

Investment, Security Design and Information¹

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Abstract

We consider the decisions of a venture capitalist who starts a new project. The prospects for the resale values of the newly created firm(s) are important features of the choice of the project. We assume that they crucially depend on the state of information when the firms are offered on the stock market. Such an information is likely to be asymmetric, which typically discourages the general public to invest in the new firms. This market imperfection influences the initial choices of the capitalist: it favors the projects where the asymmetry of information is the least pronounced (or, equivalently in our model, the risks born by the investor are smaller), and projects whose risk characteristics may be useful as a hedge to the market at large.

The paper illustrates this argument in a two stage CARA gaussian model with the following structure. In a first stage, the venture capitalist chooses her/his investment, as well as the asset structure of her/his firm(s). In the second stage, after some information has been revealed on the investment profitability, the firms are floated on the market. The stock market is competitive, with some noise that prevents prices to reveal all the available information. The first stage decisions are analyzed under rational expectations of the outcome of the stock market: we study security design (should the venture capitalist put all her/his projects into a single or several firms?), and the investment choice.

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I Introduction

We consider a venture capitalist who is willing to undertake risky projects. To finance these projects he simultaneously chooses their incorporation into one or several companies. These decisions are made at an *ex ante* stage and irreversible. It is likely that the capitalist, and possibly some other traders, benefits from some new privileged information at the time when stocks are traded. Without restrictions on insider trading, this may deter other uninformed investors to trade in the stocks. Our aim is to study, in a simple model, how the expected distortions of the functioning of the stock markets and the nature of risks to be hedged, influence the *ex ante* investment choice and security design of the venture capitalist.

To analyze these issues we consider a two-stage model. In the first stage, the firm picks a choice of projects of various risk characteristics. At the same time, it chooses how to float its investment on the stock market : while we impose that the total investment is listed on the market, the venture capitalist has the possibility to create several subsidiaries, and therefore to design several securities. We work under the simplifying assumptions often used in the finance literature : the utility functions exhibit constant absolute risk aversion, and the distributions of the project returns are assumed to have a jointly normal distribution. The securities are constrained to be linear combination of the underlying project returns and their return are normal as well.

At an *interim* stage, i.e. after the investment choice and the design of the securities but before trade, the venture capitalist and possibly part of the public receive some exogenous privileged information on the likely outcome of the risky activities. In the second stage, the securities are exchanged against the sure (numeraire) good on a competitive stock market. The description of the functioning of the market follows the principles of a rational expectations equilibrium under asymmetric information à la Grossman and Stiglitz (1980). All participants in the market are assumed to behave competitively. There are three groups of competitive agents who trade on the market. First, the risk averse venture capitalist, who has privileged information on the security returns. Second risk averse traders, called the public, with a random endowment of fundamental risks, who trade both for hedging and speculative purposes. Finally, the market makers, who are uninformed risk neutral agents, and extract all relevant information from the behavior of aggregate demand. Since the size of the endowment is unknown, they cannot separate the two sources of variability, new information or endowments shocks, from the observation of prices, and the equilibrium is not fully revealing¹. We consider two specifications of the information received by the risk averse public : either they receive the same signal as the venture capitalist, or they behave according to their prior knowledge of the model, ignoring the information revealed by the prices. Because of the risk neutral agents, the market always works, providing insurance to the risk averse traders, and prices

are equal to the expected value of the securities payoffs conditional on the publicly available information. This is in contrast with the works of Battacharya and Spiegel (1991), Bhattacharya, Reny and Spiegel (1995), where there are only risk averse agents. Then the market may collapse if the information asymmetries are too large, the potential insurers refusing to trade with the better informed ones due to a ‘lemon’ effect.

We assume that, in the first stage, the firm has rational expectations as to the outcome of the second stage. It chooses its investment and designs the securities so as to maximize its expected *ex ante* utility. As is usual in CARA gaussian models, this *ex ante* expected utility is the product of two terms. The first term corresponds to *speculative gains*. It is directly related to the insider’s informational advantage over the general public and does not depend on the investment choice of the firm, but only on the design of the securities. The second term reflects the gains from the *insurance opportunities* provided by the securities. As in Hirshleifer (1971), public information is harmful since it prevents the market to provide insurance. The corresponding insurance gains vary with the risk born by the venture capitalist, and are the larger, the lesser the information revealed in the market.

