Is shale gas a good bridge to renewables? An application to Europe

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Abstract

This paper explores whether climate policy justifies developing more shale gas and addresses the question of a potential arbitrage between shale gas development and the transition to clean energy. We construct a Hotelling-like model where electricity may be produced by three perfectly substitutable sources: an abundant dirty resource (coal), a non-renewable less polluting resource (shale gas), and an abundant clean resource (solar). The resources differ by their carbon contents and their unit costs. Shale gas extraction's technology (fracking) generates local damages. Fixed costs must be paid to develop shale gas and to obtain the drastic innovation allowing to deploy the clean resource on a large scale. Climate policy takes the form of a carbon budget. We show that, at the optimum, a more stringent climate policy does not always go together with an increase of the quantity of shale gas extracted, and that banning shale gas extraction most often leads to bring forward the development of the clean resource, but not always. We calibrate the model for Europe in order to determine whether shale gas should be extracted and in which amount, and to evaluate the effects of a moratorium on shale gas use.

Keywords: shale gas, global warming, non-renewable resources, energy transition.

JEL Classification: H50, Q31, Q35, Q41, Q42, Q54.

1 Introduction

Motivation

In France, the Jacob law of July 13th, 2011 banned hydraulic fracturing ("fracking"): "*Under the Environment Charter of 2004 and the principle of preventive and corrective action under Article L. 110-1 of the Environment Code, exploration and exploitation of hydrocarbon liquids or gas by drilling followed by hydraulic fracturing of the rock are prohibited on the national territory.*" Moreover, the exploration licences held by companies like the American Schuepbach or the French Total were cancelled. Schuepbach complained to the court that this law was unfair and unconstitutional, but the Constitutional Court confirmed the ban on October 8th, 2013, saying that the Jacob law conforms to the constitution and is not disproportionate. By the same time, French President François Hollande said France will not allow exploration of shale gas as long as he is in office.

This position, although supported by a majority of the population[1], may seem puzzling. France is the only one of the European Union's 28 countries besides Bulgaria to ban shale gas. The ban is grounded on two types of strong environmental arguments, that need to be examined closely. First, fracking is considered as dangerous and environmentally damaging. It pumps water, sand and chemical under high pressure deep underground to liberate the gas that is trapped in the rock. The main dangers are for surface water (through the disposal of the fracturing fluids) and groundwater (through the accidental leakage of fracking fluids from the pipe into potable aquifers). Also, seismic vibrations caused by the injection of water underground is feared. Finally, there are concerns over landscape, as the number of wells may be very important and their layout very dense. Second, it is argued that what should be done in the face of global warming is to reduce drastically the use of fossil fuels, not to find new ones, which will have the effect of postponing the transition to clean renewable energy. To these arguments, shale gas supporters answer that natural gas is less polluting than other fossil fuels (oil, and particularly coal), and that its substitution to coal and oil should be encouraged on environmental grounds. Anyway, coal resources are so large that they are more than sufficient by themselves to overtake

[1] IFOP survey, Sept. 13th, 2012: 74% of the respondents are opposed to shale gas exploitation; BVA survey, Oct. 2nd, 2014: 62%. Note that this is greater than the opposition to nuclear energy, which provides most of France's electricity.

any reasonable constraint on atmospheric carbon concentration. Adding to these resources new unconventional fossil fuel reserves is not an issue, as far as they help leaving ultimately more coal under the ground. Indeed, it seems impossible to fight global warming effectively without substantially reducing the use of coal, what shale gas could allow. According to the International Monetary Fund (2014), "*Natural gas is the cleanest source of energy among other fossil fuels (petroleum products and coal) and does not suffer from the other liabilities potentially associated with nuclear power generation. The abundance of natural gas could thus provide a "bridge" between where we are now in terms of the global energy mix and a hopeful future that would chiefly involve renewable energy sources.*"

The contrast between the position held by France and the situation of the United States is stunning. United States is at date the first natural gas producer in the world. Shale gas has risen from 2% of domestic energy production a decade ago to nearly 40% today (IMF, 2014). It has profoundly modified the energy mix: shale gas is gradually replacing coal for electricity generation. Coal-fired power plants produced more than half of the total electricity supply in 1990, and natural gas-fired power plants 12%; in 2014, the figures were respectively 39% and 28%; in July 2015, the monthly natural gas share of total U.S. electricity generation (35%) surpassed the coal share (34.9%) (Energy Information Administration, 2015). CO_2 emissions have been reduced by 10% between 2007 and 2013. This reduction may be due to many other factors, but gas to coal substitution has certainly played a significant part[2]. This substitution is at the heart of the Obama's administration climate policy. Of course, in France, exploiting shale gas would not be appealing from the point of vue of climate change, because it would substitute to nuclear energy, not coal.

Objective

This paper does not pretend to examine all aspects of this complex problem. Our objective is to explore whether climate policy justifies developing more shale gas, and to address the question of a potential arbitrage between shale gas development and the transition to clean energy, when

[2] According to the Economic Report of the President 2013, "*... actual 2012 carbon emissions are approximately 17 percent below the "business as usual" baseline. (...) of this reduction, 52 percent was due to the recession (...), 40 percent came from cleaner energy (fuel switching), and 8 percent came from accelerated improvement in energy efficiency (...).*" See also Feng *et al.* (2015).

environmental damages, both local and global, are taken into account. More precisely, we seek to answer two questions which relate to the role of shale gas as a “bridge fuel” between coal and renewables. The first question is whether a more ambitious climate policy should involve more shale gas extraction and under which condictions. A corollary is the question of a potential arbitrage between the global damage due to climate change and the local damage due to fracking. The second question is whether authorizing shale gas extraction, compared to a moratorium, should lead to postpone the switch to clean energy and under which condictions. This question is important, as one of the main arguments in favour of fracking is that it gives the world time to obtain the drastic innovations enabling a large scale deployment of clean renewables at an acceptable cost[3].

Sketch of the model

To answer these questions, we construct a Hotelling-like model where electricity may be produced by the means of three perfectly substitutable energy sources: an abundant dirty resource, coal, a non-renewable less polluting resource, shale gas, and an abundant clean resource, solar, provided that appropriate fixed costs are paid for. The two fossil resources differ by their carbon contents and hence their potential danger for the climate (shale gas is less CO_2-emitting than coal), and the local damages their extraction causes (shale gas is more damaging, due to the fracking technology[4]). The costs of electricity generation by the two fossil resources also differ:

[3]This argument is widespread, see for instance the speech by Edward Davey, UK Secretary of State, for Energy and Climate Change on shale gas exploration, on september 9th 2013: “Gas will buy us the time we need over the coming decades to get enough low carbon technology (...).” (https://www.gov.uk/government/speeches/the-myths-and-realities-of-shale-gas-exploration) and oil and gas magnate T. Boone Pickens argues on his website, "Natural gas is not a permanent solution to ending our addiction to imported oil. It is a bridge fuel to slash our oil dependence while buying us time to develop new technologies (...)."

[4]The model may be very easily adapted to consider more generally natural gas in all its forms –conventional and unconventional, rather than specifically shale gas. Indeed, the local damage may be seen as a differential cost or local damage caused by gas extraction and combustion compared to coal, and may be negative. Coal extraction is environmentally damaging (isses of land use, waste management, water pollution etc.), and coal combustion as well (local air pollution). Besides, coal mining has been a very dangerous activity in the past, and still remains so in many developing countries. However, the public attention is at the moment focused on local damages due to shale gas extraction. Moreover, one could interpret some of the past coal externalities as now being part of private costs due to regulation.

shale gas is cheaper than coal. Exploration and development allow to build the shale gas reserves that will be extracted (Gaudet and Lasserre, 1988). A fixed R&D cost must be paid before solar production begins. It is the price to pay to obtain the drastic innovation necessary to overcome the intermittency problem. It is decreasing in time due to technical progress. Following Chakravorty *et al.* (2006a, 2006b), climate policy takes the form of a ceiling under which atmospheric CO_2 concentration must be kept. Agents derive their utility from the consumption of electricity. The social planner seeks to maximize the intertemporal welfare, taking account of the climate constraint.

Literature and contribution

Chakravorty *et al.* (2008) explore thoroughly the question of the ordering of extraction of two fossil resources, differing by their unit extraction cost but also by their pollution content, in presence of an expensive clean backstop. Van der Ploeg and Withagen (2012) and Coulomb and Henriet (2014) consider a three resources setting, and emphasize the role of the CO_2-emitting resource less polluting than coal. These papers neither model exploration enabling to find fossil resources, nor introduce R&D in clean energy, whereas the arbitrage between these two types of investments is at the heart of our paper. Dasgupta *et al.* (1982) and Henriet (2012) introduce a fixed R&D cost prior to the use of the clean backstop, but the former does not consider climate policy whereas the latter incorporates a pollution constraint but only one fossil resource. We wonder in this paper whether shale gas is a good bridge between the present situation where electricity is mainly generated by the combustion of very polluting fossil resources, and a future where electricity generation would be clean, knowing that this clean future may not happen immediately because of the high fixed cost society has to pay before solar energy can be deployed on a large scale. Hence our contribution is not about the ordering of resource use in electricity generation, but first about the optimal quantity of shale gas to be developed and extracted, and second about the optimal date of transition to clean energy. We investigate how these two variables change as a function of the stringency of climate policy, and what are the effects of a moratorium on shale gas extraction.

Results and intuitions

We study first the effects of strengthening climate policy, when shale gas extraction is allowed.

In all events, it makes the switch to clean energy happen earlier. From an initial point where climate policy is very lenient, becoming more severe always leads to a phasing-out of coal, up to a point where only shale gas is used in electricity generation; then, becoming even more severe leads to the eviction of shale gas by clean energy. When the local damage of fracking is high, tightening climate policy leads to increase the quantity of shale gas developed during the phasing-out of coal, and to extract it earlier. However, when the local damage is small, a more stringent climate policy may have the counterintuitive effect of reducing the quantity of shale gas developed during the phasing-out of coal, if the advantage of shale gas over coal in terms of CO_2 emissions is small or electricity demand very elastic.

The intuition behind these results is the following. A more severe climate policy obliges to emit less. There are two solutions to do so: switch to a less emitting fuel, or increase the price of fossil energy to make demand decrease. The first option, which amounts to developing more shale gas, is not interesting when shale gas is economically attractive i.e. when a lot of it is already developed, and when its advantage over coal in terms of carbon content weak. The second option is all the more attractive since demand is elastic.

We then show that, compared to a moratorium, authorizing shale gas extraction leads in most cases, and in particular when the local damage due to the fracking technology is large, and/or the price elasticity of electricity demand is low, to postpone the switch to clean renewables. However, when the local damage is low, authorizing shale gas extraction can actually lead to bringing forward the transition to clean energy, provided that shale gas is polluting enough or demand elastic enough. The intuition behind these results is the same as before, namely, for the counterintuitive part, that when gas is economically attractive and only marginally less emitting than coal, or if electricity demand is very elastic, banning gas significantly raises energy costs, which causes a drop in demand that allows society to postpone the transition to clean energy. We also show that, when the social planner incorporates a climate damage function in her program, i.e. when she also chooses the level of the ceiling of atmospheric carbon concentration, authorizing shale gas extraction leads in most cases, and in particular when the local damage due to the fracking technology is large, to a lower ceiling. However, when the local damage is low, authorizing shale gas extraction can actually lead the planner to choose higher climate damages. This result may seem counterintuitive. The intuition is the following. In the case of small local damages, shale gas is not only less polluting than coal, but also cheaper. Emissions induced by shale gas might in fact be higher those of coal, as long as shale gas is consumed in greater quantity than coal. As a result, forbidding the use of a shale gas may make it less difficult to comply with a climate constraint.

We calibrate the model for Europe, which makes sense because in 2014 coal still accounts for 26% of electricity generation, and perform simulations. The results are extremely sensitive to the value of the local damage of fracking, for which we have no solid empirical estimation. If it represents 75% of shale gas unit cost, we obtain that for a carbon budget corresponding to a 3°C temperature increase, only 5.7% of total European shale gas resources should be extracted. A moratorium on shale gas development, together with the enforcement of the ceiling, entails an increase of 1.8% of energy expenditures and a decrease of 3.6% of intertemporal welfare compared to the reference scenario, and brings forward by 2 years the switch to solar energy. For a small local damage of 25% of shale gas unit cost, it is optimal to phase out coal immediately and develop all European shale gas resources. In this case the moratorium has very big negative effects. Whatever the local damage, the counterintuitive case, in which the moratorium leads to posptpone the transition to clean energy, is very unlikely to occur in our simulations.

Outline

The remaining of the paper is as follows. Section 2 presents the model and the optimal solution. Section 3 studies the sensitivity of the optimal solution to the stringency of environmental policy. Section 4 studies the consequences of a moratorium on shale gas extraction. Section 5 presents illustrative simulations concerning electricity generation in Europe. Section 6 concludes.

2 The model

2.1 Assumptions

We consider an economy where electricity is initially produced by coal-fired power plants, and where two other energy sources, shale gas and solar, may be developed and used in electricity generation as well. Coal is supposed to be abundant but very polluting. Shale gas is non-renewable, and also polluting but to a lesser extent. Solar is abundant and clean. The three resources are perfect substitutes in electricity generation.

	reserves		resources	
	EJ	GtC	EJ	GtC
conventional oil	4 900 – 7 610	98 – 152	4 170 – 6150	83 – 123
unconventional oil	3 750 – 5 600	75 – 112	11 280 – 14 800	226 – 297
conventional gas	5 000 – 7 100	76 – 108	7 200 – 8 900	110 – 136
unconventional gas	20 100 – 67 100	307 – 1026	40 200 – 121 900	614 – 1 863
coal	17 300 – 21 000	446 – 542	291 000 – 435 000	7 510 – 11 230
total	51 050 – 108 410	1002 – 1940	353 850 – 586 750	8 543 – 13 649

Reserves are those quantities able to be recovered under existing economic and operating conditions; resources are those whose economic extraction is potentially feasible. Resource data do not include reserves.

Table 1: Estimates of fossil reserves and resources, and their carbon content. Source: IPCC WG III AR 5, 2014, Chapter 7 Table 7.2

The label d for "dirty" stands for the dirty resource, namely coal. The pollution intensity of coal is θ_d: the extraction and use of one unit of coal leads to the emission of θ_d unit of CO_2 ("carbon" thereafter). The marginal long term production cost of electricity with coal is c_d. It is supposed to be constant. This cost includes the extraction cost, but also capital costs and operating and maintenance costs[5]. The extraction rate of coal is $x_d(t)$. Coal is abundant: resources under the ground are so large that scarcity is not an issue (see Table **??**).

The label e for "exhaustible" stands for shale gas. Its pollution intensity is θ_e, with $\theta_e \leq \theta_d$. Indeed, Heath *et al.* (2014), performing a meta-analysis of the literature to date, obtained that emissions from shale gas-generated electricity are approximately half that of coal-generated electricity, and that emissions from unconventional gas-generated electricity are roughly equivalent to those of conventional gas[6] (see Table **??**). The most recent estimates by IPCC are consistent

[5]This cost is in fact the levelized cost of electricity (LCOE) generated by coal-fired power plants. According to the US Energy Information Administration, it represents the per-kilowatt hour cost (in real dollars) of building and operating a generating plant over an assumed financial life and duty cycle. See EIA (2014a).

