Rage against the matching: fairness and inequalities in a market design experiment of daycare assignments in France

Julien Combe[1], Arthur Heim[2]

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[1] Assistant Professor of Economics, CREST-École Polytechnique
[2] Research and evaluation officer, department of statistics, research and studies, National family allowance fund (Cnaf) PhD Candidate, Labour and public economics research unit - Paris School of Economics (EHESS). arthur.heim@cnaf.fr, arthur.heim@psemail.eu

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Abstract

The choice of childcare alternatives is a central decision which affects several key societal dimensions such as child development, mothers' labour supply, and economic and gender inequalities. While families from lower socioeconomic backgrounds tend to gain the most from formal childcare, these services are overwhelmingly used by more affluent families - a phenomenon often called the *"Matthew effect"*. In this paper, we consider access to daycare as a matching problem controlled by local authorities. Based on policymakers' definitions of the procedure, we use market design to define assignment mechanisms and we analyse the consequences of important design choices in a field experiment. The daycare assignment problem is similar to school choice, but includes two additional features: multidimensional constraints that cover weekdays and diversity constraints, typically age groups. Policymakers' design choices affect the definition and range of stable matchings. Our algorithms deliver student optimal *fair* assignments (SOFA) in the different versions of the problem. From 2020 to 2023, we assigned daycare slots to families in nine urban districts in France. Our objectives were twofold: i) to provide automated assignment mechanisms with desirable properties and ii) to introduce random variation in assignment probabilities for future work on causal effects of accessing daycare. We use two case studies to demonstrate that our assignments meet their intended objectives and compare different assignments. Using a change in priorities in one case study and counterfactual simulations of alternative priority scores, we show that i) giving larger weights to some group (e.g. dual earners) increases their assignment probabilities and share in daycares, ii) increasing priorities with time since registration strongly penalises single parents, in part because iii) dual-earner couples compete for early entry and strategically register as soon as possible, as incentives. iv) Being the largest group and also receiving high social weights, their strategies crowd-out parents who cannot register that early and create inequalities of opportunities even among strategic parents, correlating assignment probabilities with birth month. Other analyses show that diversity constraints may create sharp discontinuities in assignment probabilities that are unrelated to priority scores. Our results provide clear evidence of the mechanisms that contribute to the Matthew effects in childcare, and they are mostly political choices. However, our tools can be used to satisfy other distributional objectives and achieve higher transparency in the assignment process.

- **JEL Classification Numbers** : D47, A13, D82, D63, D78, A13, J13, I38, H42
- **Keywords**: Market design, fairness, childcare, daycare assignment, early childhood, inequalities, social investment, France

Résumé

L'accueil des jeunes enfants affecte plusieurs dimensions sociétales clés, telles que les inégalités de développement de l'enfant, la participation au marché du travail des mères, ainsi que les inégalités sociales, économiques et de genre. Alors que la recherche souligne que ce sont les familles issues de milieux socio-économiques moins favorisés qui bénéficient le plus des modes d'accueil formels, ce sont surtout les plus favorisées qui y ont recours - un phénomène souvent appelé *"effet Matthieu"*. Dans cet article, nous considérons l'accès en crèche comme un appariement centralisé organisé par les autorités locales. En nous basant sur les définitions du problème par les décideurs politiques, nous proposons des modèles pour définir des mécanismes d'affectation et leurs propriétés. Nous analysons ensuite les conséquences des choix de conception dans le cadre d'une expérience de terrain. Le problème est similaire au choix d'école, mais comprend des contraintes multidimensionnelles - comme les jours de semaine -, et des contraintes de diversité - typiquement des groupes d'âge. Les choix des décideurs politiques affectent la définition et la gamme des appariements stables. Nos algorithmes fournissent des affectations optimales équitables pour les familles (SOFA) selon la définition des décideurs. De 2020 à 2023, nous avons affecté les places en crèche de neuf larges collectivités territoriales en France. Nos objectifs étaient doubles : i) fournir des mécanismes d'affectation automatisés aux propriétés désirables et ii) introduire une variation aléatoire dans les probabilités d'affectation pour des travaux futurs sur les effets causaux de l'accès en crèche. À partir de 2 études de cas, nous montrons que nos affectations ont les propriétés attendues, et comparons les alternatives et ajustements en commission. Nous nous appuyons sur une réforme des priorités dans une étude de cas pour simuler des affectations contrefactuelles avec des scores de priorité alternatifs. Nous montrons que i) donner plus de poids à certains groupes (couples bi-actifs en particulier) augmente leurs probabilités d'affectation et leur part dans les crèches, ii) inclure l'ancienneté de la demande dans les priorités pénalise fortement les familles monoparentales, en partie parce que iii) les couples bi-actifs sont en compétition pour les entrées précoces et s'inscrivent stratégiquement dès que possible, conformément aux incitations. iv) Étant le groupe majoritaire et parmi les plus priorisés au départ, leurs stratégies évincent les parents qui ne peuvent pas s'inscrire aussi tôt et créent des inégalités d'opportunités même parmi les parents stratégiques, corrélant les probabilités d'affectation avec le mois de naissance. D'autres analyses montrent que les contraintes de diversité peuvent créer des discontinuités nettes dans les probabilités d'affectation qui ne sont pas liées aux scores de priorité. Nos résultats documentent clairement les mécanismes qui contribuent à un effet Matthieu dans l'accueil du jeune enfant, et ils sont principalement des choix politiques. Cependant, nos outils peuvent être utilisés pour satisfaire d'autres objectifs de distribution et atteindre une plus grande transparence dans le processus d'affectation.

- **Codes Journal of economic literature** : D47, D82, D63, D78, A13, J13, I38, H42
- **Mots clés**: Design de marché, petite enfance, crèche, inégalité, investissement social, France.

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> "*Many of us chose economics because, ultimately, we thought science could be leveraged to make a positive change in the world. There are many different paths to get there. Scientists design general frames, engineers turn them into relevant machinery, and plumbers finally make them work in a complicated, messy policy environment. [...] A feature unique to economics is that scientists, engineers, and plumbers all talk to each other (and in fact are often talking to themselves — the same economist wearing different hats)*" — (Duflo 2017, "The Economist as Plumber.").

> "*Whether economists will often be in a position to give highly practical advice on design depends in part on whether we report what we learn, and what we do, in sufficient detail to allow scientific knowledge about design to accumulate.*" —(Roth 2002,"The Economist as Engineer: Game Theory, Experimentation, and Computation as Tools for Design Economics")

This Chapter is co-written with Julien Combe. It is part of our long-term research project called ISAJE *Investissement social dans l'accueil du jeune enfant* joining efforts between French National Family Allowance Fund (Cnaf) École polytechnique and Paris School of Economics. This work would not have been possible without their financial, human and administrative support. We specifically thank Saad Loutfi for his great work on the various assignment waves and Jeanne Moeneclaey, Virginie Gimbert, Florence Thibault, Bernard Tapie, Lucie Gonzalez and Vincent Mazauric for their trust and support throughout the project. We also thank Olivier Noblecourt and Jean-Benoit Dujol for their political support. We owe a great deal to the *Conseil d'agglomération* of Valence Romans Agglo but above all to its Early Childhood Department, which allowed us to start this experiment, enabled this data analysis, promoted our work to other areas and gave us its unfailing confidence. Special thanks go to Julie Vivant and her team, with whom we had an excellent working experience. We also thank the early childhood department and incumbents of the other cities that participated in this ambitious experiment. We are grateful to Anne Boring, Karen Macours and Camille Terrier for helpful comments on a first version. Special thanks to Marc Gurgand for is constant and empowering support. We would like to thank Pierre Boyer, Antoine Bozio, Julien Grenet and Delphine Roy and the Institut des Politiques Publiques (IPP) - Paris School of Economics for providing free-of-charge additional staff during this experiment. Special credits and gratitude go to to Paul-Emmanuel Chouc and Vincent Verger (IPP), for their fantastic work on various assignment waves, Agathe Eupherte, Raja Ahmed Taleb and Marion Goglio for their remarkable research assistant work. We also thank the members of the ISAJE scientific committee for helpful guidance through this work and especially Orla Doyle and Sylvana Cote. This work greatly benefited from the careful proofreading and valuable feedback provided by Alex Galitzine, to whom we express our sincere gratitude. Final thanks go to Laudine Carbuccia, Quentin Daviot, Marine De Montaignac, Cécile Ensellem, Gautier Maigne, and many other colleagues and friends for helpful discussions and support over the years.

I Introduction

Public provision of daycare is a central tool for policymakers, particularly within the European Union. Early childhood care and education policies are framed as crucial social investments[1] aimed at breaking the cycle of poverty, addressing gender inequalities, enhancing social mobility, and boosting labour market productivity (Morel, Palier, and Palme 2012a; Van Lancker 2013; Kvist 2015; Hemerijck and Huguenot-Noël 2022). These goals rely primarily on the externalities of early childcare policies through both the technology of skills formation and parents labour market outcomes (Connelly and Kimmel 2003; Tekin 2007; Bauernschuster and Schlotter 2015; Zoch 2020; Doyle 2020). The positive impact of access to childcare services, particularly on children from disadvantaged backgrounds, has been documented in numerous studies[2]. However, the use of these services is skewed in favour of higher-income families (Lancker and Ghysels 2012; Petitclerc et al. 2017). This phenomenon is known in the social investment literature as the "*Matthew effect*" and raises important questions whose answers have different policy implications. Is the *Matthew effect* due to factors on the demand side ? Is-it because of local demographics ? Parents preferences ? Do they depend on cultural norms around parenthood ? Or is is because of supply-side constraints such as limited provision, costs or institutional settings ? So far, evidence suggests that supply side constraints play a larger role in explaining the Matthew effect than demand side factors (Farfan-Portet, Lorant, and Petrella 2011; Abrassart and Bonoli 2015; Pavolini and Van Lancker 2018 ; Carbuccia, Thouzeau, et al. 2023).

This study is about what lies in between: *how* parents access formal childcare, and what policymakers can do to effectively reach their ambitious goals. While there is a large and growing body of research on the benefits of early childhood interventions, the *engineering* of childcare policies does not always yield the high and long term returns science promised (Cantillon 2011; Ghysels and Van Lancker 2011; Vandenbroucke and Vleminckx 2011). Our main hypothesis is that the *Matthew effect* arises, partly at least, because markets for childcare are rationed and not properly designed to account for the externalities of childcare. Because of rationing and heterogeneity in the effects of childcare on different populations, "*who gets what and why*" matters (Roth 2015).

In this paper, we consider access to daycare as a matching problem where demand meets supply through a centralised assignment mechanism organised by local authorities. Based on policymakers' definition of the problem, our main research questions are:

To what extend is an optimal daycare assignment mechanism possible ? What are the consequences of different design choices ?

Our main contributions are twofold. First, we provide models and algorithms for different versions of the daycare assignment problem with well understood properties. Applications provide policymakers new tools to assign parents their most preferred daycare centre following priorities with diversity and multidimensional constraints. Second, we use two case studies to analyse the effects of diversity constraints, priorities and assignment mechanisms on inequalities in daycare access across different dimensions.

At the onset, we developed these models and collected these data to answer completely different research questions. Indeed, we wanted to run a randomised experiment to measure the effect of accessing daycare on children development, mothers' labour market participation and so on. However, one does not simply fully randomise access to formal childcare when assignment is centralised by local authorities[3]. In addition to the usual barriers to experiments, there are many different providers over which parents have preferences, local authorities are in charge of the assignment and have policy objectives of their own. There is simply no standard experimental design that easily accommodates these constraints. Instead, we followed Abdulkadiroglu et al. (2017) and made our *"research design meet market design"*: we modelled and automated daycare assignment procedures with an embedded

[1] The European agenda embraced the term in 2013 with the introduction of the "Social Investment Package" (SIP). This framework aimed to redirect Member States' social policies toward lifelong social investment. The SIP defined social investments as measures that "strengthen people's skills and capacities and enable them to participate fully in the labour market and society. Priority areas include education, quality childcare, healthcare, training, employment support, and reintegration" (Commission 2015). This definition underscores the core principles of social investment: investing in individuals' human capital and promoting labour market participation (Deeming and Smyth 2015)

[2] See for instance Nores and Barnett (2010), Barnett (2011) or Kholoptseva (2016). More recently van Huizen and Plantenga (2018) systematically synthesised quasi-experimental literature assessing the effects of access to childcare services between 2005 and 2017 across developed countries.

[3] Although it has been done in Rio (Brazil) and the effects of the reform have been studied by Attanasio (2022).

randomised experiment in the *plumbing*. By doing so, we minimised disruption to the ecological conditions of access to childcare, we garnered support from territories without encroaching on local policies, putting us in a unique position of *marketmaker* and *scientist* to better understand market structures, forces and consequences. Last but not least, demands with the same priority levels were randomly ordered in the assignment procedure, generating experimental variations for future causal analysis.

In 2019, we started a long-term research programme and partnership called ISAJE (Social Investment in Early Children's Lives and Education) with the aim of providing high-quality evidence on the effects of childcare policies. From 2020 to 2023, we convinced nine large urban municipalities and local authorities to participate in an experiment that would automate their daycare assignment procedure with random tie-breaks[4]. One territory stayed for four years and provided data for most of the empirical part of this research. Two others participated for three years, three stayed for two years, and three others for one year. Ultimately, approximately 20 000 demands went through our automated procedures for 5 000 daycare slots. Within these 20 000 demands, between 2 000 and 3 000 applicants were subjected to a lottery. The next steps of this research project will be to match these databases with monthly administrative records from the National family allowance fund (Cnaf) on all parents of children under 3 in the area and build an administrative panel for the next ten years.

In this research, it is important to note that our main goal was to be able to use the data for subsequent causal evaluations. Therefore, our theoretical contribution is incremental and mostly based on existing, well defined procedures. In practice, we met policy-makers, daycare providers, heads of early childhood departments in local councils and so on. We asked what they were trying to achieve with their assignment procedure and how they were proceeding. We noticed that the French daycare market had some familiar features. In particular, i) assignments are centralised by local authorities, ii) parents register throughout the year and submit preferences over daycare centres, iii) most offers are available in September when older children move to preschools and iv) local governments organise a main assignment committee during spring to assign these slots.

The daycare assignment problem is therefore very much like static school choice problems for which well defined solutions and many successful applications already exist (Abdulkadiroğlu and Sönmez 2003). In this literature, the goal is typically to define *stable* matchings and algorithms to find them. *Stability* is a property that we want a matching to respect in order to justify to parents *why* the assignment is as it is. The theoretical motivation for concentrating on sets of stable matchings is that if the market outcome is unstable, there are agents or pair of agents who have the incentive to circumvent the match (Roth 2002). In some instances, scholars and/or policymakers already use results from the market design literature to provide mechanisms for childcare assignment (see our review in section II).

However, the definition of stability highly depends on the constraints of the problem. As Kominers, Teytelboym, and Crawford (2017) note, "*successful market design solutions are bound to vary across markets because real-world settings have distinct (and sometimes unexpected) objectives, constraints, and trade-offs*". For a given market and definition of stability, a stable matching may not exist, or there can be several. There is a range of possible designs that can produce very different *outcomes*, *i.e.* in terms of the distribution of assignments, as well as a set of *consequences*. As Li (2017a) writes, a consequence is richer than an outcome, it is a "*description of the effects of the market on the world*". Designing marketplaces for daycare requires weighing and solving the trade-offs between different objectives and constraints, and models and theory help to make clear what (some of) those trade-offs are.

In our settings, many local institutions collect preferences over week days and assign files accordingly[5]. This simple change in the way preferences are collected have important consequences on the definition of stability. Policymakers are familiar with the details of their environment, and yet often do not know how to state their objectives in precise terms or realise they are conflicting. However, market design provides "*guidance without prescribing an entire ethical theory*" (Li 2017a). Many policy objectives depend on what is feasible. For instance, a policymaker might think that it is morally obligatory to implement a system that is Pareto optimal[6] for families

[4] Importantly, local authorities were contacted for this purpose. The research was about evaluating the impacts of childcare, not market design. Most of those who participated showed strong interest in these research questions and understood what causal evaluation required and accepted randomisation as a condition for participation.

[5] In the absence of specific days, one child takes one seat, which is an indivisible and unitary good. However, in settings with specific days, a student can take some days and the rest may be assigned to another student with complementary preferences, or not. A seat is no longer indivisible and unitary.

[6] In this context, a Pareto efficient assignment implies that the distribution of childcare slots cannot be improved in a way that benefits at least one participant without harming another participant.

and respect priorities. But this is not possible (Abdulkadiroğlu and Sönmez 2003) so it cannot be a moral or political constraint. When assignment is done by allocating slots over weekays, recent developments on matching with multidimensional constraints for childcare (Kamada and Kojima 2023) and refugee resettlements (Delacrétaz, Kominers, and Teytelboym 2023) provide different definitions of stability and algorithms to find stable assignments with other desirable properties. Importantly, these papers show that no algorithm can respect priorities[7] and be non-wasteful. Again, both cannot be achieved so it cannot be a policy objective. This recent literature provides well fitted theoretical solutions for the daycare assignment problem with multidimensional constraints.

However, real life implementation adds another layer of complexity. Indeed, another important feature in the French system is the existence of diversity constraints in all childcare centres and assignment procedures. Diversity constraints imply that capacities within daycare centres are divided into *buckets*, *i.e.* bundles of capacities with attached priority rules over *groups*. For instance, some capacities will be reserved for children aged 6-12 months only, and others for children aged 12 to 24 months. These diversity constraints can be *hard* when buckets only accept one group, like in the previous examples. Sometimes, constraints are *soft* and define a preference order over groups. For instance, a bucket may accept children aged 12 to 24 months but also older children if there is enough capacity to accommodate them. While the most common forms of diversity constraints are age groups, policymakers and/or daycare providers define many different forms of diversity constraints[8].

These diversity constraints play an important role in the lack of transparency in the daycare assignment procedures. Indeed, assignment committees do not simply sort files by priorities in a daycare, they consider assignment within buckets, which definitions vary within and across daycare centres, generating more or less competition across groups and thus, variations in assignment probabilities. In a given matching, there may be children with low priority scores assigned to a daycare that higher priority parents wanted but were rejected from. Thus, the stability notion cannot be solely based on priority scores, it must account for the partition of capacities into buckets and priorities within and across buckets.

Providing a stability notion in a case with multidimensional and diversity constraints is therefore both relevant theoretically and necessary to ensure transparency and due process.

Our theoretical contributions We define daycare assignment marketplaces (DAM) as three elements policymakers must choose :

1) A version of the problem : whether they consider demands with specific days or not ;
2) A partition of capacities in *buckets* i.e. bundle of capacities with attached priority rules in each daycare to define diversity constraints ;
3) A definition of *fairness* : with priority scores and whether the mechanisms should eliminate justified envy, tolerate some small deviations or consider initially feasible demands only.

Building on the work of Ehlers et al. (2014) in a case without day slots assignment, Kamada and Kojima (2023) and Delacrétaz, Kominers, and Teytelboym (2023) in a case with day slots assignment, we propose stability notions for different versions of the daycare assignment problem with diversity constraints. Our main result – presented in Theorem 2 – then states that for each DAM, we can find the unique student optimal fair assignment (SOFA), that respects the *chosen* definition of envy-freeness. We also provide an additional definition of stability over *initially feasible demands*. From a SOFA with all demands, we show in Theorem 1 that removing initially infeasible demands is weakly preferred by all parents and thus, Pareto-improves the assignment they receive. In practice, this theorem proves most useful in second rounds where there are many more empty days in buckets.

[7] with different definitions in the two papers.

[8] e.g. for parents whose schedules vary over the months or with late-night shifts, those in an active labour market policy involving childcare arrangement (*crèche AVIP*), quotas for children with disabilities, etc.

Field experiment The second part of our research focuses on the practical application of these automated daycare allocation mechanisms. In this study, we did not have a direct hand in defining the priority criteria, and in some instances, we were not privy to the exact methodology used to calculate scores. At the time of writing, confidentiality agreements prevent us from using all data. However, we have anonymised data sharing agreements with two local administrations. We primarily use data from one large urban centre for which we have four years of operation. Importantly, the datasets are rich enough for a comprehensive understanding of the criteria, weights, and formula for priority score, and encompass a reform of the latter in the middle of the time frame. This mechanism is based on weekday demands with flexible constraints over age groups. The second case study is only one assignment based on the school choice version of the problem. The datasets contain less informations but this setting is interesting for its large discrepancy in relative supply across age groups.

Our findings are based on four key empirical assessments strongly inspired by the 4-step auditing of algorithm proposed by Kasy and Abebe (2021). First, we examine the properties of our algorithms and compare the outcomes of various mechanisms with the final assignment. Second, we leverage a reform of priorities in the middle of the time frame of Case Study I to measure its effects and that of alternative priority rules on inequalities and segregation between social groups. Third, we investigate the impact of strategic registration and its consequences on inequalities by birth month. Fourth, we use data from Case Study II to demonstrate the critical role of bucket elements and distribution.

Using data from the 2023 rounds, we first start by showing that our assignments are indeed family optimal and envy-free and usually not *too wasteful*. Then, we compare outputs of different algorithms with the final offers sent to families. This year, the committee followed our proposed assignment in 92.3 % of all cases, so there are deviations. A significant share comes from the inability to ensure joint assignments of children from the same family in the same facility. This is one of the main drawbacks of this assignment procedure. However, these slight deviations go beyond grouping children of the same family: they unmatch and move few files to also increase the number of children assigned. We find this paradox particularly interesting because while we did offer to use a weaker definition of envy-freeness to increase the number of seated children, they refused and opted instead for hand-made deviations to achieve a similar goal.

In this market, policymakers took great care in defining a multi-criteria priority score accounting for children's and parent's social situations and needs, family structures and employment, in particular. But parents do not know precisely how it is built, nor the value of their own scores. The public documentation enumerates the main criteria including time since application and encourages parents to contact the early childhood department as soon as the pregnancy is official[9].

The importance of time since registration in priorities is such that any disturbance in the distribution of registration or timing of birth can have important effects on the distribution of assignments within and across social groups. In this experiment, the Covid-19 pandemic caused delays in both registrations and birth, reducing variations in priorities as well as the share of satisfied double earner couples. In reaction, policymakers boosted priorities of parents in employment such that in 2022, only 30% of single parents with no job had higher priority scores than the 10% dual earners with lowest priorities.

We use our tools to simulate assignments with the former weights, no priority for time since registration and no priority at all and compare average assignment change by social group from individual pair difference between actual and counterfactual assignment. These *what if* scenarii show that the reform worked: it increased the proportion of dual-earner couples and active single parents at the expense of other demographic groups and increased segregation within daycares. It also shows that without time since registration, conditional assignment probabilities across group would have been roughly the same, except single parents who are disproportionately hurt by this criteria. In this setting, the other weights mostly reduce assignment probabilities of mothers without a job. For instance, if there were no priorities in the 2023 round but only preferences, capacity constraints by age and random ordering of applicants, the assignment probabilities of single-earner couples and single parent with no job would be 12 pp higher, while dual earner couple's would be 7pp lower. Such simulations offer valuable insights for policymakers, facilitating the development of more equitable weighting schemes.

Because of labour constraints, dual earner couples often require and decide their childcare arrangement earlier. In this setting, they represent 66% of the demand and all receive the second highest social weights. Other criteria affect

[9] The declaration of pregnancy to the social security system is generally made by the doctor (general practitioner or gynaecologist) or midwife before the end of the 3rd month of pregnancy. Note that not all assignment committees allow parents to register that early.

few individuals and time since registration explains more than 3/4 of variations in priority. Early registration is a dominant strategy bounded by declaration of pregnancies. Consistent with these incentives, we observe massive bunching of registration 5 months before birth for dual earner couples. However, such a strategy is out of reach for parents whose situation changes later (separation, lay-offs, hiring and so on) or simply for those who think of childcare later during their pregnancy. Consistently, there is no early bunching among single parents and other couples. While these two features already strongly favour stable couples in employment who anticipate their needs, policymakers give dual earner couples the second highest social weight, and down-weight families where at least one does not work, further accentuating social inequalities in daycare access.

Moreover, waiting for the decision of the assignment committee requires alternative childcare arrangement until they are accepted, and outside options if they are not, especially for active parents. Paradoxically, the more they wait, the more likely they are to be seated and the more likely they are to refuse the committee's proposition. Ultimately, congestion is artificially inflated by parents incentivised to register early and anticipate their needs months before birth and very far away from accessing daycares. By the time they receive an offer, many have changed their mind, or wait for a complete choice set to make a decision. These parents impose negative externalities on rejected less prioritised parents and cause part of the market to unravel.

Because pregnancies occur throughout the year but assignment committees occur in April, early-registration does not have the same pay-off for strategic parents who get pregnant in spring or autumn. The conditional assignment probability by registration month depends on the density of applications, which is lowest in the summer months, mostly for children expected in the last quarter. Therefore, conditional on registration, children born in December have the highest assignment probability of all.

However, other rules may also create variations in assignment probabilities by birth months which may favour children in other seasons. In particular, the definition of the bucket plays a pivotal strategic role in shaping assignment distributions. In Case Study II, most seats are only open for children born after the 1st January. Using a regression discontinuity design (Calonico, Cattaneo, and Titiunik 2014), we show that this sharp difference in available supply across age groups creates a sharp discontinuity in the assignment probabilities, but not in priority scores or density of the forcing variable. Our findings indicate that the probability of assignment for registered parents is heavily influenced by the synchronisation of birth dates, but through distinct mechanisms, here precisely reversed. In this setting, time since registration did not affect priorities, but policymakers supplied fewer seats for children born in the previous calendar year, As a result, children born in the first two quarters had higher assignment probabilities, while those born at the end of the year experienced much lower probabilities. In the first case study, unequal access was mostly caused by strategic early registration by the largest group, which increased their priority and resulted in higher assignment probabilities for children born in the last quarter who were registered around three months after conception. Conversely, children born in the first two quarters experienced lower assignment probabilities.

Policy implications Thinking of the market structure for childcare helps understanding *why* the assessment of universal childcare policies doesn't always yield the anticipated outcomes and *how* the socio-economic gap in early childcare enrolment occurs. In our examples, the Matthew effect is clear and stems from political choices on key design elements. We discuss this aspect more thoroughly in Section VI.

Once policymakers define the elements of the DAM, we can provide parents with their favourite choice among assignments, all the while respecting priorities and diversity constraints following the definition of *fair* assignment. However, this definition of *fairness* is very narrow (Kasy and Abebe 2021). It only encompasses *procedural justice*, *i.e.* due process and equal treatment of individuals. Yet the perception of justice encompasses other aspects and in particular, *distributional justice*, *i.e.* the "*allocation of positive and negative outcomes in a decision context and whether they are distributed equitably or deservedly amongst the affected population given their circumstances, performance or contributions*" (Binns et al. 2018). Perceived levels of justice of a decision outcome are separate from purely self-serving rationalisations of a decision outcome; *"an individual might be negatively affected by a decision whilst still thinking it is just"* (Binns et al. 2018). Conversely, if priority rules are not perceived as fair, then neither will the assignments that respect them (Fenech and Skattebol 2019).

The initial experimental phase provided an opportunity to demonstrate the relevance and effectiveness of automated procedures for different versions of the daycare assignment problem. They can also be used to offer new possibilities.

For instance, it is easy with buckets to allow parents in a daycare to try and change for another one without derogatory rules These solutions have proven effective in automating allocation committees while ensuring a realistic but controlled environment for a "natural" randomised experiment. However, the willingness to participate in the research does not reflect the general acceptability of market design solutions. In this experiment, the automation often occurred with minimal information provided to families, primarily for reasons related to the scientific integrity of the research protocol[10]. However, the *official* adoption of our DAM is likely to elicit different reactions from families and childcare providers. This point relates to an important aspect of market design: *repugnance* (Roth 2007; Satz 2010).
Disgust for certain types of transactions can pose a genuine constraint on markets and their design, similar to technological constraints or requirements for incentives and efficiency. In the early childhood care market, there are transactions that are considered unethical, such as attempting to win favours with elected officials in order to be selected. Ensuring envy-free assignments is a way to reduce such repugnant transactions[11].

In certain contexts, some market structures may be more objectionable than others. The very idea that access to childcare can be determined by something other than a human choice can be particularly startling to some people. In general, the combined use of priorities and lotteries in an automated process may be seen as a departure from traditional decision-making mechanisms, potentially challenging established trust, power dynamics and political considerations. Parents of different social background have different views on fairness and social justice (Hvidberg, Kreiner, and Stantcheva 2022).

By ensuring procedural justice, our daycare assignment mechanism moves fairness concerns over *distributional justice*. Ensuring transparency may only be feasible if the definition of the elements of the DAM align with societal values and reflect social justice principles[12]. Importantly, the definition of buckets is part of the definition of envy-freeness and relative supply constraints may be more important than priority scores in their distributional effects. When we initiated the project, we faced strong resistance from the French Mayors' Association (AMF). They saw this as a potential interference in their rights and power as elected representatives. Others objected to the project because they feared that automating processes would put jobs in local early childhood services at risk or that daycare providers would lose their bargaining power.

The adoption of such technology is therefore not merely an issue of optimisation, efficiency, or even transparency: it implies considerations of equal opportunities, social justice, and the functioning of democracy. In this regard, scaling up raises the question of societal reception to the adoption of a new technology and the conditions under which acceptance, mistrust, or rejection may occur. Before thinking of scaling up, perhaps the first question a democracy should ask is: "*who gets to pick the objective function of an algorithm?*" which is intimately connected with the political economy question of who has ownership and control rights over data and algorithms (Kasy and Abebe 2021; Schmauder et al. 2023; Albright 2023; Bohren, Hull, and Imas 2022; Kasy 2023).

The rest of the paper is structured as followed. In section II, we briefly describe the French market for childcare and review the international literature on daycare assignment. Section III presents the experimental framework and main features of our two case studies. Section IV presents our models for daycare assignment mechanisms. We start with a simple school choice model, then introduce multidimensional constraints with solutions from Kamada and Kojima (2023) and Delacrétaz, Kominers, and Teytelboym (2023) and finally we define daycare assignment mechanisms with diversity constraints. Section V is our empirical analysis of a daycare assignment mechanism over four years. We conclude section VI on the implications of our work and future perspectives.

[10] Parents were informed that the city hall took part in a research on early childcare as bare minimum.
[11] We had informal feedbacks from some civil servants of several places that direct interventions from elected officials were dramatically reduced since the adoption of our assignment procedure.
[12] For a debate in feminist economics on these issues, see *e.g.* Albelda, Himmelweit, and Humphries (2004) or Thomson (2009).

II Families, policies and markets for childcare

II.1 The market for childcare in France

The French system offers parents a range of support measures, including parental leave, tax credits for childcare spending, family benefits, publicly or privately funded daycare centres, and subsidies for childminders. In 2020, the French government spent € 6.6 billion for collective childcare, € 4.6 Billion for individual care (childminders), € 1.7 billion for tax credit (Ishii et al. 2023). If all of these resources were fully utilised, centre-based childcare could take 471,000 children (20% of children under three), and childminders 744,000 (33% of children under three). Overall, the total coverage rate of childcare for children under three is about 60%, and the total direct public spending on childcare in 2020 was €14.3 billion. In Appendix A, we describe the main types of available childcare services along with the aids for families with children under 3.

Early-childhood policies Formal childcare services are accessible to children as early as three months of age, and preschool education has been made mandatory by the age of three years in 2019[13]. Table A.2 presents the evolution of the main childcare arrangements for children under 3 from 2002 to 2021 based on representative survey reported in Caenen and Virot (2023). 20 years ago, 70% of children were mainly cared for by parents but now, almost half of all children under three are in other childcare arrangements. Childcare centres saw their share grow, now serving 1/5 of children under 3. There is a strong rationing in access to centre based childcare, 40% report daycare as their favourite choice but only 25% access one (Laporte, Crépin, and Hilairet 2019). Childminders face fewer constraints, with 1/3 of families considering them their favourite and the same proportion using their services. Moreover, 40% of families combine various solutions, while 56% rely on parental care, essentially mothers out of the labour force.

Local authorities manage access to public daycares The process of allocating daycare slots is primarily overseen by local authorities, with limited regulation or guidance on the assignment process. The law only mandates higher priority for applicants with disabled children, those in social and professional integration processes, and families referred by social services[14]. For the majority of applicants, each municipality determines their own criteria. Decisions are typically made by committees comprising elected officials, daycare managers and civil servants from early-childhood services.

Allocation committees function as centralised institutions that bring together applicants and daycare providers, operating as marketplaces designed to match supply and demand. While prices, or childcare costs, are relevant factors in these marketplaces, they do not solely determine daycare placements. Decision-making involves not only applicants but also local agencies and childcare providers, who have their own preferences regarding which children to accept. Parents also have alternative options, such as private childcare or taking care of their children themselves. To our knowledge, Herman (2017) is the only researcher, besides ourselves, who analysed the daycare assignment procedures in France. She reports three monographs and compare the different organisations, justifications, and perceptions from various types of agents involved, including policymakers, social workers, parents, and more. Local policy choices determine what is considered a priority beyond the legally defined criteria. Herman identifies two contrasting approaches: *formalism* in wealthier areas and *need-based* assignment in other areas.

The institutional setting, available information, and the complexity of forms and procedures have a significant impact on the smooth operation of markets (Pais, Pinter, and Veszteg 2011; Chen and He 2021). For parents, finding suitable childcare can be a costly and stressful process (Schüller and Steinberg 2022). The complexity of application processes may create optimisation frictions, influencing individuals' ability to make informed decisions, and either raising or lowering administrative barriers. Different information sets or media convey signals that can nudge some parents in or out of the applicant pool. In the childcare market, experiments show that detailed information and guidance and/or human support for applications have large positive impacts on registration and access for low-income families[15] (Weixler et al. 2020b; Zangger and Widmer 2020; Hermes et al. 2021, 2022).

[13] although before the law, 96% of children aged 3 where already in preschool.

[14] *article L. 214-7 du Code de l'action sociale et des familles.*

[15] This motivated another experimental research project joint with Carbuccia, Barone, et al. (2023), whose details are available on https://www.socialscienceregistry.org/trials/9901.

Priority scores and political signals In local administrations, time and efficiency constraints necessitate the use of simple and quick evaluation rules, typically in the form of priority scores. In the face of increasing demands, heterogeneous situations from registration to daycare entry, setting priority rules becomes complex. In 2018, the government assigned the French Mayor Association (AMF) to providing "*a framework for transparent and fair assignment of daycare seats*"[16]. The reports recommend to "*choose relevant priority criteria based on shared assessments, aligning them with the region's specifics*." Elected officials can endorse a charter to commit to these guidelines. However, they lack details on the allocation process, leaving implementation questions unanswered. The recommendations remain vague, with limited normative guidance or political accountability.

Most surprisingly, the assignment procedure *i.e.* *how* to assign daycare seats once priorities are set is barely mentioned in the AMF's report[17]. The only part that refers to assignment procedures is about computer tools[18]. And yet, it is well known that there can be many assignments that respect priorities with very different welfare implications. For instance, the student-proposing or college-proposing deferred acceptance algorithms (Gale and Shapley 1962) find stable assignments, but the first is the best outcome for all students and the latter is the worst. The first one is strategy-proof for students while the second is not. The choice of assignment method is crucial, and each method has its advantages and disadvantages. Transparency is a necessary condition for *fair* assignments but by no means a sufficient one.

Because there are no clear guidance on assignment mechanisms, priority scores appear as the only available tool to regulate daycare access. They further serve as political signals, informing voters about early childhood policy implementation through the prioritisation or deprioritisation of certain groups. Neimanns (2022) explores the political economy of social investment and analyses the link between childcare preferences and voting behaviour. Using survey data from in eight European countries, his main analysis models the link between childcare spending support and voting intentions by income, conditional on other measures of political values and a set of context fixed effects. He shows that the higher up the income distribution individuals are, the more tightly voting behaviours connect to preferences towards childcare. Therefore, although lower-income individuals might be the strongest supporters of additional public childcare spending, left-wing and right-wing political parties have incentives to target reforms or define priority rules favouring more affluent voters because those voters' preferences translate more directly into actual votes. Moreover, recent work on politicians' perception of voters' preferences show that most politicians strongly believe voters' preferences are more conservatives than they actually are (Pilet et al. 2023). This result is true across all political groups, and may further amplify the previous implications. Finally, subsidies depend on household incomes and richer parents pay larger fees, adding another incentive to favour more affluent families.

Priorities also signal to parents their chances of obtaining a seat and affect the composition of the demand (Ünver, Bircan, and Nicaise 2018). For many territories, they are of paramount importance, and some have taken great care in their definition. The design of a priority score depends on the selected characteristics and their associated weights. Families may have multiple combinations of these characteristics, making the political dimension of selecting these features and assigning their weights particularly salient. However, in other territories, the priority score is less explicit. It usually serves as an internal tool for implementing a policy, indicating the considered criteria but not how they are considered.

Ultimately the daycare assignment procedures lack transparency and means to ensure due process of all applicants. Despite the undeniable shortage of daycare slots, perceived opaque or convoluted assignment procedures can exacerbate the perception of a more severe scarcity. This can erode trust in the procedure – even if it is entirely regular – and can lead to dissatisfaction among parents, tension between parents and administrators, and unnecessary social and political costs.

[16] The mission led by Élisabeth Laithier (2018) suggests the establishment of committees for fair assessments, informing families about timelines, encouraging their participation, and providing tailored support.
[17] The dedicated section insists on a clear calendar, on organising assignment committees.
[18] Laithier (2018): Pages 10 and 11: "*The use of decision-support tools may therefore be appropriate, if it can leave room for human expertise. […] However, the use of these tools would not be sufficient for the exhaustive allocation of daycare slots, insofar as they would not be able to grasp the specific nature of each situation, and could leave out families who do not meet the criteria*". Authors translation.

II.2 Market designs of childcare marketplaces

Economists tend to adopt fairly expansive definitions of what counts as a transaction and (thus) what constitutes a market. In general, they are the institutions that organise transactions. Markets can be designed and their features affect "who trades, what is traded, and the terms of those trades" (Li 2017a). Roth (2018) defines marketplaces as "*infrastructures, rules, and customs through which information is exchanged and transactions are made [that] can be relatively small parts of large markets. Participants may have large strategy sets, i.e., many options available to them beyond those available in any particular marketplace*".

Market Design combines theoretical and empirical methods to build, analyse, and enhance such institutions (Roth 2018). It has led to various applications and policy successes over the past two decades[19]. The daycare assignment problem is theoretically and empirically close to the school choice problem for which there are well defined solutions and many large-scale applications. Their success are tightly linked to the theoretical properties of assignments which in practice, convey useful normative values.

Market design offers normative criteria Li (2017a) discusses ethics in market design and argue that "*Market design needs value judgements: we seek to design markets that function well, not badly, and 'well' and 'badly' have normative content*". He advocates for what he calls *informed neutrality*, *i.e.* using criteria from the market design literature to formalise many small value judgements. Typically, a matching that respects priorities is said to be *envy-free* which is often interpreted as a *fair* assignment in the literature. *Envy-freeness* facilitates *transparency* because it makes it easier to justify each individual assignment following priorities. Mechanisms that make it a dominant strategy to reveal true preferences are called "*strategy-proof*", because "participants don't have to make strategic calculations about what others are doing, they just have to decide what they like" (Roth 2018). Strategy-proofness is desirable from a normative perspective because it "*levels the playing field*" by preventing strategic players from improving their assignment at the expense of non-strategic players (Pathak and Sonmez 2008). Yet, even when a mechanism is strategy-proof, it may not be *obviously strategy-proof* (Li 2017b). Information and transparency affect participants' strategies. Details on mechanisms that make strategy-proofness salient increase truth-telling (Guillen and Hakimov 2018) but perception of risk may lead some participants to still try to game the system although there is nothing to gain (Hassidim, Romm, and Shorrer 2018). Conversely, strategies may be set up outside of the marketplace or through other markets. When place-based priorities are in place, residential mobility can also be part of parents strategies (Bjerre-Nielsen et al. 2023).

These notions of fairness take the objective of the algorithm's owner or designer as a normative goal. That is, Mayors and early-childhood department owns the power of defining the objective function of the algorithm. In our paper, we call them *policymakers* and emphasise the attached responsibility coming with that power. For Kasy and Abebe (2021), fairness provides a framework to critique the unequal treatment of individuals i with the same *'merit'*, where merit is defined in terms of *social weight*. They discuss three limitations of fairness-based perspectives under a procedural notion of justice: "*they legitimise inequalities justified by merit, rather than questioning the status quo; they are narrowly bracketed and do not adequately engage with the impact of algorithms on pre-existing inequalities; and that they do not consider within-group inequalities, leading to intersectional concerns*".

