



## Firm-network characteristics and economic robustness to natural disasters

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### ABSTRACT

This article proposes a theoretical framework to investigate economic robustness to exogenous shocks such as natural disasters. It is based on a dynamic model that represents a regional economy as a network of production units through the disaggregation of sector-scale input–output tables. Results suggest that disaster-related output losses depend on direct losses heterogeneity and on the economic network structure. Two aggregate indexes – concentration and clustering – appear as important drivers of economic robustness, offering opportunities for robustness-enhancing strategies. Modern industrial organization seems to reduce short-term robustness in a trade-off against higher efficiency in normal times.

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## 1. Introduction

Recent natural disasters have raised a growing concern about the response of local economies to large exogenous shocks. Clearly our ability to assess the total long-term cost of a large scale event, such as Katrina's landfall, is very poor. Not only is it very difficult to evaluate direct losses due to a natural disaster, i.e. the repair or replacement cost of the assets damaged or destroyed, but it is even more difficult to evaluate indirect losses, including the output losses that are the consequence of direct losses.

Output losses are, in part, direct consequences of the disaster and the resulting capital losses. The recent Icelandic volcano eruption interrupted air transport for a week effectively halting all production in the air transport sector. Similarly, a factory damaged by a hurricane cannot function until rebuilt or repaired. But output losses are also due to complex interactions between businesses. In particular, they arise from production bottlenecks through supply-chains of suppliers and producers. For instance, the production in firms relying on just-in-time imports by plane was perturbed because of the

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volcanic ash (e.g., Saltmarsh, 2010). Other examples show how problems in one firm can have major impacts on its clients (see e.g., Sheffi, 2007). These bottlenecks may even be made more likely by the modern production organization (e.g., production on demand, just-in-time delivery, small or absence of stocks, limited number of suppliers) that makes each production unit more dependent on the ability of its suppliers to produce in due time the required amount of intermediate goods. The assessment of the cost of these interactions is all the more difficult because they are highly variable (from one event to another, from one region to another) and depend deeply on the economic structure and on the shock specificities. It seems obvious, for instance, that a given amount of damages would have more serious consequences if concentrated in a key sector (e.g., electricity production and distribution) than if these damages are spread more homogeneously among sectors.

Sector-scale interactions after disasters have been the topic of intense modeling effort (Rose et al., 1997; Brookshire et al., 1997; Cochrane, 2004; Okuyama, 2004; Okuyama and Chang, 2004; Rose and Liao, 2005; Hallegatte, 2008). However, in these studies the economy is described as an ensemble of economic sectors which interact through an input–output table. Hallegatte (2008) suggests such representation of the economy may be insufficient to model disaster consequences, especially when small businesses are involved. Business interruptions and production losses due to production bottlenecks after a disaster can arise from many small-scale mechanisms including supplier failures, lifeline and transportation perturbations, customers or workers being unable to reach the production location, or bankruptcy of individual businesses. These mechanisms are very difficult to represent at the sector scale and need a much more detailed analysis. A few authors have investigated this issue using probabilistic approaches (e.g., Haimès and Jiang, 2001; Anderson et al., 2007), but a detailed analysis has been mainly done for individual supply-chains, not at a macro-economic scale (see e.g., Chopra and Sodhi, 2004). Ripple-effects within economic networks have been the topic of intense research but this research focuses on credit contagion and systemic risk in the financial system (e.g., Giesecke and Weber, 2006; Nier et al., 2007; Martínez-Jaramillo et al., 2010).

These complex interactions between firms are likely to be an important source of nonlinearity and a model that does not take them into account is at risk of underestimating indirect and output losses. For instance it is interesting to note that the 2004 hurricanes did not have the widespread consequences on Florida's economic system that hurricane Katrina had in Louisiana. Of course, there were large differences between these events: consequences of the 2004 hurricanes were more limited, were due to wind instead of flood and did not involve a large-scale evacuation. But the most important factor was probably the fact that Katrina affected the *systemic functioning* of Louisiana and New Orleans' economies by disrupting the economic system in such a way that even businesses that did not suffer any damage could not function normally. These disruptions made economic production almost impossible and therefore lead to an almost complete collapse of the local economy. By comparison the losses due to the 2004 hurricanes were important but were spread over a wider area and did not impair Florida's whole economic production ensuring an easier and more rapid reconstruction.

It seems there is a threshold between the 2004 losses in Florida and the 2005 losses in New Orleans. In the former case, losses remained below a critical level, did not badly affect crucial sectors and the economy remained able to function almost normally in spite of these losses. In the latter case, losses exceeded a critical level, and the economic system was basically paralyzed by the losses, making the reconstruction very difficult.<sup>1</sup> The main aim of this paper is to provide insights on this threshold and its drivers.

To do so, this article proposes a disaggregated dynamic input–output (IO) model in which the economy is described as a network of interactions between production units (PUs), in line with Delli Gatti et al. (2005), Battiston et al. (2007), Weisbuch and Battiston (2007), and Coluzzi et al. (2010). A crucial hypothesis is that prices do not adapt rapidly after a disaster and do not enable the necessary coordination among PUs to reach a first best production level, equivalent to an economic general equilibrium. As a consequence, the analysis of production dynamics is done out of equilibrium. The analysis of this type of dynamics was the topic of Romanoff and Levine (1993), who introduced the Sequential Interindustry Model (SIM) framework to go from the classic static IO model to the dynamic model (see a more recent application in Okuyama and Chang, 2004). Our analysis parallels their approach and supplements it by introducing the structure of the production network in the model, and investigating the consequences on disaster costs.

Section 2 presents the features of our disaggregated input–output model. Results are given in Section 4, with a sensitivity analysis that highlights which characteristics of the economic network are most important to assess the robustness to exogenous shocks. Section 5 concludes, suggests and leads for future research.

## 2. ARIO-network, a model at the production-unit scale

It seems clear from various case studies (McCarty and Smith, 2005; Tierney, 1997; Albala-Bertrand, 1993; Boarnet, 1998; Rose et al., 1997; Rose and Liao, 2005; Cho et al., 2001) that it is necessary to take into account both direct and indirect losses to evaluate accurately the economic impact of a natural disaster. Indirect losses represent a large share of total losses (see Gordon et al., 1998), it has been suggested that they increase nonlinearly with respect to direct losses (Hallegatte, 2008). Moreover, the understanding of these nonlinear effects would explain why and to what extent regional

<sup>1</sup> See an analysis of the link between reconstruction capacity and disaster total cost in Hallegatte et al. (2007).

economies behave differently when they face a disaster: why does the economy sometimes recover rapidly, while in other cases, like after Katrina, the recovery is slow and needs extensive support?

The ARIIO-network model describes how each PU from each sector uses input from other PUs to produce goods or services, following an approach comparable with the SIM approach proposed by Romanoff and Levine (1993) but at a firm level instead of a sector level. Indeed, this disaggregated model explicitly takes into account the network of production units in line with Delli Gatti et al. (2005), Battiston et al. (2007), and Weisbuch and Battiston (2007) except our paper innovates in the modeled network built to be consistent with sector scale regional IO table. The model represents each firm relying on regional suppliers and clients; a decrease in a firm production can result in ripple effects through chains of suppliers and clients.

In the real world, the extent of these ripple effects depends on whether there are alternative producers elsewhere and on how much time the perturbation lasts. But it also depends on adjustment mechanisms. For instance, when a supplier is not able to produce enough, the production of its client does not automatically decrease, because it can adapt and maintain production: (i) it may be possible for clients to import intermediate goods from outside the damaged area; or (ii) clients may find an alternative local producer who is able to produce more than its usual production and replace the failing one; or (iii) clients may have enough stock to wait for its suppliers to restore their activity.

Many important mechanisms are disregarded in this first attempt to take into account network effect in disaster cost assessment as we focus on the impact on production systems. We do not model impacts on households with the corresponding effect on final demand and labor supply. We consider a closed production system, without imports and exports. We also disregard the reconstruction process, assuming that production units are damaged forever. Therefore we investigate how the economic system can adjust to the definitive loss of PUs without trying to reproduce the full dynamics of a disaster. Moreover, we assume PUs do not create new connections in disasters aftermath, i.e. they do not have access to new clients and suppliers. In other terms, the PU network is considered fixed over the considered timescales. PU can however adjust their demands to their various suppliers, increasing their demands on suppliers that are still able to produce.