[6]Notice that whereas the combustion of natural gas is without controverse less CO_2 emitting than the combustion of coal, methane leakage from the shale gas supply chain could be high enough to offset the benefits. Heath *et al.* (2014) do not take into account methane leakage in their analysis because of the wide variability of estimates (0.66–6.2% for unconventional gas, 0.53–4.7% for conventional gas).

coal	shale	unconventional	conventional
980	470	460	450

Table 2: Median estimate of life cycle GHG emissions (g CO_2eq/kWh) from electricity generated using coal or different types of natural gas. Source: Heath *et al.*, 2014

	direct emissions	life-cycle emissions
	min / median / max	min / median / max
coal PC	670 / 760 / 870	740 / 820 / 910
gaz – combined cycle	350 / 370 / 490	410 / 490 / 650

Table 3: Emissions of selected electricity supply technologies (gCO_2eq/kWh). Source: IPCC WG III AR 5, 2014, Annex III Table A.III.2

with these results (see Table **??**). The long term marginal production cost of electricity using shale gas is c_e. As for coal, this includes the fuel extraction cost, other operating and maintenance costs and capital costs. We make the assumption that $c_e < c_d$ (see Energy Information Administration, 2014a and Table **??**). The extraction of shale gas causes a local marginal damage d, supposed to be constant. This damage is due primarily to the technology employed to extract shale gas, namely hydraulic fracturing. It has been at the center of the discussions on shale gas development, around the world and in France in particular. According to the review by Mason *et al.* (2015), the literature to date offers very few empirical estimates of these negative externalities. Before beginning to extract shale gas, it is necessary to incur an upfront exploration cost. The total quantity of reserves X_e available after exploration and development is endogenous, and proportional to the exploration investment: $X_e = f(I)$, with $f'(.) > 0$ and $f''(.) < 0$. This can also be written $I = E(X_e)$, with $E'(X_e) > 0$ and $E''(X_e) > 0$, as in Gaudet and Lasserre (1988). We suppose that the exploration cost must be paid at the beginning of the planning horizon, even though the actual extraction of shale gas may be postponed to a later date[7]. The extraction rate of shale gas is $x_e(t)$.

[7]This reflects the fact that society explores first, and then can choose to extract gas at a later date. This assumption allows to get rid of problems of concavity of the value function appearing when exploration and exploitation of shale gas reserves are performed at the same date.

	levelized capital cost	fixed O&M	variable O&M including fuel	transmission investment	total
conventional coal	60	4.2	30.3	1.2	95.6
natural gas-fired combined cycle	14.3	1.7	49.1	1.2	66.3
solar PV	114.5	11.4	0	4.1	130
solar thermal	195	42.1	0	6.0	243

Table 4: US average levelized cost of electricity (2012 \$/MWh). Source: EIA, 2014a

The label b for "clean backstop" stands for solar energy. The long term marginal production cost of electricity with solar is c_b. We do not make any *a priori* assumption about the magnitude of c_b compared to c_d and c_e. Renewable energy can be developed at a current[8] R&D cost $F(t)$. It is supposed to be decreasing in time, because of technical progress: $F'(t) < 0, F''(t) > 0$ (Dasgupta *et al.*, 1982). The type of R&D we have in mind produces drastic innovations that allow to rely on renewable energy alone for electricity generation. These innovations solve the intermittency problem inherent to renewable energies such as solar and wind. They allow to develop for instance large scale electricity storage device and enhanced electric grid[9]. As emphasized by Heal (2010), until renewable storage is possible, there will be a continuing need for coal. However, global policy should aim at a zero-emission future, which is the only way not to exceed the carbon budget. Storage will thus likely be essential. Baker *et al.* (2013) state that "the key technological challenges in grid integration relate to improvements in storage and grid controls", but they do not consider storage when reviewing the economics of solar PV in the short to medium term,

[8] We have assumed that society chooses the date at which it pays the fixed cost of developing solar energy, we could have assumed that this fixed cost is also paid at date 0 but that the cost that must be paid for innovation to take place at date t is $CF(t) = F(t)e^{-\rho t}$. We prefer the first interpretation because we have in mind massive investment expenditures to build equipments.

[9] We do not include electricity generation from renewables before the date at which electricity can be produced with renewables alone. It is straightforward to modify the model to account for the use of renewables before this date, for a given installed capacity, by simply assuming a time varying demand for fossil fuels, which reflects that electricty from renewables before the date of the transition cannot be produced on demand. On the contrary, all gas power plants and most coal power plants have been originally designed or can be modified for flexible output: the ability to "ramp" on an hourly basis to much less than full output, and "cycle" on and off on a daily basis. See Martinot (2016) for flexible coal power plants in Germany.

because of its current high cost. Gowrisankaran *et al.* (2015) find that intermittency represents a very high social cost in the value of solar energy. The fixed investment cost in our model is a rough way to represent these long term innovations, allowing for a zero-carbon future. The production rate of solar energy is $x_b(t)$.

The combustion of the two polluting resources generates carbon emissions that accumulate in the atmosphere. $Z(t)$ is the atmospheric concentration of carbon. Its change over time is given by:

$$\dot{Z}(t) = \theta_e x_e(t) + \theta_d x_d(t)$$

We suppose that there is no natural decay of carbon, which is an acceptable assumption, considering the large uncertainties surrounding the natural absorption process and its potential weakening as temperature increases.

Finally climate policy is modeled as a cap on the atmospheric carbon concentration $\overline{Z}$, following the strand of literature initiated by Chakravorty *et al.* (2006a, 2006b). The cap or ceiling $\overline{Z}$ can also be interpreted as the carbon budget available to society.

Electricity produced at date t is $x(t) = x_d(t)+x_e(t)+x_b(t)$. Agents derive their utility directly from the consumption of electricity. Let $u\,(x(t))$ be the utility function at date t, with u twice continuously differentiable, strictly increasing and strictly concave, and ρ the social discount rate, assumed to be constant. The social planner chooses the extraction and production rates $x_d(t)$, $x_e(t)$, $x_b(t)$, the amount of shale gas developed X_e, and the date T_b at which the R&D investment for solar energy is made which maximize the discounted sum of utilities minus costs, under the resource constraint and the climate constraint.

In order to solve the general problem, we first assume that date T_b is given, and we compute the constrained optimal path. We obtain the value of the problem for each T_b, and we maximize this value over T_b.

2.2 Ordering resource use

For a given date T_b, the social planner's maximization gives $V(T_b)$ where:

$$
V(T_b) = \begin{cases}
\max_{x_d(t),x_e(t),x_b(t),X_e} \Big\{ \int_0^\infty e^{-\rho t} \Big[u\left(x_d(t)+x_e(t)+x_b(t)\right) \\
-c_d x_d(t) - (c_e+d)x_e(t) - c_b x_b(t) \Big] dt - E(X_e) - F(T_b)e^{-\rho T_b} \Big\} \\
\text{subject to:} \\
\int_0^\infty x_e(t)dt \leq X_e & (1) \\
\int_0^\infty (\theta_d x_d(t) + \theta_e x_e(t))dt \leq \overline{Z} - Z_0, \quad Z(0) = Z_0 \text{ given} & (2) \\
x_d(t) \geq 0,\ x_e(t) \geq 0, \text{ and } x_b(t) \begin{cases} \geq 0 \text{ if } t \geq T_b \\ = 0 \text{ otherwise} \end{cases} & (3)
\end{cases}
$$

The current value Hamiltonian of the problem reads, with $\lambda(t)$ the scarcity rent associated to the stock of shale gas and $\mu(t)$ the carbon value:

$$
\begin{aligned}
\mathcal{H} &= u\left(x_d(t)+x_e(t)+x_b(t)\right) - c_d x_d(t) - (c_e+d)x_e(t) - c_b x_b(t) \\
&\quad - \lambda(t)x_e(t) - \mu(t)\left(\theta_d x_d(t) + \theta_e x_e(t)\right)
\end{aligned}
$$

The first order necessary conditions of optimality are:

$$u'(x_d(t)+x_e(t)+x_b(t)) \leq c_d + \theta_d \mu(t) \tag{4}$$

$$u'(x_d(t)+x_e(t)+x_b(t)) \leq c_e + d + \lambda(t) + \theta_e \mu(t) \tag{5}$$

$$u'(x_d(t)+x_e(t)+x_b(t)) \leq c_b \tag{6}$$

with equality when the energy on the left-hand side corresponding to the cost on the right-hand side is actually used, and

$$\dot{\lambda}(t) = \rho\lambda(t) \tag{7}$$

$$\dot{\mu}(t) = \rho\mu(t) \text{ before the ceiling} \tag{8}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)X_e(t) = 0 \tag{9}$$

$$\lim_{t\to\infty} e^{-\rho t}\mu(t)Z(t) = 0 \tag{10}$$

to which we must add:

$$E'(X_e) = \lambda_0 \tag{11}$$

This last equation is a transversality condition at date 0 (Gaudet and Lasserre, 1988). It states that costs of exploration for finding shale gas reserves must be paid up to the point where the exploration cost of a marginal unit of reserve $E'(X_e)$ is equal to the value of this reserve under the ground, which is the initial scarcity rent λ_0.

Following Chakravorty *et al.* (2006a, 2006b) and the subsequent literature, it is easy to see that at the optimum the three energy sources are used successively, the stock of shale gas developed, X_e, is exhausted, the ceiling is reached at the date of the switch to clean energy, T_b, solar is developed only when it starts to be used, at T_b (Dasgupta *et al.*, 1982).

We have supposed that the marginal cost of production of electricity with shale gas is lower than the one with coal: $c_e < c_d$. However, because of the existence of the local damage caused by shale gas extraction, the full marginal production cost for shale gas $c_e + d$ may be lower or higher than the marginal production cost for coal c_d. We successively study the two cases of a large and a small marginal local damage.

2.2.1 Large local damage

By large local damage we mean that the local damage more than compensates the gain in terms of production cost due to the use of shale gas instead of coal in electricity generation: $d > c_d - c_e$. Hence if the total marginal cost is taken into account, coal is cheaper than shale gas. However, shale gas has an advantage over coal as regards carbon emissions.

The price[10] path is potentially composed of three phases (see Chakravorty *et al.*, 2008, or Coulomb and Henriet, 2014).

In phase 1, coal is used in quantity $X_d = \frac{\overline{Z} - Z_0 - \theta_e X_e}{\theta_d}$, between dates 0 and T_e, at a price:

$$p_d(t) = c_d + \theta_d \mu_0 e^{\rho t} \tag{12}$$

with μ_0 such that: $\int_0^{T_e} x_d(t)dt = \int_0^{T_e} D\left(p_d(t)\right)dt = X_d$, where $D(.) = u'^{-1}(.)$ is the demand function.

In phase 2, shale gas is used in quantity X_e, between dates T_e and T_b, at a price:

$$p_e(t) = c_e + d + (\lambda_0 + \theta_e \mu_0)e^{\rho t} \tag{13}$$

[10] Of course, as we are considering a central planner problem, the term "price" is used simply but inaccurately to denote the marginal utility of electricity consumption.

with λ_0 such that: $\int_{T_e}^{T_b} x_e(t)dt = \int_{T_e}^{T_b} D\left(p_e(t)\right)dt = X_e$. T_e, the date of the switch from coal to shale gas, is endogenously determined by the continuity of the energy price at date T_e: $p_d(T_e) = p_e(T_e)$, i.e.

$$c_d + \theta_d \mu_0 e^{\rho T_e} = c_e + d + (\lambda_0 + \theta_e \mu_0) e^{\rho T_e} \tag{14}$$

In phase 3, the clean backstop is used at the constant price:

$$p_b(t) = c_b \tag{15}$$

from date T_b onwards.

One (or two) of these phases may not exist. For instance, in the absence of any constraint on the atmospheric carbon concentration (when $\overline{Z} \to \infty$), CO_2 emissions do not matter and, as coal is available in infinite amount with no fixed cost, it will be used alone forever. As soon as $\overline{Z}$ is finite however, there will be a switch to solar at some point. But is it useful to introduce shale gas as well? Clearly, if θ_e is close to θ_d, shale gas, which is more costly than coal, because of the local damage and the upfront development cost, and equally polluting, will never be used. On the other hand, if θ_e is close to zero and the ceiling constraint very tight, it may happen that shale gas is exploited from the beginning of the trajectory at the expense of coal.

2.2.2 Small local damage

In this case, $d < c_d - c_e$. The advantage of shale gas in terms of production costs dominates. Shale gas is also less polluting than coal. It will be used immediately in electricity generation. But it may be the case that we return to coal, more costly and more polluting than shale gas, later on, because shale gas is scarce while coal is abundant.

Again, the price path is potentially composed of 3 phases.

In phase 1, shale gas is used in quantity X_e, between dates 0 and T_d. Its price is given by (**??**), with $(\lambda_0 + \theta_e \mu_0)$ such that: $\int_0^{T_d} x_e(t)dt = \int_0^{T_d} D\left(p_e(t)\right)dt = X_e$.

In phase 2, coal is used in quantity X_d, between dates T_d and T_b. Its price is given by (**??**), with μ_0 such that: $\int_{T_d}^{T_b} x_d(t)dt = \int_{T_d}^{T_b} D\left(p_d(t)\right)dt = X_d$. T_d, the date of the switch from shale gas to coal, is endogenously determined by $p_e(T_d) = p_d(T_d)$, i.e.

$$c_e + d + (\lambda_0 + \theta_e \mu_0) e^{\rho T_d} = c_d + \theta_d \mu_0 e^{\rho T_d} \tag{16}$$

In phase 3, the clean backstop is used at price c_b (see (**??**)) from date T_b onwards.

Here again, one of these phases may not exist. For instance, absent climate policy ($\overline{Z} \to \infty$) shale gas, the cheapest source of energy, is used first, then coal is used forever. Solar is never developed.

2.3 Optimal switch to clean energy

We now obtain the optimal date of the switch to clean energy, by solving:

$$\max_{T_b} V(T_b)$$

We show in Appendix **??** that there exists a unique maximum T_b^*.

2.3.1 Large local damage

When $d > c_d - c_e$, the optimal date of the switch from shale gas to solar, T_b^*, solves (see Appendix **??**):

$$\pi_b - \pi_e\left(T_b^*\right) = \rho F(T_b^*) - F'(T_b^*) \tag{17}$$

where:

$$\pi_b = u\left(x_b\right) - c_b x_b \tag{18}$$

$$\pi_e\left(T_b\right) = u\left(x_e(T_b)\right) - (c_e + d + (\lambda_0 + \theta_e \mu_0)e^{\rho T_b})x_e(T_b) \tag{19}$$

Equation (**??**) shows that at the optimal date of the switch from shale gas to solar the marginal benefit of the switch (on the left-hand side of the equation), in terms of current net surplus, is equal to its marginal cost (on the right-hand side), in terms of additional R&D fixed cost. Since $\rho F(T_b^*) - F'(T_b^*) > 0$, the current net surplus jumps upwards at the date of the switch, which requires that the electricity price jumps downwards.