Merit is an important *narrative* to justify inequalities and priorities but 1) views on *deservingness* vary and can be changed (Stantcheva 2021; Hvidberg, Kreiner, and Stantcheva 2023) and 2) this narrative often serves "*merit as reward*", while more economic definitions would emphasise *merit as effectiveness*" (Durlauf 2008; Sethi and Somanathan 2023). Eliaz and Spiegler (2020) develop a model for competitive narratives, which she defines as a "*causal model that maps actions into consequences, weaving a selection of other random variables into the story*". Policymakers' narrative can be seen as a causal model, and may have testable implications. In the case of algorithmic decision, Kasy and Abebe (2021) propose to use causal inference and social weights to formally assess the fairness, equality and balance of power of algorithmic decisions. We take a similar approach in section V when we analyse the effects of different elements of priorities on inequalities.

[19] e.g. assignment of resident doctors to hospitals (Roth 1984), the matching of children to schools (Abdulkadiroğlu and Sönmez 2003) and teachers to schools (Combe, Tercieux, and Terrier 2022), social housing (Waldinger 2021), Kidney exchange programs (Akbarpour et al. 2020) and so on.

Daycare assignment mechanisms borrowed from the school choice literature Childcare allocation mechanisms vary across countries, reflecting diverse approaches. When assignment are centralised, research has drawn inspiration from the school choice literature, applying related findings to daycare allocation. For instance, Carlsson and Thomsen (2015) simulate daycare assignment using the student-proposing deferred acceptance algorithm (SPDA) on German data (Gale and Shapley 1962). Similarly, Herzog and Klein (2018) consider three German daycare marketplaces which are decentralised and sometimes uncoordinated. They propose dynamic versions of the *immediate acceptance* mechanism (Also called the *Boston algorithm*). Reischmann, Klein, and Giegerich (2021) implemented dynamic versions of SPDA specifically designed for decentralised contexts, and provides empirical evidence from two cities. Veski et al. (2017) evaluate the welfare effect of using a centralised assignment mechanism using SPDA instead of the previous decentralised system in Harku, Estonia. Kennes, Monte, and Tumennasan (2014) analyse the daycare assignment mechanism in Aarhus, Denmark, which is common to most Danish municipalities, including Copenhagen. The market is such that the oldest unassigned child is given high priority in a daycare where no current capacity restriction exists - a concept called "child care guarantee". Children who are already assigned have the highest priority in those places in the subsequent period. Apart from age, some characteristics give higher priorities, such as children with special needs, siblings in the same daycare, immigrant children in need of special assistance in daycare.

Some settings use lottery-based assignment procedures which researchers leveraged as natural experiments to evaluate childcare policies and allocation processes. For example, Rio de Janeiro reformed its daycare market in 2008, introducing a lottery system to enhance transparency and Attanasio (2022) shows this shift positively influenced labour earnings and child development. Similar lotteries have been employed in Oslo, Norway, involving randomised slot assignments (Drange and Havnes 2015), and in Bologna, Italy, where applicants express preferences and are allocated slots based on a Family Affluence Index (Ichino, Fort, and Zanella 2019). However, being focused on evaluation research questions, these research provide little details on the assignment procedures.

New approaches to centralised assignment Kamada and Kojima (2023)[20] propose a theoretical framework for matching markets under multidimensional constraints. In their study, they employed data from two municipalities in Japan to simulate various scenarios to assign childcare with their algorithms. Market designs are not only applied to school choices and sometimes research on a different topic can be used to solve problem in others. In this case, we found a clear analogy between the recent work by Delacrétaz, Kominers, and Teytelboym (2023) on refugee resettlement and the daycare assignment problem. Indeed, both involve navigating multidimensional constraints related to family characteristics and needs (such as the number and age of children), the availability of suitable facilities across different locations, and the establishment of fair prioritisation criteria. We discuss both models in details in section IV and build our main results from theirs.

[20] At the time we began, a working paper was available at Kamada and Kojima (2019).

III A multi-site market design experiment

The following section presents the setting of our ongoing experiment and the data we use in the empirical framework. The project started in spring 2019 where we first actively engaged with local representatives and childcare service providers to gain a comprehensive understanding of how daycare assignment processes were structured in diverse settings. We organised group meetings, visited about twenty city-halls, interviewed professionals at various levels and observed the proceedings of two assignment committees. We started assigning daycare seats for the first time in April 2020 and recruited new cities over the years, guided by sample size requirements for causal evaluation of heterogeneous treatment effects. The project is ongoing and because of that, our data access are provisionally limited (See details in Appendix B).

In this paper, our analyses focus on two large urban centres, chosen for two reasons: first, one consider preferences over week-days, while the other does not. Second, their data and characteristics are well-fitted to analyse the effects of the main features of these markets: the roles of priorities and diversity requirement in daycare accessibility inequalities.

III.1 Case Study I: A large marketplace with demands over weekdays and diversity constraints

Most of the analysis focuses on data from Valence Romans Agglo (VRA or Case Study I hereafter), an urban community in the Drôme department, southeastern France. It involves 54 municipalities and 223,630 inhabitants in 2020, half of which living in the two main cities[21].

A) Context

This marketplace provides access to 33 collective daycare centres, known as "*multi-accueils*". Families can apply through two central registration points (*guichets uniques*). Additionally, they propose two *crèches familiales* where accredited childminders provide care. Figure C.13 displays a map of the area, plots the daycares and the density of demands in 2023. It shows that this marketplace is very wide[22] and daycare centres are mostly located in the two largest cities and close suburbs. Most demands also come from these urban centres. Our collaboration with VRA spanned from spring 2020 to 2023, covering four years of operation.

In this setting, parents can register throughout the year, but the assignment committees are conducted in two rounds, typically in April and May for entries in September. At the end of March, the first round[23] of assignments takes place. A small allocation committee then reviews the proposals and makes any necessary adjustments. Families are offered a first-round assignment or placed on a waiting list. Assigned parents must promptly confirm their registration by scheduling appointments with their designated daycare centre. A second allocation round is held to fill any remaining vacancies, including those from the first round and those that became available due to withdrawals or changes in scheduling. This list includes families who were rejected in the first round, those who declined the first-round offer but remained on the waiting list, and new applicants. In September, a small committee meets to update any vacant placements and allocate them to families at the top of the waiting list, if possible. The specific procedures for these ad-hoc committees are not publicly available. Last, the early childhood department also has an emergency (temporary) process in place for parents who suddenly require childcare arrangements due to unforeseen circumstances such as divorce, violence, or job changes. Additionally, they offer occasional care services.

Our experiment started in 2020 and we ran assignments for four years. Obviously, 2020 was not the easiest year to start off with. First, there were local elections in March that prevented current incumbents to sign any contract over the next term. Then, the COVID-19 pandemic broke-out, delaying (among other things) the second ballot. However,

[21] https://www.insee.fr

[22] Going from the centre of Valence to the centre of Romans-Sur-Isère is a 30 min car trip or 40 min train.

[23] A small number of high-priority files do not participate in this marketplace. These files typically involve parents or children with specific needs, such as disabilities, social support or surveillance from child protection services, or underage parents. Social workers meet before the assignment committees to determine how to handle these special cases. Only after this pre-assignment round do we know the capacities to be assigned.

the *Communauté d'Agglomération* was not affected by the elections and maintained its commitment to the project just as the pandemic was beginning. Paradoxically, this crisis period really helped to prove the usefulness of our automated assignment as it freed civil servants in the early childhood department from this tedious task in a moment where their time was a scarce resource. Despite the pandemic *per se* and working remotely, they were in charge of organising childcare for *essential workers*, helping parents in distress, dealing with unemployed childcare workers and more. Furthermore, organising the assignment committee remotely was technically difficult at that time. This first success then convinced other municipalities to join the programme and the early childhood department provided positive testimonies to other municipalities.

B) Data and notable features

Data and main variables of interest We use two sets of data:

- Family datasets: they contain households and children IDs, reported preferences, priority score and sociodemographic variables used in its formula. These tables also contain results of the assignment, propensity scores and assignments from alternative simulations and approximate GPS coordinates[24].
- Supply datasets: They contain daycare ids and, number of seats for each age group over weekdays and age limits. It indicates which age group accepts children from the older or younger groups.

The family datasets contains one line per demand, and there are several demands for the same child and from the same family for different children in different assignment waves. We stack all family datasets and build our main variables of interest: age at registration, social groups and distances to reported daycares. The committee defines 5 social groups defined as dual earner couples, single earner couples, active single parents, inactive single parents and inactive couples. They give each group different weights and they can have other characteristics considered in other criteria. We do not have other socio-demographic variables. Since parents can register as soon as the pregnancy is official, child-age at registration is used as a proxy of anticipation and may be different across social groups.

We have several simulations of alternative assignments for 2022 and 2023. For 2022, we simulated assignments with priorities defined like in 2021, with the new priorities without the one for time since registration and fully random priorities. For 2023, we run similar simulations and also implement alternative mechanisms.

We also build a dataset of all ranked preferences and results of the assignment. In practice, this dataset allows to track each assignment and justify decisions individually. We use this dataset to showcase the properties of our proposed solutions.

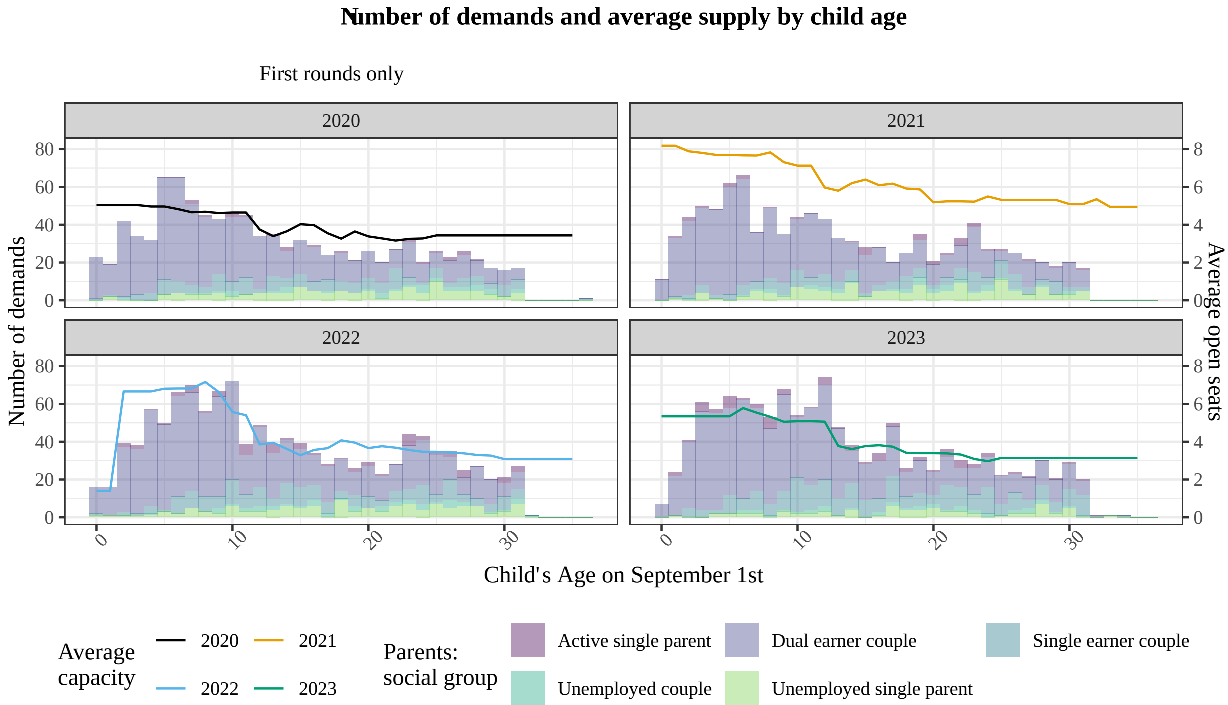
On the supply side: capacities by weekdays for different age groups As discussed in section II, daycares often have age groups with designated area and capacities which may vary from one daycare to another. In this setting, there are up to five age groups in a daycare and the supply datasets report capacities by age groups and over weekdays. Moreover, some age groups have flexible definition and can accept children from other groups, if there are enough capacity. The problem involve eligibility rules and a nested priority ordering for some buckets of capacities sorting files by age group then priorities.

Table C.3 in the Appendix summarises the aggregated results from various allocation committees. During the four spring committees, daycare centres offered an average equivalent of 415 placements for the first round. In this setting and like in all cities, daycares offer much more seats for younger children than older ones.

One way to apprehend the differences in access based on children's ages is to count, for each age value in September, how many buckets are open in all daycares and how many slots they offer on average over weekdays. By summing the number of slots in all buckets, we obtain the "potential" of open slots by age. By dividing the potential by the number of sections that open it, we obtain the average potential seats in each daycare for a child of a given age. Figure 1 presents these estimates along with an histogram of demands. On average, for first rounds, children have between 5 to 7 slots offered if they are under 12 months old, and around 3 or 4 if they are older.

[24] The GPS coordinates comes from the GoogleMaps API from a list of addresses and have been rounded for privacy purpose. We rounded latitude and longitude at the $100^t h$ degrees which creates points every 100 meters.

Figure 1: The share of open seats by age follows the distribution of age in the demand



Sources: ISAJE, Case Study I − 2020 : 2023 − first rounds only.
Note: − Histograms (left scale): number of demands by children age in September, by social group. Binwidth of 1 month.
− Solid lines (right scale): Ratio of the sum of the average number of days offered in all buckets accepting children of this age to the number of buckets opening a slot.

On the demand side: 1140 demands ranking up to 8 daycares reporting preferences over weekdays Table C.4 in the Appendix reports the number of demands for each round. In each year of the experiment, we processed an average of 1144 applications in the first rounds and 834 in the second rounds. Between 2020 and 2021, the demand increased by 5%, but it was in 2022 that the progression was the strongest: +17% compared to 2021. In 2023, there are 25% more requests than in the 2020 Committee. Table C.5 and Figure C.14 show the evolution of demands by social groups and how the proportion of each category evolves each year. Most demands come from dual earner couples while there are very few cases of single parents working. Over the years, the number and share of couples with only one parent working and single parents increased.

Families can rank up to 8 daycare centres, but only 1.1% of families provide a fully complete list. Figure C.18 in the Appendix shows that 1/4 of families reported only one choice and half request only 3. This means that parents' choices are not limited by the 8 preferences. Figure C.19 in the Appendix shows the average proportion of demands over weekdays for each year of assignment and by main social groups. Figure C.20 shows the cumulative distribution of the number of days reported. Overall, more than 50% of all demands register for 4 or 5 days, there are less demands for Wednesdays and significant heterogeneity across social groups and over the years.

We provide extra analysis of preferences in Appendix C.V with additional comments. Figure C.21 plots the *heat map* of the average number of applications over years for each daycare and preference order rank. The 5 largest daycare centres are also the 5 most demanded across all ranked choices. In the map presented in Figure C.22, we show the flows from parents' home towns to each requested daycare centres. The two urban centres serve as major hubs, concentrating nine of the most sought-after and, consequently, most congested daycare centres. Parents out of the two large urban centres tend to choose the closest daycare as first choice and then others in the main urban centres. In Figure C.23, we present the distribution of distances to various types of childcare centres. Registered parents live close to the most demanded daycare centres, especially the most congested ones. This is endogenous selection at play and we do not observe the distribution for parents who did not apply. We can only say that among those registered, most live very close to their chosen daycares.

Priority criteria and weights In this setting, the application forms and website do not provide parents with much information on how the assignment committee works. They encourage parents to "*contact early childhood services as soon as the pregnancy is official. Daycare seats are allocated according to availability and a number of criteria: residency, family situation (social or health-related), family structure and employment and time since registration. Fees depend on incomes and household composition*[25]". However, parents do not know how these criteria are weighted and used and children age is not reported. At best, it is implicitly implied that availability could depend on children age.

In practice, the early childhood department defines a weighted sum of 10 variables built from these criteria. In Appendix C.IV, we describe precisely how the score has been built and evolved. In brief, family situation defines 3 criteria: based on parent health, job search or training ; children health or disability and when child protection services are involved ; siblings - additional priority for multiple demands (e.g. twins) or when a sibling is already in one daycare. Family structure and employment define the 5 variables classifying social groups and strongly favour dual earner couples and single parents working. The last two variables are time since registration (in months) and living in the urban area.

The priority score is a constraint of the daycare assignment problem which guides us toward assignment free of *justified envy*. These criteria and weights have been voted by incumbents from the 54 municipalities and elected officials take pride in this consensus[26]. We asked elected incumbents and heads of early childhood department why these criteria and weights where chosen, and if they considered them "fair". First, territorial inequalities were central to the debates and they voted for equal priorities for all parents in the urban area. Second, they emphasise the balance between prioritising "*families with higher needs*" while rewarding parents who anticipate and those who wait longer. *Deservingness* justifies the weights, the residency bonus and time since registration. Anticipation and willingness to wait are perceived as a signal of higher needs and a mean for parents to increase their chance of receiving a seat by registering earlier than their competitors. We call *social weights* the part of the priority scores without time since registration as they reflect policymakers valuation of their social situation.

Parents do not know their social weights but know that registration is open as soon as the pregnancy is official and time since registration increase priorities. For parents, striving to be earlier than others can be a dominant strategy when the system is partly "first come, first served". The readier they are, the better, as registration can start on the third month of pregnancy. These rules can also affect the demand at the extensive margin $i.e.$ discourage applications from parents whose pregnancy is poorly timed with regards to the assignment schedules. At the intensive margin, densities of registration relative to the child birth can shed light on strategic behaviours.

The consequences of these strategic behaviours on the distribution of assignments depends on how much time since registration explains the variations in priorities. The remaining variance depends on joint-distribution of family situations and social groups. For the latter, Table C.6 in the Appendix presents the distribution of all criteria by social groups over the four years of the experiment. Family situations actually concern relatively few demands, although, in proportion, couples with no jobs are more likely to have children or parent priorities. It is striking to see that the social group with the highest priority – working single mothers – has very few demands although their number increases over the years. An important feature of this market is that dual earner couples represent 66% of the demand and only 14% of them have family situation priorities.

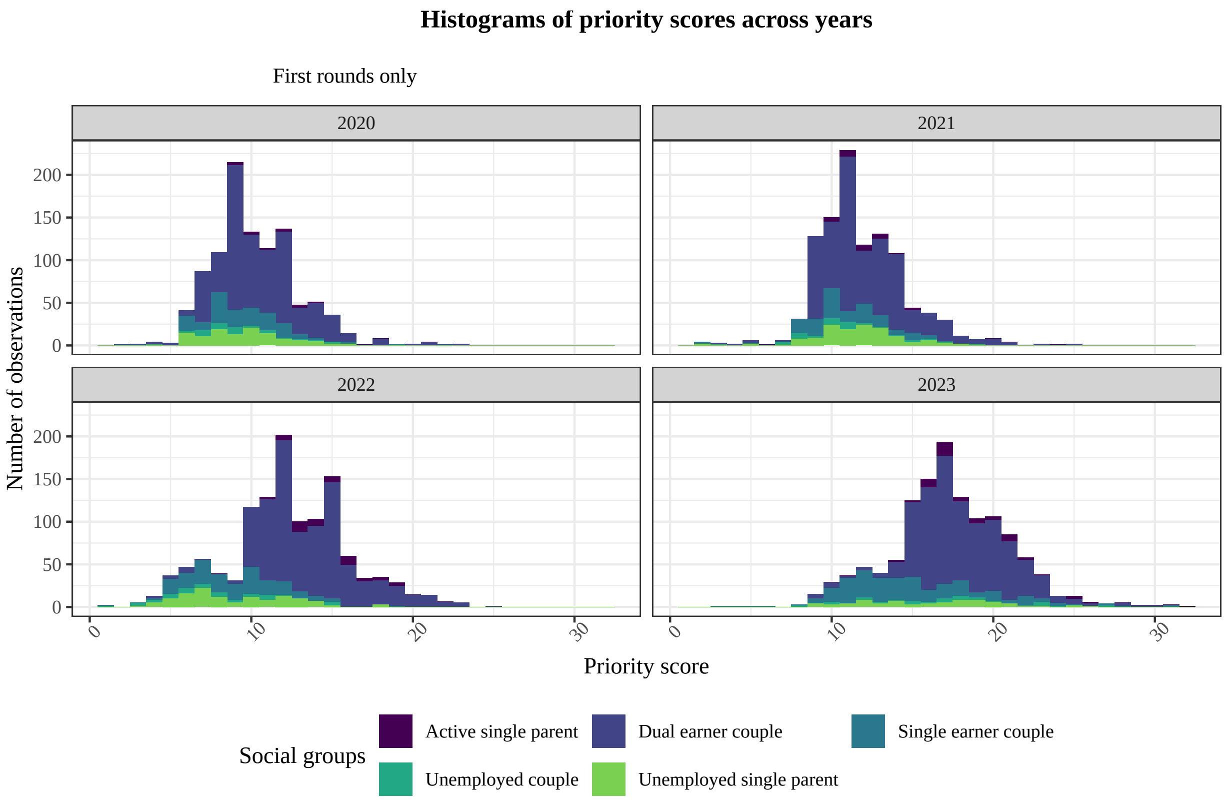
Figure C.16 in the Appendix presents the scatterplot between months since registration and the priority scores. We simply fit an OLS regression to estimate the share of variance explained. In 2020, 78% of the variability in the priority score come from time since registration. The criteria used are coarse and without time since registration, there would be few large groups of priority ties. Early registration is a strategic parameter that breaks ties within social priority groups Timing of registration is therefore a highly strategic parameter for parents, especially for dual earner couples. They represent the majority of demands and mostly compete against each other for early entry. Early registration is their main strategic lever to increase their chance of being seated.

[25] Own translation of the short description of assignment committees available on the city-hall website.

[26] The president, Nicolas Daragon, was first elected in 2016 and re-elected in 2020. Member of *Les Républicains*, the historical right-wing party, he has been the mayor of Valence since 2014.

The 2022 priority weigth reform In 2022, priorities were reformed, dramatically increasing weights for dual earner couples and active single parents. From 2022 onwards, weights for dual-earner couples have been multiplied by 2.6 going from 3 to 8 points, those of active single-parents by 2.25 going from 4 to 9. Inactive single parent families and couples now receive 2 or 3 points, while unemployed couples still receive 1 point. Figure 2 presents the distribution of scores for the first rounds of assignment for all four years, stacked by social group. In 2020 and 2021, the construction of scores is similar, and the two distributions are close. There are significantly more dual-earner couples than all other situations combined in all the allocation rounds. From 2022 onwards, densities shift for dual earner couples to higher priorities. In subsection V.1, we investigate the consequences of this change by simulating alternative assignments and measuring the effects on segregation.

Figure 2: Evolution of the distributions of priority scores over the years by social group



Sources: ISAJE, Case Study I − 2020 : 2023 − first rounds only.
Stacked histograms of priority scores by social group. New weights from 2022 for 'Dual earner couples' and 'Active single parents'.

III.2 Case study II: A simple school choice problem with diversity constraints

The previous case study considers the daycare assignment problem with preferences and capacity over weekdays although only a quarter report less than 4 days. In other settings, policymakers opted for a simpler version of the problem considering only "*full-time*" demands in the assignment procedure. Demands for fewer days are only matched after the main assignment took place on the remaining capacities. In these settings, our experiment only concerned the full-time assignment problem, leaving the part-time market aside. Our second case study is one of those.

The main conceptual difference is that without preference over weekdays, one child takes one seat and the problem is well known. Our contract with this local administration does not allow to reveal much details and the main point of this second case study is to emphasise that i) policymakers choose the definition of the problem, ii) this affects the structure of the market and parents options iii) diversity constraints have more influence on assignment probabilities than priorities. In this setting they cause sharp inequalities by date of birth.

Context In this setting, parents can register from the 6^{th} month of pregnancy through a *rendez-vous* at the city-hall and rank up to 5 daycares for 4 or 5 days. Like in VRA, they are informed of the criteria used in the assignment process but do not know how. The main difference with VRA is that i) they do not use time since registration, but a small bonus for those who update their files on time, and families whose demands for another child have been rejected in previous years. ii) they have more options for various criteria, but the number of ties is high (Figure 3).

Daycares are divided into age groups with strict eligibility rules. In VRA, several buckets allowed children from other age groups, it is not the case here. Moreover, that year, the first age group has the same definition in all daycare: children born on that calendar year. Groups for older children may vary.

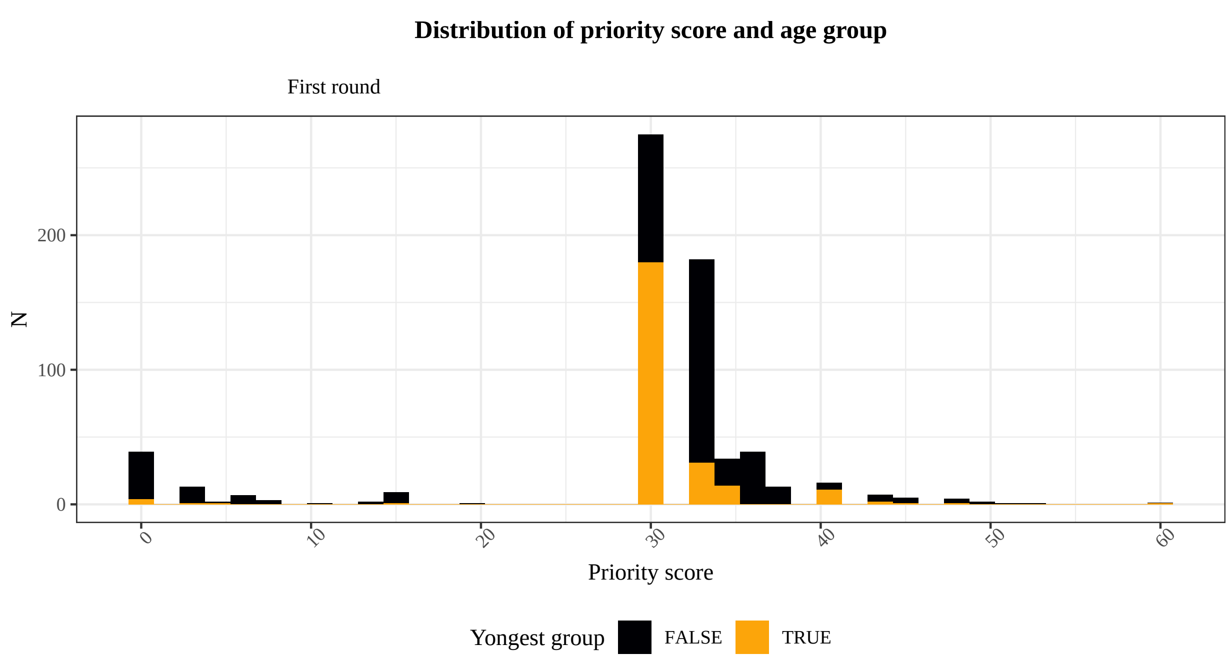
Data and descriptive statistics We use data from the *supply* file of case study II which contains 15 daycares, buckets birth date limits and capacity. The *Family* file of case study II contains 657 demands with family ids, date of registration and date of birth, ranked preferences priority scores and criteria. It also contains the results of the assignment, the lottery realisation and propensity scores.

Figure 3 shows the distribution of priorities colour-coded for children born that year and older ones. The priority score is essentially divided into three groups:

- Priority under 30 represents 12% of the demands. Their shared trait is that they don't work, and their children are not born that year ;
- Priority of 30 represents 42% of the demands and corresponds to the minimal score for parents who work. They are 3/4 with children born that year.
- Priority higher than 30 represents 46% of the demands and includes parents who work with extra criteria. Most of them were not born that year.

There are lots of ties for parents of the main group with no specific characteristics: dual earner couples, 3/4 of them with children born on the first semester of that year.

Figure 3: The most frequent priority is 30, those are mostly children born that year.



Sources: ISAJE − Case study II. Histogram of priority scores. Youngest group defined as born after January 1st that year.
Notes: Binwidth of 1.5. 12% of the demands are below 30, 40% of the demands have a priority of 30.

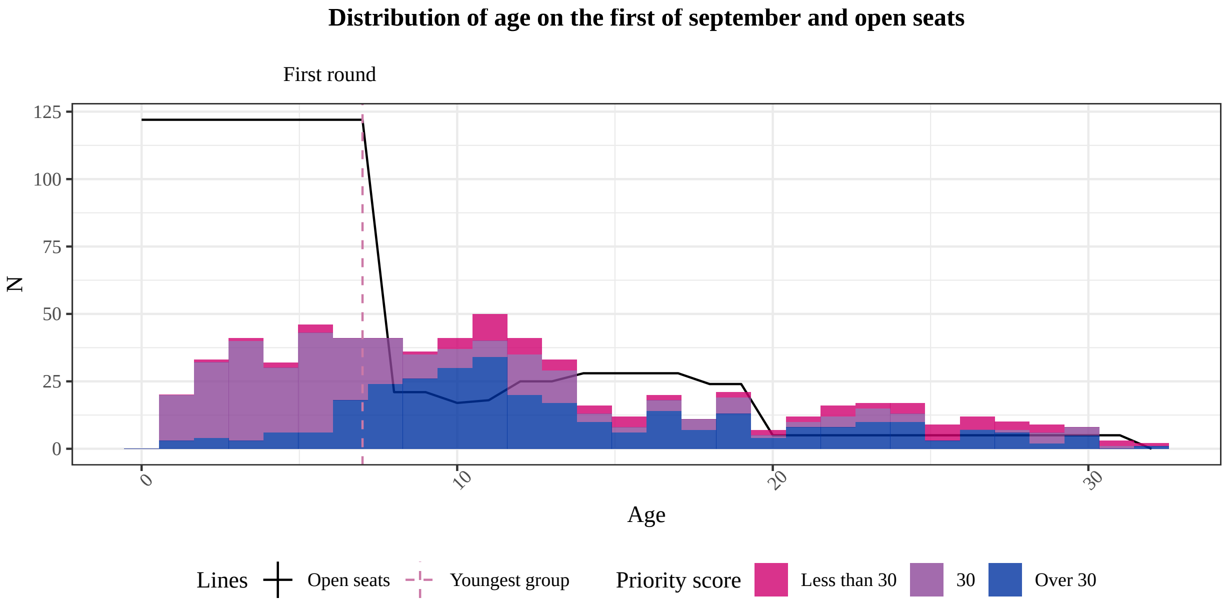
That year, daycares offer 165 seats among which 122 are available only for children born this calendar year (younger than 8 months in September). We compute the total number of open seats across daycares by children's age in

September. We plot the result in Figure 4 with the histogram of children's age in September by priority group. This figures shows that:

1) There is a sharp discontinuity in the number of offered seats for children older than 8 months in September but roughly the same number of demands on either side of the discontinuity ;
2) Eligible demands for the most available seats are mostly working parents with no other priorities ;
3) Children with priorities higher than 30 are mostly older than 8 months and have far fewer open seats.
4) Those with priority lower than 30 (who do not work) are also mostly over 8 months and compete with those with highest priority over few seats.

We come back to these two case studies in section V.

Figure 4: Young children get 100 more seats, demands and priority are evenly distributed across age limits



Sources: ISAJE − Case study II. Histogram of age on September 1st, stacked by priority groups.
Notes: 12% of the demands are below 30, 40% of the demands have a priority of 30.
Open seats are computed as the sum of slots across all buckets with open seats for children that age.
Youngest group indicates the common age threshold across daycares that define the youngest group.

So far, there are two versions of the daycare assignment problem. The difference is a political choice of simplification and sequencing. Case study II consider two separate markets: one for full-time and one for part-time. Full time demands is the main marketplaces and the others are used to fill the gap. Case study I consider demands and capacity constraints over weekdays and combine as many demands as possible while following priorities and capacity constraints. In practice, many rules stems from practical constraints and we adapt our algorithms and codes to local specifics. An important shared feature is that seats in daycare are reserved for specific age groups. This makes it hard for policymakers to justify assignment using the scores. Indeed, because of age groups, children with the same score asking the same daycare may have different outcomes because they are not eligible over the same bucket. Building from these features and constraints, we now move to our theoretical models.

IV Models for the daycare assignment problem

We consider any daycare marketplace where a local authority is in charge of assigning slots to families through a centralised assignment mechanism. For consistency with the school choice literature we call applicants "students" and daycare centres "schools" throughout the paper. Like school choice, application processes involve submitting an ordered list of **preferences** and reporting a set of characteristics used to sort students with **priority levels** and set eligibility status. For now, we consider individual applications[27]. Recall that our goal is not to provide new theories but adapt and possibly improve existing work so that they could be applied in the field. Our models are very general although they reflect our empirical settings. Like assignment committees, they i) take agents participating in the daycare marketplace and their characteristics as fixed, and ii) do not consider the problem of self selection, where preferences come from, and the role of outside options or dynamics.

The goal of this section is to provide a formal definition of **daycare assignment marketplaces**, and for that, we need to i) enumerate the **elements** of the problem, ii) introduce appropriate **stability** notions, iii) define **algorithms** to find stable matchings, if they exist and iv) enumerate their **properties**.

We opt for a pedagogical approach and start by introducing notations and take the student proposing deferred acceptance as a benchmark in subsection IV.1. Subsection IV.2 adapts the problem with diversity constraints and solve the simplified problem without demands over weekdays. In subsection IV.3, we consider the problem with demands over weekdays and we present our main result in subsection IV.4. Finally, we show the properties of our assignment in action and compare the outcomes of our various algorithms in subsection IV.5.

IV.1 Notations

We denote I the set of individuals, where $i \in I$ is a specific student. Student i is characterised by their **type** θ_i which captures all the relevant information for the assignment of individual i. We denote by Θ the set of possible types of each individual and by $\boldsymbol{\theta} := (\theta_i)_{i \in I} \in \Theta^N$ the vector of individuals' types. There are S schools indexed by integers $s = 0, 1, \dots, S$ where $s = 0$ is the "*null school*", *i.e.*, being unassigned.

Each applicant $i \in I$ has a (strict) **preference order**[28] $\succ_i$ over the set of schools,[29] where $s \succ_i s'$ indicates that family i prefers school s over school s'. Applicant $i \in I$ in school $s \neq 0$ is assigned a **priority level**[30] $\rho_{is} \in \{-\infty, 0, 1, \dots, K\}$. Priority levels are **given** by municipalities. They typically use scalars summing and weighting a (small) set of criteria. If $\rho_{is} > \rho_{i's}$, applicant i has a higher priority than applicant i' in school s. Priorities are usually coarse and there can be many applicants with the same priority applying for a school. If $\rho_{is} = -\infty$, applicant i is ineligible in school s. If $\rho_{is} = 0$, applicant i has no particular priority for the school s. A school s is said to be **acceptable** to family i if $s \succ_i \emptyset$.

A school s has q_s^t seats available at day t. We let $q_s = (q_s^t)_{t \in T}$ be the vector of **capacities** of school s and $\mathbf{q} := (q_s)_{s \in S}$ be the one for all schools. Schools have capacity and diversity constraints. They can come in many forms but we can define constraints very broadly. A **constraint** at school s is a non-empty collection $\mathcal{F}_s \subseteq 2^I$ of sets of families. We say that a subset $I' \subseteq I$ is **feasible** at s if $I' \in \mathcal{F}_s$ and it is **infeasible** otherwise.

In the matching literature, a policymaker typically uses a **matching mechanism** ϕ, also called **matching algorithm**. An algorithm simply maps each element of a problem P to a matching (μ) of these individuals to schools.

27 Putting aside twins and families with multiple demands of children of different ages following current practices. Parents in such situations usually receive higher priorities.

28 These preferences are lexical *i.e.* they describe preference *hierarchies* and the idea is that (under certain conditions), an agent's rankings of their possible choices can be represented as if they assigned a level of utility.

29 Note that the schools contain the "null school" so that an applicant can prefer to be unassigned than being assigned to some schools.

30 Our notations define priorities in a reversed way to Abdulkadiroglu et al. (2017) for whom a lower ρ gives a higher priority. We make this choice to better reflect the practices of municipalities that generally use a weigted combination of criteria or where a higher level gives a higher priority. In practice, we always redefine priorities with lexicographic notation to nest different priority levels. For instance, if there is a score defined in tens, we will use the hundreds to code the age priorities which are superimposed on the score within daycares.

Stochastic assignment as a research design Priority levels typically create *ties* among applicants at a given school. To select among such candidates, we use a **single random tie-breaker**. For each individual i, we draw a random number ϵ_i from the uniform distribution $\in [0, 1]$ and define π_{is}, the **applicant score** at school s as a combination of priority level and random tie-break. For each realisation of the tie-breaker, an algorithm returns a matching. For a given problem, a mechanism generates a distribution of probabilities over possible matchings, which is referred to as a **stochastic assignment**. A stochastic assignment generates a matrix $\mathcal{P}$ of size $|I| \times S$ where the entry p_{is} represents the probability that applicant i is assigned to school s. $\mathcal{P}$ is key to later estimate the effects of accessing daycares or estimate compliers' characteristics. We literally embed randomised experiments at various steps of the algorithm and *harvest* these variations to identify causal effects of interest. In Appendix F.III, we provide a formal proof that our stochastic assignment methodology can be employed for estimating design-based propensity scores and later be used for causal evaluation.

The school choice problem: a brief summary In the so called *school choice problem*, demands and capacities are **unidimensional**, each school s can admit at most q_s students, and we denote $\mathbf{q} := (q_1, ..., q_S)$ as the vector of capacities. Applicant i's type is a combination of their preferences and priorities, that is, $\theta_i = (\succ_i, \rho_i)$ and Θ is just the product set of all possible preference profiles and vectors of priority levels. In this case, the policymaker faces the **assignment problem** $P_{1:1}(I, S, \mathbf{q}, \boldsymbol{\theta})$. The main features are preferences from parents and schools/policymakers through priorities and capacity constraints. As exposed in section II, this problem is well known in the school choice literature and already implemented for the daycare assignment problem in various settings. An important definition is the stability notion for this problem.

Definition 0.1 (Stability in school choice)**.**

Given a matching problem $P_{1:1}(I, S, \mathbf{q}, \theta)$, a matching μ is said to be **stable** if there exists no **blocking pair** (i, s) of an individual and a school s.t. $s \succ_i \mu_i$, so individual i also prefers school s and either:

- $|\mu_s| < q_s$ and $\rho_{is} > -\infty$: school s is not full and individual i is eligible to school s.
- $\exists\ i' \in \mu_s$ s.t. $\pi_{is} > \pi_{i's}$: school s is assigned another individual i' which belongs to an applicant with a lower score than i at s.

If such pair exists, some applicants will be able to "complain" about this assignment. If the complaint is that there are empty seats left in a school, we say that the matching is **wasteful**. If the complaint is due to the existence of an applicant with lower priority assigned to the school, we say that the matching has **justified envy**. In this setting, a stable matching is simply a matching that is **non-wasteful and envy-free**.

There may be many stable matchings. Among these, one can show that there exists one that is the most preferred for all applicants. This can be found using the *Student Proposing Deferred Acceptance* (SPDA) algorithm proposed by Gale and Shapley (1962) and defined in this setting in Algorithm 1 in the Appendix. The SPDA has many attractive theoretical properties. It is is the **only mechanism** that is **stable** *i.e.* non-wasteful and envy-free, and **strategy-proof** *i.e.* applicants have an incentive to truthfully report their preferences[31](Abdulkadiroğlu and Sönmez 2013).

Moreover, the SPDA algorithm can also be seen as a cut-off adjusting function[32] and Abdulkadiroglu et al. (2017) derive their propensity score formula from this definition in a continuum economy. At each round of provisional assignment, the rank (or priority) of the last provisionally accepted applicant defines an admission cut-off. At any round, applicants with a rank lower than the cut-off defined in the previous round are rejected. Applicants with a rank higher than the cut-off will increase the cut-off to the rank of the last feasible provisional assignment and so on.

Considering the two cases presented in section III, it is clear that this does not fit the first setting (VRA) because of preferences over days. However, Case study II is very similar and only adds age groups constraints. As we will see, the school choice problem with diversity constraints can be transformed into a $P_{1:1}$ problem and solved using SPDA.

[31] Precisely, it is a dominant strategy for them to be truthful: whatever the report of other applicants is, one cannot benefit of reporting a preference list other than their true preferences.

[32] See theorem 1 in Kamada and Kojima (2023) and the characterisation of the space of cut-off profiles as a finite lattice and the characterisation of *stable* matching as fixed-points.