In spite of its limited scope, this model already provides interesting insights on the influence of the network structure on economic robustness.

### 2.1. A disaggregated model of the production system

As in Hallegatte (2008), the aim of this work is to take into account limited production capacities and both forward and backward ripple-effects within the economic system. But here we focus on the role of the economic networks, representing the economy as a set of interconnected production units. In Hallegatte (2008), the economy was modeled as a set of homogeneous economic sectors. This assumption is equivalent to a very special case of our PUs network structure in which all PUs interact with every other PU; we refer to this special-case network as the “full network” in the following article. In addition, contrary to Hallegatte (2008), we focus on the impact of the shock and assume that no reconstruction is taking place. We also introduce the role of inventories in the production process.

In this model, like in Battiston et al. (2007), we represent the independent behavior of each PU. Each PU acts according (i) to demand, depending on orders it receives from its clients; (ii) to input availability depending on supplier production and inventory levels; and (iii) to its own internal production constraints.

We assume that  $P$  is the vector of outputs of the different PUs and  $A$  is the PU-IO matrix, i.e. the matrix that describes the quantity each PU is purchasing from other PUs to produce one unit of good. As already stated, we do not model here households or reconstruction and assume that final demand is not impacted by the disaster and remains constant.

The production is used to satisfy demand of intermediate goods and final demand. At equilibrium, the production of PU  $i$  is given by

$$P_i = \sum_j A(j,i)P_j + C_i \quad (1)$$

where  $A(j,i)$  is the amount of good from PU  $i$  needed by PU  $j$  in order to produce one unit of good and  $C_i$  is the final demand addressed to PU  $i$ . The equilibrium equation is then

$$P = A'P + C \quad (2)$$

where  $C$  is the vector of final demands and  $P$  the equilibrium production. Classically, the optimal production is

$$P^0 = (I - A')^{-1}C \quad (3)$$

$P^0$  would be the production if the production capacities were not bounded and if there were no inventories. However, in the present model we will consider the production capacity of each PU (of course, a PU cannot produce more than what it has been designed to produce) and the impact of inventories and input availability on demands.

#### 2.1.1. Inventories and demand model

We define  $D_i(t)$ , as the total demand to the  $i$ th PU at the time  $t$ . This demand consists of final demand and of PU-to-PU demand (i.e. intermediate consumption demand). The PUs produce commodities by drawing from their commodity

inventories. They then have to order new inputs to their suppliers in order to restore their inventories. The inventory level at the end of each time step is used to determine the demand to suppliers.

We assume that the  $i$ th PU has an inventory  $S(i,s)$  of the commodity  $s$ , produced by PUs from sector  $s$ . The demand from the  $i$ th PU to the  $j$ th PU, belonging to  $i$ 's suppliers<sup>2</sup> and producing commodity  $s$ , is designed to restore the inventory  $S(i,s)$  to a level equal to a given number of days  $n_{i,s}^{opt}$  of intermediate consumption, at the production level needed to satisfy total demand. As an example, consider an automobile factory having a stock of tires allowing for 10 days of production at the pre-event production pace. If the demand to this plant increases by 10%, the plant will increase its stock of tires, so it will still have a stock of tires allowing for 10 days of production at the pace needed to satisfy this increased demand. The target stock of tires, therefore, will also be increased by 10%.

The orders  $O(i,j)$  from the  $i$ th PU to the  $j$ th PU, when the PU  $i$  has enough stock of commodity  $s$  to produce exactly  $D_i(t-1)$  during  $n_{i,s}$  days, read<sup>3</sup>

$$O(i,j) = A(i,j)D_i(t-1) + \frac{A(i,j)D_i(t-1)}{\tau n_{i,s}}(n_{i,s}^{opt} - n_{i,s}) \tag{4}$$

where  $D_i(t-1)$  is the demand directed toward the PU  $i$  (by all its clients) at the previous time step,<sup>4</sup> in line with Romanoff and Levine (1993). To produce one unit of good, the PU  $i$  needs an amount  $A(i,j)$  of inputs from the PU  $j$ . So, the first term of the right-hand side of Eq. (4),  $A(i,j)D_i(t-1)$ , is the amount of commodity needed by the  $i$ th PU to satisfy the demand at the previous time step  $D_i(t-1)$ . The second term of the RHS of Eq. (4) represents the orders that make the inventory converge toward its equilibrium value, i.e. toward  $n_{i,s}^{opt}$  days of consumption. Whenever the total stock of commodity  $s$  is not enough to produce  $D_i(t-1)$  during  $n_{i,s}^{opt}$  days, i.e. whenever  $n_{i,s} < n_{i,s}^{opt}$ , this term is positive and decreasing with  $S(i,s)$ . The term  $\tau n_{i,s}$  is a characteristic time of the inventory restoration. If  $n_{i,s}$  is large, it is not urgent to restore inventories. In the following, we assume that  $\tau = 1$ .

This modeling provides the total demand directed toward each PU  $j$  at the time step  $t$ , by adding all demands from individual PUs, plus final demand  $C_j^5$ :

$$D_j(t) = C_j + \sum_i O(i,j) \tag{5}$$

### 2.1.2. Production model

Without constraint, each PU  $j$  would produce at each time step  $t$  the exact level of demand  $D_j(t)$ . But production can be lower than demand either (i) because production capacity is insufficient; or because (ii) inventories are insufficient as a result of the inability of other PUs to produce enough (forward propagation). The production capacity of each PU depends on its stock of productive capital (e.g., factory, equipments), and on the direct damages to the firm capital (e.g., a firm that suffers from disaster damages can produce less).

The capacity and supply constraints are described by the following relationships:

- Limitation by production capacity: independently of its suppliers, the production capacity  $P_i^{cap}$  of the  $i$ th PU reads

$$P_i^{cap} = P_i^{ini}(1 - \Delta_i) \tag{6}$$

where  $P_i^{ini}$  is the pre-event production of this PU, assumed equal to the normal production capacity. The variable  $\Delta_i$  is the reduction in productive capacity due to the disaster, directly because of the disaster (e.g., the volcanic ash case) or through capital destruction.

- Limitation by supplies: practically, it is assumed that all PUs have inventories of the goods or services produced by each of their suppliers, and that production by all PUs is done using inventories only. Production can thus be limited by insufficient inventories. The inventory of commodity  $s$  owned by PU  $i$  is written  $S(i,s)$ . In the pre-event situation, the PU  $i$  needs an amount  $A(i,j)$  from each PU  $j$  to produce one unit of its commodity. So, it consumes a total amount  $A_{tot}(i,s)$  of commodity  $s$ , which is equal to

$$A_{tot}(i,s) = \sum_{j \in \text{sector } s} A(i,j) \tag{7}$$

As a consequence, with an amount of available inventories  $S(i,s)$  for the commodity  $s$ , the maximum production is limited at

$$P_i^s(t) = \frac{S(i,s)}{A_{tot}(i,s)} \tag{8}$$

Some commodities, for instance those produced by manufacturing sectors, can be stocked, while it is almost impossible to stock electricity. As a consequence in a black-out all PUs depending on electricity will stop producing. On the other

<sup>2</sup> Of course, if  $A(i,j)=0$ , then if the PU  $i$  is not a client of the PU  $j$  and the demand from PU  $i$  to PU  $j$  is nil.

<sup>3</sup> In this equation and in the following ones,  $A(i,j)$ ,  $O(i,j)$ ,  $n_{i,s}$  and  $S(i,s)$  depend on the time step  $t$ , but we omit it for simplicity.

<sup>4</sup> If this demand was larger than the production physical capacity constraint of PU  $i$  ( $P_i^{cap}$ ) then the orders are set to produce this maximal production level, i.e.  $D_i(t-1) = P_i^{cap}$  in Eq. (4).

<sup>5</sup> This total demand is equivalent to the *desired output* of Battiston et al. (2007).

hand, if a PU from the manufacturing sector is damaged its client will have the possibility to draw from their inventories to produce at least a fraction of their usual production for a certain period of time. These stocks are measured in the number of days of pre-event consumption by the PU. For instance, an automobile factory may have a stock of tires allowing it to produce cars during 15 days at the pre-event production pace. For simplicity, non-stockable goods – like electricity – are modeled assuming their inventories cannot be larger than what is needed to produce during one day (the model time step). It means, if electricity is shut down, production in the affected area will stop from the day following.