Notice that T_b^* is strictly positive if and only if:

$$\rho F(0) - F'(0) \geq \pi_b - \pi_e(0) \tag{20}$$

that is if the marginal fixed cost of innovating in clean energy at date 0 is higher than the marginal benefit of adopting innovation at date 0. Basically, this condition is satisfied when the fixed cost at date 0 $F(0)$ and the unit cost of solar energy c_b are high. We suppose in the sequel that it is the case.

Equations (**??**), (**??**), (**??**), (**??**) and (**??**) characterize the optimal solution when the sequence of energy use is coal (from 0 to T_e), shale gas (from T_e to T_b^*) and solar, i.e. when the three phases identified above exist.

We compute in Appendix **??** the conditions under which one of the two first phases does not exist, given that the last phase (solar) always exists as soon as some climate policy is introduced. We show that there exist two values of the carbon ceiling, $\overline{Z}_1$ and $\overline{Z}_2$, such that if $\overline{Z} \geq \overline{Z}_2$ (lenient climate policy) shale gas is not developed, if $\overline{Z}_1 < \overline{Z} < \overline{Z}_2$ (intermediate climate policy) coal and gas are used successively, and if $\overline{Z} \leq \overline{Z}_1$ (stringent climate policy) it is optimal to phase out coal completely at once.

2.3.2 Small local damage

When $d < c_d - c_e$, the optimal date of the switch from coal to solar, T_b^*, solves:

$$\pi_b - \pi_d\left(T_b^*\right) = \rho F(T_b^*) - F'(T_b^*) \tag{21}$$

where π_b has been defined above, and:

$$\pi_d\left(T_b\right) = u\left(x_d(T_b)\right) - (c_d + \theta_d \mu_0 e^{\rho T_b}) x_d(T_b) \tag{22}$$

The interpretation of equation (**??**) is similar to the one given in the case of a large local damage.

Similar to the case of a large local damage, T_b^* is strictly positive if and only if:

$$\rho F(0) - F'(0) \geq \pi_b - \pi_d(0) \tag{23}$$

We suppose in the sequel this condition satisfied.

Equations (**??**), (**??**), (**??**), (**??**) and (**??**) characterize the optimal solution when the sequence of energy use is shale gas (from 0 to T_d), coal (from T_d to T_b^*) and solar (from T_b^* onwards).

As shale gas is cheaper and less polluting than coal, necessarily $c_e + d + \theta_e \mu_0 < c_d + \theta_d \mu_0$ $\forall \mu_0$. Hence $\exists \lambda_0 > 0$ s.t. $p_e(0) < p_d(0)$, meaning that there always exists scope for shale gas exploration and extraction.

Now, it is possible to switch directly from shale gas to solar, and leave coal forever in the ground? We show in Appendix **??** that it is the case if climate policy is stringent enough, and more precisely, if the carbon ceiling is lower than a threshold $\overline{Z}_3$ that we characterize.

2.4 Summary of results

These results are summarized in the following Proposition:

Proposition 1 *When the local damage is large and climate policy lenient, coal is used alone to get to the ceiling. It is not optimal in this case to explore and develop shale gas. When environmental policy becomes more stringent, shale gas replaces coal at some point before the ceiling. For an even more stringent environmental policy, coal is completely evicted by shale gas.*

When the local damage is small shale gas is always developed, and its extraction begins immediately. If climate policy is lenient, shale gas is replaced by coal at some point before the ceiling, because it is abundant whereas shale gas is scarce and costly to develop. However, if climate policy is stringent, coal is completely phased out.

3 A more stringent climate policy

We now perform exercises of comparative dynamics to see how the optimal solution is modified when environmental policy becomes more stringent. In particular, we wonder whether climate policy justifies developing more shale gas, and making the transition to solar earlier. We show that, in the case of small local damage, a more stringent ceiling might in fact lead to extract less shale case.

3.1 Large local damage

In the case of a large local damage and when the three resources are used ($\overline{Z}_1 < \overline{Z} < \overline{Z}_2$), we show in Appendix **??** that:

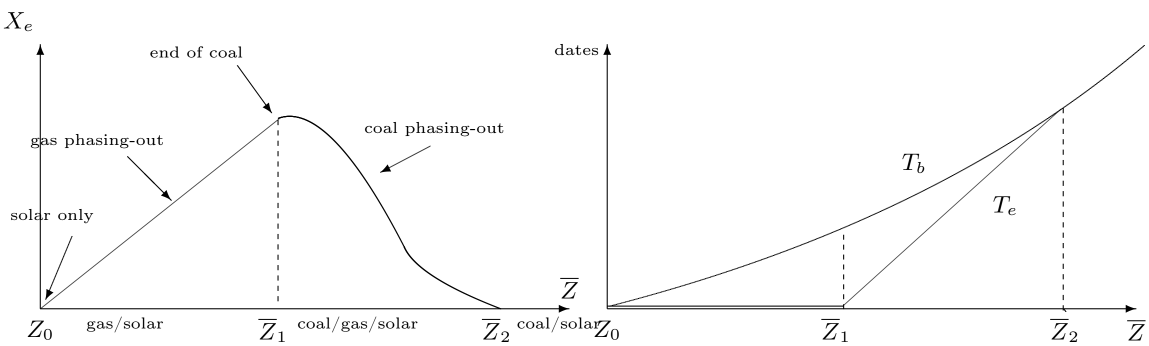
$$\frac{\partial T_e}{\partial \overline{Z}} > 0, \qquad \frac{\partial T_b}{\partial \overline{Z}} > 0, \qquad \frac{\partial X_e}{\partial \overline{Z}} < 0$$

As environmental policy becomes more stringent, the use of shale gas becomes more interesting because of its lower carbon content. This advantage on the climate point of view overcomes more and more the local damage drawback and the exploration cost that has to be paid prior to exploiting shale gas. It becomes therefore optimal to use shale gas earlier and to develop it in a greater amount.

A more severe climate policy also makes the switch to solar energy happen earlier. The reason is the same as for shale gas: the advantage of solar from the climate point of view overcomes more and more the fixed cost.

Clearly, in this case, the effect of a more stringent climate policy is to partially or even totally evict coal and replace it by more shale gas before the ceiling, and also to make the transition to clean energy happen sooner. The eviction of coal is completed when $\overline{Z} = \overline{Z}_1$.

When initial climate policy is even more stringent ($\overline{Z} < \overline{Z}_1$), only shale gas is used before the ceiling, in quantity $X_e = (\overline{Z} - Z_0)/\theta_e$. Now, a more severe climate policy leads to a decrease of the quantity of shale gas extracted. Shale gas is replaced by solar energy (see Figure **??**).



Quantity of shale gas X_e Dates T_e and T_b

Figure 1: Strengthening climate policy; large local damage

3.2 Small local damage

Likewise, a comparative dynamics exercise yields in the case of a small local damage (see Appendix **??**):

$$\frac{\partial T_d}{\partial \overline{Z}} < 0, \qquad \frac{\partial T_b}{\partial \overline{Z}} > 0$$

Remember that in this case it is optimal to develop shale gas first. Then, quite intuitively, when environmental policy becomes more stringent, the date of the switch to coal is postponed while the date of the switch to solar is brought forward. However, the effect of a more stringent climate policy on the amount of shale gas reserves developed is ambiguous in general. The relative carbon content of gas and the elasticity of demand are key elements. The two polar cases where shale gas is not polluting at all and shale gas is as polluting as coal lead to very different outcomes (see Appendix ??):

$$\text{if } \theta_e = 0,\ \frac{\partial X_e}{\partial \overline{Z}} < 0$$
$$\text{if } \theta_e = \theta_d,\ \frac{\partial X_e}{\partial \overline{Z}} > 0$$

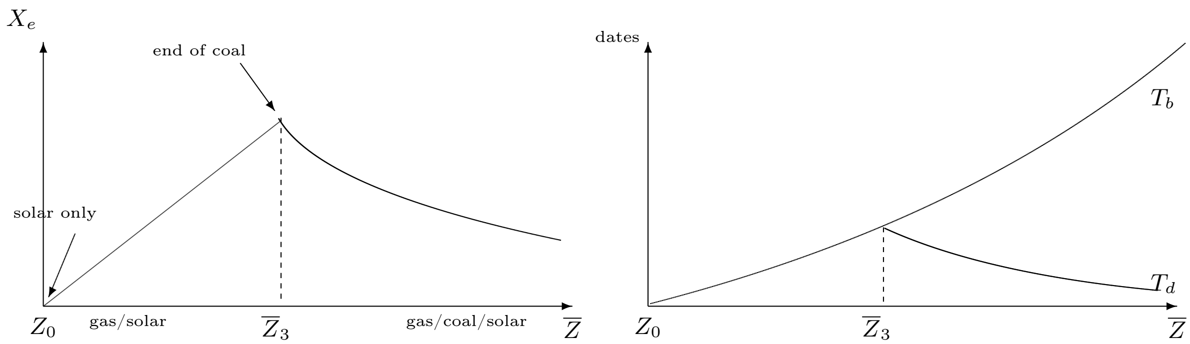
When shale gas is not polluting at all, the more stringent climate policy is, the more shale gas is developed (see Figure ??, upper panel). The total marginal variable cost of shale gas is smaller than the one of coal because the marginal local damage is small; furthermore, shale gas is not polluting. The only reason why coal is not completely evicted is the costly initial exploration investment needed to develop shale gas. However, when shale gas is as polluting as coal, imposing a climate policy does not favour shale gas: the more stringent climate policy is, the less shale gas is developed (see Figure ??, lower panel).

In the general case, when gas is polluting but less polluting than coal, we show in Appendix ?? that if the price elasticity of electricity demand is small enough[11], the more stringent climate policy, the more shale gas is extracted (see Figure ??, upper panel). The intuition is the following. A more severe climate policy obliges to emit less. There are two solutions to do so: decrease fossil energy consumption by making its price increase, or switch to a less emitting fuel. In the plausible case where the price elasticity of electricity demand is low, only the second option is left. The economy resorts to fuel switching, which means using more shale gas and switching to solar earlier. As we have already noticed, switching to solar is costly, and there are powerful incentives to postpone the switch as much as possible, namely technical progress and discounting. When demand is elastic, the first option is all the more interesting since the carbon content of shale gas is low and shale gas is cheap (see Figure ??, lower panel).

The previous results are summarized in the following Proposition:

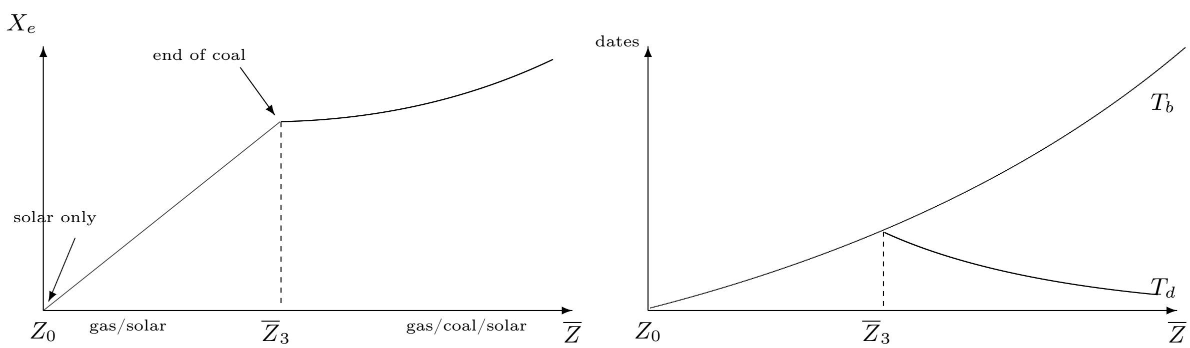
Proposition 2 *Tightening climate policy always brings forward the transition to clean energy. When the local damage is large, it also leads to an increase of the quantity of shale gas developed, at the expense of coal, up to the point where coal is completely phased out; then the quantity of*

[11] The empirical literature shows that this is actually the case. See Alberini *et al.* (2011), Table 1 pp. 871, for a survey of recent estimates of price elasticities of residential electricity consumption.



Quantity of shale gas X_e Dates T_d and T_b

$\theta_e = 0$ or small price elasticity of demand



Quantity of shale gas X_e Dates T_d and T_b

$\theta_e = \theta_d$ or large price elasticity of demand

Figure 2: Strengthening climate policy; small local damage

shale gas decreases. When the local damage is small, it may on the contrary lead to reduce the quantity of shale gas developed, if demand is elastic or the advantage of shale gas over coal in terms of carbon emissions is not large enough.

4 A moratorium on shale gas extraction

Shale gas has been advocated as a bridge fuel to smooth the transition from polluting coal to emission-free renewable energy. One of the main question that arises is whether the extraction of shale gas should be used to buy time to make the more arduous shift to even cleaner forms of energy, or if its use does not justify postponing the transition to clean energy. To answer this question we compare the optimal energy transition analyzed above with an energy transition constrained by a moratorium on shale gas exploitation, for a given climate policy. Under the moratorium, the planner is left with two options for electricity generation: coal and solar energy. The solution obtained is of course sub-optimal. The moratorium imposes a cost on society in terms of intertemporal welfare[12].

4.1 The timing of the switch to clean energy: moratorium vs optimum

If there is a moratorium on shale gas extraction, then only coal is used until the ceiling is reached. The extraction path is the same as described in Dasgupta *et al.* (1982), with a fixed quantity $\overline{Z}/\theta_d$ of coal used.

For a given date T_b, the social planner's program reads:

$$\widetilde{V}(T_b) = \begin{cases} \max\limits_{x_d(t),x_b(t)} \left\{ \int_0^\infty e^{-\rho t} \left[u\left(x_d(t) + x_b(t)\right) - c_d x_d(t) - c_b x_b(t) \right] dt - F(T_b) e^{-\rho T_b} \right\} \\ \text{subject to } \int_0^\infty x_d(t) dt \leq \frac{\overline{Z}}{\theta_d} \end{cases}$$

For a given T_b, the constrained optimal path is characterized by the following properties (see Dasgupta *e*t al., 1982). First, it is never optimal to develop solar before its date of use, so that the carbon budget $\overline{Z}$ is reached at date T_b. Prior to innovation, for $t \leq T_b$, only coal is used and

[12] Note nevertheless that it leads to the optimal solution in the case where the development of shale gas is actually not optimal, that is when the local damage is large and climate policy lenient (more precisely, $\overline{Z} > \overline{Z}_2$). In this case, the moratorium is inconsequential.

its price is equal to $c_d + \theta_d \mu_0 e^{\rho t}$. After the innovation, the price is equal to the marginal cost of producing solar c_b.

The optimal date of the switch to clean energy solves:

$$\max_{T_b} \widetilde{V}(T_b)$$

We show in Appendix **??** that $\widetilde{V}(T_b)$ has a unique maximum $\widetilde{T}_b$ which satisfies:

$$\pi_b - \left[u\left(x_d(\widetilde{T}_b)\right) - (c_d + \theta_d \mu_0 e^{\rho \widetilde{T}_b}) x_d(\widetilde{T}_b)\right] = \rho F(\widetilde{T}_b) - F'(\widetilde{T}_b) > 0 \quad (24)$$

There is a jump in prices at the (unique) optimal innovation date.

