Fundamental Volatility and Financial Stability[*]

Gabriel Desgranges[†] Stéphane Gauthier[‡]

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Abstract

Financial investors choose the capital they invest into risky firms based on the return they expect. The actual return depends on fundamental shocks and the aggregate investment, which gives rise to beauty-contest issues. The paper characterizes how the ability of investors to solve these issues relates to the amount of fundamental volatility. It exploits this link to provide a quantitative assessment of the contribution of fundamentals to market volatility. Volatility would be driven by fundamentals in most markets. However out-of-equilibrium beliefs significantly contribute to observed volatility in markets of the financial sector.

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[†]`gabriel.desgranges@cyu.fr`; ThEMA, Cergy Paris Université, 33 bd du Port 95000 Cergy Pontoise; France.

[‡]`stephane.gauthier@univ-paris1.fr`; Paris School of Economics, Université Paris 1 Panthéon-Sorbonne and Institute for Fiscal Studies, 48 bd Jourdan 75014 Paris, France.

1 Introduction

In the rational expectations approach, market volatility is entirely driven by economic fundamentals and prices provide traders with reliable signals on values. Following Shiller [23] and LeRoy and Porter [18], however, the literature emphasizes the existence of excess volatility on top of that implied by fundamentals. Behavioral, sentiments and other belief-based exuberance factors also contribute to excess volatility (see Barberis [3] for a recent survey) and blur price signals. Our paper builds a simple theoretical model where markets are subject to fundamental shocks but sometimes sentiments matter. A quantification exercise suggests that fundamental shocks would be the main source of volatility. Still, sentiments matter in a small subset of markets, mostly within the financial sector in bad times. They would then account for one-third of observed volatility in the whole economy.

It is known that the traditional dividend discount approach does not fully account for stock market fluctuations. Exogenous amplification mechanisms, e.g., habit formation (Campbell and Cochrane [9]), shocks on long-run expectations (Bansal, Kiku and Yaron [2]) or disaster risks (Barro [4]) have been explored to explain magnified volatility, albeit fundamentally driven. Non-fundamental volatility, on the other hand, can originate from overconfidence and over-reaction to news (Daniel et al. [12]), herd behavior and information externalities (Park and Sabourian [21]), or bubbles arising in the presence of coordination issues (Abreu and Brunnermeier [1]). Bordalo, Gennaioli, La Porta and Shleifer [7] provides recent empirical evidence on non-fundamental volatility.

Our paper considers a theoretical benchmark where volatility is entirely driven by fundamentals in the rational expectations equilibrium. Nevertheless it allows for the possibility of non-rational out-of-equilibrium outcomes. Out-of-equilibrium volatility depends on both fundamentals and behavioral factors captured by non-rational beliefs.

Financial investors invest in firms that compete in the good market. Their decisions are based on the forecasts they make about firms' returns. The actual returns are eventually determined by the aggregate investment of all investors and exogenous fundamental shocks. In this setup, therefore, the returns are determined through a beauty contest between investors, as they need to forecast the aggregate investment to accurately predict returns. The rational expectations equilibrium obtains when investors successfully solve the contest by guessing others' investment decisions. Otherwise, in case of failure, out-of-equilibrium beliefs of investors matter.

The criterion for delineating conditions for success or failure to guess others' decisions relies on iterative elimination of non-best responses. Following Guesnerie [16] we argue that a rational expectations equilibrium more likely occurs if it is the only solution surviving iterative elimination. It is then the unique rationalizable solution in the game theory terminology. This approach shares common features with level-k thinking, as in e.g., Nagel [20], and conditions for a unique rationalizable solution often closely relate to those found for stability under adaptive learning (Marx [19]).

The key sufficient statistics for the occurrence of an equilibrium is the sensitivity of the return to the investment. An equilibrium obtains if and only if this sensitivity is low enough; that is, the return does not depend much on investors' decisions. The intuition is that investors can then form accurate predictions about an almost fixed return without guessing precisely others' investment decisions. Rational expectations are perfectly correlated with fundamentals, which makes the equilibrium outcome determined by fundamentals only. If, instead, the sensitivity is high, it becomes much more difficult to form accurate predictions about the return. In case of failure, the out-of-equilibrium outcome displays a lower correlation between fundamentals and expectations, as out-of-equilibrium beliefs contribute to explain volatility of the returns.

A crucial comparative static property for the empirical analysis is that the sensitivity threshold below which the equilibrium should occur is increasing in investors' risk aversion and the magnitude of fundamental volatility. Intuitively, risk-averse investors become very reluctant to adjust their portfolio when they face high fundamental volatility. The resulting inertia in the investment behavior makes the aggregate investment more easily predictable. Equilibrium occurrence therefore is consistent with higher return sensitivities in the presence of high fundamental volatility. This is this feature that we exploit in the empirical illustration to recover fundamental volatility: one can infer from accurate predictions formed by investors operating in high-sensitivity markets that fundamental shocks are of high magnitude.

IBES data show that high-sensitivity markets more likely fall out-of equilibrium. It contains information about capital and actual returns, allowing us to estimate sensitivities. It also reports returns predicted by financial analysts; the difference between actual and predicted returns yields prediction errors. The data exhibits a positive correlation between estimated sensitivities and prediction errors that is consistent with the theoretical analysis.

Based on this finding we consider markets with a low sensitivity as more likely to have reached an equilibrium. In order to identify these markets, we estimate a threshold sensitivity such that every market with a sensitivity below this threshold is considered as in equilibrium while all the others stand out-of equilibrium.

We find 88 percent of the markets in the low-sensitivity group. In these markets, volatility of firms' returns would be mostly fundamentally-driven. In the remaining markets, fundamental volatility would only represent two-third of the actual return volatility; the other one-third would accordingly come from non-fundamental beliefs. A finer decomposition suggests that non-fundamental volatility especially matters in two markets of the financial sector, investment banking and insurance, during periods of global macroeconomic slowdowns.

The paper is organized as follows. Sections 2 and 3 lay down our theoretical setup. We first focus in Section 2 on the simple case where investors allocate their portfolio between a safe asset and a single risky asset whose return depends on the profit of a single firm in the good market. This setup highlights how the return sensitivity interacts with fundamental volatility in making plausible the occurrence of an equilibrium. It is enough for a reader to follow the arguments used in the empirical illustration. Section 3 extends the analysis to the case of a multiplicity of firms competing in the good market, thus a multiplicity of risky assets. Competition is found detrimental to financial market stability by making out-of equilibrium outcomes more likely. The empirical illustration is in Section 4.

2 Monopoly in the good market

2.1 Setup

There are two periods and two markets, a financial market and a good market. In the first period, the good market remains closed while in the financial market a continuum $[0, 1]$ of infinitesimal identical investors with CARA preferences allocate their initial wealth across two assets. One asset yields a given safe net return $R > 0$. The other asset is risky. Its return $\tilde{r}(k)$ is interpreted as the return on equity of a single firm operating in the good market. It depends on the aggregate investment k into the firm that is made by all investors. It will be determined in the second period in the good market.

In the second period, the financial market is closed. The firm uses the capital invested in the previous period to produce a final good, and sells its production in the good market. Given k, the profit of the firm is $\pi(k) + \tilde{\varepsilon}k$, where $\pi(k)$ stands for the mean profit and $\tilde{\varepsilon}$ is a Gaussian noise with mean 0 and variance σ^2 which affects capital cost. The profit is entirely redistributed to the investors. The return $\tilde{r}(k)$ thus satisfies

$$1 + \tilde{r}(k) = \frac{\pi(k) + \tilde{\varepsilon}k}{k} = \frac{\pi(k)}{k} + \tilde{\varepsilon} \overset{\text{def}}{=} 1 + r(k) + \tilde{\varepsilon}, \tag{1}$$

where $r(k)$ represents the mean return of capital investment.

We assume:

Assumption 1. *The return $r(k)$ is non-negative and decreasing in k, $r(k) \geq 0$ and $r'(k) < 0$.*

Assumption 1 is satisfied if the profit $\pi(k)$ is increasing concave in k and inaction is possible, $\pi(0) = 0$.

Assumption 2. *The return satisfies $r(0) > R$.*

If Assumption 2 is not satisfied, then the safe asset always dominates the risky asset, and no investor allocates capital into the firm. Assumption 2 holds if $\pi'(0)$ is large enough, e.g., under some Inada condition making the marginal productivity of the firm arbitrarily high for a low amount of capital.

Let W_0 be the initial wealth of an investor. For a given aggregate capital k, the final wealth of investor i is $(1 + R)(W_0 - k_i) + (1 + r(k) + \tilde{\varepsilon})k_i$ when she invests k_i into the firm. In the CARA-Gaussian setup, the amount of capital k_i maximizes

$$(r(k) - R)k_i - \frac{a}{2}\sigma^2 k_i^2 \tag{2}$$

where a is the coefficient of absolute risk aversion of the investor. Hence, the best-response of investor i to the aggregate capital k is

$$k_i = BR(k) \overset{\text{def}}{=} \frac{r(k) - R}{a\sigma^2}. \tag{3}$$

The best-response function $BR(k)$ is decreasing in k. An investor who expects a higher aggregate investment into the firm, and thus a reduced mean return of the risky asset, reallocates her wealth toward the safe asset.

A Nash equilibrium in the financial market is a collection $(k_i^*)_{i\in[0,1]}$ such that $k_i^* = BR(k^*)$ for every investor i, where

$$k^* = \int_0^1 k_i^* \, \mathrm{d}i$$

stands for the aggregate equilibrium capital stock. Our assumption of identical investors implies that the best-response function does not depend on i, and so every investor i chooses the same investment $k_i^* = k^*$ in equilibrium. Therefore, by summation of (3) over investors, the aggregate investment k^* in equilibrium is a fixed point of the best-response function, i.e., $k^* = BR\,(k^*)$.

We have:

Lemma 1. *Under Assumptions 1 and 2 there exists a unique Nash equilibrium. In this equilibrium every investor invests a positive amount of capital* k^* *satisfying*

$$k^* = \frac{r\,(k^*) - R}{a\sigma^2}. \tag{4}$$

Proof. The left-hand side of (4) is increasing in k^* while Assumption 1 implies that its right-hand side is decreasing in k^*. There is consequently at most one Nash equilibrium. By Assumption 2 the right-hand side is positive for $k^* = 0$, and by Assumption 1 it is smaller than the left-hand side for high enough values of k^*, which implies the existence of a fixed point k^* of the best-response function. The result follows. □

The fact that the return of the risky asset is determined by the aggregate investment chosen by all investors implies a beauty-contest issue. In order to make an optimal investment decision, every investor must succeed to formulate an accurate guess about the return $r(k)$, and so the aggregate capital k invested by all investors. When they are successful, the only source of return volatility is the fundamental shock $\tilde{\varepsilon}$ which affects capital cost.

