

Income Composition Inequality*

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Abstract

The present paper proposes a methodology to examine the relationship between the functional and personal distribution of income. To this end, it introduces the concept of *inequality in income composition*. If we decompose total income into *two* factors, such as capital and labour income, then income composition inequality is the extent to which the income composition is distributed unevenly in a group of people. Inequality in income composition is maximal when individuals at the top and at the bottom of the income distribution separately earn the two different types of income. On the contrary, it is minimal when each individual earn the same composition of the two factors. This article proposes an indicator to measure income composition inequality. The sign of this indicator determines the condition of transmission for the rising share of income from any source to increase overall income inequality. This methodology is then applied to several European countries on basis of EU-SILC data.

JEL-Classification: C430, E250

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1 Introduction

The study of the relationship between the functional and personal distribution of income has seen a revival of interest over the past two decades (Atkinson, 2009, Piketty, 2014). In this article we argue that, in order to determine a formal link between the two distributions, we need to introduce a novel inequality concept, that we call *income composition inequality*. Moreover, this article introduces a summary statistics for its measurement. The need for introducing and then measuring income composition inequality can be motivated by the very development of the literature and of economies over time.

Already in 1997, Atkinson argued that to understand the drivers of inequality, the economic theory of the distribution of income is in need of further development (Atkinson, 1997, p. 317). He argues that the current priority should be to bring the several existing contributions on this theory together into a single framework (p. 317). He also argues that, among the different aspects which affect the dynamics of the distribution of income, the relationship between the functional and personal distribution should feature prominently (p. 298).

Such relationship binds a macroeconomic phenomenon with a microeconomic one.¹ Brandolini (1992) claims that this link connects economic systems and people, and it is provided by what he calls “entitlement rules”.² As Glyn (2011) points out, unfair entitlement rules may lead the employer’s profit rate to grow faster than the employee’s wage rate. Moreover, unfair entitlement rules are likely to trigger political tensions between different interest groups. Income inequality needs therefore to be analyzed with an eye for the multidimensional nature of the typologies of income. Unsurprisingly, the laws which regulate the distribution were considered to be the principal problem in political economy by the classical author Ricardo (Ricardo, 1911).³

¹In a later article, Atkinson wrote that one reason for studying this link is that “there is at present and evident disjuncture between the macroeconomic measures of economic performance and the perceptions by citizens as to what is happening to their incomes?” (Atkinson, 2009, p. 5).

²According to Brandolini, the entitlement rules are “rules stating who has the right to receive a given type of income and which proportion of it” (Brandolini, 1992, p. 3).

³We report the famous statement by Ricardo, which says: “the produce of the earth - all that is derived from its surface by the united application of labour, machinery and capital, is divided among

Several contributions have recently explored the empirical nature of the link between the functional and personal distribution. The most popular one has been made by [Piketty \(2014\)](#), who analyzes the long-run evolution of the functional distribution and of the top income shares at the international level. In his framework, Piketty considers top income shares as measures of income inequality.⁴ His landmark book *Capital in the Twenty-First Century* (2014) is an attempt to combine the different data sources available, such as fiscal data, survey data and national accounts in a systematic way.⁵ One of the most important findings from his research is that the capital share of income has increased in many developed countries over the last decades (see also [Piketty, 2015](#)). Furthermore, Piketty shows that the capital income share tends to move together with the capital-income ratio in the long run. Given that inequality in capital income is generally greater than inequality in labor income, the rising share of capital income in net product leads to a greater inter-personal inequality. This result highlights the positive relationship between the functional and personal distribution of income from a historical perspective. Another empirical contribution on the matter is the article by [Bengtsson and Waldenström \(2018\)](#), who find evidence of a “strong, positive link [between the functional and personal distribution of income] that has grown stronger over the past century” by means of a novel historical cross-country database they personally assembled. However, they do not believe this relationship to remain stable over time, insofar as it can be contingent on production technology, the structure of personal income, and the institutional context. [Francese and Mulas-Granados \(2015\)](#), based on an analysis that covers up to 93 countries between 1970 and 2013, find instead that the distribution of income between labor and capital has not been a major factor in explaining income inequality. The two previous works provide evidence that, as [Milanovic \(2017\)](#) puts it, “the link is not as simple and unambiguous as it seems”.

