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## Free Choice of Unfunded Systems: A Preliminary Analysis of a European Union Challenge

Gabrielle Demange

Most European countries have set up a mandatory unfunded pension scheme, often called *first pillar*, financed through contributions levied on wages. Although this common characteristic is crucial, the systems significantly differ in two dimensions at least. First, although benefit rules have evolved, systems can still be classified as they were at their set up. Some are mostly “Bismarckian” with individuals’ pensions that are earnings-related, while others are mostly “Beveridgean” with flat pensions. Second, the level of the mandatory contributions (hence, the level of the pension benefits) varies significantly across countries. For example, this level represented in 2003 roughly 9 percent of the gross domestic product in the United Kingdom, 16.5 percent in France, 19.5 percent in Germany, and 32.7 percent in Italy.<sup>1</sup> Thus, the *redistribution* carried out within a generation and the *level* of the contributions are two major characteristics that differentiate European systems. Currently, the minimal contributing period necessary to give pension rights is long, thereby limiting the portability of the systems. This limitation constitutes a barrier to workers’ mobility, which may slow down labor integration, a major objective of the European Union (EU).

There are various ways to diminish the impact of such barriers. One is harmonization. Given the current differences in the systems and the problems of transition, agreement on a common system or even on steps toward convergence can only be slow. Another somewhat indirect but potentially powerful way to influence social security systems is free choice. By *free choice*, I mean to let any EU citizen to choose the system of any EU country *without* moving. Owing to the differences in the social security taxes and the benefit rules, free choice could trigger a drastic change in the allocation of individuals between the various systems. Would all systems survive? What would be the impact on efficiency, redistribution, and ultimately on citizens’ welfare?

The purpose of this chapter is to explore these questions in as simple a model as possible while still accounting for the basic features just outlined. The analysis is limited to two countries with identical fundamentals. Economies are modeled as overlapping-generation models to discuss the tradeoff between physical investment and direct intergenerational transfers (such as those performed by unfunded systems). To account for intragenerational redistribution, workers within a generation differ in their productivity. The growth rate of population and rate of return on investment are exogenous and constant over time.

An unfunded social security system is in place in each country and is mandatory for its citizens. A system is characterized by two parameters—the contribution rate on earnings and the Bismarckian factor that determines the intragenerational redistribution operated by the system.<sup>2</sup> Even though the economies are identical, these parameters may differ in the two countries to account of the stylized facts referred to above. I investigate the situation in which the citizens of both countries can freely choose either system without having to move.

What effect may free choice have? Roughly speaking, the choice of an individual is determined by comparing the rates of return expected from each system (Aaron, 1966). Two factors influence this comparison. Not surprisingly, one factor is related to the efficiency of intergenerational transfers (Samuelson, 1958; Gale, 1973). If the growth rate of the population is less than the rate of return on investment, for example, efficiency considerations favor the system with the lower contribution rate. The second factor is the redistribution operated within each system. In contrast to efficiency, redistribution affects individuals in a differential way according to their earnings. Furthermore, the *effective* redistribution within a system is influenced not only by its design (Bismarckian factor and contribution rate) but also by the distribution of earnings of its contributors (even a Beveridgean system operates no redistribution if earnings are all equal). This is a crucial point for understanding free choice since the contributors to each system are no longer determined by nationality.

Under free choice, individuals' choices affect the redistribution levels within each system, which in turn determine individuals' choices. A simple example illustrates this interaction. Let contribution rates be equal in the two countries, and let one system be Beveridgean and the other be Bismarckian. Initially, in the absence of liquidity constraints, workers with wages smaller than average are better off in the Beveridgean system than in the Bismarckian. At the opening of the systems,

low-income workers presumably choose the Beveridgean system, and wealthy workers choose the Bismarckian one (as is surely true if they base their choice on the initial situation). If this is the case, however, the average contributors' earnings to the Beveridgean system will diminish (and that to the Bismarckian will raise). As a result, the effective redistribution within the Beveridgean system decreases, and the initial incentives to choose it is reduced.

To assess the full impact of free choice, this chapter considers a steady-state equilibrium. The wage distribution of the contributors to each system is constant overtime, determined by the choices of individuals who correctly expect the returns of each system for them (the so-called rational expectations hypothesis). I show that a (not necessarily unique) equilibrium always exists. Furthermore, several types of equilibria may occur, depending on whether one or both systems are active and which system is chosen by the high-income workers. I investigate how the various parameters—characteristics of the systems, population growth rate, return to investment, wage dispersion—influence the equilibrium type.

If both systems cannot be active in equilibrium, one system will be selected in the long run by all citizens and the other will be de facto eliminated. How to interpret this result? To suppose, as in this chapter, that the opening of the systems would take place without any adjustment in their characteristics is not very realistic. The result nevertheless suggests that if the systems cannot both be active, then adjustments must be sufficiently fast or a system will be eliminated. Allowing for fast adjustments may be an even more unrealistic assumption owing to the current important differences between systems and the strong resistance to reforms.

The question addressed by this chapter is political. The opening of systems limits the margins of maneuvering of a country. The analysis however differs from the approach referred to as the political approach to social security. The purpose of this literature, as initiated by Browning (1975), is to explain the characteristics of a system by considering various decision-making processes, such as planner, median voter, lobbies (see, for example, the review by Galasso and Porfeta, 2002). This paper clearly differs since its goal is to analyze the interaction between different systems taking the characteristics of the systems as given, inherited from the past. On this respect, the closest analysis to ours is that of Casarico (2000). She also looks at the specific problem of integration and pension systems, with a focus that is somewhat

complementary to ours. A more precise comparison is given after the analysis of the model.

Finally, the impact of national pay-as-you-go (PAYG) systems on the individuals' decisions to migrate has been examined by several authors (e.g., Hombourg and Richter, 1993; Breyer and Kolmar, 2002). Individuals must contribute to the system where they live and may differ only by a migration cost. Thus, in contrast with our analysis, redistribution is not an issue, and the driving force explaining why people move is the differences in (endogenous) population growth and interest rate across countries.

The chapter introduces the model and determines the initial situation when a mandatory system is in place within each country. It then studies equilibrium configurations when the two systems are opened to the citizens of both countries and considers some dynamics before gathering proofs in the final section.

### The Model

I consider two countries, denoted by  $A$  and  $B$ , with the same economy but with a different pension system.

#### The Economy in a Country

The economy in each country is described by the same overlapping-generations model, with a structure close to that of Diamond (1965). Each generation lives for two periods, there is a single good that can be either consumed or invested, and population grows at a constant rate  $g - 1$ . An individual works only during the first period of his life, with an inelastic supply normalized to 1.

The technology of production is linear with marginal productivities of capital and labor that are constant over time. The quantity of goods available at date  $t$  results from the return on investment at the previous period and from labor of the current young generation. An amount  $s$  of capital invested in period  $t - 1$  will produce  $rs$  units of goods in period  $t$ , where  $r$  is the exogenous return to investment. As for labor, workers differ in their productivity/wages  $w$ . A  $w$ -worker denotes someone who produces and earns  $w$ . Wages are distributed on  $[w_{min}, w_{max}]$  with a mean denoted by  $\bar{w}$ . Thus the total quantity of good available at date  $t$  per head of old agents is  $g\bar{w} + rs_{t-1}$  if  $s_{t-1}$  was the average quantity invested in period  $t - 1$ . The distribution of wages is assumed to be continuous and constant across generations.