IV.2 The problem with diversity constraints

School are often structured to welcome children of certain ages in specific areas which defines **diversity constraints** within schools in the form of **eligibility or priority rules** attached to subsets of capacities. Daycare providers define their capacities for different **groups** of children. Groups broadly define categories (low-income families, etc.) that the policymaker would like to balance in each assignment[33]. We introduce new elements that we call "**buckets**" of capacity to designate bundle of capacities within daycares with attached eligibility and ordering over groups. This term is also used by Abdulkadiroglu et al. (2017) to designate reserved seats in the Denver Public School assignment. In practice, buckets often refer to age sections, but we prefer this more general notion.

A) Additional notations with diversity constraints

There are two main differences with $P_{1:1}$:

1) Each student $i \in I$ has a *group* $g_i \in G$ where G is the set of all possible groups. We denote g and G' as respectively a generic element and a subset of G. Following our previous notations, groups are now part of a student's type $\theta_i = (\succ_i, (\rho_{is})_s, g_i)$;
2) Policymakers and/or daycare providers define **buckets of capacity** with **eligibility** and **priority rules** by groups. Eligibility rules mean that some capacities restrict access to students of groups g only. Priority rules mean that they will favour students of group g but other groups g' may be accepted too, but only if there are no student of group g that can be accommodated in these capacities. This defines two sorts of diversity constraints:
    - **Soft quotas** are seats who accept students of groups g first but other groups can be accepted if there is enough capacity to accommodate them.
    - **Hard quotas** only accept students of group g. The others are not eligible.

Let us formally define the notion of buckets of capacities.

Definition 0.2 (Buckets of capacities)**.**

There is a set B of **buckets**. A bucket $b \in B$ is characterised by elements $(s_b, \succeq_b, c_b)$ where:

- $s_b \in S$ is the school of bucket b. We let $B_s \subseteq B$ be the buckets which belong to school s.
- $\succeq_b$ is the weak priority ordering of bucket b over $G \cup \{\emptyset\}$ where $\emptyset$ will be used to define the eligibility of a group for bucket b.
- $c_b := (c_b^t)^t$ represents the capacities of bucket b for each day t.

For our daycare problem, a bucket b can typically correspond to an age group in a daycare and the ordering $\succeq_b$ defines which age groups are eligible and prioritised. To fix ideas, assume that groups $G := \mathbb{N}$ correspond to the age (in months) of a child. If a bucket b of daycare s represents the number of seats that are proposed to children from 3 to 9 months old, if there are seats left, we authorise children from 10 to 18 months old to apply, then $\succeq_b$ is such that: if $g, g' \in \{3, \dots, 9\}$ or $g, g' \in \{10, \dots, 18\}$, then $g \sim g' \succ_b \emptyset$. If $g \in \{3, \dots, 9\}$ and $g' \in \{10, \dots, 18\}$ then $g \succ_b g' \succ_b \emptyset$. All $g > 18$ are not eligible so $\emptyset \succ_b g$.

[33] Groups can also designate special needs or specific types of accommodations. For instance, few seats with longer hours for parents with late shifts, children with disabilities. Since 2019, there are additional *diversity* subsidies for daycares with a share of low income and disabled children higher than a threshold.

a) A redefinition of priorities Policymakers still use a priority score ρ_{is} to sort applicants according to their criteria. However, assignments must respect the diversity constraints first. This means that they do not simply sort students by priorities in a school but within buckets. Now, priorities are defined differently in buckets depending on groups.

To fix ideas, consider a school s with at least two buckets j and k in B_s with $b_s^j(g)$ and $b_s^k(g')$ ordering groups in each bucket. Consider two students i of group g and i' of group g' such that[34] $\rho_{is} > \rho_{i's}$; $\mu_{i'} = s$; $s \succ_i \mu_i$. In other words, two students of different groups where i prefers school s to their assignment and i' has a lower priority score $\rho_{i's}$ in that school than student i. Based on the priority score only, i' appears to have justified envy.

In practice, committees are uncomfortable with such situations. They cannot justify an assignment based on the priority score ρ_{is} only. Yet they sometimes make the definition of the priority score ρ_{is} an important political tool. Weights and characteristics used act as a political signal to parents. The latter hold assignment committees accountable for respecting these rules. Unfair assignments can be politically costly for incumbents. But transparency without a stability notion with diversity quota can backfire. As Li (2017a) writes, « *A policy-maker may be familiar with the details of their environment, and yet not know how to state their ethical requirements in precise terms.* ». This is where formal market models of assignment committees bring more than fast and convenient computer programmes.

If we can define stability notions with diversity constraints and algorithms that can find such stable matchings, policymakers will be able to justify assignments and ensure transparent and due process of all applications. To do that, we make clear the transformation of priorities following the ordering of groups within a bucket.

Definition 0.3 (Priorities within buckets)**.**

For each bucket b of school $s = s_b$ and student i, we construct $\tilde{\pi}_{ib}$ the priority of student i in bucket b as follows:

- If $\emptyset \succ_b g_i$, then $\tilde{\pi}_{ib} = -\infty$ so that i is not eligible to b.
- If $g_i \succ_b \emptyset$, then $\tilde{\pi}_{ib} = \sum_{g \in G : g_i \succ_b g} \max_i(\rho_{is} + 1) + \underbrace{\rho_{is} + \epsilon_i}_{\pi_{is}}$

In plain words, we consider students in the daycare they apply and redefine priorities within buckets starting with eligibility. When students are eligible, we add to the priority score ρ_{is} and tie-break ϵ_i the **highest priority score** $\rho_{is} + 1$ **as many times as there are groups ranked lower** than the group g_i in that bucket *i.e.* $g_i(b_s)$. There are other functions we could have used. They only need to preserve the **lexicographic nested structure of priorities** with diversity constraints.

b) Mapping student preferences to buckets Finally, students report their preferences $\succ_i$ over schools but they will be assigned to buckets. Thus, to run an algorithm which matches students to buckets, we need to map preferences over schools into preferences over buckets.

Definition 0.4 (Preference order over buckets)**.**

For each school s, fix an arbitrary ordering $\gg_s$ over buckets in B_s. For each student i, we define $\tilde{\succ}_i$ as an ordering over $B \cup \{\emptyset\}$ as follows: for $b, b' \in B$:

- if $s_b \succ_i s_{b'}$, then let $b \tilde{\succ}_i b'$
- if $s_b = s_{b'}$ and $b \gg_s b'$, then let $b \tilde{\succ}_i b'$

In plain words, the preferences over buckets follow an arbitrary order within schools but otherwise respect preferences over schools. The only role of the ordering over groups is to tie-break how students apply over different buckets in a given school[35].

[34] We neglect tie-break here but in practice, stability is enforced over π_{is}.

[35] In the literature, this ordering is referred to as a *precedence order*. This ordering could have an impact on the final assignment. However, it is unclear whether one should follow an alternative definition of ordering. In our empirical work, we simply follow the order provided by municipalities, usually starting from lower age groups to older.

The previous definitions allowed to introduce buckets, quotas and new objects for priorities and preferences over buckets and we can define the general problem of the policymaker:

Definition 0.5 (The problem with diversity constraints)**.** With soft quotas, policymakers face a problem composed of **Students** I, **Schools** S **Buckets** B and their elements, **Capacities q**, **Student types** $\boldsymbol{\theta}$. Formally, $P_{DC}(I, S, B, \mathbf{q}, \boldsymbol{\theta})$

Intuitively, the careful redefinition of priorities and preferences transform the sophisticated problem with diversity constraints into a problem where students apply to buckets and each bucket is reinterpreted as a *tiny school* with its own set of capacities, eligibility and priority rules. Student types $\boldsymbol{\theta}$ may be extended to account for preferences over days and capacities **q** be defined over weekdays.

Our strategy is simple, we take a problem like P_{DC}, apply definitions 0.3 and 0.4 to create new priorities and preferences with buckets. We then use well defined algorithm on the modified problem, retrieve this assignment and map assignment in buckets back into schools capacities. We start with the simplified problem where policymakers only consider full-time demands like in Case Study II.

B) Stable matchings for the school choice problem with diversity constraints

We denote $P_{s:1}(I, S, B, \mathbf{q}, \theta)$ the school choice problem with diversity constraints. Again, this problem is well defined in the school choice literature and we adapt the work of Ehlers et al. (2014) to our setting. For a school s, a group g and a matching μ, we denote by μ_s^g the set of students of group g assigned to s under matching μ. Each school has a capacity q_s divided into quotas $\underline{q_s^g}$ and our *trick* is to define buckets b_s as **unitary seats**. We create b_s buckets and for each group g, we take $\underline{q}_s^g$ buckets with priority ordering $g \succeq_b g' \cup \emptyset$. In words, there are now **quotas** for groups g ; hard quotas only accept students of group g, soft quotas may weakly or strongly prefer students g over g'. Let B_s^g the set of buckets in school s reserved for group g. We have $|B_s^g| = \underline{q}_s^g$ and for $b \in B_s^g$, $g \succ_b \emptyset \succ_b g'$ for $g' \neq g$.

In plain words, there are individual seats. Each has their own priority order and a seat gives a priority to a given group of students. We now give a definition of stability in this version of the problem. The intuition is the same as before: we want to eliminate potential cases where students can have justifiable envy. We focus on the case with **soft quotas** $i.e.$ individual buckets with ordering over groups. We briefly discuss the case when all buckets are hard quotas with the properties.

Definition 0.6 (Stability with soft quotas in the problem without days)**.** For a matching μ, we let

$$\mu_s^g := \{i : \mu_i = s \text{ and } g_i = g\}$$

We say that a matching μ is **stable with soft quotas** if for any applicant $i \in I$ with $g := g_i$ and any school s s.t. $s \succ_i \mu_i$ either:

1. $\rho_{is} = -\infty$: *Applicant i is unacceptable at school s*, **or**
2. $|\mu_s^g| \geq \underline{q}_s^g$: *Quota is filled* **and**
   (a) $\pi_{i's} > \pi_{is} \forall i' \in \mu_s$ with $g_{i'} = g_i$
   (b) $\pi_{i's} > \pi_{is} \forall i' \in \mu_s \quad s.t. \quad g_{i'} := g' \quad$ **and** $\quad |\mu_s^{g'}| > \underline{q}_s^{g'}$

In the case without quotas, we only have to respect the priority levels of applicants and fill each school. Now, with soft quotas, the only reason to violate the priority level of an applicant at a school is to respect the diversity quota at that school. This is exactly what Condition 2 of the definition imposes.

Let us detail all the cases hidden in Condition 2. If applicant i from group g prefers school s to their assignment, then:

- if the diversity quota of school s for group g is not reached, a student of group g would be able to complain. This is excluded by the condition $|\mu_s^g| \geq \underline{q}_s^g$ in the definition.
- If there is a student assigned to school s who belongs to the group of student g and who would have a worse priority at that school than student i, then the latter would be able to complain. This is excluded by condition 2a of the definition (note that the higher the value of π_{is} the higher the priority).
- Last, if we assign strictly more students at school s from another group g' than the objective of the quota and that one of these students has a worse score than student i, then the latter would be able to complain. Indeed, there would be no reason to give the seat to this student with a score worse than i because the diversity objective is already met. This case is ruled out by condition 2b of the definition.

SPDA with diversity constraints Again, many matchings can be stable with soft quotas. We use a generalisation of the SPDA algorithm proposed by Ehlers et al. (2014) and presented in Algorithm 4 in the Appendix. It simply redefines the problem into a 1:1 marriage problem *à la* Gale and Shapley (1962) by transforming preferences and priorities over and within buckets, then runs SPDA on the modified version. An important theorem in school choice is that "*a college admissions problem is stable if and only if the corresponding matching of its related marriage problem is stable*" (Abdulkadiroğlu and Sönmez 2013). Ehlers et al. (2014) show that this algorithm is **strategy-proof with soft quotas** and reaches the **student optimal stable matching with soft quotas**[36].

When policymakers define the problem with soft quotas and without days, we can solve and return the only assignment that is the most preferred by all families that strictly respects priorities and diversity constraints. In theory, this algorithm is strategy-proof so parents have an incentive to reveal their true preferences. The definition of buckets with group rankings allows to accommodate sophisticated constraints (through eligibility rules and local priorities over buckets) and to define target distributions over groups g in each daycare. Stability means that the assignment is envy-free and non-wasteful with soft quotas. However, if the definition of buckets is too narrow, assignments may be *artificially* stable and leave empty seats nonetheless. This happened many times in practice and while some adjusted buckets for re-running the assignment, others stood by the initial definition accepting empty seats and fewer satisfied family. Buckets are highly sensitive parameters and we think policymakers should consider them more strategically.

From a normative standpoint, stability ensures *fairness* and *transparency* by allowing individual justification of every assignment. It is also *safe* to participate in the market and reveal true preferences since there is no gain in trying to game the algorithm. We used this algorithm for Case Study II and other municipalities. As practitioners, we know this definition is a simplification of the *real problem* where parents' needs and daycares' capacities vary with days. The solution focus on a constrained market where parents are only eligible if they require full-time access, and part-time demands are considered after. This affects the composition of the demand at the extensive margin or may create other strategic behaviours. If the number of days reported is not binding, some could fill full-time demands and renegotiate their contracts once they receive an offer. At the same time, it can be hard for parents to anticipate their true needs so enforcing initial demands could make participation too risky for some. Conversely, using an assignment that directly accounts for preferences and capacities over weekdays could solve part of these problems. We now move to models with diversity quotas and multidimensional constraints.

[36] Ehlers et al. (2014) work with more general constraints than ours. They allow for upper and lower quotas. The algorithm presented here can be shown to be a special case when the upper and lower quotas coincide.

IV.3 Models with multidimensional constraints

In the situation without specific days required, one child takes one seat. A seat is an indivisible and unitary good. In settings with specific days, a student can take some days and the rest may be assigned to another with complementary preferences. A seat is therefore no longer indivisible and unitary. What changes is that childcare providers submit their capacities by days and parents rank daycares and their expected needs over weekdays. While a slot may represent a physical bound in a daycare (literally a cradle for each child), they are not tied to a particular child and different children can be assigned (and use) the slot in different days. Then, the allocation committees set priorities over applicants and consider combining the days of different seats to accommodate as many children as possible. The more general problem also involves diversity constraints and for that, our solution is to use the explicit redefinition of priorities and preferences using buckets to map the problem into a version with well defined solutions. In our case, the latter stem from the refugee matching framework of Delacrétaz, Kominers, and Teytelboym (2023) and of the constrained matching framework of Kamada and Kojima (2023)[37]. We start by introducing the new elements of the problem and a first important fact in problems with multidimensional constraint.

A) New elements in the problem with preferences over weekdays

The problem involve new elements in student types θ to map preferences over days, and changes in the definition of capacities. Each student now requires a set of days d_i. Assume that there are T days and that $d_i \in \{0,1\}^T$ where $d_i(t) = 1$ if applicant i needs to be assigned to a school day t.[38] We denote by $d = (d_i)_{i\in I}$ the vector of the **days required** by the applicants[39].

A school s now reports q_s^t seats available at day t. We let $q_s = (q_s^t)_{t\in T}$ be the vector of capacities of school s and $\mathbf{q} := (q_s)_{s\in S}$ be the one for all schools. In this framework, a matching μ is **feasible** if $\forall s \in S$ and $\forall t \in T$, $\sum_{i\in\mu_s} d_i^t \leq q_s^t$. Equivalently, $\mu_s \in \mathcal{F}_s \ \forall \ s$. In plain words, a matching is feasible if for each day, the total number of assigned students to school s who required that day does not exceed the capacity of that day.

Recall that in the model without demands over weekdays, the stability notion is based on envy-freeness and non-wastefulness. However, as noted by Delacrétaz, Kominers, and Teytelboym (2023) and Kamada and Kojima (2023), it is easy to see that such stable matching might not exist in a setting with days.

Example 0.1 (Non existence of a stable matching)**.**

Assume there are 2 applicants i, i', one school s and that $\rho_{is} > \rho_{i's}$. Assume only 2 days ($T = 2$) and that s only has one seat available at day $t = 1$ and zero seats at day $t = 2$. Say i requires to be assigned both days but i' only requires to be assigned at day $t = 1$. In this example, there are no stable matching because

- There is no feasible matching that matches i to s since there is no seat at day $t = 2$.
- Matching i' to s would lead i to envy i' since i' has a lower priority at s.
- Leaving s empty is also not an option since, in that case, the matching would be wasteful since one can feasibly match i' to s.

A stronger result of both Kamada and Kojima (2023) and Delacrétaz, Kominers, and Teytelboym (2023) is that there are **no algorithm** that is both **envy-free** and **non-wastefull**. However, it is easy to show that an envy-free (but potentially wasteful) matching always exists. For instance, the trivial matching where nobody is matched to any

[37] In the terminology of the former, each day would be a *service* and our model would be a special case since we assume that each applicant requires only one seat per day whereas in the framework of Delacrétaz, Kominers, and Teytelboym (2023), a refugee family can require more than one unit of a service.

[38] In the daycare assignment of many municipalities, the days are typically the 5 work days, i.e. Monday to Friday. If one allows for half-days occupation then one can define $T = 10$ and interpret a given t has a half day.

[39] Here, we assume that preferences over schools and days are separated objects. A more general model would require to have preferences over the pairs schools and days assigned. We chose not to model it that way even though it is with loss of generality. First, many municipalities ask the parents to report exactly as in our model so that we chose to match their current practice. Second, negative results are already present and would probably be even more important in a more general model.

school is envy-free but obviously wasteful. The question is whether we can find matchings that are envy-free and not *too* wasteful. One can think that the condition of *envy* that we require may be strong. Indeed, i can block with s against a lower ranked applicant i' even though it would be impossible to feasibly match i to s while removing i' since they may require different days. This is indeed an important discussion of the literature of constrained matchings[40]. However, many different "natural" notions also fail to exist. The two aforementioned papers propose different notions of envy-freeness with different algorithm.

B) Chosing the right fairness notion

To understand the main issue with the definition of envy-freeness and the differences between Kamada and Kojima (2023) and Delacrétaz, Kominers, and Teytelboym (2023), we start with a simple example.

Example 0.2 (Reducing wastefulness)**.** In a daycare, there are four open slots every day except on Tuesdays. Say the family with highest priority asks all 5 weekdays and the next three families request different days than Tuesday (for instance, four days excluding Tuesday). The first family cannot be accepted because there is no open slots on Tuesdays, but the three others could. If we abide priorities, no family can be accepted and this daycare remains empty. This situation is bad for daycare providers and rejected families so policymakers may want to accept the three feasible families despite causing justified envy to the first.

a) Chosing weak-envy freeness Recall that the definition of a feasible matching in the multidimensional case is that $\forall s \in S$ and $\forall t \in T$, $\sum\limits_{i \in \mu_s} d_i^t \leq q_s^t$ i.e. there are enough capacities in each day in each school. Delacrétaz, Kominers, and Teytelboym (2023) introduce the notion of **weak accomodation** of applicants:

Definition 0.7 (Weak accomodation)**.**

School s can **weakly accommodate** applicant $i \in I$ alongside families $F \subseteq I \setminus i$ if $\forall t \in T$, either:

1. $d_i(t) = 0$ or
2. $d_i(t) + \sum_{f \in F} d_f(t) \leq q_s^t$

In plain words, this definition relaxes the original concept of feasibility by only taking into account the days in which i will take at least one unit of capacity. It has a **cumulative feature** considering every students that ever applied. If the aggregated sizes of applicants in F exceed the capacity of daycare s for some days, i cannot be accommodated alongside F; however, it may still be possible to weakly accommodate i if i 's size in that dimension is zero. In the example above, the three families can be weakly accommodated in this daycare as $d_i(t) = 0$ when t is Tuesday and $d_i(t) + \sum_{f \in F} d_f(t) \leq q_s^t$ for all other days.

By contrast, they define **strong justified envy** as a matching that allows **weak accomodation** but otherwise, respects priorities. Formally:

Definition 0.8 (Strong justified envy)**.**

Given a matching μ, applicant i in I has **strong justified envy** over family $i' \neq i$ with assignment $\mu_{i'} = s$ such that:

1. $\mu_{i'} \succ_i \mu_i$: student i prefers the school obtained by i'.
2. $\pi_{is} \geq 0$: i is eligible for seats in s and
3. $\pi^s_{i's} < \pi_{is}$ and
4. μ_s cannot weakly accommodate i' alongside all families with higher priority than i' at s that weakly prefer μ_s to their current match.

[40] We refer the reader to the discussion of Kamada and Kojima (2023) and other related papers of the two authors.

In this definition, we don't want a student i to complain that they are not in a school s they prefer to their current assignment μ_i when there exists student j with $\mu_j = s$ with $\pi_{js} < \pi_{is}$ i.e. lower priority[41] than student i in that school over days that *would have allowed* i *to be accepted*. Weak justified envy is the case where there exists student i' with $\mu_{i'} = s$ and $\pi_{i's} < \pi_{is}$ but there is no conflict over days between i and i'. Then we can define a matching that eliminates strong justified envy:

Definition 0.9 (Weakly envy-free assignment)**.** A matching μ is **weakly envy-free** if no family strongly envies another family.

Weak justified envy corresponds to the situation where we tolerate justified envy only towards those weakly accommodated. Weak envy-freeness relaxes envy-freeness by allowing some arguably innocuous priority violations.

b) Chosing strong envy-freeness Example 0.2 suggests that weak envy-freeness is a rather efficient way to limit wastefulness, with small deviations from priorities. The idea is that we can explain to such parents that accommodating these other students did not prevent them from receiving a seat. In some situations, policymakers may rather have larger vacancies than to deviate from priorities. There can be many reason for that, from transparency and accountability motives to organisational choices to offer occasional care on these remaining vacancies. It is therefore a **political choice** which affects the definition of a *fair* assignment.

In our setting, Kamada and Kojima (2023)[42] showed that it is possible to define matchings that are *student optimal* and *envy-free*.

Definition 0.10 (Student optimal fair matching)**.**

A matching μ is the **student-optimal fair matching** (SOFM) if :

1. μ is feasible, individually rational, eliminate justified envy **and**
2. $\mu_i \succeq \mu'_i \forall i \in I$ and every μ' that is feasible, individually rational, and fair.

Where **justified envy** is defined as any violation of priorities with tie-break. Formally:

Definition 0.11 (Justified envy)**.** An applicant i of group g in a matching μ has **justified envy** if $\exists s$ s.t. $s \succ_i \mu_i$ and $\exists i'$ of group $g \in \mu_s$ with $\pi_{i's} < \pi_{is}$.

In other words, definition 0.10 simply relaxes the *non-wastefulness* requirement, strongly enforces priorities and characterises the most preferred assignment. The notion of *optimal* matching does not mean that there is no other assignment that families would prefer to μ but that among those that eliminate justified envy, μ is the only one that is weakly preferred by all parents. Note that this definition includes student groups in order to be used with diversity constraints. In the setting without buckets, groups are irrelevant or equivalently, all demands are of the same group.

C) Algorithm for student optimal envy-free assignment

Together with the previous definition, Kamada and Kojima (2023) propose an algorithm - the *Cutoff Adjustment Mechanism* (CAM) - that always returns the SOFM. We formally present the CAM in Algorithm 2 in the Appendix. Intuitively, every student applies to their favourite school for which they have not been rejected yet. In each school, we sort the demand by rank $\tilde{r}_{is}$. The main *engine* is an **adaptive cut-off function** that checks feasibility in each school in every steps. At each round, every school that does not respect the feasibility constraint increases the cut-off by one, thus excluding a student that cannot be accommodated. At one point, the constraint is satisfied in every school and the assignment is final.

[41] with tie-break

[42] They consider a much more general setting than ours. They give a sufficient condition on their feasibility condition such that an optimal envy-free matching always exists. One can check that their condition is respected in our framework. We again refer the reader to Kamada and Kojima (2023).

Kamada and Kojima (2023) show that this algorithm stops in a finite number of steps and returns the **unique student-optimal envy-free** matching[43].

As we mentioned earlier, the matching returned by the CAM algorithm can be wasteful, $i.e.$ there might be an applicant who prefers a school which still has empty seats that would allow this applicant to be feasibly matched to that school (see example 0.2 above). However, by doing so, it would violate the priority of another applicant, who also prefers that school (which would in turn violate envy-freeness). The impossibility does not say whether the wastefulness "is large" or not, in particular it might be the case that, in practice, the amount of empty seats left is relatively small.

D) Algorithm for student-optimal weakly envy free assignments

Together with the notion of weak-envy freeness, Delacrétaz, Kominers, and Teytelboym (2023) propose the *Knapsack deferred acceptance* (KDA) mechanism[44] for refugee resettlement which we adapt to our setting in algorithm 3 in the Appendix.

This algorithm returns the **unique family-optimal weakly envy-free matching** (SOWFM). The main difference with the CAM algorithm and resulting matching lies in the interpretation of the concept of justified envy. The CAM procedure provides assignments that strictly adhere to priorities, even if it means leaving a potentially high number of seats vacant. KDA relaxes the definition of justified envy to accommodate more families as long as the total demand does not exceed capacity in any day.

Intuitively, this algorithm has a "cumulative" feature, that is, when a school temporarily keeps students, it considers all the students who have ever applied to it, even if they were rejected in an earlier step. It will weakly accommodate students up to capacity and adjust the cut-off based on the definition of weak accommodation.

How many more families can be accommodated with KDA rather than CAM depends on the setting. Because of the iterative and dynamic nature of the algorithms, a small change in some daycare can induce large changes in the pool of assigned families. Ultimately, both algorithms cannot ensure a non-wasteful assignment and our next question is: can we increase the number of assigned families further while ensuring a fair matching ? By optimality of both algorithms, it is clear that we cannot do better with these definitions of fairness. But we can further adjust the definition and see if we can improve welfare by sitting more students without harm.

E) No strategy-proofness in the assignment over weekdays

As in our previous section, one may wonder if the applicants have an incentive to truthfully report their preferences[45] $(\succ_i)_i$ in problems with multidimensional constraints. However, Kamada and Kojima (2023) show that **no algorithm is optimal envy-free and strategy-proof** and Delacrétaz, Kominers, and Teytelboym (2023) further prove that there are **no strategy-proof and family-optimal weakly envy-free** mechanism. They also propose an alternative algorithm that is weakly envy-free and strategy-proof but does not yield the most preferred assignment. We recently adapted this algorithm to this setting but did not use it in practice.

This negative result is strong: no algorithm can give the applicants an incentive to report truthfully while always returning an optimal envy-free matching[46]. However, it "just" means that we can find one example where one family, at a particular preference profile, knowing the preference profile of others, has an incentive to misreport. The theorem does not say whether "it is easy" to manipulate for a family, whether it is always possible to manipulate and so on. For instance, in large markets, the cut-off of a school is determined by the highest-priority applicant who

[43] In Kamada and Kojima (2023) terms, our setting respects the *general upper bound* property. Thus, the algorithm returns an optimal envy-free matching.

[44] In previous versions of their work, this algorithm was named *Cascading Multidimensional Deferred Acceptance* (CMDA).

[45] Note that here, we assume that applicants are only able to misreport their preferences over schools and not their required days. Indeed, it is consistent with our primitives since we assumed that applicants have separate preferences that do not vary depending on the days assigned. One can see the days as a hard constraint for families, say their job or obligations that they cannot change. Relaxing this assumption would only strengthen the negative result on the incentive properties.

[46] Note that the optimality is needed. Indeed, the mechanism which leaves all schools empty is trivially strategy-proof and envy-free. Whether there exists a non trivial strategy-proof and envy-free matching is an open question. Delacrétaz, Kominers, and Teytelboym (2023) propose such algorithm for a weaker notion of stability, we refer the reader to their article for details.

is rejected from it, and this depends on the entire distribution of student preferences as well as school priorities. It appears unlikely that any one particular student is in a position to influence the cut-off in any significant manner. Thus, one conjecture could be that such manipulations are rare in practice or too complicated for an applicant to find but more research are needed.

IV.4 Main results

A) A Pareto improving minor adjustment

By ensuring envy-free or weakly envy-free matchings, CAM and KDA provide assignments that can be wasteful. That is the price of envy-freeness. Kamada and Kojima (2023) tried to define *feasible envy* as a criteria for stable matching $i.e.$ considering that those with infeasible demands cannot envy family with lower priorities that are feasible. They show that a stable matching may not exist. What about *initially feasible envy* ? Could we say that a demand that cannot be satisfied *ex-ante* should not be considered in the application ? What does it change if we do that ?

a) Definitions and intuition Let us formally define **initial feasibility**:

Definition 0.12 (Initially feasible application)**.**

We say that a school s is **initially feasible** for family i if before starting the assignment:

- **case without soft quotas**
$$\nexists\, t \ s.t. \ d_i^t > q_s^t$$
- **case with soft-quotas**
$$\nexists\, t, g \ s.t. \ d_i^t > \underline{q_s}^{tg}$$

In plain words, school[47] s is initially feasible if there is no seat/day that family i demands that is not available at school s[48]. Note that since $d_i^t \leq 1$, it implies just that $q_s^t = 0$. In particular, if $\forall s, t, q_s^t > 0$ then the problem is initially feasible.

From there, we define a new criterion of stability that we call **initially feasible envy**:

Definition 0.13 (Initially feasible envy)**.**

We say that applicant i has **initially feasible envy** towards i' if there exists a school (or bucket) s s.t. $s \succ_i \mu_i$ and

1. school s is initially feasible for i and
2. there exists applicant $i' \in \mu_s$ with $\pi_{i'}^s < \pi_i^s$.

If either of the two conditions 1) and 2) does not hold, then there is no feasible envy. In particular, this definition tolerates justified envy of applicants who are not initially feasible. This deviation is *deemed* harmless because these applicants could never have been accepted in this daycare.

To get an insight from what we intend to achieve, we motivate the following result with Example .5 in Appendix F.II. This solution was developed to limit vacancies in second rounds of assignment. Indeed, there are often many daycares with empty seats on some days and we saw that parents who could not be seated *ex-ante* were blocking feasible assignments. This example suggests that if we tolerate infeasible envy, then we may (weakly) improve the assignment of all applicants while ensuring no initially feasible envy.

We want to see whether this particular result is valid in the general case. In other words: can we prove that the SOFM with initially feasible demands is weakly better than the SOFM with full preferences for all individuals ? Does this result also apply to weakly envy-free assignments ?

[47] Or the group/seats g at school s

[48] Or at the bucket g at school s

b) Matchings free of initially (weakly) feasible envy improve assignment for everyone Consider a daycare assignment mechanism with multidimensional constraints, its problem $P = (I, S, B, \mathbf{q}, \underline{q}, \theta)$ with student preferences $\succ_i$. Let $\phi := P \rightarrow \mu$ be the student optimal fair assignment (SOFA) returned by algorithm ϕ. Let $\bar{\succ}_i$ denotes initially feasible demands. Then we have the following theorem:

Theorem 1. *The assignment μ' obtained with the same mechanism ϕ on* **initially feasible demands Pareto dominates** *μ. We call this equilibrium the* **Student optimal initially feasible fair assignment** *(SOIFFA)*

See proof in Appendix F.II.

Theorem 1 shows that we can assign a (weakly) better outcome to all parents if before the assignment we adjust their preference order and remove demands that cannot be satisfied *ex-ante*. In the same spirit, Kesten (2010) proposes an efficiency-adjusted deferred acceptance mechanism (EADAM), that allows a student to consent to waive a certain priority that has no effect on his or her assignment. Here, policymakers modify parents reported preferences and cause them no harm, but may help many others benefit as a consequence. Although it seems like a harmless *low hanging fruit* to reduce vacancies and satisfy more families, agents may value the idea that all their choices were considered in the assignment and on the other side, policymakers may be reluctant to such interventions. On the other hand, since everyone gets a *weakly* preferred outcome, the adjustment (weakly) improves welfare for all agents. In a setting where the number of choices is not restricted, this adjustment can make participation to the assignment mechanism *safer* in the sense that if a choice is doomed to be rejected it is discarded *ex-ante* and does not negatively affect others. This result is almost ineffective in the first rounds of assignment when there are lots of seats for every day in almost every bucket. In the second rounds, this theorem proves useful.

B) General solutions to the daycare assignment problem

So far, we did not consider student groups and buckets in the definition of the problem but empirical application requires a solution for the problem with multidimensional and diversity constraints. Again, our overall strategy is to transform the problem with diversity constraints in an equivalent and well defined problem that can be solved with CAM or KDA. Before, the "*trick*" was to break diversity requirement into unidimensional unitary buckets. Now, since each school has multidimensional capacities, *i.e.* one for each day, this is not possible anymore. Moreover, we don't want unitary buckets since our goal is precisely to try and combine as many demands as we can over weekdays. In fact, we need a definition of buckets that contains **as many capacities as possible**.

Buckets are defined by policymakers and daycare providers and *should* correspond to empirical constraints (a specific room, equipment, a staff-to-children ratio and so on). They constitute the bundle of capacities across days over which we can combine demands following priorities. **Capacities across buckets cannot be combined**. For our purpose here, we do as before: a bucket b possesses elements $(s_b, \succeq_b, c_b)$ but where now capacities c_b are defined over days so that $c_b(t)$ is the number of seats in bucket b for day t. We let $\tilde{\mu}$ be a matching of students to buckets.

Example 0.3 (Capacities in different buckets)**.** Consider a school s with at least two buckets k and l in B_s and let $c_k = (1,1,0,0,0)$ and $c_l = (0,0,0,1,1)$. Student i asks $d_i(t) = (1,1,0,1,1)$ and both buckets accept student i although not in the same order[49]. In our definition, student i will apply to each bucket and regardless of the definition of envy we choose, i cannot be accommodated in any bucket. However, in school s, there appears to be enough capacity to accommodate i across buckets. In our setting we assume that if capacities cannot be bundled in the same bucket, that is because a child cannot be some days in one bucket, and the other days in another, even if student i is eligible to both buckets.

[49] Otherwise, they would have been aggregated together.

a) Stability definitions in the problem with multidimensional constraints and soft quotas Without buckets, we introduce the **strong envy-freeness** definition of Kamada and Kojima (2023) and the **weak envy-freeness** notion of Delacrétaz, Kominers, and Teytelboym (2023). The main difference lies in the definition of feasibility. where the latter allows *weak accommodation* and the former does not. Both rely on priorities. Our Theorem 1 adjust the set of preferences to limit externalities but rely on the same definition otherwise. All the following results can use initially infeasible demands. Let us give a formal definition of envy-freeness with diversity quotas:

Definition 0.14 (Strong justified envy with diversity quotas)**.** Given a matching $\tilde{\mu}$ of students to buckets for the problem, applicant i in I from group $g_i = g$ with assignment $\tilde{\mu}_i = b$ has *strong justified envy with diversity quotas* over family $j \neq i$ of group $g_j = g'$ with assignment $\tilde{\mu}_j = b'$ if and only if

1. $s_{b'} \succ_i s_b$: *Student i prefers the school of the bucket of j* **and**
2. $\tilde{\pi}_{ib'} \geq 0$: *Student i is eligible for seats in b'* **and**
3. $\tilde{\pi}_{ib'} > \tilde{\pi}_{jb'}$: i has higher priority score in bucket b' than student j and $\tilde{b'}$ cannot weakly accommodate j alongside with all families with higher priority than j at b' that weakly prefer $s_{b'}$ to the school of their assigned bucket.

The definition is simply a version of weak envy-freeness in a matching of students to buckets where only the preferences over schools matter to define whether a student has a strong justified envy.

b) Assignment mechanisms for problems with multidimensional constraints and diversity quotas Policymakers have important choices to make daycare assignment marketplaces functional and we want to emphasise what it means in terms of fairness, optimality and strategy proofness. Our main message can be wrapped up in Definition 0.15 and Theorem 2:

Definition 0.15 (Daycare assignment marketplace (DAM))**.**

A **daycare assignment marketplace** (DAM) is built upon three elements policymakers must **define**:

1. A **version of the problem** : considering demands with specific days or not ;
2. A **partition of capacities** into buckets ;
3. A **definition of *fairness*** : whether they want an envy-free or weakly envy-free assignment, on initially feasible demands or all demands.

The definition of a DAM fixes the problem $P := (I, S, B, \mathbf{q}, \underline{\mathbf{q}}, \theta)$ which follows definition 0.5. Using definitions 0.3 and 0.4 to create new priorities and preferences with buckets, we map elements of P in a simpler problem. The definition of fairness also imply the target properties on assignment μ and therefore, the appropriate mechanism ϕ.

Hence, once policymakers fix a DAM, we propose the following theorem:

Theorem 2. *For a given daycare assignment marketplace (DAM), there exists a unique assignment that is the most preferred by all families, respects priorities, multidimensional and diversity constraints. We call this particular assignment the* **Student optimal fair assignment** *(SOFA) which can be found by one of the three algorithms.*

Proof. The proof directly follows from the definition of the problem with diversity quotas and the properties of the mechanisms used. In particular, the careful definition of buckets, priorities and preference order over buckets in each model maps the original problem into one that fits the definition of the input for each algorithm. Their properties presented earlier directly apply to the problem with diversity quotas and our definition of envy-freeness with diversity quotas.

More precisely:

1. The SPDA algorithm returns the only non-wasteful strategy-proof SOFA for DAM with diversity constraints.

2. The CAM algorithm returns the only SOFA for DAM with multidimensional constraints, diversity constraints and fairness defined by envy-freeness.

3. The KDA algorithm returns the SOFA for the DAM with multidimensional constraints, diversity constraints, and fairness defined by weak-envy freeness.

4. From an assignment with CAM or KDA on a DAM with multidimensional constraints and diversity constraints, the same mechanism on initially feasible demands Pareto dominates the assignment with all demands.

QED

Definition 0.15 emphasises how the design of an assignment depends on a set of political decisions. The definition of the problem has consequences on what is feasible, on achievable properties and so on. When policymakers define a DAM, we provide clear solutions with traceable assignments. All models return student-optimal matchings. There are no better assignment that at least one family prefers that respects the **chosen** definition of *fairness*. This is true with our definition of buckets as we cannot feasibly accommodate students without breaking the definition of buckets and the associated group ranks.

When one defines a DAM, many details will have consequences. In particular, the partition into bucket is a very sensitive task. An envy-free DAM and a weak-envy free DAM will not yield the same outcome, even if the rest of the problem is identical.

c) Extension: allowing mobility of already assigned families In practice, some parents assigned in previous committees may want to participate in the assignment mechanism again in order to obtain another daycare. Some settings allow such mobility although they usually struggle to define a fair way of doing so.

The use of the assignment with diversity constraints makes this operation easy[50]. One simply requires these parents to submit their preferences and municipalities to set their level of priority in the childcare they demand and flag these files in the data. Then, we proceed as follow:

1) Add the daycare they come from as their last school ranked
2) Add the capacity they would use in this daycare to quotas and buckets.
3) Set these student's priority over this bucket so that they (and only they) have the highest priority if they end-up unassigned to preferred school.
4) Run the algorithm corresponding to the DAM.

Combe, Tercieux, and Terrier (2022) discuss in details the properties of a similar assignment mechanism to assign teachers in middle and high schools in France. In practice, we only encountered one municipality that allowed such transfers.

d) Main limit: multiple demands While our models are designed to accommodate individual applicants (students), the reality often involves families submitting multiple demands simultaneously. Twins are easily accommodated as they usually are of the same type and receive priority bonuses. To foster joint assignment, we pick the highest lottery realisation within the family, and assign them the highest (+ $1/10^6$). That way, they always follow each other in the demand sets and separation only occurs when they end up last in the bucket. Assignment committees usually manage to *push the walls* and accommodate them. Otherwise, they unmatch them and they go again in the next round. The problem is harder when families have children of different ages and submit several choices. Handling this situation becomes challenging due to variations in competition across different daycares and the unique constraints each bucket imposes. This practical constraint is not fully considered in our model, and

[50] With initial endowments and priorities, it is well known that there is no matching that is both individually rational and stable in the sense that no agent has justified envy. To ensure the compatibility between the two notions of IR and stability, the concept of stability has been relaxed to exclude blocking pairs caused by a daycare slot that is assigned to its initial owner. A condition known as μ_0 $stability$. See Combe and Schlegel (2021)

further research is required to address it effectively. In practice, these challenges have significant consequences, as committees must sometimes deviate from theoretically optimal fair assignments. Even minor deviations can result in substantial justified envy, making it difficult to individually justify the assignments. We discuss these practical implications in the following section.

In subsection IV.5, we illustrate the tension between *fairness* and *wastefulness* by verifying the student-optimal and envy-freeness properties of our assignments and compare outcomes of different algorithms.

