Random taxation and fiscal discrimination

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April 26, 2024

Introduction

- Various forms of ordeals are used to relax incentive constraints in the presence of asymmetric information.
- Interested in introducing random noise in the redistribution system.
  - Empirical side : rationing
- Hellwig (2007) benchmark: Random transfers designed to the poor relax IC if the rich ‘envy’ the poor. Socially useful only if the rich are more risk averse than the poor. Does not seem to be the relevant case.
- Still, a puzzling numerical example in Strausz (2004) where the mimicked should face random noise, though they are the most risk averse.
- This paper: in the same vein as Strausz (2004), with bunching and a reversal of incentives.

General framework

- A continuum of agents indexed by $\theta \in \Theta = \left[\theta^{\text{inf}}, \theta^{\text{sup}}\right]$, θ private information to the agent, cdf F with positive density f.
- Preferences $u(c, \theta) - y$, with (c, y) a pair of after and before-tax income.
  - u increasing in c and θ, and concave in c
  - Spence-Mirrlees condition $u_c'(c, \theta)$ is decreasing in θ.
    High types put less value on receiving the good.
    Does not fit standard versions of the optimal income tax setup.
  - Example: $u(c, \theta) = \ln(c + \theta)$, with θ some 'other' income.

Optimal Rawlsian policy

- A policy is a menu $(\tilde{c}(\theta), \tilde{y}(\theta))_{\theta \in \Theta}$ of income lotteries.
- The optimal policy maximizes

$$V(\theta^{\inf})$$

subject to

$$\int_{\Theta} \mathbb{E}[\tilde{c}(\theta) - \tilde{y}(\theta)] \, \mathrm{d}F(\theta) \leq 0 \tag{FC}$$

and

$$V(\theta) = \mathbb{E}[u(\tilde{c}(\theta), \theta) - \tilde{y}(\theta)] \geq \mathbb{E}[u(\tilde{c}(\tau), \theta) - \tilde{y}(\tau)] \tag{IC}$$

for all (θ, τ) in $\Theta \times \Theta$.

- A deterministic policy is a menu $(c(\theta), y(\theta))_{\theta \in \Theta}$ of certain incomes.
- No role for before-tax income randomness. Set $(y(\theta))_{\theta \in \Theta}$ deterministic.

Randomness and fiscal discrimination

- The optimal policy yields $V(\theta^{\inf}) \leq$ the one from the 'relaxed' policy where IC are replaced with to $V'(\theta) = \mathbb{E}\left[u'_\theta(\tilde{c}(\theta), \theta)\right]$ for all θ.
- A 'relaxed' policy yields a level of welfare

$$\int_\Theta \mathbb{E}[W(\tilde{c}(\theta), \theta)] \, \mathrm{d}F(\theta)$$

- Let $c^*(\theta) = \arg\max_c W(c, \theta)$ so that

$$\int_\Theta \mathbb{E}[W(\tilde{c}(\theta), \theta)] \, \mathrm{d}F(\theta) \leq \int_\Theta W(c^*(\theta), \theta) \, \mathrm{d}F(\theta).$$

- If $(c^*(\theta))_{\theta \in \Theta}$ satisfies IC, then the optimal policy is deterministic. That is, bunching in the deterministic optimum is necessary for useful (after-tax) income randomization. Fiscal discrimination.

Small income noise

- Suppose that each type gets (c^*, y^*) in the optimal deterministic policy.
- Randomize after-tax income to get the lottery $\tilde{c}(\theta) = c^* + \tilde{\varepsilon}(\theta)$ designed for type θ. Adjust y^* to $y(\theta)$.
- Restrict to 'small noise' where
  - realizations of $\tilde{\varepsilon}(\theta)$ are close to 0.
  - $\mathbb{E}[\tilde{\varepsilon}(\theta)] = \mathrm{var}[\tilde{\varepsilon}(\theta)] = \lambda v(\theta)$,
    $\lambda > 0$ close to 0 and $v(\theta) \geq 0$ bounded from above.

- The utility derived from the after-tax income is

$$\mathbb{E}\left[u\left(c^{*}+\tilde{\varepsilon}(\tau),\theta\right)\right] \simeq u\left(c^{*},\theta\right)+\lambda S\left(c^{*},\theta\right) v(\tau)$$

  where

$$S\left(c^{*},\theta\right)=u_{c}^{\prime}\left(c^{*},\theta\right)\left(1-\frac{A\left(c^{*},\theta\right)}{2}\right),$$

$$A(c,\theta)=-\frac{u_{cc}^{\prime\prime}(c,\theta)}{u_{c}^{\prime}(c,\theta)}.$$

- $S\left(c^{*},\theta\right)$ approximates the change in utility derived by type θ agent following a unit increase in her after-tax income.
  It may be thought of as a (marginal) valuation for the good.
- We have $S\left(c^{*},\theta\right)<1$
- For standard specifications, $S\left(c^{*},\theta\right)>0$ and $S_{\theta}^{\prime}\left(c^{*},\theta\right)<0$.
  True for, e.g., $u(c,\theta)=\ln(c+\theta)$. 'Economically' relevant case.
  In this specification, $A_{\theta}^{\prime}(c,\theta)<0$. Most favored = most risk averse.