Individuals' preferences bear on consumption levels when young and old, denoted by  $c^j$  and  $c^o$  (there is no altruism motive) and are strictly increasing in each argument. Preferences may be heterogeneous.

I assume away liquidity constraints. This assumption allows one to conduct the analysis by working with *intertemporal wealth* only without specifying preferences. Let us consider an individual who receives labor income net of contributions  $(1 - \tau)w$  in the first period of his life and expects to receive a pension benefit  $\pi$  in the second period (dropping unnecessary time index). He faces the following successive constraints:

$$c^y + s = (1 - \tau)w \quad \text{and} \quad c^o = sr + \pi, \quad (5.1)$$

where  $s$  is an investment if positive and a loan if negative. This imply

$$c^y + c^o/r = (1 - \tau)w + \tau \frac{\pi}{r}. \quad (5.2)$$

In other words, the discounted value of consumption levels is equal to the intertemporal wealth, defined as the value of net labor income plus the discounted rights to pension. Conversely, in the absence of constraint on  $s$ , the intertemporal constraint (5.2) describes all feasible consumption plans: (5.1) and (5.2) are equivalent.<sup>3</sup>

Thus, *in the absence of liquidity constraints*, the welfare of an individual varies as his intertemporal wealth. As a result, the impact of a pay-as-you-go system on an individual's welfare can be analyzed through the impact on wealth. Similarly, the choice between two systems is determined by comparing the wealth values expected from contributing to either system.

### Remarks

1. Growth in productivity/wages can be introduced in the usual way by interpreting  $g - 1$  as the growth rate of the aggregate wage bill.
2. The assumption of a linear technology excludes endogenous variations in productivity, as would obtain with a nonlinear production function. Related to this, the absence of liquidity constraints makes sense only if aggregate savings are positive (see also note 5).
3. The ratio workers to retirees, equal to  $g$ , is exogenous. This ratio is sensitive to some policies, especially to social security. Labor participation changes over time, owing to changes in legislation affecting the

choice of retirement date or the number of working hours, for instance. Also it has been suggested that life expectancy and fertility are influenced by social security (see Philipson and Becker, 1983, and De la Croix and Doepke, 2003, for instance). These aspects are not addressed here.

### Characteristics of a Pension System

In the initial situation, a pension system is in place in each country and is mandatory for its citizens. Once systems are opened, each young individual will be able to choose between the two systems. This section describes the functioning of a system without specifying who contributes to it (and dropping unnecessary country index).

A system is *unfunded* (PAYG), characterized by two parameters specifying the contribution rate  $\tau$  and the redistribution Bismarckian factor  $\alpha$ .

Contributions are levied on wages, with a constant rate  $\tau$ : a young  $w$ -worker contributes  $\tau w$ . By construction, the system is balanced. Thus, at date  $t$ , given  $\bar{w}_t$  the average wage level of the contributors to the system and  $g_t$  the number of contributors per pensioner, the *average* pension benefits per pensioner  $\bar{\pi}_t$  is equal to

$$\bar{\pi}_t = \tau g_t \bar{w}_t. \quad (5.3)$$

The Bismarckian factor determines the benefit rule, which relates the pension benefits of a specific pensioner to the contributions he made in the previous period. Let us consider a pensioner at  $t$  who earned  $w$  at period  $t - 1$  while the average wage over the contributors to the system was  $\bar{w}_{t-1}$ . He thus contributed  $w/\bar{w}_{t-1}$  times the average level of contributions. If the Bismarckian factor is  $\alpha$ , the pensioner receives benefits given by<sup>4</sup>

$$\pi_{w,t} = \left( \alpha \frac{w}{\bar{w}_{t-1}} + (1 - \alpha) \right) \bar{\pi}_t. \quad (5.4)$$

A pensioner whose contribution was equal to the average contribution per capita  $w = \bar{w}_{t-1}$  receives benefits equal to the average benefits per pensioner  $\bar{\pi}_t$ , whatever value for  $\alpha$ . Note that for  $\alpha = 0$ , all pensioners receive this level independently of the amount of their previous contributions: the system is Beveridgean. At the opposite, a Bismarckian system obtains for  $\alpha = 1$ , since pension benefits are proportional to contributions. Thus, for  $\alpha$  between 0 and 1, the system combines a Bev-

eridgean system and a Bismarckian one. This is a crude description of the current systems, which are much more complex (see, for example, the Whitehouse 2003 report on nine OECD countries).

### Pension Systems

Initially, each young worker contributes to the mandatory pension system of his country. I consider the steady-state situation in which the system is in place and is expected to remain in place. After examining a country, I draw some brief comparisons between distinct systems.

#### National Systems

Let us consider a country with a system characterized by the parameters  $(\tau, \alpha)$ . While in place, the average level of wages of the contributors is  $\bar{w}$ . Also, the numbers of contributors per pensioner is equal to  $g$ . This gives  $g_t = g$ , and  $\bar{w}_{t-1} = \bar{w}_t = \bar{w}$  at a steady state. Therefore, from expressions (5.3) and (5.4), a  $w$ -worker will receive a pension benefit equal to  $[\alpha w + (1 - \alpha)\bar{w}]\tau g$ . Plugging this value into (5.2) gives the value for intertemporal wealth:

$$W(w) = \left[ 1 + \tau \left( \frac{g}{r} - 1 \right) \right] w + \tau \frac{g}{r} (1 - \alpha) (\bar{w} - w). \quad (5.5)$$

Note that in the absence of a PAYG system, wealth would be simply equal to the wage  $w$ . Thus, the system has a positive impact on an individual if  $W(w)$  is larger than  $w$ . To highlight the impact of each characteristic, it is convenient to define

$$R = 1 + \tau \left( \frac{g}{r} - 1 \right) \quad \text{and} \quad D = \tau \frac{g}{r} (1 - \alpha). \quad (5.6)$$

With this notation, wealth writes as  $W(w) = Rw + D(\bar{w} - w)$ .

The factor  $R$  can be described as the *rate of return* of the system at the steady-state situation: whatever value for  $\alpha$ , average wealth is equal to  $R\bar{w}$ , whereas without a PAYG system, it would be equal to average wage  $\bar{w}$ . Furthermore, in the absence of redistribution, the wealth of a  $w$ -worker is given by  $Rw$  for any wage  $w$ . Thus, each individual benefits from the system<sup>5</sup> if the rate  $R$  is larger than 1—that is, if the growth rate of the population is larger than the rate of return on investment, and each one is hurt by the system in the opposite case of a rate  $R$  less than 1. In other words, *at steady states, a Bismarckian PAYG system makes*

every individual better off if  $g > r$  and everyone worse off if  $g < r$ . The distinction between the two cases is well known since Gale (1973), who referred to them respectively as the Samuelson and classical cases.