- Taking into account both production capacity and limited inventories, the maximum production level of the  $i$ th PU is

$$P_i^{max}(t) = \text{Min}(P_i^{cap}(t), \text{Min}_s(P_i^s(t))) \quad (9)$$

Actual production  $P_i^a$  is then given by

$$P_i^a(t) = \text{Min}(P_i^{max}(t), D_i(t)) \quad (10)$$

$P_i^a(t)$  is the vector of actual production by each PU taking into account the two production constraints.<sup>6</sup> These constraints then propagate into the economy: if a firm proves unable to produce enough to satisfy demand, it will both (i) ration its clients, and (ii) demand less to its suppliers. These two effects, forward and backward propagations, affect the entire economy.

### 2.1.3. Market modeling, rationing scheme and inventory dynamics

When a PU is not able to produce enough to satisfy the demand, in the absence of an optimal price response to restore production–demand equality, producers have to ration the clients. To model this effect it is necessary to introduce a rationing scheme.

In our framework, in the absence of market equilibrium, demand can be larger than actual production (in such a case  $P_i^a = P_i^{max}$ , see Eq. (10)):

$$D_i = \sum_{j=1}^N O(j,i) + C_i \geq P_i^a$$

Despite the incompatible demand and supply the actual sales and purchases must be balanced

$$D_i^* = \sum_{j=1}^N O^*(j,i) + C_i^* = P_i^a$$

Therefore some agents must be rationed. The rationing scheme gives the sales and purchases of each agent depending on demands and supplies of all the agents. In the present case, since we are interested in disasters, there is only underproduction and the suppliers can sell all their production while clients only get a fraction of their demand. We have assumed that the rationing scheme is a *proportional rationing scheme*<sup>7</sup> (Bénassy, 1984); the rationing fraction is equal for each client (PUs and final consumers).

$$O^*(j,i) = O(j,i) \cdot \text{Min}\left(1, \frac{D_i^*}{P_i^a}\right) \quad (11)$$

$$C_i^* = C_i \cdot \text{Min}\left(1, \frac{D_i^*}{P_i^a}\right) \quad (12)$$

Of course, since all PUs from a sector  $s$  produce the same commodity  $s$ , if a PU  $j$  has two suppliers from sector  $s$  (the PUs  $i_1$  and  $i_2$ ) the inventory of commodity  $s$  is restored thanks to orders from  $j$  to both  $i_1$  and  $i_2$ . In the model, the actual sales of PU  $i$  to PU  $j$  ( $O^*(j,i)$ ) are those that increase the commodity  $s$  inventories of the PU  $j$  from one time step to the next one:

$$S(j,s)(t+1) = S(j,s)(t) + \Delta t \left[ \sum_{i \in \text{sector } s} O^*(j,i) - A_{tot}(j,s) P_j^a(t) \right] \quad (13)$$

where  $\Delta t$  is the model time step, the term  $O_{ij}^*$  is the increase in inventory thanks to purchases from supplier  $i$ , and the last term is the decrease in inventory due to the commodity consumption needed to produce the amount  $P_j^a(t)$ .

### 3. Random economic network

The disaggregated model proposed here is based on a synthetic production-unit input–output (PU-IO) table, developed from a sector-scale input–output table and from simple network characteristics (e.g., number of PUs per sector, number

<sup>6</sup> This actual production is the equivalent of the *effective output* of Battiston et al. (2007).

<sup>7</sup> The problem of this rationing procedure is that it can theoretically be manipulated: an agent can declare a higher demand to increase his transactions. In the present study, we assume that PUs declare their true demand, that is to say the amount of intermediate good they actually need to satisfy their own demand.

of suppliers per PU, number of clients per PU). Compared with previous works at the firm level (e.g., Battiston et al., 2007), the innovation here is that we created a PU-IO table which is consistent with aggregated IO data at the sector level and with network characteristics chosen in an ad hoc manner (in the absence of data on economic network characteristics).

Many disaggregated tables can be consistent with sector-scale IO table and some network characteristics. To avoid results to be biased by the choice of a peculiar network structure, this section presents a method to generate uniformly random matrices and investigates the model behavior with these different matrices.

### 3.1. Building the disaggregated IO-matrix

#### 3.1.1. IO-matrix features

The initial IO table is the 15-sectors IO table for the U.S., from the Bureau of Economic Analysis. The sectors are (1) agriculture; (2) mining and extraction; (3) utilities; (4) construction; (5) manufacturing; (6) wholesale trade; (7) retail trade; (8) transportation; (9) information; (10) finance; (11) business services; (12) education; (13) arts and food; (14) other services; (15) government. From this national table, a regional table for Louisiana has been built, using Gross State Product for Louisiana (see details in Hallegatte, 2008) and simple assumptions about the proportion of each sector production exported outside the region.

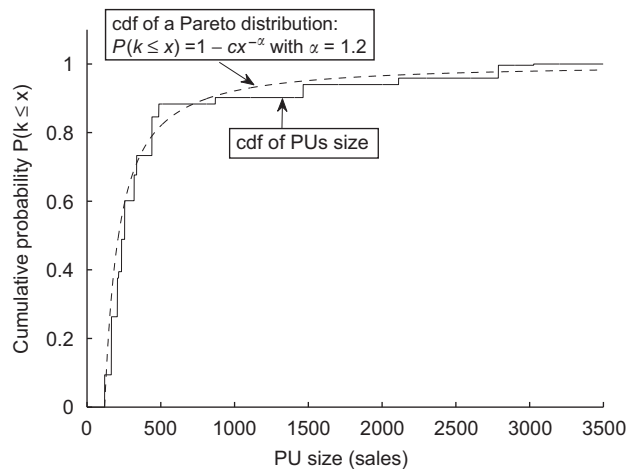
Building the disaggregated PU-IO table from this sector-scale table requires first knowledge of the number of PUs in each sector or, if the full economy cannot be modeled, the proportion of these PUs for each sector relative to the total number. The Census Bureau provides the number of establishments per sector and their size distribution in Louisiana in 2004 (see [www.census.gov](http://www.census.gov)). It is assumed that each establishment is a PU. The proportion of PUs per sector according to these data is given in Table 1.

Using this information on the relative number of PUs for each sector, we set a number of PUs for each sector, so that the final network is a  $500 \times 500$  PU's matrix. The sectoral IO table is then expanded into a PU-IO table, which describes the exchanges between all PUs of the local economy. Obviously, this PU-IO table is much larger than the sectoral one. Also, this table contains mostly zeros, since most PUs have no direct relationship with each other.

It is assumed that all PUs from a given sector produce the same commodity and have the same size. On the other hand, PUs from different sectors have different sizes (for instance utility PUs are much larger than retail sector PUs). PUs size cumulative distribution is plotted in Fig. 1, along with the cumulative distribution of a power law (parameter 1.2). Even though all PUs from a given sector are identical, the size distribution is heterogeneous because each sector has a different

**Table 1**  
The proportion of production units among sectors.

Sect.								
	Agric.	Min.	Util.	Constr.	Manuf.	Whol.	Retail.	Transp.
Prop.	1.9%	1.9%	0.4%	7.5%	3.8%	5.6%	16.9%	3.8%
Sect.								
	Info.	Finance	Busi., Serv.	Edu., Health	Food, Art	Serv.	Gov.	
Prop.	1.9%	11.2%	11.2%	11.2%	9.4%	9.4%	3.9%	



**Fig. 1.** Cumulative distribution function of PUs sizes and power law cumulative distribution function with parameter 1.2.

size and includes a different number of PUs. The size distribution resembles a power law, even though the network is too small to conclude in a robust manner that the distribution is a power law. This feature is in line with results from empirical analyses of firms network (e.g., Fujiwara and Aoyama, 2010).

All PUs produce and exchange intermediate consumption goods and services, and produce final consumption goods and services for local demand. As all PUs from a given sector are identical, they all receive the same quantity of inputs from each sector. However, they can order this quantity from only one PU or from several PUs from this sector, depending on the network structure. Similarly, all PUs from a given sector sell the same quantity of good to each sector. Thus, we impose a constant sum over each column and line in each block describing the exchanges between each pair of sectors. Since all PUs in the network need inputs from all sectors, the PUs from the sectors that only include a few PUs are “hubs”, connected to many other PUs.