Reversal of incentives

- Incentives require $V'(\theta) \geq 0$ for all θ.
- IC write: for all τ and θ,

$$\begin{aligned} V(\theta) \simeq u\left(c^*,\theta\right) + &\lambda S\left(c^*,\theta\right) v(\theta) - y(\theta) \\ &\geq u\left(c^*,\theta\right) + \lambda S\left(c^*,\theta\right) v(\tau) - y(\tau) \end{aligned}$$

$$\Leftrightarrow U(\theta) = \lambda S\left(c^*,\theta\right) v(\theta) - y(\theta) \geq \lambda S\left(c^*,\theta\right) v(\tau) - y(\tau)$$

Lemma 2. IC are satisfied if and only if

$$U'(\theta) = \lambda S'_\theta\left(c^*,\theta\right) v(\theta)$$

and $v(\theta)$ is non-increasing for all θ.

- Now, $S'_\theta\left(c^*,\theta\right) \leq 0 \Rightarrow U'(\theta) \leq 0$, so that $U'(\theta)V'(\theta) \leq 0$.

Welfare improving randomization

Proposition 1. The random menu improves upon the deterministic optimum if and only if, for $v(\theta)$ non-increasing,

$$\int_{\Theta} \phi(c^*, \theta) v(\theta) \, \mathrm{d}F(\theta) > 0$$

where $\phi(c, \theta) = S(c, \theta) - 1 - m(\theta) S'_{\theta}(c, \theta)$, and $m(\theta) = [1 - F(\theta)]/f(\theta)$ is the Mills ratio.

- The sign of $\phi(c, \theta)$ is ambiguous:
  - $S(c^*, \theta) - 1 < 0$
  - $-m(\theta) S'_{\theta}(c, \theta) \geq 0$ (if $S'_{\theta}(c, \theta) \leq 0$).

Intuition from the perturbation approach

- The tax paid by a θ agent is $y(\theta) - [c^* + \lambda v(\theta)]$.
- Since $U(\theta) = \lambda S(c^*, \theta)\, v(\theta) - y(\theta)$, the total collected tax is

$$\int_{\Theta} [\lambda S(c^*, \theta)\, v(\theta) - U(\theta) - c^* - \lambda v(\theta)] \;\mathrm{d}F(\theta).$$

- Consider a change $\mathrm{d}v > 0$ in $v(\theta)$ for all types between θ and $\theta + \mathrm{d}\theta$.
- Behavioral effect (takes $U(\theta)$ as given)
  - Every type θ directly concerned by the reform works more.
    This increases her before-tax income by $\lambda S(c^*, \theta)\, \mathrm{d}v$.
    The total tax resources increase by $\lambda S(c^*, \theta)\, f(\theta) \mathrm{d}v \; \mathrm{d}\theta$.
  - The higher variance goes with a higher transfer.
    This costs $\lambda \mathrm{d}v$ per agent, hence a total cost of $\lambda f(\theta) \mathrm{d}v \; \mathrm{d}\theta$.
  - Total tax change of $\lambda [S(c^*, \theta) - 1]\, f(\theta) \mathrm{d}v \; \mathrm{d}\theta < 0$, a net social cost.

- Mechanical effect (Adjust rents in $U(\theta)$ for incentive reasons)
  - $\mathrm{d}U'(\theta) = \lambda S'_\theta\,(c^*, \theta)\,\mathrm{d}v < 0$ for types directly concerned by the reform.
    Lower rents can be given to high types, implying a change in total collected tax equal to $-(1 - F(\theta))\lambda S'_\theta\,(c^*, \theta)\,\mathrm{d}v\ \mathrm{d}\theta > 0$.
- The sum of the two effects is $\lambda\phi(c^*, \theta)f(\theta)\mathrm{d}v\ \mathrm{d}\theta$.

Randomness must bear on the most favored

- By Proposition 1, useful randomization iff, for some $v(\theta)$ non-increasing,
$$\int_{\Theta} \phi(c^*, \theta) v(\theta) \, \mathrm{d}F(\theta) > 0.$$
- If $\phi(c^*, \theta)$ is decreasing in θ. Then satisfied iff $\phi(c^*, \theta^{\text{inf}}) > 0$. Then, for some $\theta^* > \theta^{\text{inf}}$, set $v(\theta) > 0$ for all $\theta < \theta^*$ and $v(\theta) = 0$ otherwise.
- More generally, relying on Myerson's (1981) priority rule, there is some non-increasing function $\bar{\phi}(c^*, \theta)$ such that:

  Proposition 2. The random menu improves upon the deterministic optimum if and only if
$$\bar{\phi}(c^*, \theta^{\text{inf}}) > 0.$$
  Then, there exists $\theta^* \geq \theta^{\text{inf}}$ such that the highest amount of extra taxes from after-tax income randomizations obtains by setting $v(\theta) > 0$ and non-increasing for all $\theta < \theta^*$, and $v(\theta) = 0$ for all $\theta \geq \theta^*$.