2.2 Financial stability

Beauty-contest issues are assumed to be already solved in (4). Every investor expects the aggregate capital k^* and reacts by choosing the individual investment k^* that self-validates her a priori belief.

Suppose instead that every investor only knows that the aggregate capital k stands between some lower bound $k^{\inf}(0)$ and upper bound $k^{\sup}(0)$, with $k^{\inf}(0) \le k^* \le k^{\sup}(0)$. This assumption does not exclude the possibility that investors expect k^* but it allows for some departure from the equilibrium. If, for instance, investors under-estimate the amount of capital invested into the firm, $k < k^*$, then they expect a return above $r(k^*)$, and so they select in (3) a too high investment k_i greater than k^*. The actual aggregate capital, which is above k^*, eventually appears as inconsistent with their initial belief. Wrong beliefs then constitute an additional source of return volatility that complements fundamental shocks in $\tilde{\varepsilon}$.

A simple argument may help investors narrow the set of possible aggregate capital values from the initial interval $I(0) = [k^{\inf}(0), k^{\sup}(0)]$. Indeed the investment k_i of every investor i has to be between $BR(k^{\sup}(0))$ and $BR(k^{\inf}(0))$. This implies an aggregate capital in the new interval

$$I(1) \overset{\text{def}}{=} \left[BR(k^{\sup}(0)), BR(k^{\inf}(0))\right] \cap I(0)$$

that includes k^*. The same argument can be reproduced from $I(1)$. And, after τ steps of this reasoning, every investor knows that the aggregate capital belongs to

$$I(\tau) \overset{\text{def}}{=} \left[BR(k^{\sup}(\tau-1)), BR(k^{\inf}(\tau-1))\right] \cap I(\tau-1).$$

The interval $I(\tau)$ consists of the aggregate capital values surviving τ steps of elimination of non-best responses. Therefore it includes k^*.

The sequence of intervals $(I(\tau))_{\tau \ge 0}$ is decreasing and thus converges to a limit set including k^*. The Nash equilibrium is said to be 'stable' if the limit set consists of k^* only. The equilibrium is 'unstable' otherwise.

In case of stability, investors relying on the iterative inferences described above eventually expect a return $r(k^*)$. Each of them accordingly invests $k^* = BR(k^*)$ into the firm. The aggregate capital is k^*, which self-confirms their initial prediction. By (4) the volatility of the return only relates to fundamental shocks.

In case of instability, it is less likely that investors expect the equilibrium to occur since any amount of capital in the limit set may occur as well, including non-equilibrium outcomes $k \neq k^*$. Departures from the equilibrium give rise additional volatility coming on top of fundamental shocks.[1]

[1]The process can be viewed as relying on Common knowledge of rationality, the best-

In the sequel, we set $k^{\inf}(0)$ and $k^{\sup}(0)$ close to k^*. Investors may not be perfectly aware of the equilibrium but they are assumed to have a precise enough knowledge about it. Then, the equilibrium is stable if and only if the best-response function is locally contracting at k^*, i.e., $|BR'(k^*)| < 1$. From (3), we have:

Proposition 1. *The equilibrium is stable if and only if*

$$-r'\left(k^*\right) < a\sigma^2. \tag{5}$$

When $|r'\left(k^*\right)|$ is small, the return does not vary much with the aggregate investment. This makes it easier to predict even though the aggregate capital is imperfectly known initially. The inequality (5) provides us with $r'\left(k^*\right)$ as a sufficient statistics for equilibrium stability. It will be exploited in this way in the empirical illustration.

The optimal investment behavior described in (3) also makes clear the role played by $a\sigma^2$ as a threshold in the right-hand side of (5). It shows that a high $a\sigma^2$ implies a soften reaction of the individual investment following a change in the expected return. Investors are more likely to assess correctly the aggregate investment, hence the return.

2.3 Linear-quadratic specification

The best-response function (3) also shows that a high $a\sigma^2$ reduces the equilibrium amount of capital investment, thus implying a higher return in the right-hand side of (5). In this Section, we re-express the condition for stability in terms of exogenous parameters for a particular textbook specification of the good market. The monopolist faces a linear (inverse) demand where consumers are ready to pay the unit price $P(q) = A - Bq$ to buy q units of the good, and its cost of producing q units of the good is $Cq^2/(2k)$. The parameters A, B and C are all positive real numbers. For this specification, the profit of the firm is

$$\pi(k) = \max_q \left(A - Bq\right) q - \frac{Cq^2}{2k} = \frac{1}{2}\frac{A^2 k}{2Bk + C}. \tag{6}$$

response function (3), and the initial restriction $\left[k^{\inf}(0), k^{\sup}(0)\right]$. The limit set consists of the aggregate capital levels consistent with these common knowledge assumptions. This amounts to interpret the model as a game between investors and to compute the rationalizable aggregate capital levels of this game. The equilibrium is stable when it is the unique rationalizable aggregate capital level.

Using (1) and (6), the return is

$$r(k) = \frac{\pi(k)}{k} - 1 = \frac{1}{2}\frac{A^2}{2Bk + C} - 1. \tag{7}$$

Given k, it increases with the profit $\pi(k)$ of the firm, i.e., when the willingness to pay A of the consumers is high, the sensitivity B of the price is low (which allows the monopolist to rise its production without sacrificing on price) and the cost is low (as reflected in a low C).

The return in (7) fits Assumption 1. Assumption 2 writes

$$\frac{A^2}{2C} > 1 + R. \tag{8}$$

Compared to the general setup in Section 2.1, an additional feature of the linear-quadratic specification is to yield a return convex in k. Therefore small sensitivities, $r'(k)$ close to 0, obtain for high amounts of capital; a large firm size favors market stability.

The equilibrium k^* is the unique solution of (4), which rewrites

$$1 + R + a\sigma^2 k^* = \frac{1}{2}\frac{A^2}{2Bk^* + C}. \tag{9}$$

The right-hand side of (9) gives the return that the firm is willing to pay for k^* units of capital. It can be viewed as the inverse demand of capital from the firm. The left-hand side instead corresponds to an inverse supply of capital, as it gives the return required for investors to offer k^*. This return is high when the return R of the safe asset is high, when investors are strongly risk averse (a is large) and fundamental shocks make profit and return highly volatile (σ^2 is large). The equilibrium in the financial market is depicted in Figure 1, with the capital inverse demand (in blue) and supply (in red) functions that appear in both sides of (9) denoted by $r^{\mathrm{d}}(k)$ and $r^{\mathrm{s}}(k)$, respectively.

Using (7), the stability condition (5) becomes

$$-r'(k^*) = \frac{A^2 B}{(2Bk^* + C)^2} < a\sigma^2. \tag{10}$$

Consistent with the partial equilibrium interpretation of the financial market in Figure 1, the left-hand side of this inequality is the (absolute value of the)

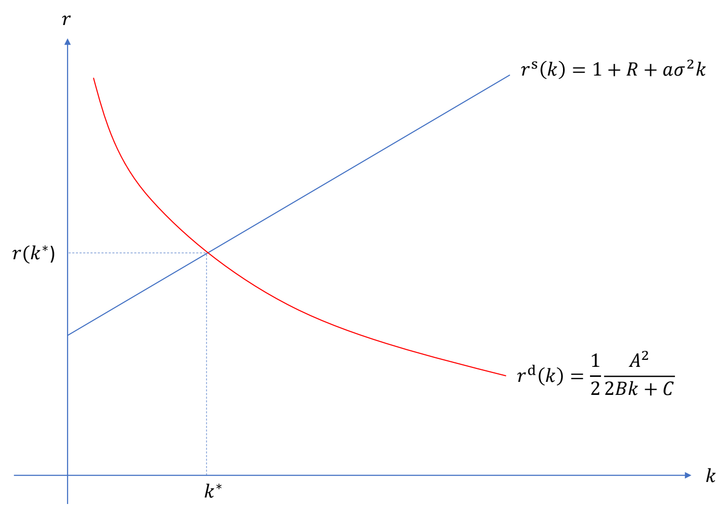


Figure 1: Equity market

slope of the inverse demand $r^{\mathrm{d}}(k)$ for capital, while the slope of the inverse supply $r^{\mathrm{s}}(k)$ of capital appears in its right-hand side. The stability condition thus holds if the (absolute value of the) slope of the inverse demand is smaller than the slope of the inverse supply. This reading makes the condition closely reminiscent of the one found by Guesnerie [16] in a competitive partial equilibrium setup.

The intuition for the stability condition is easy to grasp from Figure 1. A slope of the inverse demand close to 0 corresponds to a high sensitivity of the demand of capital to its return, which fixes the equilibrium return independently of the level of capital. A high slope of the inverse supply instead fixes the value of the equilibrium capital independently of the return. In both configurations investors can readily predict either the return $r(k^*)$ or the aggregate capital k^* itself. In Figure 1, these circumstances correspond to a flat inverse demand $r^{\mathrm{d}}(k)$, while the inverse supply $r^{\mathrm{s}}(k)$ instead moves toward a vertical line, at least close the equilibrium point $(k^*, r(k^*))$.

Combining (9) and (10) allows us to eliminate the endogenous equilibrium capital k^* from the stability condition. This leads to the following result:

Proposition 2. *Consider a monopoly producing q units of a final good at cost $Cq^2/(2k)$ when endowed with k units of capital. Suppose that it faces an inverse demand $P(q) = A - Bq$ in the good market. The equilibrium capital* *stock k^* is stable if and only if*

$$\frac{B}{C} < \frac{1}{2}\frac{a\sigma^2}{1+R}. \tag{11}$$

Proof. The stability condition (10) writes

$$k^* > \bar{k} \stackrel{\text{def}}{=} \frac{1}{2}\left(\frac{A}{\sqrt{aB\sigma^2}} - \frac{C}{B}\right).$$

The left-hand side of (9) is increasing in k. Its right-hand side is decreasing in k. Both sides are equal at k^*. Thus, $k^* > \bar{k}$ is equivalent to

$$1 + R + a\sigma^2\bar{k} < \frac{1}{2}\frac{A^2}{2B\bar{k}+C}.$$

Replacing $\bar{k}$ with its expression proves the result. □

Condition (11) shows that stability improves for a low safe return R, a high risk aversion and significant fundamental noise captured by $a\sigma^2$, and a low B/C ratio.

A low return R implies a high investment k^* into the risky asset. In the linear quadratic specification, demand of capital locally becomes sluggish; with $r(k)$ decreasing convex, $r'(k^*)$ gets close to 0 for high k^*.