In the presence of redistribution, the analysis remains valid on average since average wealth is  $R\bar{w}$ . In addition to  $Rw$ , the wealth of a  $w$ -worker is affected by a term stemming from redistribution  $D(\bar{w} - w)$ , which is positive for wages less than the average and negative otherwise. As a result, even if  $g < r$ , a system can nevertheless be beneficial to some low-income workers or, at the opposite, even if  $g > r$ , a system can be detrimental to some high-income workers. Since the redistribution term is proportional to  $D$ , the factor  $D$  determines the *extent* of the redistribution. Note that  $D$  depends not only on the Bismarckian factor but also on the contribution rate and the ratio  $g/r$ .

### Comparing Systems

Even though the two countries, denoted by  $A$  and  $B$ , have the same economy (identical population growth, return to investment, and wage distribution), their systems may differ significantly and hence have a different impact on citizens welfare. The characteristics of the system in country  $I = A, B$  are denoted by  $(\tau^I, \alpha^I)$ . From (5.6), the rate of return to system  $I$  and the extent of the redistribution are given by

$$R^I = 1 + \tau^I \left( \frac{g}{r} - 1 \right) \quad \text{and} \quad D^I = \tau^I \frac{g}{r} (1 - \alpha^I), \quad (5.7)$$

and the wealth of a  $w$ -worker in country  $I$  is

$$W^I(w) = R^I w + D^I (\bar{w} - w). \quad (5.8)$$

In the Samuelson case  $g > r$ , the system that has the highest rate of return is the one with the largest contribution rate. In the classical case, it is the opposite. From now on, the system that has the highest rate of return will be referred to as the *more efficient* system. This is justified as follows.

In the absence of redistribution, the welfare of citizens in different countries but with the same wage  $w$  is easily compared through their wealth  $R^I w$ . Hence, at the steady state with Bismarckian systems, a  $w$ -worker is better off in the country that has the highest rate of return  $R^I$ .

In the presence of redistribution, the *average* wealth of the citizens is larger in the country with the largest return. Thus, with adequate transfers, all contributors to the less efficient system could be made

better off by changing their contribution rate to that of the other country (that is, decreasing it in the classical case and increasing it in the Samuelson case). Note that there is an important difference between the two cases if one considers, instead of steady states, the transition from the less to the more efficient system. Whereas in the Samuelson case, every individual can be made better off, in the classical case  $g < r$  surely some individuals have to be hurt in a transition toward the more efficient system (by similar arguments as used in the seminal paper by Gale, 1973).

### Equilibrium under Free Choice

This section considers the situation in which each country opens its social security system to any citizen of the other country. More precisely, each young worker must contribute to a social security system but can freely choose between the two systems without moving. The choice is made once when young.

To choose between system  $A$  or  $B$ , a  $w$ -worker evaluates the wealth that he expects from each. Let us spell out this evaluation. The pension that is anticipated from a system depends on the wage level of its current and next contributors and on the growth rate of the number of contributors. Current wages determine the future redistributive gains or losses within a system, and next wages together with the growth rate determine the level of pension benefits. Let  $w_{t-1}^I$  and  $w_t^I$  be the anticipated average wage of the young contributors to system  $I$  at the current period  $t - 1$ , and let the subsequent period  $t$ , and  $g_t^I - 1$  be the anticipated growth rate of the number of contributors. According to (5.3) and (5.4), a  $w$ -worker will expect pension benefits equal to

$$\left[ \alpha^I \frac{w}{w_{t-1}^I} + (1 - \alpha^I) \right] \tau^I g_t^I w_t^I$$

from contributing to system  $I$ . This yields the level of intertemporal wealth

$$(1 - \tau^I)w + \tau^I \frac{g_t^I}{r} [\alpha^I w + (1 - \alpha^I)w_{t-1}^I] \frac{w_t^I}{w_{t-1}^I}. \quad (5.9)$$

We look for a *stationary equilibrium*, which requires two conditions: (1) in each system, the number of contributors grows at a constant rate equal to that of the population, and the average wage of the

contributors is constant over time, and (2) individuals base their choices on these variables, which are correctly expected. Before making this definition more precise, it is convenient to analyze the choices of individuals who have (not necessarily correct) stationary expectations.

### The System That Is More Favorable to High Income

The analysis of individuals' choices assuming *stationary and identical expectations* leads to a simple typology of the systems. Under stationary expectations, individuals expect the same types of workers to choose the systems at the current and next period. Thus, they expect the number of contributors to each system to grow as the population  $g_t^I = g$  and the average wage of the contributors to each system to be constant overtime  $w_t^I = w_{t-1}^I$  for  $I = A, B$ . Let us denote by  $w^I$  this constant expectation. From (5.9) and using the expressions (5.7) of  $R^I$  and  $D^I$ , the intertemporal wealth expected by a  $w$ -worker from contributing to system  $I$  is

$$W^I(w^I, w) = R^I w + D^I(w^I - w). \quad (5.10)$$

The choice of a system by a  $w$ -worker is made accordingly by comparing the wealth values obtained from  $A$  and  $B$ —namely, by the sign of

$$\begin{aligned} W^A(w^A, w) - W^B(w^B, w) \\ = [(R^A - D^A) - (R^B - D^B)]w + D^A w^A - D^B w^B. \end{aligned}$$

The key point is that this expression is linear with respect to wage  $w$  and that expected levels  $w^A$  and  $w^B$  affect only the level and not the slope. Assume the slope to be positive. (In the sequel, we exclude the degenerate case in which the slope is null.) If an individual prefers  $A$  to  $B$ , then all those who earn more than him also prefer  $A$  to  $B$ . This leads to the following definition:

**Definition 5.1** System  $A$  is said to be more favorable than system  $B$  to high-income workers if

$$[R^A - D^A] - [R^B - D^B] > 0,$$

which, replacing the  $R^I$  and  $D^I$  by their expressions, is equivalent to

$$\tau^A \left(1 - \frac{g}{r} \alpha^A\right) < \tau^B \left(1 - \frac{g}{r} \alpha^B\right). \quad (5.11)$$

System  $A$  is less favorable to high-income workers than  $B$  if the inequalities are reversed. Under stationary and identical expectations, the workers who choose the system that is more favorable to high-income workers are those whose wage is larger than a given threshold.

Which system is the more favorable is determined by the difference in efficiency (as measured by  $R^A - R^B$ ) relative to the difference in the extent of redistribution (as measured by  $D^A - D^B$ ). If both systems are Bismarckian, there is no redistribution whatsoever, and the system the more favorable than the more efficient one to high-income workers. More interesting is the case of systems that differ in their redistribution. It is worth recalling that in Europe systems with rather flat benefits tend to be associated with low contribution rates. Consider the case where the system with the smaller Bismarckian factor (say,  $B$ ) has the smaller contribution rate:  $\alpha^A > \alpha^B$  and  $\tau^A > \tau^B$ . From (5.11), the system the more favorable to high-income workers is the one with the lowest product  $\tau^I(1 - \alpha^I g/r)$ . Thus, under neutrality (for example,  $g = r$ ), the more Bismarckian system  $A$  is not necessarily the more favorable to high-income workers. As the ratio  $g/r$  decreases, the more inefficient a PAYG system is, and the more likely it is that the system with the lower contribution rate is the more favorable to high-income workers.