We consider the first stage decisions of security design and of investment choice separately. Given an investment choice, we first ask whether it is better for the venture capitalist to have a single asset, incorporating the whole project, or to separate the firm into several subsidiaries, each to be floated on the market. It turns out, not surprisingly, that the answer depends on the precise shape of the demand of securities of the public, which determines the information revealed by prices. In the first case that we study, when the risk averse public receives the same signal as the investor and behaves rationally, the best choice to maximize the insurance gains is to have a single firm, whatever the risk characteristics of the investment. A lot of information then is anyhow revealed by the price, and introducing more securities could only worsen the situation from the capitalist’s viewpoint. In the second case, when the risk averse public is uninformed and does not extract information from the price, the venture capitalist’s best choice of asset(s) is less easy to characterize. When the shape of the covariance matrix of the fundamental risks is much changed by the signal, his/her informational advantage allows in some circumstances, depending on the precise shape of the hedging demand of the risk averse public, to do best by diversifying her/his activity into several subsidiaries, which then reduces the information contained in the prices. The above results concern the design of securities to maximize the *insurance gains*. When one also takes into account the *speculative gains*, diversifying becomes more advantageous and may be profitable, even when the risk averse public has the same information as the capitalist. Speculative gains depend on the informational advantage of the firm, which typically may spread along other directions than the return on the aggregate single firm² Finally, for

given risk characteristics, the investor chooses a level of investment that increases with the insurance provided by the market, i.e. decreases with the information revealed by prices. We take a first look at the risk composition of investment that maximizes the insurance gains. It seems to depend, in a rather complicated way, both on the informational content of the signal available at the time of trade and on the hedging needs of the risk averse public.

The paper touches a number of themes that have received a lot of attention in the recent literature. First, security design has been considered from a variety of viewpoints. Allen and Gale (1988) assume that the designer of the securities differentiates his products to best fit the insurance needs of the buyers, subject to a transaction cost that depends on the number and shape of the securities he puts on the market. For Duffie and Jackson (1989), market organizations, whose revenues are based on a fee proportional to trade volume, facing some operating costs, create markets in a non-cooperative setup. Both of these works assume symmetric information, and the insurance needs of the potential buyers create the incentives for designing securities. At the opposite, in Boot and Thakor (1993), the motivation for designing securities is not to improve risk sharing – all agents are risk neutral – but to provide a better dissemination of information. Here, as in Demange and Laroque (1994, 1995), we neglect the transaction costs studied in these previous works, we focus the attention on the trade-off between insurance and information: as pointed out in a celebrated paper of Hirshleifer (1971), advanced information on a risky project precludes risk sharing.

Second, the paper is related to the literature on initial public offerings, initiated with the seminal work of Leland and Pyle (1977), and with the pros and cons of insider trading (see e.g. Leland (1995)). Here the information is revealed after the investment decision, and insider trading is a nuisance. It reduces the insurance provided by the market, and because it arrives too late, it does not help to select the projects.

Finally, the design of securities is linked with the incompleteness of markets (see e.g. Demange and Laroque (1995), Rahi (1995)). While the introduction of differentiated subsidiaries would be in the general interest of the public, we find here that it is not always in the interest of the venture capitalist, due to the adverse effects of the information revealed on the new markets. This points to another motive, aside from transaction costs or lack of liquidity (Allen and Gale (1994), Cuny (1993)), for the incompleteness of markets.

The paper is organized as follows. The model is laid down in the second section. Section 3 then describes the functioning of the stock market under asymmetric information, building upon the Grossman-Stiglitz and Battacharya-Spiegel models, when the agents have risky initial endowments. Section 4 computes the *ex ante* utility expected by the venture capitalist from her/his trades in the securities and derives some properties of the optimal security design associated to the capitalist's investment choice. Section 5 studies the optimal investment choice.

All proofs are gathered at the end of the paper in Section 6.

II The model

The economy has a single good. At the outset, the venture capitalist can invest in a number of projects whose returns are random. A level of investment K will yield $\mu(K) + \sigma(K)' \tilde{v}$, where \tilde{v} is a zero mean, unit variance normal vector of dimension k , and $\mu(K)$ and $\sigma(K)$ are well behaved functions of \mathbb{R}_+ into, respectively, \mathbb{R}_+ and \mathbb{R}_+^k .

We consider a sequence of three dates $t = 1, 2, 3$.