There are two different channels through which $a\sigma^2$ influences stability. First, a higher $a\sigma^2$ corresponds to a steeper inverse supply, which favors stability. This also reduces investment k^* (which is this detrimental to stability) but this effect appears dominated.

The role played by B/C can be understood as follows. A high cost parameter C makes the slope of the inverse demand of capital close to 0; a sluggish demand is good for stability. It also implies a low capital value k^* at the equilibrium point but, as above, this effect is dominated. Finally, a low price sensitivity B of demand for the good produced by the monopoly translates into a high monopoly price: large profits lead to a high investment k^*, which is stabilizing.

3 Competition in the good market

We now extent the setup used in Section 2 to study the impact of competition in the good market on financial stability. Competition among several producers is associated with a multiplicity of risky assets, which allows investors to diversify better their portfolio.

3.1 Setup

There are $J \geq 1$ firms competing in the good market. The portfolio of investor i consists of the safe asset and J risky assets. Let k_{ij} be the amount of capital allocated to firm j by investor i. The total amount invested in the jth risky asset is

$$k_j = \int_0^1 k_{ij} \, \mathrm{d}i. \tag{12}$$

Let $\mathbf{k}_{-j}$ be the vector of aggregate capital invested into the $J-1$ firms other than j. The profit $\pi(\mathbf{k}_j) + \tilde{\varepsilon}_j k_j$ of firm j depends on the whole profile of investment where $\mathbf{k}_j = (k_j, \mathbf{k}_{-j})$. The random shocks $(\tilde{\varepsilon}_1, \ldots, \tilde{\varepsilon}_J)$ are i.i.d and every $\tilde{\varepsilon}_j$ is a Gaussian noise with mean 0 and variance σ^2. Firms are assumed to only differ in the capital they receive from investors and the realized shocks; the function π is the same for every firm.

We also assume:

Assumption 3. Firms' anonymity. *The function π is symmetric in $\mathbf{k}_{-j}$ (the value $\pi(k_j, \mathbf{k}_{-j})$ does not depend on the order of the components in $\mathbf{k}_{-j}$).*

The return of the capital invested into firm j is $\tilde{r}_j(\mathbf{k}_j)$ satisfying

$$1 + \tilde{r}_j(\mathbf{k}_j) = \frac{\pi(\mathbf{k}_j) + \tilde{\varepsilon}_j k_j}{k_j}.$$

It follows a Gaussian distribution with mean

$$\mathbb{E}[\tilde{r}_j(\mathbf{k}_j)] = \frac{\pi(\mathbf{k}_j)}{k_j} - 1 \stackrel{\text{def}}{=} r(\mathbf{k}_j) \tag{13}$$

and variance σ^2. There is zero correlation between the returns because the shocks $\tilde{\varepsilon}_j$ are assumed to be independent across firms.

In order to adapt Assumptions 1 and 2 to the oligopoly case, we consider the return $r(\mathbf{k})$ of firm j evaluated at $\mathbf{k}_j = \mathbf{k} = (k, \ldots, k)$. From firms' anonymity Assumption 3, the partial derivatives of the return of firm j w.r.t. the capital k_z held by any competitor $z \neq j$ are all equal when evaluated at $\mathbf{k}$. The two partial derivatives of its return w.r.t. its own capital k_j and w.r.t. another firm's capital k_z are denoted

$$r_1'(\mathbf{k}) = \frac{\partial r}{\partial k_j}(\mathbf{k}), \quad \text{and } r_2'(\mathbf{k}) = \frac{\partial r}{\partial k_z}(\mathbf{k}) \text{ for all } z \neq j.$$

Assumption 4. *The return $r(\mathbf{k})$ is non-negative, and it has negative derivatives, $r_1'(\mathbf{k}) < 0$ and $r_2'(\mathbf{k}) < 0$ for all $\mathbf{k}$.*

Assumption 5. *The return at $\mathbf{k} = (0, \ldots, 0) = \mathbf{0}$ satisfies $r(\mathbf{0}) > R$.*

The return of a firm decreases with its own capital. In addition, a higher capital allocated to its competitors penalizes it through a lower profit. As a result, the return $r(\mathbf{k})$ is decreasing in k, i.e.,

$$r_1'(\mathbf{k}) + (J-1)r_2'(\mathbf{k}) < 0$$

for all k. Table 5 in the empirical illustration in Section 4 suggests that Assumption 4 is met for most firms covered by IBES data.

A CARA investor i chooses a portfolio $(k_{i1}, \ldots, k_{iJ})$ maximizing

$$\sum_j (r(\mathbf{k}_j) - R) k_{ij} - \frac{a\sigma^2}{2} \sum_j k_{ij}^2$$

taking as given the distribution $(k_1, \ldots, k_J)$ of aggregate capital across firms. Her optimal investment into firm j thus is

$$k_{ij} = BR(\mathbf{k}_j) \overset{\text{def}}{=} \frac{r(\mathbf{k}_j) - R}{a\sigma^2}. \tag{14}$$

The right-hand side of (14) involves a prediction $\mathbf{k}_j$ of investors about the distribution of aggregate capital across firms. It is associated with an expected mean return $r(\mathbf{k}_j)$, which is in turn used for deciding which individual investments k_{ij} should be made. By summation over investors, the aggregate capital of firm j is

$$k_j = BR(\mathbf{k}_j).$$

The actual distribution of capital therefore relates to the distribution of capital $(k_1, \ldots, k_J)$ expected by investors. This is the form taken by the beauty-contest faced by financial investors.

A Nash equilibrium is a vector $(k_1^*, \ldots, k_J^*)$ satisfying

$$k_j^* = BR(k_j^*, \mathbf{k}_{-j}^*)$$

for every firm j. In such an equilibrium, the actual and expected distributions of capital coincide. In the sequel, we restrict our attention to symmetric equilibria, where every firm j ends up with the same amount $k_j^* = k^*$ of capital. The corresponding capital distribution is denoted by $\mathbf{k}^* = (k^*, \ldots, k^*)$.

Proposition 3. *Under Assumptions 4 and 5, there is a unique symmetric equilibrium. In this equilibrium, the capital stock k^* satisfies*

$$k^* = \frac{r(\mathbf{k}^*) - R}{a\sigma^2}. \tag{15}$$

Proof. The characterization (15) obtains from (14) where $(k_1, \ldots, k_J)$ is replaced with $(k^*, \ldots, k^*)$. By Assumption 4, the right-hand side of (15) is decreasing in k. Since the left-hand side is increasing in k there is at most one symmetric equilibrium. Existence follows from Assumption 5 and the fact that the left-hand side tends to infinity asymptotically. □

3.2 Financial stability

Suppose that each investor knows that for every firm j the aggregate capital k_j belongs to the interval $I_j(0) = [k_j^{\inf}(0), k_j^{\sup}(0)]$ comprising k^*, with $k_j^{\inf}(0)$ and $k_j^{\sup}(0)$ close to k^*. Reproducing the argument made in Section 2.2, all investors know after τ steps that k_j belongs to the interval

$$I_j(\tau) \overset{\text{def}}{=} [BR(\mathbf{k}_j^{\sup}(\tau-1)), BR(\mathbf{k}_j^{\inf}(\tau-1))] \cap I_j(\tau-1)$$

for every firm j and step $\tau \geq 1$. The sequence of intervals converges to $\mathbf{k}^*$ if and only if the best-response mapping $\mathbf{k} \to (BR(\mathbf{k}_1), \ldots, BR(\mathbf{k}_J))$ is locally contracting at $\mathbf{k}^*$. This is the case if the $J \times J$ Jacobian matrix $\mathbf{J}(\mathbf{k}^*)$ of this mapping evaluated at $\mathbf{k}^*$ has all its eigenvalues in the unit circle. In this case, the equilibrium is stable. Otherwise, it is unstable.

Lemma 2. *Under Assumptions 4 and 5, the Jacobian $\mathbf{J}(\mathbf{k}^*)$ has only two different eigenvalues,*

$$\lambda_1 = \frac{r_1'(\mathbf{k}^*) - r_2'(\mathbf{k}^*)}{a\sigma^2}, \quad \text{and } \lambda_2 = \frac{r_1'(\mathbf{k}^*) + (J-1)r_2'(\mathbf{k}^*)}{a\sigma^2}.$$

Proof. The Jacobian is the $J \times J$ matrix

$$\frac{1}{a\sigma^2}\mathbf{Dr}(\mathbf{k}^*)$$

with $\mathbf{r}(\mathbf{k})$ being the J-vector $(r(\mathbf{k}_1), \dots, r(\mathbf{k}_J))$ of the returns of the J firms. The anonymity Assumption 3 implies that $\mathbf{Dr}(\mathbf{k}^*)$ has every diagonal entry equal to $r_1'(\mathbf{k}^*)$ and every-off diagonal entry equal to $r_2'(\mathbf{k}^*)$. This special form implies that λ_1 and λ_2 are the two only eigenvalues of the Jacobian (λ_1 with multiplicity $J-1$ and λ_2 with multiplicity 1). □

Lemma 2 shows that the stability analysis relies on two baseline reallocations of capital across firms.

The $J-1$ eigenvectors associated with λ_1 have one component equal to 1, another component equal to -1 and the $J-2$ remaining components equal to 0. Each one corresponds to a deviation from the equilibrium distribution $\mathbf{k}^*$ where one unit of capital is transferred from some firm j to another firm j'. When investors expect such a deviation to occur, their actual investment into j and j' changes by $-\lambda_1$ and λ_1 respectively. This specific capital reallocation maintains unchanged the total resource allocated to risky assets, and yields an asymmetric firm size distribution.

The eigenvector associated with λ_2 is the unit vector $(1, \dots, 1)$. The expected deviation from $\mathbf{k}^*$ now consists of a one-unit additional investment into every firm. Investors react by reducing their investment in every firm by $|\lambda_2|$ units of capital (λ_2 is negative by Assumption 4). This reallocation respects the initial symmetry of the capital distribution and redirects investors resources from the safe asset to the risky firms.

Proposition 4 shows that the uniform expansion of capital driven by λ_2 is the only one to be relevant for stability.