We now compare the optimal switch to clean energy at the optimum and under the moratorium.

4.1.1 Large local damage

We show that in this case, the clean substitute always arrives sooner with a moratorium, implying that shale gas is used to postpone the costly switch to clean renewables.

By definition of T_b^* and $\widetilde{T}_b$,

$$\left.\frac{\partial \widetilde{V}(T_b)}{\partial T_b}\right|_{T_b^*} > 0 \Leftrightarrow \widetilde{T}_b > T_b^*$$

We show in Appendix **??** that it is the case if and only if $c_e + d + (\lambda_0^* + \theta_e \mu_0^*) e^{\rho T_b^*} > c_d + \theta_d \widetilde{\mu}_{0,T_b^*} e^{\rho T_b^*}$, where $\widetilde{\mu}_{0,T_b^*}$ denotes the shadow price of carbon in the moratorium case when the switch to solar occurs at date T_b^*, and other notations are straightforward. This inequality means that the moratorium postpones the clean transition if and only if the price of energy at the optimal date of the switch to solar T_b^* is lower with a moratorium than without. It would imply that the whole price path would be lower under the moratorium. This cannot be the case. Otherwise, extraction would be higher at each date with a moratorium than without, which contradicts the fact that the ceiling $\overline{Z}$ should not be violated in both cases.

As a conclusion, in the case of a large local damage we necessarily have: $\widetilde{T}_b < T_b^*$.

4.1.2 Small local damage

We show that in this case, the switch to solar does not always occur sooner with a moratorium, implying that shale gas should not necessarily be used to postpone the costly switch to clean renewables. In some cases, shale gas is only used to consume more energy at each date.

If there is a moratorium on shale gas extraction, then only coal is used until the ceiling is reached, in fixed quantity $\overline{Z}/\theta_d$.

Again, wer have:

$$\left.\frac{\partial \widetilde{V}(T_b)}{\partial T_b}\right|_{T_b^*} > 0 \Leftrightarrow \widetilde{T}_b > T_b^*$$

We show in Appendix **??** that:

$$\left.\frac{\partial \widetilde{V}(T_b)}{\partial T_b}\right|_{T_b^*} > 0 \Leftrightarrow \widetilde{\mu}_{0,T_b^*} < \mu_0^*$$

As μ_0 is the social cost of carbon, this inequality means that the climate constraint has to be "more difficult" to satisfy at the optimum than in the moratorium case.

Let us look at three extreme cases: gas is not polluting at all, gas is as polluting at coal, demand is inelastic.

Assume first that $\theta_e = 0$ and that $\widetilde{\mu}_{0,T_b^*} < \mu_0^*$. Then more coal is extracted between dates T_d and T_b^* in the moratorium case than at the first best. Before T_d, only gas in extracted at the first best. As we have assumed that $\theta_e = 0$, this entails no carbon emissions. In the moratorium case, between 0 and T_d, coal is consumed and carbon is emitted. As a result, emissions are higher at all dates between 0 and T_b^* in the moratorium case than at the first best. This contradicts the fact that the ceiling $\overline{Z}$ should not be violated in both cases. Hence, if $\theta_e = 0$, $\widetilde{T}_b < T_b^*$.

Assume now that $\theta_e = \theta_d$ and that $\widetilde{\mu}_{0,T_b^*} > \mu_0^*$. Then less coal is extracted between dates T_d and T_b^* in the moratorium case than at the first best. By definition of date T_d, the prices of gas and coal are equal at this date, which implies: $c_e + d - c_d = (\theta_d \mu_0^* - (\lambda_0^* + \theta_e \mu_0^*))\, e^{\rho T_d} < 0$. This in turn implies: $c_e + d - c_d < \theta_d \mu_0^* - (\lambda_0^* + \theta_e \mu_0^*) < 0$. If moreover we have $\widetilde{\mu}_{0,T_b^*} > \mu_0^*$, then for all t in $[0, T_d]$, $c_e + d + (\lambda_0^* + \theta_e \mu_0^*) e^{\rho t} < c_d + \theta_d \widetilde{\mu}_{0,T_b^*} e^{\rho t}$. This last equation implies that extraction of coal in the moratorium case is lower than extraction of gas at the optimum between dates 0 and T_d. Together with $\theta_e = \theta_d$, this gives that there are more carbon emissions between 0 and T_d also at the optimum than in the moratorium. This contradicts the fact that the ceiling $\overline{Z}$ should not be violated in both cases. So that, if $\theta_e = \theta_d$, $\widetilde{T}_b > T_b^*$.

If the price elasticity of demand is nil, then $\widetilde{\mu}_0 > \mu_0^*$. Otherwise, the overall demand would be higher on the moratorium extraction path and energy consumption would be more polluting.

This contradicts the fact that the ceiling $\overline{Z}$ should not be violated in both cases. This result can be generalized to a small enough price elasticity of demand (see Appendix **??**).

The previous results are summarized in the following Proposition:

Proposition 3 *If the local damage is large, a moratorium on shale gas exploitation always brings forward the transition to clean energy, compared to the optimum. If the local damage is small, the moratorium only brings forward the transition to clean energy if shale gas is clean enough compared to coal, or if the price elasticity of electricity demand is low. Otherwise, the moratorium can actually postpone the transition to clean energy.*

The intuition behind these results is the following. There are two reasons why one could want to extract shale gas. The first one is that it is cheaper than coal and the second one is that it is less polluting. In the case of a large local damage, shale gas is not actually cheaper than coal, so that its only advantage is that it is less polluting. The only reason to use shale gas is thus to buy time to decrease the cost of the switch to clean energy, by the combined effects of discounting and technical progress. It is actually optimal to do so. Hence, with a moratorium on shale gas, the switch to solar occurs sooner than without. Things are quite different when the local damage is small. Then, shale gas is cheaper than coal, and one may want to use it in order to consume more energy, even absent any climatic constraint. This incentive introduces a new effect that plays in the opposite direction, and is all the stronger since the price elasticity of demand is high. Then, if the cost of shale gas is small enough and the price elasticity of demand high enough, extracting shale gas leads to an increase in energy use and a ceiling reached more rapidly. The switch to clean energy happens sooner whitout a moratorium than with it.

4.2 Optimal ceiling: moratorium vs optimum

Let us now assume that climate change induces damages which depends on the maximum amount of carbon concentration. The damage function is thus $H(\overline{Z})$, increasing and convex. This damage function can be justified on two grounds. The first reason to think that this damage function is well suited to describe the climate change impacts that are taken into account by a social planner is that IPCC reports as well as international negociations focus on the determination of the "carbon budget", i.e. the amount of carbon dioxyde that can be emitted if we are to have a likely chance of averting the most dangerous of climate change impacts[13]. The carbon budget seems to be a good indicator to describe climate change impacts. Second, one can think of $H(\overline{Z})$ as an adaptation cost. For instance, the height of the dikes that should be built because of climate change depends on the maximum sea level rise, which in turns only depends on the maximum amount of pollution in the atmosphere. Assuming that the adaptation cost $H(\overline{Z})$ is paid upfront, because dikes are built once for all, this means that below $\overline{Z}$ there are no damages from climate change, whereas above $\overline{Z}$, most areas will be rapidly flooded, inducing catastophic damages. This cost can also include other adaptation measures such as for instance purchase of air conditioning or anticipary migration. The last reason for choosing this damage function is that it simplifies greatly the algebra.

The planner chooses the optimal ceiling, or optimal carbon budget. The first order condition with respect to the ceiling at the optimum gives:

$$H'(\overline{Z}) = \mu_0^* \tag{25}$$

We wonder here whether the optimal damage is higher with a moratorium than at the optimum. It amounts to comparing the shadow cost of pollution under a ceiling constraint, in the moratorium case and at the optimum.

4.2.1 Large local damage

In the case of a large local damage, we show that the moratorium leads the planner to choose higher global damages. This comes from the fact that it is more difficult to stay below a given ceiling in the moratorium case, because gas is not available to replace coal. The proof is in Appendix **??**.

4.2.2 Small local damage

In this case it is straightforward that the moratorium has the same effect on the optimal ceiling than on the optimal arrival date of clean energy. With a moratorium constraint, the social

[13] The 2015 Paris Agreement includes a two-headed temperature goal: "holding the increase in the global average temperature to well below 2°C above pre-industrial levels and pursuing efforts to limit the temperature increase to 1.5°C". The impacts associated with these two carbon targets have been studied recently in Schleussner *et* al. (2016).

planner chooses a ceiling that may be higher or lower than at the optimum. This result may seem counterintuitive. In the case of small local damages, shale gas is not necessarily used because it is less polluting than coal, but also because it is cheaper. To get a better understanding, note that it may be the case that emissions induced by shale gas in optimum at some date, are higher than the pollution that would happen with coal at price c_d, because shale gas is cheaper than coal. As a result, forbidding the use of a cheap polluting resource may decrease the shadow cost of pollution, which reflects the difficulty to comply with the constraint $Z \leq \overline{Z}$, compared to a case without this same constraint.

The previous results are summarized in the following Proposition:

Proposition 4 *If the local damage is large, a moratorium on shale gas exploitation always leads to higher global damages, compared to the optimum. If the local damage is small, the moratorium only lead to higher global damages if shale gas is clean enough compared to coal, or if the price elasticity of electricity demand is low. Otherwise, the moratorium can actually lead to lower global damages.*

5 Simulations

We perform in this section illustrative simulations. We use standard functional forms: a quadratic utility function, a solar R&D cost decreasing at a constant rate due to exogenous technical progress, and a quadratic shale gas exploration cost:

$$
\begin{aligned}
u(x) &= ax - \frac{b}{2}x^2 \Longrightarrow D(p) = \frac{a-p}{b} \\
F(t) &= F_0 e^{-\gamma t} \\
E(X_e) &= \frac{\varepsilon}{2} X_e^2
\end{aligned}
$$

We calibrate the model as far as possible to the European case, making the assumption that the unit costs of the three energy sources in electricity generation are equivalent in the US and in Europe, and that the marginal cost of shale gas exploration and development would be the same in Europe as in the US.

5.1 Calibration

Unit costs c_d, c_e and c_b are in \$/MWh, and are drawn from the US levelized cost of electricity from EIA (2014a), see Table **??**.

Emission coefficients θ_d and θ_e are in tCO_2eq/kWh and come from Heath *et al.* (2014), see Table **??**.

The exogenous rates of discounting and technical progress on the cost of R&D are arbitrarily[14] taken equal to $\rho = 0.02$ and $\gamma = 0.03$.

The initial carbon concentration in the atmosphere is $Z_0 = 400$ ppm, which amounts[15] to 3120 10^9 tCO_2. According to the IPCC SRES scenarii[16], around 50% of total emissions is projected to come from electricity generation. Around 11% of the greenhouse gases emitted worldwide in 2012 come from the European Union. Hence other things being equal, increasing total atmospheric carbon concentration by 150 ppm to reach 550 ppm CO_2 (i.e. reaching a 3°C target) corresponds to a European sectoral ceiling in electricity generation of $\overline{Z} = Z_0 + 150 * 0.5 * 0.11 = 408$ ppm $= 3183$ 10^9 tCO_2.

The fixed cost of developing a clean technology at date 0, F_0, is assumed to be the investment necessary to solve the intermittence problem inherent to renewable energy such as solar energy and wind power (for instance, large scale electricity storage device and enhanced electric grid). This investment is calibrated using the French Environment and Energy Management Agency report[17] (ADEME, 2015). This cost is the sum of the network capacity cost, the network fixed cost, the electricity storage system and pumped storage power stations costs. It amounts to 17 329 Million €/year. With $\rho = 2\%$, $F_0 = 17\ 329/0.02 \simeq 866.45\ 10^9$ \$.

Demand is calibrated using the assumptions that:

- absent climate policy, electricity is produced by coal-fired power plants; hence $p = c_d = 95.6$ \$/MWh;
- the price elasticity of demand at this price is taken equal to 0.25 (see Alberini *et al.*, 2011).

[14] Sensitivity analysis around $\rho = 0.02$ and $\gamma = 0.03$ show that the results do not change significantly.

[15] Using the fact that 1 ppmv = 2.13 GtC = 2.13*3.664 $GtCO_2$ = 7.8 $GtCO_2$.

[16] http://www.ipcc.ch/ipccreports/sres/emission/index.php?idp=118#533

[17] www.ademe.fr/sites/assets/documents/rapport100enr_comite.pdf.

See Table 4 in the Appendix of the report.

Hence $a = \frac{1.25}{0.25} * 95.6 = 478$.

According to the World Development Indicators 2015, consumption per capita of electric power in the Euro area in 2011 is 6.5 MWh and the population of the Euro area in 2011 is 337 Million. This gives $b = 0.174\ 10^{-6}$. Note that the elasticity of demand is not constant, and is equal to -0.45 for a price of 150$, and -0.14 for a price of 60$.

To calibrate the marginal cost of shale gas exploration, we use data on US shale wells:

- The US shale gas production is given by the EIA Natural Gas Weekly Update[18]. We get monthly data from Jan. 2000 to Feb. 2015 for the major shale gas plays in billion cubic feet/day. We convert the data in MWh, take the average over the period Jan. 2008–Feb. 2015 and multiply by 365 to obtain an average annual production of the major plays in MWh. The four most productive plays are Marcellus (PA & WV), Haynesville (LA & TX), Fayetteville (AR) and Barnett (TX).

- We consider that the total cost of shale gas use in electricity generation is $E(X_e) + c_e X_e$. The corresponding marginal cost is then $E'(X_e) + c_e$ i.e., according to our specifications, $\varepsilon X_e + c_e$. We obtain this cost from Sandrea (2014), which gives the HH price[19] of US plays in $/Mcf. We sort the previous four shale gas plays by increasing HH price and cumulate the corresponding productions and obtain the parameters of the marginal cost function.

We obtain $\varepsilon = 0,051\ 10^{-9}$.

We check that the amount of shale extracted in a reference scenario (i.e. without any ceiling constraint) is consistent with data on shale gas reserves in urope. According to EIA, Europe is estimated to have 615 trillion cubic feet of technically recoverable resources of shale gas (see EIA, 2013) i.e. $180\ 10^9$ MWh. With the previous calibration, for $d = 0$ (no local damage of shale gas) and $\overline{Z} \to \infty$ (no climate policy) we get $X_e = 160\ 10^9$ MWh. The order of magnitude is correct: absent environmental externalities, if the levelized cost of producing electricity with shale gas is lower than the one with coal, it is optimal to substitute shale gas to coal at the beginning of

[18]http://www.eia.gov/naturalgas/weekly/

[19]Fig. 1b p.4, "Basin Economics for various US plays (single well) shale gas" gives the current HH price for different plays (the Henry Hub price is the pricing point for natural gas futures contracts traded on the New York Mercantile Exchange and the OTC swaps traded on Intercontinental Exchange).

the horizon, and the quantity of shale gas that will be extracted is exactly equal to the stock available under the ground.