IV.5 Delivering the SOFA: empirical illustration

We focus on the last year of assignment of VRA, the first case study presented in sub-section III.1 in section III. VRA defines the DAM with demands over weekdays, groups by age (rounded in months) on September the 1st. In 2020, they chose a definition of *fairness* based on envy-freeness and from 2021 onward, they opted for assignment on initially feasible demands. We coded all solutions of Theorem 2 in Matlab and used them to assign daycare seats instead of assignment committees. The main testable properties of our assignment are **envy-freeness** on **initially feasible demands** and **optimality** accounting for **diversity constraints**.

One way to show these properties at work is to rebuild the demand set of each bucket, retrieve the cut-off of the last accepted file and observe each file's status[51]. The properties have implications for the content of this table. In particular, families with rankings above the threshold of the last admission can only be allocated to this section or to a daycare centre higher up on their preference list. Families ranked below this threshold cannot be accommodated in this bucket to prevent justified envy. If the daycare facility has flexible age groups, they might find placement in another bucket of the same school eventually. Otherwise, they will either be allocated to a different daycare centre (the highest one on their preference list where they are eligible) or placed on the waiting list. In practice, this step serves both as a verification and means to build justifications for each assignment[52].

A) Student optimal fair matching and transparency

For every buckets $b \in B$, let $\underline{c}_s$ be the **cut-off** of bucket b which is an integer in $\{0, \dots, N\}$ where $N = |I|$ is the number of applicants. Once ties are broken and the scores $\tilde{pi}_{is}$ at bucket s are obtained, order the applicants by decreasing order of score and let $\tilde{r}_{ib}$ be the **rank of applicant** i in this ordering.

Given a collection of cut-offs $c := (c_b)_B$, let:

- $A_i(c) := \{b \in B : \tilde{r}_{ib} \leq c_b\}$ be the set of **accessible buckets** for applicant i.
- $D_b(b) := \{i \in I : b \in A_i(c)$ and $b \tilde{\succeq}_i b'$ for $b' \in A_i(c) \backslash \{b\}\}$ be the **demand set** of bucket b at the cut-offs c.

In plain words, accessible buckets are those where the rank of an applicant i in that bucket is lower than the cut-off ; the demand set is composed of all applicants in their most favourite accessible bucket.

Figure 5 presents a proportional stacked bar chart illustrating demand sets rank relative to the last admitted file in each bucket. This chart verifies and emphasises the key features of our models. For policymakers, these properties can be summarised in Fact 1.

Fact 1. *Each registered family was assigned to their most preferred daycare centre from a set of options where they met the assignment criteria. This allocation was determined by considering initially feasible demands, which were assigned by preference order following priorities within age groups and subject to capacity constraints over different days.*

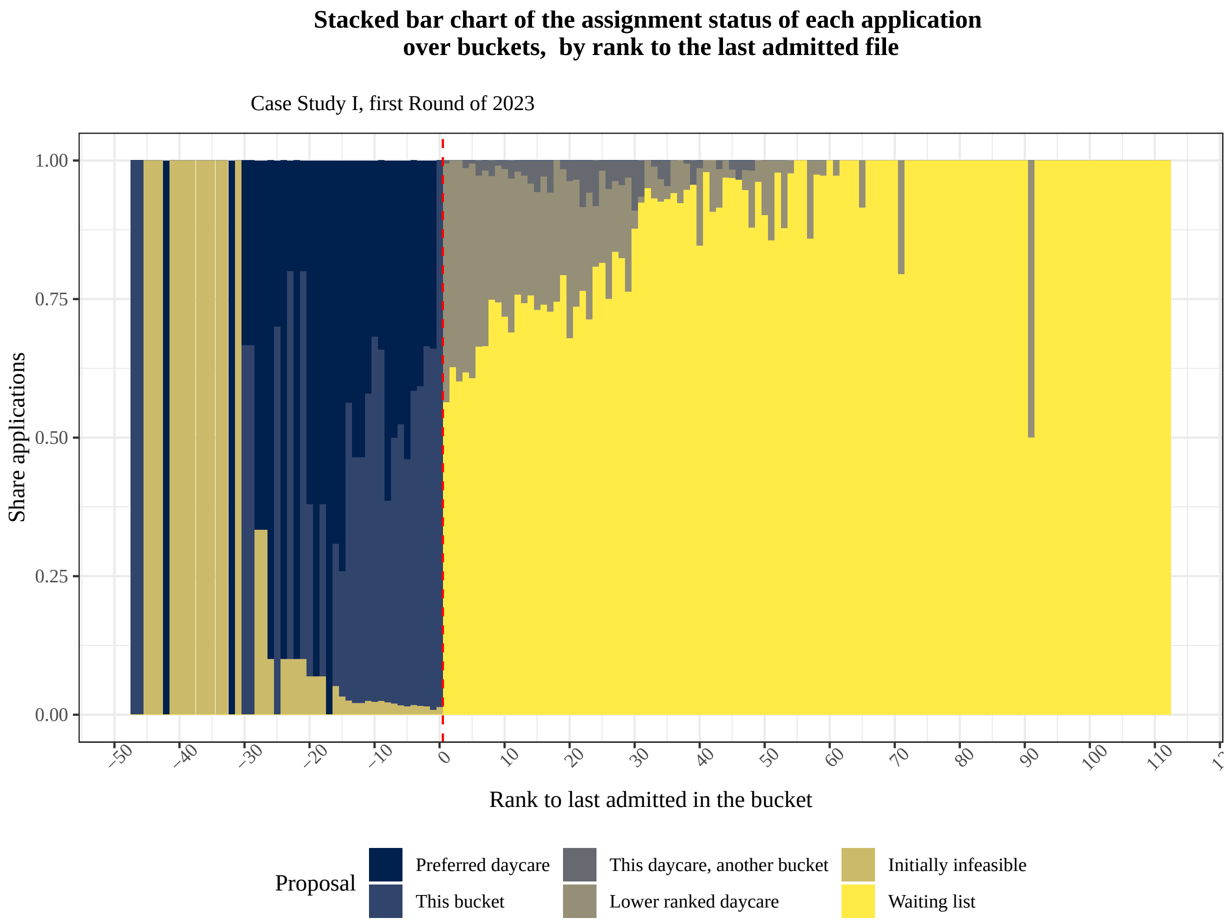
[51] In practice, we start by selecting the 1267 records from the assignment file of this round and pivot it over each reported choice. This file is made of 5005 reported choices. Next, we match each choice to the corresponding buckets. Using age bounds and flexibility criteria between age groups, we determine which choices are eligible for each section and exclude those that are not. Once this step is completed, we have a database of 3740 choices over buckets. From there, we can reconstruct the actual priorities by ranking families within each bucket based on the priority age group, then the score, and finally the lottery number. We retain the order of files in each bucket and keep the ranking of the last admitted in each section.

[52] Each, year we provide an Excel file with one sheet per daycare. In each sheet, we provide the demand set over each bucket and all relevant information to justify assignment. In particular, we show the decreasing number of available seats for each day, and the assignment of every file.

While initially infeasible demands are less common in first-round assignments, they still exist. Some age categories with older children have certain days with zero available capacities and some initially infeasible preferences were removed before the assignment. Had we retained them, 6 fewer children would have been assigned. In the second round, where zero capacities are more prevalent, the welfare improvement of SOIFFA over SOFA becomes much more salient. During that year, our assignments increased from 100 to 115 when considering initially feasible demands, which represents a 15% rise in admissions.

This last finding underscores the significance of a precise definition of *fairness* in the daycare assignment marketplace. This is especially crucial if policymakers aim to minimise inefficiency while upholding the principles of due process.

Figure 5: Student-optimal initially feasible fair matching with diversity constraints and initially feasible demands: an illustration



Sources: ISAJE, Case Study I − 2023, First round only.
Notes: This figure is based on initially feasible family preferences. Using age group and score definitions, we calculate the rank of each applicant in each bucket where they applies.
These calculations allow us to verify the properties of the algorithm.
Negative values are above the last admitted applicant's in this bucket.
Among higher ranked families, applications are either in a preferred daycare or in this bucket.
Below, families are either assigned to another daycare, to another bucket of the same daycare when buckets are flexible, or placed on the waiting list.

B) Assignments with alternative procedures and final assignments

The CAM algorithm is the most rigorous with priorities and tolerates no weak accommodation. Allowing deviations for initially infeasible demands improves assignments for everyone (SOIFFA) and can theoretically seat more families. However, the assignment can be wasteful. We illustrate the leftovers by days across daycares after the assignment we just presented in Figure D.25 in the Appendix. There are many days lefts but there are few within buckets and mostly on Wednesdays, the least demanded day. Another interesting question is to compare the assignment with *weak envy freeness* with KDA.

Assessing the cost of envy-freeness Table 1 compares the SOIFFA with alternative assignments[53] and the final offer.

The SOFM with all demands is the most restrictive regarding priorities and tolerates no justified envy. It yields 402 assignments and 865 in the waiting list. Removing initially infeasible demands adds 6 families, 2 more first choices, 3 third choices, and one fourth choice. In this market, allowing weak accommodations on all demands gives the same number of seated families as removing initially infeasible demands. However, KDA serves more second choices and weakly dominates the SOIFFA.

Table 1: Comparison of assignments with different procedures and the final assignment

Results	SOFM All demands	SOFM Initially feasible	KDA all demands	Brut force	Final assignment
N 1st choice	238	240	240	264	**258**
N 2nd choice	85	85	87	98	**96**
N 3rd choice	46	49	48	63	**52**
N 4+ choice	33	34	33	42	**33**
Waiting list	865	859	859	800	**825**
Total offer	402	408	408	467	**439**
Total	1,267	1,267	1,267	1,267	**1,264**

Sources: ISAJE, Case Study I - 2023, first round only.
Notes: Columns 2, 4, 5 are simulations based on the same files.
Column 1 is the SOFA obtained using the CAM algorithm on all demands.
Column 2 is the SOIFFA delivered to the early childhood department.
Column 3 is the SOFA obtained using the KDA algorithm on all demands.
Brut force simulations assign as many feasible demands as possible without following priorities. They give an upper bound of the total number of children that could be assigned without priorities.
Final assignment indicates the proposals made to the families.

[53] SOFM uses the CAM algorithm, the SOFA returned by KDA uses weak-envy freeness to define fairness.

Respecting priorities induces some rather large vacancies. Filling as many vacancies as possible by brute force[54], one could have assigned 467 children. Fairness *cost* 59 vacancies that could have been filled. However, this assignment has absolutely no property apart from leaving less (but not necessarily zero) empty seats.

However, the assignment committee observes these vacancies, and in the final assignment, they assign 31 more files than the SOIFFA. Some of these changes are quite inevitable, particularly because in the current use, we are not able to guarantee that multiple demands by the same parents are assigned the same daycare. In this example, 29 of the modified applications correspond to multiple demands. Since the procedure still assigns a maximum number of applications, moving around thirty of them forces the reassignment of others. In the end, the early childhood committee modified the allocation of 94 applications.

Adjustments and deviations from envy-free assignments Table D.9 in the Appendix compares the deviation from the SOIFFA in the final assignment. The committee added 47 files from the waiting list to assign them a place and improved the assignment of 17 files. However, to make these adjustments, they had to revoke assignments for 15 families and downgrade 15 others to less-preferred childcare centres in their preference order. These adjustments inevitably introduce justified envy, and not just from those who were moved, but anyone with lower priority in the demand set. Therefore, the assignment no longer respects the priorities established by the territory and we cannot justify the allocation of moved files based on the rules of the procedure. These adjustments have implications for transparency. However, we believe that this issue can be resolved by defining conditions under which the committee can deviate from priorities. One possible solution would be to establish a voted, public, binding ethical charter. This would allow the allocation committee to make deviations from priorities for specific and legitimate reasons, such as "not separating twins", while restoring transparency to the decisions. The solutions we propose remain tools supposed to support decision-making. They efficiently handle large demands while eliminating technical challenges related to priority compliance, age groups, and capacity constraints. However, access to childcare may require more tailored decision-making in some cases.

V Market design in the field: *Who gets what and why ?*

In what follows, we analyse the functioning of our assignment procedures in the field, with results from the two case studies introduced in section III. We use our unique position of *plumber, engineer and scientists* (Duflo 2017) to analyse the consequences of their main features. First, we focus on the Case Study I to analyse the role of priorities using simulated assignments with different priority rules. Then, we detail the consequences of time since registration. Finally, we use the second case study to show the effects of buckets.

As discussed in section II, The evaluation of a market design require normative criteria. It hinges not only on its consequences but also on whether these consequences are desirable, undesirable, intentional or not. In the face of any consequence, we may be called upon to make normative judgements. Are we morally permitted to bring it about? Are we morally obliged to bring it about? As noted by Li (2017a), "*informed neutrality requires us to engage with reasonable ethical theories, not to resolve which one is correct, but to formalize the features that they regard as morally relevant*". In this research, our *moral compass* is the Matthew effect and our analysis aim at understanding how DAM foster or mitigate it.

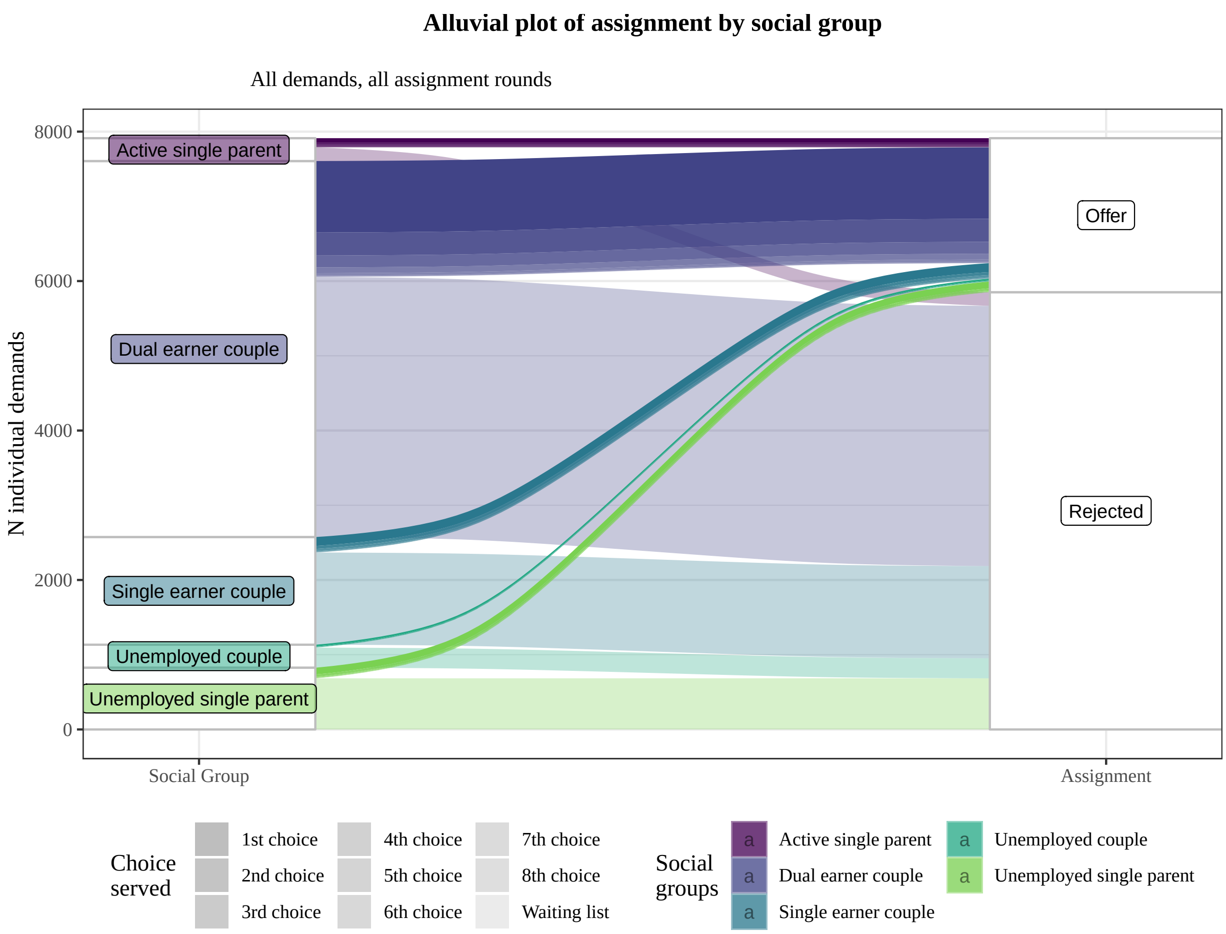
[54] The brute force algorithm simply fills as many children as possible proceeding as followed. 1) Sort Buckets, 2) **For** each bucket **do** : i) Order families. ii) Match if seats available. iii) Move to the next family. 3) Move to the next bucket. 4) Repeat Until Done. **EndFor**

V.1 Case study I: Who gets what ? General overview

Main results of the assignments Each year, we processed an average of 1144 applications in the first rounds and 834 in the second rounds (See table D.7 in the Appendix). In this first year, we allocated seats to 400 families, which accounted for approximately 40% of the total demand. About two-thirds of seated families received their first choice. The outbreak of the pandemic resulted in minimal new registrations for the second round, during which we successfully assigned an additional 111 children. Over the years, the competition intensified, demand surged in 2022 and despite more available capacities, the proportion of accepted families got lower and lower. On average, 36% of demands were assigned a seat in first rounds.

In terms of social composition, 64% of all files come from dual earners, 18% single-earner couples and 4% couples where neither work (See Table C.5 in Appendix A). In this market, active single parents represent 4 % of all demands, and single parents with no job 10 %. Figure 6 summarises the distribution of assignments across all rounds and years by social group. Overall, double earner couples get 75% of all seats while representing 64% of all files.

Figure 6: Flows of assignment by social group



Sources: ISAJE, Case Study I − 2020 : 2023.
Note:Number of assigned and rejected files in every assignment over the four years by social group.

Table D.8 present the average characteristics of seated and waitlisted families, showing significant differences in virtually all included measures. Families who submit only one wish, on average, have a 6-point lower probability of being admitted. On the other hand, those who submit at least three wishes are 10 points more likely to receive an offer than those who rank fewer daycares. Admitted families tend to enroll much earlier, with a significant number doing so before delivery. For instance, admitted children are, on average, one month older than those on the waiting list. Moreover, children who are admitted in September have been registered for an average of 13 months, which is

3.5 months longer than those on the waiting list, on average. It is also worth noting that the average child age at the time of registration is 2.5 months younger among assigned families than among waitlisted ones. Additionally, 65% of admitted children were enrolled before birth, compared to 47% of rejected children.

Interpretations in the context of the pandemic Among other things, the economic uncertainty brought about by the Covid-19 pandemic had large impacts on family dynamics[55]. The INSEE (2021) collection show that the first lock-down led to a significant decline in births. This is especially true for places heavily affected by the health aspects of the pandemic but less so economically, such as VRA. Births between December and February in this region decreased by 13.37% compared to the three previous years' average for the same period. There was a slight increase in births, by only 2.55%, from March to May[56].

In our report for the local administration, we describe in details an unexpected effect of the pandemic that changed the distribution of registrations and birth dates, causing a reduction in differences between social groups in the 2021 assignment[57]. Figure C.17 in the Appendix plots the cumulative distribution to show stochastic dominance. In 2021, with these enrollment lags, the empirical cumulative distribution of priorities for single parents and dual-working couples overlaps on the full support of priorities and 1/4 of family without work have higher priorities than half of dual earner couples. Said differently, the scores no longer allow to sort groups along policymakers preferences. Facing more complains from working families, they revised it in 2022, giving a massive priority boost to dual-working couples and active single-parent families.

V.2 The 2022 priority reforms and its consequences

The 2022 reform simply consists in adding 5 more points to dual earner couples and single parents working leaving everything else the same. The former went from 3 to 8 and the latter from 5 to 9, while cumulating priorities of other criteria. In 2022, the mean priority score for other group is 8.3. Both groups have thus been given higher priorities than the average priorities among other parents. However, parents were not informed of this change and could not anticipate its effects. In spring 2022, the new weights were simply implemented in our algorithms and we were not involved or informed before we received the data. The boost is so massive that **only 30 % of single parents without a job have a higher priority than the 10 % dual earners with the lowest priorities**. However, the 2022 demand is much larger than the previous years, with more demands from couples with one parent working and single parents, more often for part times for both groups (See Figure C.20 in the Appendix). Most of them being waitlisted, a large share remained registered. Assuming that the *ranking* of weights across social groups can be interpreted as policymakers' preferences, Table D.8 (in the Appendix) shows that the differences between assigned and waitlisted families satisfy policymakers' preferences on average. Yet, they were disappointed again by the distribution of assignments in 2023 as the share of assigned working parents shrank, despite a large increase in their priorities in 2022. In what follows, we analyse how and why by answering a series of *what if* questions on the role of priorities.

[55] For instance, Barbuscia et al. (2023) show large increases in separation rates, especially among young couples, in the six months following the initial lock-down. Individuals who were unemployed or had lower incomes before the pandemic are more likely to separate immediately after the lock-down. Degraded financial situation for men is associated with a higher risk of separation throughout this period. Lu et al. (2022) reveal a continuous rise in anxiety and depression symptoms during lock-downs in France. A small group of people experienced a significant and long-lasting deterioration in their mental and physical health. Breton et al. (2021) document major structural demographic changes, including a decrease in births, marriages, immigration, and an increase in deaths.

[56] Among families registered for daycare in our data, the drop and rebound are almost of the same magnitude.

[57] In brief, registration were not possible during the first lock-down and no national political orientation regarding the reopening of daycare centres were made before late August 2020. There are far fewer families registered between March and August compared to the other years and therefore less variation in priorities. Furthermore, the pandemic affected the timing of births. 9 months after the first lock-down, there are far fewer children born compared to other months but many pregnancies were simply postponed and there is a surge of children born in spring 2021.

A) What if there were no or other reforms ? Simulated counterfactual analysis

In this setting, if we had been given different priority parameters, we would have treated the assignment problem the same way with these alternative priorities. Policymakers could have asked the effects of alternative priorities before voting on the 2022 weights[58]. In the Neyman-Rubin framework, causal analysis is often seen as a *missing data* problem where only one potential outcome can be observed (Imbens 2020). Other approaches of causality use theory to either draw causal graphs or structural econometric models that impose sufficient condition for identification (J. J. Heckman and Pinto 2022). Here, the economic model is the actual model. We can actually **know** the missing individual counterfactual outcomes by simply running our algorithm with alternative priority rules, holding everything else constant. In particular, we keep the same lottery realisation so that ranks only vary with the change we test. In particular, we ask:

1) **What if the score was not changed ?** We simulate a "Control" assignment with the former weights with the 2022 data;
2) **What if there were no priorities at all, only age groups ?** We simulate a "Random" assignment using only the random tie breaker to sort applicants for assignments in 2022 and 2023;
3) **What if priorities stopped considering time since registration ?** We simulate a "Social" assignment using all priority criteria but time since registration for assignments in 2022 and 2023;

Our strategy ends-up very close to the model of Kasy and Abebe (2021) on the causal impact of algorithms on inequalities. They propose a 4-step algorithm audit using distributional decomposition: i) Normative choices: what are the relevant outcomes, measures of welfare and/or inequalities, and so on ii) Calculation of influence function: using a machine learning algorithm, iii) Causal effect estimations and iv) Counterfactual assignment probabilities.

They require influence function because their framework is based on predictive algorithms. In our setting, we do not need these estimations and can directly predict individual counterfactuals instead. As normative choice, we analyse how the assignment probabilities conditional on social groups evolve and later, we analyse segregation.

We take a simple potential outcomes notation and let $Y_i = Y_i(T)$ be the observed assignment with the reform. Let $Y_i(c) \quad c \in \{C, R, S\}$ be the counterfactual assignment in the Control, Random and Social scenario. We note $D_i = \mathbb{1}(d = c)$ and X denote social groups. For each social group x We want to estimate :

$$\mathbb{E}[Y_i(c) - Y_i(T) | X = x]$$

$I.e.$ the expected difference in assignment probability between the observed and counterfactual situation. For simplicity[59], we define Y as a dummy for being assigned. To estimate these, we simply build a stacked database for each values of D and actual and counterfactual assignments and analyse the differences like a match-pair design. We have two observations for each pair of individuals i, we simply estimate the following equation separately by social groups using OLS and cluster robust standard errors at the pair level:

$$Y_{idx} = \alpha_i + \beta_x \mathbb{1}(d = c) + \varepsilon_{idx} \quad \forall i \text{ s.t. } X_i = x \tag{1}$$

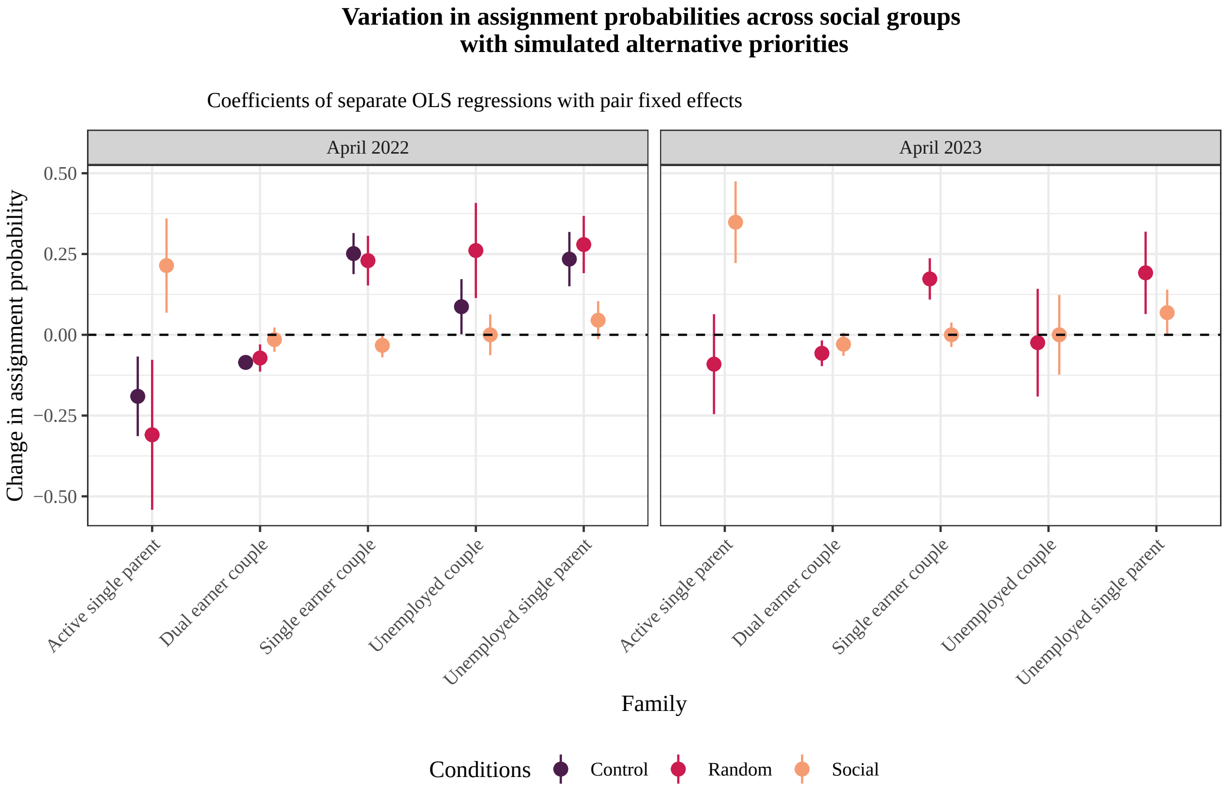
Where β_x retrieves a weighted average of within individual difference in assignment from observed to simulated. The cluster robust variance accounts for the correlation within individuals. Since nothing else changes between the two conditions but the priority parameter we manipulate, we interpret these estimates as the **effects** of the priority we test. For instance, when we compare the average difference between actual and social scenario for active single parents, we measure the average effect of *suddenly* removing the early registration premium on the average probability of a single parent to be assigned. Confidence intervals are clustered by pair of individual and used to gauge variability due to group size using large sample approximation for simplicity[60]. We present the results of these estimations for 2022 in Figure 7.

[58] In fact, they did ask for such simulations after the first year. At that time, the debate was about setting priorities for parents asking daycare in the same city they live. The question was about territorial segregation and our analysis showed that such change would i) drastically reduce assignment probabilities of parents living in cities without a daycare and 2) strongly favour residents of the two largest cities. This reform project was dropped after we presented our evidence.

[59] We could test more precise hypotheses like better or worse assignments, same assignment and so on. While these are interesting for policymakers, we don't think they would bring much in this research paper.

[60] Note that a reform where parents would know that time since registration does not matter would affect the composition of the demand. The external validity of these estimates strongly depends on the self-censoring and strategic timing this criteria generate among parents who consider daycare. An important question that we will be able to adress matching these data with those of the National Family allowance fund.

Figure 7: Effects of priority criteria on assignment probability by social groups



Sources: ISAJE, Case Study I − 2022 : 2023 − first rounds only.
Notes: We simulated assignments using alternative definition of priority scores while all other parameters are held constant.
We stacked simulations in a database and present the coefficients of separate regressions by social groups of a dummy for assignment over scenario dummies and individual fixed effects. Confidence intervals are clustered at the individual level.
'Actual' assignment is the reference, 'Control' is the scenario without the reform i.e. the 2021 weights,
'Random' is the scenario without priorities i.e. students are sorted by decreasing lottery realisation with age buckets,
and 'social' use the priorities without weights for time since registration. In 2022, it uses reformed weights.
We use cluster robust standards errors at the individual level to build 95% confidence intervals.

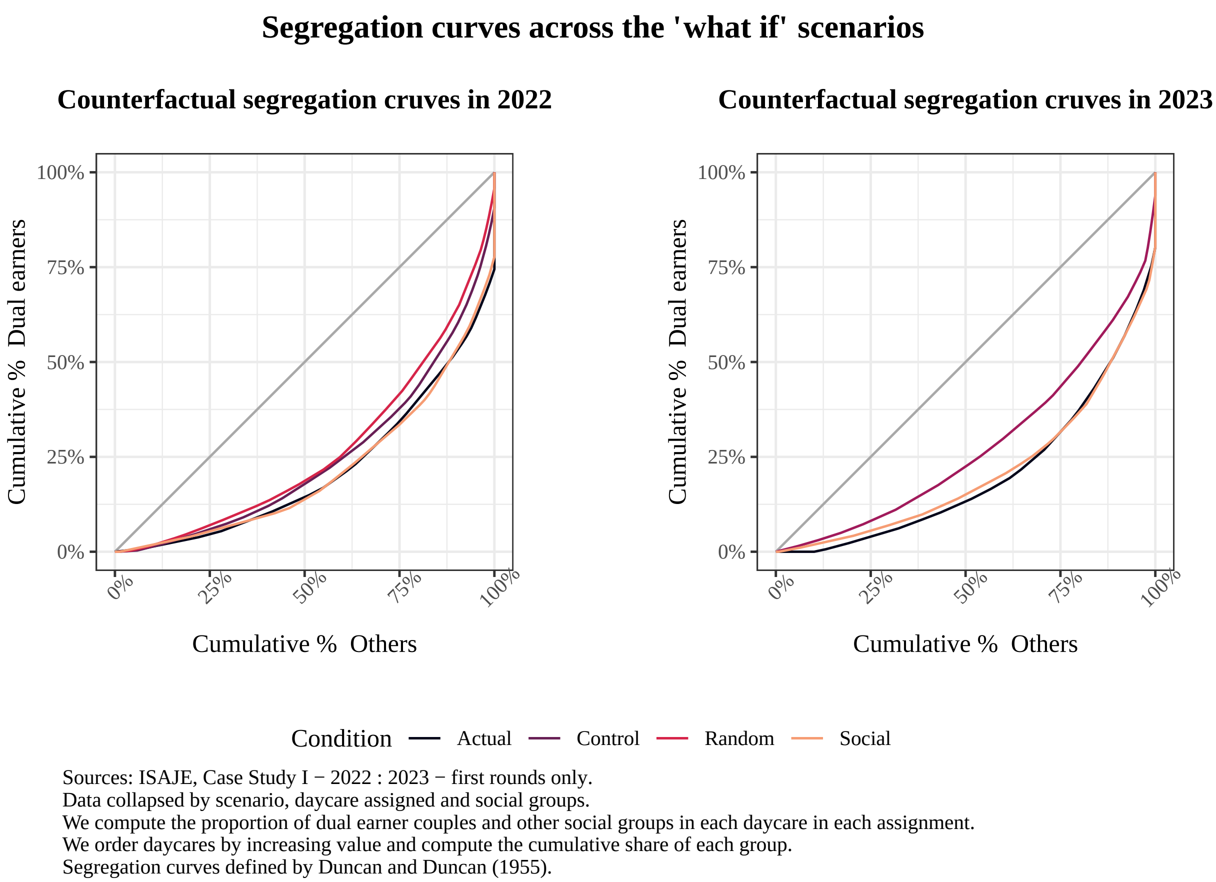
First, the comparison of assignments with "Control" shows that the reform increased the share of dual earner couples and active single parents, but at the expense of all other groups. Second, the results for the simulation without priorities ("Random") are very close to the control ones, except unemployed couples. Third, removing time since registration ("Social") has almost no aggregated effects across social groups but for single parents. Working single parents are most heavily penalised by time since registration while the effect is smaller for single parents without a job. Both results also hold true on the 2023 data.

Such simulations provide valuable insights for policymakers and can help build better weighting schemes. Our goal is not to criticise the politics of favouring working parents but our tools can help answer whether it is effective, who benefits, who loses, and by how much. The reform is highly effective the year of its implementation. However, time since registration priorities do not affect assignment probabilities across groups except reducing that of single parents. In 2023, time since registration reduces the effect of the reform on the group with the highest social weight.

B) The effects of priorities on segregation within daycares

The conditional assignment probabilities and effects of the alternative priorities are estimates over all demands but they may not be the relevant measure. Perhaps policymakers' are more interested in the distribution of social groups among those accepted and within daycares. Segregation measures describe such differences in the distribution of social groups across daycares, and there are many indicators capturing different aspects of the same phenomenon (Mora and Ruiz-Castillo 2011). First, we simply uses segregation curves[61] proposed by Duncan and Duncan (1955) in Figure 8. The left panel for 2022 shows that the reform increased segregation. That was the intended effect. We also see that the segregation curve of assignments without time since registration in the priority overlaps almost perfectly. On average, the priority scores would yield the same level of segregation between dual earner couples and other groups without time since registration. Without the reform, the segregation curve would have been very close to that of assignment without priorities. The right panel shows that in 2023, the two segregation curves with priorities are way below that of assignment without priorities showing that priorities do increase segregation. Without time since registration, the assignment would be slightly less segregated but otherwise keep the higher advantage for dual earner couples.

Figure 8: Effects of priority criteria on segregation



Finally, we look at segregation within daycares using the new visualisation method of *"segplots"* proposed by Elbers and Gruijters (2024). Intuitively, it sorts daycares on the x axis based on their level of segregation measured by a local version of the Theil Information Index (H index). Binwidth are proportional to the number of assigned families and the proportions represent the share of each social group in each daycare. In the bottom panel, we also

[61] A segregation curve represents the cumulative fraction of dual earner couples (on the y axis) and the cumulative fraction of other type of households (on the x axis) when groups are ordered from those with low values of the relative share of each group to those with high values of the ratio (Hutchens 1991). Like Lorenz curve, perfect integration implies a segregation curve on the 45° line. A segregation curve 'dominates' another if it lies at no point below and at some point above the other. The distribution associated with the dominant curve is ranked as more equal.

report each daycare h index and the overall H index. We use this new method to represent segregation over social groups across daycares for the actual and 'what if' scenarii in 2022 (Figure D.26), and the actual segregation over the four years (Figure D.27). Results are presented in the Appendix. The comparison of segregation across 'what-if' assignment and the actual assignment shows that i) segregation is highest with the reform and many daycare only get children from dual earners. Removing time since registration allows more active single parents to be assigned but the segregation remains high. Conversely, segregation without reforms and without priorities both give much more diversity in most daycare and the H index is 10 points lower than with the reform. The new priority weights for dual earner couples increased segregation in 2022 compared with simulated counterfactuals. We can see that segregation remains high in 2023 and was much lower in 2020 and 2021 (Figure D.27).

Interpretations The prioritisation of dual earner couples and active single parents in 2022 led to an increase in their assignment probabilities and a higher degree of segregation within daycares. Policymakers were content with this distribution, and the 2023 assignments have maintained a similar level of segregation across daycares. Our research reveals that when time since registration is removed from the equation, the assignment probabilities remain consistent for all groups, except single parents, who are disproportionately crowd-out because of more recent registrations. On the other hand, our findings suggest that time since registration primarily reallocates assignments within social groups. As dual earner couples make up two-thirds of all demands, their sorting has a significant impact on the placement of other groups. Given this, the timing of registration is a crucial factor for these couples, as we will illustrate in the following subsection.

V.3 Parents' needs and strategic timing of registration

In this section, we study the strategic use of time since registration across social groups and their consequences on assignment probabilities across birth months. This criteria is very often used in the childcare market. It is simple, easy to understand and to explain, and *queuing* usually is the way to go through administrative process. It also shares the responsibility of priorities with parents. Without time since registration, priority scores are coarse and policymakers let parents compete by signalling anticipation and willingness to wait. In a way, it reduces policymakers accountability because *late* parents can only blame themselves.

Our main argument is that time since registration is a highly unfair criteria mostly penalising vulnerable families, especially single parents through two main mechanisms. First, strategic parents create negative externalities on other families ; second, groups with lowest social priorities are typically those who cannot anticipate their needs and are penalised twice.

Strategic registration: Bunching registrations at pregnancy declaration Dual earner couples often request childcare earlier and mostly compete among themselves through early registration. They represent the majority of the demand and only active single parents have higher social weights. Parents, while aware of the four-priority criteria[62], have no precise knowledge about *how* these criteria are used. What they do know is that time since registration is important and that they can register from the declaration of pregnancy[63]. Note that this is a local feature. In many other marketplaces, registrations are typically accepted after the 7th month, or sometimes after birth. Regardless, when one uses time since registration for priorities, a lower bound for anticipation always exists and latecomers are penalised by design. Knowing that, parents face two significant challenges:

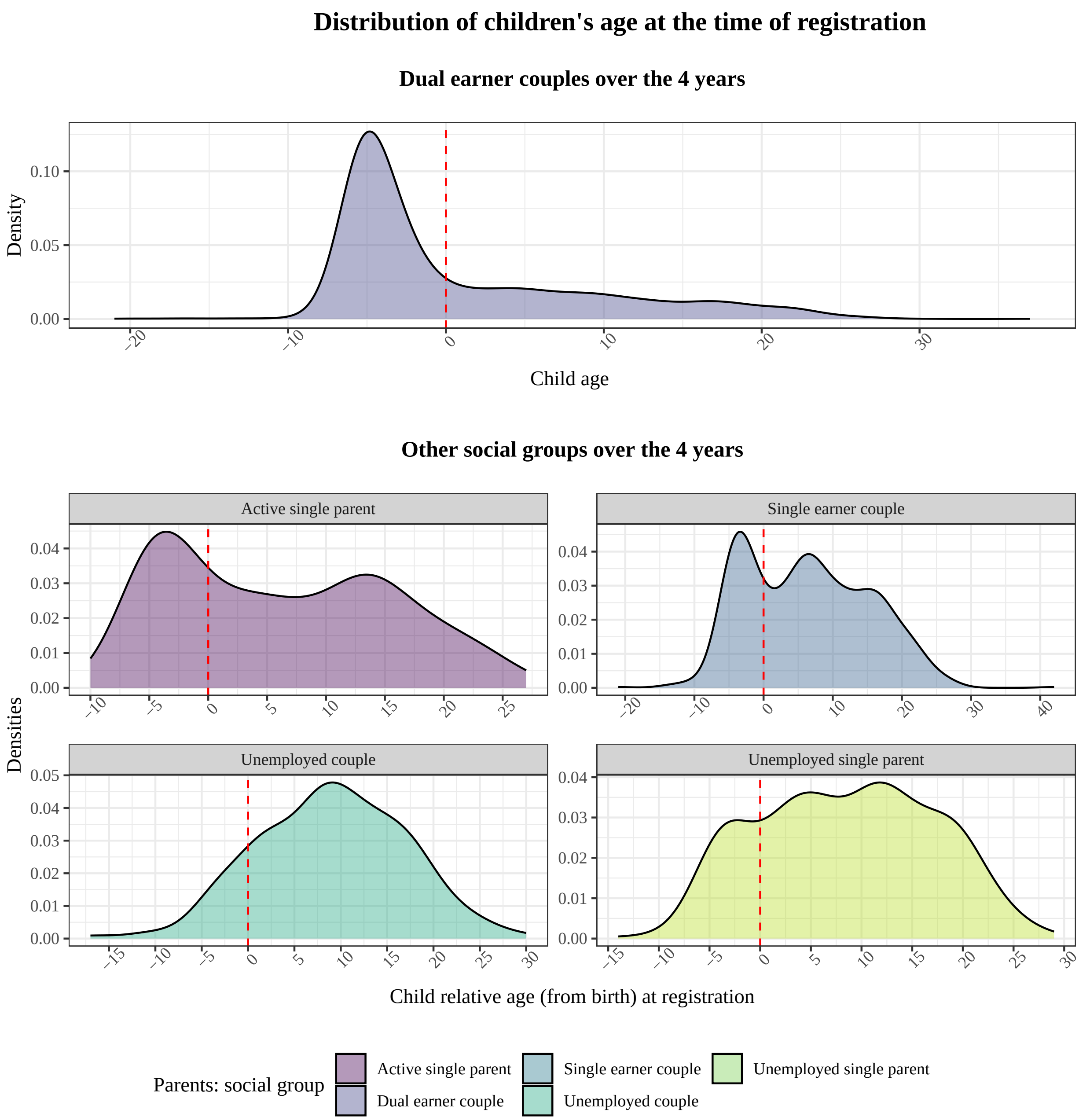
1. **Strongly anticipating their needs**: Parents are compelled to make decisions about their childcare arrangements at a remarkably early stage or risk being crowd-out by other strategic parents.
2. **Being able to wait**: Between birth and the assignment committee, parents require other childcare arrangements (including parental care) and wait for the committee's decision, with no guarantee of placement even when they registered early.