Proposition 4. *Under Assumption 4,* $|\lambda_1| < |\lambda_2|$*. Therefore stability obtains if and only if* $|\lambda_2| < 1$*, i.e.,*

$$-\left(r_1'(\mathbf{k}^*) + (J-1)r_2'(\mathbf{k}^*)\right) < a\sigma^2. \tag{16}$$

Proof. Assumption 4 implies that $\lambda_2 < 0$. In addition, we have:

$$\lambda_2 < \lambda_1 \Leftrightarrow r_2'(\mathbf{k}^*) < 0,$$

and

$$\lambda_2 < -\lambda_1 \Leftrightarrow 2r_1'(\mathbf{k}^*) + (J-2)r_2'(\mathbf{k}^*) < 0.$$

Both inequalities hold true because of Assumption 4. This shows that $a\sigma^2\lambda_2$ is the eigenvalue of $\mathbf{Dr}(\mathbf{k}^*)$ with the highest modulus. The expression of λ_2 in Lemma 2 gives the result. □

Stability does not relate to portfolio reallocations that break symmetry in the capital distribution. The expected return changes by $r_1'(\mathbf{k}^*)+(J-1)r_2'(\mathbf{k}^*)$ when investors expect that one additional unit of capital will be invested in each firm. Proposition 4 appears as a straight generalization of the stability result obtained in Proposition 1 in the monopoly case: the sensitivity of the return to uniform capital expansions should not exceed the threshold $a\sigma^2$ for the equilibrium to occur.

Remark 1. In our setup firms are supposed to use somewhat passively the amount of capital chosen by investors. Appendix C considers firms that demand capital as a function of its unit cost $r(k)$. The extra flexibility due to capital demand elasticity makes financial stability more difficult to achieve.

3.3 Linear-quadratic specification

To account for the endogenous determination of the equilibrium capital, we return to the particular specification considered in Section 2.3, switching from a monopoly to the case of Cournot competition in the good market. The inverse demand for the good is $P(Q) = A - BQ$ where Q represents the aggregate quantity of goods. The production cost is $Cq^2/(2k)$ for every firm, with q the production of an individual firm.

Given k_j every firm j produces

$$q(Q_{-j}, k_j) = \arg\max_q (A - B(q + Q_{-j}))q - \frac{Cq^2}{2k_j} \tag{17}$$

where Q_{-j} is the aggregate production of firms other than j. For a given capital distribution $(k_1, \ldots, k_J)$, we compute the profit of every firm in the unique Cournot equilibrium in the good market. This yields the return $r(k_j, \mathbf{k}_{-j})$ of every firm j. It satisfies Assumptions 4 and 5 so that by Proposition 3 there exists a unique symmetric equilibrium. A detailed derivation is in Appendix A. The stability condition in Proposition 4 rewrites as follows:

Proposition 5. *There exists a function $\bar{\beta}(J)$ decreasing in J and taking values between 0 and $1/(2J)$ such that stability obtains if and only if*

$$\frac{B}{C} < \bar{\beta}(J) \frac{a\sigma^2}{1+R}. \tag{18}$$

In addition,

$$\frac{1+r(\mathbf{0})}{1+R} < 4(J-1)^2 \Rightarrow \bar{\beta}(J) = \frac{1}{2J}. \tag{19}$$

Proof. See in Appendix A. □

The condition (18) for financial stability appears as a slight variation on the one found when only the monopolist was raising capital: the only change is the new coefficient $\bar{\beta}(J)$ that replaces 1/2 in (11). It entirely captures the role played by the number J of competitors on stability. For plausible values of the returns, the ratio in the left-hand side of (19) should be close to 1, and thus $\bar{\beta}(J)$ is $1/(2J)$ for all $J > 1$. In this case, the fundamentals B/C, $a\sigma^2$ and R play as in Proposition 2.

The novel feature is the finding that an increased number of firms makes financial stability more difficult to achieve. This 'destabilizing competition' property is consistent with the idea found in the one-firm case that low market power (a high B) is destabilizing. In the financial market, an additional firm corresponds to a new risky asset, which allows investors to achieve better diversification opportunities. Such opportunities have the same flavor as a lower risk aversion a or a lower volatility of fundamental shocks σ in investors' behavior: what facilitates asset reallocations makes the capital distribution more difficult to predict.

Remark 2. Propositions 4 and 5 focus on the beauty-contest issue faced by investors assuming that the good market has reached an equilibrium. In Desgranges and Gauthier [13] we have analyzed stability of a Cournot equilibrium associated with a given (exogenous) arbitrary distribution of capital across firms. Appendix B shows that stability in the good market is met in the linear-quadratic specification associated with a uniform capital distribution.

4 An illustration from IBES data

We now document the link between the sensitivity of the return and prediction errors, and exploit it to disentangle fundamental and behavioral belief-based components of the equity market volatility. We rely on predictions of professional forecasters compiled in the IBES (Institutional Brokers Estimate System) database managed by the Wharton School, University of Pennsylvania.[2]

IBES information includes the net asset value (`nav`) and the return on equity (`roe`) of U.S. listed companies. The `nav` provides us with a measure of total equity of a firm, and will be used as a proxy for the capital invested by stockholders. The `roe` equals total net income divided by total equity. If one interprets the net income as the difference between the profit π and capital k, then the `roe` matches the return $r = \pi/k - 1$ of the theoretical model.

In IBES the predictions are available at the granular level of an individual analyst $\times$ firm $\times$ time. The analyst referenced 80474 for instance reported on October 29, 2015 a prediction of 9.4 per cent for the 2015 return on assets (the forecast period end is December 31, 2015) of the company Talmer Bancorp (TLMR). The data also records a realized return of 10.14 per cent over this period, so that the prediction error is 0.74 percentage point below the actual return. Predictions are collected from sell-side institutions (mostly brokers) while actual realizations are gathered from diverse sources, including press releases, company websites and public filings. We interpret the process iterated elimination of non best investment decisions as part of a short-run process and accordingly rely on analysts' forecasts formed within the same quarter as the forecast period quarter.[3]

The illustration proceeds as follows. Section 4.1 gives some descriptive statistics on IBES. Section 4.2 reports estimates of the sensitivity of the return obtained by regressing `roe` on `nav`. In Section 4.3 we match these estimates with prediction errors. We find a positive correlation between the two variables, which is consistent with the theory. In view of this correlation, Section 4.4 uses Propositions 1 and 4 to recover the standard error σ of

[2]Analysts' forecasts are commonly used in accounting and finance research as a proxy for investor beliefs. However, there is mixed empirical evidence regarding the correlation between these two variables. In the study by Gennaioli et al. [15], earnings forecasts provided by Wall Street financial analysts are found to be closely aligned with expectations formed by Chief Financial Officers of major US corporations. On the other end of the spectrum, Walther [24] instead suggests a low correlation between the two variables. According to Clement [10] investor beliefs would only partially adjust to analysts' forecasts; Ramnath [22] suggests that analysts who perceive investors as overestimating (underestimating) the precision of their forecasts could underreact (overreact) to new information.

[3]The IBES variable `fpi` is set to 6.

fundamental shocks.

4.1 Descriptive statistics

The good market refers to the finest available 10-digit Thomson Reuters Business Classification (`trbc`). The classification distinguishes 630 markets allocated to 10 large sectors defined by the coarsest 2-digit level of the classification: Energy (`trbc` 50), Basic materials (`trbc` 51), Industrials (`trbc` 52), Consumer cyclicals (`trbc` 53), Consumer non-cyclicals (`trbc` 54), Financials (`trbc` 55), Healthcare (`trbc` 56), Technology (`trbc` 57), Telecommunication services (`trbc` 58), Utilities (`trbc` 59).

We use all observations for the 20-year period 2002:2021, but most information is from 2009 onwards. There are $42,892$ company $\times$ time period observations with completed pairs of positive `nav` and `roe`.[4] They are from 453 out of 630 markets.

Table 1: IBES SECTORAL RETURNS ON EQUITY

2-digit `trbc` sector	Number of companies	Number of markets[b]	Average `nav`[c]	Average `roe`[d,e]	Number of analysts[e]	Number of predictions[e]	Average `roe` prediction[d,e]
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Energy (`trbc` 50)	284	27	84,430	14	166	7,872	23
Basic materials (`trbc` 51)	145	40	16,407	19	116	2,358	11
Industrials (`trbc` 52)	420	78	12,428	23	350	9,396	20
Consumer cyclicals (`trbc` 53)	406	117	10,104	23	273	9,644	16
Consumer non-cyclicals (`trbc` 54)	111	37	17,199	22	76	1,315	14
Financials (`trbc` 55)	933	55	77,962	11	696	103,119	10
Healthcare (`trbc` 56)	216	33	30,360	16	114	1,756	10
Technology (`trbc` 57)	526	50	39,458	20	354	12,887	17
Telecommunication services (`trbc` 58)	39	7	73,725	16	49	957	10
Utilities (`trbc` 59)	64	9	44,456	8	25	783	5

Notes:
b. 10-digit level of the Thomson Reuters Business Classification.
c. Net asset value expressed in millions USD. Mean of the sum of company `nav` per market and time period.
d. Expressed in per cent.
e. Data from the subsample of firm $\times$ time period with completed positive `nav` and `roe`.

Table 1 shows data broken down for the 10 sectors of the 2-digit classification. Telecommunication services and Utilities appear marginal in the data: they involve few companies operating in few markets and subject to low attention from analysts. To some extent, Consumer non-cyclical, Healthcare

[4] We keep IBES observations with a filled identity of the analyst, a filled predicted and actual value of `roe`, and a filled value of `nav`. We discard observations corresponding to forecasts formed after the public release of the actual value of the variable. A time period is defined by its forecast end period (the IBES variable `fpedats`).

and Basic materials resemble these two sectors. At the other extreme of the spectrum is Financials, a sector with a high amount of capital, relatively low returns, and receiving the greatest attention from the analysts.

Most returns stand above 15-20 per cent, a level which is usually considered as good by practitioners. In general the predictions are below actual realizations, with an under-estimation from 1 to 3 percentage points in sectors that analysts closely follow. The 1-point smallest difference is found for Financials.

4.2 Return sensitivity to capital

The sensitivity $r_1'(\mathbf{k}^*) + (J-1)r_2'(\mathbf{k}^*)$ is the change in the return following a marginal change in the aggregate capital distributed uniformly across firms. For each 10-digit `trbc` market m, the aggregate capital $\texttt{nav}_{mt}$ is computed as the sum of net asset values of all the companies subject to some prediction about the `roe` in period t. We denote by $\texttt{roe}_{jmt}$ the return of (`ticker`) firm j operating in market m at time t. An estimate of the sensitivity obtains from the specification

$$\texttt{roe}_{jmt} = \beta \log(\texttt{nav}_{mt}) + \texttt{time}_t + \texttt{market}_m + \varepsilon_{jmt}. \tag{20}$$

Time $\texttt{time}_t$ and market $\texttt{market}_m$ fixed effects are used to account for the static single-market framework used in the theoretical analysis. The estimate $\hat{\beta}$ gives the percentage point change in the return following a one-percent increase in aggregate capital. Table 2 reports OLS estimation results of (20). The return decreases and, given the log specification, it is convex with aggregate capital.

