existing contracts and temporary general equilibrium, Harvard University Discussion Paper 246, 1972.
9. S. GROSSMAN AND J. STIGLITZ, On the impossibility of informationally efficient markets, IMSSS Technical Report No. 259, Stanford University, April 1978.
10. M. HARRIS AND A. RAVIV, Optimal incentive contracts with imperfect information, *J. Econ. Theory* **20** (1979), 231-259.
11. E. HELPMAN AND J. LAFFONT, On moral hazard in general equilibrium theory, *J. Econ. Theory* **10** (1975), 8-23.
12. J. HIRSHLEIFER, The private and social value of information and the reward to inventive activity, *Amer. Econ. Rev.* **61** (1971), 561-574.
13. L. HURWICZ, On informationally decentralized systems, "Decision and Organization" (C. B. McGuire and R. Radner, Eds.), Chap. 14, North-Holland, Amsterdam, 1972.
14. R. KIHLESTROM AND M. PAULY, The role of insurance in the allocation of risk, *Amer. Econ. Rev.* **61** (1971), 371-379.
15. E. PRESCOTT AND R. TOWNSEND, On competitive theory with private information, presented at the Econometric Society Summer Meetings, Montreal, 1979.
16. R. RADNER, Competitive equilibrium under uncertainty, *Econometrica* **36** (1968), 31-58.
17. A. RAVIV, "Mathematical Models for Optimal Insurance Policy Selection," Ph.D. dissertation, Northwestern University, 1975.
18. S. ROSS, The determination of financial structure: The incentive-signaling approach, *Bell J. Econ.* **8** (1977), 23-40.
19. M. ROTHSCHILD AND J. STIGLITZ, Increasing risk, I: A definition, *J. Econ. Theory* **2** (1970), 225-243.
20. S. SHAVELL, On moral hazard and insurance, Harvard Institute of Economic Research, Discussion Paper 557, 1977; forthcoming in *Quart. J. Econ.*
21. M. SPENCE AND R. ZECKHAUSER, Insurance, information and individual action, *Amer. Econ. Rev.* **61** (1971), 380-387.
22. R. WILSON, Informational economies of scale, *Bell J. Econ.* **6** (1975), 184-195.
23. R. WILSON, On the theory of syndicates, *Econometrica* **36** (1968), 119-132.