three classes of the community, namely, the proprietor of the land, the owner of the stock or capital necessary for its cultivation, and the labourers by whose industry it is cultivated . . . To determine the laws which regulate this distribution is the principal problem in Political Economy” ([Ricardo, 1911](#) [1817], p. 1 in 1911 edition).

⁴The advantage of considering top income share as a measure of income inequality is that they can be easily compared both across countries and cross time.

⁵Piketty himself states that his book is primarily about the history of the distribution of income and wealth ([Piketty, 2015](#)).

On a technical level, few works have attempted to precisely measure the strength of this link. In his recent work, [Milanovic \(2017\)](#) argues that in a context of rising share of capital income the level of income inequality grows only under two conditions: (i) a high level of inequality in capital income and (ii) a high and positive association between capital-rich and overall income-rich people. These two conditions, operationalized by the Gini of capital income and the correlation coefficient between capital and total income respectively,⁶ suggest an important theoretical connection between factor shares and income inequality. Particularly, the correlation coefficient between capital and total income, which is an elasticity of inter-personal income Gini to changes in capital income share, may act as an intuitive and simple measure of such link. However, this correlation coefficient does not formally determine the condition of transmission of changes in the functional distribution into income inequality, as it will be discussed later in the article. [Atkinson and Bourguignon \(2000\)](#) and [Atkinson \(2009\)](#) approach the measurement of this link by decomposing the squared coefficient of variation of income, where there are two types of income: wage income and capital income.⁷ In such way, they manage to show the condition under which an increase in the capital income share is transmitted into an increase in overall income inequality, as measured by the standard deviation of income. Another way of measuring the association between capital and labor have also recently been proposed by [Atkinson and Lakner, 2017](#). The authors study the association between capital and labor by constructing a rank-based measure of association which is a discrete approximation of the copula density. All these methods, however, do not aim at precisely measuring the strength of this link, nor to create a single summary statistics for such purpose. [Atkinson and Lakner, \(2017\)](#), for instance, do not precisely discuss under which specific joint distributions of capital and labor the strength of the link is maximal and minimal. On the other end, [Atkinson and Bourguignon \(2000\)](#) do

⁶These two variables emerge from the Yitzhaki-Lerman decomposition of the Gini coefficient ([Yitzhaki and Lerman, 1984](#)).

⁷Specifically, the coefficient of variation of income V^2 can be written as a function of the capital share of income π , of the inequality of wage income V_w and capital income V_k , and of the correlation ρ between wage income and capital income: $V^2 = (1 - \pi)^2 V_w^2 + \pi^2 V_k^2 + 2\pi(1 - \pi)\rho V_w V_k$. Now, if we define λ as the relationship between wage income dispersion and capital income dispersion, then a rise in the capital share of income is transmitted into personal income inequality only when the following condition is satisfied: $\pi > \frac{1-\lambda\rho}{1+\lambda^2-2\lambda\rho}$.

not provide any summary statistics that can be used to measure the strength of this relationship.

As previously stated, the purpose of this article is to fill this gap by proposing a metric which captures the strength of the link between the functional and personal distribution of income. To this end, the article first introduces a novel inequality concept, labeled *income composition inequality*.

If we decompose total income into *two* factors, such as capital and labour income, then income composition inequality is the extent to which the income composition is distributed unevenly across the population. Inequality in income composition is *maximal* when individuals at the top and at the bottom of the income distribution separately earn the two different types of income. On the contrary, it is *minimal* when each individual earns the same composition of the two factors.