- Set $v(\theta) = v > 0$ for $\theta \leq \theta^*$, and $v(\theta) = 0$ otherwise. Useful randomization iff there is $\theta^* \in \left[\theta^{\text{inf}}, \theta^{\text{sup}}\right]$

$$\frac{1 - S(c^*, \theta^{\text{inf}})}{1 - S(c^*, \theta^*)} < 1 - F(\theta^*).$$

- The LHS is decreasing from 1 to some positive value, and the RHS is decreasing from 1 to 0.
- Never true at $\theta = \theta^{\text{sup}}$. All agents should never be exposed to risk.
- Sufficient condition: locally true at $\theta = \theta^{\text{inf}}$,

$$\frac{S'_\theta(c^*, \theta^{\text{inf}})}{1 - S(c^*, \theta^{\text{inf}})} < -f(\theta^{\text{inf}}).$$

  - $S(c^*, \theta^{\text{inf}})$ is high (close to 1) and $S'_\theta(c^*, \theta^{\text{inf}})$ is highly negative.
  - $f(\theta^{\text{inf}})$ is small (close to 0). In the uniform case, $\theta^{\text{sup}} - \theta^{\text{inf}}$ is high.
- The most favored agents like the good (captured by $S(c^*, \theta)$) while the others do not, and there is high dispersion in the 'other' income (captured by θ). Reminiscent of Weitzman (1977).

Non-uniform partial bunching

Proposition 3. In the case where there are $n \geq 1$ bunching intervals, useful randomization iff

$$\sum_{i=1}^{n} \int_{\bar{\theta}_i^*}^{\bar{\theta}_{i+1}^*} \phi\left(c_i^*, \theta\right) \, \mathrm{d}v(\theta) \, \mathrm{d}F(\theta) > 0$$

for some non-increasing profile of after-tax income variance $(\mathrm{d}v(\theta))$ close to 0.

Remark. Partial bunching. Proposition 3 also applies for $\bar{\theta}_{n+1}^* < \theta^{\text{sup}}$, i.e., in the absence of bunching at the top of the distribution. Then, one can set $\mathrm{d}v(\theta) = 0$ for all types that are not concerned by bunching, $\theta \geq \bar{\theta}_{n+1}^*$.

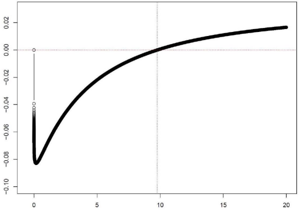
A parametric example

- $u(c, \theta) = \ln(c + \theta)$ and $F(\theta) = 1 - \exp\left[1 - (1 + \lambda\theta^b)^a\right]$ (Generalized Weibull distribution).
- For $a = 0.5, b = 0.05$ and $s = 0.5$, the optimal deterministic income tax schedule consists of a single income pair $(\bar{c}^*, \bar{y}^*)$ offered to every type $\theta \leq \bar{\theta}^*$ while all the other types are asigned the optimal relaxed income pair $(c^*(\theta), y^*(\theta))$.
- Find $\bar{c}^* = 4.02$ and $\bar{\theta}^* = 74.87$, with $F(\bar{\theta}^*) = 23.88$ percent,

$$c^*(\theta) = \frac{1 + (1 + 4m(\theta))^{1/2}}{2} - \theta$$

- Useful randomness if there is $\bar{\theta}$ such that

$$-F(\bar{\theta}) - \frac{1-F(\bar{\theta})}{\bar{c}^* + \bar{\theta}}\left(1 - \frac{1}{2\left(\bar{c}^* + \bar{\theta}\right)}\right) + \frac{1}{\bar{c}^*}\left(1 - \frac{1}{2\bar{c}^*}\right) > 0.$$



- Satisfied iff $\bar{\theta} \in \left[9.76, \bar{\theta}^*\right]$, with $F(9.76) = 7.33$ percent. Global maximum reached at $\bar{\theta} = 73.83$.
- randomness should concern between at least the bottom 7.33 and at most 23.88 percent of types.

Conclusion

- Possible to expand redistribution to the most risk averse agents through randomized taxes and transfers.
- Argument involves fiscal discrimination that is not possible in the deterministic case
- Deterministic options are offered to high types (those with the lowest risk aversion) while random bundles are designed for low types (those socially favored, with the greatest risk aversion).
- Trade-off between a social gain from a change in the direction of incentives, now aligned with the social objective, versus social cost from the risk faced by the most favored types.
- Provide various conditions for useful randomization, satisfied in a parametric example.