Utility and Contrast in Evidence Accumulation Models

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Short abstract:

Evidence accumulation models can combine choice and response time data to better measure preferences when decision-makers make mistakes. Common evidence accumulation models, such as the Decision Diffusion Model or the Linear Ballistic Accumulator, fit equally well the joint distributions of choice and response times (C&RT) observed empirically for a given choice set. However, they generate diverging quantitative predictions about the effect of changing the utility of an alternative. In this paper, I clarify theoretically how utility enters these models. I characterize evidence accumulation models by their range - the set of C&RT distributions that they can generate - and their contrast - the extent to which increasing the utility of one alternative slows down the choice of another. Common evidence accumulation models have a similar range, but a drastically different contrast. One key implication is that any evidence accumulation model can be used for measuring utility as long as its contrast is properly calibrated. I propose a tractable framework for this aim and give general conditions under which it is applicable. Overall, this paper contributes to bridging the gap between the concepts of mathematical psychology and their use in empirical economic research.

Extended abstract:

There is an increasing interest in the use of response times in the analysis of economic decisions. Response times, on their own or combined with choice data, have been shown to predict utility in various contexts [30] [5] [1] [2]. Most of these studies rely on evidence accumulation models. These models describe decision-making as a noisy process of information acquisition over time that continues until a threshold level of evidence is reached and determines a choice. Prominent examples of evidence accumulation models are the Drift-Diffusion Model (DDM) [21, 23], the Linear Ballistic Accumulator (LBA) model [7] and the Leaky Competing Accumulator (LCA) model [29]. The LBA belongs to the subclass of horse-race models [20], in which each alternative is represented by an independent accumulation process and the first to reach the threshold is chosen.

Utility does not enter as a parameter in the original version of these models, which stem from psychology and cognitive science. It is generally assumed that each alternative-specific utility affects the rate of the evidence accumulation, a parameter that describes the pace at which positive evidence is gathered. However, the calibrated values of this proxy for alternative-specific utility have been shown to vary greatly depending on which evidence accumulation model is used and how exactly it is specified [10] [22]. This is a methodological issue of the utmost importance for economists who would like to use evidence accumulation models to measure utility. Interestingly, practionners have noted that, in spite of their diverging parameters, common evidence accumulation models seem to generate comparable distributions of response and time, a phenomenon that the literature has called mimicry [17] [11]. The correspondance across models between parameters that generate similar predictions is still not well understood and has been studied so far mostly through numerical simulations.

In this paper, I provide a theoretical solution to the problem of comparing drift rates across evidence accumulation models. My main contribution is a theoretical characterization of evidence accumulation models by their range - the set of C&RT distributions that they can generate - and

their contrast – the extent to which increasing the utility of one alternative slows down the choice of another. This characterization applies to a wide class of models that features the symmetric DDM, the LBA model and several of their popular variants. Common evidence accumulation models have a similar range, which explains the possibility to mimicry – see Figure 1. But they also have a drastically different contrast, which explains why and how their drift rates differ – see Table 1. In a nutshell, the DDM is equivalent to a LBA model in which the drift rate of an alternative is negatively affected by the utility of the others. The extent to which this contrast - absent from the LBA and very strong in the DDM - is present in actual behaviour is an empirical question.

I also introduce a tractable framework to tune the contrast of existing evidence accumulation models. I derive general conditions under which this methodology is applicable to 1) calibrate the contrast from experimental data and 2) measure alternative-specific utility using the calibrated model. An important implication is that those who want to use evidence accumulation models for measuring utility can safely ignore the debates on which model is the best, pick the most convenient model with a correct range and calibrate its contrast appropriately. When I calibrate the contrast of a LBA on data generated by a DDM, I find the theoretically predicted value for the contrast parameter and I observe that the predictions of two models no longer diverge as some alternative-specific utility varies – see Figure 2. I contribute directly to a theoretical literature at the intersection of economics and cognitive science. One strand of the literature, closer from decision-theory, has offered some axiomatization of the DDM [12] [3] and related it with classical random utility models [9][14][13]. Another strand of the literature, closer to mathematical psychology, has shown some weak equivalence between DDM-like models and horse-race models [20] [16] and tried to derive analytically horse-race counterpart of the DDM [22] [10] [26] [18].