[62] Time since registration, residence, family's situation and employment and relationship status together

[63] Throughout every communication regarding daycare assignment procedures, parents are advised to contact the relevant administration *"as soon as their pregnancy is officially declared"*.

Indeed, for parents who anticipate their childcare needs, early registration clearly emerges as the dominant strategy, if they can afford waiting. Additionally, early registration during pregnancy reduces the needs (and costs) for childcare during the waiting period for the committee's response. Our sample is self-selected and if parents follow this dominant strategy, there should be a large bunching of registrations around 5 months before birth. This is precisely what we observe in the top panel of Figure 9 for dual-earners.

Figure 9: Strategic timing of registration of dual earners



Sources: ISAJE, Case Study I − 2020 : 2023 − first rounds only.
notes: Top panel present kernel density estimates of children age at time of registration and expected entry in september for dual earner couples.
Bottom panels present kernel density estimates of children age at time of registration for other social groups. For every estimates:
− One observation is kept for each child presented to one or other of the two committees,
− The variable analysed is constructed as the difference, rounded in months,
between the date of birth (actual or expected) and the date of registration.

A stable couple's privilege Parents in employment usually have to think and decide on their childcare arrangement early, sometimes even before conception. For dual earner couples choosing daycare, early registration is a dominant strategy and parents are incentivised to do so. These parents register young children for an early entry and this result is neither surprising nor negative *per se*.

However, this is a stable couple's privilege, as one can see in the distribution for other social groups in the bottom panels of Figure 9 and the empirical cumulative distributions in Figure D.24 in the Appendix. Over 60% of dual earners are registered before the birth, and over 50% registered more than three months before birth. This is more than twice as much as single-earner couples or single mothers in employment, for whom there is only a small bump of early registration.

This is pure endogenous selection at play as we only observe registered parents, and not parents who did not register. There are two obvious explanations for these differences: on the one hand, one parent may chose to care for their child themselves for some time and use childcare later. On the other hand, couples and jobs are unstable and break-ups and lay-offs are more likely to occur after the 3rd month of pregnancy. Both affect childcare affordability, needs, and ability to wait for the committee's decision. These parents cannot play the same strategy as dual earner couples and with born children, waiting is more expensive as it require alternative childcare arrangement. Moreover, they register older children and are competing over other buckets. Those have fewer available seats and so parents of older children face worse odds. The weights of social groups are highest for single parents working, but dual earner couples only have 1 point less. For them, registration a quarter earlier is enough to have higher priorities than single parents working. After 2022, single parents with no job need to be registered one year earlier than dual earner couples to reach their baseline priorities. Same for single earner couples and even longer for couples without jobs.

In the end, these weights create large inequalities of opportunities by allocating more seats for younger children mostly demanded by dual earner couples, giving this group the second highest social weight while it is the largest share of demands, and the opportunity to largely influence their priorities by applying as soon as they know of the pregnancy. This strategy is only accessible to stable families: stable couples, with stable jobs, who knew about their childcare needs and when to register, most likely before getting pregnant. For parents that *cannot* decide that early, they compete over fewer seats, opportunities are lower and furthermore, social weights strongly penalise parents whose childcare needs may be related to the events that lead them to be single-earner couple or single parent, and not dual earner couple. Social weights recognise the inability to wait of working single parents but the latters are still hurt by time since registration, as we have previously shown. The low weights for unemployed parents makes it very hard for them to access daycare. Furthermore, their job opportunities may involve atypical, flexible and unpredictable working hours that can be hard to reconcile with childcare services with typical opening hours. Allowing registration from the pregnancy declaration may have seemed like a harmless choice but it shapes the composition of the demand. It creates strong inequalities of opportunities for parents who *cannot* decide that early : in particular precarious couples with precarious jobs. The work of Pavolini and Van Lancker (2018) on the *Matthew effect* conclude from cross country analysis that structural constraints in childcare provision matter everywhere and tend to limit the uptake of childcare especially for children growing up in disadvantaged circumstances. We come back to this point in the discussion.

A) Other consequences of time since registration

With the previous results, we underlined that the majority of the largest group strategically register at the time of pregnancy to have higher priorities and that waiting is costly, especially for parents who decided on their childcare arrangement after the child is born. We show that the first also creates inequalities of opportunities even for strategic parents and generate variations in assignment probabilities by month of birth. The second requires parents to have outside options and paradoxically, increase inefficiencies by having more high priority parents choosing their outside options to the proposed assignment.

More inequalities of opportunities Even among dual earner-couples, these design choices generate negative consequences at equilibrium. The majority of demands are double earners and most play the dominant strategy. However, pregnancies occur throughout the year, and the *early-registration premium* decreases with pregnancies declared closer to the committee. As a result, parents who are well-synchronized have the opportunity to increase their priorities, which is not feasible for those whose pregnancy is declared closer to the allocation committee, unless they can wait until the following year. In the end, there are inequalities of opportunities even for strategic parents.

We discuss two situation in Example .4 in the Appendix. Well synchronised pregnancies offer parents with early childare needs an unfair advantage compared with parents with similar needs but whose pregnancy occurs at another moment. The same strategy does not yield the same "*return*" for parents whose children are born at different times of the year, or at a higher cost through a longer wait. And in this case, the child will be in a different age group, thus other buckets. In this market, daycare centres offer fewer seats for older children. Early registration priority gains can therefore be offset by capacity constraints at the bucket level, and for these very "*patient*" families, the outcome is still very uncertain. At equilibrium, these are inequalities of opportunities resulting from initial early-registration incentives which correlate assignment probabilities with pregnancy declaration and thus, month of birth. Figure D.28 in the Appendix displays the conditional assignment probabilities and 95% confidence intervals by registration and birth months[64]. The figure also provides a histogram and the admission status of children admitted on each date on the right scale.

There are substantial variations in admission probabilities by registration date, shifting from approximately 1/3 for registrations in the spring to 2/3 for registrations during the summer months (Example .4). Unsurprisingly, there is a negative correlation with the number of applicants. Admission probabilities also fluctuate with the child's birth date, with a pattern staggered by 5-6 months and of slightly lesser magnitude. The number of demands from children born in different months remains relatively constant from January to August. However, there are fewer demands from children born in the last quarter, with the associated assignment probabilities being the highest. For these parents, the declaration of pregnancy, occurring approximately 3 months before the summer, coincides with a notably reduced number of registrations. It is plausible that summer vacations postpones registration or makes them harder if offices close, compelling some to postpone their registration until September or even forgo it. Moreover, parents whose child is born in the last quarter of the year have to wait the most before getting a slot which may deter some parents from choosing daycare centres, as suggested by the lower number of registration.

Consequences: Unraveling The associations between registration months, birth dates, and admission probabilities are inherent consequences of the overarching market organisation. However, the early childhood department has informed us of increasing challenges, including a rise in refusals, numerous changes between requested days and actually signed contracts, and more. We have no data to measure this phenomenon yet, but the early childhood department currently considers this problem sufficient to motivate structural adjustment of the DAM. The time since application criterion creates opportunities to manipulate the assignment probability but requires a rather long waiting and without insurance of receiving a slot. Parents have to find alternative childcare solutions while they wait for a slot (including caring for their children themselves) and in case of rejection. Consequently, families enter the market with more or less outside options. This situation means that at the time of the committee, the list of preferences does not reflect the complete preferences of families. Some have outside options, the subjective rank of which is unknown in relation to the preferences expressed. If the committee offers a daycare that parents consider less preferable than their outside option, they will certainly refuse it. The problem is

[64] They are calculated over children as principal statistical units and for all the rounds they could participate in, from 2020 to 2023.

that priorities favour oldest demands, those whose preferences, situation or needs are most likely to have changed. Thus, because of outside options and preference updates, some with assignment probability are also the most likely to refuse them. In the current context, where families must confirm their registration shortly before the committee, many only withdraw after receiving an offer. This information asymmetry on outside options therefore causes part of the market to unravel[65]. In other words, this organisation results in:

1) Inflated demand;
2) Exacerbated congestion;
3) Seats being assigned to families whose childcare needs are already met – or can (more) easily be – rendering them more likely to withdraw their applications;
4) Over-rejection of single parents.

Although the organisation of a second round aims at partially rectifying this inefficiency, there is indeed a negative externality of withdrawing families on others. This situation underscores the importance of fostering sincere preferences and questions the relevance of *envy-freeness* as a stability criteria when envy is based on such inefficient and unfair rules.

V.4 Heterogeneous relative supply across age groups: lessons from Case study II

We saw in the subsection III.2 of section III that this marketplace features a sharp discontinuity in the total number of vacancies open for children born around January 1st. This DAM uses a simplified version of the problem and we implement the SPDA algorithm with sharp diversity constraints by birth dates. We want to estimate the effect of the sharp and common definition of the higher age bound of buckets and associated drop in number of open seats on assignment probabilities. We use a regression discontinuity design using time (in days) between January 1st 2023 and the child birthdate as forcing variable and analyse the jump in assignment probabilities and scores. We follow standard practice and let binwidth selection and polynomial order be data driven following Cattaneo and Titiunik (2022). We also run a McCrary (2008) test of continuous densities of birth dates around the cut-off and present the results in Figure E.29 in the Appendix. All models are estimated using the *'rdrobust'* package.

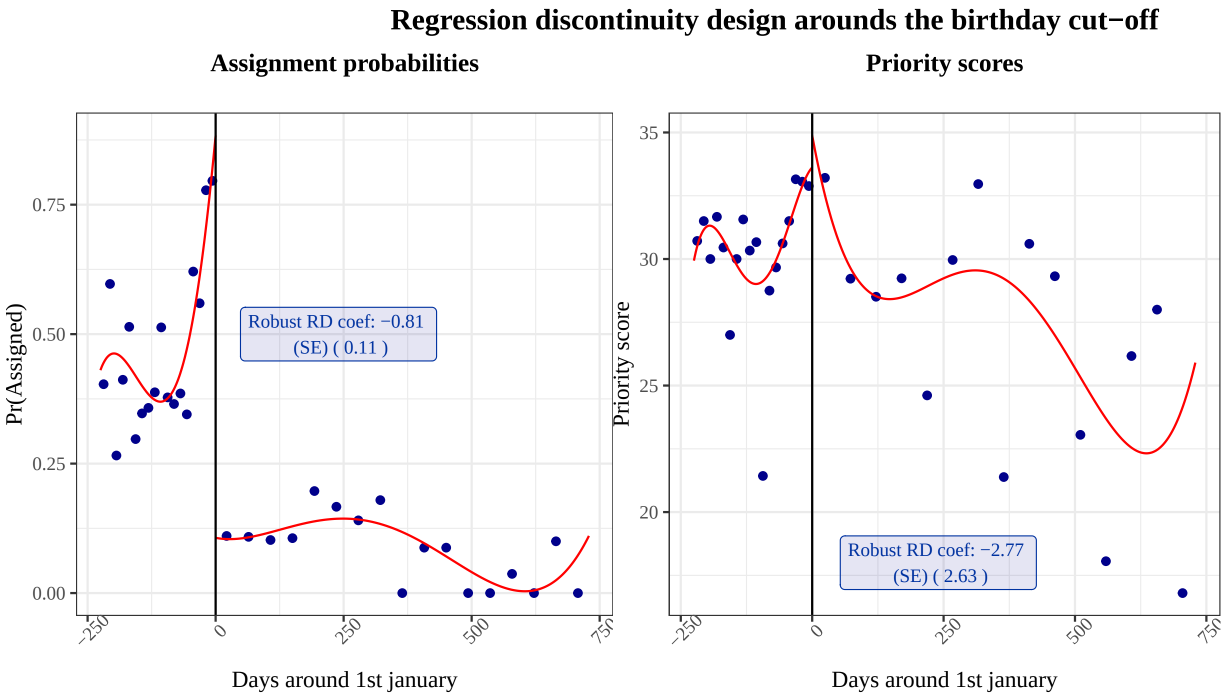
The main results are presented in Figure 10, with the discontinuity in assignment probability on the left and on priority scores on the right. As expected, the shortage of open seats for children born the calendar year before creates a sharp discontinuity in assignment probabilities. However, the priorities are not discontinuous around the threshold.

Interpretations The algorithm follows the definition of stability with diversity constraints and Theorem 2 imply that this is the best assignment for parents among those that respect priorities and diversity constraints. The matching is stable with diversity constraints and the *fairness* interpretation depends on their precise definition. Theoretically, those are the elements $(s_b, \succeq_b, c_b)$ that define the bucket structure and they are part of the definition of the problem. Although capacity constraints depend on actual physical constraints, buckets are strategic for both policymakers and daycare providers. The inequalities in assignment probabilities by date of birth only stems from a repartition of capacities across age groups that does not match the age structure of the demand. Policy makers knew that the relative size of age group mattered but did not realise parents differing only by a few days between their children's birth could face such unequal odds. The definitions of buckets are no less important than priority scores for shaping the distribution of assignment and generating inequalities. Priority weights change assignment probabilities across parents with different characteristics and are, by definition, intentional inequalities following *distributional justice* concerns. We showed they can cause *unfair inequalities* when groups with high social weights (single parents) are penalised by parents using a strategy (registration at birth declaration) they cannot use. Policymakers also control the relative supply they provide to different groups.

Finally, our two case studies show that assignment probabilities among registered parents strongly depends on well synchronised birth, but through a completely different mechanism. In Case Study I, unequal access by birthdate was

[65] Unravelling refers to the phenomenon where offers to families are rejected, accelerated, and require rapid adjustments, among other complexities, see e.g. Roth (2018)

Figure 10: Large discontinuity in assignment probabilities, no discontinuity in score



Sources: ISAJE, 'Family' files, Case Study II.
Notes: Data driven regression discontinuity with evenly spaced binscatters using the rdrobust package.

caused by strategic early registration from the largest group who could increase their priorities. It results in larger assignment probabilities for children born in the last quarter, registered around 3 months after conception. Children born in the two first quarters have smaller assignment probabilities. In case study II, time since registration does not affect priorities but policymakers supply less seats for children born in the previous calendar year. Thus, the effects of birth dates are reversed. Assignment probabilities are higher for children born in the two first quarters and much smaller for children from the end of the year. These results point the importance of all elements of the DAM, and the crucial role of buckets' elements and agents strategies in shaping the distribution of assignments.

VI Discussions

We started this project to provide high quality evidence on the effects of accessing daycare and for that, the first ordeal was to convince cities to let us randomise daycare assignments. To circumvent political resistance and legitimate ethical concerns, we learnt how committees assigned daycare slots. We asked how to define daycare marketplaces and the sort of optimality criteria we could define, and analysed the consequences of different design. We noticed similarities and contrasts with the school choice literature (Ehlers et al. 2014), and extend results from Kamada and Kojima (2023) and Delacrétaz, Kominers, and Teytelboym (2023) to define mechanisms tailored to each version of the problem we encountered. We then implemented these algorithms in 9 large urban district in France for up to four years and used data from two of them. We now discuss what we learnt, what these findings can tell about inequalities in access to formal childcare, and important policy implications.

How well do our model perform in the field ? One thing that all markets challenge participants to do is to decide what they like and one of our main contribution is to offer policymakers a *menu* of definitions and mechanisms to build daycare assignment marketplaces. Our work emphasises the trade-offs between different goals and lets them know the theoretical implications of their choices. Once policymakers define their DAM, the proposed procedures offer several distinct advantages over current practices in the field.

1) These computer-based tools can swiftly process a large number of applications, resulting in time savings for those involved in organising and participating in the assignment committees.
2) Our models make clear what can and cannot be done and the trade-off of each assignment mechanism. From a normative perspective, they ensure *fair* assignments in the sense that they eliminate *justified envy* based on a clear and chosen definition. Policymakers choose whether they want to strictly respect priorities, or accommodate more children by allowing weak accommodation and/or only consider initially feasible demands.
3) All assignment decisions are traceable and can be explained to every families. For each assignment we provide a table for every daycare that justifies every decision[66]. Transparency and accountability can be achieved, if policymakers choose to do so.
4) By ensuring that priorities are always respected and individually justifiable, they can create a public good[67]: they ensure *procedural justice* and can move ethical and justice concerns to *distributive justice*. Since priorities are always respected, society can debate on what constitutes fair (priorities) inequalities. For that, one requires *informational justice* i.e. that all participants have access, know and understand the rules.
5) Policymakers can define target assignment distributions by choosing the shares of seats they want to give to certain groups. Our models are flexible enough to consider many constraints. In particular, the definition of (soft) diversity quotas is a powerful tool to ensure that the assignment satisfies some distributional requirements. Without our procedure, that kind of policy objectives is very hard to achieve. Or at least, hard to justify.
6) These tools can also be used to simulate the effects of changing some parameters and inform policymakers using *what if* scenarii.

Good designs do not ensure well functionning markets In the first case study, we match demands over weekdays based on strong envy-freeness on initially feasible demands over flexible age groups and ranked through a priority score. The latter is made of three components: i) A disadvantage for demands outside the area, ii) social weights over groups based on the joint distribution of employment and relationship status summed with few additional weighted socio-demographic situations, and iii) time since registration, open from pregnancy declaration. The latter encourages early registrations and explains most variations in priorities. The majority group - dual earner couples - often require childcare arrangement earlier and most strategically register as early as possible. Using simulation of counterfactual assignment with alternative priorities, we showed that weighting social groups can effectively change assignment probabilities. However, allowing parents to manipulate their priorities through early registration strongly penalises single parents.

This is in part because early registration cannot be a strategy for those who became single parents later in their pregnancy. These inequalities are further aggravated by high social weights for dual earner couples who can anticipate more, wait longer and at a lower cost. Over the four years, single parents and couples where only one parent works have only seen their situation worsen. Strategic timing of registration does not have the same return when pregnancies are declared later in the year and they create inequalities of opportunities even among strategic parents through large variations in assignment probabilities by date of birth. In the second case study, the distribution of capacities across age groups is highly unbalanced and far from the corresponding demand. The sharp discontinuity in assignment probabilities by date of birth is another illustration of inequalities of opportunities stemming from the bucket structure.

[66] These tables are built from the database used to make the figure 5, add the initial capacities (by day) and cumulative use.

[67] According to Colquitt et al. (2001), justice perceptions can be broken down into several heuristics. Procedural justice concerns the processes, logic and deliberation behind a decision. Distributive justice concerns the allocation of positive and negative outcomes in a decision context and whether they are distributed equitably or deservedly amongst the affected population given their circumstances, performance or contributions. Interactional justice concerns the extent to which the affected individual is treated with dignity and respect by the decisionmakers. Finally, informational justice pertains to the information and explanations provided for decisions: are they candid, thorough, and tailored to individual needs?

A daycare assignment marketplace, like any market, is a social construct influenced by various factors that can either facilitate smooth transactions or give rise to frictions, inefficiencies, selection biases, and more. The choices made by policymakers matter in every aspect. Priority scores is not their only tool but it is a powerful one to enforce higher assignment probabilities for those with valued characteristics. However, buckets are even more crucial because they define who can access which capacities, how much capacities they receive and how to chose applicants. These buckets are part of the definition of the stability notion and as such, they should be defined with care.

Can daycare assignment be fair ? What would it means ? The trade-off between fairness and wastefulness is an inherent aspect of the daycare market. Our work offers policymakers different options regarding the concept of "*fairness*", enabling them to mitigate wastefulness without compromising a robust definition of envy-freeness. However, this definition depends on many parameters that affect the entire market structure. Inequalities of opportunity emerge at equilibrium and may be hard to justify when the definition of priorities is not tied to some notion of social justice. For instance, rejecting a set of families because one family registered early and cannot be served is arguably more contentious than when families are declined because it was not possible to accommodate a disadvantaged child.

Sometimes the problem is more about the *perception of fairness* rather than *fairness* as defined by absence of justifiable envy (Bullock, Williams, and Limbert 2003; Van Lancker and Ghysels 2016; Hufe, Kanbur, and Peichl 2020; Trump 2020). Perceived levels of justice are separate from purely self-serving rationalisations of a decision outcome. The very existence of priorities means accepting some have more chances of acceptance than others. It is a sensitive political question. However, most municipalities do not elicit preferences from their population to build priorities and face little accountability. The agreement between policymakers and daycare providers on the definition of buckets is central and yet, its strategic and fairness implications are absent from the debate. Even when criteria and weights are well defined and reflect a political objective, few are accessible and known to parents and in most cases, criteria are presented without their weights.

The instructions that accompany a mechanism (e.g., priority rules or strategy-proofness) are an important part of its design (Roth 2018). For instance, Hassidim, Romm, and Shorrer (2018) identify a substantial fraction of applicants in the Israeli Psychology Master's Match who misrepresent their preferences even though the mechanism is SPDA, which is strategy-proof. They further show that academically weaker applicants are more likely to misunderstand the instructions provided by the clearinghouse, explaining why they are more likely to misreport their preferences. Traditional incentives of more well-off families for using formal care (i.e., dual careers and few informal care opportunities) are often less salient for underprivileged families. It may thus be necessary to emphasise other motives and make registration safe. If parents intend to enroll their children, the lack of transparency may hinder the parents' action and make the enrollment process too costly (Carbuccia, Thouzeau, et al. 2023).

The choice of childcare among other strategies Securing a slot strongly depends on institutional settings such as priorities in daycare assignment mechanisms, congestion, access to alternative options and more. Importantly, some parents react to the market structure, but not all of them, and sometimes at the expense of more vulnerable families. There are several similar results in the school choice literature, starting with strategic preferences reporting and the externalities on sincere families, usually from lower economic background (Pathak and Sönmez 2008). Strategies of more affluent families are not limited to these. Fack and Grenet (2010a) showed that high quality schools raise housing prices in Paris in the previous residence-based assignment system. A very recent working paper by Bjerre-Nielsen et al. (2023) showed that the introduction of a residence-based admission criteria in the school choice system in Denmark doubled address changes and only in areas where the incentive to manipulate is high-powered. In the childcare market, Steinberg and Kleinert (2022) analyse the timing of registration for childcare testing rational choice theory and find consistent estimates with their theoretical predictions. In this setting, *bunchers* are optimising and their strategies are not limited to the timing of registration. Other decisions or outcomes are related. For instance, Bernal and Keane (2010) estimate quasi-structural models of the joint decision of childcare choice and labour market participation to measure the effects on children development. Their estimates focus on single mothers in the USA and use the Welfare reform of 1996 to instrument childcare use. Their results show large negative effects of the joint decision to work and use childcare on children cognition. Allègre, Simonnet, and Sofer (2015) model simultaneously labour market participation and type of childcare choice using a selection model *à la* J. Heckman

(1990) and estimate a multivariate probit on French data from 2002. Their results show that monetary incentives matter for chidcare use, but not which childcare is used. They also show that tax-rate matters only on labour market participation at the intensive margin. However this model lacks clear identification and omits important variables. In France, daycare centres have an objective comparative advantage in that they are, in general, the most affordable childcare solution. They also have a shared perceived comparative advantage over children socialisation, and researchers have documented that parents prefer collective childcare to childminders (Bouteillec, Kandil, and Solaz 2014; Cartier et al. 2017 ; Caenen and Virot 2023). Pora (2020) uses large but staggered increases in daycare provision across municipalities and time to estimate the effect of these apparent supply shocks on mothers' labour market participation. On average, he finds no effect on labour market participation but documents a crowding-out effect on registered childminders. These results suggest a general equilibrium effect in which women attached to the labour market shifts from childminders to daycare and manage to secure access for their children. In our setting, parents who work are much more likely to be seated because of priority rules and strategic behaviour, which supports the general equilibrium hypothesis and results of Pora (2020).

Surprisingly, this simple supply/demand/equilibrium reasoning is very much overlooked. However, it helps to think of how policymakers may improve welfare and where interventions are likely to make things worse. There may also be *optimisation frictions*: tedious application processes and complex tax subsidies schemes, among others, which may lead parents to perceive childcare as out of reach (Ünver, Bircan, and Nicaise 2018; Weixler et al. 2020a; Hermes et al. 2021, 2022; Valant and Weixler 2022). Policymakers, daycare providers and parents have to make sequences of nested decisions regarding childcare and every choice has an opportunity cost. In particular, families from disadvantaged backgrounds face greater challenges in securing a place for their children in high-quality daycare centres, limiting their access to crucial developmental opportunities (Gambaro, Stewart, and Waldfogel 2015). A crucial question is: how large is the opportunity cost of the Matthew effect ? $I.e.$ what is the welfare loss due to "*the observation that the benefits of government spending on social policy disproportionally accrue to middle- and upper-class relative to other social groups*" ? (Pavolini and Van Lancker 2018)

The Matthew effect as a political choice Single parent families hold a pivotal place in the discourse surrounding social investment. Policies for reconciling work and family life, such as formal childcare arrangements, aim to both enable parents to work and contribute to children's healthy development, ultimately reducing social inequalities from childhood (Van Lancker 2013). We showed that in these two settings, policymakers strongly influence the assignment distribution and favour more affluent households and early entry in childcare. Time since registration negatively affects single parents and unemployed parents. The current priority system strongly advantages stable families: stable couples, with stable jobs, who knew about their childcare needs and when to register, most likely before getting pregnant.

If we think of the ambitious goals for early childcare policies in the social investment narratives (Morel, Palier, and Palme 2012b; Hemerijck and Huguenot-Noël 2022), this sort of assignment distribution is very regressive. Two recent meta-analysis show that access to childcare services have large benefits for children and the positive effects are primarily concentrated among children from socio-economically disadvantaged backgrounds (van Huizen and Plantenga 2018; Schmutz 2024).

These results have strong economic and fairness implications. From an economic perspective, the Matthew effect creates large efficiency losses by reduces crucial development opportunities for low-income families. Another way of thinking of this problem is to think of the matching surplus and opportunity costs such priority generates for parents, policymakers, and society. A first important thing to note is that current weights generate actual private benefits while opportunity costs are mostly uncertain and in the long run. Neimanns (2022) argue that there are short-term electoral gains to favour more affluent families and in practice, they do favour more affluent families. Moreover, fees increase with incomes so it is also cheaper to favour them. Affluent families also benefit from collective childcare who are in general cheaper than childminders or private childcare services. Their career may be less affected by the *child penalty* $i.e.$ the gender gap in the labour market after the birth of the first child[68].

When policymakers favour working parents, it acts both as a reward for hard working and deserving mothers and a mean of reducing the child penalty. However, the counterfactual for these parents may not have been parental care

[68] The seminal work of Kleven, Landais, and Søgaard (2019) analyses career trajectories using retirement data in Denmark and found a persistent effect over 20 years for women, with an approximately 20% decrease in employment, income, and other indicators. There is no penalty for men, but for women, the loss is more pronounced in families with traditional values.

because these parents have other outside options. Conversely, low-income families who do not get a seat usually cannot afford alternative formal childcare arrangement. This in turn may be responsible for larger child penalty at the bottom of the income distribution. Meurs and Pora (2020) use an event-study design *à la* Kleven, Landais, and Søgaard (2019) to estimate the *child penalty* In France. They find a persistent 20% decrease in labour market participation over 10 years, a 40% decrease in income, a 10% decrease in hours worked, and a declining trend in wage rates for women following childbirth. For men, there is no economic penalty associated with the arrival of a child; in fact, their incomes increase in the years following childbirth[69]. An innovative aspect of this research is the analysis of gender penalty by income deciles. Their results indicate that the penalty is much more pronounced and persistent at the lower end of the income distribution, whereas it disappears completely for women at the higher end of the distribution. Bazen, Xavier, and Périvier (2022) estimate similar models focusing on cohorts of new entrants in the labour market and find that the child penalty is much higher for mothers with lower education. The latter group is disproportionately represented among single mothers. Last, because of gender norms and marital specialisation, Bonnet, Garbinti, and Solaz (2021) document large gender gaps in standard of living after divorce and massive labour market re-entry by previously inactive women. However, access to childcare is often a necessary condition to look for, and eventually get and hold a job (Gorey 2009). Labour market effects also depends on job opportunity and market tightness, travel distance and institutions such as flexible hours (Flèche, Lepinteur, and Powdthavee 2020; Le Barbanchon, Rathelot, and Roulet 2020; Chung 2020). Together, these evidence show that the Matthew effects largely depends on political choices and its cost on society is largely carried by women an children in low income groups.

Can we scale-up ? Should we ? An ambiguity between *market design* and *computer science* lays in the way users or policymakers think of the role of such mechanism. In our fieldwork, some municipalities were interested in our algorithm because they reduce the amount of back-office work and considered it as an updated IT system. Others valued the envy-free and transparency properties of our designs and even communicated lottery realisations to parents to justify assignments. In general though, the automation occurred with rather minimal information provided to families, primarily to preserve the integrity of the research protocol[70] and avoid political backlash. As we have seen, mechanisms are appropriate for DAM based on our models of economic agents. We understand they can react to different designs or features. The official adoption of our tools will likely elicit different reactions from families, early childhood professionals and policymakers.

Local power dynamics are important and incumbents value their autonomy. Even the inter ministerial mission assigned to an elected member of the Association of French Mayor (AMF) could not provide recommendation on *which* criteria *could/should* be used for priorities (Laithier 2018). Their political opposition has been made quite clear. After publishing this report encouraging transparency, meeting us several times, AMF sent a letter[71] to every mayors part of the early-childhood group deterring them from joining our project. Our approach was both seen as a threat to the self-government of territories and was amalgamated with IT tools:[72] "*This approach, which consists of replacing the human dimension and the finesse of the work carried out by elected representatives, in particular in the committee responsible for allocating daycare seats, with an algorithm, is in total contradiction with Laithier (2018).*" and "*The AMF's fear is that the research carried out by the CNAF will eventually lead to the generalisation of the algorithm for allocating crèche places imposed by the family branch on managers of early childhood education establishments (EAJE) or, at the very least, that co-financing will be conditional on its use.*"

The political aspect of this research is not secondary. We changed market structures. We are accountable for which family received a slot and which did not. To respect ethics and scientific integrity, our work was discussed and approved by an international scientific committee with researchers from different fields (sociology, economics, education science and so on). As researchers, we must acknowledge our responsibility and part of it is to report what we learnt. While we faced resistance in the early stage, we received a lot of positive feedbacks from those we worked

[69] also see the recent publication by France Stratégie signed by Dherbecourt and Flamand (2023)

[70] Parents were informed that the city hall took part in a research on early childcare as bare minimum.

[71] See the link on their website : https://www.amf.asso.fr/

[72] Original text is : "Cette démarche consistant à remplacer la dimension humaine et la finesse du travail réalisé par les élus, notamment en commission d'attribution des places en crèche, par un algorithme est en totale contradiction avec le vade-mecum" and "La crainte de l'AMF est qu'à terme la recherche menée par la CNAF aboutisse à la généralisation de l'algorithme pour l'attribution des places en crèche imposée par la branche famille aux gestionnaires d'établissements d'accueil du jeune enfant (EAJE) ou, à tout le moins, que des co-financements soient conditionnés à son utilisation."

with, such as informal confirmations of reduced interventions from elected officials or less formal complaints from families.

Depending on the quality of the tool offered, the adoption of technology can also lead to endogenous changes in committee organisation systems. Many territories opt for holding 1 to 4 allocation committees, often constrained by logistical issues that would be immediately addressed with a suitable tool. The problem would then be quite different from how we have conceptualised it. Dynamic assignments are fairly distinct theoretical problems[73]. An important feature that guided our models and assignment mechanisms is that capacities are mostly released at once, in September, and then, organising the convergence of supply and demand at a specific time allows for more choices for parents and managers. Market thickness offers participants a public good through larger *menus* for both sides of the market but as Roth (2015) points out, "*Part of making a market thick involves finding a time at which lots of people will participate at the same time. But gaming the system when the system is "first come, first served" can mean contriving to be earlier than your competitors.*" These strategic players also are those with outside options which may cause the market to unravel. Family choices are heavily driven by the existence of multiple outside options for care and the results on school choice and outside options are scarce (Akbarpour et al. 2022). Recent evidence show that families do have heterogeneous beliefs about childcare access and quality (Boneva et al. 2022). One central question is to know whether participation is driven by preferences *per se* or misbeliefs of certain types of agents (e.g., low income families). Currently, there are no studies on the impact of misbeliefs of parents for participation decision into a school choice type of market. Finally, these models are partial equilibria and the actual participants' strategy may involve other choice sets outside of this market, dynamic interactions with other markets or itself (when families remain for more than a year). Parents may have preferences over a larger set of options that are not represented in this market (or any market). While this has little impact on our modelling assumptions to design assignment mechanisms, it has tremendous implications in real life settings, especially when outside options are not accessible to all parents and a market may be noxious because some of the transacting parties are vulnerable (Li 2017a).

Scaling up creates new challenges. Details matter and designing well functioning marketplaces may require inputs from different types of agents. We think scaling-up should involve public institutions (Municipalities, CAF, PMI,…) and citizens, researchers and engineers, so that the proposed services perform *well* based on normative criteria policymakers should define. Early-childhood education and care is seen as one of the most important policies to reduce inequalities and more. This provides clear objectives and we can define measures to assess whether daycare assignment mechanisms do well. Our work would benefit from the expertise of researchers from other related disciplines, such as organisational sociologists to investigate needs, reluctances, and reactions among parents, policymakers and childcare providers. Eliciting preferences to define *fair* priority rules would require the involvement of psychologists, behavioural scientists and sociologists. Fostering access to daycare for certain population may require additional interventions which could involve associations and other local organisations.

In the end, these daycare assignment mechanisms can open a path towards easier and more transparent assignment mechanisms, and provide new tools to define distributional objectives. It definitely opens new research agendas by generating new data that so far, have not often been accessible to researchers. The embedded lotteries can help answer important policy relevant research questions. For instance, unravelling causes many inefficiencies and adjustment but we can identify compliers characteristics, at least at the margin. Ultimately, we have enough random variation across all marketplaces we worked with to create a sample of about 20 000 demands among which 3 000 were subject to random assignments. Our next work will be to assess the effect of accessing daycare on parents' labour market decisions and marital stability.

[73] See *e.g.* Kennes, Monte, and Tumennasan (2014) and Grenet, He, and Kübler (2022)

Appendix

A A short description of the French family benefits system for parents of young children

A.I Types of childcare available in France

Daycare centres are usually public or strongly depend on public funds but there are some private for-profit centres and other arrangements. Here are the different types commonly found:

- **Public Childcare Centres** (*Crèches*): Public childcare centres, often referred to as "*crèches*," are operated and funded by local municipalities or public institutions. They provide full-day care for children, typically between the ages of three months and three years. Crèches are regulated and staffed by qualified professionals. There are laws on the number of children by adults that vary according to children's ability to walk and binding regulations on care practices, nutrition, environment,... They are mostly publicly funded and parents' fee depend on their income and family situation.
- **Private Childcare Centres**: Private childcare centres are operated by independent organisations or individuals. These centres often offer similar services to public crèches, but they may have different pricing structures or cater to specific age groups. Moreover, some employers can either build their own daycare centres for their staff (and sometimes accept applicants from non-staff parents) or "buy" seats in other centres. These often exists in some public administrations (hospital and universities) and large private firms.
- **Micro-Crèches**: Micro-crèches are small-scale childcare centres that typically accommodate a limited number of children, usually up to 10. These centres provide a more intimate and home-like environment with a focus on individualised care.
- **Childminders** (*Assistantes Maternelles*): Chidlminders, known as "*assistantes maternelles*," are registered and licensed childminders who offer childcare services in their own homes. They care for up to four children, providing nurturing and personalised settings. They are directly hired by parents who receive subsidies (CMG, see below) and tax credits. Some are also hired by city-halls
- **Parent-Run Co-Operatives**: Parent-run co-operatives are childcare arrangements where a group of parents collaboratively organises and manages a childcare facility. Parents are involved in the decision making and operation of the cooperative, ensuring a high level of parental involvement.
- **Nannies** (*Nounous*): Nannies are private childcare providers who directly work for individual families. They provide personalised care for children in their families' homes and offer flexible scheduling to meet the family's specific needs. Nannies are typically employed by the family and may live-in or work on a part-time basis.

A.II Aids directed specifically to parents with children under 3

In France, PAJE (*Préstation d'Accueil du Jeune Enfant*) is a comprehensive family benefit programme aimed at supporting parents in the early stages of their child's life. PAJE provides financial assistance and benefits to families with children under the age of three. The programme includes several components:

- *Prime à la Naissance*: the premium at birth or on adoption is paid, depending on income[74], on the birth of a child, or at the time of the adoption of a child under 20 years old. For eligible households, it amounts to € 1 019.40 paid on the seventh pregnancy month or at the time of adoption.

[74] In 2023, for single-earner couples, the eligibility threshold of the premium at birth is € 33 040 of yearly income 2 years earlier for the 1st child, € 39 648 for the second and € 47 578 for the third. For single parents or double-earner couples, these thresholds are respectively €43 665, €50 273 and € 58 203. For families with more than 3 children, thresholds increase by € 7 930 for every additional child, whatever the household situation.

Table A.2: Change in the main type of childcare for children under 3 on weekdays between 2002 and 2021

	2002	2007	2013	2021	Counterfactual†: if parents had their first choice
Parents	70	63	61	56	36
Grandparents or other family members	4	4	3	3	2
Childminder or Shared Childcare Facility (MAM)	13	18	19	20	23
Early Childhood Education and Care (EAJE)	9	10	13	18	35
Other forms of childcare arrangements[1]	4	5	5	3	4

[1] e.g. in-home care, school, shared childcare facility (MAM), friend, neighbor, babysitter, or other non-family member, kindergarten, after-school care, leisure centre, or specialised establishment.
† In the survey, the ask families what their first choice whould have been, regardless of what they are using now.
* *Note:* The week is counted from Monday to Friday, from 8 a.m. to 7 p.m.
** *Reading:* In 2021, 56% of children under 3 years old are primarily cared for by their parents from Monday to Friday, from 8 a.m. to 7 p.m., compared to 70% of children of the same age in 2002.
*** *Source*: DREES (French Directorate for Research, Studies, Evaluation, and Statistics), surveys on childcare arrangements for young children, Metropolitan France, children under 3 years old.

- *Allocation de Base (AB)*: the basic allowance follows the payment of the premium at birth or on adoption and takes two values (full or partial), depending on income[75]. It is paid from the month following the birth until the last day of the calendar month before the child's 3rd birthday (or 3 years from the month following the adoption, up to a maximum child age of 20).

- *Prestation partagée d'éducation de l'enfant, PreParE* The shared allowance for the children's education allows one or both parents to stop or reduce their activity to take care of their children under 3 years old (under 20 years old in case of adoption). For the first child, each parent can take up to 6 months up to the child first birthday or the full year for single parents. For families with more than two children, they can take up to 24 months each up to the youngest child's third birthday. When parents entirely stop working, they are eligible for a € 428,71 monthly allowance. If they work up to 50

- *Complément de libre choix du mode de garde (CMG)*: The free choice of activity supplement is paid to the household or person who directly employs someone to take care of a child who is under 6 years old or place the child in a micro-crèche.