### Example

To illustrate this point, let us consider the case of France ( $A$ ) and the United Kingdom ( $B$ ). The tax rate in the United Kingdom is roughly half that in France,  $\tau^A/\tau^B \approx 2$ . Also, the U.K. system is much more redistributive than the French system.<sup>6</sup> According to some data, the parameters  $\alpha^A = 0.8$  and  $\alpha^B = 0.2$  are reasonable. The threshold value of  $g/r$  that determines whether  $A$  is more favorable to high income than  $B$  is  $1/1.4 \approx 0.7$ . This gives the following:

- For  $g/r > 1$ ,  $A$  is more efficient and more favorable to high-income workers than  $B$ ;
- For  $1 > g/r > 0.7$ ,  $A$  is less efficient but more favorable to high-income workers than  $B$ ; and
- For  $g/r < 0.7$ ,  $A$  is less efficient and less favorable to high-income workers than  $B$ .

Thus, the U.K. system, although much more redistributive than the French system, can be more favorable to high-income workers thanks to its low contribution rate. This is especially true if PAYG systems are perceived as inefficient.

Not surprisingly, the ratio of growth rate to investment return plays a crucial role. Which value for this ratio is reasonable? This is a delicate question because it is not clear which return should be chosen for  $r$ . A period here represents roughly thirty years. If one takes for  $r$  the return on the stock market since the end of World War II and for  $g$  the projected growth rate of aggregate wage bill, the compounding effect will give a low value for  $g/r$ . This is, however, related to the equity premium puzzle. If, indeed, individuals are risk averse and ready to pay a high-risk premium, then one should take for  $r$  a much smaller value than the stock-market return. Also, a PAYG system provides retirees with an annuity, thereby insuring them against some of the risks of living into old age. Making insurance compulsory avoids the usual problems encountered in markets with asymmetric information. As documented by various studies, the premium associated to the longevity risk is roughly 5 percent (see Brown, Mitchell, and Poterba, 2001). To account for this premium, an extra return on a PAYG system could be introduced. Due to these difficulties and the uncertainty on future, I discuss in next section equilibria for different values of  $g/r$ .

### Equilibrium

Given an anticipated average wage  $w^I$  of the contributors to each system  $I$ , let  $w^*$  be the wage level defined by  $W^A(w^A, w^*) - W^B(w^B, w^*) = 0$ . To fix the idea, assume system  $A$  to be more favorable to high-income workers. The individuals who choose system  $A$  are those who earn more than  $w^*$ . Note that the threshold  $w^*$  may not be in the range of wages. If  $w^* \leq w_{min}$ , for example, all individuals choose  $A$ , and if  $w^* \geq w_{max}$ , all choose  $B$ .

Individuals' anticipation on contributors' average wages determine their choices, which in turn determine the realized wages. To get an equilibrium, anticipation and realization must be consistent. This leads to the following definition:

**Definition 5.2** Let system  $A$  be more favorable than  $B$  to high-income workers. An equilibrium is determined by average wages  $w^A$  and  $w^B$  and  $w^*$  that satisfy  $W^A(w^A, w^*) - W^B(w^B, w^*) = 0$  and the following expectations conditions:

- If  $w^*$  is in  $]w_{min}, w_{max}[$ :  $w^A = E[w|w \geq w^*]$ ,  $w^B = E[w|w \leq w^*]$ , then both systems are active, and the equilibrium is called an *AB-equilibrium*;
- If  $w^* \leq w_{min}$ :  $w^A = \bar{w}$ ,  $w^B = w_{min}$ , then only system *A* is active, and the equilibrium is called an *A-equilibrium*;
- If  $w^* \geq w_{max}$ :  $w^A = w_{max}$ ,  $w^B = \bar{w}$ , then only system *B* is active, and the equilibrium is called a *B-equilibrium*.

For an active system, the expectation condition says that the anticipation of the average wage of the contributors to an active system is correct equal to its expectation conditional on individuals' choices.<sup>7</sup> For an inactive system, the condition needs a justification: since there are no contributors, the conditional expectation of their average wage is not well defined. Hence individuals' behaviors are supported by some beliefs about this wage.<sup>8</sup> In the above definition, the beliefs are those justified by a perfect equilibrium argument. If the set of contributors to *B* is small, for example, it is formed by the individuals whose wages are close to the minimum. Taking the limit, if only *A* is active, the belief on  $w^B$  is the minimum wage. A similar argument justifies that with only *B* active, the belief on  $w^A$  is set to the maximum wage.

Before going further, it is helpful to note that *the system the less favorable to high-income workers is eliminated whenever it is also the less efficient*. The intuition is clear. Suppose there is an *AB-equilibrium*, and consider a  $w^*$ -worker who is indifferent between *A* and *B* (a similar argument shows that a *B-equilibrium* does not exist by considering workers with wage  $w_{max}$ ). By choosing *A*, the worker would benefit from the larger efficiency return provided by *A*. Furthermore, since the wage  $w^*$  is not greater than  $w^A$ , he can only benefit from redistribution in *A* instead of being penalized by it in *B*: he definitely prefers *A* to *B*, a contradiction. The analysis is more complex if, from the point of view of low-income workers, efficiency and redistribution benefits enter into conflict. The following proposition characterizes the equilibrium configurations in function of the parameters:

**Proposition 5.1** Let system *A* be more favorable than *B* to high-income workers. There exists

- An *A-equilibrium* if and only if

$$W^A(\bar{w}, w_{min}) - R^B w_{min} \geq 0 \Leftrightarrow R^B - R^A \leq D^A \frac{\bar{w} - w_{min}}{w_{min}}; \quad (5.12)$$

- A  $B$ -equilibrium if and only if

$$W^B(\bar{w}, w_{max}) - R^A w_{max} \geq 0 \Leftrightarrow D^B \frac{w_{max} - \bar{w}}{w_{max}} \leq R^B - R^A; \quad (5.13)$$

- An  $AB$ -equilibrium if either an  $A$  and a  $B$  equilibrium both exist and (5.12) and (5.13) hold or no one exists and neither (5.12) nor (5.13) holds.

It follows that an equilibrium always exists. As already said, it can only be an  $A$  equilibrium if  $A$  is more efficient: (5.12) holds but not (5.13) if  $R^B \leq R^A$ . Otherwise, the tradeoff between efficiency and redistribution for low-income or top-income workers determines equilibrium configurations. To see this, let us explain how the equilibrium conditions are obtained. To check whether  $A$  alone can be in equilibrium, assume that  $A$  is chosen by every worker. The average wage  $w^A$  is equal to the overall mean  $\bar{w}$ . To form an equilibrium, it suffices that workers whose wages are close to the minimum level  $w_{min}$  have no incentives to subscribe to  $B$ . This gives condition (5.12), which results from the following tradeoff. By subscribing to  $B$ ,  $w_{min}$ -workers lose all the redistribution benefits in  $A$  without getting any in  $B$  (because wages in  $B$  are roughly identical), but they also benefit from the larger return in  $B$  (assumed to be more efficient). If the loss outweighs the efficiency gain, an  $A$  equilibrium is obtained. Since the larger the ratio  $\bar{w}/w_{min}$  is, the larger the loss in redistributive benefits, a low value for the minimum wage makes it more likely that an  $A$  equilibrium will exist. Similarly, a large value for the extent of the redistribution in  $A$  (subject to  $A$  being more favorable to high-income workers) makes it more likely that an  $A$  equilibrium will exist.