The parameters used for the simulations are given in Table **??**.

c_d	c_e	c_b	F_0	θ_d	θ_e	ρ	γ	ε	a	b	Z_0
95.6	66.3	130	866.45 10^9	0.98	0.47	0.02	0.03	0.051 10^{-9}	478	174 10^{-9}	3120 10^9

Table 5: Calibration parameters

5.2 Reference scenario

We suppose that the European sectoral ceiling in electricity generation is $\overline{Z} = 408$ ppm = 3183 10^9 tCO_2 (see above).

In the case of a large marginal local damage, taken equal to 3/4 of the unit cost of shale gas: $d = 66.3 * 3/4 = 26.52$ \$/MWh, it is optimal to switch from coal to shale gas in $T_e = 30$ years, and from shale gas to solar in $T_b = 34$ years. Very few shale gas is extracted: we obtain $X_e = 7.8$ 10^9 MWh, whereas technically recoverable resources of shale gas in Europe are estimated to 138 10^9 MWh; hence only 5.7% of the total resources are developed. The price path is represented on Figure **??**.

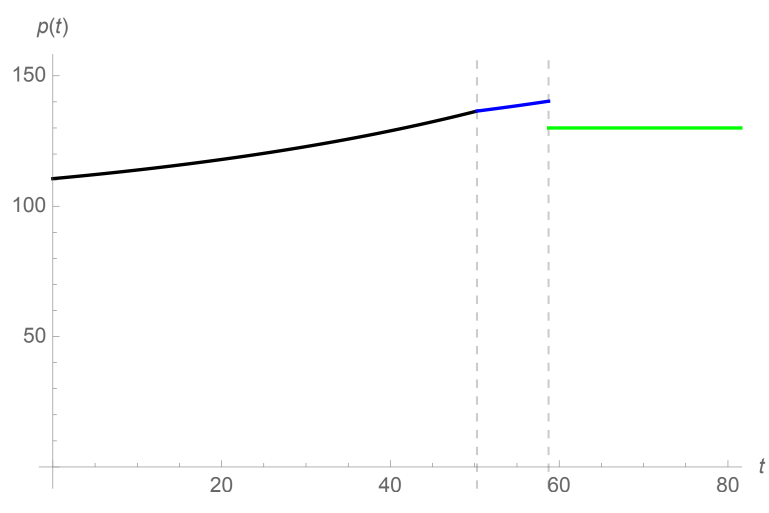


Figure 3: Price path in the reference scenario when the marginal local damage is large (black=coal, blue=shalegas, green=solar)

In the case of a small marginal local damage, that we take by symmetry equal to 1/4 of the unit cost of shale gas: $d = 66.3 * (1/4) = 16.575$ \$/MWh. For this level of damage coal is completely evicted by shale gas. The solution is thus extremely sensitive to the magnitude of the marginal local damage. When the marginal damage is small, it is basically optimal to develop all European shale gas reserves, and to substitute shale gas to coal right now. The transition to solar energy will take place in about 60 years[20]. When the marginal local damage is high, the quantity of shale gas developed as well as the date of the switch to solar decrease rapidly when the damage increases.

5.3 The trade-off between local and global damages

Figure **??** shows iso–X_e curves in the plane $(\overline{Z}, d)$. For the parameters given above, the local marginal damage is small if $d < c_d - c_e = 29.3$, large otherwise. Follow for instance the iso–X_e curve for $X_e = 100$ from the right to the left. First, the climate constraint is lenient and the local damage small. Shale gas is used first in electricity generation, then coal then solar. As we move to the left on Figure **??**, the same quantity of shale gas developed corresponds to a more and more stringent climate constraint and an increasing level of the local damage. The quantity of coal used is lower and lower and the switch to solar occurs earlier and earlier. Coal is progressively evicted by solar. When the local damage becomes larger than the threshold value of 29.3, materialized on Figure **??** by the horizontal dotted line, coal becomes used first in electricity generation, now before shale gas. When the threshold $\overline{Z}_1$ is met, coal is completely evicted, and the economy switches directly from shale gas to solar.

5.4 A moratorium on shale gas development

We have shown in section **??** that the moratorium could bring forward or postpone the switch to solar energy when the local damage is small. We perform simulations of the moratorium using the parameters of the reference scenario (given in Table **??**) and considering a small local damage $0 < d < c_d - c_e$. We find that the moratorium always brings forward the transition to clean energy, for all possible values of the (small) local damage. For a local damage of $d = 16.575$\$/MWh

[20]Remember the carbon budget we consider in the simulations corresponds to an average increase of temperature of 3°C. A target of 2°C is already impossible to satisfy.

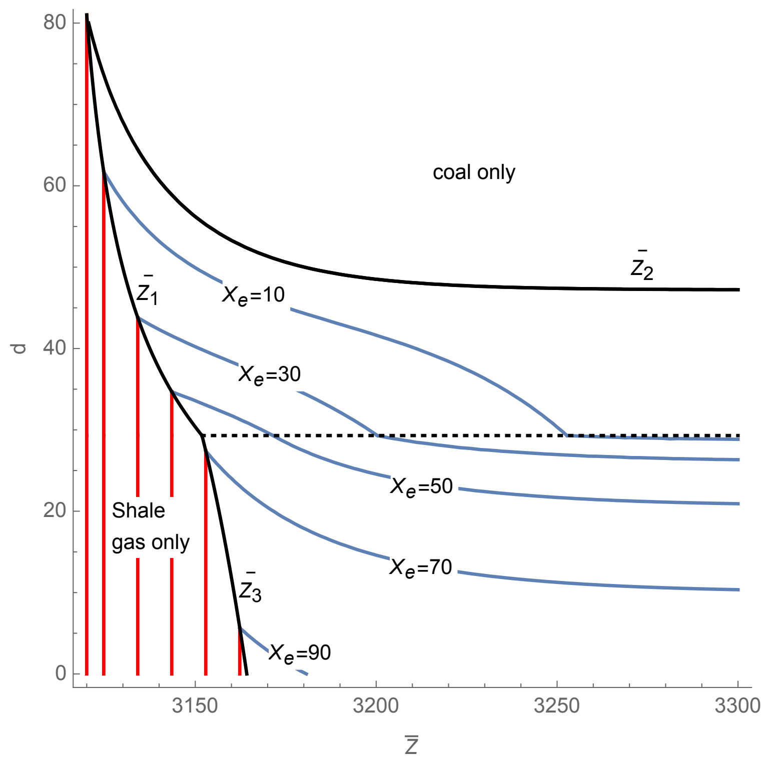


Figure 4: Iso-X_e lines

and $\theta_e = 0.47$, we obtain that the switch to solar occurs 31 years earlier with a moratorium and intertemporal welfare decrease by 33.5%. The negative effect of the moratorium is thus massive. The quantity of shale gas developed in optimum is large, so that the moratorium induces large effects.

With this value of the local damage, the moratorium would have the counterintuitive effect of postponing the switch to clean energy only if $\theta_e \geq 0.99\theta_d$. We perform another simulation with a higher elasticity of demand, $\epsilon = 1$ (evaluated at price c_d) . This gives $a = 191.2$ and $b = 43$ 10^{-9}. Using the other parameters of the reference scenario (given in Table **??**) and considering a small local damage $0 < d < c_d - c_e$, we also find that the moratorium always brings forward the transition to clean energy, for all possible values of the (small) local damage. For a local damage of $d = 16.575$$ / MWh, the moratorium postpones the switch to clean energy only if $\theta_e \geq 0.93\theta_d$, keeping everything else equal. These simulations show that, according to our calibration of the European case, the moratorium is very unlikely to have counterintuitive effects on the date of switch to clean energy or on the optimal ceiling. The date of switch would be postponed and the optimal ceiling would be higher with a moratorium only if shale gas happened to be almost as polluting as coal and/or if the elasticity of demand was very large.

For a large local damage $d = 66.3 * (3/4)$, we obtain that the switch to solar occurs 2 years earlier with a moratorium energy expenditures increase by 1.8% and intertemporal welfare decrease by 3.6%. As the quantity of shale gas optimally developed for this level of the damage is very small, the effect of the moratorium is very moderate.

6 Conclusion

This paper has explored one particular aspect of the complex problem posed by unconventional gas: does climate policy justify developing more shale gas, and what is the consequence for the switch to clean energy? We have developed a model whose assumptions are appropriate to study this question, but do not allow us to address other aspects, among which two seem particularly important.

First, the economy we consider here is a closed economy, which makes it impossible to study the potential leakage effect of an asymmetric climate policy. In a companion paper (Daubanes *et al.,* 2016), we consider an open economy with two zones, one producing coal and shale gas and implementing a carbon ceiling constraint, the other one producing coal only and having no climate policy. Coal production of the first zone may be exported to the other one. We address the following questions. (1) Faced with a more stringent climate constraint, should the shale gas producing economy increase its gas production? (2) Does this strategy decrease or increase global emissions?

Second, our model is a partial equilibrium of the electricity sector. However, shale gas supporters in the US put forward that it has allowed to create jobs, relocate some manufacturing activities, lower the vulnerability to oil shocks, and impact positively the external balance (IMF, 2014). Hence, the general equilibrium effects of shale gas exploitation should be analyzed.

Some other aspects of the shale gas question are worth studying, among which, in no particular order: the reasons why in France, not only the *exploitation* of shale gas is banned, but also the *exploration* of potential reserves; the impact of the subsoil property rights regimes on the decision to develop shale gas; the NIMBY effects of shale gas extraction in densely populated areas; etc. The question of the value of local damages associated with extraction should also receive attention, as this value may not be exogenous but instead depend on the investment in technology to reduce local damages. These aspects are left for future research.

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Appendix

A Optimal switch to clean energy

A.1 Large local damage

In this case, coal is used first, and then shale gas. Hence gas is used just before the switch to solar. Using the envelope theorem, the marginal benefit of delaying innovation can be written as:

$$\begin{aligned}\frac{\partial V(T_b)}{\partial T_b}e^{\rho T_b} &= \left[u\left(x_e(T_b)\right) - (c_e + d + (\lambda_0 + \theta_e\mu_0)e^{\rho T_b})x_e(T_b)\right] - \left[u\left(x_b\right) - c_b x_b\right] - (F'(T_b) - \rho F(T_b)) \\ &= \pi_e(T_b) - \pi_b + (\rho F(T_b) - F'(T_b)) \qquad (26)\end{aligned}$$

Functions $T_b \to (\rho F(T_b) - F'(T_b))$ and $T_b \to \pi_e(T_b)$ are decreasing with T_b. It follows the assumptions made on $F(.)$ for the first one ($F''(.) < 0$). For the second one, we have:

$$\begin{aligned}\pi_e'(T_b) &= \left[u'\left(x_e(T_b)\right) - (c_e + d + (\lambda_0 + \theta_e\mu_0)e^{\rho T_b})\right]\frac{\partial x_e(T_b)}{\partial T_b} - \frac{\partial(\lambda_0 + \theta_e\mu_0)e^{\rho T_b}}{\partial T_b}x_e(T_b)\\ &= -\frac{\partial(\lambda_0 + \theta_e\mu_0)e^{\rho T_b}}{\partial T_b}x_e(T_b) < 0\end{aligned}$$

as the final price of shale gas $c_e + d + (\lambda_0 + \theta_e\mu_0)e^{\rho T_b}$ increases with T_b. Hence $\frac{\partial V(T_b)}{\partial T_b}e^{\rho T_b}$ is continuous and decreases with T_b. It is positive for T_b such that $x_e(T_b) = x_b$[21] and strictly negative when T_b goes to $+\infty$.

As a result, $V(T_b)$ has a unique maximum T_b^* which satisfies:

$$\pi_b - \pi_e(T_b^*) = \rho F(T_b^*) - F'(T_b^*)$$

Functions $T_b \to (\rho F(T_b) - F'(T_b))$ being decreasing with T_b and function $T_b \to \pi_b - \pi_e(T_b)$ increasing with T_b, $T_b^* \geq 0$ if and only

$$\rho F(0) - F'(0) \geq \pi_b - \pi_e(0)$$

A.2 Small local damage

The same reasoning applies, except that in this case, shale gas is used first, then coal. Hence coal is used just before the switch to solar occurs. $V(T_b)$ has a unique maximum T_b^* which satisfies:

$$\pi_b - \pi_d(T_b^*) = \rho F(T_b^*) - F'(T_b^*) > 0$$

$T_b^* \geq 0$ if and only

$$\rho F(0) - F'(0) \geq \pi_b - \pi_d(0)$$

B Thresholds

B.1 Large local damage

If shale gas is used alone, and coal is left under the ground, then the values of λ_0, μ_0, T_b and X_e must solve the system composed of equations (**??**), (**??**), (**??**) and

$$\theta_e X_e = \overline{Z} - Z_0 \tag{27}$$

[21] T_b such that $x_e(T_b) = x_b$ is the lowest possible T_b satisfying that solar production begins at the date of innovation (see Dasgupta *et al.*, 1982).

which replaces (**??**). Moreover, to ensure that there exists no incentive to introduce coal at date 0, the initial price of shale gas $p_e(0)$ must be below the initial price of coal, $p_d(0)$, i.e. we must have

$$(\theta_d - \theta_e)\mu_0 \geq c_e + d - c_d + E'(X_e) \tag{28}$$

If the solution of the above system is such that this condition is satisfied, then shale gas is used alone to get to the ceiling. There exists a threshold value of the ceiling $\overline{Z}_1$ under which only shale gas is used. It is solution of the system composed of equations (**??**), (**??**), (**??**), (**??**) and (**??**), this last equation being taken as an equality.

If coal is used alone to get to the ceiling, then the values of μ_0 and T_b must solve the following system:

$$\theta_d \int_0^{T_b} x_d(t)dt = \overline{Z} - Z_0 \tag{29}$$

$$[u(x_b) - c_b x_b] - \left[u(x_d(T_b)) - (c_d + \theta_d \mu_0 e^{\rho T_b}) x_d(T_b)\right] = \rho F(T_b) - F'(T_b) \tag{30}$$

where equation (**??**) is the combination of equations (**??**) and (**??**) for $X_e = 0$, and equation (**??**) is equation (**??**) in the case $X_e = 0$. Moreover, we must make sure that there is no incentive to extract shale gas: the final price of coal $p_d(T_b)$ must be lower than the price of the first unit of shale gas that could be extracted at date T_b, $c_e + d + \theta_e \mu_0 e^{\rho T_b}$. Hence we must have:

$$(\theta_d - \theta_e)\mu_0 e^{\rho T_b} \leq c_e + d - c_d \tag{31}$$

meaning that the marginal gain in terms of pollution of switching from coal to shale gas, evaluated at the carbon value at date T_b, is smaller than the marginal cost of the switch. If the solution of the above system is such that this condition is satisfied, then shale gas is never extracted. There exists a threshold value of the ceiling $\overline{Z}_2$, such that if $\overline{Z} \geq \overline{Z}_2$ shale gas is not developed. $\overline{Z}_2$ is solution of the system composed of equations (**??**), (**??**) and (**??**), this last equation being written as an equality.

For an intermediate ceiling $\overline{Z}$ such that $\overline{Z}_1 < \overline{Z} < \overline{Z}_2$, the three phases exist.

Note that these two thresholds cannot coincide, except if $T_b = 0$, which cannot be the case, by assumption.