The eligibility and specific details of each component of PAJE vary based on factors such as family income, the number of children, and the type of childcare chosen.

In addition to the PAJE program, there are several other aids available to parents with children under three in France. These include:

- Aide Personnalisée au Logement (Personalised Housing Assistance): This financial aid helps families with housing costs, including rent or mortgage payments. Eligibility and the amount of assistance depend on factors such as income, family composition, and housing expenses.

- Allocation de Rentrée Scolaire (Back-to-School Allowance): Although primarily aimed at school-aged children, this annual allowance provides financial assistance to low-income families to help cover the costs of school supplies and equipment. It can be applicable to families with children entering early childhood education.

[75] The full monthly payment of AB amount to € 184.81 and the partial payment is € 92.40 in 2023. Single earner couples with one child get the full payment if their yearly income is less than € 27 654, €33 185 for two children and €39 822 for three. The thresholds for partial AB are the same as those for the premium at birth. For single parents or dual-earner couples, the thresholds for the full AB are €36 546 for one, € 42077 for two and € 48 714 for three children. With more children, thresholds increase by €6 637 for each additional child.

- Allocations Familiales (Family Allowance): This universal benefit is provided to families with at least two children and helps support the general costs of raising children. The amount of the allowance varies based on the number of children and their ages.
- Aide au Temps Libre (Leisure Time Assistance): This benefit aims to facilitate the participation of children in recreational and cultural activities. It can help cover the costs of sports clubs, music lessons, cultural outings, and other extracurricular activities.
- Couverture Maladie Universelle Complémentaire (CMU-C) or Complémentaire Santé Solidaire (CSS): These are healthcare coverage programs that provide access to medical services, including for children under three. They offer assistance to low-income households who may not have adequate health insurance coverage.

B The ISAJE research project

General idea and constraints The research design of ISAJE relies on the implementation of an algorithm to allocate daycare seats to families and retrieve those who were subject to randomisation in the process. The share of demands subject to a lottery can be small relative to the whole population and strongly depends on the coarseness of the priority rules. We defined a set of constraints to search for cities. They had to

1. Use well defined priority rules that do not depend (or slightly depend) on tight criteria such as "date-since-application", or who were willing to give them up for the experiment;
2. Allocate daycare seats through a main allocation committee ;
3. Are willing to substitute an algorithm to their committee, and commit to its results in large;
4. Have a large number of demands so that the experimental sample is not too small;
5. Are significantly rationed, measured by the ratio of theoretical capacity over the total number of children under 3.

As a rule of thumb, we restricted potential prospects to local administrations whose theoretical capacity is above 600 seats. We started prospecting municipalities in 2019 and stopped in 2023. We worked with 9 large urban cities, some for several years. We are running another round of assignments in 2024 for a subset of cities who asked for this service to be continued.

Data limitations because of ongoing research Starting this project, our primary research objective was to analyse the impact of daycare access on families using counterfactuals derived from comparison groups within the same local labour market. To achieve this goal, we convinced 9 large urban centres to participate in this research project. The most important aspect for the evaluation was to ensure that enough assignments and rejections would be random and traceable, while limiting disruption in the ecological conditions. The models we present in section IV provide solutions compatible with existing practices, a necessary condition to gain support from incumbents without infringing on local policies. In practice, we operated from 2020 to 2024 as subcontractors so that city-halls remained data controllers, as per GDPR. We charged no fees and provided detailed feedbacks and policy recommendations based on data analysis. For some, we also ran simulations, showcasing another advantage of our tools: simulating assignments with counterfactual priority rules to inform policymakers of possible distributional consequences.

Because of our subcontractor status we cannot use all these data for research yet. However, Valence-Romans Agglo enabled us to keep fully anonymous datasets. Another agreed we could report results using their data without disclosing the source. Our analysis focuses on priorities and how parents react, and the role of age groups and diversity requirement.

B.I Marketing the ISAJE experiment

Figure B.11: Flyer to recruit new territories, P1

Isaje une recherche sur l'accueil du jeune enfant

Une ambition : adosser l'investissement social dans l'accueil du jeune enfant à une démarche scientifique

L'accueil des jeunes enfants est une politique déterminante pour la société française et mobilise de nombreux acteurs, au premier rang desquels se trouvent les collectivités territoriales et les Caisses d'Allocations familiales (Caf). Il participe à la réduction des inégalités femmes-hommes et, parce qu'il intervient à un moment clé du développement des enfants, est susceptible de réduire les inégalités d'acquisition de compétences liées à leur origine sociale.
Les acteurs publics jouent donc le rôle d'investisseurs sociaux. Ils s'intéressent au développement de l'enfant et anticipent que les efforts d'aujourd'hui généreront des gains économiques et éviteront des coûts sociaux futurs. La crédibilité de cette démarche repose sur l'existence de travaux scientifiques permettant de mesurer les effets des actions d'investissement social et les gains induits pour la société. C'est pourquoi nous lançons **le projet de recherche ISAJE pour « investissement social dans l'accueil du jeune enfant ».**

Le projet de recherche Isaje

Ses objectifs

L'objectif de cette recherche est de mesurer **les effets de l'obtention d'une place en établissement d'accueil du jeune enfant** (EAJE) sur trois dimensions :

- Le développement des enfants dans les compétences langagières, cognitives, motrices, socio-émotionnelles ;
- Les conditions de ressources des familles et l'activité professionnelle des parents ;
- Le bien-être subjectif des parents et le quotidien du ménage.

Elle s'attache en outre à **mesurer si ces impacts sont différents suivant l'origine sociale des enfants.**
L'originalité de cette recherche et son intérêt majeur résident dans sa capacité à identifier des relations « de cause à effet » entre le fait d'avoir accès à un mode d'accueil collectif et la situation de l'enfant et de sa famille. Les études permettant de montrer les effets des modes d'accueil sur le développement des enfants sont rares en France, et ce sont alors souvent quelques études emblématiques étrangères qui sont citées pour en justifier l'intérêt.

Un partenariat institutionnel et scientifique

Ce projet est porté par la **Caisse nationale des Allocations familiales** (Cnaf) avec le soutien de la **Délégation interministérielle à la lutte contre la pauvreté des enfants et des jeunes.**
La méthodologie de la recherche est conçue par deux chercheurs[1] de l'**École d'économie de Paris (PSE)** et de l'**École polytechnique** avec l'aide d'un conseil scientifique pluridisciplinaire et international composé de chercheurs, d'institutions et de l'équipe de recherche[2]. Sa gouvernance est assurée par un comité de pilotage dans lequel les territoires engagés dans la démarche sont représentés. Ce projet est proposé en étroite collaboration avec les Caf concernées.

[1] Arthur Heim (Économiste, doctorant à PSE) et Julien Combe (Économiste, École polytechnique).
[2] Les chercheurs sont : Sylvana Coté (Psychologue, University of Montreal), Orla Doyle (Économiste, University College Dublin), Denis Fougère (Économiste Sciences Po Paris), Claude Martin (Sociologue, EHESP Rennes), Yusuke Narita (Économiste, Yale University), Laurent Toulemon (Démographe, Ined Paris), les institutions représentées sont la Cnaf (Florence Thibault), la DREES (Emilie Raynaud), l'INJEP (Thibault de Saint-Pol / Aude Kerivel), et le projet est porté par Marc Gurgand (PSE), Arthur Heim (PSE-Cnaf), Julien Combe (École polytechnique), Jeanne Moeneclaey (Cnaf).

Figure B.12: Flyer to recruit new territories, P2

Comment ça marche ?

Le processus d'attribution des places

L'accueil d'un enfant en crèche est le résultat d'un processus complexe impliquant d'une part, des préférences et des contraintes du côté des familles, et du côté de l'offre, un nombre de places souvent limité, dont l'accès est défini par les critères de priorité établis localement. Ainsi, le processus d'attribution induit des différences de profils entre les enfants accueillis en crèche et ceux qui ne le sont pas. Dès lors la comparaison de ces deux groupes ne permet pas d'identifier les effets liés à l'accueil en lui-même. Le projet ISAJE vise à répondre à cet enjeu en organisant un suivi de familles statistiquement identiques dont une partie est accueillie en crèche et l'autre est en liste d'attente. Le projet porte sur des territoires marqués par un excès de demande de places en crèche par rapport à l'offre disponible, de sorte que la recherche n'implique en aucun cas de priver des enfants d'une place d'accueil.

Evidemment, chaque territoire a ses propres critères pour l'attribution des places et il faut tenir compte des préférences exprimées par les familles. Le projet de recherche ISAJE propose donc **une solution technique prenant en compte les souhaits des familles et les critères de priorisation des communes tout en permettant la réalisation de la recherche, qui suppose d'introduire de l'aléa dans l'attribution des places.** Cette solution concerne uniquement la commission d'attribution liée à la recherche (*a priori* printemps 2021).

Le projet de recherche mobilise une procédure automatique d'affectation utilisant des critères explicites et transparents, fournis par les territoires, qui garantit une stricte égalité de traitement à profil donné. Il accorde ainsi **aux plus prioritaires le meilleur choix possible parmi les vœux formulés faisables.** De plus, cette procédure permet de prendre en compte la complexité de la réalité : elle tient compte des demandes par jours spécifiques, peut introduire des places réservées (pour renforcer la mixité sociale par exemple), ou avoir un traitement adapté aux dossiers sensibles (situation de handicap par exemple).

Des enquêtes auprès des parents et des enfants

Une fois les attributions de places réalisées, **un échantillon de familles** bénéficiaires et non-bénéficiaires d'une place **sera suivi au cours des trois années suivantes.** Des enquêtes seront régulièrement menées auprès des parents et une évaluation des compétences des enfants à 3 ans sera réalisée par une ou un psychologue, psychomotricienne ou psychomotricien.

Valence-Romans agglo : un territoire pilote

En 2020, l'agglomération de Valence-Romans s'est portée volontaire pour participer au pilote de ce projet. La commission d'attribution du printemps 2020 portait sur 1014 demandes pour environ 380 places à temps plein réparties dans 43 structures (dont 12 assistantes maternelles), divisées en 5 groupes d'âges. Les familles exprimaient jusqu'à 8 vœux (3 en moyenne) avec des jours spécifiques. Les priorités des familles sont définies par un score construit à partir des caractéristiques sociodémographiques des familles auquel un numéro au hasard est ajouté de sorte à départager les dossiers ayant le même niveau de priorité.
En utilisant la procédure ISAJE, 400 familles ont vu leurs demandes satisfaites, soit 39 % des demandes. Parmi elles, 64 % ont obtenu leur premier choix, 17 % leur second. En comparaison avec ce qu'aurait fait la commission : 83 % de dossiers auraient eu une affectation identique. Une première enquête téléphonique sera réalisée au printemps 2021 auprès de l'ensemble des parents ayant demandé une place en crèche.

ISAJE a besoin de vous !

La recherche portera sur 3 000 à 5 000 enfants et concerne toutes les familles demandant une place dans n'importe quel établissement collectif géré par la commission d'attribution des territoires engagés dans la recherche. **Sa réussite nécessite la participation active d'une trentaine de territoires disposant de plusieurs centaines de places d'accueil et acceptant d'intégrer le projet pour les commissions d'attribution du printemps 2021.** Cela pour garantir une précision statistique suffisante et la représentation des communes de France dans leur diversité.
Voilà pourquoi nous comptons sur votre mobilisation et espérons que vous rejoindrez cette expérience inédite aux enjeux importants. Vous inscririez ainsi votre territoire dans une démarche d'innovation démontrant votre investissement pour la transparence dans l'attribution des places en EAJE et l'égalité des chances dès les premières années de la vie.

Contact : arthur.heim@cnaf.fr

Cnaf- 32 avenue de la Sibelle - 75685 Paris Cedex 14

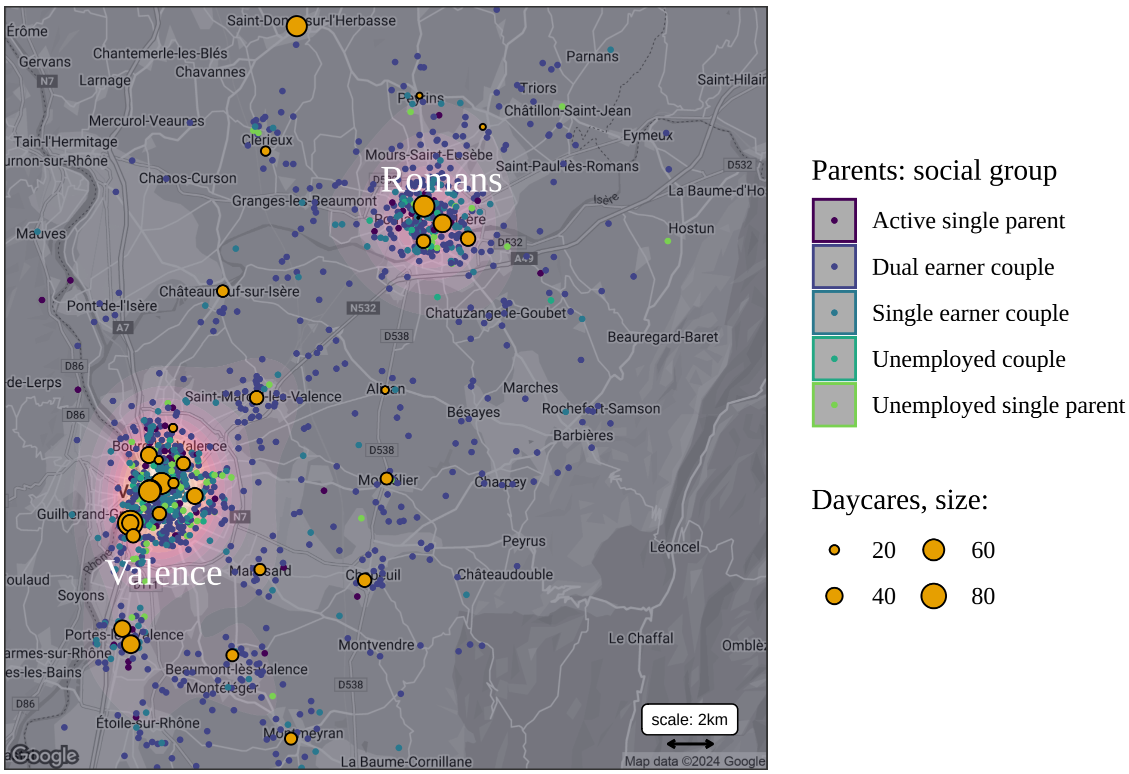
C The market for childcare in Valence Romans

C.I 52 cities in the same daycare markets

Figure C.13: Map of the daycare market in Valence Romans Agglo

Map of the demand, density and daycare's capacities

Demands for the first round in 2023



Sources: ISAJE, Case Study I − 2023, first rounds only.
Notes: Social groups are defined by the local administration with the 'Professional status' criteria. Positions are jittered by .01 degree.
Capacities are those reported on the early−childhood brochure for 2023.

C.II Supply

Table C.3 summarises the aggregated results from various assignment committees. During the four spring committees, daycare centres offered an average equivalent of 415 placements for the first round.

Following the assignment, which could leave vacancies, and considering the decisions of parents accepted in the first round, we initiated a second round with an average equivalent of 124 placements. It's worth noting that most second-round placements predominantly result from family withdrawals rather than vacancies after the procedure.

In 2020, daycare centres supplied an equivalent of 399 placements. However, this first committee occurred amid the initial lock-down, and until August 25th, 2020, the full reopening of creches was uncertain. In 2021, there were nearly 20 fewer placements than in 2020 but more second-round placements. This indicates a higher number of refusals for placements offered in the first round. In 2022, the number of available placements increased significantly. Simultaneously, we will explore how demand also rose substantially that year. Lastly, in 2023, 429 placements were offered.

Table C.3: Available Slots, Number of Sections, and daycares in Different assignment Committees

Year	N daycares		N Buckets		N places	
	1st Round	*2nd Round*	*1st Round*	*2nd Round*	*1st Round*	*2nd Round*
2020	33	29	95	63	399	113
2021	30	31	67	64	380	140
2022	35	34	98	70	451	115
2023	35	32	107	77	429	128
Average					415	124

Sources: ISAJE, Case Study I - 2020 : 2023.
Notes: In 2021, in the first round, childminders were assigned to family daycare centres. For other committees, the early childhood department assigned childminders in advance for families requesting corresponding family daycare centres.

The administration offers guidance in a leaflet and their website[76] allows to map various childcare arrangements. Figure C.13 displays all daycare centres and family childcare in the 54 cities participating in the daycare assignment mechanism. Most childcare facilities are located in the two largest cities Valence and Romans-Sur-Isere.

During the first three years, the number of slots offered to children in the first buckets increased, but there were fewer slots for older children. This situation reversed in 2023.

[76] https://geo.valenceromansagglo.fr

C.III The demand

In each year of the experiment, we processed an average of 1144 applications in the first rounds and 834 in the second rounds. Between 2020 and 2021, the demand increased by 5%, but it was in 2022 that the progression was the strongest: +17% compared to 2021. In 2023, there are 25% more requests than in the 2020 committee.

Table C.4: Attendance at committees from 2020 to 2023

CAMAEn	**Numbers**		
	Total Requests	**Children Presented**	**Families**
April 2020	1,014	1,014	920
June 2020	694	694	642
April 2021	1,064	1,064	991
June 2021	810	810	769
April 2022	1,233	1,233	1,140
June 2022	911	911	860
April 2023	1,267	1,267	1,177
May 2023	919	919	867
TOTAL 1th Round	**4,578**	**4,257**	**3,347**
TOTAL 2th Round	**3,334**	**3,166**	**2,649**
TOTAL unique	**7,912**	**4,681**	**3,551**

Sources: ISAJE, Case Study I - 2020 : 2023 - first rounds only.
Notes:
- The total number of requests counts the registered files, i.e., the number of children.
- Family count identifies the number of unique families at each committee.
- The totals identify the number of unique children and families present.

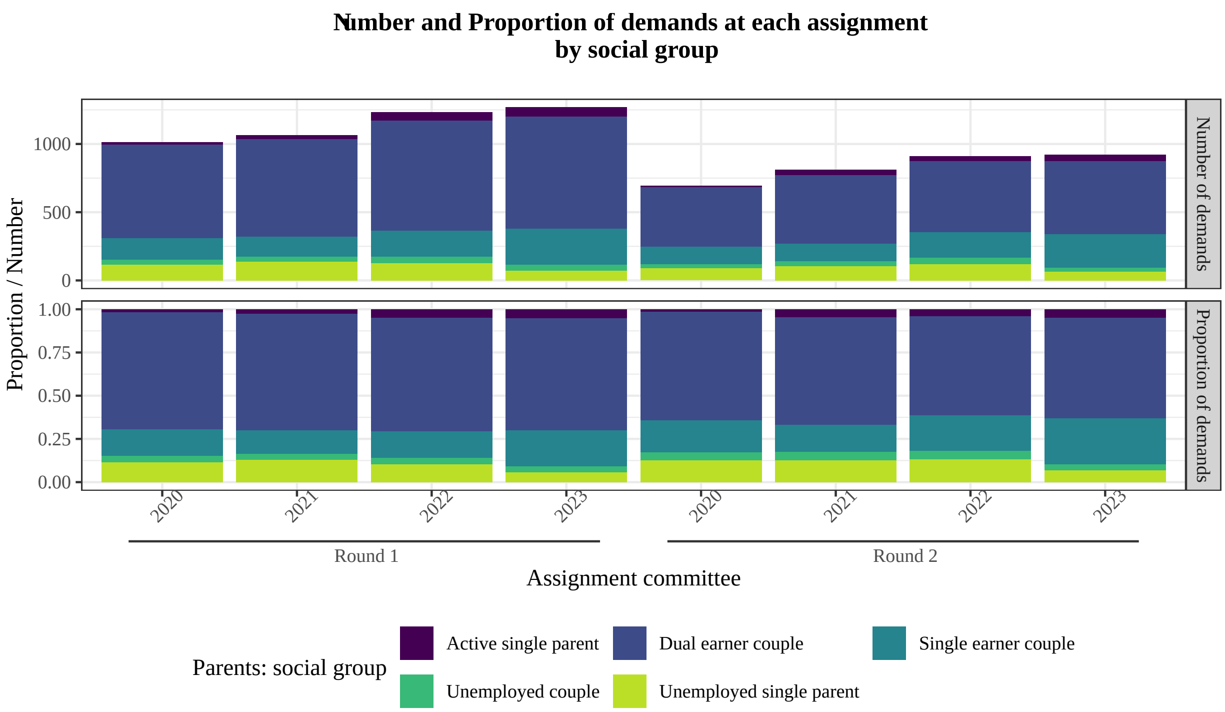
A) The composition of the demand by social group

Table C.5: Composition of the demand by social group across years

	Assignment							
	2020		**2021**		**2022**		**2023**	
	Round 1	*Round 2*	*Round 1*	*Round 2*	*Round 1*	*Round 2*	*Round 1*	*Round 2*
Active single parent	19 (2 %)	11 (2 %)	30 (3 %)	37 (5 %)	61 (5 %)	37 (4 %)	66 (5 %)	45 (5 %)
Dual earner couple	686 (68 %)	436 (63 %)	715 (67 %)	505 (62 %)	809 (66 %)	523 (57 %)	821 (65 %)	536 (58 %)
Single earner couple	156 (15 %)	128 (18 %)	145 (14 %)	126 (16 %)	191 (15 %)	185 (20 %)	266 (21 %)	244 (27 %)
Unemployed couple	36 (4 %)	31 (4 %)	36 (3 %)	39 (5 %)	46 (4 %)	47 (5 %)	41 (3 %)	31 (3 %)
Unemployed single parent	117 (12 %)	88 (13 %)	138 (13 %)	103 (13 %)	126 (10 %)	119 (13 %)	73 (6 %)	63 (7 %)
TOTAL	**1014**	**694**	**1064**	**810**	**1233**	**911**	**1267**	**919**

Sources: ISAJE, Case Study I - 2020 : 2023.
Number and proportion of applicants from each social group in each demand set.

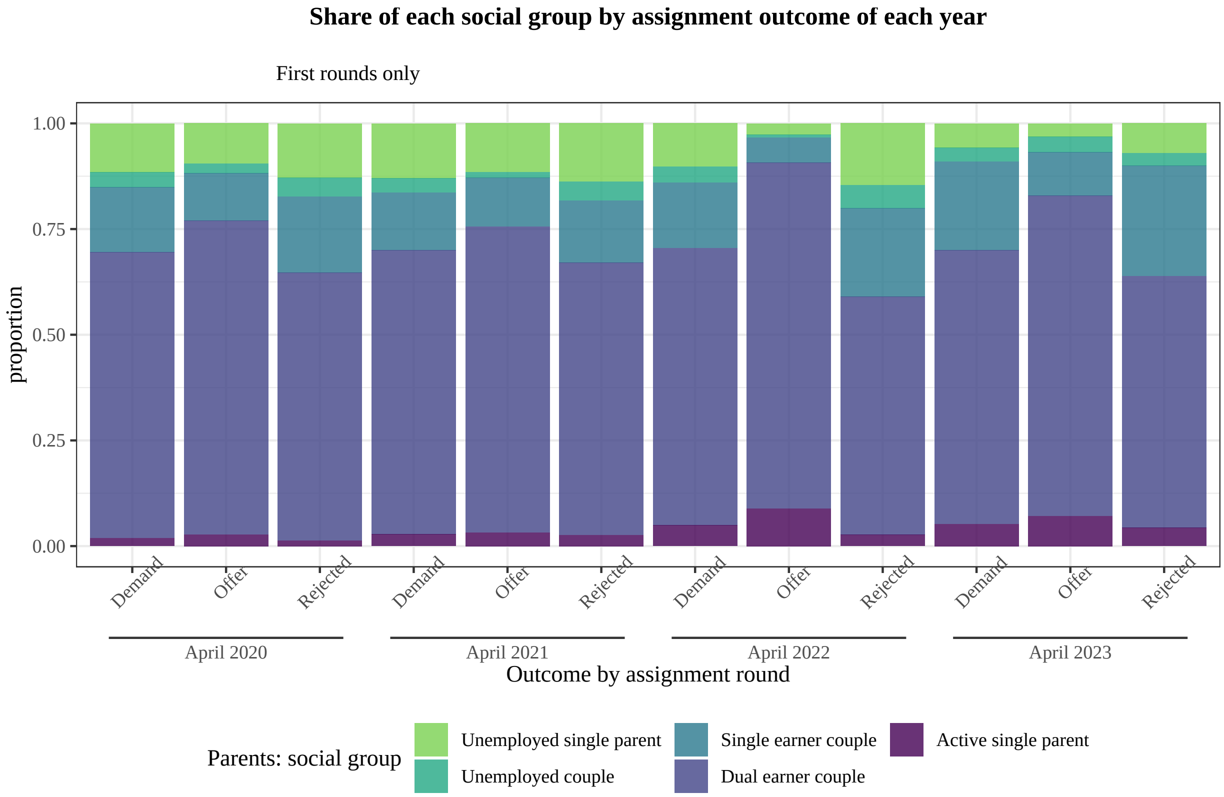
Figure C.14: Demand on the rise



Sources: ISAJE, Case Study I − 2020 : 2023.
Notes:
− Top panel: The number of demands by social group.
− Bottom panel: The proportion of demands at each assignment.

B) Cumulative share of early registration

Figure C.15: Variation of the social composition of assigned and waitlisted groups over the years



Sources: ISAJE, Case Study I − 2020 : 2023 − first rounds only.

C.IV Definition of priorities

A) Criteria

i) Time since registration The committee grants 1/2 point per month since registration. For the research, we needed to adjust these priorities to have enough randomised families. We negotiated with the municipality to retain the criterion but to no longer add 1/2 point per month, instead granting 1.5 points every 3 months. This way, we maintained the general *philosophy* of rewarding families who plan ahead and those who have been waiting for a long time. We just made these benefits less gradual. In 2023, the score uses half a point per month again.

ii) Residence: The committee grants a 4-point bonus for residents of the agglomeration and 0 otherwise. Note that the agglomeration accepts demands from individuals living outside, which is not the case in some territories participating in ISAJE. However, there are almost no family from outside the agglomeration. From 2021 onwards, this weight has been raised to 6 points, requiring parents from outside the agglomeration to wait a year before having the same priority level as residents. What this results in is that parents from outside the agglomeration are *de facto* banned.

iii) Family situation (social or health-related) The *family situation* is taken into account in three variables, and the weights can therefore be added to the others.

- Criteria related to the **parents' situation** can provide up to 4 points, awarded to unemployed parents actively searching for a job (2 points), undergoing job relocation or moving in the territory (1 to 2 points), facing long-term illness, disability, or social support (2 to 4 points).
- Criteria related to the **children's situation** pertain to their health and disability (2 to 4 points) or to the family receiving children related social support (1 to 4 points).
- Criteria related to "**siblings**" grant a 4-point bonus for demands for family reunification in daycare centres, *i.e.*, for families with a child already enrolled in a facility, and an additional 2 points for families with multiple demands (*e.g.* simultaneous demands for twins or siblings).

iv) Social groups: family structure and employment The criteria for *the family's professional situation* combine *activity* and *household structure* to divide demands into 5 mutually exclusive categories:

1) Single-parent in employment receive 4 points, the maximum.
2) Dual earner couples receive 3 points.
3) Single earner couples receive 2 points.
4) Single-parent without employment receive 2 points.
5) Couples with unemployed parents receive 1 point, the minimum.

This criteria defines what we call *social groups*. Table C.5 in Appendix A reports the number and share of each social group every years.

For the following years, the early childhood department revised the weights for some criteria. From 2022 onwards, the priorities for active parents have been dramatically increased. Weights for dual-earner couples have been multiplied by 2.6 going from 3 to 8 points, those of active single-parents by 2.25 going from 4 to 9. Inactive single-parent families and couples now receive 2 or 3 points, while unemployed couples still receive 1 point.

B) Distribution of other priority criteria across groups

Table C.6: Distribution of priority criteria across social groups over each year

	Social groups	**Share**						**N**
		children priority	*parents priority*	*multiple demands priority*	*sibling group priority*	*Resident priority*	*early registration (>9 months)*	
2020	*Active single parent*	4.5	36.4	0.0	0.0	100.0	13.6	**22**
	Dual earner couple	2.1	5.1	3.2	3.1	99.5	24.5	**746**
	Single earner couple	12.0	21.0	0.0	1.8	100.0	16.2	**167**
	Unemployed couple	23.7	57.9	0.0	0.0	100.0	13.2	**38**
	Unemployed single parent	4.0	21.6	8.0	3.2	99.2	17.6	**125**
2021	*Active single parent*	6.8	15.9	0.0	4.5	100.0	0.0	**44**
	Dual earner couple	3.0	5.2	4.0	4.8	99.7	12.0	**724**
	Single earner couple	6.4	19.2	1.3	3.8	100.0	5.8	**156**
	Unemployed couple	13.3	37.8	11.1	2.2	100.0	4.4	**45**
	Unemployed single parent	9.1	35.0	7.0	2.1	95.1	9.1	**143**
	Active single parent	0.0	18.2	15.2	0.0	100.0	13.6	**66**
	Dual earner couple	0.5	3.3	9.0	3.1	98.4	14.8	**815**
	Single earner couple	6.0	25.1	14.1	1.0	100.0	11.1	**199**

Sources: ISAJE, Case Study I - 2020 : 2023.
Notes : We identify files at the children level to avoid counting repeated demands for the same child. Totals indicate the number of unique application per children over all years.

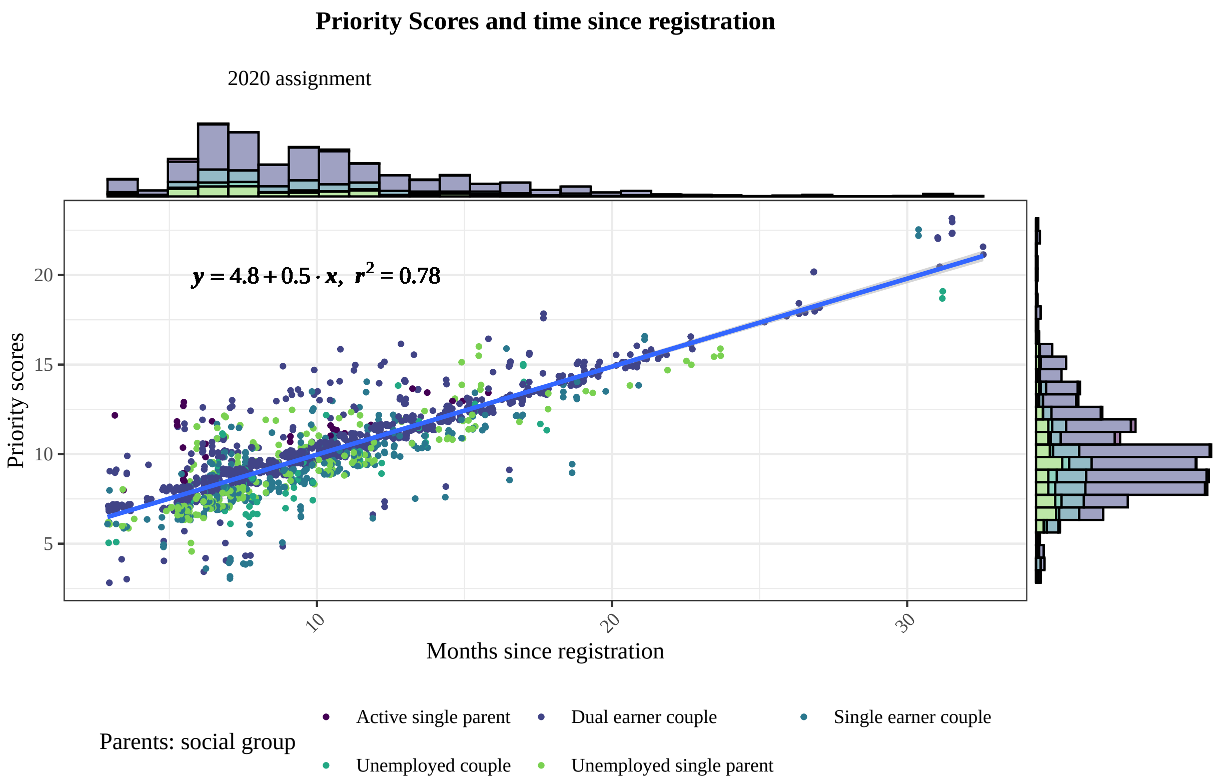
Table C.6: Distribution of priority criteria across social groups over each year

	Social groups	**Share**						**N**
		children priority	*parents priority*	*multiple demands priority*	*sibling group priority*	*Resident priority*	*early registration (>9 months)*	
2022	*Unemployed couple*	4.9	41.5	7.3	0.0	97.6	7.3	**41**
	Unemployed single parent	11.7	39.1	14.1	1.6	99.2	7.0	**128**
	Active single parent	9.1	19.7	6.1	0.0	100.0	7.6	**66**
	Dual earner couple	2.0	4.8	6.4	2.8	98.9	17.3	**784**
	Single earner couple	16.8	29.0	13.4	3.1	100.0	21.0	**262**
2023	*Unemployed couple*	53.5	72.1	27.9	0.0	100.0	25.6	**43**
	Unemployed single parent	38.8	61.2	20.9	0.0	100.0	10.4	**67**

Sources: ISAJE, Case Study I - 2020 : 2023.
Notes : We identify files at the children level to avoid counting repeated demands for the same child.
Totals indicate the number of unique application per children over all years.

C) Variation of priorities with time since registration in 2020

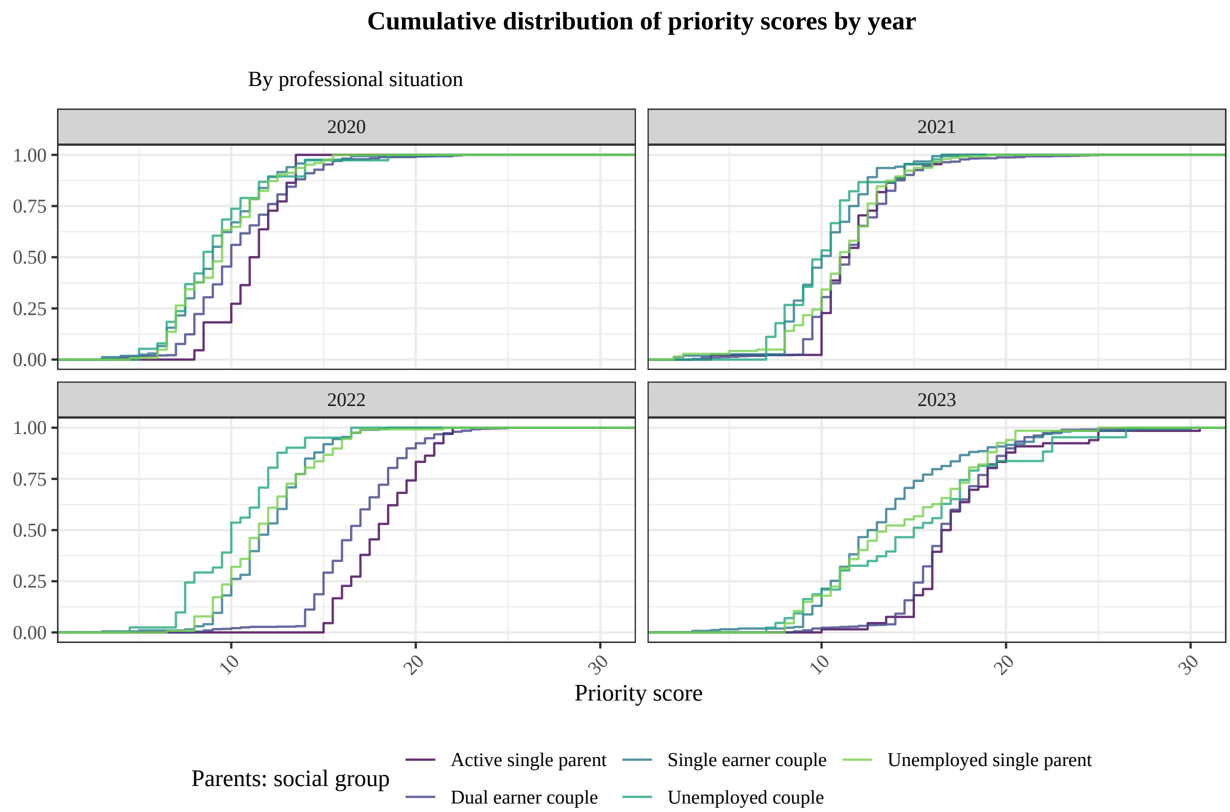
Figure C.16: Variation of priority scores with time since registration in 2020



Sources: ISAJE, Case Study I − 2020, first round only.
Notes: We present the scatterplot of the priority score against the number of months since registration at the time of the committee. The line is the prediction of the linear regression associated with the scatterplot. The colors represent social groups. We add the marginal distributions of both variables at the outset, color coded by social groups.

D) Evolution of the cumulative distribution of priorities over the years

Figure C.17: Structural changes in the composition of demand and priorities



Sources: ISAJE, Case Study I − 2020 : 2023 − first rounds only.

C.V Preferences

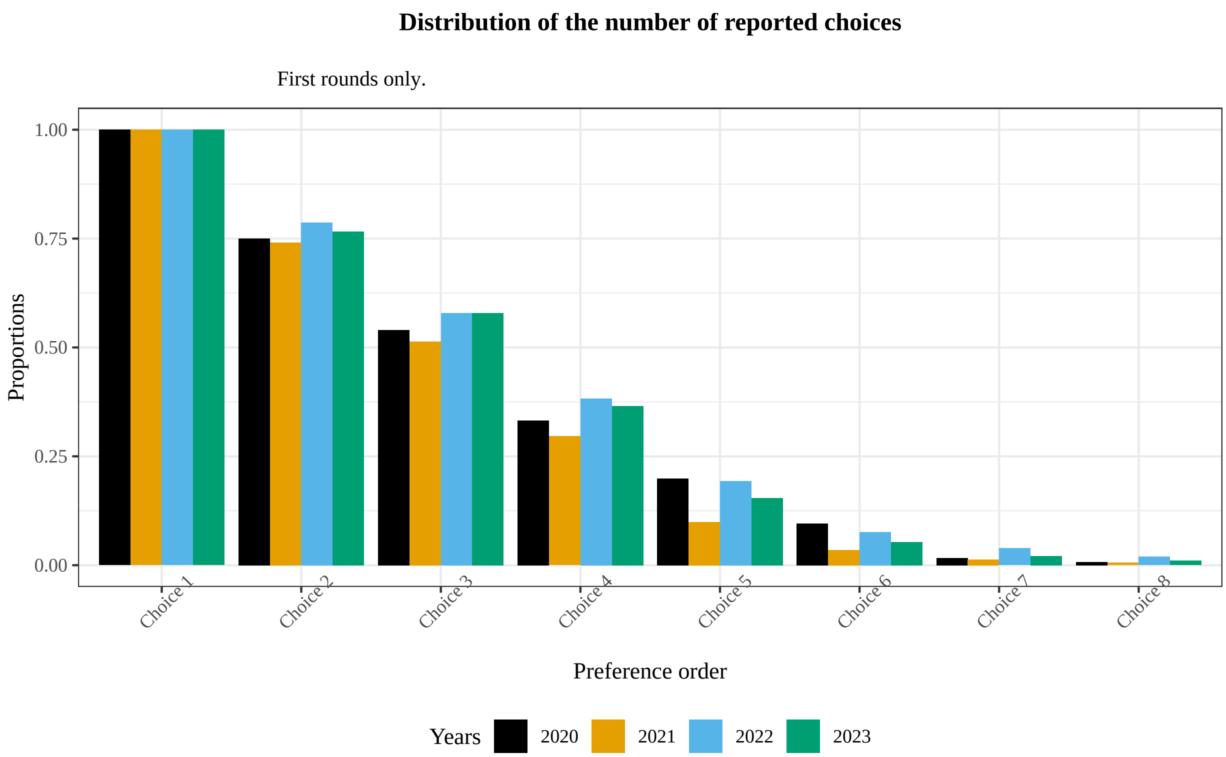
Parents can rank up to 8 wishes and figure C.18 shows their distribution. Most parents report one or two choices. This means that parents' choices are not limited by the 8 preferences, and families find few options acceptable. The committee tries to allocate families from their most preferred to least preferred options, and those who provide fewer preferences have fewer opportunities to be qualified. It all depends on how strong the competition is in the age groups of these childcare facilities.