Similarly, a  $B$  equilibrium is obtained if workers whose wages are close to the maximum level  $w_{max}$  have no incentives to subscribe to  $A$ , which gives condition (5.13). The condition is strong: it requires that top-income workers are better off by subscribing to system  $B$  applied to the whole population rather than by subscribing to  $A$  without redistribution loss. As the ratio  $w_{max}/\bar{w}$  and the extent of the redistribution in  $B$  increase, the redistribution losses for a top-income worker subscribing to  $B$  rather than to  $A$  outweighs the efficiency gains:  $B$  alone is in equilibrium only for a small enough ratio and small value for  $D^B$ .

Note that the above arguments are valid whatever the assumption on the distribution of earnings, whether continuous or not: only

the incentives of the top- or bottom-income workers matter.<sup>9</sup> This insight is likely to be quite robust and to extend to more general benefit rules.

According to this discussion, as the range of wages is enlarged, the redistribution effects become predominant and determine the equilibrium. Workers who most benefit from redistribution and those who are the more penalized by it are both encouraged to choose the system that is more favorable to high-income workers: for sufficiently low  $w_{min}$  and sufficiently large  $w_{max}$ , only condition (5.12) holds. This gives the following corollary:

**Corollary** If the range of wages is sufficiently large, then only the system that is more favorable to high-income workers is active at equilibrium.

Instead, various equilibrium configurations are possible when the dispersion of wages is not too large and the system the less favorable to high income is the more efficient. Even the three types of equilibrium can simultaneously exist. As long as redistribution or efficiency is not dominant factor, the incentives conditions, as given by (5.12) and (5.13) are to some extent independent: one bears on incomes at the bottom, and the other at the top. This is illustrated by the following example.

#### Example (Continued)

Consider again the illustrative case in which  $\alpha^A > \alpha^B$  and  $\tau^A > \tau^B$ . We know that when one system is both more efficient and more favorable to high-income workers than the other, it is only active at equilibrium. It immediately follows that only the more Beveridgean system  $B$  is active if pay-as-you-go systems are sufficiently inefficient, while it is the more Bismarckian system  $A$  if they are sufficiently efficient.<sup>10</sup> Taking the values of the France–United Kingdom example above, this gives the following:

- For  $g/r > 1$ , an  $A$ -equilibrium obtains because system  $A$  is more efficient and more favorable to high income than  $B$ ;
- For  $g/r < 0.7$ , a  $B$ -equilibrium obtains because  $A$  is less efficient and less favorable to high-income workers than  $B$ .

It remains to determine what happens when efficiency and redistribution enter into conflict, which occurs here for  $1 > g/r > 0.7$ :  $A$  is

more favorable to high-income workers but less efficient than  $B$ . The case  $g/r = 0.8$  is illustrated in figure 5.1.

In the top graph, the dashed line represents  $R^B w - W^A(\bar{w}, w)$  as a function of  $w$ . According to (5.12), there is an  $A$  equilibrium if it is negative at  $w_{min}$ —that is, if  $w_{min} < a \approx 0.6$ . Similarly, the normal line represents  $W^B(\bar{w}, w) - R^A w$ , and according to (5.13) there is a  $B$  equilibrium if it is positive at  $w_{max}$ , which gives  $w_{max} < b \approx 1.5$ . The difference in wealth at the initial situation  $W^A(\bar{w}, w) - W^B(\bar{w}, w)$  is also represented (the thick line). This is useful for subsequent welfare comparisons and for understanding the initial incentives to choose one system rather than another. Here the difference is increasing in  $w$  because  $A$  is more favorable than  $B$  to high-income workers. Also it is negative at the mean value  $\bar{w}$  because  $B$  is more efficient than  $A$  and  $W^I(\bar{w}, \bar{w}) = R^I \bar{w}$ .

The bottom graph summarizes the equilibrium types as a function of the range of earnings keeping the mean constant. When the range is small ( $w_{min} > 0.6$  and  $w_{max} < 1.5$ ), there is only a  $B$  equilibrium: efficiency effects are dominant. One checks that for these values, at the initial situation, all citizens in  $A$  would prefer system  $B$ : the difference  $W^A(\bar{w}, w) - W^B(\bar{w}, w)$  is negative in the relevant range for  $w$ . This is always true, as is shown in proposition 5.2. As the range is increased, we move to the northwest and only the  $A$  equilibrium remains: the redistribution effects become dominant.

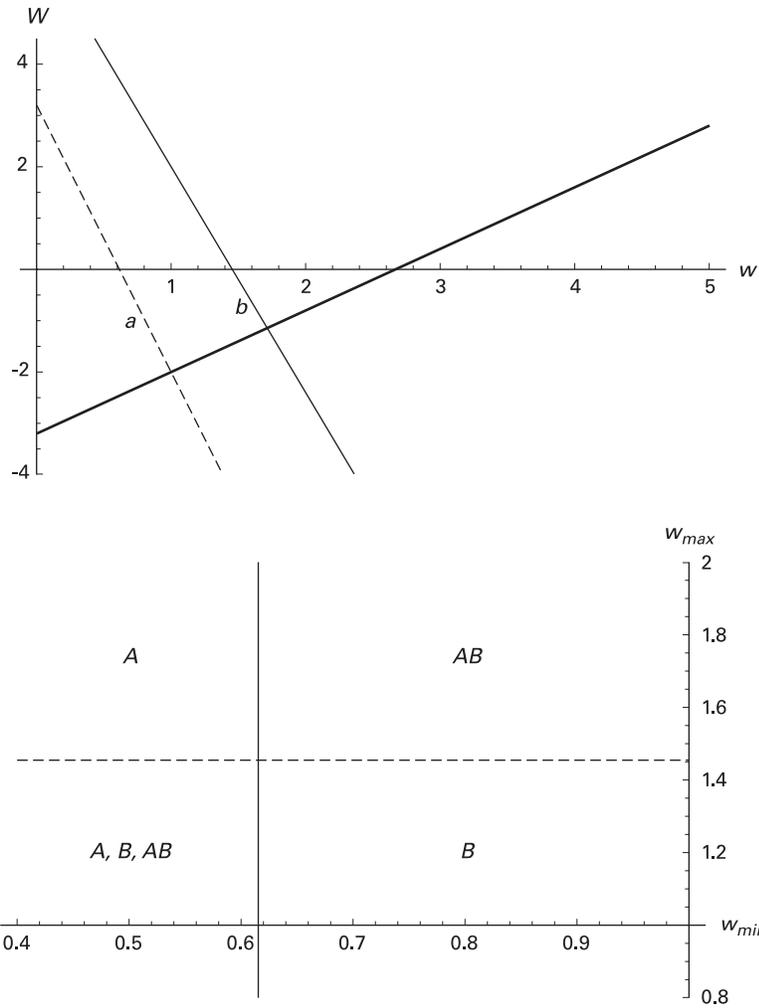
### Welfare

In light of these results, one may wonder whether introducing free choice is beneficial. To avoid considering too many cases, I discuss the situation in which an  $I$  equilibrium is obtained (I do not consider an  $AB$  equilibrium). Then the winners or losers are easily determined. To see this, note that any  $w$ -worker gets an intertemporal wealth equal to  $W^I(\bar{w}, w)$ . Thus, citizens in country  $I$  are not affected by the reform, while in  $J$  the losers (respectively, winners) are those for whom the intertemporal wealth  $W^I(\bar{w}, w)$  was larger (respectively, smaller) than  $W^I(\bar{w}, \bar{w})$ .