We define the PU-IO connection matrix  $C$ , which is composed of positive integers and describes which clients from each sector buy intermediate goods from which suppliers in each other sector, and in which proportion. The coefficient  $C_{ij}$  is positive if the  $i$ th PU buys goods from the  $j$ th PU, and zero otherwise. To generate PU-IO matrices that are consistent with the sector-scale IO table and our disaggregation assumptions, we need to generate random graphs in which the size of each PU is fixed as well as the total input it purchases from each sector and total output it sells to each sector. In other words, we need to generate matrices such that the sum of the values on each row and column is fixed within each “sector-to-sector block” describing the exchanges between two sectors.

### 3.1.2. Generation method

Generating a uniformly randomized set of matrices with specific constraints is a challenging issue, and there is no universal method. For the particular case of matrices corresponding to a graph with a fixed weighted degree distribution – i.e. with fixed values of the sum over the lines and columns – one can use the *configuration model* (Newman, 2003). But our case is more complex, since the constraints are not applied to the entire matrix, but for each block corresponding to sales from a sector to another one. Moreover, in Section 4, networks with specific characteristics will be created (e.g., with different values of clustering or concentration), and this cannot be accommodated directly by the configuration model. We propose an alternative approach which is described below.

As in the Maslov–Sneppen algorithm (see Maslov and Sneppen, 2002; Zlatic et al., 2009), the basic idea is to choose random pairs of links and swap their extremities. Starting from a graph obeying a specific weighted degree distribution it is possible to produce any graph respecting this distribution, with equal probability, by iterating the following Markov-chain method:

- We draw two random couples of values  $(i, j)$  and  $(i', j')$ , such that the PUs  $i$  and  $i'$  and the PUs  $j$  and  $j'$  are in the same sector (i.e. the couples  $(i, j)$  and  $(i', j')$  are in the same sector-to-sector block of the matrix) and that coefficients  $C(i, j)$  and  $C(i', j')$  are greater than or equal to 1 (i.e. the PU  $j$  sells some of its production to the PU  $i$ , and the PU  $i'$  sells some of its production to the PU  $j'$ ).
- We modify the matrix in the following way:

$$\begin{pmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & C(i, j) & \dots & C(i, j') & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & C(i', j) & \dots & C(i', j') & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \rightarrow \begin{pmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & (C(i, j)-1) & \dots & (C(i, j')+1) & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & (C(i', j)+1) & \dots & (C(i', j')-1) & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

In the corresponding graph, this modification is a “link swap”, i.e. an exchange of the extremities of two links. Each swap is accepted provided that  $i \neq i'$ ,  $i \neq j'$ ,  $i' \neq j$  and  $j \neq j'$ . These conditions ensure that swaps have economic sense (see Vitali and Battiston, 2011).<sup>8</sup> For PU  $i$ , a swap represents a change of suppliers from a given sector (if initially  $C(i, j) = 1$  and  $C(i, j') = 0$ ) or a change in the proportion supplied by two given suppliers from this sector (if initially  $C(i, j) > 1$  or  $C(i, j') > 0$ ).

This process does not affect the sum of the lines and columns in each sector-to-sector block of the matrix, therefore respecting our constraints of consistency with the sector-scale IO table. Moreover, this process imposes a maximum on the number of non-zero components in the matrix, i.e. a maximum number of PU-to-PU connections. The process yields thus a maximum matrix density, consistent with a reasonable number of suppliers and clients per PU. In addition, according to Taylor (1980), we know that this method can generate any weighted and directed matrix satisfying these constraints. By iterating this procedure, we perform a random walk in this set of matrices: such a Markov process tends to lose memory of the starting point and to generate random elements. A description of this procedure in the unweighted and directed case can be found in Rao et al. (1996) or Miklós and Podani (2004); the statistical uniformity of this process is discussed in Roberts (2000).

A flaw of this method is that the number of steps necessary to reach an arbitrary element is unknown. In such a situation, the use of experimental criteria is widespread (e.g., Gkantsidis et al., 2003). In practice, a typical geometric characteristic of the graph is chosen, and its evolution is measured throughout the process. When the value of the chosen

<sup>8</sup> Contrary to Vitali and Battiston (2011), swaps creating multiple links are authorized as the weight of link  $(i, j)$  represents the relative size of supplier  $j$  for PU  $i$ .

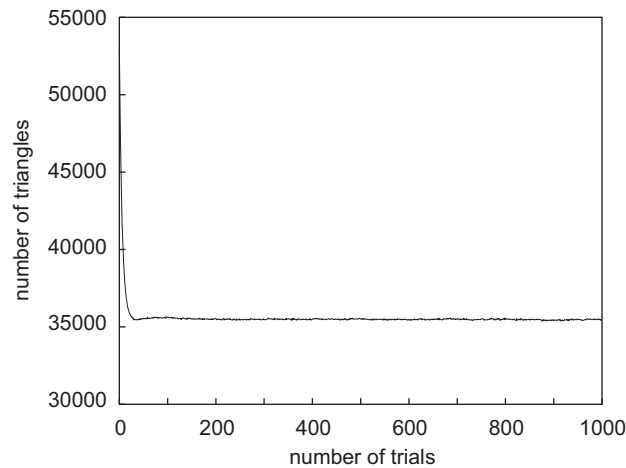


Fig. 2. Change in the number of triangles during the randomization process.

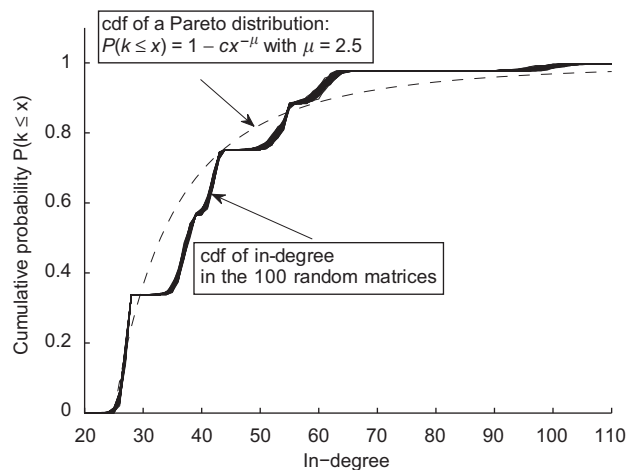


Fig. 3. Cumulative distribution function of PUs in-degrees for the 100 random matrices and power law cumulative distribution function with parameter 2.5.

observable remains stable, the memory of the initial point of the Markov-chain is considered lost, and the element is supposed random.

### 3.1.3. Generating random IO matrices

In our case we start from an ad hoc PU-IO connection matrix which has the desired properties: all nodes from one sector have the same characteristics (i.e. the same quantity of input from each sector and the same quantity of output to each sector) so that each block has fixed sums over lines and columns.

We then implement the method described above. We follow the convergence of the Markov-chain using the number of triangles on the whole matrix to assess whether the process has reached its steady state and lost memory of the initial matrix. Fig. 2 represents the evolution of this measure throughout the randomization process and shows its stabilization after a certain amount of swaps.

The degree distribution of the generated matrices is plotted in Fig. 3 with the cumulative distribution function of a power law (with parameter 2.5). Fujiwara and Aoyama (2010) find that the distribution of the degrees of the Japanese production network is well fitted by a power law, implying that the network is scale free, like many large real-world networks. The network we consider here, which represents only a regional economy and includes only 500 PUs, is too small to conclude rigorously that it exhibits a power law degree distribution.<sup>9</sup> However, in-degree, out-degree and undirected degree distributions resemble power-law distributions consistent with the findings of Fujiwara and Aoyama (2010).

<sup>9</sup> Around three decades (or orders of magnitude) of the degree distribution would be necessary to consider that it is well fitted by a power-law, while our regional economic network has degrees lying between 24 and 266.



The generated matrices and the corresponding graphs exhibit classical features of real-world networks:

- They are sparse: the maximum number of weighted links is 7.9% of the total number of possible links.
- The degree is heterogeneously distributed, as can be seen in Fig. 3.
- The matrices contain a giant connected component which gathers most of the nodes (all of them in our case), and the distance between two nodes of this component is of the order of  $\log(N)$ —which is one of the main characteristics of small-world networks (Watts and Strogatz, 1998).