In the sequel we allow for greater heterogeneity in the sensitivities by estimating 10-digit `trbc` market-specific sensitivities (β_m) from the specification

$$\texttt{roe}_{jmt} = \sum_m \beta_m \log(\texttt{nav}_{mt}) + \texttt{time}_t + \texttt{market}_m + \varepsilon_{jt}, \tag{21}$$

The (unweighted) average of the $\hat{\beta}_m$ coefficients is -2.71 in the subsample of markets that excludes the bottom and top 2.5 per cent of the $\hat{\beta}_m$ distribution, which are considered as outliers. In this subsample, there are 56 over 429 markets where $\hat{\beta}_m$ is positive significant at the 5 per cent level, thus violating Assumption 1. This is especially concerning in Consumer cyclicals, with 20 affected markets over 117. Instead the correlation is positive in only 2 over 55 markets in Financials.

Table 2: Return sensitivity

	Return on equity $\texttt{roe}_{jmt}$			
	(1)	(2)	(3)	(4)
Net asset value $\log(\texttt{nav}_{mt})$	-1.437^{***}	-1.437^{***}	-1.372^{***}	-1.354^{**}
	(0.130)	(0.114)	(0.105)	(0.619)
Constant	31.283^{***}	31.283^{***}		
	(1.415)	(1.418)		
Observations	42,892	42,892	42,892	42,892
R^2	0.0028	0.0028	0.0075	0.0381
Standard error		Robust	Robust	Robust
Fixed effect			Time[a]	Time & Market[b]

Notes: *** (resp., ** and *) 1 (resp., 5 and 10) per cent level.
a. Forecast period end (`fpedats` IBES variable).
b. 10-digit `trbc` level.

4.3 Return sensitivities and prediction errors

We complement IBES information on predicted and actual returns of firm j operating in market m at time t with our estimated sensitivities $\hat{\beta}_m$. This yields a $149,565$ observation sample at the granular level of a prediction made by a given analyst a at time τ about the return of firm j operating in market m (defined at the 10-digit `trbc` level) at time t. For every such observation, we compute the relative prediction error

$$e_{jmt}^{a\tau} = \left| \frac{\mathbb{E}^{a\tau}\left[\texttt{roe}_{jmt}\right] - \texttt{roe}_{jmt}}{\texttt{roe}_{jmt}} \right|$$

where $\mathbb{E}^{a\tau}\left[\texttt{roe}_{jmt}\right]$ is the prediction made by analyst a at time τ on $\texttt{roe}_{jmt}$. The normalization by $\texttt{roe}_{jmt}$ is made to avoid size effects in the presence of heterogeneous returns across markets. All errors are computed as positive deviations of predicted from observed returns, as the theory does not address whether or not predictions exceed realizations.

Applying (20) to a market with J firms and aggregate capital $K = Jk$, we have

$$\beta = \frac{dr}{dK/K} = \frac{dr}{dk/k} = k\frac{dr}{dk} = \frac{K}{J}\left(r_1'\left(\mathbf{k}^*\right) + \left(J-1\right)r_2'\left(\mathbf{k}^*\right)\right).$$

In a market m with J_m firms, an estimate of the sensitivity can consequently be set to

$$\hat{r}'_{mt} = \frac{J_m \hat{\beta}_m}{\texttt{nav}_{mt}}, \tag{22}$$

where $\hat{\beta}_m$ obtains from the estimation of the market-specific variant (21).

Preliminary insights into the relationship between the sensitivity of returns and the prediction errors made by analysts can be grasped from the model

$$\log(e^{a\tau}_{jmt}) = \gamma \log(|\hat{r}'_{mt}|) + \delta \Delta^{a\tau}_{jmt} + \texttt{time}_t + \texttt{market}_m + \texttt{broker}^{a\tau}_{jmt} + \varepsilon^{a\tau}_{jmt},$$

where the additional control $\Delta^{a\tau}_{jmt}$ is the number of days (within a quarter) between the moment τ when the forecast $e^{a\tau}_{jmt}$ on the return of firm j is made and the forecast end period t when the actual roe is realized.[5] The $\texttt{broker}^{a\tau}_{jmt}$ fixed effect applies to the broker employing the analyst forming the prediction. It is used to control for analysts heterogeneity that is absent from the theoretical modeling. The coefficient of interest γ gives the change in per cent of the prediction error $e^{a\tau}_{jmt}$ following a 1 per cent increase in the sensitivity $|\hat{r}'_{mt}|$.

Table 3: Prediction errors and return sensitivity

	Prediction error $\log(e^{a\tau}_{jmt})$		
	(1)	(2)	(3)
Return sensitivity (γ)	0.055***	0.040***	0.039***
	(0.021)	(0.019)	(0.019)
Time to realization (δ)			0.028***
			(0.008)
Observations	142,086	136,258	136,258
R^2	0.202	0.199	0.199
Standard error	Robust	Robust	Robust
Cluster	Broker	Broker	Broker
Fixed effects	Time & Market & Broker	Time & Market & Broker	Time & Market & Broker

Notes: *** (resp., ** and *) 1 (resp., 5 and 10) per cent level.

Various estimates of this coefficient are reported in Table 3, with robust standard errors clustered at the broker level (the sampling design involves forecasts collected from sell-side institutions). We abstract from the top 5

[5] It may be more in line with the theoretical model to refer to the sensitivity $\hat{r}'_{m\tau}$ at time τ when the forecast is formed. This has little impact on the results since both t and τ refer to the same quarter.

per cent of observations with highest errors, which are considered as outliers. This yields a $142,086$ observation sample, which is used in Column (1). Columns (2) and (3) instead consider the subsample of markets where the sensitivity is not positive significant, which is in line with the theoretical model. High sensitivities go with high prediction errors: a 1 per cent increase in sensitivity is associated with a 0.04 per cent increase in prediction error. This accords with the view that high sensitivities of the return to capital make accurate predictions more difficult to achieve. Appendix E.2 provides additional information on the link between sensitivities and errors. It shows that the positive correlation is closely related to the inclusion of broker fixed effects. This may reflect the fact that the heterogeneity among brokers' forecasts leads to a form of compensation between their prediction errors.

4.4 Quantifying fundamental volatility

The theoretical model suggests two strategies to quantify the volatility of fundamental shocks. The first one exploits the fact that volatility only reflects fundamentals in an equilibrium and that observed and equilibrium returns more likely coincide if the equilibrium is stable. The second strategy recovers fundamental volatility measured by σ^2 from the specific form taken by the sensitivity threshold $a\sigma^2$ above which analysts make greater prediction errors. The implementation of these two strategies rely on the knowledge of this threshold.

4.4.1 IBES economy-wide estimates

We first assume that the threshold $a\sigma^2$ is the same for all markets and periods. Its value obtains from the estimation of the two-regime model

$$e_{jmt}^{a\tau} = \zeta \texttt{stab}_{mt} + \delta \Delta_{jmt}^{a\tau} + \texttt{time}_t + \texttt{sector}_s + \texttt{broker}_{jmt}^{a\tau} + \varepsilon_{jmt}^{a\tau} \qquad (23)$$

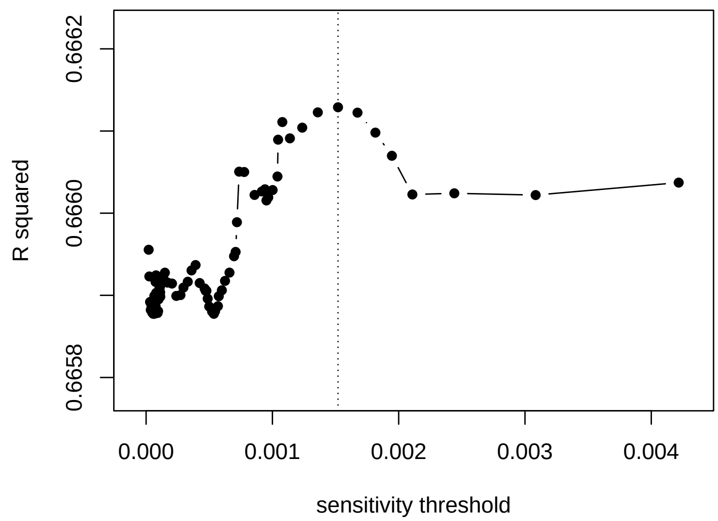
where $\texttt{stab}_{mt}$ is a dummy variable that classifies market m at time t as falling into a low versus high regime of prediction errors. The dummy is 0 if the sensitivity $|\hat{r}'_{mt}|$ is below some cut-off $\bar{r}$, and 1 otherwise. The choice of the cut-off is to be described below.

The ζ coefficient gives the additional error made in the regime where the sensitivity falls above $\bar{r}$. In our static theoretical model with a single final good market, no error is made in the low-sensitivity regime. More generally, if expectations are rational, errors should be non-predictable given information known when the forecast was made. Fixed effects in (23) are thought of as capturing such available information, possibly unobserved by the econometrician. They imply that prediction errors are typically non-zero in the regime where $|\hat{r}'_{mt}| \leq \bar{r}$, which makes (23) consistent with the finding that survey data often reject the assumption of non-predictable forecast errors (Bordalo et al. [6]). That is, given the fixed effects and the control $\Delta^{a\tau}_{jmt}$ for heterogeneity in the forecast horizon, the average forecast error is zero in the regime where the sensitivity falls below $\bar{r}$, hence with $\texttt{stab}_{mt} = 0$.

Specification (23) assumes that markets $\times$ time period observations with low sensitivities display no additional forecast errors beyond those specific to the quarter when the prediction is made, the 2-digit TRBC sector $\texttt{sector}_s$ where firm j operates, and the broker responsible for the prediction. If information circulates within the broker, it is appropriate to include a control for the broker rather than the financial analyst. It is not clear, however, whether the sector fixed effect accounts for information at the finer market m or firm j level. The general flavor of the results in this illustration remains unchanged in the variants performed in Appendix E.2 with finer fixed effects. These variants however suggest a critical role for the broker/analyst control.

We now come to the choice of the cut-off value $\bar{r}$. The OLS estimation of (23) is performed for $\bar{r}$ scanned over percentiles of the $|\hat{r}'_{mt}|$ distribution. The threshold $a\sigma^2$ is set to the value of $\bar{r}$ that yields the best fit of (23). Figure 2 plots the R-squared in the vertical axis against the percentile of the $|\hat{r}'_{mt}|$ distribution in the horizontal axis. The variant with the highest R-squared obtains for $\bar{r} = 1.519 \times 10^{-3}$, which corresponds to the 88th percentile of the $|\hat{r}'_{mt}|$ distribution. In this variant the estimate $\hat{\zeta}$ is 0.021 (t-value 2.265), i.e., prediction errors are 2.1 percentage points higher in the high-sensitivity regime where $|\hat{r}'_{mt}| > 1.519 \times 10^{-3}$.