B.2 Small local damage

If shale gas is used alone to get to the ceiling, then λ_0, μ_0, T_b and X_e must solve the system composed of equations (??), (??), (??) and:

$$[u(x_b) - c_b x_b] - \left[u(x_e(T_b)) - (c_e + d + (\lambda_0 + \theta_e \mu_0)e^{\rho T_b})x_e(T_b)\right] = \rho F(T_b) - F'(T_b) \tag{32}$$

Moreover, the final price of shale gas $p_e(T_b)$ must be lower than the price of the first unit of coal that could be extracted at date T_b, $p_d(T_b)$, i.e. we must have:

$$(\theta_d - \theta_e)\mu_0 e^{\rho T_b} > c_e + d - c_d + E'(X_e)e^{\rho T_b} \tag{33}$$

meaning that the cost in terms of pollution of switching to coal instead of going directly to solar is higher than the advantage in terms of production costs. It happens for values of the ceiling below $\overline{Z}_3$ defined by (??), (??), (??), (??) and (??) taken as an equality.

For $\overline{Z} > \overline{Z}_3$, the three resources are used.

C The effects of a more stringent climate policy

C.1 Large local damage

In this case, equations (??) and (??) may be written as:

$$\int_{T_e}^{T_b} x_e(t)dt = X_e \tag{34}$$

$$\int_0^{T_e} \theta_d x_d(t)dt + \int_{T_e}^{T_b} \theta_e x_e(t)dt = \overline{Z} - Z_0$$

Using (??), this last equation reads:

$$\int_0^{T_e} x_d(t)dt = \frac{1}{\theta_d}\left(\overline{Z} - Z_0 - \theta_e X_e\right) \tag{35}$$

Totally differentiating system (??), (??), (??), (??) and (??) yields:

$$x_e(T_b)dT_b - x_e(T_e)dT_e + \int_{T_e}^{T_b} dx_e(t)dt = dX_e$$

$$x_d(T_e)dT_e + \int_0^{T_e} dx_d(t)dt = \frac{1}{\theta_d}\left(d\overline{Z} - \theta_e dX_e\right)$$

$$[\theta_d\mu_0 - (\lambda_0 + \theta_e\mu_0)]\,\rho dT_e + (\theta_d - \theta_e)d\mu_0 - d\lambda_0 = 0$$

$$\begin{aligned}
&-\left[u'\left(x_e(T_b)\right) - (c_e + d + (\lambda_0 + \theta_e\mu_0)e^{\rho T_b}\right] dx_e(T_b) + ((d\lambda_0 + \theta_e d\mu_0) + (\lambda_0 + \theta_e\mu_0)\rho dT_b)e^{\rho T_b}x_e(T_b) \\
&= \left(\rho F'(T_b) - F''(T_b)\right) dT_b
\end{aligned}$$

$$d\lambda_0 = E''(X_e)dX_e$$

As

$$\begin{aligned}
x_d(t) = D(p_d(t)) &\Rightarrow dx_d(t) = D'(p_d(t))dp_d(t) = D'(p_d(t))\theta_d e^{\rho t}d\mu_0 \\
x_e(t) = D(p_e(t)) &\Rightarrow dx_e(t) = D'(p_e(t))dp_e(t) = D'(p_e(t))e^{\rho t}\left(d\lambda_0 + \theta_e d\mu_0\right)
\end{aligned}$$

the first two equations read equivalently:

$$x_e(T_b)dT_b - x_e(T_e)dT_e + \left[\int_{T_e}^{T_b} D'(p_e(t))e^{\rho t}dt\right](d\lambda_0 + \theta_e d\mu_0) = dX_e$$

$$x_d(T_e)dT_e + \left[\int_0^{T_e} D'(p_d(t))e^{\rho t}dt\right]\theta_d d\mu_0 = \frac{1}{\theta_d}\left(d\overline{Z} - \theta_e dX_e\right)$$

Besides,

$$\begin{aligned}
&\dot{D}(p_d(t)) = D'(p_d(t))\dot{p}_d(t) = D'(p_d(t))\theta_d\mu_0\rho e^{\rho t} \\
&\Rightarrow \int_0^{T_e} D'(p_d(t))e^{\rho t}dt = \frac{1}{\theta_d\mu_0\rho}\int_0^{T_e}\dot{D}(p_d(t)dt = \frac{1}{\theta_d\mu_0\rho}\left[D(p_d(T_e)) - D(p_d(0)\right] = \frac{x_d(T_e) - x_d(0)}{\theta_d\mu_0\rho}
\end{aligned}$$

and

$$\int_{T_e}^{T_b} D'(p_e(t))e^{\rho t}dt = \frac{x_e(T_b) - x_e(T_e)}{(\lambda_0 + \theta_e\mu_0)\rho}$$

Hence the first two equations read:

$$-x_e(T_e)dT_e + x_e(T_b)dT_b - dX_e + \frac{x_e(T_b) - x_e(T_e)}{(\lambda_0 + \theta_e\mu_0)\rho}(d\lambda_0 + \theta_e d\mu_0) = 0$$

$$x_d(T_e)dT_e + \frac{\theta_e}{\theta_d}dX_e + \frac{x_d(T_e) - x_d(0)}{\mu_0\rho}d\mu_0 = \frac{1}{\theta_d}d\overline{Z}$$

Using the equality between marginal utilities, the fourth equation simplifies, and we obtain easily:

$$A \times \begin{pmatrix} dT_e \\ dT_b \\ dX_e \\ d\lambda_0 \\ d\mu_0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\theta_d} \\ 0 \\ 0 \\ 0 \end{pmatrix} d\overline{Z}$$

with

$$A = \begin{pmatrix} -x_e(T_e) & x_e(T_b) & -1 & \frac{x_e(T_b)-x_e(T_e)}{(\lambda_0+\theta_e\mu_0)\rho} & \theta_e \frac{x_e(T_b)-x_e(T_e)}{(\lambda_0+\theta_e\mu_0)\rho} \\ x_e(T_e) & 0 & \frac{\theta_e}{\theta_d} & 0 & \frac{x_e(T_e)-x_d(0)}{\mu_0\rho} \\ [\lambda_0+(\theta_e-\theta_d)\mu_0]\,\rho & 0 & 0 & 1 & \theta_e-\theta_d \\ 0 & (\lambda_0+\theta_e\mu_0)\rho x_e(T_b)+z_1 & 0 & x_e(T_b) & \theta_e x_e(T_b) \\ 0 & 0 & -z_2 & 1 & 0 \end{pmatrix}$$

where

$$z_1 = -\left(\rho F'(T_b) - F''(T_b)\right) e^{-\rho T_b} > 0$$
$$z_2 = E''(X_e) > 0$$

Hence:

$$\begin{aligned}
&\rho\theta_d\mu_0(\lambda_0+\theta_e\mu_0)\det A \\
&= \theta_d \left[\underbrace{(x_e(T_e)-x_e(T_b))}_{>0} x_d(0)\theta_d\mu_0 + \underbrace{(x_d(0)-x_e(T_e))}_{>0} x_e(T_b)\,(\lambda_0+\theta_e\mu_0) \right] z_1 z_2 \\
&+ \rho \left\{ \left[\underbrace{(\theta_e x_e(T_b)-\theta_d x_d(0))}_{<0} \theta_e\mu_0 - x_d(0)\theta_d\lambda_0 \right] \underbrace{(\lambda_0+(\theta_e-\theta_d)\mu_0)}_{<0} + x_e(T_e)\theta_d\lambda_0^2 \right\} z_1 \\
&+ \rho\theta_d x_d(0) x_e(T_e) x_e(T_b) \theta_d\mu_0(\lambda_0+\theta_e\mu_0) z_2 \\
&+ \rho^2\theta_d(\lambda_0+\theta_e\mu_0)x_e(T_b) \left[x_e(T_e)\lambda_0^2 - x_d(0)(\lambda_0+\theta_e\mu_0) \underbrace{(\lambda_0+(\theta_e-\theta_d)\mu_0)}_{<0} \right]
\end{aligned}$$

i.e. $\det A > 0$.

$$A^{-1} \times \begin{pmatrix} 0 \\ \frac{1}{\theta_d} \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\rho\theta_d\mu_0(\lambda_0+\theta_e\mu_0)\det A} \times$$

$$\begin{pmatrix} \mu_0\left(\lambda_0+\theta_e\mu_0\right)\left[\frac{\theta_d}{\lambda_0+\theta_e\mu_0}\left(x_e(T_e)-x_e(T_b)\right)z_1z_2+\rho z_1(\theta_d-\theta_e)+\rho x_e(T_b)\left(x_e(T_e)z_2\theta_d+\rho(\theta_d-\theta_e)\left(\lambda_0+\theta_e\mu_0\right)\right)\right] \\ -\rho x_e(T_b)\mu_0(\lambda_0+\theta_e\mu_0)\left[-x_e(T_e)\theta_d z_2+\rho\theta_e\underbrace{\left(\lambda_0+(\theta_e-\theta_d)\mu_0\right)}_{<0}\right] \\ -\rho\mu_0\left[-x_e(T_b)z_1\theta_e\underbrace{\left(\lambda_0+(\theta_e-\theta_d)\mu_0\right)}_{<0}+x_e(T_e)\theta_d\lambda_0\left(z_1+\rho x_e(T_b)(\lambda_0+\theta_e\mu_0)\right)\right] \\ -z_2\rho\mu_0\left[-x_e(T_b)z_1\theta_e\left(\lambda_0+(\theta_e-\theta_d)\mu_0\right)+x_e(T_e)\theta_d\lambda_0\left(z_1+\rho x_e(T_b)(\lambda_0+\theta_e\mu_0)\right)\right] \\ -\rho\mu_0\left(\lambda_0+\theta_e\mu_0\right)\begin{bmatrix} \frac{\theta_d\mu_0}{\lambda_0+\theta_e\mu_0}(x_e(T_e)-x_e(T_b))z_1z_2+x_e(T_b)z_1z_2-\rho z_1\left(\lambda_0+(\theta_e-\theta_d)\mu_0\right) \\ -\rho x_e(T_b)\left[-x_e(T_e)\theta_d\mu_0z_2+\rho(\lambda_0+\theta_e\mu_0)\left(\lambda_0+(\theta_e-\theta_d)\mu_0\right)\right] \end{bmatrix} \end{pmatrix}$$

As $\det A > 0$, we deduce:

$$\frac{\partial T_e}{\partial \overline{Z}} > 0, \quad \frac{\partial T_b}{\partial \overline{Z}} > 0, \quad \frac{\partial X_e}{\partial \overline{Z}} < 0, \quad \frac{\partial \lambda_0}{\partial \overline{Z}} < 0, \quad \frac{\partial \mu_0}{\partial \overline{Z}} < 0$$

C.2 Small local damage

In this case, equations (**??**) and (**??**) may be written as:

$$\int_0^{T_d} x_e(t)dt = X_e \tag{36}$$

$$\int_{T_d}^{T_b} x_d(t)dt = \frac{1}{\theta_d}\left(\overline{Z}-Z_0-\theta_e X_e\right) \tag{37}$$

Totally differentiating system (**??**), (**??**), (**??**), (**??**) and (**??**) yields:

$$x_e(T_d)dT_d + \frac{x_e(T_d)-x_e(0)}{(\lambda_0+\theta_e\mu_0)\rho} = dX_e$$

$$x_d(T_b)dT_b - x_d(T_d)dT_d + \frac{x_d(T_b)-x_d(T_d)}{\theta_d\mu_0\rho} = \frac{1}{\theta_d}\left(d\overline{Z}-\theta_e dX_e\right)$$

$$-((d\lambda_0+\theta_e d\mu_0)+(\lambda_0+\theta_e\mu_0)\rho dT_d)e^{\rho T_d}x_e(T_d)+\theta_d(d\mu_0+\mu_0\rho dT_d)e^{\rho T_d}x_d(T_d)=0$$

$$\theta_d(d\mu_0+\rho dT_b)e^{\rho T_b}x_d(T_b) = \left(\rho F'(T_b)-F''(T_b)\right)dT_b$$

$$d\lambda_0 = E''(X_e)dX_e$$

Using $x_e(T_d) = x_d(T_d)$, we obtain:

$$A \times \begin{pmatrix} dT_d \\ dT_b \\ dX_e \\ d\lambda_0 \\ d\mu_0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\theta_d} \\ 0 \\ 0 \\ 0 \end{pmatrix} d\overline{Z}$$

with

$$A = \begin{pmatrix} x_d(T_d) & 0 & -1 & \frac{x_d(T_d)-x_e(0)}{(\lambda_0+\theta_e\mu_0)\rho} & \theta_e\frac{x_d(T_d)-x_e(0)}{(\lambda_0+\theta_e\mu_0)\rho} \\ -x_d(T_d) & x_d(T_b) & \frac{\theta_e}{\theta_d} & 0 & \frac{x_d(T_b)-x_d(T_d)}{\mu_0\rho} \\ \left[-\theta_d\mu_0 + (\lambda_0+\theta_e\mu_0)\right]\rho & 0 & 0 & 1 & -(\theta_d-\theta_e) \\ 0 & y_1 & 0 & 0 & \theta_d x_d(T_b) \\ 0 & 0 & -E''(X_e) & 1 & 0 \end{pmatrix}$$

where

$$y_1 = -\left(\rho F'(T_b) - F''(T_b)\right)e^{-\rho T_b} + \rho x_d(T_b)\theta_d\mu_0 > 0$$

Let's denote

$$y_2 = E''(X_e)\left[x_d(T_d)\theta_d\mu_0 + x_e(0)\left(\lambda_0 + (\theta_e-\theta_d)\,\mu_0\right)\right]$$

According to (**??**), we have:

$$\lambda_0 + (\theta_e - \theta_d)\,\mu_0 = (c_d - (c_e + d))\,e^{-\rho T_d} > 0$$

which implies that y_2 is also positive.