To fix the idea, let  $A$  be more favorable to high-income workers. Then, using that  $W^I(\bar{w}, \bar{w}) = R^I \bar{w}$ , the following inequalities hold:

$$(R^A - R^B)\bar{w} > W^A(\bar{w}, w) - W^B(\bar{w}, w) \quad \text{for } w < \bar{w}, \quad \text{and} \quad (5.14)$$

$$(R^A - R^B)\bar{w} < W^A(\bar{w}, w) - W^B(\bar{w}, w) \quad \text{for } w > \bar{w}. \quad (5.15)$$



**Figure 5.1**

Parameters:  $\tau A/\tau B = 2$ ,  $\alpha^A = 0.8$ ,  $\alpha^B = 0.2$ ,  $g/r = 0.8$ ,  $w = 1$ .

*Top graph:* The horizontal axis represents  $w$ , the vertical one differences in wealth, multiplied by 100. The thick line represents  $W^A(w, \cdot) - W^B(w, \cdot)$ , the dashed line  $R^B w - W^A(w, w)$  (there is an  $A$ -equilibrium if  $w_{min} < a \approx 0.6$ ) and the normal line:  $W^B(w, w) - R^A w$  (there is a  $B$ -equilibrium if  $w_{max} < b \approx 1.5$ ).

*Bottom graph:* Equilibria as function of  $w_{min}$  and  $w_{max}$ .

If  $A$  is not less efficient than  $B$ , there is a  $A$  equilibrium only. Also, surely in country  $B$  all citizens whose income is larger than  $\bar{w}$  are made better off from (5.15) since  $R^A - R^B \geq 0$ . Other citizens also are better off—maybe all if efficiency gains are large enough relative to redistribution effects.

Assume now that  $A$  is less efficient than  $B$ . If a  $B$  equilibrium obtains, the opening of the systems is beneficial without ambiguity, as stated by the following proposition:

**Proposition 5.2** The system that is more favorable to high-income workers can be eliminated only if, at the initial situation, all individuals are better off with the other system. If  $A$  is more favorable to high-income workers, a  $B$  equilibrium exists only if

$$W^B(\bar{w}, w) \geq W^A(\bar{w}, w) \quad \text{for any } w \text{ in } [w_{min}, w_{max}] \quad (5.16)$$

and thus leads to a (weak) Pareto improvement over the initial situation.

Condition (5.16) is strong, but it is not a sufficient condition for a  $B$  equilibrium to exist. The range of wages must be small enough. Consider the example again. If  $1.5 < w_{max} < 2.8$  and  $w_{min} < 0.6$ , only the  $A$  equilibrium exists, but  $B$  applied to the whole population gives a larger wealth to every one than  $A$ .

Instead, if the  $A$  equilibrium obtains, some  $B$  citizens must lose since the less efficient system is now in place in country  $B$ . In particular, from (5.14) and the fact that  $R^A < R^B$ , all workers who earn less than the average wage are worse off. Since the average wage is typically larger than the median value, more than a majority of workers in  $B$  are made worse off by the reform. How can this happen? The dynamics contemplated below help to explain this point. Assuming that initially all workers who prefer  $B$  choose it, system  $A$  becomes more attractive, which triggers new choices, which may eventually lead to all choosing  $A$ . It can even happen that all workers in  $B$  are hurt at a  $A$  equilibrium. In that case, implementing  $B$  for everybody would be weakly Pareto improving. This occurs if (5.16) is met but the  $A$  equilibrium obtains. This is surely the case if the  $A$  equilibrium is unique ( $1.5 < w_{max} < 2.8$  and  $w_{min} < 0.6$  in the example).

To sum up, even though the model is very simple, the welfare impact of the opening of the systems largely depends on the situation at hand. If redistribution losses or gains are too important, there are few chances that a Pareto improvement will be obtained.

At this point, it is worth comparing our analysis with Casarico (2000). In two countries with identical economies, compulsory pension systems are in place, unfunded in one country and fully funded in the other. Redistribution plays no role (the analysis is conducted with a representative individual in each country). The chapter focuses on the impact of the differences in pension systems on capital integration when capital becomes fully mobile, labor remaining immobile. Production is carried out through a neoclassical production function. Owing to the different pension systems, investments (and hence their returns) differ in the two countries before capital integration. Capital integration has a welfare effect because the return to investment is equalized across countries. While this effect is clearly absent in our analysis, Casarico does not allow workers to choose a system. If they could, they would all choose the more efficient (since there is no redistribution).

### Dynamics

By keeping the tax rate constant, the balance of a system is ensured through adjustments in pension benefits. As a result, a system can be thought of as a defined-contribution one. In a correct expectations framework, as just considered, it is also a defined-benefit one. It may no longer be true under some dynamics. Dynamics are determined by expectations and information. At the time that workers have to choose a system, they are concerned with the wage level of the current and next contributors to each system. The current ones determine the future redistributive gains or losses within a system, and the next ones give the level of pension benefits. Starting from the initial situation in which each system in place in a country is exclusively for the citizens, I consider here the evolution of the systems driven by myopic expectations.

Myopic expectations mean that workers at time  $t$  who do not know yet the choice of their contemporaries and descendants expect that they will perform the same choice as the previous generation. For example, initially, all workers assume that the average contributors' wage is identical in each system and is equal to  $\bar{w}$ . A threshold value  $w_0^*$  for wages is determined, according to which all workers with income larger (respectively, smaller) than the threshold choose  $A$  (respectively,  $B$ ), still assuming  $A$  is more favorable than  $B$  to high-income workers. Afterwards, the evolution of the system is described as follows. Let  $w_{t-1}^*$  be the threshold value between  $A$  and  $B$  at time  $t - 1$ .

Workers at time  $t$  observe this value. Under myopic expectations, they expect the choice of current and next contributors to remain unchanged. Thus, they expect the average wage level of the current and next contributors to each system to be given by

$$w_{t-1}^A = E[w|w \geq w_{t-1}^*], \quad w_{t-1}^B = E[w|w \leq w_{t-1}^*]. \quad (5.17)$$

It follows that a  $w$ -worker at date  $t$  evaluates the wealth generated by system  $I$  as  $W^I(w_{t-1}^I, w)$  defined by (5.10) and chooses between  $A$  and  $B$ , accordingly. A new threshold  $w_t^*$  level is determined and so on.