A first approach to quantifying fundamental uncertainty relies on a strict interpretation of the theoretical model: fundamental volatility coincides with the empirical `roe` volatility in the markets $\times$ time period falling in the low-sensitivity (and low prediction errors) regime. The empirical `roe` standard



The horizontal axis reports the return sensitivities $|\hat{r}'_{mt}|$. The vertical axis reports the R^2 of OLS regressions where the dummy $\texttt{stab}_{mt}$ is built referring to every percentile of the sensitivity distribution. That is, the dummy is 1 if the sensitivity $|\hat{r}'_{mt}|$ computed in (22) is above the percentile, and 0 otherwise. We rely on the $136,385$ observation subsample excluding markets with a positive significant $\hat{\beta}_m$ and use robust standard errors clustered by broker. The dotted straight line corresponds to the percentile for which the highest R-squared obtains. The specification with the highest R^2 obtains for the 88th percentile, which corresponds to a value of 1.519×10^{-3}. Relying on this best fit specification we set the threshold $a\sigma^2$ to this value.

Figure 2: Selecting the sensitivity threshold

error is 7.436 in this regime,[6] so that one should set

$$\sigma = 7.436.$$

The empirical `roe` standard error is 11.107 in the high-sensitivity regime. The contribution of non-fundamental factors to volatility would thus be $11.107 - 7.436 = 3.671$, i.e., one-third of the observed volatility in this regime. Although 88 per cent of market $\times$ time period observations fall in the low-sensitivity regime, non-fundamental volatility remains significant in the whole $136,385$ observation sample, where the empirical `roe` standard error is 8.684. The difference $8.684 - 7.436 = 1.248$ still corresponds to 15 per cent of the observed volatility.

A second approach for assessing the role played by fundamentals exploits the expression of the threshold in Propositions 1 and 4,

$$a\sigma^2 = 1.519 \times 10^{-3}. \tag{24}$$

Using the median estimate $a = 3.4 \times 10^{-5}$ obtained by Cohen and Einav [11], the formula (24) leads to

$$\sigma^2 = \frac{1.519 \times 10^{-3}}{3.4 \times 10^{-5}}.$$

This gives a standard error $\sigma = 6.684$ close to the standard error of 7.436 found from the straight computation of the empirical `roe` standard error for the markets in the low-sensitivity regime. Both approaches thus do not seem inconsistent.

The variants of the specification (23) reported in Appendix E.2 suggest no substantial change in the share of market $\times$ time period observations that are assigned to the low-sensitivity regime if different fixed effects are used to control for the available information. However the close match between the recovered fundamental volatility from the two approaches no longer holds. The most significant changes follow from relying on firm or market granularity, rather than the final good sector, for controlling for information held

[6] The granularity of IBES data implies many observations on the same firm over the same forecast period, thus with the same realized `roe`. A straight computation on IBES data would thus yield an artificially low `roe` standard error in periods when a given firm is subject to many forecasts. To limit such biases, we first compute the average `roe` for every 10-digit TRBC market and forecast period year, and then the standard error of this average for every market. The reported standard error is a (unweighted) average of these (forecast period year) market standard errors.

on the entity subject to the forecast. Fine granularity leads to a fall of the share of market × time period observations in the low-sensitivity regime to one-third approximately. As analysts or brokerages often specialize in certain industry sectors 'of expertise' (Hwang et al. [17]), the sector granularity seems to be the most relevant in practice.

4.4.2 IBES sectoral estimates

To get insights into the heterogeneity of fundamental shocks across sectors, we have reproduced the same exercise as in Section 4.4.1 for each 2-digit TRBC sector separately. Column (1) of Table 4 follows the first approach by reporting the empirical `roe` standard error in the low-sensitivity markets for every sector. Column (2) gives the empirical `roe` standard error for all (low and high-sensitivity) markets in every sector. Column (3) follows the second approach where the fundamental standard error σ_s equals the ratio of the recovered sector s threshold and Cohen and Einav [11] risk aversion a estimate.

Table 4: SECTORAL FUNDAMENTAL VOLATILITY

2-digit `trbc` sector	`roe` standard error – stable markets only –	`roe` standard error – all markets –	σ_s	a_s
	(1)	(2)	(3)	(4)
Energy (`trbc` 50)	6.71	8.65	2.26	3.87×10^{-6}
Basic materials (`trbc` 51)	5.87	6.00	26.92	7.14×10^{-4}
Industrials (`trbc` 52)	22.89	10.59	1.96	2.49×10^{-7}
Consumer cyclicals (`trbc` 53)	13.86	10.85	23.55	9.82×10^{-5}
Consumer non-cyclicals (`trbc` 54)	7.20	7.54	39.52	1.02×10^{-3}
Financials (`trbc` 55)	2.20	4.53	0.70	3.40×10^{-6}
Banking services (`trbc` 551010)	4.05	4.15	5.63	6.57×10^{-5}
Investment banking and services (`trbc` 551020)	2.95	4.98	0.26	2.62×10^{-7}
Insurance (`trbc` 553010)	0.94	3.65	1.72	1.14×10^{-4}
Real estate operations (`trbc` 554020)	11.72	11.89	25.43	1.59×10^{-4}
Residential and commercial REITs (`trbc` 554030)	2.78	3.20	2.52	2.79×10^{-5}
Collective investments (`trbc` 555010)	3.82	3.70	11.37	3.01×10^{-4}
Healthcare (`trbc` 56)	6.15	12.42	0.61	3.29×10^{-7}
Technology (`trbc` 57)	9.03	8.08	7.01	2.04×10^{-5}
Telecommunication services (`trbc` 58)	6.52	6.61	25.93	5.38×10^{-4}
Utilities (`trbc` 59)	3.99	4.78	2.15	9.82×10^{-6}

Note: Values of σ_s in Column (3) use the mean (economy-wide) estimate $a = 3.4 \times 10^{-5}$ in Table 5 in Cohen and Einav [11]. REIT (Real Estate Investment Trust).

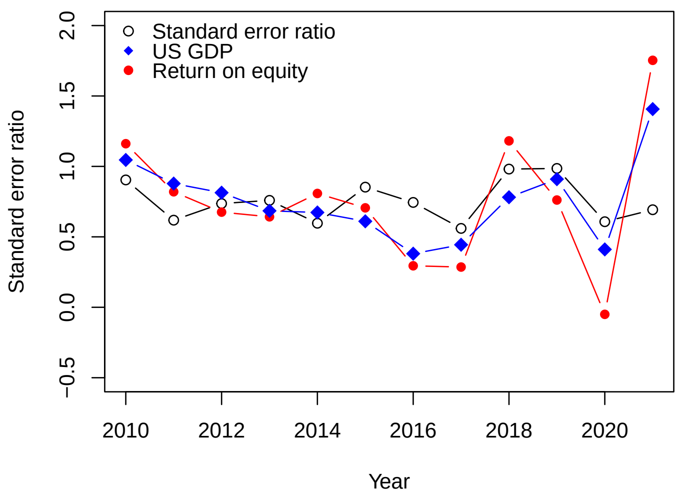
If fundamental volatility contributes to the overall sectoral volatility, the estimated `roe` standard errors in Column (1) should stand below those in Column (2). This is satisfied in all sectors but Industrials, and to a lesser extent Technology and Consumer cyclicals. In most sectors, fundamental volatility nearly coincides with the observed volatility. The main exception is Financials, which would drive the departures from rational expectations observed in the whole economy. Fundamental volatility in Financials would only represent half of the observed volatility. Reproducing the same exercise at the finer 6-digit TRBC level within Financials points toward Insurance and Investment banking and services as registering the greatest rationality mismatch.

The σ_s figures in Column (3) perform poorly for most sectors. They often stand above those in Column (2), and they exhibit weak correlation with those in Column (1). Column (4) assesses the assumption of a risk aversion a similar in each sector. An estimate of the sectoral risk aversion a_s obtains for every sector s by dividing the best fit sectoral $\bar{r}_s$ threshold with the squared figure in Column (1) for this sector, which is considered as as reliable measures of fundamental sectoral volatility. The resulting estimates are reported in Column (4). Their magnitude does not seem out of range when compared to those in Cohen and Einav [11]. There would exist significant heterogeneity in risk aversion across sectors. The figures suggest that analysts operating in sectors where non-fundamental volatility matters tend to display low risk aversion.

4.4.3 IBES annual estimates

The economy-wide estimates of fundamental volatility in Section 4.4.1 are partly driven by variability over time. We have performed the decomposition of the observed volatility for each year separately. For every year we recover the threshold $a\sigma^2$ and compute the ratio of the `roe` standard error in the stable markets (with a return sensitivity below this threshold) to the `roe` standard error in all markets. From 2010 there are between $9,768$ and $12,829$ observations in the $136,385$ observation sample every year; very few observations concern the before 2009 period. Figure 3 depicts in black the ratio of the two standard errors from 2010.

It also plots the (rescaled) detrended annual average return on equity (in red) and US GDP (in blue) from the Bureau of Economic Analysis. It shows a clear correlation between the three variables that suggests that non-fundamental factors especially matter during periods of economic slowdowns, i.e., of downward deviations of the GDP and returns from their long-run trends. This is consistent with the view in Bordalo et al. [8] that analysts are too pessimistic in bad times. Over the short-run (quarterly) forecast hori-



Black dots have in the vertical axis the ratio of the `roe` standard error in low-sensitivity markets over the `roe` standard error in all markets. It proxies the contribution of fundamentals shocks to observed volatility: it is 1 if the observed volatility is entirely fundamentally-driven. We recover a single threshold $a\sigma^2$ for each year t separately using the methodology described in Section 4.4.1. Low-sensitivity markets are those where the return sensitivity $|\hat{r}'_{mt}|$ stands below the threshold for year t. Standard errors are computed for every sector and year, and then averaged across sectors. Figure 3 also plots the deviations from the long run trend of the average return on equity in red and the GDP in blue for the United States. Both are rescaled so that only their evolution over time is meaningful.

Figure 3: Annual fundamentally-driven volatility

zon that we use, departures from rationality in bad times do not seem to come along with too optimistic forecasts made in good times. Instead, some behavioral asymmetry in line with Francis and Philbrick [14] would prevail, yielding volatility in more favorable economic conditions mostly fundamentally-driven.

5 Conclusion

Financial investors were considered as facing beauty-contest issues when they choose how much to invest into firms. We rely on the circumstances where they more likely succeed to solve this contest to provide a quantitative assessment of fundamental volatility. Fundamentals would be the main source of volatility in most markets; non-fundamental factors would matter specific markets of the financial sector, especially during periods of economic downturn.