At date 1, the capitalist decides to go public, by incorporating the whole project into a single firm, or by setting up several firms. His decision is described by the stocks returns $\tilde{\varepsilon}$ of the different firms he sets up. We assume that any return is a linear combination of the underlying securities. Therefore, when the number of firms is n , the relationship between the vector of stock returns and the underlying project returns is described by a matrix A of dimension $(k \times n)$:

$$\tilde{\varepsilon} = A' \tilde{v}.$$

It will not be useful to have more than k firms and, without loss of generality, the matrix A is normalized, so that $A'A = I_n$. We impose the constraint that the whole firm is incorporated on the market so that :

$$\sigma(K)' \tilde{v} = S' \tilde{\varepsilon},$$

for some $(n \times 1)$ matrix S .

At date 2, the venture capitalist privately observes an advanced signal $\tilde{\theta}$ on the likely outcome of his undertakings. $\tilde{\theta}$ has a jointly normal distribution with the fundamental risks, has zero mean and unit variance.

At date 3, stock markets are open. Apart from the capitalist, risk neutral rational traders operate, as well as traders whose random wealth may be correlated with \tilde{v} .

The main decisions are taken at date 1, where securities are designed and at date 3 where trade takes place. We first investigate the outcome of the stock market, since the design of the securities depend on the gains the investor expects to realize on the market.

III The stock market

The stock market allows the venture capitalist to insure some of the risk he bears, and possibly to benefit from his private information. The investment decisions have been made and to simplify notation we drop the argument K in μ and σ .

III.1 The venture capitalist

The venture capitalist's initial wealth is equal to $\mu + S'\tilde{\varepsilon}$. The portfolio choice of the capitalist is *conditional on the signal* $\tilde{\theta}$. The capitalist's wealth, when he buys a portfolio X , can be written as :

$$\tilde{W} = \mu + S'\tilde{\varepsilon} + X'(\tilde{\varepsilon} - p).$$

Assuming CARA preferences, with a coefficient of risk aversion equal to a , the investor maximizes :

$$E\tilde{W}|_{\tilde{\theta}} - \frac{a}{2}\text{var}\tilde{W}|_{\tilde{\theta}},$$

i.e. :

$$X'(E\tilde{\varepsilon}|\theta - p) - a(X + S)'\text{var}(\tilde{\varepsilon}|\theta)(X + S).$$

Therefore his security demand is equal to :

$$X(\theta, p) = -S + \frac{1}{a}\text{var}(\tilde{\varepsilon}|\theta)^{-1}(E\tilde{\varepsilon}|\tilde{\theta} - p). \quad (11.1)$$

It is composed of two parts : the hedging demand, $-S$, and the speculative demand which is proportional to the expected return, i.e. to the spread between the expected value and the price. In a risk neutral market without asymmetric information, the speculative demand is null : the capitalist sells his whole risky project without paying any risk premium.

III.2 Traders with wealth correlated with the investment project

There are a large number of small traders, with CARA preferences. Apart from their initial wealth they all share the same information I_T . At the time of trade, a trader knows the fraction of his wealth that is correlated with \tilde{v} . Therefore the aggregate security demand³ of these traders at date 3 is given by the standard formula :

$$x(p, y) = \text{var}(\tilde{\varepsilon}|I_T)^{-1} \left(\frac{E\tilde{\varepsilon}|I_T - p}{b} - \text{cov}[(\tilde{\varepsilon}, \tilde{v})|I_T] y \right), \quad (11.2)$$

where b denotes the aggregate risk aversion coefficient of these traders, $y'\tilde{v}$ the fraction of their endowment that is correlated with \tilde{v} . Traders know their own endowment only and perceive the aggregate value \tilde{y} as a random variable, which is drawn from a k dimensional gaussian distribution of zero mean and variance $\text{var}(\tilde{y})$, and uncorrelated with the investment returns.

As for the information structure I_T we shall consider two polar cases :

1. (*Informed public*) The small traders share the same information as the venture capitalist, $I_T = \tilde{\theta}$.

2. (*The public is uninformed and does not extract information from the price*)
 Small traders do not extract information from the observation of the price: I_T is reduced to the prior information of the model, $E\tilde{\varepsilon}|I_T = 0$, $\text{var}(\tilde{\varepsilon}|I_T) = I_n$.