While we use capital and labor as income sources in this paper, it is important to stress that the study of income composition inequality can be applied to analyze the joint distribution of *any pair* of income (or wealth) components, such as net income and taxes, saving and consumption, and financial and non-financial assets, among others.

A high level of income composition inequality is associated with a strong relationship between the functional and personal distribution of income. The underlying intuition is straightforward: if the rich earn all the capital income in the economy, then an increase in the capital income share rises the income of the rich. Analogous reasoning can be proposed to show that under a high level of income composition inequality the functional distribution of income can be seen as a measure of income inequality.

To measure income composition inequality we design a specific summary statistics. Such statistics is constructed by means of concentration curves for income source. Similarly to the concentration curves proposed by [Kakwani \(1977a, 1977b\)](#), or to the pseudo-Lorenz curves proposed by [Fei et al. \(1978\)](#), the concentration curve for income source is the cumulative distribution of income source across the population with individuals being indexed by their total income ranking. Differently from Kakwani's concentration curve (and from Fei et al.'s pseudo-Lorenz curve), however, this curve does not sum to one, but to the total level of the factor share.

To design this summary statistics, in addition to the concentration curve for income source, we need two additional concentration curves: the *zero-* and the *maximum-concentration curves*. The zero-concentration curve characterizes a population in which income factors are evenly distributed with respect to both the overall level of income inequality and total factor shares in the economy. This curve provides a benchmark of zero inequality in income composition, as the egalitarian line does for inequality in income.⁸ The maximum-concentration curve, instead, provides a benchmark of maximum inequality in income composition. We show that under the zero-concentration condition the Gini coefficient does not depend on the functional income distribution.

Visually, when the concentration curve lies above the zero-concentration curve, then the income source is concentrated at the bottom of the income distribution. On the contrary, when the concentration curve lies below the zero-concentration curve, then the income source is concentrated at the top of the income distribution.

Following the way in which the Gini coefficient is computed,⁹ we define this indicator as the difference between the concentration curve for a given income source and its zero-concentration curve, suitably normalized.

This indicator can be looked at in two ways. Firstly, from a technical perspective, it can be considered an elasticity of personal income inequality to fluctuations in the functional income distribution. In other words, it mathematically links the functional and personal distribution of income. Secondly, from a political economy perspective, it measures the “degree of capitalism” of a given social system. Specifically, following the classification proposed by [Milanovic \(2017\)](#), under the maximum-concentration condition a society can be considered a case of classical capitalism, while under the zero-concentration condition it can be regarded as a case of new capitalism.¹⁰ For instance, a fall in the indicator

⁸The egalitarian line is the bisector with which the Lorenz curve is normally compared to assess the level of income inequality between individuals. Interestingly, while within the context of inequality in income the egalitarian line is the same for all population sets, within the context of inequality in income composition the zero-concentration curve is specific to each population set. Indeed, two population sets with different income inequality levels, or rather with different distributions between the capital and labor share, display different benchmarks of zero inequality in income composition.

⁹The Gini is computed by taking the difference between the Lorenz curve for income and the egalitarian line, suitably normalized

¹⁰To be perfectly in line with Milanovic’s own framework, we should refer to a *new capitalism 2* under the zero-concentration condition.

suggests that the corresponding society is moving towards becoming a new form of capitalism, in which individuals have multiple sources of income at their disposal and where there is a weaker relationship between functional and personal distribution of income (and the other way around).

This article is structured as follows. Section 2 introduces the general framework and the indicator. Section 3 derives the indicator in a two-person economy and describes its mathematical properties. Section 4 applies the proposed framework to several European countries, finding that income composition inequality is positive for all countries analyzed. Section 5 concludes.