**Proposition 5.3** Let system  $A$  be more favorable than  $B$  to high-income workers. Assume myopic expectations. Dynamics converge to (1) the (unique)  $A$  equilibrium if  $A$  is more efficient than  $B$ , and (2) in one step to the  $B$  equilibrium if it exists. Also dynamics surely converge to an equilibrium if  $D^B E[w|w \leq x] - D^A E[w|w \geq x]$  is non-decreasing.

In the first two cases convergence always occurs. Note that the  $B$  equilibrium Pareto dominates the  $A$  equilibrium if both exist (by proposition 5.2). Therefore a good equilibrium is selected. In other cases, an additional assumption is needed. To understand why, consider the opening of the systems in which workers choose the systems assuming the same distribution within each one. Since those who choose  $B$  are less wealthy than those who choose  $A$ , the redistribution within the system  $B$  is diminished and within  $A$  increased: more individuals will choose  $A$  next period. The monotonicity assumption ensures that the set of workers who choose  $A$  will grow. The assumption is satisfied if  $D^A$  is sufficiently low ( $A$  Bismarckian, for instance) or if wages are uniformly distributed.<sup>11</sup>

### Concluding Remarks

Even though the model is simple in many dimensions, it helps to highlight some features that are likely to be quite robust. First, the analysis shows that the system that is preferred by high-income workers is not necessarily the more Bismarckian one. Both the levels of the contribution rates and the efficiency or inefficiency of unfunded systems play important roles. In particular, in situations in which unfunded are perceived as very inefficient, the system with the lower contribution rate is preferred. Second, a large dispersion of wage earnings eliminates the system that is less favorable to high-income workers even if it is

the more efficient: the redistribution effects become dominant for the workers who most benefit from redistribution or for those who are the most penalized by it. Third, free choice does not necessarily lead to select the more efficient system. In some cases, the new situation may be Pareto dominated by the initial one.

The analysis has been conducted under strong simplifying assumptions. It should be extended in several directions to test the robustness of the results. Production and endogenous factor prices could be introduced. Uncertainty about production and population growth would make comparisons between the rates of return of the systems less trivial and more realistic. Liquidity constraints, which are likely to be binding on low-income workers, should be taken into account.

Finally, to incorporate political elements into the analysis would be interesting. It would require describing the adjustments of the systems confronted with the impact of free choice, even if such adjustments are slow. The analysis would then be similar in some aspects to that of fiscal competition (see, e.g., Epple and Romer, 1991; Wildasin, 2003). A basic concern is whether factor mobility, as dictated by the European construction, necessarily undermines redistributive policies. Not surprisingly, the literature on taxation between areas—regions, jurisdictions, countries—and on the limitation on redistribution due to the mobility of capital or labor is vast and growing (see the survey of Cremer and Pestieau, 2003).

### Proofs

Recall that under the assumption of stationarity, the wealth of a  $w$ -worker contributing to system  $I$  with expectations  $w^I$  is given by (5.10)

$$W^I(w^I, w) = R^I w + D^I(w^I - w).$$

*Proof of Proposition 5.1* Let us determine the conditions under which there is an active equilibrium with only  $A$ . In this case,  $w^A = \bar{w}$  and  $w^B = w_{min}$ . An equilibrium is obtained if and only if an individual whose wages are minimum is not incited to choose system  $B$ . This is written as  $W^A(\bar{w}, w_{min}) - W^B(w_{min}, w_{min}) \geq 0$ . Using the expression of wealth (5.10), this gives the inequality (5.12).

In a similar way, a situation with only  $B$  active forms an equilibrium if an individual whose wage is maximal is not incited to choose system  $A$ . Since  $w^A = w_{max}$  and  $w^B = \bar{w}$ , this gives  $W^A(w_{max}, w_{max}) - W^B(\bar{w}, w_{max}) \leq 0$  which yields (5.13).

It remains to consider an  $AB$  equilibrium. Let the function  $\phi$  be defined on  $[w_{min}, w_{max}]$  by

$$\phi(x) = W^A(E[w|w \geq x], x) - W^B(E[w|w \leq x], x). \quad (5.18)$$

By continuity of the distribution of wages, the function  $\phi$  is continuous. An equilibrium with two active systems is associated with  $w^*$  in  $]w_{min}, w_{max}[$  that satisfy  $\phi(w^*) = 0$ . Note that (5.12) is equivalent to  $\phi(w_{min}) \geq 0$  and (5.13) to  $\phi(w_{max}) \leq 0$ . By continuity of  $\phi$ , it follows that if no  $I$  equilibrium exists, there is an interior  $w^*$  such that  $\phi(w^*) = 0$ . Similarly, if both equilibria exist and the inequalities are strict, there is also an  $AB$  equilibrium. ■

*Proof of Proposition 5.2* As just seen, a  $B$  equilibrium exists iff  $W^B(\bar{w}, w_{max}) - W^A(w_{max}, w_{max}) \geq 0$ . Since  $W^A(w_{max}, w_{max}) = R^A w_{max} \geq W^A(\bar{w}, w_{max})$  (because of the redistribution loss), this gives  $W^B(\bar{w}, w_{max}) - W^A(\bar{w}, w_{max}) \geq 0$ , which implies  $W^B(\bar{w}, w) - W^A(\bar{w}, w) \geq 0$  for any  $w$  (because  $B$  is less favorable to high-income workers). ■

*Proof of Proposition 5.3* To simplify notation, set  $\Delta = R^A - R^B - D^A + D^B$ . The assumption that system  $A$  is more favorable to high-income workers than  $B$  is written as  $\Delta > 0$ . The choice of a  $w$ -worker at  $t$  is made on the basis of the difference in expected wealth

$$W^A(w_{t-1}^A, w) - W^B(w_{t-1}^B, w) = \Delta w + D^A w_{t-1}^A - D^B w_{t-1}^B, \quad (5.19)$$

where the values  $w_{t-1}^A$  and  $w_{t-1}^B$  are the average wages of the contributors to each system observed at time  $t - 1$ . By positivity of  $\Delta$ , the function is increasing in  $w$ . Thus, all workers whose wage is larger than  $(D^B w_{t-1}^B - D^A w_{t-1}^A) / \Delta$  choose  $A$ , and the others choose  $B$ . If this wage value is larger than  $w_{max}$ , then everybody chooses  $A$ , and if it is smaller than  $w_{min}$ , then everybody chooses  $B$ . To define the threshold  $w_t^*$ , it is convenient to consider the projection  $P$  on the interval  $[w_{min}, w_{max}]$ :

$$\begin{aligned} P(x) &= w_{min} \quad \text{if } x < w_{min}, \\ &= x, x \in [w_{min}, w_{max}], \\ &= w_{max} \quad \text{if } x > w_{max}. \end{aligned}$$

The threshold  $w_t^*$  is defined by

$$w_t^* = P((D^B w_{t-1}^B - D^A w_{t-1}^A) / \Delta). \quad (5.20)$$

The values  $w_{t-1}^A$  are  $w_{t-1}^B$  expected at  $t$  are based on observed at time  $t - 1$ . Thus, at the opening of the systems  $t = 0$ , both are equal to the average wage—

$$w_{-1}^B = w_{-1}^A = \bar{w} \quad (5.21)$$

—and afterwards  $t > 0$ , they satisfy

$$w_{t-1}^A = E[w|w \geq w_{t-1}^*] \quad \text{and} \quad w_{t-1}^B = E[w|w \leq w_{t-1}^*]. \quad (5.22)$$

Conditions (5.20), (5.21), and (5.22) define  $w_0^*$  and  $w_t^*$  as a function of  $w_{t-1}^*$ . A fixed point  $w^*$  of the sequence gives an equilibrium. This is clear if  $w^*$  is in  $]w_{min}, w_{max}[$ . Otherwise—say, if  $w^* = w_{max}$  is a fixed point—then knowing that everybody chooses  $B$  at  $t - 1$ , everybody makes the same choice at  $t$ : condition (5.13) is fulfilled and a  $B$  equilibrium is obtained. Similarly,  $w_{min}$  is a fixed point iff there is an  $A$  equilibrium.