III.3 Price formation

The price is fixed on the market through competitive risk neutral traders. They are rational and use all public information I_P , which includes the price, their knowledge of the correlation of the underlying risks with the securities, the signal structure, and more importantly the shape of the security demands (1) and (2). Risk neutrality implies :

$$p = E\tilde{\varepsilon}|I_P.$$

Denoting their demands as $z(p)$ the equilibrium condition may be written as :

$$X(\theta, p) + x(p, \tilde{y}) + z(p) = 0.$$

Inspection of (1) and (2) shows that the observation of the price is informationally equivalent to that of :

$$\tilde{\gamma} = E\tilde{\varepsilon}|\tilde{\theta} + a\text{var}(\tilde{\varepsilon}|\tilde{\theta})\text{var}(\tilde{\varepsilon}|I_T)^{-1} \left(\frac{E\tilde{\varepsilon}|I_T}{b} - \text{cov}[(\tilde{\varepsilon}, \tilde{v})|I_T] \tilde{y} \right).$$

If the risk averse public is informed (case 1), we have :

$$\tilde{\gamma} = E\tilde{\varepsilon}|\tilde{\theta} - \frac{ab}{a+b}\text{cov}[(\tilde{\varepsilon}, \tilde{v})|\theta] \tilde{y},$$

while in case 2 :

$$\tilde{\gamma} = E\tilde{\varepsilon}|\tilde{\theta} - a\text{var}(\tilde{\varepsilon}|\theta)\text{cov}(\tilde{\varepsilon}, \tilde{v}) \tilde{y}.$$

In both cases the price is given by:

$$p = E[\tilde{\varepsilon}|\gamma].$$

IV Optimal security design

The capitalist's incentives to design one or several securities depend on the utility he expects from the trades.

IV.1 The *ex ante* utility of the venture capitalist

The CARA gaussian structure of the market allows to compute the ex-ante utility u of the capitalist at the end of date 1, when he has decided on the security structure, before he has received the signal. Let u_0 be the capitalist's utility level if s/he is not allowed to transact (or equivalently if s/he does not go public). Since utility is negative, u_0/u increases with the utility level.

Proposition 1 : *the ex ante utility level of the venture capitalist is given by :*

$$\frac{u_0}{u} = \left[\frac{\det \text{var} \tilde{\varepsilon} | \tilde{\gamma}}{\det \text{var} \tilde{\varepsilon} | \tilde{\theta}} \right]^{1/2} \exp \left\{ \frac{a^2}{2} \text{var}[\sigma' \tilde{v} | \gamma] \right\}. \quad (11.3)$$

The two terms correspond respectively to speculative and insurance gains.

The ratio of determinants is larger than 1, since the private information $\tilde{\theta}$ is more precise than the public information $\tilde{\gamma}$. It corresponds to the speculative gains of the investor : it increases his ex-ante utility level by a factor which is independent of the amount of capital invested.

The term $\text{var}[\sigma' \tilde{v} | \gamma]$ corresponds to the gain of utility associated with the insurance. It is maximal when the market gets no information at all: the capitalist may get rid of all risks without paying any risk premium. The more information the market gets, the lower this gain: this is known as the information effect. Accordingly the information revealed to the market is the crucial determinant of these gains.

We now study in turn the consequences of this formula for security design, given an investment choice, and for the choice of investment. We are far from having a complete solution to the problem, and the results that we present are partial. We shall mainly focus on the circumstance where the investment is 'large', so that the most important factor is the insurance gains, and we can neglect the speculative gains. For that purpose the following expression for the variance term in the insurance gains is useful :

$$\text{var}[\sigma' \tilde{v} | \gamma] = \sigma' \sigma - \sigma' \text{var}(E\tilde{v} | \theta) A (\text{var} \gamma)^{-1} A' \text{var}(E\tilde{v} | \theta) \sigma. \quad (11.4)$$

IV.2 Security design

Proposition 2 : *In order to maximize his insurance gains, the venture capitalist forms a single firm if the risk averse traders are informed. If they are uninformed he takes into account their hedging needs.*

The crucial point when the risk averse public is as well informed as the venture capitalist is that, whatever securities are issued, the signal

$$E[\sigma' \tilde{v} | \theta] - \frac{ab}{a+b} \sigma' \text{var}(\tilde{v} | \tilde{\theta}) \tilde{y}$$

on the overall investment may be extracted from the prices⁴. Therefore setting up only one security with the overall investment return $\tilde{\varepsilon} = \sigma' \tilde{v}$ always minimizes the revelation of information, and therefore maximizes the gains from insurance.

The situation is different when the public is uninformed. In fact, from (11.4), all the differences between the two cases come from the term $A(\text{var}\gamma)^{-1}A'$. The information that is revealed by the security collinear to the overall investment becomes :

$$S' \tilde{\gamma} = E[\sigma' \tilde{v} | \theta] - a\sigma' \text{var}(\tilde{v} | \theta) A A' \tilde{y}.$$

It now depends (through AA') on the other securities that can be exchanged on the market. One cannot exclude that issuing more securities affects the demand in a such way that less information is revealed.