Finally, we need to analyse the requested days by families. For this, we estimate the average proportion of requests for each day in the first rounds of the committees by professional situation. Figure C.19 presents these results in separate panels for each year.

In 2020 and 2021, aside from differences in average proportions between groups, the patterns of requested days in the week are similar between groups, with only Wednesday being consistently less requested. Starting in 2022, the requested days evolve for some groups: in 2021, unemployed couples and single-parent families mainly request days at the beginning of the week and fewer towards the end of the week. The following year, we see similar requests to 2020 and 2021.

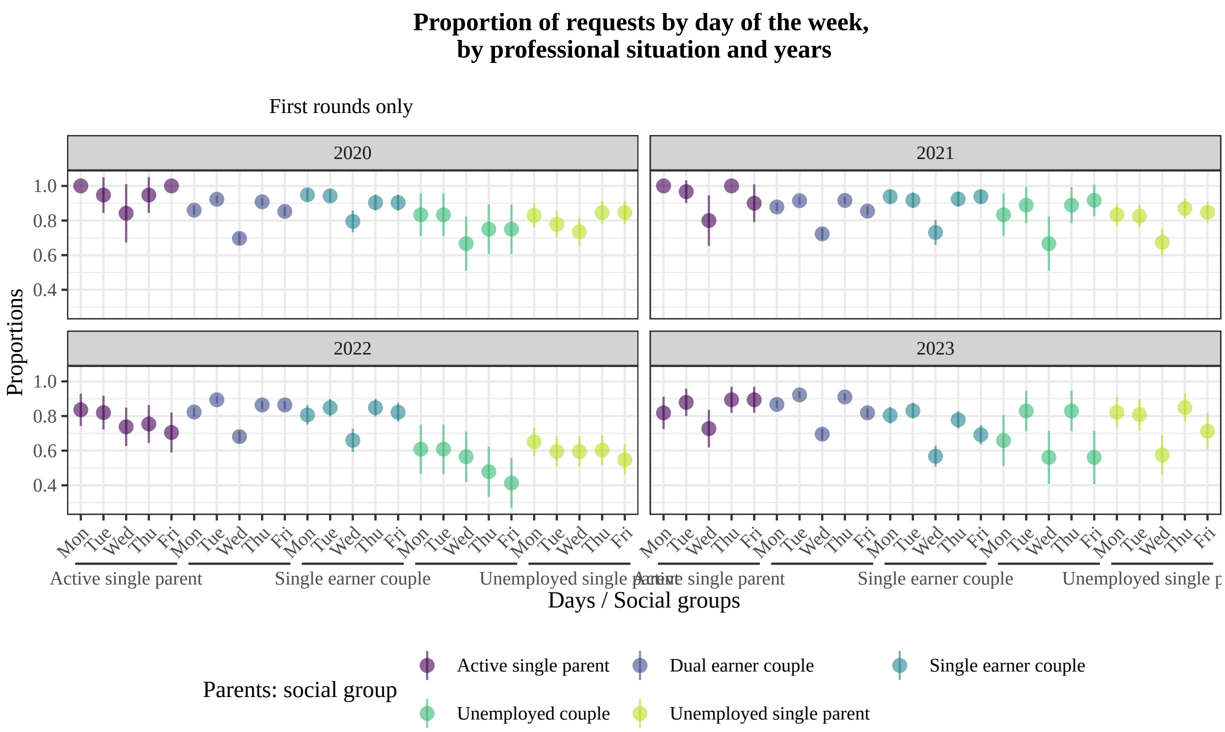
For all families, Wednesday remains the least requested day, and working families request more days than other families.

Figure C.18: 50% of families report 3 choices or less



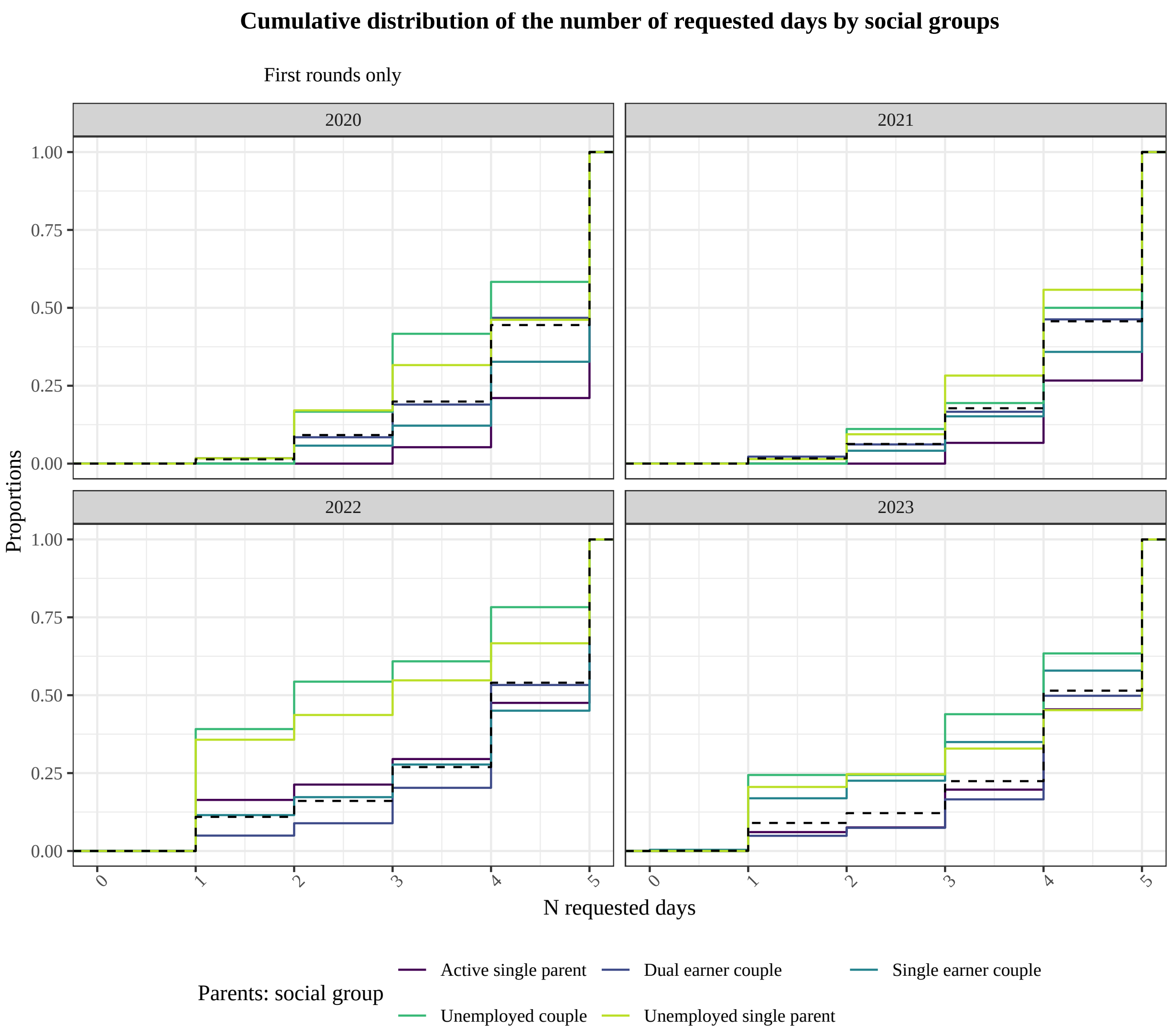
Sources: ISAJE, Case Study I − 2020 : 2023 − first rounds only.
Notes: In 2020, 75% of applications ranked at least 2 daycares

Figure C.19: Changing patterns in the demands over weekdays over years and social groups



Sources: ISAJE, Case Study I − 2020 : 2023 − first rounds only.
Notes: − Points indicate the average proportion of requests by day, committee year, and social groups. Error bars represent the 95% confidence interval.

Figure C.20: 2022 : higher demands for few days among those without a jobs



Sources: ISAJE, Case Study I − 2020 : 2023 − first rounds only.
Notes: empirical cumulative distribution of the number of requested days by social group.
Dashed line represents the ECDF over all demands.

A) Parents reported preferences in a *wide* marketplace

Figure C.21 plots the *heat map* of the average number of applications over years for each daycare and preference order rank. This visualization shows the 5 largest daycare centres are also the 5 most demanded across all ranked choices. Most facilities are located in the two main cities (Valence and Romans-Sur-Isère) and distance to other daycare centres may be large. In the map presented in Figure C.22, we show the flows of reported choices with proportional width by $\log^{10}$ flow size, mapped from parents' home towns to each daycare centres, colour-coded by rank. The two urban centres serve as major hubs, concentrating nine of the most sought-after and, consequently, most congested daycare centres. Parents out of the two large urban centres tend to choose the closest daycare as first choice and then others in the main urban centres. In Figure C.23, we present the distribution of straight-line distances to various types of childcare centres. Registered parents live close to the most demanded daycare centres, especially the most congested ones. This is endogenous selection at play, and one should not infer that distance deter parents. Only that conditional on registration, parents live very close to their chosen daycares. It is likely that parents moved-in partly because of nearby childcare infrastructures (Lawrence, Root, and Mollborn 2015). Residential mobility is influenced by amenities for children, childcare and school in particular[77].

Parents do not seem constrained by the number of options. Many could report more choices but choose not to. So which daycare centres do they chose ?

Figure C.21 plots the *heat map* of the average number of applications over years for each daycare and preference order rank. We rank them from the most demanded – across all choices – to the least. We also add the publicised childcare capacity to the left and the total demand by capacity. This visualization shows the "*most popular*" daycare centres, their capacity and at what rank they are chosen. In this marketplace, the 5 largest daycare centres are also the 5 most demanded across all ranked choices. However, *clé des champs* is much more reported as a first choice and far less as other choices. *Pablo Neruda* is the highest second choice.

To better understand these patterns, we first proceed with a geographical analysis. Recall that this market pools 54 cities over 940 km^2. Most facilities are located in the two main cities (Valence and Romans-Sur-Isère) and distance to other daycare centres may be large. In the map presented in Figure C.22, we show the flows of reported choices with proportional width by $\log^{10}$ flow size, mapped from parents' home towns to each daycare centres, colour-coded by rank. We also display some note-worthy establishments : the top 5 most demanded and the top 5 most congested daycare centres.

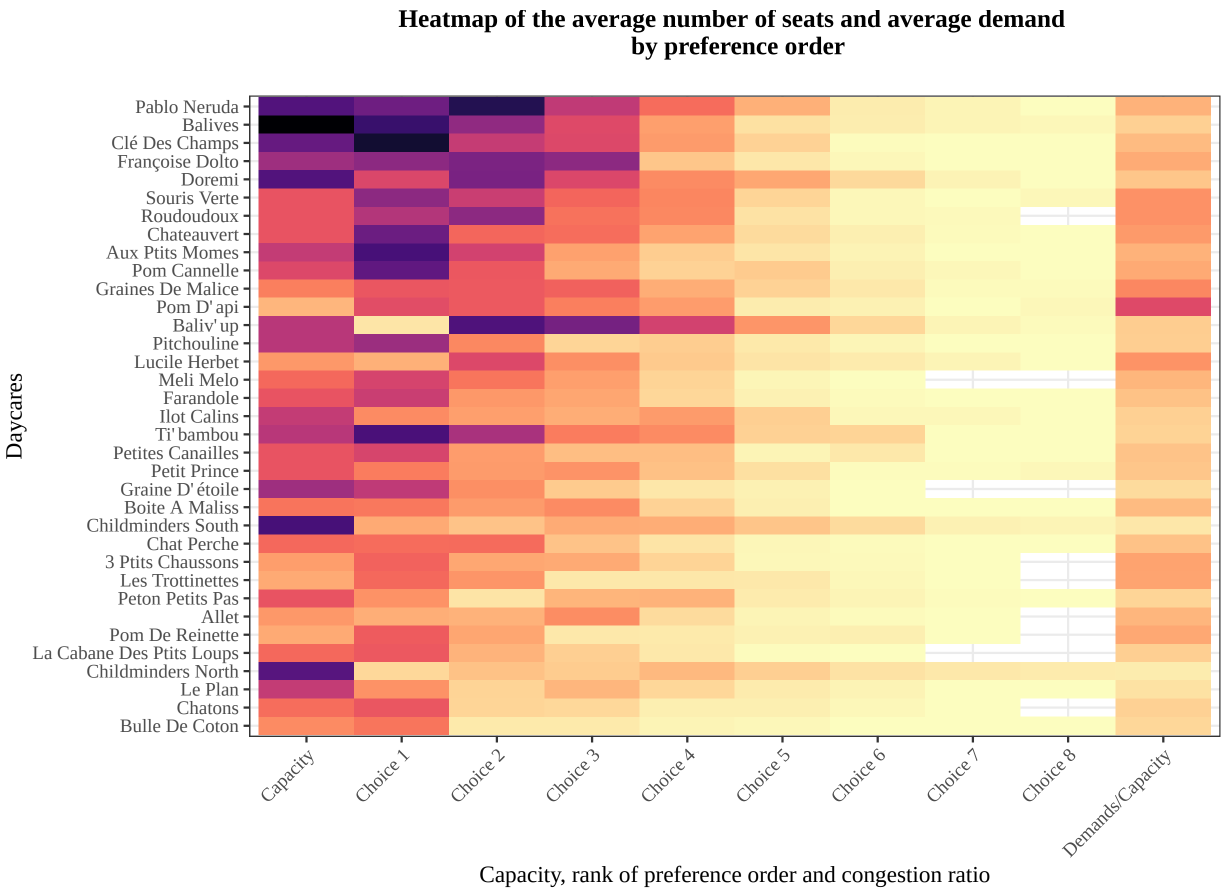
This map reveals three compelling phenomena. First, it's clear that the two urban centres serve as major hubs, concentrating nine of the most sought-after and, consequently, most congested daycare centres. Second, despite being the largest, these daycare centres are often ranked as 3rd or higher choices by parents residing away from the urban centres. Parents out of the two large urban centres tend to choose the closest daycare as first choice and then others in the main urban centres. Third, the third most demanded childcare centre, *Clé des Champs*, located to the north, predominantly attracts first-choice demands from parents in Romans-Sur-Isère. This highlights the significance of the geographic distribution of daycare centres in shaping demand and the strategies parents employ. However, this map does not fully capture the dynamics, hiding those of parents residing within the urban centres.

In Figure C.23, we present the distribution of straight-line distances to various types of childcare centres. The top panels display the distribution for the five most demanded and five most congested daycare centres. In the lower-left panel, we depict the distance distributions for parents who request "crèches familiales," which consist of childminders appointed by the administration. While these childcare facilities are typically located in the children-parents area ("Lieu Parent Enfant"), our focus is primarily on the distribution of choices within this context. As already observed, demand for childminders mostly comes from low rank choices reflecting parents preferences for collective childcare (Cartier et al. 2017). The final panel illustrates the distance distribution for other childcare options.

The distribution of the demand for top 5 most demanded daycares is bimodal only because of *clé des champs* but otherwise, all distribution shows that registered parents live close to the most demanded daycare centres. That is even more striking among the most congested childcare centres.

[77] Avery and Pathak (2021) model the externality of introducing a school choice market on the housing market and describe complex equilibria. In France, Fack and Grenet (2010b) showed that the quality of public schools in Paris increases house prices around.

Figure C.21: Preferences, capacity and congestion

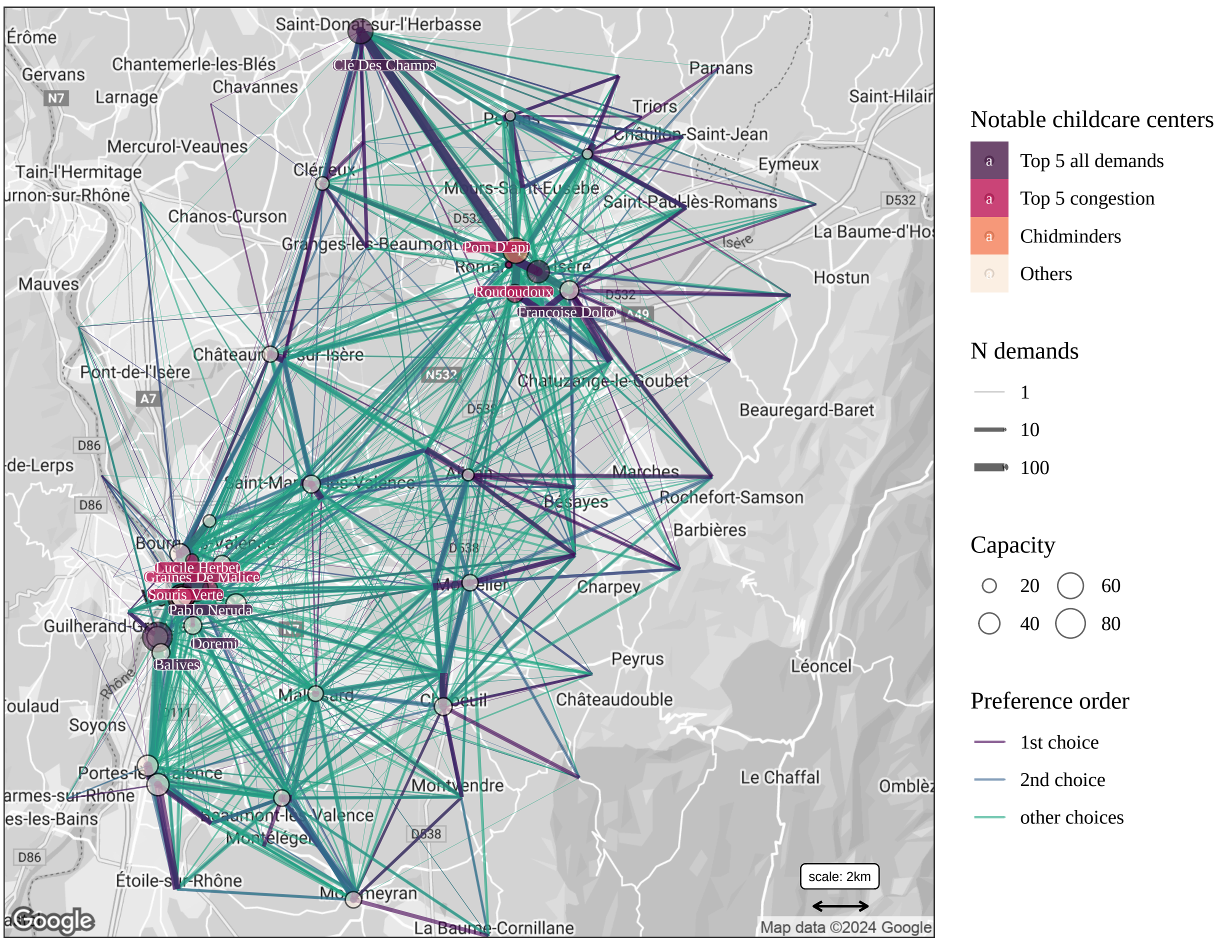


Sources: ISAJE, Case Study I − 2020 : 2023 − first rounds only.
Notes:
− Capacity as reported on the early−childcare page of the adminstration website.
− Average number of applications submitted each year to each daycare by preference order.
− Ratio of the mean total demands over theoretical capacity.

Figure C.22: Demand flows from different cities by preference order

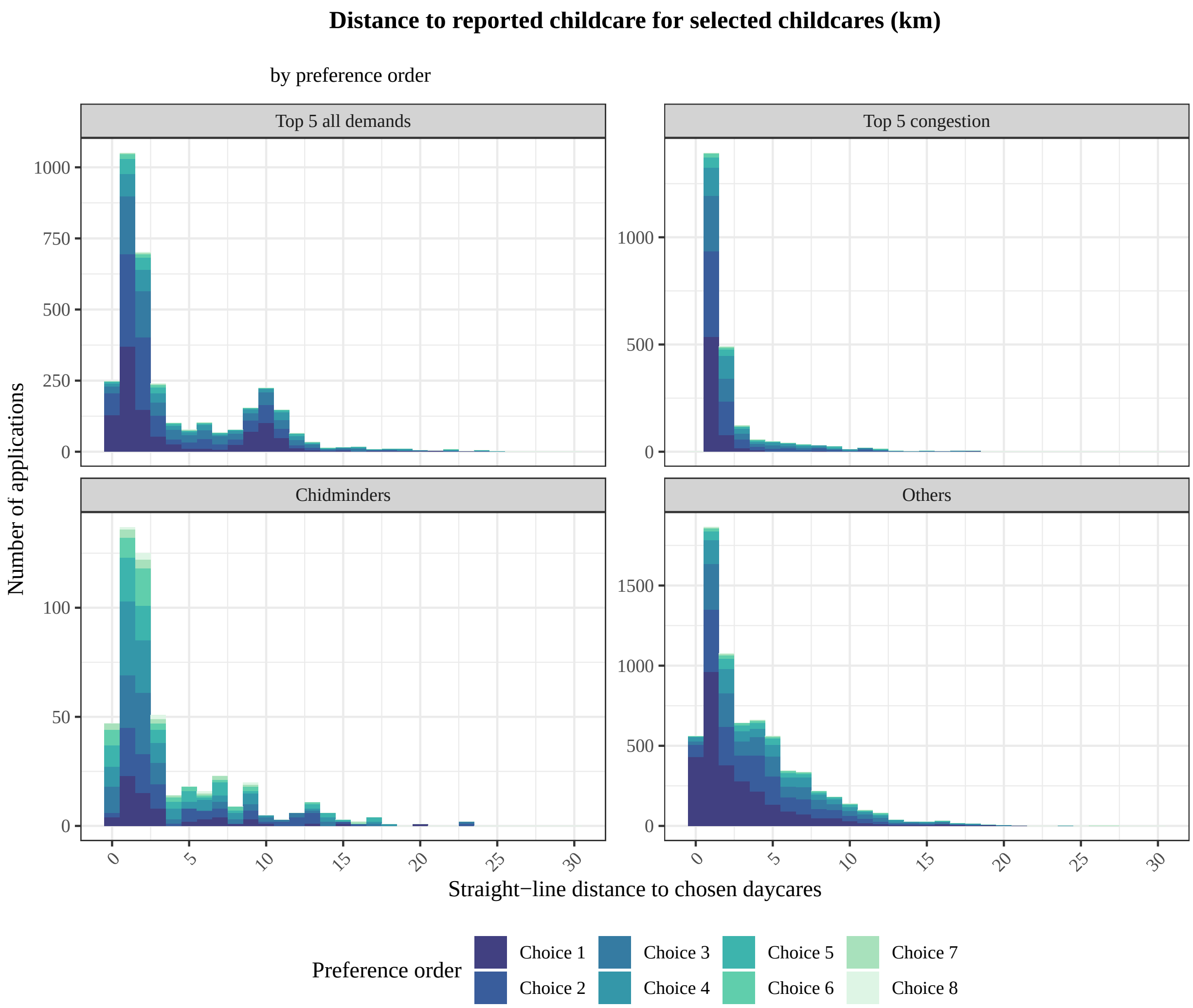
Flow map of demands to ranked daycares by preference order

Demands for the first rounds



Sources: ISAJE, Case Study I − 2020 : 2023 − first rounds only.
Capacities are those reported on the early−childhood brochure for 2023.
All demands are grouped by their home town. Flows are proportional to the log10 of the number of files from the same place.

Figure C.23: Distribution of the distance from home to reported daycare centres by rank and for different group of childcares



sources: Sources: ISAJE, Case Study I − 2020 : 2023 − first rounds only.
Note: Distance from home to reported daycares by group of daycares and rank in the preference order.
Childminders designates 'crèches familiales' and their distance is less meaningful.

D Descriptive statistics on the assignments

D.I Table of aggregate results

Table D.7: Results of automated assignment in VRA over the four years of experiment

		Total					Proportions			
Year	**Round**	Number of demands	Maximum open days	Minimum open days	Average offered seats	Number of assigned families	% Offer	% 1st choice	% 2nd choice	% 3rd choice
2020	1	1,014	440	344	392	400	39.4	64.0	17.2	11.0
	2	694	162	80	111	98	14.1	58.2	18.4	12.2
2021	1	1,064	427	333	380	373	35.1	67.6	18.0	9.4
	2	810	212	97	140	108	13.3	50.9	18.5	15.7
2022	1	1,233	509	401	442	450	36.5	60.9	24.2	10.2
	2	911	175	75	109	109	12.0	52.3	16.5	13.8
2023	1	1,267	488	382	429	408	32.2	58.8	20.8	12.0
	2	919	197	91	128	115	12.5	49.6	25.2	11.3

Sources: ISAJE, Case Study I - 2020 : 2023.
The proportions of choices served are calculated among those accepted.
Reading: In 2023, in the first round, the procedure offers a spot to 408 out of 1267 registered applications, which is 32.2% of demands satisfied, among which 58.8% received their first choice.

D.II Comparison of Accepted and Waitlisted Families' Profiles

Table D.8 presents the average values of various participants' characteristics over the 4 years of the experiment, based on their admission status with the SOFM of that year. We keep only one observation per child.

The results presented in this table are informative. First, in general:

- There are significant differences in virtually all included measures. The only ones with no difference concern the number of requested days.
- The characteristics weighted in the priority scores increase the probabilities of admission.

Regarding family strategies:

- Families who submitted only one wish, on average, have a 6-point lower probability of being admitted.
- Families who submitted at least 3 wishes are 10 points more likely to receive an offer than those who ranked less daycares.
- Admitted families enroll much earlier, for a large part before delivery.
  - Admitted children are 1 month older.
  - Children admitted in September have been registered for an average of 13 months, 3.5 months longer than those in the waiting list, on average.
  - The average child age at the time of registration is 2.5 months younger among assigned families than among waitlisted ones.
  - 65% of admitted children were enrolled before birth, compared to 47% of rejected children.

These observations highlight the direct impact of parental anticipation on the outcomes.

Regarding professional situations, dual-working couples and active single-parent families are significantly more represented among the admitted families than among those on the waiting list. However, for other types of families, the differences are reversed.

The last row of the table indicates the share of families whose assignment depends on the outcome of the lottery, which is necessary for the ISAJE research[78]. Overall, the assignment is largely determined by priorities and capacity constraints, as only 20% of admitted families have an assignment dependent on the lottery draw, and only 10% of those on the waiting list. This confirms that the experimental conditions did not have a significant effect on the course of the assignment committees. We did not disrupt much the assignment by introducing lotteries.

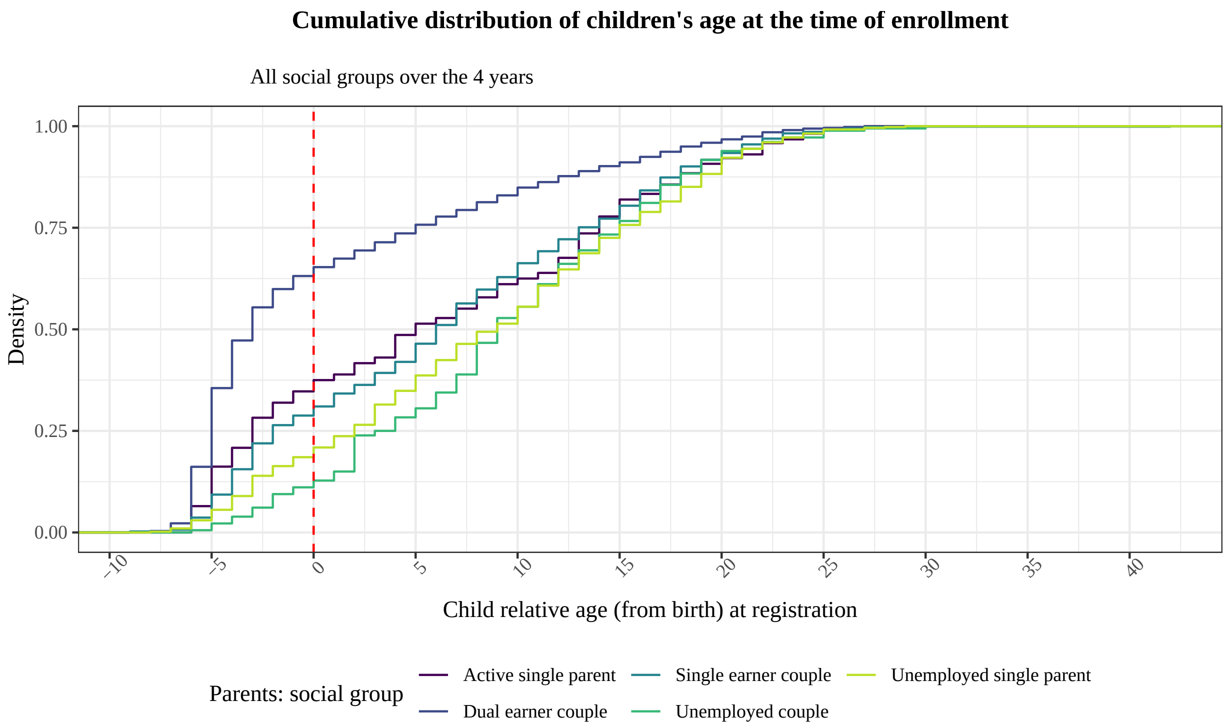
[78] It is obtained by simulating 1,000,000 assignments while fixing the entire problem and changing only the lottery draw. For each record, we then calculate the proportion of these one million simulations in which it is admitted or on the waiting list.

Table D.8: Comparison of assigned and waitlisted families over the 4 years in the experiment

	Offer (N=1462)		Rejected (N=3219)		Diff. in Means	Std. Error
	Mean	*Std. Dev.*	*Mean*	*Std. Dev.*		
Child Age (months)	13.39	7.99	12.21	8.74	-0.99***	0.29
Number of Preferences	3.22	1.66	2.87	1.58	-0.38***	0.06
Share with Only One Preference	0.19	0.39	0.24	0.43	0.06***	0.01
Share with at Least 3 Preferences	0.63	0.48	0.54	0.50	-0.10***	0.02
Average Number of Days Requested	4.16	1.06	4.10	1.22	-0.06	0.04
Share Requesting 5 Days	0.49	0.50	0.53	0.50	0.03*	0.02
Share Requesting at Least 3 Days	0.20	0.40	0.23	0.42	0.02	0.01
Average Months Since Registration	13.06	4.60	8.82	3.71	-3.55***	0.13
Average Child Age at Registration	0.77	8.60	3.80	9.07	2.54***	0.31
Share Registered Before Birth	0.65	0.48	0.47	0.50	-0.16***	0.02
Share Dual earner couple	0.76	0.43	0.61	0.49	-0.15***	0.02
Share Single earner couple	0.09	0.29	0.20	0.40	0.11***	0.01
Share Unemployed couple	0.02	0.13	0.04	0.20	0.03***	0.00
Share Unemployed single parent	0.07	0.25	0.11	0.32	0.05***	0.01
Share Active single parent	0.06	0.23	0.04	0.18	-0.03***	0.01
Share Random	0.20	0.40	0.11	0.31	-0.08***	0.01

* = p<.1, ** = p<.05, *** = p<.01
Sources: ISAJE, Case Study I - 2020 : 2023 - first rounds only.
The reference for all date calculations is September 1st.
Note: We present means and standard deviations in each group and the mean difference and associated standard error, accounting for stratification by assignment round.

Figure D.24: Cumulative empirical distribution of the age of the child at month of registration



Sources: ISAJE, Case Study I − 2020 : 2023.
notes:
− One observation is kept for each child presented to one or the other of the two committees,
− The variable analysed is constructed as the difference, rounded in months,
between the date of birth (actual or expected) and the date of registration.

D.III Wastefulness: Illustration with the First Round of the 2023 committee

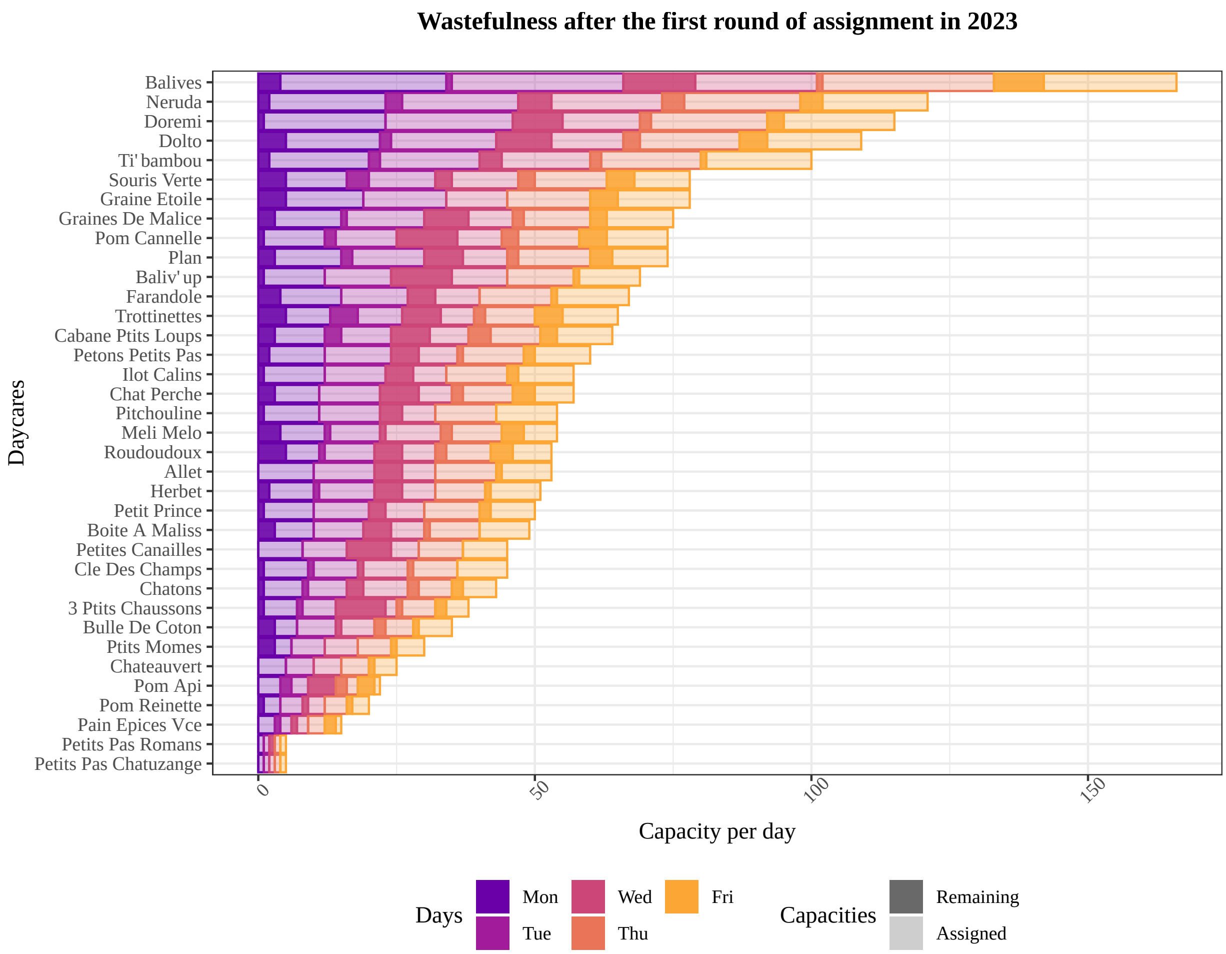
As explained earlier, our algorithms allow applications to be ranked within each bucket based on priority scores, and we continuously check if there are enough capacities in each day to accommodate the received applications. With the CAM algorithm, as soon as a day reaches its capacity threshold, the subsequent applicant that requires that day sets the final cutoff. If we use an alternative definition of *fairness* and allow weak accommodation, more children may still be accepted. However, it's important to note that the majority of demands are for full-time slots, with only a few part-time demands that can be combined (Especially in 2022, see Figure C.20 in the Appendix). Consequently, once the most requested days can no longer accommodate new demands, some buckets are left with available capacities on specific days.

In Figure D.25, we illustrate the wastefulness across daycare centres in the 2023 first round. The majority of capacities are successfully assigned, but there are still some vacant spots in all facilities. Notably, given the lower demand for Wednesdays (Figure C.19 in the Appendix), most remaining slots are on that particular day. During the 2023 round, rejections primarily result from capacity exhaustion on Tuesdays and Thursdays.

These available spots become part of the pool for the second round after families receive notification of the results approved by the assignment committee. This validation process includes confirmation with the early childhood service upon notification. More importantly, it entails scheduling appointments with the directors of the childcare facilities. During these contractual negotiations, family needs can be adjusted, and any days left vacant due to withdrawals and adjustments are reintroduced in the second round.

Essentially, the "*waste*" in the first round finds a second chance for assignment in the second round. Moreover, the spots from the second round are repurposed in September to facilitate new assignments, especially for providing occasional care. This coordination between two "*markets*," each catering to different needs, allows for more efficient resource utilisation. Nonetheless, one might question whether we can achieve even more efficiency by employing alternative methods, such as the KDA procedure.

Figure D.25: Used and remaining capacities after the first round of assignment in 2023



Sources: ISAJE, Case Study I 2023.
Notes: We sum the available slots per day in the sections and subtract those assigned.

D.IV Comparison of the final assignment with results of the algorithm

Table D.9: Deviation from the family optimal fair matching in the final assignment

SOIFFA	**Final assignment**				
	Same as SOIFFA	*Unmatched*	*Waitlist reassigned*	*Lower ranked choice*	*Higher ranked choice*
1 choice	217	6		14	
2 choice	73	2		1	9
3 choice	41	2			3
4 choice	14	1			4
5 choice	9	1			
6 choice	1				1
7 choice	2				
Waiting List	813	3	47		
TOTAL	**1,170**	**15**	**47**	**15**	**17**

Sources: ISAJE, Case Study I - 2023, first round only.
Notes: This table compares the actual assignment with the proposal from the SOIFFA. The first column indicates how many files received the same offer. In the following columns, we indicate how many families were moved from the initial assignment in the corresonding row.
217 families had their first choice with the algorithm, but 6 should have had their first choice and were placed on the waiting list by the allocation committee. 47 families were placed on the waiting list in the SOIFFA and received a proposal instead.