We have

$$\begin{aligned}
&-\rho\theta_d\mu_0(\lambda_0+\theta_e\mu_0)\det A \\
&= \rho x_d(T_b)^2\theta_d^2\mu_0\left\{\rho(\lambda_0+\theta_e\mu_0)(\lambda_0+(\theta_e-\theta_d)\mu_0) + E^{''}(X_e)\left[x_d(T_d)\theta_d\mu_0 + x_e(0)(\lambda_0+(\theta_e-\theta_d)\mu_0)\right]\right\} \\
&+ y_1\rho\left\{x_d(T_d)\theta_d\lambda_0^2 + x_e(0)\theta_e^2\mu_0(\lambda_0+(\theta_e-\theta_d)\mu_0) - x_d(T_b)\theta_d(\lambda_0+\theta_e\mu_0)(\lambda_0+(\theta_e-\theta_d)\mu_0)\right\} \\
&+ y_1 E^{''}(X_e)\theta_d\left\{x_e(0)(\lambda_0+\theta_e\mu_0)\left(x_d(T_d)-x_d(T_b)\right) + x_d(T_b)\theta_d\mu_0(x_e(0)-x_d(T_d))\right\}
\end{aligned}$$

It is straightforward that the terms of the first and third lines are positive. Let look at the term of the second line:

$$y_1\rho\left\{x_d(T_d)\theta_d\lambda_0^2 + x_e(0)\theta_e^2\mu_0(\lambda_0 + (\theta_e - \theta_d)\mu_0) - x_d(T_b)\theta_d(\lambda_0 + \theta_e\mu_0)(\lambda_0 + (\theta_e - \theta_d)\mu_0)\right\}$$

Dividing by $y_1\rho > 0$, it has the sign of:

$$\begin{aligned}&\lambda_0^2(\theta_d x_d(T_d) - \theta_d x_d(T_b))\\ &+ \lambda_0\mu_0(\theta_e^2 x_e(0) + \theta_d^2 x_d(T_b) - 2\theta_e\theta_d x_d(T_b))\\ &+ \mu_0^2\theta_e(\theta_e^2 x_e(0) + \theta_d^2 x_d(T_b) - \theta_e\theta_d x_d(T_b) - \theta_e\theta_d x_e(0))\end{aligned}$$

It is straightforward that $\lambda_0^2(\theta_d x_d(T_d) - \theta_d x_d(T_b)) > 0$. Moreover

$$\lambda_0\mu_0(\theta_e^2 x_e(0) + \theta_d^2 x_d(T_b) - 2\theta_e\theta_d x_d(T_b)) = \lambda_0\mu_0 x_d(T_b)(\theta_d - \theta_e)^2 + \lambda_0\mu_0\theta_e^2(x_e(0) - x_d(T_b)) \quad (38)$$

and

$$\mu_0^2\theta_e(\theta_e^2 x_e(0) + \theta_d^2 x_d(T_b) - \theta_e\theta_d x_d(T_b) - \theta_e\theta_d x_e(0)) = \mu_0^2\theta_e(\theta_d - \theta_e)(\theta_d x_d(T_b) - \theta_e x_e(0)) \quad (39)$$

so that regrouping the last two terms (**??**) and (**??**), one gets :

$$\begin{aligned}&\lambda_0\mu_0\left(\theta_e^2 x_e(0) + \theta_d^2 x_d(T_b) - 2\theta_e\theta_d x_d(T_b)\right) + \mu_0^2\theta_e\left(\theta_e^2 x_e(0) + \theta_d^2 x_d(T_b) - \theta_e\theta_d x_d(T_b) - \theta_e\theta_d x_e(0)\right)\\ &= \lambda_0\mu_0 x_d(T_b)(\theta_d - \theta_e)^2 + \lambda_0\mu_0\theta_e^2(x_e(0) - x_d(T_b)) + \mu_0^2\theta_e(\theta_d - \theta_e)(\theta_d x_d(T_b) - \theta_e x_e(0))\\ &= \lambda_0\mu_0 x_d(T_b)(\theta_d - \theta_e)^2 + \lambda_0\mu_0\theta_e^2(x_e(0) - x_d(T_b)) + \mu_0^2\theta_e(\theta_d - \theta_e)((\theta_d - \theta_e)x_d(T_b) - \theta_e(x_e(0) - x_d(T_b)))\\ &= \lambda_0\mu_0 x_d(T_b)(\theta_d - \theta_e)^2 + \lambda_0\mu_0\theta_e^2(x_e(0) - x_d(T_b)) + \mu_0^2\theta_e(\theta_d - \theta_e)^2 x_d(T_b) - \mu_0^2\theta_e^2(\theta_d - \theta_e)(x_e(0) - x_d(T_b))\\ &= \mu_0 x_d(T_b)(\theta_d - \theta_e)^2(\lambda_0 + \theta_e\mu_0) + \mu_0\theta_e^2(x_e(0) - x_d(T_b))(\lambda_0 + \mu_0(\theta_e - \theta_d))\end{aligned}$$

which is positive. As a result:

$$\det A < 0$$

We also obtain:

$$A^{-1}\times\begin{pmatrix}0\\ \frac{1}{\theta_d}\\ 0\\ 0\\ 0\end{pmatrix} = \frac{1}{\theta_d(\lambda_0 + \theta_e\mu_0)\det A}\begin{pmatrix} y_1\left[E''(X_e)(x_e(0) - x_d(T_d))\theta_d + \rho\left(\theta_d - \theta_e\right)\left(\lambda_0 + \theta_e\mu_0\right)\right]/\rho\\ -x_d(T_b)\theta_d\left[\rho(\lambda_0 + \theta_e\mu_0)\left(\lambda_0 + \left(\theta_e - \theta_d\right)\mu_0\right) + y_2\right]\\ y_1\left[x_d(T_d)\theta_d\lambda_0 - x_e(0)\theta_e\left(\lambda_0 + \left(\theta_e - \theta_d\right)\mu_0\right)\right]\\ y_1 E''(X_e)\left[x_d(T_d)\theta_d\lambda_0 - x_e(0)\theta_e\left(\lambda_0 + \left(\theta_e - \theta_d\right)\mu_0\right)\right]\\ y_1\left[\rho(\lambda_0 + \theta_e\mu_0)\left(\lambda_0 + \left(\theta_e - \theta_d\right)\mu_0\right) + y_2\right]\end{pmatrix}$$

As $\det A < 0$, we deduce:

$$\frac{\partial T_d}{\partial \overline{Z}} < 0, \quad \frac{\partial T_b}{\partial \overline{Z}} > 0, \quad \frac{\partial X_e}{\partial \overline{Z}} \text{ ambiguous}, \quad \frac{\partial \lambda_0}{\partial \overline{Z}} \text{ ambiguous}, \quad \frac{\partial \mu_0}{\partial \overline{Z}} < 0$$

$\frac{\partial X_e}{\partial \overline{Z}}$ and $\frac{\partial \lambda_0}{\partial \overline{Z}}$ have the same sign as $x_e(0)\theta_e(\lambda_0 + (\theta_e - \theta_d)\mu_0) - x_d(T_d)\theta_d\lambda_0$. It is negative when $\theta_e = 0$, and positive when $\theta_e = \theta_d$.

D Low price elasticity of demand

Step 1. Expenditure $pD(p)$ is continuous and increasing with p. From Lagrange theorem, denoting $p_{T_b} \equiv p(T_b)$ and $x_{T_b} = D(p(T_b))$ there exists a price $p_i \in]c_b, p_{T_b}[$ such that:

$$p_{T_b} x_{T_b} = c_b x_b + (D(p_i) + p_i D'(p_i))(p_{T_b} - c_b)$$

The elasticity of demand at price p_i is $\epsilon_i = -\frac{p_i D'(p_i)}{D(p_i)}$ so that the above equation can be rewritten as:

$$\frac{x_{T_b}}{D(p_i)} = \frac{c_b x_b}{p_{T_b} D(p_i)} + (1 - \epsilon_i)\left(1 - \frac{c_b}{p_{T_b}}\right)$$

or equivalently:

$$\frac{x_{T_b}}{D(p_i)} - 1 = \frac{c_b}{p_{T_b}}\left(\frac{x_b}{D(p_i)} - 1\right) - \epsilon_i\left(1 - \frac{c_b}{p_{T_b}}\right)$$

As $\frac{x_{T_b}}{D(p_i)} - 1 < 0$ and $\frac{c_b}{p_{T_b}}\left(\frac{x_b}{D(p_i)} - 1\right) > 0$, denoting $\epsilon = \max_i(\epsilon_i)$, it comes that:

$$\frac{x_{T_b}}{D(p_i)} - 1 = O(\epsilon) \tag{40}$$

$$\frac{x_b}{D(p_i)} - 1 = O(\epsilon)\frac{p_{T_b}}{c_b} \tag{41}$$

Similarly, using Lagrange theorem between prices c_e and c_b, one gets, with $p_j \in]c_e, c_b[$:

$$\frac{x_b}{D(p_j)} - 1 = O(\epsilon) \tag{42}$$

$$\frac{x_{c_e}}{D(p_j)} - 1 = O(\epsilon)\frac{c_b}{c_e} \tag{43}$$

So that, if the price elasticity of demand is such that $\epsilon\frac{cb}{c_e} = O(\zeta)$, then $\frac{x_b}{D(c_e)} - 1 = O(\zeta)$.

Step 2. Recall that:

$$(u(x_b) - c_b x_b) - (u(x_{T_b}) - p_{T_b} x_{T_b}) = \rho F(T_b) - F'(T_b) \tag{44}$$

$\rho F(T_b) - F'(T_b)$ is decreasing with T_b as $F'' > 0$, so that $\rho F(T_b) - F'(T_b) < \rho F\left(\frac{\overline{Z}}{\theta_d x_{c_e}}\right) - F'\left(\frac{\overline{Z}}{\theta_d x_{c_e}}\right)$. Using equation (**??**), it comes that $\forall c_b, c_e$, there exists ϵ such that $\rho F(T_b) - F'(T_b) \leq \rho F\left(\frac{\overline{Z}}{\theta_d x_b}\right) - F'\left(\frac{\overline{Z}}{\theta_d x_b}\right)$. Equation (**??**) thus implies that:

$$(u(x_b) - c_b x_b) - (u(x_{T_b}) - p_{T_b} x_{T_b}) \leq \rho F\left(\frac{\overline{Z}}{\theta_d x_b}\right) - F'\left(\frac{\overline{Z}}{\theta_d x_b}\right)$$

so that

$$0 \leq p_{T_b} x_{T_b} - c_b x_b \leq \rho F\left(\frac{\overline{Z}}{\theta_d x_b}\right) - F'\left(\frac{\overline{Z}}{\theta_d x_b}\right)$$

so that

$$0 \leq \frac{p_{T_b} x_{T_b}}{c_b x_b} - 1 \leq \frac{\rho F\left(\frac{\overline{Z}}{\theta_d x_b}\right) - F'\left(\frac{\overline{Z}}{\theta_d x_b}\right)}{c_b x_b}$$

and thus

$$1 \leq \frac{p_{T_b}}{c_b} \leq \left[1 + \frac{\rho F\left(\frac{\overline{Z}}{\theta_d x_b}\right) - F'\left(\frac{\overline{Z}}{\theta_d x_b}\right)}{c_b x_b}\right] \frac{x_b}{x_{T_b}}$$

Substituting the equation above in equation (**??**), it comes that:

$$\frac{x_b}{D(p_i)} - 1 \leq O(\epsilon)\left[1 + \frac{\rho F\left(\frac{\overline{Z}}{\theta_d x_b}\right) - F'\left(\frac{\overline{Z}}{\theta_d x_b}\right)}{c_b x_b}\right] \frac{x_b}{x_{T_b}}$$

which can be rewritten, multiplying both sides by $\frac{x_{T_b}}{x_b}$:

$$\frac{x_{T_b}}{D(p_i)} - \frac{x_{T_b}}{x_b} \leq O(\epsilon)\left[1 + \frac{\rho F\left(\frac{\overline{Z}}{\theta_d x_b}\right) - F'\left(\frac{\overline{Z}}{\theta_d x_b}\right)}{c_b x_b}\right]$$

For an arbitrarily small ζ, one can find ϵ such that $\epsilon\left[1 + \frac{\rho F\left(\frac{\overline{Z}}{\theta_d x_b}\right) - F'\left(\frac{\overline{Z}}{\theta_d x_b}\right)}{c_b x_b}\right] \leq \zeta$. As a result, if $\epsilon\left[1 + \frac{\rho F\left(\frac{\overline{Z}}{\theta_d x_b}\right) - F'\left(\frac{\overline{Z}}{\theta_d x_b}\right)}{c_b x_b}\right] = O(\zeta)$, then, using equation (**??**): $\frac{x_{T_b}}{x_b} = \frac{x_{T_b}}{D(p_i)} + O(\zeta) = 1 + O(\zeta)$.

So that, $\forall \zeta, c_e, c_b, x_b, \overline{Z}$, there exists ϵ such that, if the elasticity of demand is always below ϵ then $\forall p \in [c_e, p_{T_b}]$,:

$$\frac{x_p}{x_e} = 1 + O(\zeta)$$

For a small local damage, we have shown in Appendix **??** that $\frac{dX_e}{d\overline{Z}}$ has the sign of $x_e(0)\theta_e(\lambda_0 + \theta_e - \theta_d)\mu_0 - x_d(T_d)\theta_d\lambda_0$. Using that, for a sufficiently low elasticity of demand $x_e(0) = x_d(T_d) + O(x_e(0)\zeta)$, it comes that $\frac{dX_e}{d\overline{Z}}$ has the sign of $-x_e(0)((\theta_d - \theta_e)(\lambda_0 + \theta_e\mu_0) + O(\zeta\theta_d\lambda_0)) < 0$.

E Switch to clean energy in the moratorium case

Using the envelope theorem:

$$\frac{\partial \widetilde{V}(T_b)}{\partial T_b} e^{\rho T_b} = \left[u\left(\widetilde{x}_d(T_b)\right) - (c_d + \theta_d \widetilde{\mu}_0 e^{\rho T_b})\widetilde{x}_d(T_b)\right] - \pi_b - \left(F'(T_b) - \rho F(T_b)\right) \tag{45}$$

E.1 Large local damage

To avoid confusions between the optimum and the moratorium case, let us denote in this Appendix $x_e^*(t)$ the optimal extraction of shale gas, μ_0^* the optimal initial shadow price of carbon and λ_0^* the optimal initial scarcity rent of shale gas (for a date of the switch to solar T_b^* by definition of the optimum); remember that the variables in the moratorium case are denoted with a $\widetilde{}$; and denote for instance by $\widetilde{\mu}_{0,T_b}$ the initial shadow price of carbon in the moratorium case for a date of the switch to solar T_b.

According to equation (**??**), in the case of a large local damage, T_b^* is such that:

$$\pi_b - \left[u\left(x_e^*(T_b^*)\right) - (c_e + d + (\lambda_0^* + \theta_e \mu_0^*)e^{\rho T_b^*})x_e^*(T_b^*)\right] = \rho F(T_b^*) - F'(T_b^*) \tag{46}$$

Introducing equation (**??**) in equation (**??**), it comes that:

$$\begin{aligned}\left.\frac{\partial V(T_b)}{\partial T_b}\right|_{T_b^*} e^{\rho T_b^*} = &\left[u\left(\widetilde{x}_{d,T_b^*}(T_b^*)\right) - (c_d + \theta_d \widetilde{\mu}_{0,T_b^*} e^{\rho T_b^*})\widetilde{x}_{d,T_b^*}(T_b^*)\right] \\ &- \left[u\left(x_e^*(T_b^*)\right) - (c_e + d + (\lambda_0^* + \theta_e \mu_0^*)e^{\rho T_b^*})x_e^*(T_b^*)\right]\end{aligned}$$

This expression is strictly positive if and only if:

$$c_e + d + (\lambda_0^* + \theta_e \mu_0^*)e^{\rho T_b^*} > c_d + \theta_d \tilde{\mu}_{0,T_b^*} e^{\rho T_b^*} \tag{47}$$

At the date T_e^* of the switch from coal to gas at the optimum, coal and gas price are equal. Hence:

$$c_e + d - c_d = \left(\theta_d \mu_0^* - (\lambda_0^* + \theta_e \mu_0^*)\right) e^{\rho T_e^*} > 0 \tag{48}$$

As $T_b^* > T_e^*$, it implies that $c_e + d - c_d < \left(\theta_d \mu_0^* - (\lambda_0^* + \theta_e \mu_0^*)\right) e^{\rho T_b^*}$. Using this inequality and assuming that inequality (**??**) holds, we get that

$$\mu_0^* > \widetilde{\mu}_{0,T_b^*} \tag{49}$$

This last inequlity implies that more coal is extracted between dates 0 and T_e^* in the moratorium case than in first best. Equality (**??**) and inequality (**??**) imply that

$$c_e + d + (\lambda_0^* + \theta_e \mu_0^*) e^{\rho T_e^*} > c_d + \theta_d \widetilde{\mu}_{0,T_b^*} e^{\rho T_e^*} \tag{50}$$

Together with Assumption (**??**), inequality (**??**) implies that for all t in $[T_e^*, T_b^*]$, $c_e + d + (\lambda_0^* + \theta_e \mu_0^*) e^{\rho t} > c_d + \theta_d \widetilde{\mu}_{0,T_b^*} e^{\rho t}$. This last equation implies that extraction of coal in the moratorium case is higher than extraction of gas at the optimum between dates T_e^* and T_b^*, implying more pollution between 0 and T_e^* also in the moratorium case than in the optimum. There is a contradiction: emissions are higher at all dates in the moratorium case than at the optimum, which contradicts the fact that the ceiling $\overline{Z}$ should not be violated in both cases.