In Fig. 3 the degree distribution does not vary much across the 100 random matrices, this is due to the strong set of constraints on the matrices. Most of the matrices satisfying this set of constraint exhibit the kind of degree distribution shown in Fig. 3. We cannot be sure, however, that the real-world network corresponds to the statistically most likely matrices. As a consequence we investigate different network characteristics in Section 4.

Once such a random matrix has been generated, the corresponding PU-IO matrix is created by dividing the total amounts of sales and purchases between every two sectors, from the sectoral IO matrix, by the sum of the oriented weighted links between these two sectors. This method provides a regional IO matrix at the PU level which is consistent with: the sector-scale IO table from the Bureau of Economic Analysis; information on the number of PUs from the Census Bureau; and characteristics of real-world firm networks identified by Fujiwara and Aoyama (2010).

### 3.2. Consequence of a specific shock

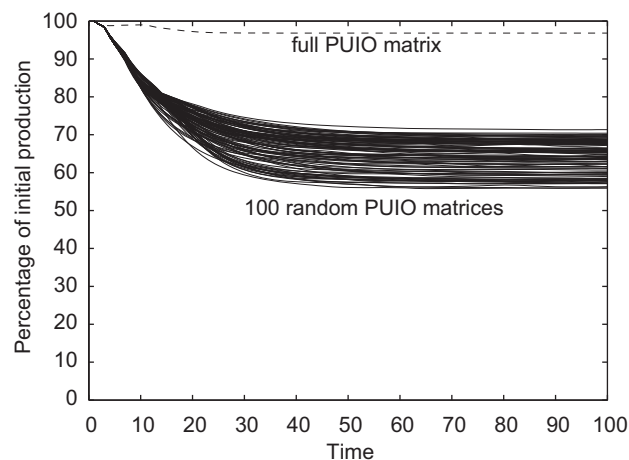
In this section, the consequence of a shock affecting two PUs, reducing their production capacity by 50% is investigated (see Section 2.1.2 and Eq. (6)). At the aggregated level this shock corresponds to a 10% reduction in the mining-sector production capacity.

The economic model was tested with this same shock on a number of random graphs. Fig. 4 represents total production, as a percentage of initial production, from date 0 until 100 days after the shock. This figure shows how the time path of the decrease in production and the final production of the whole economy is dependent upon the economic network structure. The aim of this paper is to seek some insights on the role of this structure.

In order to highlight the role of the disaggregation, the model was also tested using the “full” PU-IO connection matrix, i.e. the PU-IO connection matrix with ones everywhere. This matrix structure corresponds to a situation in which all PUs are clients and suppliers of all PUs, i.e. an economy with maximum redundancy and risk-sharing. This structure is also the one implicitly assumed in classical aggregated sector-scale IO models.

The dash line of Fig. 4 represents production with this full matrix. Indirect losses are minor because a decrease in sales and demands from the two damaged PUs are shared among all PUs. In sparse PU-IO matrices, each PU relies only on a limited number of suppliers from each sector and has a limited number of clients: if one of these suppliers or clients is unable to produce, the PU suffers from a catastrophic decrease in its inputs or demand. As the economy is out of equilibrium and PUs do not change their suppliers and clients the shock spreads through the economy. In the case of the full matrix, the shock is smoothed by the redundancy of suppliers and clients acting as an insurance against the risk of a shock.

Our analysis differs from many other analyses of network robustness. Indeed, many real-world networks (like the world wide web or the airlines network) are considered robust to random attacks but vulnerable to targeted and coordinated attacks (see Cohen, 2002). This differentiated vulnerability arises from the fact that a few of their nodes (hubs) have a lot of connections, while most nodes have very few connections. Since a node can function if it remains connected



**Fig. 4.** Total production of the economy from date 0 until 100 days after the shock (50% reduction in production capacity of two PUs of the mining and extraction sector) for 100 random matrices and for the “full” PU-IO connection matrix.

by at least one link to the rest of the system, an accident or attack to a node that has few connections has little effects on the entire system. These networks are vulnerable to an attack only if it is targeted against one of the hubs, and a random accident has little probability to affect hubs.

In our model, the network also has large hubs (e.g., the PUs of the utility sector), but the network is nevertheless vulnerable to random shocks. This vulnerability arises from the economic model, in which the existence of remaining links between two sectors is not always sufficient to ensure functioning of the production system. Because of limited production capacities, one link between two sectors is not enough to maintain the necessary level of supply to the client sector, contrary to models in which there are no flow limits between two nodes. Because of inability of PUs to adapt by creating new links over the short term, even a shock on a small PU with few connections can have dramatic consequences on the entire system. This analysis can be compared with the analysis by Nier et al. (2007) on the impact of connectivity on financial stability.

### 3.3. Sensitivity to the inventory level

This section investigates the impact of initial inventory level on robustness of the economy. Fig. 5 shows response of 100 random matrices to the same shock, but with different initial inventory levels in each PU (2, 4, 7, and 10 days), except in utility and transport sectors in which initial inventory levels are only one day. Clearly, increasing inventories from 2 to 10 days slows down the overall reduction in production and enhances the robustness to disasters, since inventories make it possible for PUs to keep producing in immediate disaster aftermath, even if their suppliers are disabled. In the real world, this buffer effect leaves more time for PUs to transfer their demand to alternative suppliers and for recovery and reconstruction to take place. Larger inventories, therefore, decrease the likelihood of a disaster-related economic collapse.

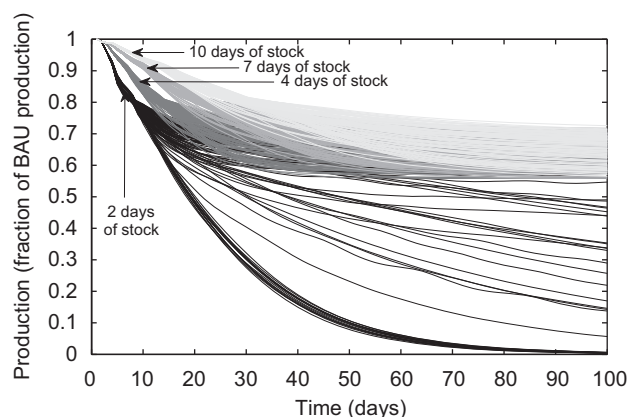
### 3.4. Heterogeneity of losses

Modeling disaggregated economy makes it possible to compare disasters with different patterns and impact distributions. For instance, one can compare a disaster that affects strongly a few producers (e.g., a flood affects only businesses in the flood plain within a region) to another one that affects all producers of a region, but with more limited consequences (e.g., a wind storm that affects all businesses in a region).

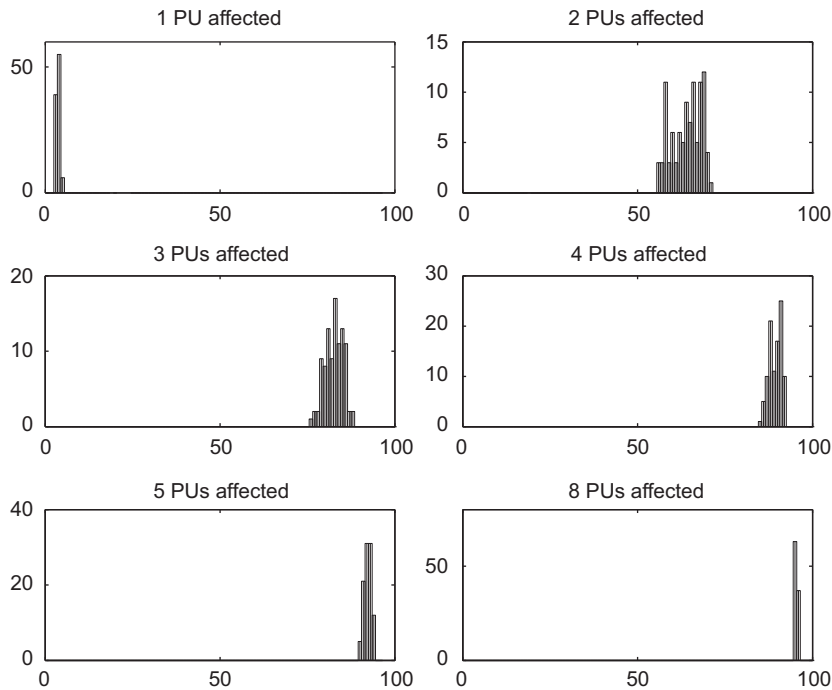
To do so, Fig. 6 shows model simulations of impact on the whole economy of a 10% reduction in the production capacity of the mining and extraction sector, which includes 10 PUs in our model. But this reduction in production capacity affects only one PU (which becomes totally unable to produce) or is distributed among 2, 3, 4, 5 or 8 PUs. In this last case the eight affected PUs only lose 12.5% of their production capacity and the shock is much more homogeneous than in the former cases. It is interesting to note the latter case in which most PUs for a sector are affected is implicitly assumed when working at the sector scale.