Our theoretical benchmark makes strong homogeneity and symmetry assumptions. The finding that more competing firms reduce the likelihood of a symmetric equilibrium suggests insights into the potential impact of considering asymmetric equilibria. Indeed, if investors were to choose to invest their capital into the same firm, the resulting monopoly would present the highest chance of achieving a stable equilibrium. Asymmetry in firm size, viewed as similar to a reduced number of competitors, would favor the stability of the Nash equilibrium.

An intuition for destabilizing impact of competition goes through increased possibilities of portfolio diversification. Accounting for some correlation among the returns of different assets lessens possible diversification, which should contribute to stabilizing the equilibrium.

Finally, heterogeneity in risk aversions should destabilize the equilibrium. Indeed, in the CARA-Gaussian case, individual investment is a convex function of the coefficient of risk aversion a. By Jensen inequality this implies that the aggregate investment is more sensitive to expected returns in the economy where risk aversion is more dispersed.

Traditional tests for rational expectations are based on the assumption of non-predictable errors given the available information. However, it remains unclear what specific controls should be included to account for such information. In our empirical illustration, we rely on fixed effects to account for such information. However, it is possible that fixed effects may not effectively capture certain types of information, such as public releases of future policy news that would not enter time fixed effects, or market-specific information that is diluted within sector fixed effects. Volatility within the low sensitivity regime may therefore not accurately reflect fundamental volatility.

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Appendices

A Proof of Proposition 5

We consider the linear-quadratic specification of the model described in Section 3.3. For any given distribution of capital $(k_1, ..., k_J)$, we first compute the Cournot equilibrium and the associated profits. It is convenient to introduce auxiliary notations for

$$\phi_j = \phi(k_j) = \frac{Bk_j}{C + Bk_j},$$

and

$$\Phi = \sum_j \phi_j.$$

Lemma 3. *For any given distribution of capital $(k_1, ..., k_J)$, there exists a unique Cournot equilibrium. At the Cournot equilibrium, the profit of every firm j is*

$$\pi\left(k_j, \mathbf{k}_{-j}\right) = \frac{\left(1 + \phi_j\right)\phi_j}{2B}\left(\frac{A}{1 + \Phi}\right)^2. \tag{25}$$

Proof. Profit maximisation for firm j leads to

$$q_j = \phi_j\left(\frac{A}{B} - Q\right).$$

Summing over j gives

$$Q = \frac{A}{B}\frac{\Phi}{1 + \Phi},$$

and so

$$q_j = \frac{A}{B}\frac{\phi_j}{1 + \Phi}.$$

This shows that there is a unique equilibrium $(q_1, ..., q_J)$. The equilibrium profit is

$$\pi\left(k_j, \mathbf{k}_{-j}\right) = \left(A - BQ\right) q_j - \frac{Cq_j^2}{2k_j}.$$

Using the above values of q_j and Q proves the Lemma. □

Lemma 3 implies that the mean return of the capital of firm j is

$$1 + r(k_j, \mathbf{k}_{-j}) = \frac{\pi(k_j, \mathbf{k}_{-j})}{k_j} = \frac{(1+\phi_j)\phi_j}{2Bk_j}\left(\frac{A}{1+\Phi}\right)^2.$$

It is decreasing in both k_j and every k_z $(z \neq j)$. Hence Assumption 4 holds true. If every firm j is endowed with the same amount $k_j = k$ of capital, the return writes

$$1 + r(\mathbf{k}) = \frac{1}{2}\frac{A^2(2Bk + C)}{((J+1)Bk + C)^2}. \tag{26}$$

Therefore

$$1 + r(0, \ldots, 0) = \frac{A^2}{2C}, \tag{27}$$

and Assumption 5 rewrites as (8) in the single-firm case. Since Assumptions 4 and 5 are met, Proposition 3 shows that there exists a unique symmetric equilibrium. The equilibrium capital k^* is given by (15) and (26),

$$1 + R + a\sigma^2 k^* = \frac{1}{2}\frac{A^2(2Bk^* + C)}{((J+1)Bk^* + C)^2}. \tag{28}$$

By symmetry, the profit function $\pi(k_j, \mathbf{k}_{-j})$ has only two different partial derivatives evaluated at $(k^*, \ldots, k^*)$. The derivative w.r.t. to k_j is denoted π_1 and the derivative w.r.t. to any other component k_z is denoted π_2.

Lemma 4. *Let $\phi^* = \phi(k^*)$. The partial derivatives of $\pi(k_j, \mathbf{k}_{-j})$ evaluated at $(k^*, \ldots, k^*)$ are*

$$\begin{aligned}
\pi_1 &= \frac{C}{2B^2(k^*)^2}\left(\frac{(1+2\phi^*)(1+J\phi^*) - 2(1+\phi^*)\phi^*}{(1+J\phi^*)}\right)\left(\frac{A\phi^*}{1+J\phi^*}\right)^2, \\
\pi_2 &= -\frac{C}{B^2(k^*)^2}\frac{(1+\phi^*)\phi^*}{(1+J\phi^*)}\left(\frac{A\phi^*}{1+J\phi^*}\right)^2.
\end{aligned}$$

Proof. The expression (25) defines π as a function of $(\phi(k_1), \ldots, \phi(k_J))$. We have then (with straightforward notation)

$$\pi_1 = \frac{\partial \pi}{\partial \phi_j}\frac{d\phi}{dk}, \quad \pi_2 = \frac{\partial \pi}{\partial \phi_z}\frac{d\phi}{dk}.$$

Routine computations of these two derivatives give the expressions of π_1 and π_2 stated in the Lemma. □

Combining the above Lemma with the stability condition (16) in Proposition 4 gives

Lemma 5. *The equilibrium is stable iff*

$$\frac{2B\left(JC+\left(J+1\right)Bk^{*}\right)}{\left(C+\left(J+1\right)Bk^{*}\right)\left(C+2Bk^{*}\right)}\left(a\sigma^{2}k^{*}+1+R\right)<a\sigma^{2}. \tag{29}$$

Proof. We have

$$r_{1}=\frac{\pi_{1}k^{*}-\pi^{*}}{k^{*2}},\quad r_{2}=\frac{\pi_{2}}{k^{*}}.$$

The stability condition (16) writes

$$A^{2}B\frac{\left(J+1\right)Bk^{*}+JC}{\left(\left(J+1\right)Bk^{*}+C\right)^{3}}<a\sigma^{2}.$$

Using the equilibrium condition (28), we have the expression stated in the lemma. □

We can now prove the following proposition, which is a more detailed version of Proposition 5.

Proposition 6. *There is* $\bar{\beta}\in\left[\max\left(0,\frac{3-J}{2(J+1)}\right),\frac{1}{2J}\right]$ *such that stability obtains iff*

$$\frac{B}{C}<\bar{\beta}\frac{a\sigma^{2}}{1+R}. \tag{30}$$

We have $\bar{\beta}$ *decreasing in* J*. It is defined as follows:*

- *If*

$$\frac{A^{2}}{2C\left(1+R\right)}\geq4\left(J-1\right)^{2}$$

then, $\bar{\beta}$ *is the unique solution in* $\left[\max\left(0,\frac{3-J}{2(J+1)}\right),\frac{1}{2J}\right]$ *of the equation*

$$\frac{A^{2}}{2C\left(1+R\right)}\beta\left(\beta-\frac{3-J}{2\left(J+1\right)}\right)^{2}-\left(J^{2}-1\right)\left(\frac{1}{J+1}-\beta\right)^{3}=0$$

- *If*
$$\frac{A^2}{2C(1+R)} \leq 4(J-1)^2 \tag{31}$$
then $\bar{\beta} = \frac{1}{2J}$

Proof. Condition (29) rewrites
$$\left(J - 3 + 2(J+1)\frac{B}{C}\frac{1+R}{a\sigma^2}\right)\frac{B}{C}k^* < 1 - 2J\frac{B}{C}\frac{1+R}{a\sigma^2}. \tag{32}$$

We distinguish between 3 cases:

- If
$$2(J+1)\frac{B}{C}\frac{1+R}{a\sigma^2} < 3 - J,$$
then the LHS in (32) is negative and the RHS in (32) is positive. Condition (32) holds true.

- If
$$1 + \frac{1}{J} > 2(J+1)\frac{B}{C}\frac{1+R}{a\sigma^2} > 3 - J$$
then the LHS and the RHS in (32) are positive. Condition (32) rewrites $k^* < \hat{k}$ with
$$\hat{k} = \frac{1 - 2J\frac{B}{C}\frac{1+R}{a\sigma^2}}{\left(J - 3 + 2(J+1)\frac{B}{C}\frac{1+R}{a\sigma^2}\right)\frac{B}{C}}.$$
Using the equilibrium condition (28), the same argument as in Proposition 2 shows that the stability condition $k^* < \hat{k}$ rewrites
$$\frac{A^2}{2C(1+R)}\beta\left(\beta - \frac{3-J}{2(J+1)}\right)^2 < \left(J^2 - 1\right)\left(\frac{1}{J+1} - \beta\right)^3. \tag{33}$$
with $\beta = \frac{B}{C}\frac{1+R}{a\sigma^2} \in \left[\frac{3-J}{2(J+1)}, \frac{1}{2J}\right]$. The LHS is increasing in β and the RHS is decreasing in β. The condition holds true at the lower bound $\beta = \frac{3-J}{2(J+1)}$. The condition holds true at the upper bound $\beta = \frac{1}{2J}$ iff (31) holds true. Therefore,

  - either condition (31) holds true, and (33) is satisfied for every $\beta \in \left[\frac{3-J}{2(J+1)}, \frac{1}{2J}\right]$,

– or condition (31) does not hold true, and (33) is satisfied iff $\beta < \bar{\beta}$.

- If
$$2\,(J+1)\,\frac{B}{C}\frac{1+R}{a\sigma^2} > 1 + \frac{1}{J},$$
then the LHS in (32) is positive and the RHS in (32) is negative. Condition (32) does not hold.

Combining the 3 cases gives the stability conditions stated in the proposition. Lastly, differentiating the equation defining $\bar{\beta}$ shows that $\frac{d\bar{\beta}}{dJ} < 0$. □

B Stability in the good market

The linear-quadratic specification in Section 3.3 assumes that beauty-contest coordination issues only concern investors in the financial market while a Cournot-Nash equilibrium is reached in the good market. Given k_j and a prediction Q_{-j} made on the aggregate production of firms other than j, firm j chooses a production $q_j = q(Q_{_j}, k_j)$ where the mapping $q(\cdot)$ is defined in (17) in the main text. Firms thus face in the good market a problem analogous to investors: they need to predict the aggregate production of others. Given a distribution of capital $(k_1, \ldots, k_J)$, every firm j produces

$$q_j^* = q(Q^* - q_j^*, k_j), \quad Q^* = \sum_{\ell=1}^{J} q_\ell^* \tag{34}$$

in a Nash equilibrium. The Nash equilibrium quantities depend on the distribution of capital. Desgranges and Gauthier [13] give a condition for stability of the Cournot-Nash equilibrium in the good market for a given arbitrary distribution of capital. In the linear-quadratic specification of the model, when every firm uses the same amount k of capital, this condition reads

$$(J-3)\,\frac{B}{C}k < 1. \tag{35}$$

We are interested into the circumstances where (35) is met when k is set to the equilibrium value k^* satisfying (28).