A simple example may help to understand why. Suppose that the fundamental risks are two-dimensional and that the project is given by $\sigma' = (1, 0)$. The small traders' endowments are concentrated on the other branch of the fundamental risk, i.e. $y_1 = 0$. If there is only one security $\tilde{\varepsilon} = \tilde{v}_1$, their hedging demand, without privileged information, is null, so that the private information θ is entirely revealed to the market, and $E[\sigma' \tilde{v} | \gamma] = E[\sigma' \tilde{v} | \theta]$. If the two securities, with returns respectively equal to \tilde{v}_1 and \tilde{v}_2 , are marketed, small traders may now hedge their risks, by taking a position on the second security. Then the public receives two signals, γ_1 and γ_2 , and in general cannot perfectly extract the values of $E[\tilde{v}_1 + \tilde{v}_2 | \theta]$. More precisely

$$\gamma = E\tilde{v} | \theta - a \text{var}(\tilde{v} | \theta) \tilde{y}$$

so that

$$E[\sigma' \tilde{v} | \gamma] = E[\sigma' \tilde{v} | \theta] - a\sigma' \text{var}(\tilde{v} | \theta) \tilde{y}.$$

When $\sigma' \text{var}(\tilde{v} | \theta) \tilde{y}$ has a non zero variance, i.e. conditioning with respect to θ changes the shape of the covariance matrix of the fundamental risks, the investor is better off issuing two securities rather than one. As in Allen and Gale (1994), the demand side of the market is important in the design of the security.

Proposition 2 deals only with the insurance gains. It is of interest to know whether speculative gains could change the nature of the results. In particular, in case 1, when the risk averse public receives the same signal as the venture capitalist, can prospective speculative gains induce the capitalist to float several securities on the market, to benefit from her/his privileged information? The answer to this question is yes. A simple intuition is that the investor has nothing to lose by trading a security that does not reduce her/his insurance gains. Consider as above an example where fundamental risks are two-dimensional and the investment project is given by $\sigma' = (1, 0)$. Assume that the capitalist has two uncorrelated advanced signals, each of which brings information on \tilde{v}_1 or \tilde{v}_2 only. For example $\tilde{\theta}_i = v_i + \tilde{v}_i$, where each \tilde{v}_i is independent of all other variables. Assume furthermore that \tilde{y}_1 and \tilde{y}_2 are independent. If the two securities are

floated, the two markets are independent. The investor gains on the first security are identical whether the second market exists or not⁵ while s/he gains from his superior information by trading on the second security.

V Investment choice

We now consider the optimal investment choice. It of course depends on the specification of the functions $\mu(K)$ and $\sigma(K)$. To make things as simple as possible and to concentrate on the choice of the risk profile, we suppose that the expected return by unit of investment is constant equal to some strictly positive μ . Therefore $\mu(K) = \mu K$ for any K . At the opposite the venture capitalist can choose any risk profile for her/his investment :

for any σ such that $\sigma'\sigma = 1$, any K , there is a project with $\sigma(K) = \sigma K$.

It is easy to determine a property on the optimal level of capital stock.

Proposition 3 : *Let the risk characteristics of the investment, σ , and the security design, A , be given. Then the optimal capital stock of the venture capitalist is given :*

$$K = \frac{\mu}{a\text{var}(\sigma'E\tilde{v}|\gamma)}.$$

The investment level increases with the expected return μ and decreases with the investor risk aversion a . When no information passes through the prices, $\text{var}\sigma'E\tilde{v}|\gamma = 0$, and the optimal stock of capital is infinite : the capitalist can fully hedge the risks without bearing any risk premium on the (risk neutral) market while earning a positive return μ on each unit invested. In all other circumstances, the investment level is well defined and decreases with the information revealed on the market.

There remains to understand the optimal risk characteristics of the venture capitalist's investment. We consider the case where the investment level is large, so that the speculative gains (the first term in (11.3)) are negligible compared to the insurance gains. From Proposition 3, we see that the best direction for investment is the one that allows for the largest insurance possibilities on the market : it should combine the information brought by the signal θ , with the hedging needs of the public.