2 Methodology

2.1 The Concentration Curve for Income Source

Suppose we have a fixed population of n individuals, each endowed with income Y_i with $i = 1, \dots, n$. We can define each individual's income share as $y_i = \frac{Y_i}{Y}$, where $Y = \sum_{i=1}^n Y_i$ is the total income of the population. Total income is divided into two sources, capital (Π) and labor (W), so that $Y = \Pi + W$ and hence $y = 1 = \pi + w$, where $\pi = \frac{\Pi}{Y}$ and $w = \frac{W}{Y}$ are the capital and labor shares in income, respectively. Consider the following decomposition of individual i 's income:

$$y_i = \alpha_i \pi + \beta_i w, \quad (1)$$

where $\alpha_i = \frac{\Pi_i}{\Pi}$ and $\beta_i = \frac{W_i}{W}$ are the relative shares of capital and labor of individual i , such that $\sum_{i=1}^n \alpha_i = \sum_{i=1}^n \beta_i = 1$ and Π_i and W_i represent i 's total amount of capital and labor. Assume that $y_i \leq y_{i+1} \forall i = 1, \dots, n-1$ and $y_0 = 0$, so that individuals are indexed by their income ranking. We can define $p = \frac{i}{n}$ as the proportion of the population with income less than or equal to y_p , so that $p \in \mathbb{Q} := [0, 1]$. Let $\mathcal{L}(\mathbf{y}, p) = \sum_{j=1}^i y_j$, with $i = 1, \dots, n$, be the Lorenz curve for income corresponding to the distribution \mathbf{y} .¹¹ We

¹¹We are defining the Lorenz curve here as in [Shorrocks \(1983\)](#).

can define the concentration curve for capital, $\mathcal{L}(\boldsymbol{\pi}, p)$, corresponding to the distribution $\boldsymbol{\pi}$, as follows:

$$\mathcal{L}(\boldsymbol{\pi}, p) = \pi \sum_{j=1}^i \alpha_j \quad \forall i = 1, \dots, n. \quad (2)$$

Similarly, the concentration curve for labor, $\mathcal{L}(\mathbf{w}, p)$, corresponding to the distribution \mathbf{w} , is:

$$\mathcal{L}(\mathbf{w}, p) = w \sum_{j=1}^i \beta_j \quad \forall i = 1, \dots, n. \quad (3)$$

The two curves describe the cumulative distribution of capital and labor across the population with individuals being indexed by their income ranking. It is hence possible that an individual with a higher capital share be ranked below someone with a lower capital share, if the income of the latter is above that of the former (formally, we can find a pair (i, j) s.t. $\alpha_i > \alpha_j$ and $y_i < y_j$). Additionally, note that when $i \rightarrow n$ (or $p \rightarrow 1$) then $\mathcal{L}(\boldsymbol{\pi}, p) \rightarrow \pi$ and $\mathcal{L}(\mathbf{w}, p) \rightarrow w$, where $\pi, w \leq y$. The concentration curves for income source can also be regarded as *pseudo-Lorenz curves* (Fei et al., 1978) scaled down to the level of the related income share.

According to the previous decomposition of individual income, we can write as follows:

$$\mathcal{L}(\mathbf{y}, p) = \mathcal{L}(\boldsymbol{\pi}, p) + \mathcal{L}(\mathbf{w}, p) \quad \forall i = 1, \dots, n. \quad (4)$$

The Lorenz curve for income $\mathcal{L}(\mathbf{y}, p)$, for every p , can therefore be decomposed into the sum of the two previously-defined concentration curves. Now, we can write the Gini coefficient, \mathcal{G} , as follows:

$$\mathcal{G} = 1 - \frac{1}{n} \left(\sum_{i=1}^n \left[\mathcal{L} \left(\boldsymbol{\pi}, \frac{i}{n} \right) + \mathcal{L} \left(\boldsymbol{\pi}, \frac{i-1}{n} \right) + \mathcal{L} \left(w, \frac{i}{n} \right) + \mathcal{L} \left(w, \frac{i-1}{n} \right) \right] \right). \quad (5)$$

Figure 1 plots an example of $\mathcal{L}(\mathbf{y}, p)$ (the blue curve) and $\mathcal{L}(\boldsymbol{\pi}, p)$ (the red curve) for a population of size $n = 10$. Total income is equally split between capital and labor, hence $\pi = w = \frac{1}{2}$.