The heterogeneity of losses was found to play a central role in the indirect losses. When direct losses are more homogeneously distributed among PUs from one sector (e.g., lower right figure), total losses are drastically lower than when only one PU suffers all damages (upper left figure). More generally, for a fixed amount of direct damages, the total loss of the economy is decreasing with the number of affected PUs. In the present case, the same amount of losses leads to output reduction (in the 100 days following the shock) lying between 3% and 98% depending on the heterogeneity of losses. This result suggests that strongly concentrated disasters (like floods) may cause larger output losses than “broader” disasters that affect homogeneously large regions (like wind storms or heat waves).

It is useful to recall here the extreme assumptions of our model: all the random graphs are connected, PUs do not create new supplier or client connections and have no access to imports in the disaster aftermath, and there is no reconstruction



**Fig. 5.** Production from date 0 until 100 days after the shock (two PUs damaged: 10% reduction in production capacity of the mining and extraction sector) as a fraction of the BAU production, depending on the number of days of stock.



**Fig. 6.** Histograms of the sum of production from date 0 until 100 days after the shock (10% reduction in production capacity of the mining and extraction sector) as a fraction of the BAU production, depending on the number of PUs affected and for 100 random matrices.

(losses in production capacity are assumed permanent). In such an extreme situation, when one PU is totally destroyed, consequences on the entire economy are dramatic and production eventually collapses. The clients of the affected PU cannot shift to alternative suppliers, their production is therefore reduced and cannot supply fully their clients. The shock thus spreads through the whole economy until it collapses. The ability of PUs to increase their demand to their other suppliers (provided that the commercial link pre-existed) is not sufficient to adapt the economic system to the production interruption: the creation of new links is necessary. On the other hand, when losses are homogeneously distributed, each PU suffers from a limited loss, reduces slightly its production, and total loss is eventually quite small.

Regardless of modeling caveats, heterogeneity in losses among PUs and the redundancy of suppliers and clients are key parameters to explain robustness of the economy in our model. Such characteristics of the shock and of the economy cannot be represented using sector-scale aggregated IO models, justifying our approach. Moreover, it appears from our simulations that aggregated IO models stand for an especially favorable case where losses are equally distributed among PUs and where each PU is linked to all other PUs.

It is also found that the influence of the network structure depends on heterogeneity of losses: the result variance in Fig. 6 is small when few or most PUs are affected, and large when direct losses are distributed among a median number of PUs. When one or few PUs are affected the economy collapses anyway, so the network structure is irrelevant. When all PUs are affected, the model behaves like a sector-scale model and the network structure is similarly unimportant. However, in intermediate cases the network structure is important to the assessment of output losses. For instance, when two PUs are affected indirect losses do not have the same magnitude depending on whether the two affected PUs have common clients or not. Indeed, if one PU has two damaged suppliers the overall effect on its production is worse since it has to compensate the loss of two suppliers instead of one by demanding more from remaining ones.

However, the structure of graphs in our sample of 100 random matrices has a relatively small effect on the overall robustness of the economy, especially when compared with other parameters (e.g., heterogeneity of losses). This is particularly the case when only one PU suffers from all losses. But the network matrices generated by our method are only a random subset of all possible matrices. This sample is very small and may not include low-probability structures that are possible and very different from most likely ones. There is indeed no reason to believe the real economic network is one of the most likely ones. To investigate different network structures, an alternative generation process is suggested in the next section, to investigate networks with “targeted” characteristics.

#### 4. Results on “targeted” network structures

As shown in Section 3.1.3, the networks generated from the random process share some characteristics which appear to have a high probability of occurrence. To explore low-probability matrices that may correspond more closely to real-world

economic networks this section investigates specific network structures. To do so, it is first necessary to build these specific networks.

The specific network characteristics investigated here are *concentrations* (What is the role of the redundancy in suppliers and clients?), *clustering* (What happens if the clients of a PU are also the suppliers of its suppliers?) and *connectedness between subregions* (Is it useful to connect with many links different regions?).

These characteristics are interesting because, in the real world, these parameters can be related to strategic choices on the organization of local economies. For instance, an agglomeration in which PUs interact only with their geographic neighbors (which can be the case in a local economy or with vertical-integration industrial organizations) can be compared with an agglomeration in which each area is specialized in a particular economic activity (specialized economy like financial districts or horizontal-integration industrial organization). Similarly, the redundancy in suppliers and clients and connectiveness between subregions depends on PUs choices; they may decide to spread risks by having many suppliers and clients, possibly located in different regions.

To investigate these questions, a method is first proposed to produce a sample of random matrices with a particular value of concentration or clustering. The next subsection investigates the links between subregions. The robustness of all produced economic networks is then assessed using the same shock, namely a 100% reduction in the production capacity of one PU in the mining and extraction sector (equivalent to the upper left panel of Fig. 6).

#### 4.1. Concentration

The rationale behind the analysis of concentration is that redundancy of suppliers and clients can act as a risk-sharing mechanism against the risk of a shock on a supplier or client. If each PU relies on very few PUs, any shock cannot be smoothed by redundancy of suppliers and each PU cannot compensate for the loss of one supplier by increasing demand on many other ones.

##### 4.1.1. Network building

Concentration seeks to measure reliance of clients on a few suppliers. It is built on the same principle as the Herfindahl index:

- We call  $w_{ij} = M_{ij}$  the weight of a supplier  $j$  (from sector  $Y$ ) in the supply of a PU  $i$  (from sector  $X$ ). The sum of all these weights is fixed for all  $i$  in sector  $X$ :

$$\mathcal{W}_{iY} = \sum_{j \in S_Y} w_{ij}$$

- We then define the sector  $Y$  concentration for buyer  $i$  as

$$h_{iY} = \sum_{j \in S_Y} \left( \frac{w_{ij}}{\mathcal{W}_{iY}} \right)^2$$

- Then the concentration of the block  $(X,Y)$  is the weighted sum per buyer:

$$H_{XY} = \frac{\sum_{i \in S_X} h_{iY} \cdot \mathcal{W}_{iY}}{\sum_{i \in S_X} \mathcal{W}_{iY}}$$

- And finally total concentration will be the weighted sum of all-block concentrations:

$$H = \frac{\sum_{(X,Y)} h_{XY} \cdot \mathcal{W}_{XY}}{\sum_{X,Y} \mathcal{W}_{XY}}$$

A high concentration index for block  $(X,Y)$  indicates that the sector  $X$  relies on a few firms from sector  $Y$ . A high aggregate concentration index indicates that each PU depends on a few clients and that each sector relies on a few suppliers from all other sectors. Concentration is also a way to take into account the importance of weights distribution which has been emphasized in relation to transportation networks by [Barrat et al. \(2004\)](#).

Creating matrices with a high level of concentration can be done with a two-step method:

- First, as in the production of random matrices, a sequence of link swaps is carried out, but each swap is made only if it increases value of concentration index  $H$ . This process is carried out until reaching a specific value  $H_0$  of  $H$ .
- Second, a homogenization phase is conducted, with a sequence of random link swaps, to produce a random graph with a concentration index of  $H_0$ . For a fuller description of a standard procedure of homogenization see [Tabourier et al. \(2010\)](#).

Resulting PU-IO matrices have a concentration index ranging from 35% to 70%. Interestingly, in conjunction with data on real economic networks, the same technique would allow the construction of PU-IO matrices consistent with sector-scale IO tables and with observed network characteristics. Fig. 7 shows the in-degree distribution for these PU-IO matrices. We show that targeting the concentration index allows us to reach matrices with various degree distributions, whereas it would be very unlikely to produce such matrices with a fully random process.