Proposition 4 : *Consider a market where the risk averse traders are informed (case 1). The risk characteristics of the investment project that maximizes the insurance gains from the market is a normalized vector σ , $\sigma'\sigma = 1$,*

which satisfies :

$$\frac{2(\text{var}E\tilde{v}|\theta)\sigma}{\sigma'(\text{var}E\tilde{v}|\theta)\sigma} - \frac{(\text{var}E\tilde{v}|\theta + \frac{a^2b^2}{(a+b)^2}\text{var}\tilde{v}|\theta\text{vary}\text{var}\tilde{v}|\theta)\sigma}{\sigma'(\text{var}E\tilde{v}|\theta + \frac{a^2b^2}{(a+b)^2}\text{var}\tilde{v}|\theta\text{vary}\text{var}\tilde{v}|\theta)\sigma} = \sigma.$$

The choice of the investment risk characteristics is the solution of a non convex program, and the above proposition only gives the first order condition associated with this program. The first order condition can be rewritten so that σ appears to be an eigenvector of a linear combination of $\text{var}E\tilde{v}|\theta$ and of $\text{var}\tilde{v}|\theta\text{vary}\text{var}\tilde{v}|\theta$, with respective weights depending on the values of the two corresponding quadratic forms along the (unknown) direction σ . Since $\text{var}E\tilde{v}|\theta = I_k - \text{var}\tilde{v}|\theta$ this condition finally depends on two matrices, the matrix $\text{var}\tilde{v}|\theta$ which describes the information contained in the signal, and the covariance matrix of the endowments, vary . The optimal direction may be a complicated function of the information motive and of the demand for hedging of the public. A simple situation occurs when the two above matrices have the same set of eigenvectors. The first order condition then says that the optimal direction is one of these eigenvectors. Going back to the expression for the insurance gains, if we let V_i , $0 \leq V_i \leq 1$, and Y_i denote respectively the eigenvalues of $\text{var}\tilde{v}|\theta$ and vary associated with the i th eigenvector, the optimal direction is the value of i that minimizes :

$$\frac{(1 - V_i)^2}{1 - V_i + \frac{a^2b^2}{(a+b)^2}V_i^2Y_i}.$$

Given Y , this expression is minimized for the largest possible value of V . Also, given V , one wants to have the largest possible Y . In words, when the hedging demand of the public is uniformly distributed ($\text{vary} = YI_k$), the venture capitalist chooses the direction where the signal reveals the less information (largest eigenvalue of $\text{var}v|\theta$). Conversely, when the information is uniformly revealing in all directions ($\text{var}v|\theta = VI_k$), the preferred investment is the one that best satisfies the hedging demand (largest eigenvalue of vary).

VI Proofs

Proof of Proposition 1 : Given θ and p , substituting the expression of the security demand (11.1) yields :

$$E\tilde{W}|\tilde{\theta} - \frac{a}{2}\text{var}\tilde{W}|\tilde{\theta} = \mu(K) + S'E\tilde{\varepsilon}|\theta - S'(E\tilde{\varepsilon}|\theta - p) + \frac{1}{2a}(E\tilde{\varepsilon}|\tilde{\theta} - p)'\text{var}(\tilde{\varepsilon}|\theta)^{-1}(E\tilde{\varepsilon}|\tilde{\theta} - p),$$

or

$$E\tilde{W}|\tilde{\theta} - \frac{a}{2}\text{var}\tilde{W}|\tilde{\theta} = \mu(K) + S'p + \frac{1}{2a}(E\tilde{\varepsilon}|\tilde{\theta} - p)'\text{var}(\tilde{\varepsilon}|\theta)^{-1}(E\tilde{\varepsilon}|\tilde{\theta} - p). \quad (11.5)$$

We compute the mathematical expectation of $\exp - a\{E\tilde{W}|\tilde{\theta} - \frac{a}{2}\text{var}\tilde{W}|\tilde{\theta}\}$ over the distributions of θ and p , or equivalently over the distributions of θ and y . Recall that $p = E[\tilde{\varepsilon}|\gamma]$ and that $\tilde{\gamma} = E\tilde{\varepsilon}|\tilde{\theta} + AV\tilde{y}$ for a suitable definition of the matrix V . Lemma 1 shows that the price p , and therefore the cost from hedging, $S'p$, is independent of the speculative demand of the venture capitalist.