D.V Assignment probabilities by social groups

Table D.10: Probability of receiving an offer by social group

	sample				
	Full sample	*2020*	*2021*	*2022*	*2023*
Active single parent	0.52*** (0.04)	0.58*** (0.11)	0.40*** (0.09)	0.66*** (0.06)	0.44*** (0.06)
	[0.45, 0.60]	[0.36, 0.79]	[0.22, 0.58]	[0.53, 0.78]	[0.32, 0.56]
Dual earner couple	0.41*** (0.01)	0.43*** (0.02)	0.38*** (0.02)	0.45*** (0.02)	0.38*** (0.02)
	[0.39, 0.43]	[0.39, 0.47]	[0.34, 0.41]	[0.42, 0.49]	[0.34, 0.41]
Single earner couple	0.21*** (0.02)	0.29*** (0.04)	0.30*** (0.04)	0.14*** (0.03)	0.16*** (0.03)
	[0.18, 0.24]	[0.21, 0.36]	[0.22, 0.37]	[0.09, 0.20]	[0.11, 0.21]
Unemployed couple	0.20*** (0.04)	0.25*** (0.07)	0.14** (0.07)	0.07 (0.05)	0.37*** (0.09)
	[0.13, 0.27]	[0.11, 0.39]	[0.01, 0.27]	[-0.02, 0.16]	[0.19, 0.55]
Unemployed single parent	0.23*** (0.02)	0.32*** (0.05)	0.31*** (0.04)	0.10*** (0.03)	0.18*** (0.05)
	[0.19, 0.27]	[0.23, 0.42]	[0.23, 0.39]	[0.04, 0.15]	[0.09, 0.27]
Num.Obs.	4578	1014	1064	1233	1267
Std.Errors	by: idfam	by: idfam	by: idfam	by: idfam	by: idfam

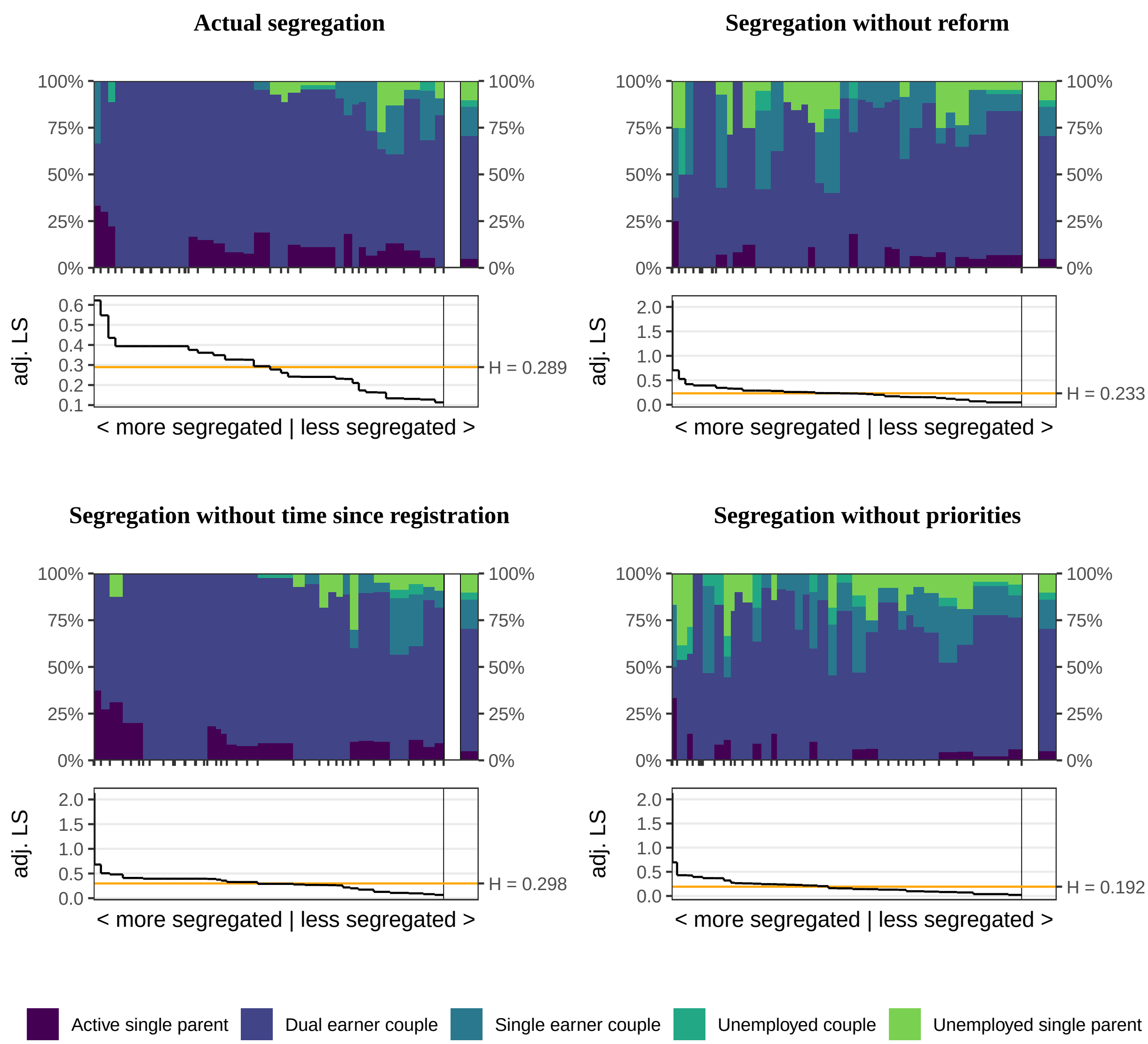
Sources: ISAJE, Case Study I - 2020 : 2023 - first rounds only.
notes: OLS estimate without constant of the proportion of families who received an offer using the automatic procedure each year. The coefficients indicate the proportion of families in each category who received an offer, the number in brackets indicates the standard error. In the first round of 2023, 37.6% of parents in a dual-earner couple were offered a place using the automatic procedure.

D.VI Analysis of segregation within daycares

A) Segplots comparing segregation across *what if* scenarii in 2022

Figure D.26: Segplots across daycares by social groups in the 2022 what if scenarii

Segplots showing patterns of segregation across daycares in the 'what if' scenarios of 2022

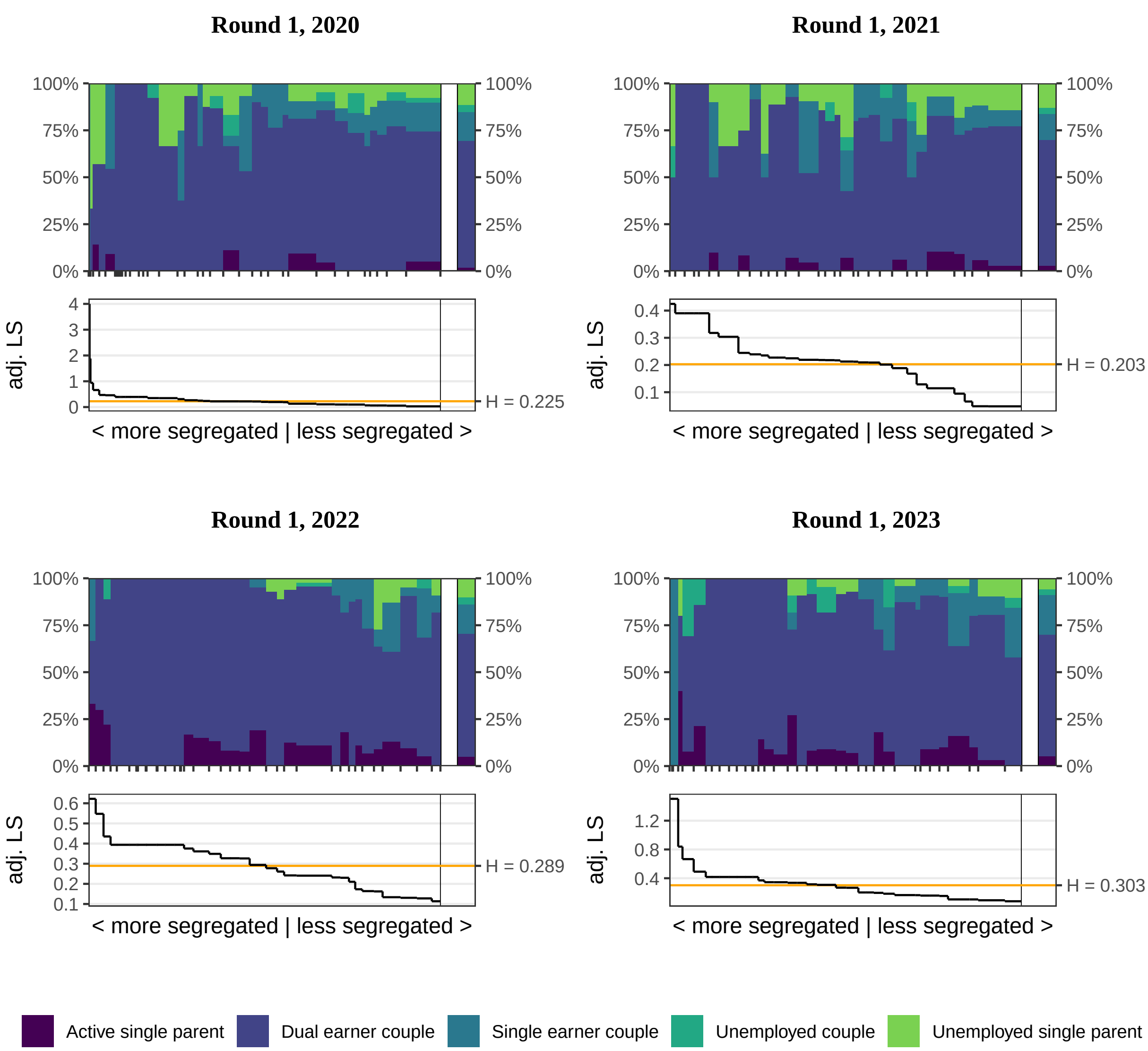


Sources: ISAJE, Case Study I − 2022, first rounds only.
Notes: Segplots are defined by Elbers and Gruijters (2024) and implemented in the package 'segregation'.
Each daycare is shown as one individual bar, where the width of the bar is proportional to the size of the daycare.
Each bar shows the distribution of social groups within each daycare.
The rightmost bar depicts the 'reference distribution'. This is the overall group distribution in the demand, i.e., including those in the waiting list. Daycares are ordered in each segplot by decreasing local segregation scores.
The bottom figure shows the adjusted local segregation scores of each unit and the overall H index (horizontal orange line).

B) Segplots comparing segregation through the years

Figure D.27: Segplots across daycares by social groups over the years in the first rounds

Segplots showing patterns of segregation across daycares in the 'what if' scenarios of 2022



Sources: ISAJE, Case Study I − 2020 : 2023 − first rounds only.
Notes: Segplots are defined by Elbers and Gruijters (2024) and implemented in the package 'segregation'.
Each daycare is shown as one individual bar, where the width of the bar is proportional to the size of the daycare.
Each bar shows the distribution of social groups within each daycare.
The rightmost bar depicts the ...reference distribution.... This is the overall group distribution in the demand, i.e., including those in the waiting list. Daycares are ordered in each segplot by decreasing local segregation scores.
The bottom figure shows the adjusted local segregation scores of each unit and the overall H index (horizontal orange line).

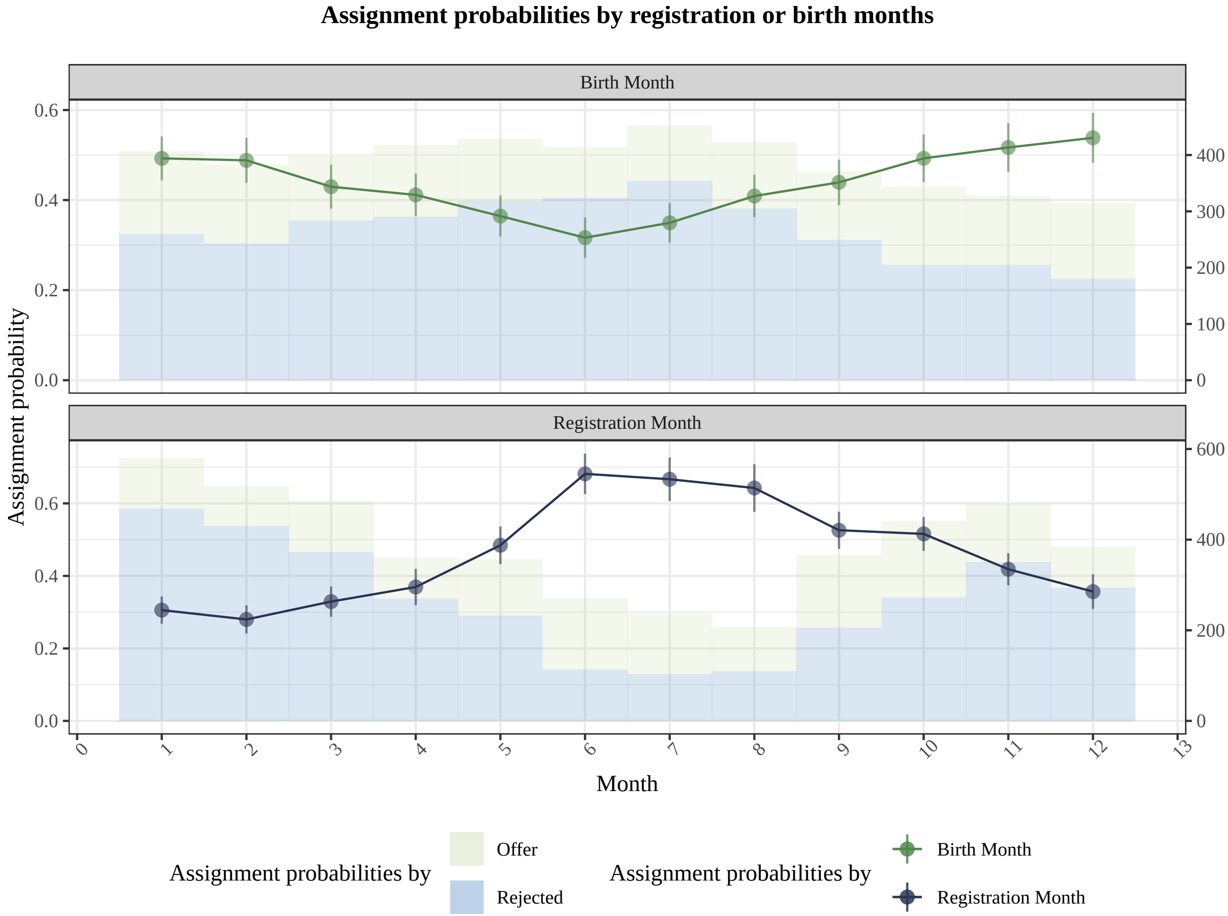
D.VII Assignment probabilities and months of registration and birth

Example .4 (The unfair advantage of well timed pregnancy)**.** Take two dual-earner couples living in the urban area, without other particular priorities[79]. A parent of the first couple becomes pregnant in February, discovers the pregnancy in April, and declares it in May. If all goes well, the child should be born in October. Given the school year, this family can expect a slot in September when the child is about 11 months old. Their dominant strategy (if they can) is to register in May. When the assignment occurs in the following April, they have been registered for 11 months, which adds 5.5 points to the 7 they have as dual-earners from the urban area. In 2020, this score puts them in the top 20% of priorities, significantly increasing their chances of obtaining a slot, provided they can wait until the child is 11 months old and do not change their mind by then.

The parent of the second couple becomes pregnant in October, discovers the pregnancy in December, and declares it in January for a child expected in July. By the start of September, this child will be about 3 months old. If this family follows the same strategy by registering as soon as they declare the pregnancy, their score will only be increased by 1.5 points in April, placing them below 55% of the demands. Their chances of obtaining a slot are therefore considerably reduced. However, if they can wait for another year until the child is 15 months old, they will have a higher priority.

[79] that is, the situation of 59.6 % of demands.

Figure D.28: Between 2020 and 2023, the probability of admission is 1/3 for families registering in January and 2/3 for those registering in June.

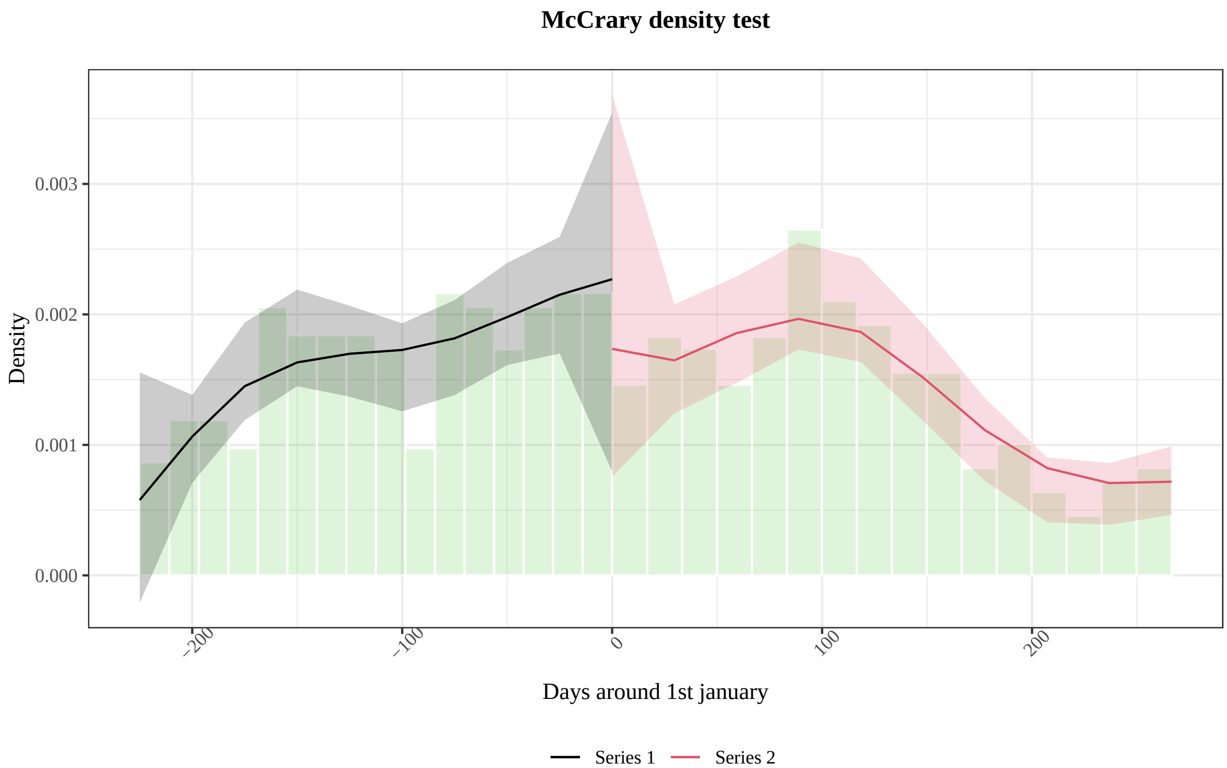


Sources: ISAJE, Case Study I − 2020 : 2023 − first rounds only.
Notes: The points indicate the proportion that received an offer based on
− the month of registration (top panel)
− the child's birth month (bottom panel)
The value of the histograms is readable on the right scale and indicates the number of observations for each month, colour−coded by admission status after the automatic procedure.
Over the period 2020−2023, children registered in June have a probability of admission of nearly 60%, while children born in October have a probability of admission of 40%.

E Additional results on Case study II

E.I McCrary density test of discontinuity

Figure E.29: No discontinuity in the densities of date of birth



Sources: ISAJE, Case Study II.
Notes: Validation test following McCrary (2008) of no discontinuity in the density around the cutoff.
Non parametric estimations following Cattaneo et al. (2014).

F Algorithms, proofs and illustrations of the main theoretical results

F.I Formal presentation of the algorithms

A) The student proposing deferred acceptance algorithm

Intuitively, we *break* daycares into individual buckets (seats) and transform preferences over daycares into preferences over buckets. This allows to define priorities for each buckets easily allowing different ordering of students within a daycare to respect diversity constraints. With SPDA, at any moment in the procedure, each bucket (seat) will keep the student that applies to this seat that has the highest local priority on that bucket. If a bucket only accepts students of group g (hard quota) and no such student applied to this seat, this bucket can end up empty. Since this wastefulness comes from eligibility rule, this matching is still stable since other student are not eligible. Soft quotas reduce the risks of such cases but again, this is a policymaker's decision.

Algorithm 1 Student Proposing Deferred Acceptance (SPDA)

Require: $(inputs := (I, S, \mathbf{q}, \theta))$

$\pi_{is} \leftarrow \rho_{is} + \epsilon_i$ ▷ Priorities with random tie-break
$k \leftarrow 1$ ▷ Initialisation
all applicants apply to their first-ranked schools.
if All schools can accomodate all applicants **then**
  assignment is final
  return $\mu(k)$
else if at least one school receives more applicants than its capacity **then**
  keep the best applicants in all schools according to their scores π_{is} up to the capacity constraint and permanently reject the others.
  Rejected applicants apply to their next favourite school[a].
  $k \leftarrow k+1$
end if
while $k \geq 2$ **do**
  Sort previously held and new applications in each school according to their scores π_{is}
  if the total number of new and previously accepted applicants is greater than its capacity **then**
    keep the best applicants in all schools according to their scores and permanently rejects the others[b]
    applicants rejected by a school in the previous step apply to their next favourite school
    $k \leftarrow k+1$
  else
    Assignment is final
  end if
end while
return $\mu(k)$

[a] So their favorite school among those which have not rejected them yet.
[b] Note that the assignment is always provisional, since schools must select the best applicants among both the new ones and the previously accepted ones. Thus, a student accepted at a given step can still be rejected if another student with a better score applies to the school. This is a key element of the algorithm.

B) The cut-off adjustment mechanism

Before describing the algorithm, let us introduce a few notations.

For a school s, let c_s be a **cut-off** of school s which is an integer in $\{0, \dots, N\}$ where $N = |I|$ is the number of applicants. Once ties are broken and the scores π_{is} at school s are obtained, order the applicants by decreasing order of score and let $\tilde{r}_{is}$ be the **rank of applicant** i in this ordering.

Given a collection of cut-offs $c := (c_s)_s$, let:

- $A_i(c) := \{s \in S : \tilde{r}_{is} \leq c_s\}$ be the set of **accessible schools** for applicant i.
- $D_s(s) := \{i \in I : s \in A_i(c) \text{ and } s \succ_i s' \text{ for } s' \in A_i(c) \backslash \{s\}\}$ be the **demand set** of school s at the cut-offs c.

In plain words, accessible schools are those where the rank of an applicant i in that school is lower than the cut-off ; the demand set is composed of all applicants in their most favourite accessible school. Cut-offs determine who can apply to a school and, given a collection of cut-offs, applicants apply to their favourite accessible school.

Algorithm 2 Cut-off adjustment mechanism

Require: *Inputs* (I, S, **q**, θ))
  $round \leftarrow 0$ ▷ *Steps 0*
  if Applicant i is ineligible at s **then**
    $\pi_{is} \leftarrow -\infty$ ▷ Or simply negative priorities[a]
  else
    $\pi_{is} \leftarrow \pi_{is} + \epsilon_i$
  end if
  $\tilde{r}_{is} \leftarrow rank(\pi_{is})$ ▷ Define the rank of each applicant in each school
  $c_s(0) = N - \sum_i \mathbb{1}(\pi_{is} < 0)$ ▷ Initialise cut-offs as the number of applicants eligible in that school
  $round \leftarrow 1$ ▷ *steps round* ≥ 1
  while $round \geq 1$ **do**
    $\mu(round) \leftarrow \mu_s(k) := D_s(c(k-1))$ ▷ Demand for school s over the cut-off at round k
    if $\mu(k)$ is feasible **then**
      Exit while
    else
      $\tilde{S}(\mu(k)) := \{s \in S : \exists t, \sum_{i \in \mu_s(k)} d_i^t > q_s^t\}$ ▷ Schools for which $\mu(k)$ is not feasible
    end if
    if $s \in \tilde{S}(\mu(k))$ **then** ▷ Adjust the vector of cut-offs $c(k)$ for schools in $\tilde{S}(\mu(k))$
      $c_s(k) = c_s(k-1) - 1$ ▷ Decrease the cut-off by one rank
    else
      $c_s(k) = c_s(k-1)$ ▷ Leave other cut-offs like they were in the previous step
    end if
    $k \leftarrow k+1$
  end while
  return $\mu(k)$

[a] Another alternative is to just delete that school from the preference list. We discuss such settings more precisely later in the paper.

C) The Knapsack deferred acceptance algorithm

The main description of KDA by their authors is different from this one. We want to emphasise the link between CAM and KDA so we use the cut-off adjustment representation of the deferred acceptance mechanism in the multi-dimensional setting. Kamada and Kojima (2023) show in their appendix that the SOFM can be obtained by this algorithm using their definition of justified envy. The main difference with the CAM algorithm is that we now use **weak-justified envy** to define the **cut-off adjustment function**. In practice, the Matlab functions are based on the definition of KDA from Delacrétaz, Kominers, and Teytelboym (2023) which is empirically *faster* than this version.

Algorithm 3 Knapsack Deferred Acceptance

Require: *Inputs* (I, S, **q**, θ))
  $round \leftarrow 0$ ▷ *Steps 0*
  if Applicant i is ineligible at s **then**
    $\pi_{is} \leftarrow -\infty$ ▷ Or simply negative priorities[a]
  else
    $\pi_{is} \leftarrow \pi_{is} + \epsilon_i$
  end if
  $\tilde{r}_{is} \leftarrow rank(\pi_{is})$ ▷ Define the rank of each applicant in each school
  $c_s(0) = N - \sum_i \mathbb{1}(\pi_{is} < 0)$ ▷ Initialise cut-offs as the number of applicants eligible in that school
  $round \leftarrow 1$ ▷ *steps round* ≥ 1
  while $round \geq 1$ **do**
    $\mu(round) \leftarrow \mu_s(k) := D_s(c(k-1))$ ▷ Demand for school s over the cut-off at round k
    if $\mu(k)$ is **weakly feasible then**
      Exit while
    else
      $\tilde{S}(\mu(k)) \leftarrow \{ s \in S : \exists t \ s.t. \ d_i(t) = 0 \mid d_i(t) + \sum_{i \in \mu_s(k)} d_i(t) > q_s^t \}$ ▷ Schools for which $\mu(k)$ is not weakly feasible
    end if
    if $s \in \tilde{S}(\mu(k))$ **then** ▷ Adjust the vector of cut-offs $c(k)$ for schools in $\tilde{S}(\mu(k))$
      $c_s(k) = c_s(k-1) - 1$ ▷ Decrease the cut-off by one rank
    else
      $c_s(k) = c_s(k-1)$ ▷ Leave other cut-offs like they were in the previous step
    end if
    $k \leftarrow k + 1$
  end while
  return $\mu(k)$

[a] Another alternative is to just delete that school from the preference list. We discuss such settings more precisely later in the paper.

D) SPDA with diversity requirements

Algorithm 4 SPDA with soft quotas

Require: *Inputs* $P = (I, S, \mathbf{q}, \underline{\mathbf{q}}, \theta)$; *Function* (*SPDA*)

procedure Break the problem in soft quotas(P)

For each school $s \in S$ and group g, create $\underline{q}_s^g$ buckets with $c_b = 1$. Let B_s^g the be the set buckets in school s corresponding to group g. Let $B := \bigcup_{s,g} B_s^g$.

Fix an ordering $\gg_s$ for each school over its buckets.

For a bucket $b \in B_s^g$, let $\succ_b$ s.t. $g \succ_b \emptyset \succ_b g'$ for $g' \neq g$.

Define $\tilde{\theta}$ such that :

i) $\tilde{\succ}_i$ is the preferences of i over B ▷ apply definition 0.4 on reported preferences to define preferences over buckets

ii) $\tilde{\pi}_{ib}$ is the score of i in bucket b ▷ apply definition 0.3 on priorities to define priorities within buckets

Let $\tilde{P} := (I, B, c, \tilde{\theta})$ be the assignment problem over buckets.

end procedure

Run SPDA on $\tilde{P}$

Output $\tilde{\mu}$ which is a matching of students to buckets

Let μ be the matching of students to schools s.t. $\mu_i = s_{\tilde{\mu}_i}$ ▷ Map matched students in each buckets to the schools and buckets

return μ

F.II Example and proof of theorem 1

Example .5 (Motivating example). Consider a very simple market with T=2 (e.g. half-time/full-time), 2 schools A and B with capacities $q_A = (1,0)$ and $q_B = (1,1)$, and three students i_1, i_2 and i_3 of the same age group with types θ summarised:

$$A \succ_{i_1} B \,;\, d_{i_1} = (1,0) \quad \rho_{1A} = 3 \,;\, \rho_{1B} = 1$$
$$A \succ_{i_2} B \,;\, d_{i_2} = (0,1) \quad \rho_{2A} = 2 \,;\, \rho_{2B} = 3$$
$$A \succ_{i_3} B \,;\, d_{i_1} = (0,1) \quad \rho_{3A} = 1 \,;\, \rho_{3B} = 2$$

Recall that the higher the ρ, the higher the priority. Normally we would add a lottery number but there is no need for it with this toy example.

We use the CAM algorithm to assign students their optimal fair matching. At the beginning, both schools start with a cut-off at 3, and at the first step, all three families apply to A. Because i_1 has top priority at A and is not feasible, we adjust the cut-off successively until the one at A reaches 0 and the daycare remains empty. At the same time at B, the different steps of the CAM algorithm leads us to a final assignment of i_2 while i_1 and i_3 are unassigned. The matching is thus $\mu = \begin{pmatrix} A & B & \emptyset \\ & i_2 & i_1,\, i_3 \end{pmatrix}$

Because i_1 is not initially feasible at her favourite daycare, she imposes a negative externality on the others and still ends up unassigned. Now, let us remove the daycares in the preference lists that are initially infeasible. The new preferences are given by:

$$B \succ_{i_1} \emptyset \,;\, d_{i_1} = (1,0)$$
$$A \succ_{i_2} B \,;\, d_{i_2} = (0,1)$$
$$A \succ_{i_3} B \,;\, d_{i_1} = (0,1)$$

Now let's run the CAM algorithm again with these preferences. The final assignment is obtained in 2 rounds with final matching $\mu' = \begin{pmatrix} A & B \\ i_2 & i_1,\, i_3 \end{pmatrix}$

In this example, removing initially infeasible demands from the preferences improves the assignment of all applicants. Now, i_2 gets her first choice and i_1 and i_3 are assigned to their second choice instead of being unassigned. This improvement comes at the cost of justified envy for applicant i_1, but this envy is based on an initially infeasible demand.

Proof of Theorem 1

Proof. Fix a problem $P = (I, S, B, \mathbf{q}, \underline{q}, \theta)$ and consider an applicant i. A policymaker defines rules for the DAM and chose a mechanism $\phi := P \Rightarrow \mu$ among CAM and KDA.

μ is the assignment using ϕ on problem P and by optimality of these algorithms, there are no stable assignment that are preferred by parents and respects the corresponding definition of envy-freeness.

Let $\succ_i$: $\{s_1, \ldots, s, \ldots, s_n\}$ be the reported preferences of i_i and let $\mu_i = s$. s can be any school in the preference list $\succ_i$ and also be $\emptyset$ if applicant i is unassigned.

Suppose that there is at least one initially infeasible demand in $\succ_i$ and let's call it s_k[80]. Let us denote $\bar{\succ}_i \equiv \succ_i \setminus \{s_k\}$ the new set of preferences without the initially infeasible school s_k.

We want to show that assignment μ with preferences $\succ_i$ is also (weakly) envy-free with $\bar{\succ}_i$. We prove by contradiction that the SOFA with initially feasible can only be better than the SOFA with full preferences. For this proof, we use fairness defined as envy-freeness and the CAM algorithm but the demonstration is the same with weak envy-freeness.

[80] Note that $s_k \neq s$. Indeed, if s_k is not feasible for i, then by feasibility of a matching with ϕ, i cannot be assigned to s_k.

Envy of applicant i in the matching with modified preferences $\bar{\succ}_i$:

1. Suppose that when i is assigned to $\mu_i = s$ using the modified preferences $\bar{\succ}_i$, she has justified envy.
2. Then $\exists\, s'\ s.\,t.\ s' \bar{\succ}_i \mu_i = s$ and $\exists\, i' \in \mu'_s$ with $\pi_{i's'} < \pi_{is'}$.
3. Since $\bar{\succ}_i \equiv \succ_i \setminus \{s_k\}$ where s_k is not feasible for i, then $s' \bar{\succ}_i \mu_i = s \Rightarrow s' \succ_i \mu_i = s$.
4. Thus, if i has justified envy when assigned to $\mu_i = s$ using the modified preferences $\bar{\succ}_i$, she also has justified envy when assigned to $\mu_i = s$ using preferences $\succ_i$. A contradiction with μ being the SOFA.

Therefore, the matching $\mu_i = s$ using the modified preferences $\bar{\succ}_i$ is also envy-free.

Envy of other applicants i' in the matching with modified preferences $\bar{\succ}_i$

1. Since only $\succ_i$ has been modified, no other preferences have been changed.
2. Therefore, if any applicant i' has justified envy with assignment μ with preferences $\bar{\succ}_i$, she also has justified envy with assignment μ with preferences $\succ_i$. A contradiction with μ being the SOFA.

Therefore, the matching μ using the modified preferences $\bar{\succ}_i$ is envy-free, completing the first part of the Proof.

Next we want to prove that this assignment is preferred to the assignment with the full set of preferences.

1. if μ is envy-free for the assignment problem P with $\bar{\succ}_i$ instead of $\succ_i$,
2. then, by optimality of the matching of the SOFM, the outcome μ' of the CAM algorithm on the same problem P but with $\bar{\succ}_i$ instead of $\succ_i$ is the student-optimal fair matching.

Thus, it is weakly preferred to μ and therefore Pareto dominates μ

QED

F.III Proof that our assignment mechanisms can be used for evaluation

Our general challenge is well summarised by Duflo (2017): "*Well-designed plumbing experiments can sometimes introduce variation that does not exist in natural conditions, and thus generate a counterfactual to illuminate theoretical mechanisms that are not easily observable in nature*". The econometrics of school choice is now mature enough to clearly define conditions under which we can use assignments using matching algorithm to measure causal effects on compliers[81].

One condition proved by Abdulkadiroglu et al. (2017) is that mechanisms need to satisfy *equal treatment of equals*. This condition is true if

1. the mechanism Φ is anonymous,
2. the distribution of probability of the lottery is symmetric

In other words, the mechanisms must not consider specific applicants differently but through priority rules and diversity quotas.

For every mechanisms we considered, we defined a lottery number ϵ_i that is $i.i.d.$ drawn from $U_{[0;1]}$. This random number is used to define the applicants' score π_{is} at schools s by combining with her priority level ρ_{is}. For each realisation of the tie-breaker, an algorithm returns a matching. For a given problem, a mechanism generates a distribution of probabilities over possible matchings, which is referred to as a **stochastic assignment**. A stochastic assignment generates a matrix $\mathcal{P}$ of size $|I| \times S$ where the entry p_{is} represents the probability that applicant i is assigned to school s s.t.

[81] For a recent review, see the chapter by Angrist, Hull, and Walters (2023) in the Handbook of economics of Education, 2023 edition.

- $\forall i \in I$, $\forall s$: $0 \leq p_{is} \leq 1$.
- $\forall i \in I$, $\sum_{s=0}^{S} p_{is} = 1$.
- $\forall s$, $\sum_{i \in I} p_{is} \leq q_s$

We have three main algorithms (and their associated soft quota version):

1. Assignment of families with SPDA soft quotas
2. Assignment of families using CAM
3. Assignment of families using KDA

for which only the definition of family types θ evolved. For problem 1, SPDA with soft quota satisfy ETE. Indeed, DA with lotteries has been shown to satisfy ETE by Abdulkadiroglu et al. (2017) and we simply break schools into seats and use SPDA. Hence the results of Abdulkadiroglu et al. (2017) apply immediately: we can compare families with the same propensity score.

For the other algorithms, we need to formally prove that they also satisfy ETE for the evaluation problem.

Assignment with days adds another component to family type θ which becomes more complex but still contains all relevant information for daycare assignment. As long as we use lotteries drawn from symmetric distributions of probabilities (which is the case for uniform distributions), the proof of Abdulkadiroglu et al. (2017) naturally extends to assignment with KDA and CAM, as shown in theorem 3.

Theorem 3. *The CAM and KDA algorithms satisfy equal treatment of equals.*

Proof. Fix a problem $P = (I, S, \mathbf{q}, \underline{q}, \theta)$, consider two applicants i and j of types θ_i and θ_j and denote ϵ_i and ϵ_j their lottery. Consider the mechanism $\Phi := P \Rightarrow \mu$ among CAM and KDA, with or without soft quotas. Denote $\mu_i(\epsilon_i) = s$ and $\mu_j(\epsilon_j) = t$ their assignments with the realised lotteries, which can be $\emptyset$ if they remain unassigned.

Suppose that individuals are of the same type $\theta_i = \theta_j = \theta$. Then $\succ_i = \succ_j$ and $D_i = D_j$ and $\rho_i = \rho_j$ and $g_i = g_j$ with soft quotas.

Since $\succ_i = \succ_j$, family i and j are ranked by the same schools (buckets) and their positions in these rankings depend on $\pi_i = f(\rho_i, g_i, \epsilon_i)$ and $\pi_j = f(\rho_j, g_j, \epsilon_j)$ where $f(\cdot)$ is common and defined within the algorithm. Since $\theta_i = \theta_j$, their rank only depends on the realised lotteries ϵ_i and ϵ_j. It is then obvious to see that if we swap their lottery and consider the ranking with $\pi'_i = f(\rho_i, g_i, \epsilon_j)$ and $\pi'_j = f(\rho_j, g_j, \epsilon_i)$, then we just swap family i and j ranks. Now, consider in particular the ranking at daycare s where i is assigned with $\mu_i(\epsilon_i)$. It is clear then that if we swap the lotteries, we swap the rank at school s_i and by design $\mu_j(\epsilon_i) = s$. Therefore Φ respects equal treatment of equals.

QED

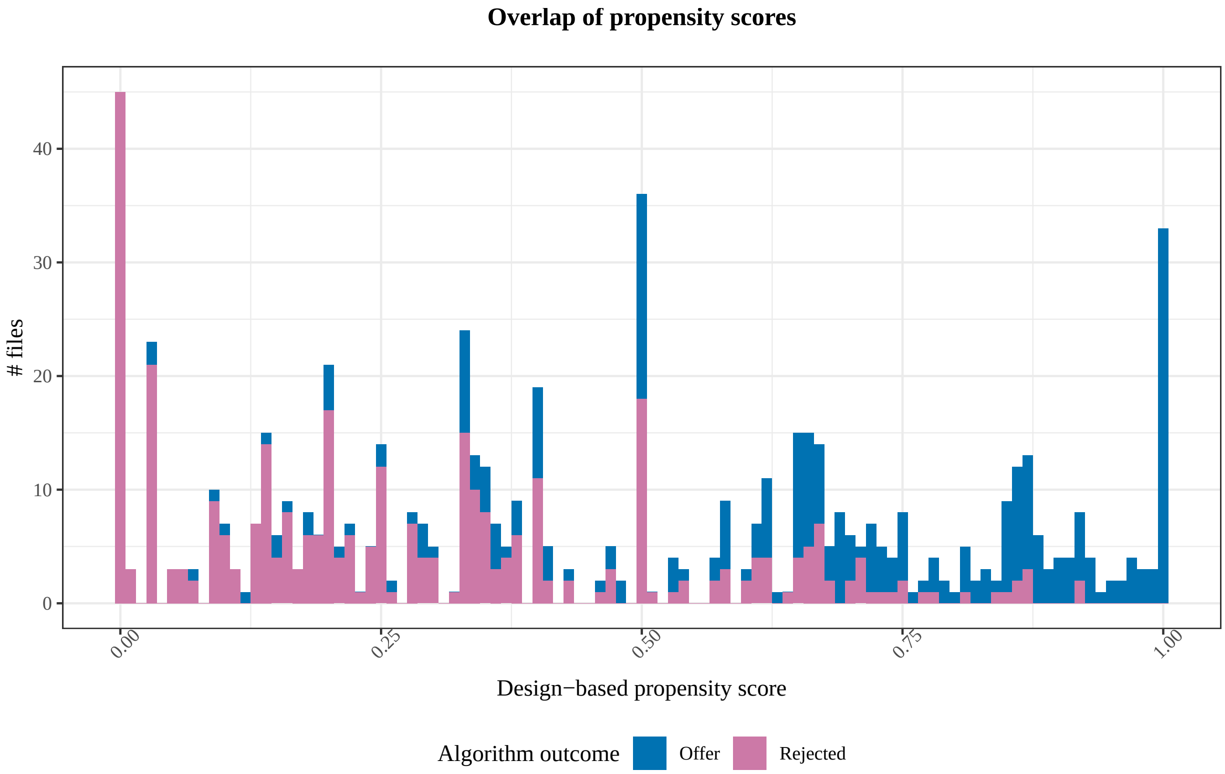
Our models provide the right setting for our evaluation problem.

Tie-breaking introduces artificial stability constraints since, after tie-breaking, schools appear to have strict preferences between students for whom they are indifferent. Abdulkadiroğlu, Pathak, and Roth (2009) showed that single tie-breaking, *i.e.* one random lottery per student and not per school, has superior welfare properties. Furthermore, there is no strategy-proof mechanism (stable or not) that Pareto-improves assignment with deferred acceptance with single tie-breaking. In practice, our random tie-break substitutes alternative criteria. Typically, assignment committees use application dates, the *quotient familial* or ad-hoc decision rules to select participants with the same priority level.

F.IV Randomness in the assignment: Illustration

In every round of assignment, we simulated a million alternative match and compute individual propensity scores as the frequency of assignment over these simulations. Over the four first rounds we implemented, 56.7% of all demands had a 0 probability of being assigned and 29.2% had a probability of 1. The remaining 14% were subject to randomisation and we present their distribution by offer in figure F.30.

Figure F.30: Distribution of propensity scores among those subject to random assignments



Sources: ISAJE, Case Study I − 2020 : 2023 − first rounds only.
Notes: We present the histogram of the propensity scores over bins of .01 for those with propensity score in]0;1[.

The distribution of design-based propensity scores for the instrument is neither smooth nor overlapping. This is important for the next steps of our work since a lot of econometric methods rely on well-behaved probability functions. Results can be sensitive to bin size and numbers, a regression function will typically put a lot of weight on bins with more observations, not where the propensity score is higher (Słoczyński 2022) and so on.

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