#### 4.1.2. Results

Fig. 8 shows the sum of production over the 100 days following the shock, as a function of the economic-network concentration index. There is a significant correlation between concentration index and robustness of the economy: the more concentrated the economy, i.e. the less redundancy in suppliers and clients of each PU, the larger the output loss. As expected redundancy in suppliers and clients acts as an insurance against risks, increasing overall economic robustness.

This result remains valid for random shocks, even though the network includes well-connected hubs and weakly connected PUs. This contradicts the idea that such networks would have a limited vulnerability to random shocks (and a high vulnerability to targeted attacks). As already stated, this difference arises from the fact that maintaining one link between two PUs or two sectors is not sufficient in our model to maintain production level, because of the limited production capacity in remaining PUs. In practice, in our model, the existence of hubs increases vulnerability because as soon as one of these hubs is affected (and it happens at one point because the graph is connected), the shock spreads rapidly across all sectors.

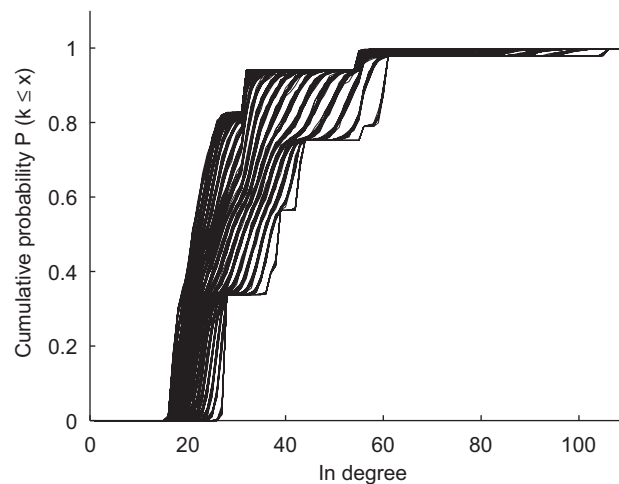


Fig. 7. Cumulative distribution function of PUs in-degrees for targeted network, concentration index ranging from 35% to 70%.

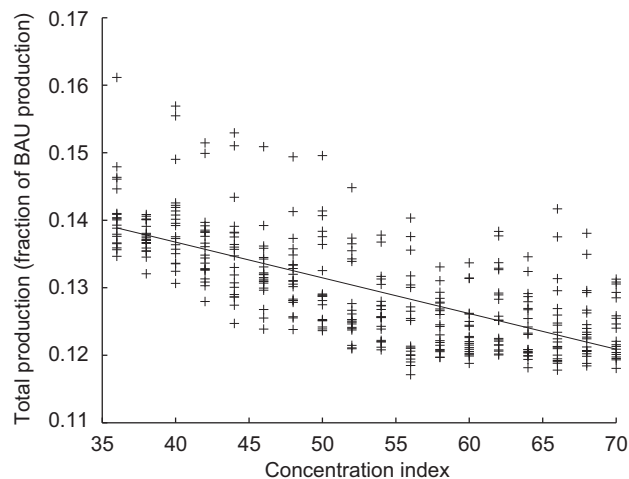


Fig. 8. Sum of production from date 0 until 100 days after the shock as a fraction of the BAU production, depending on the concentration index.

## 4.2. Clustering

The clustering index can be understood as an index of geographical interactions between PUs (see Barthélemy, 2003 for instance). Indeed, a high clustering index indicates that a PU’s suppliers are likely to also be its clients; this is consistent with a “localized” economy, in which clients and suppliers are in the vicinity, compared to a “specialized–globalized” economy. In other words, a high clustering index indicates that PUs in the same area interact primarily with each other. This type of structure is inconsistent with agglomerations in which each area is specialized in a given activity (e.g., financial districts). But most real world social networks exhibit a high clustering coefficient which is related to community formation (see Watts and Strogatz, 1998; Newman, 2003).

### 4.2.1. Network building

The clustering index used in this study is the “oriented clustering.” It is a generalization of the clustering measure, usually defined for undirected graph as the ratio

$$C_3 = 3 \cdot \frac{\#(\text{triangles})}{\#(\text{forks})}$$

i.e. the ratio between the number of triangles (whatever are the sizes of the links) over the number of “possible” triangles, the factor “3” ensuring the normalization of this quantity. Here, the following generalization is used:

$$C_3 = 3 \cdot \frac{\#(\text{oriented triangles: } A > B > C > A)}{\#(\text{oriented forks: } A > B > C)}$$

Fig. 9 provides examples of graph with different clustering indices. The left-hand graph has two oriented triangles and six oriented forks; its clustering index is equal to 1. The middle graph has one oriented triangle, and four oriented forks, and its clustering index is equal to 3/4. The right-hand graph has no oriented triangle and it has a clustering index equal to zero.

Fig. 10 shows the in-degree distribution for these PU–IO matrices. As with concentration, targeting the clustering index allows us to reach matrices with different degree distributions.

### 4.2.2. Results

The anticipated influence of clustering is unclear. On the one hand, small groups of PUs are highly vulnerable to shocks affecting one of their members. On the other hand, they are isolated from disasters affecting other groups and there is a “loss containment effect”.

Model results are shown in Fig. 11. The correlation between the clustering index and the robustness is positive and significant. The regression is still valid controlling for the concentration index indicating that this result is different from the effect of concentration on robustness. A high clustering means there are little groups of PUs interacting with each other, but quite isolated from the rest of the economy. These groups are better protected against a shock (when the shock occurs outside of their group) than when the entire sector relies on only a few suppliers.

From these results one can identify two ways of improving economic robustness: an “isolation” approach in which small groups of PUs interact as little as possible to contain disaster losses; and an “insurance” approach in which all PUs are connected to the largest possible number of PUs, to mitigate the impact of a shock affecting one PU.

According to Fujiwara and Aoyama (2010), the global clustering index of the network of Japanese firms is very low. According to our model, a low value of the clustering index induces a limited resilience of the economic system to exogenous shocks.

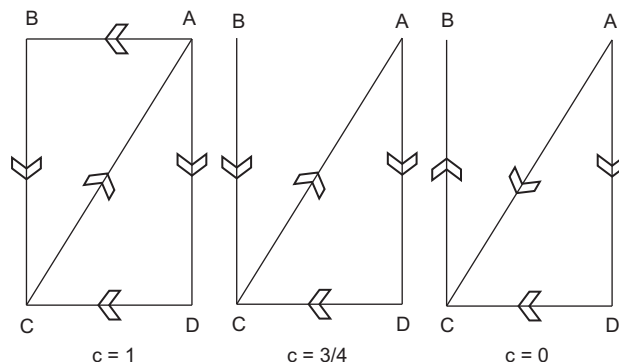


Fig. 9. Examples of clustering.

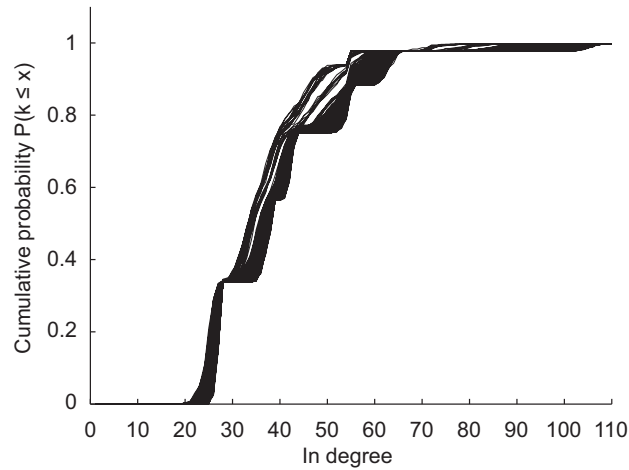


Fig. 10. Cumulative distribution function of PUs in-degrees for targeted network with a different clustering index.

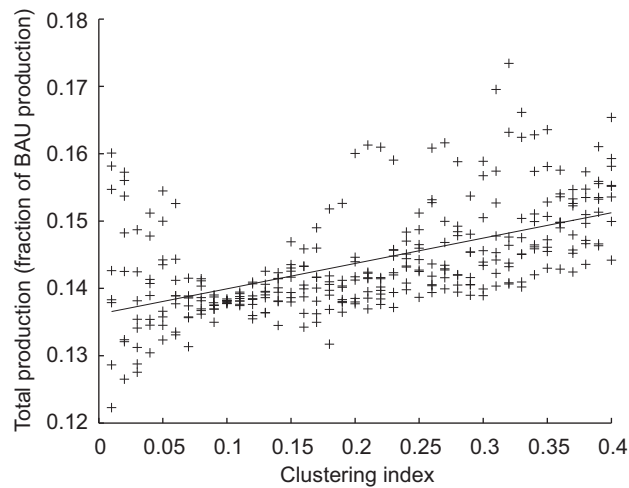


Fig. 11. Sum of production from date 0 until 100 days after the shock as a fraction of the BAU production, depending on the clustering index.