Lemma 1 :

$$\text{cov}(E\tilde{\varepsilon}|\tilde{\theta} - E\tilde{\varepsilon}|\tilde{\gamma}, E\tilde{\varepsilon}|\tilde{\gamma}) = 0$$

and

$$\text{var}(E\tilde{\varepsilon}|\tilde{\theta} - E\tilde{\varepsilon}|\tilde{\gamma}) + \text{var}\tilde{\varepsilon}|\tilde{\theta} = \text{var}\tilde{\varepsilon}|\tilde{\gamma}.$$

Proof of Lemma 1 : This follows from the fact that $E\tilde{\varepsilon}|\tilde{\theta} - E\tilde{\varepsilon}|\tilde{\gamma}$ is uncorrelated with $\tilde{\gamma}$. Indeed, one can write :

$$E\tilde{\varepsilon}|\tilde{\theta} - E\tilde{\varepsilon}|\tilde{\gamma} = (\tilde{\varepsilon} - E\tilde{\varepsilon}|\tilde{\gamma}) - (\tilde{\varepsilon} - E\tilde{\varepsilon}|\tilde{\theta}). \quad (11.6)$$

The first term on the right hand side is uncorrelated with $\tilde{\gamma}$ by definition, while the second term is uncorrelated with $\tilde{\theta}$, and uncorrelated with \tilde{y} by assumption, which implies that it is uncorrelated with $\tilde{\gamma} = E\tilde{\varepsilon}|\tilde{\theta} + AV\tilde{y}$. The same argument shows that $(E\tilde{\varepsilon}|\tilde{\theta} - E\tilde{\varepsilon}|\tilde{\gamma})$ is uncorrelated with $(\tilde{\varepsilon} - E\tilde{\varepsilon}|\tilde{\theta})$ so that the second equation follows. ■

The utility level in the absence of trade is $u_0 = -E[\exp - a\{\mu(K) + S'\tilde{v}\}]$. From Lemma 1 we get

$$\frac{u_0}{u} = E[\exp - a\{S'(\tilde{v} - p)\}] E[\exp\{\frac{1}{2}(E\tilde{\varepsilon}|\tilde{\theta} - p)' \text{var}(\tilde{\varepsilon}|\tilde{\theta})^{-1} (E\tilde{\varepsilon}|\tilde{\theta} - p)\}].$$

Lemma 2 : let Z be a zero mean k -dimensional multivariate normal variable of variance covariance matrix Σ , and let H be a symmetric matrix such that $2H + \Sigma^{-1}$ is positive definite. Then :

$$E\exp(-Z'HZ) = \frac{1}{(\det |I_k + 2H\Sigma|)^{1/2}}.$$

Proof of Lemma 2 : One just has to compute the integral, noting that :

$$-Z'HZ - \frac{1}{2}Z'\Sigma^{-1}Z = -\frac{1}{2}Z'[2H + \Sigma^{-1}]Z.$$

Using the shape of the distribution of the multivariate normal, the integral of the exponential of this expression is equal to :

$$(2\pi)^{k/2}[\det |2H + \Sigma^{-1}|]^{-1/2}.$$

The result follows. ■

Lemma 3 : (*speculative gains*)

$$E \exp - \left\{ \frac{1}{2} (E\tilde{\varepsilon}|\tilde{\theta} - E[\tilde{\varepsilon}|\tilde{\gamma}])' \text{var}(\tilde{\varepsilon}|\theta)^{-1} (E\tilde{\varepsilon}|\tilde{\theta} - E[\tilde{\varepsilon}|\tilde{\gamma}]) \right\} = \left(\frac{\det \text{var}(\tilde{\varepsilon}|\tilde{\gamma})}{\det \text{var}(\tilde{\varepsilon}|\tilde{\theta})} \right)^{1/2}$$

Proof of Lemma 3 : Let

$$Z = \text{var}(\tilde{\varepsilon}|\theta)^{-1} (E\tilde{\varepsilon}|\theta - E\tilde{\varepsilon}|\gamma), \text{ and } H = \frac{1}{2} \text{var}\tilde{\varepsilon}|\tilde{\theta}.$$

The variance of Z is given by

$$\Sigma = (\text{var}\tilde{\varepsilon}|\tilde{\theta})^{-1} \text{var}(E\tilde{\varepsilon}|\tilde{\theta} - E\tilde{\varepsilon}|\tilde{\gamma}) (\text{var}\tilde{\varepsilon}|\tilde{\theta})^{-1}.$$

Moreover $I_k + 2H\Sigma = I_k + \text{var}(E\tilde{\varepsilon}|\tilde{\theta} - E\tilde{\varepsilon}|\tilde{\gamma}) (\text{var}\tilde{\varepsilon}|\tilde{\theta})^{-1}$. Lemma 1 yields

$$I_k + 2H\Sigma = \text{var}\tilde{\varepsilon}|\tilde{\gamma} (\text{var}\tilde{\varepsilon}|\tilde{\theta})^{-1}$$

and it suffices to apply Lemma 2. ■

End of the Proof of Proposition 1 : $E[\tilde{\varepsilon}|\gamma]$ is normal with zero mean. Therefore

$$E \{ \exp - a S' E[\tilde{\varepsilon}|\gamma] \} = \exp - \frac{a^2}{2} \text{var}(S' E[\tilde{\varepsilon}|\gamma]).$$