We first prove that  $w_t^* \leq w_0^*$  for any  $t$ . Note that  $w_0^*$  is the threshold value associated with expectations  $w_{-1}^A = w_{-1}^B = \bar{w}$ . At any subsequent step, the average wage in  $A$  can only be larger and that in  $B$  can only be smaller. This implies that the incentives to choose  $A$  are larger than at the initial step. Formally note that the inequalities  $E[w|w \geq x] \geq \bar{w} \geq E[w|w \leq x]$  hold whatever  $x$ . Thus, surely  $w_{t-1}^A \geq \bar{w} \geq w_{t-1}^B$ . Since  $P$  is nondecreasing, (5.20) yields  $w_t^* \leq w_0^*$ .

Assume first  $A$  to be more efficient than  $B$ :  $R^A - R^B \geq 0$ . Since inequalities  $E[w|w \geq x] \geq x \geq E[w|w \leq x]$  always hold,  $(D^B w_{t-1}^B - D^A w_{t-1}^A) \leq (D^B - D^A)w_{t-1}^*$ . Thus, if  $D^B - D^A \leq 0$ , the argument of  $P$  in (5.20) is negative:  $w_t^* = w_{min}$  whatever  $t$ , the (unique)  $A$  equilibrium is obtained at the opening. If  $D^B - D^A > 0$ , then  $\Delta \geq D^B - D^A \geq 0$ : the argument of  $P$  in (5.20) is less than  $w_{t-1}^*$ . Thus, the sequence  $w_t^*$  decreases as long as it is above  $w_{min}$ : it converges to the  $A$  equilibrium.

Assume now that a  $B$  equilibrium exists. From proposition 5.2, everybody chooses  $B$  initially:  $w_0^* = w_{max}$ . Afterward, (5.13) gives  $w_1^* = w_{max}$ : the  $B$  equilibrium is reached. Conversely, if a  $B$  equilibrium does not exist, surely  $w_1^* < w_{max}$ . Otherwise,  $w_1^* = w_{max} \leq w_0^*$  implies that  $w_{max}$  is a fixed point of the sequence—namely, that there is a  $B$  equilibrium.

Finally, assuming that  $B$  is more efficient than  $A$ , consider the situation without a  $B$  equilibrium. Let the function  $D^B E[w|w \leq x] - D^A E[w|w \geq x]$  be nondecreasing. By induction, using  $w_1^* \leq w_0^*$ , the

sequence  $w_t^*$  decreases thus converges (since it is bounded). Furthermore, since  $w_1^* < w_{max}$  (because there is no  $B$  equilibrium), the sequence  $w_t^*$  converges to a value strictly less than  $w_{max}$ —that is, to an equilibrium of type  $AB$  or  $A$ . ■

## Notes

École des Hautes Études en Sciences Sociales at Paris-Jourdan Sciences Economiques, École Normale Supérieure, 48 Boulevard Jourdan, 75014 Paris, France, e-mail <demange@pse.ens.fr>. This chapter benefited from the detailed comments of George Casamatta, Robert Fenge, Christian Gollier, Fabien Moizeau, and Pierre Pestieau. I also thank the seminar participants of the November 2004 CESifo conference “Strategies for Reforming Pension Schemes” for their remarks.

1. Cross-country comparisons are hazardous and vary according to the definition of *social security*. In line with the objectives of the chapter, I have tried to consider only the first pillars the systems. Data for France, Germany, and Italy are taken from <<http://www.ssa.gov/policy/docs/progdsc/ssptw/2004-2005/europe/guide.html>>. The same document gives 23.8 percent for the United Kingdom, but it includes the second pillar, which is also mandatory but funded. For a description of the U.K. system, see the European Union Commission and the Council Joint Report *Adequate and Sustainable Pensions* (2003).

2. I use here the modeling of Casamatta, Cremer, and Pestieau (2000).

3. The pension may not be correctly anticipated (even though we require it to be at equilibrium). The only assumption that is needed is that the anticipation is single-valued: the young individual makes decisions as if he will receive  $\pi$ .

4. The benefit rule can also be written as

$$\frac{\pi_{w,t}}{w} = \left( \alpha + (1 - \alpha) \frac{\bar{w}_{t-1}}{w} \right) \frac{\bar{\pi}_t}{\bar{w}_{t-1}},$$

which shows how replacement rates vary with income.

5. The argument cannot be extended too much. It cannot be deduced from expression (5.6) that if  $g > r$ , then increasing the rate of contribution always leads to a Pareto improvement. Beyond a certain contribution rate, no young individual saves, which invalidates the approach by intertemporal wealth. To treat this question correctly, the return on capital must be endogenous, determined by a production function. Then the rate of return becomes larger than population growth if saving/investment is sufficiently low.

6. This remark does not account for the reform that has just been decided in France. The minimum level for pension benefits has been increased up to 85 percent of the minimal wage. For a large fraction of low-income earners, this constraint may become binding, which would make the French system more Beveridgean than previously. I thank Thomas Piketty for mentioning this point to me.

7. Note that because the wage distribution is identical in the two countries, the wage distribution in the union of the two countries is identical to that of a single country. Thus, if only  $I$  is active, then  $w^I = \bar{w}$ .

8. A referee objects that a young individual can choose a system that has no retirees (say,  $B$ ) and get pension benefits when old for free. It is true only if in next period a young person will agree to contribute to  $B$ . Looking at intertemporal wealth, which amounts to considering stationary behavior, accounts for this requirement.
9. With a discrete distribution, conditions are for an  $I$ -equilibrium to exist. The  $AB$  equilibria may be a little bit different: some may be semipooling, meaning that individuals with the same wage choose distinct systems (they must be indifferent between both).
10. The first case holds if both inequalities  $g < r$  and  $\tau^B(1 - \alpha^B g/r) < \tau^A(1 - \alpha^A g/r)$  hold, and the second holds if both inequalities are reversed.
11. Since  $A$  is less efficient and more favorable to high income than  $B$ , both inequalities  $0 > R^A - R^B > D^A - D^B$  hold. Thus, the monotonicity condition is met if the slope of  $E[w|w \geq x]$  is not larger than that of  $E[w|w \leq x]$ . This is true for a uniform distribution since the conditional expectations are, respectively, equal to  $(w_{\max} + x)/2$  and  $(x + w_{\min})/2$ .

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