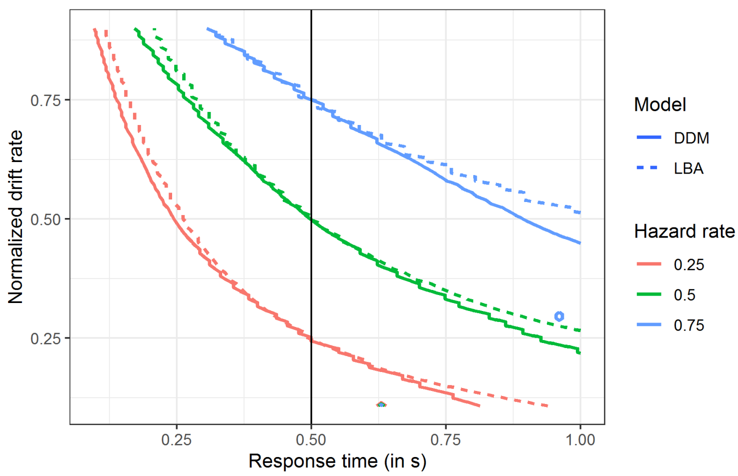
This work also hopes to answer some foundational methodological questions for future empirical research in economics based on evidence accumulation models. So far, few economists have used evidence accumulation models in empirical studies, and those who did have focused on the DDM because of its theoretical appeal. I show that the DDM imposes some constraint on the contrast that must be tested empirically and that other models that are conceptually and numerically simpler could equally well explain the evolution of the C&RT distribution when the utility of an alternative vary.

<u>Figures :</u>

Figure 1 : Illustration of the range of two evidence accumulation models.

The range is a function of two scalar arguments, the time and the normalized drift rate, that characterizes the set of all joint distributions of choice and response time that the model can generate across all possible utility of the alternatives

The fact that the range is similar for the two models explains the previous empirical observation that the DDM and the LBA are equally able to fit experimental data when the drift drate is a free parameter.



Drift rate normalization at t0 = 0.5s. The hazard rate is the value of F.

Table 1 : Theoretical expression of the contrast

A contrast mapping is a function that maps the utility of all alternatives to a scalar, alternative-specific parameter, the drift rate. My main result shows that evidence accumulation models are characterized by their range and their contrast mapping - up to a normalization of the drift rate.

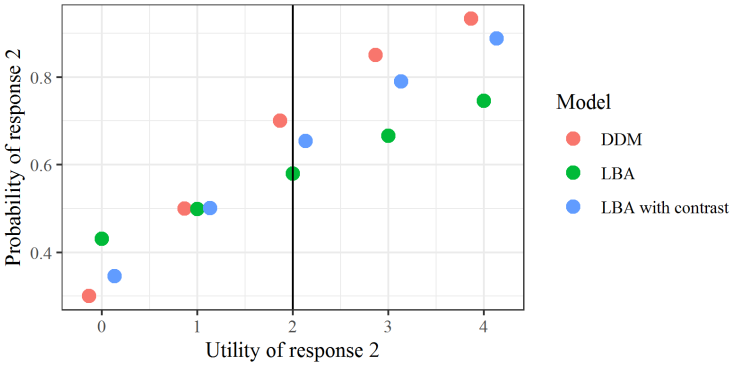
This table reports the theoretical expression of the contrast mapping for the LBA and DDM. The DDM introduces contrast between the alternatives, in the sense that the utility of one alternative affects negatively the drift rate of another, while the LBA does not. I introduce the LBA with contrast as a generic model in which the level of contrast can be calibrated

Model	Drift rate
DDM	$d_1 = u_1 - u_2$
LBA	$d_1 = u_1$
LBA with contrast	$d_1 = u_1 - a\, u_2$

Figure 2 : A LBA model with contrast replicates a DDM model

I calibrate a LBA model with contrast on data generated by a DDM with a varying utility of the alternatives. I find a contrast parameter *a* close from 1, as predicted by the theory.

Figure 2 shows that choice probability predicted by the LBA with contrast after calibration is in line with the DDM for a wide range of utility values, while the predictions of the usual DDM diverge. This is a practical implication of my main theoretical result: the LBA has a similar range with the DDM, and the LBA with contrast calibrated on DDM data also has the same contrast mapping, hence it replicates a DDM model.



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