E.2 Small local damage

Call $x_d^*(t)$ optimal extraction path of coal and μ_0^* the optimal shadow price of carbon. By definition T_b^* is such that (small local damage):

$$\pi_b - \left[u\left(x_d^*(T_b^*)\right) - (c_d + \theta_d \mu_0^* e^{\rho T_b^*}) x_d^*(T_b^*) \right] = \rho F(T_b^*) - F'(T_b^*) \tag{51}$$

As a result, introducing (**??**) in (**??**), it comes that:

$$\left.\frac{\partial V(T_b, \overline{Z})}{\partial T_b}\right|_{T_b^*} = \left[u\left(\widetilde{x}_{d,T_b^*}(T_b^*)\right) - (c_d + \theta_d \widetilde{\mu}_{0,T_b^*} e^{\rho T_b^*}) \widetilde{x}_{d,T_b^*}(T_b^*) \right] - \left[u\left(x_d^*(T_b^*)\right) - (c_d + \theta_d \mu_0^* e^{\rho T_b^*}) x_d^*(T_b^*) \right]$$

so that:

$$\left.\frac{\partial V(T_b, \overline{Z})}{\partial T_b}\right|_{T_b^*} > 0 \Leftrightarrow \widetilde{\mu}_{0,T_b^*} < \mu_0^*$$

E.3 Low elasticity of demand

Assume that $\widetilde{\mu}_{0,T_b^*} < \mu_0^*$. Then more coal is extracted between dates T_d^* and T_b^* in the moratorium case than in first best. Before T_d^*, only gas in extracted in first best and coal in the moratorium. For the ceiling constraint to be satisfied in both cases, it must be the case that:

$$\theta_e \int_0^{T_d^*} D\left(c_e + d + (\lambda_0^* + \theta_e \mu_0^*) e^{\rho t}\right) dt > \theta_d \int_0^{T_d^*} D\left(c_d + \theta_d \widetilde{\mu}_{0,T_b^*} e^{\rho t}\right) dt \tag{52}$$

We have:

$$D(c_d) \geq D\left(c_d + \theta_d \widetilde{\mu}_{0,T_b^*} e^{\rho t}\right) \geq D\left(c_d + \theta_d \widetilde{\mu}_{0,T_b^*} e^{\rho T_b^*}\right) \equiv x(\widetilde{p}(T_b^*))$$

Step 1. Expenditure $pD(p)$ is continuous and increasing with p. From Lagrange theorem, there exists a price $p_i \in]c_b, \widetilde{p}(T_b^*)[$ such that:

$$\widetilde{p}(T_b^*)D(\widetilde{p}(T_b^*)) = c_b x_b + (D(p_i) + p_i D'(p_i))(\widetilde{p}(T_b^*) - c_b)$$

The elasticity of demand at price p_i is $\epsilon_i = -\frac{p_i D'(p_i)}{D(p_i)}$ so that the above equation can be rewritten as:

$$\frac{D(\widetilde{p}(T_b^*))}{D(p_i)} = \frac{c_b x_b}{\widetilde{p}(T_b^*)D(p_i)} + (1-\epsilon_i)\left(1 - \frac{c_b}{\widetilde{p}(T_b^*)}\right)$$

or:

$$\frac{D(\widetilde{p}(T_b^*))}{D(p_i)} - 1 = \frac{c_b}{\widetilde{p}(T_b^*)}\left(\frac{x_b}{D(p_i)} - 1\right) - \epsilon_i\left(1 - \frac{c_b}{\widetilde{p}(T_b^*)}\right)$$

As $\frac{D(\widetilde{p}(T_b^*))}{D(p_i)} - 1 < 0$ and $\frac{c_b}{\widetilde{p}(T_b^*)}\left(\frac{x_b}{D(p_i)} - 1\right) > 0$, denoting $\epsilon = \max_i(\epsilon_i)$, it comes that:

$$-\epsilon < -\epsilon\left(1 - \frac{c_b}{\widetilde{p}(T_b^*)}\right) < \frac{D(\widetilde{p}(T_b^*))}{D(p_i)} - 1 < 0$$

so that:

$$\frac{D(\widetilde{p}(T_b^*))}{D(p_i)} - 1 = O(\epsilon) \tag{53}$$

$$\frac{x_b}{D(p_i)} - 1 = O(\epsilon)\frac{\widetilde{p}(T_b^*)}{c_b} \tag{54}$$

Similarly, using Lagrange theorem between prices c_d and c_b, one gets, with $p_j \in]c_d, c_b[$:

$$\frac{x_b}{D(p_j)} - 1 = O(\epsilon) \tag{55}$$

$$\frac{D(c_d)}{D(p_j)} - 1 = O(\epsilon)\frac{c_b}{c_d} \tag{56}$$

So that for any arbitrarily small ζ, if the price elasticity of demand is such that $\epsilon\frac{c_b}{c_d} = O(\zeta)$, then:

$$\frac{x_b}{D(c_d)} - 1 = O(\zeta) \tag{57}$$

Step 2. Recall that:

$$(u(x_b) - c_b x_b) - (u(D(\widetilde{p}(T_b^*)) - \widetilde{p}(T_b^*)D(\widetilde{p}(T_b^*)) = \tag{58}$$

But $\rho F(T_b^*) - F'(T_b^*)$ is decreasing with T_b^* (as $F'' > 0$), so that $\rho F(T_b^*) - F'(T_b^*) \leq \rho F\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right) - F'\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right)$. Equation (**??**) thus implies that:

$$(u(x_b) - c_b x_b) - (u(D(\widetilde{p}(T_b^*)) - \widetilde{p}(T_b^*)D(\widetilde{p}(T_b^*)) \leq \rho F\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right) - F'\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right)$$

so that:

$$0 \leq \widetilde{p}(T_b^*)D(\widetilde{p}(T_b^*)) - c_b x_b \leq \rho F\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right) - F'\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right)$$

so that:

$$0 \leq \frac{\widetilde{p}(T_b^*)D(\widetilde{p}(T_b^*))}{c_b x_b} - 1 \leq \frac{\rho F\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right) - F'\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right)}{c_b x_b}$$

and thus:

$$1 \leq \frac{\widetilde{p}(T_b^*)}{c_b} \leq \left[1 + \frac{\rho F\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right) - F'\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right)}{c_b x_b}\right] \frac{x_b}{D(\widetilde{p}(T_b^*))}$$

Substituting the equation above in equation (**??**), it comes that:

$$\frac{x_b}{D(p_i)} - 1 \leq O(\epsilon)\left[1 + \frac{\rho F\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right) - F'\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right)}{c_b x_b}\right] \frac{x_b}{D(\widetilde{p}(T_b^*))}$$

which implies, multiplying both sides by $\frac{D(\widetilde{p}(T_b^*))}{x_b}$:

$$\frac{D(\widetilde{p}(T_b^*))}{D(p_i)} - \frac{D(\widetilde{p}(T_b^*))}{x_b} \leq O(\epsilon)\left[1 + \frac{\rho F\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right) - F'\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right)}{c_d D(c_d)}\right]$$

For an arbitrarily small ζ, one can find ϵ such that $\epsilon\left[1 + \frac{\rho F\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right) - F'\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right)}{c_d D(c_d)}\right] \leq \zeta$. As a result, if $\epsilon\left[1 + \frac{\rho F\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right) - F'\left(\frac{\overline{Z}}{\theta_d D(c_d)}\right)}{c_d D(c_d)}\right] = O(\zeta)$, then, using equation (**??**):

$$\frac{D(\widetilde{p}(T_b^*))}{x_b} = \frac{D(\widetilde{p}(T_b^*))}{D(p_i)} + O(\zeta) = 1 + O(\zeta)$$

so that, $\forall \zeta, c_d, c_b, D(c_d), \overline{Z}$, there exists ϵ such that, if the elasticity of demand is always below ϵ, then $\forall p \in [c_d, \widetilde{p}(T_b^*)]$,:

$$\frac{D(p)}{D(c_d)} = 1 + O(\zeta)$$

The exact same reasoning gives that: $\forall p \in [c_e, p(T_b^*)]$:

$$\frac{D(p)}{D(c_d)} = 1 + O(\zeta)$$

A necessary condition for inequality (**??**) to hold is that:

$$\theta_e D\left(c_e + d + (\lambda_0^* + \theta_e \mu_0^*)e^{\rho T_b^*}\right) > \theta_d D(c_d)$$

But for any arbitrarily small ζ, if ϵ small enough,

$$\theta_e D(c_e + d + (\lambda_0^* + \theta_e \mu_0^*) e^{\rho T_b^*}) = \theta_e D(c_d) + O(\zeta)$$

For $\theta_d > \theta_e$, one can choose ζ such that $\theta_e D(c_d) + O(\zeta) < \theta_d D(c_d)$ so that inequality (**??**) does not hold for ϵ small enough.

F Optimal ceiling: moratorium vs optimum

We consider the case of a large local damage.

For ease of notation, call $p_b = \widetilde{p}(\widetilde{T}_b)$ the optimal final price of coal under the moratorium constraint. We first show that if this final price is given, and X_e is exogenous, the value of μ_0 decreases with X_e. At X_e, p_b given, the price path is defined by the following system of equations:

$$\theta_d \int_0^{T_e} D(c_d + \theta_d \mu_0 e^{\rho t}) dt = \overline{Z} - Z_0 - \theta_e X_e \tag{59}$$

$$\int_{T_e}^{T_d} D(c_e + (\lambda + \theta_e \mu_0) e^{\rho t}) dt = X_e \tag{60}$$

$$c_d + \theta_d \mu_0 e^{\rho T_e} = c_e + (\lambda_0 + \theta_e \mu_0) e^{\rho T_e} \tag{61}$$

$$c_e + d + (\lambda_0 + \theta_e \mu_0) e^{\rho T_b} = p_b \tag{62}$$

For ease of notation, we denote $p_{T_e} = c_e + d + (\lambda_0 + \theta_e \mu_0) e^{\rho T_e}$ and $p_0 = c_d + \theta_d \mu_0$. We make the substitution $u = T_b - t$ in the integral of equation (**??**). We get:

$$\int_{T_b - T_e}^{0} D(c_e + d + (p_b - c_e - d) e^{-\rho t}) dt = X_e \tag{63}$$

Differentiating equation (**??**) gives:

$$d(T_b - T_e) x(T_e) = dX_e \tag{64}$$

Equations (**??**) and (**??**) give $p_0 = c_d + (p_{T_e} - c_d) e^{-\rho T_e}$, so that:

$$dp_0 = dp_{T_e} e^{-\rho T_e} + (p_{T_e} - c_d) e^{-\rho T_e} (-\rho e^{-\rho T_e})$$

so that:

$$\frac{dp_0 e^{\rho T_e}}{\rho (p_{T_e} - c_d)} = \frac{dp_{T_e}}{\rho (p_{T_e} - c_d)} - dT_e \tag{65}$$

We make the substitution $u = T_e - t$ in the integral of equation (**??**). We get:

$$-\theta_d \int_{T_e}^{0} D(c_d + \theta_d \mu_0 e^{\rho(T_e - u)}) du = \overline{Z} - Z_0 - \theta_e X_e$$

which can be rewritten as:

$$\theta_d \int_{0}^{T_e} D(c_d + (p_{T_e} - c_d) e^{\rho(T_e - u)}) du = \overline{Z} - Z_0 - \theta_e X_e \qquad (66)$$

Differentiating equation (**??**), one gets:

$$\theta_d x(0) dT_e + \left(\theta_d \int_{0}^{T_e} \frac{dD(c_d + (p_{T_e} - c_d) e^{\rho(T_e - u)})}{dp} e^{-\rho u} du \right) dp_{T_e} = -\theta_e dX_e$$

which rewrites:

$$\theta_d x(0) dT_e - \left(\theta_d \int_{0}^{T_e} \frac{dD(c_d + (p_{T_e} - c_d) e^{\rho(T_e - u)})}{du} du \right) \frac{dp_{T_e}}{\rho(p_{T_e} - c_d)} = -\theta_e dX_e$$

This gives:

$$\theta_d x(0) \left[dT_e - \frac{dp_{T_e}}{\rho(p_{T_e} - c_d)} \right] + \frac{\theta_d x(T_e) dp_{T_e}}{\rho(p_{T_e} - c_d)} = -\theta_e X_e \qquad (67)$$

Using equations (**??**) and (**??**), we get:

$$\frac{dp_0 e^{\rho T_e}}{\rho(p_{T_e} - c_d)} = \frac{\theta_d x(T_e) dp_{T_e}}{\rho(p_{T_e} - c_d)} + \theta_e dX_e \qquad (68)$$

But $p_{T_e} = c_e + d + p_b + (p_b - c_e - d) e^{-\rho(T_b - T_e)}$, so that:

$$dp_{T_e} = -(p_b - c_e - d) e^{-\rho(T_b - T_e)} \rho d(T_b - T_e)$$

Using equation (**??**):

$$dp_{T_e} = -(p_b - c_e - d) e^{-\rho(T_b - T_e)} \rho \frac{dX_e}{x(T_e)} \qquad (69)$$

So that equation (**??**) can be rewritten as:

$$\frac{dp_0 e^{\rho T_e}}{\rho(p_{T_e} - c_d)} = \left[\frac{\theta_d (p_b - c_e - d) e^{-\rho(T_b - T_e)})}{\rho(p_{T_e} - c_d)} + \theta_e \right] dX_e \equiv \left[-\theta_d \frac{\lambda_0 + \theta_e \mu_0}{\theta_d \mu_0} + \theta_e \right] dX_e \equiv -\frac{\lambda_0}{\mu_0}$$

As a result $\frac{dp_0}{dX_e} < 0$, which gives that:

$$\frac{d\mu_0}{dX_e} < 0$$

So that for a given final price p_b, μ_0 is higher with a moratorium than without. This is a sufficient condition to prove that the initial shadow cost of pollution is higher in the moratorium case than at the optimum, as we showed that the final price in the moratorium case is in fact higher than in the optimum. As a result, the optimal damage is higher in the moratorium case than at the optimum.