The result follows from Lemma 3. ■

Proof of Proposition 4 : The capitalist seeks to maximize his insurance gains. Since there is a single security, proportional to the risk characteristics of the firm, using (11.4), this amounts to find the value of σ , of norm 1, which minimizes :

$$\frac{(\sigma' \text{var} E\tilde{v}|\theta \sigma)^2}{\text{var}\gamma},$$

where :

$$\text{var}\gamma = \sigma' \text{var} E\tilde{v}|\theta \sigma + \frac{a^2 b^2}{(a+b)^2} \sigma' \text{var}\tilde{v}|\theta \text{var}(\tilde{y}) \text{var}\tilde{v}|\theta \sigma.$$

To simplify notations, let :

$$B = \text{var} E\tilde{v}|\theta, \quad C = \frac{a^2 b^2}{(a+b)^2} \text{var}\tilde{v}|\theta \text{var}(\tilde{y}) \text{var}\tilde{v}|\theta.$$

B and C are symmetric positive definite matrices, and we want to minimize

$$M = \frac{(\sigma' B \sigma)^2}{\sigma' (B + C) \sigma},$$

subject to the constraint $\sigma' \sigma = 1$.

Taking logarithms, the first order condition for an extremum of M under the constraint can be written, where λ is the multiplier associated with the constraint :

$$\frac{2B\sigma}{\sigma'B\sigma} - \frac{(B+C)\sigma}{\sigma'(B+C)\sigma} = \lambda\sigma.$$

Multiplying from the left by σ' , and using the constraint shows that $\lambda = 1$. Therefore σ is an eigenvector of the matrix $\frac{2}{\sigma'B\sigma}B - \frac{1}{\sigma'(B+C)\sigma}(B+C)$, with eigenvalue equal to 1. ■

Notes

1. An important feature of all the models with asymmetric information is the ‘noise’ that prevents the uninformed traders from inferring the private information signals from the mere observation of the market prices. We use the modeling device introduced by Battacharya and Spiegel (1991) and Bhattacharya, Reny and Spiegel (1995). Here the noise is fully rational and comes from lack of information on some idiosyncratic risks. In our previous work (Demange and Laroque (1994, 1995b), like in a large part of the literature (Grossman and Stiglitz (1980), Schleifer and Summers (1990)), we postulated an exogenous price inelastic random supply of securities. Since the price and information sensitivity of the security demand of the public is a priori an important element of the investment and security design decisions, this procedure is too crude for our current purpose.

2. This is in contrast with Rahi (1996) in which a firm that is only allowed to design a single security, does not find optimal to use its privileged information. However the result is somewhat trivial because the informational set up is such that the investor’s information is always completely revealed to the market. Obviously then the capitalist has nothing to gain at trading in other directions than his own risks.

3. With obvious notation, each individual i ’s demand is :

$$x(p, y_i) = \text{var}(\tilde{\varepsilon}|I_T)^{-1} \left(\frac{E\tilde{\varepsilon}|I_T - p}{b_i} - \text{cov}[(\tilde{\varepsilon}, \tilde{v})|I_T] y_i \right).$$

4. To see this, it suffices to multiply on the left by S' the vector of signals $\tilde{\gamma}$:

$$\tilde{\gamma} = E\tilde{\varepsilon}|\tilde{\theta} - \frac{ab}{a+b}\text{cov}[(\tilde{\varepsilon}, \tilde{v})|\theta] \tilde{y},$$

and to use $S'\tilde{\varepsilon} = \sigma'\tilde{v}$.

5. Under the informational assumptions, $E\tilde{v}|\tilde{\theta} = E\tilde{v}_i|\tilde{\theta}_i$ and the matrix $\text{var}(\tilde{v}|\theta)$ is diagonal. Consider the cases with two securities $\varepsilon = v$. We get $\tilde{\gamma} = E\tilde{v}|\tilde{\theta} -$

$\frac{ab}{a+b}\text{var}(\tilde{v}|\theta)\tilde{y}$, so that, as we already know, γ_1 is unaffected by the trading on the second security. As a consequence p_1 and the capitalist's demand (see(11.1)) are also unaffected: the gains from the first security are unchanged.

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