Proposition 7. *Consider a Cournot-Nash equilibrium in the good market defined in (34) with $k_j = k^*$ for all j. This equilibrium is stable for every $J \leq 3$. For $J > 3$, it is stable if*

$$\frac{1 + r(0, \ldots, 0)}{1 + R} < 4\frac{J - 1}{J - 3}. \tag{36}$$

The right-hand side of (36) is increasing in J from a value of 12 for $J = 4$. In this specification, a higher number J of competitors is always detrimental to stability in the good market. In addition, as the ratio of the returns in the left-hand side of (36) should stand close to 1 in practice, thus much below 12, stability should always obtain in the good market.

C Capital demand

Let $c(q, k)$ be the cost supported by a firm to produce q goods from k units of capital. Its total cost include the capital cost. It equals $c(q, k) + (1 + r)\, k - \tilde{\varepsilon} k$. If the cost function is decreasing convex in capital (as is the case in the quadratic specification in Sections 2.3 and 3.3, the (interior) demand for capital of firm i obeys the first-order condition

$$c'_k(q, k) + (1 + r) = 0,$$

where $c'_k(q, k)$ denotes the partial derivative of the cost with respect to capital evaluated at (q, k). The production of the monopolist in the good market maximizes the average profit $P(q)q - c(q, k) - (1 + r)\, k$. If positive, it satisfies the familiar equality between marginal revenue and marginal cost,

$$P(q) + P'(q)q - c'_q(q, k) = 0.$$

These two first-order conditions yield an inverse demand for capital $r(k)$.

A financial equilibrium is still defined by a capital stock k^* satisfying the equality (4) in Section 2.3 but the supply from CARA investors must now meet this new demand $r(k)$, rather than the one given in (7). As a result, the equilibrium condition (4) in the linear-quadratic specification no longer expresses as in (9). It now writes

$$1 + R + a\sigma^2 k^* = \frac{1}{2}\frac{A^2 C}{(2Bk^* + C)^2}. \tag{37}$$

For the same reason, the condition for financial stability is still given by (5) in Proposition 1, but with the new demand function $r(k)$, it becomes

$$\frac{C}{B} > 2\frac{1+R}{a\sigma^2} + \left(\frac{A^2 C}{4B^2 a\sigma^2}\right)^{\frac{1}{3}}.$$

The comparison with condition (11) shows that an elastic demand of capital makes stability is more difficult to achieve.

D Sensitivity to own versus others' capital

In the main text we do not account for the difference between $r_1'(\mathbf{k}^*)$ and $r_2'(\mathbf{k}^*)$. This can be done by decomposing $\texttt{nav}_{mt}$ into $\texttt{nav}_{jmt}$ and the aggregate capital of firms other than j, which we denote by $\texttt{nav}_{-jmt}$. Table 5 shows estimation results for the enriched model where $\beta \log(\texttt{nav}_{mt})$ in (20) is replaced with $\beta_1 \log(\texttt{nav}_{jmt}) + \beta_2 \log(\texttt{nav}_{-jmt})$. There is a negative correlation between the return and both own and others' capital. The introduction of market fixed effects suggests that the return of a firm mostly relates to its own capital.[7]

E Variants with different sets of fixed effects

E.1 Prediction errors and return sensitivity

The correlation between prediction errors and return sensitivity in Table 3 is shown given time, market and broker fixed effects. Table 6 reports how the correlation is affected by the presence of these fixed effects. It highlights a crucial role for the entity making the forecast: the correlation obtains when accounting for the analyst or broker providing the forecast, but not

[7] The estimation in Table 5 uses all markets, independently of the number of competitors, by setting a value of 0^+ to $\texttt{nav}_{-jmt}$ if there is a single firm in market m. A possible interpretation for this procedure is that monopolies are potentially subject to entry of some small competitors. A different treatment of monopolies instead considers every market as closed, without any entry or exit of firms. In this case, sensitivities can be obtained by performing the estimation on two distinct subsamples. The first subsample consists of monopolies and a priori sets $\beta_2 = 0$. The second subsample has markets with several competitors. The (not-reported) results for this alternative treatment do not significantly differ from those in Table 5.

Table 5: OWN VERSUS OTHERS' CAPITAL

	Return on equity $\texttt{roe}_{jmt}$			
	(1)	(2)	(3)	(4)
Own net asset value $\log(\texttt{nav}_{jmt})$	-0.897^{***}	-0.897^{***}	-1.031^{***}	-2.072^{***}
	(0.175)	(0.341)	(0.351)	(0.643)
Others' net asset value $\log(\texttt{nav}_{-jmt})$	-1.452^{***}	-1.452^{***}	-1.370^{***}	0.068
	(0.135)	(0.114)	(0.092)	(0.044)
Constant	-8.949^{**}	-8.949^{***}		
	(3.525)	(2.320)		
Observations	42,892	42,892	42,892	42,892
R^2	0.0035	0.0035	0.0084	0.0401
Standard error		Robust	Robust	Robust
Fixed effects			Time	Time & Market

Notes: *** (resp., ** and *) 1 (resp., 5 and 10) per cent level.

otherwise. This property seems consistent with the reasoning process used in our theoretical analysis based on rationalizability, where a low sensitivity helps traders to formulate accurate forecasts. The results in Column (3) only differ from those in Table 3 by the control for the firm subjected to the prediction, which is the firm itself in Table 6 rather than the sector where this firm operates in 3. They show a similar magnitude of the correlation in the two cases. It looses precision with firm fixed effects, however, which may be due to the lower heterogeneity in return sensitivities at the firm level.

Table 6: PREDICTION ERRORS AND RETURN SENSITIVITY

	Prediction error $\log(e^{a\tau}_{jmt})$			
	(1)	(2)	(3)	(4)
Return sensitivity (γ)	0.020	0.013	0.032^{*}	0.029^{*}
	(0.043)	(0.019)	(0.018)	(0.017)
Time to realization (δ)	0.003	0.008	0.031^{***}	0.025^{***}
	(0.029)	(0.022)	(0.007)	(0.006)
Observations	136,258	136,258	136,258	136,258
R^2	0.112	0.189	0.267	0.285
Standard error	Robust	Robust	Robust	Robust
Cluster	Broker	Broker	Broker	Broker
Fixed effects	Time & Market	Time & Firm	Time & Firm & Broker	Time & Firm & Analyst

Notes: *** (resp., ** and *) 1 (resp., 5 and 10) per cent level.

E.2 Quantification of fundamental versus behavioral volatility

We have reproduced the same exercise as in Section 4.4.1 using variants of the specification (23) which differ according to the fixed effects that are accounted for. The first columns in Table 7 indicate the fixed effects entering (23). The last two columns report the best fit cut-off $\bar{r}$ (identified as the product $a\sigma^2$) and the share (in percent) of (10-digit TRBC level) market $\times$ time period observations falling in the low-sensitivity regime where volatility is fundamentally-driven.

Table 7: FIXED EFFECTS FOR UNPREDICTABLE ERRORS

	Time		Forecaster		Final good				
	Day	Year	Analyst	Broker	Firm	Market	Sector	Threshold	Percentile
1								1.197×10^{-6}	2
2	X							1.078×10^{-3}	85
3		X						1.078×10^{-3}	85
4		X	X					5.575×10^{-3}	97
5		X		X				1.078×10^{-3}	85
6		X		X	X			8.820×10^{-5}	40
7		X		X		X		8.819×10^{-5}	36
8		X		X			X	1.673×10^{-3}	90

Notes: Robust standard error clustered by broker.
All estimations are made on the 136, 258 observation sample (which does not include observations in the top 5 percent highest errors).
The forecast horizon control $\Delta^{a\tau}_{jmt}$ is always included.
For runtime reasons the results in line 9 obtain from scanning over 5 percentiles.

Line 1 of Table 7 shows that in the absence of fixed effects in (23) all markets are found subject to non-fundamental volatility only. This specification can be interpreted as a situation where forecasts would be formed without referring to the available information. In this respect, the finding of a negligible role of fundamentals suggests some validity for the classification procedure in two regimes based on (23) since then most forecasts would plausibly be subject to behavioral factors.

Lines 2 and 3 introduce time fixed effects, which are defined by the day in Column (1) and the year when the forecast is formed in Column (2). The granularity of time fixed effects is found to be irrelevant for classifying markets into low or high-sensitivity regimes. It may be that most information used for professional forecasters to provide predictions of returns on equity over a quarterly time period is of low frequency. For runtime considerations, the remaining variants in Table 7 are all based on the broad year-level granularity.

Columns (3) and (4) control for the entity making the forecast through, e.g., privileged information or idiosyncratic forecasting processes. Switching from the individual analyst to the broker implies no significant change in the volume of markets classified in the low or high-sensitivity regime, given time controls in Lines 4 and 5. Both controls point to a restricted influence of non-fundamental factors. This is consistent with a smooth within broker circulation of information held by analysts, a feature that is in line with empirical evidence about information sharing and spillovers within brokers (Hwang et al. [17]).

Columns (5) to (8) discuss the role played by the granularity of the control for the entity that is subjected to the forecast given year and broker fixed effects. In the data predictions are made on firms' returns. Our theoretical setup abstracts from firm heterogeneity and applies to some return common to every competitor in the market (defined at the 10-digit TRBC level) or sector (2-digit TRBC level). The results suggest a lesser role played by fundamental factors (proxied by a lower recovered cut-off value) when the control is narrowly defined, at the firm or market rather than the sector. Specialization of professional forecasters by industry sectors (Hwang et al. [17]) seems to make the sector granularity better grounded.

In general, the results in the last two columns point to three main alternatives: either (1) almost all markets are classified as out-of equilibrium (thus subject to non fundamental sources of volatility), or (2) about one-third of the markets fall in the stable equilibrium regime, or (3) all markets but 10 percent fall in this regime. The magnitude of the cut-off level is 10^{-2} or even 10^{-3} lower in the first two alternatives. These low magnitudes are difficult to reconcile with the absolute risk aversion in Cohen and Einav [11]. They also imply that the observed volatility in the low-sensitivity regime is inconsistent with the recovered standard error σ at the economy-wide level used in Section 4.4.1. The formulation used as a benchmark in the main text is similar to the one in line 8 of Table 7.