#### 4.3. Blocks connections

In order to strengthen the “isolation” hypothesis and investigate role of subregion connections, an additional set of simulations has been carried out with a  $1000 \times 1000$  IO matrix built from the aggregation of two  $500 \times 500$  matrices representing two subregions. We have gradually increased the coupling between the two subregions by increasing the numbers of links between these two subregions at each simulation step, reproducing more or less integrated subregions (Fig. 12).

##### 4.3.1. Network building

First, two random  $500 \times 500$  matrices  $C_1$  and  $C_2$  are built following the process described in Section 3.1. These two matrices are then aggregated as a block-diagonal matrix, to obtain a  $1000 \times 1000$  matrix  $m$ . Then, connections between the two diagonal blocks are introduced in the following way.

We select  $i \in X$  and  $j \in Y$  in the subregion 1 such that  $C_1(i, j) > 0$ , and  $i' \in X$  and  $j' \in Y$  in the subregion 2 such that  $C_2(i', j') > 0$ . The ends of these links are then swapped:

$$C(i, j) \rightarrow m(i, j) - 1$$

$$C(i', j') \rightarrow m(i', j') - 1$$

$$C(i, j') \rightarrow m(i, j') + 1$$

$$C(i', j) \rightarrow m(i', j) + 1$$

We repeat this procedure until the desired number of links is reached, checking that we are swapping new links.

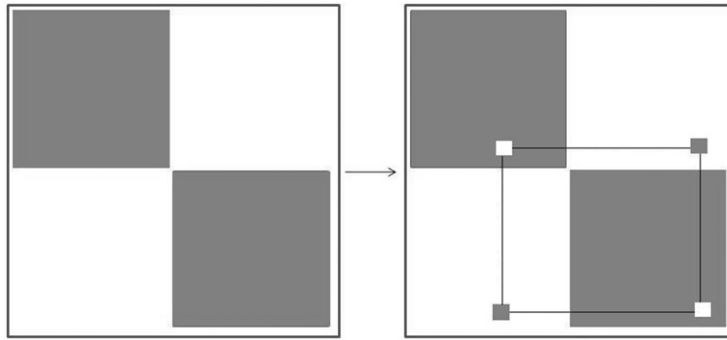


Fig. 12. Building of the connected matrix from the two blocks matrix.

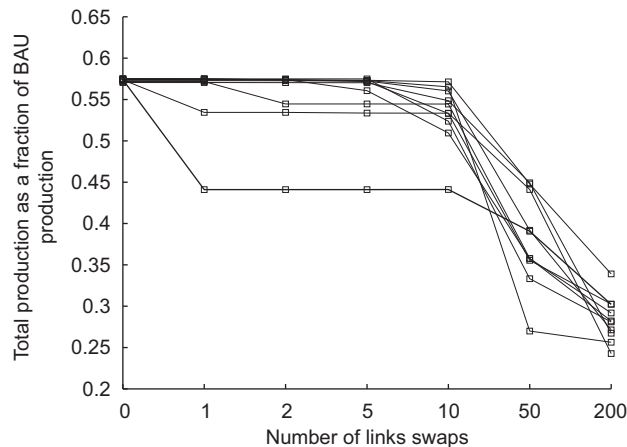


Fig. 13. Sum of production from date 0 until 100 days after the shock as a fraction of the BAU production, depending on the number of connections between the two blocks.

#### 4.3.2. Results

The shock is still a 100% loss in the production capacity of one PU from the mining sector. Results are reproduced in Fig. 13 as a function of the number of connections between the two blocks. The process has been conducted for 10 different couples of  $500 \times 500$  matrices. Each square in the figure corresponds to the sum of the production of the economy over the 100 days that follow the shock. Each line stands for one original couple of  $500 \times 500$  matrices, which are connected through an increasing number of links (on the x axis). When the  $1000 \times 1000$  matrix is formed by two independent blocks, the total loss in the economy is exactly the loss of one  $500 \times 500$  matrix. As the number of connections increases between the two blocks, the loss becomes larger.

The effect on subregion connections is easier to predict: when subregions are connected, each region becomes vulnerable to shocks affecting another region and, since the number of links does not change, there are no benefits from an “insurance” effect. Results confirm this intuition: not only is the subregion which is not directly affected by the shock but also the economy as a whole is worse off with these connections.

## 5. Conclusion

We do not pretend to assess actual output losses with our disaggregated model. Its assumptions are too simplistic to do so and crucial mechanisms have been left out of this first-step analysis: the ability of producers to create new connections in disaster aftermath, i.e. to find new suppliers or new clients; the role of imports to replace disabled regional producers; the role of reconstruction to restore initial production capacity and the disaster impact on final demand through consumption behavior and investments. Some of these mechanisms would amplify losses; some would dampen losses. Overall, the absence of reconstruction and complete rigidity of the economic network are acceptable assumptions only over very short terms, and lead to a large overestimation of total losses.

However, we claim that our analysis makes a contribution as it justifies the development of a more disaggregated approach to the modeling of economic consequences of natural disasters, or any other punctual shock to the economy. Indeed, we have showed classical IO models may be too optimistic as they represent the most favorable case in which risks



and losses are optimally shared among all producers, i.e. a case in which each firm is a client and a supplier of all other firms and in which direct losses are uniformly distributed among all firms. In spite of the simplification of our model, the mechanisms that are identified in our simulations appear important and need to be taken into account.

A disaggregated approach is necessary to evaluate cost amplifications due to heterogeneity of losses and business interactions within the production network. In particular, our results indicate that the output losses due to a disaster would depend on interaction between geographical disaster footprint and economic sector localizations. For equivalent aggregated losses a localized disaster affecting strongly few producers – like a flood – would lead to larger short-term output losses than a widespread disaster affecting a broad region with more limited impacts—like a wind storm or a heat wave. The output loss is also potentially larger if all producers from a sector are located in the same location, since output losses are larger if many producers from the same sector are affected.

Two strategies to improve robustness are suggested by the analysis of various network structures. The first one is an “isolation” strategy, in which many small groups of producers are as isolated as possible from all other groups. This strategy increases robustness, reducing the disaster impact on all groups not directly affected. The second strategy is the “insurance” one, in which producers have as many suppliers and clients as possible to be able to compensate the loss of one supplier and to limit consequences of the loss of one client. In addition, larger inventories are logically found to increase robustness, as it allows producers to keep producing temporarily even when their suppliers are disrupted.

The various strategies can be related to different industrial organizations which appear to have different robustness. For instance, a “localized” economy, in which groups of clients and suppliers are located close to each other, seems to be more robust than a “specialized–globalized” economy in which clients and suppliers are more homogeneously spaced. The modern industrial organization, with few suppliers and small inventories, also seems to increase vulnerability. It is an open question whether the loss in robustness is more than compensated for by an increased efficiency in normal time.

Since the model can reproduce disaster-provoked economic collapses, it may be used to investigate existence and location of thresholds in terms of disaster losses. For instance, this approach may be able to discriminate between limited economic consequences of the 2004 hurricanes in Florida and widespread economic consequences of Katrina in New Orleans in 2005. It seems that heterogeneity of losses is an essential parameter in the assessment of the risk of an economic systemic failure. Since flood losses are more heterogeneous than wind losses, it is interesting to note that Katrina was mainly a flood event while the 2004 hurricanes caused losses mainly because of high wind. The model remains too simple to provide stronger statements on this point, and additional developments will be necessary to go further in this direction.

Finally, network structure appears important to the robustness of the economy. Beyond the insights on which network structures are more robust than others, the model can help identify new economic parameters that are not available in statistic data bases in spite of their potential interest. For example, it would be useful to measure concentration, clustering and redundancy indexes for actual economic networks. It would make it possible to use the methods suggested in this paper to generate economic networks that are better representations of the real economy.

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