The concentration curves allow us to understand whether a given income source is concentrated primarily at the bottom or at the top of the income distribution. Given the interdependence of the two concentration curves (i.e., when one source is concentrated at

Concentration Curves for Income Source

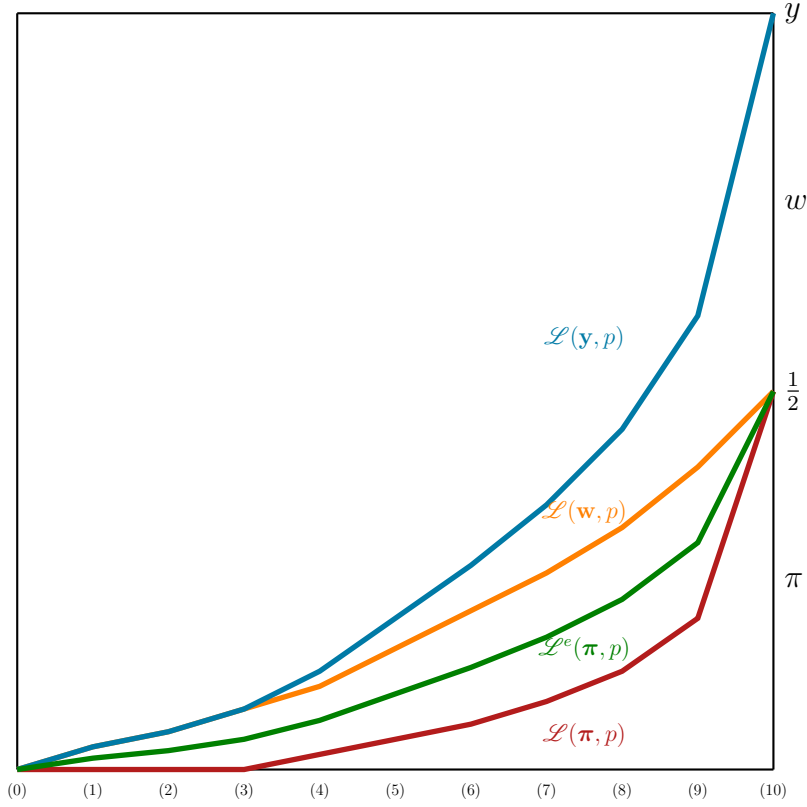


Figure 1: A graphical representation of the concentration curve for capital $\mathcal{L}(\pi, p)$, the concentration curve for labor $\mathcal{L}(\mathbf{w}, p)$, the Lorenz curve for income $\mathcal{L}(\mathbf{y}, p)$ and the zero-concentration curve $\mathcal{L}^e(\pi, p)$ with 10 individuals (or groups) and equal sources of income in the economy ($\pi = w = \frac{1}{2}$). As it can be noticed, for each population decile p the Lorenz curve for income $\mathcal{L}(\mathbf{y}, p)$ equals the sum of the concentration curve for capital $\mathcal{L}(\pi, p)$ and the concentration curve for labor $\mathcal{L}(\mathbf{w}, p)$. In addition, given that $\pi = w$, the two zero-concentration curves coincide: $\mathcal{L}^e(\pi, p) = \mathcal{L}^e(\mathbf{w}, p) \forall p$.

the top the other is concentrated at the bottom), a single curve is sufficient to analyze the joint distribution of capital and labor. However, to precisely assess the extent to which capital and labor are polarized across the income distribution, two benchmark conditions must be defined: the *zero-* and *maximum-concentration conditions*. On the basis of these two conditions, the corresponding *zero-* and *maximum-concentration curves* are hence introduced.

2.2 The Zero-Concentration Curve

In this Section, we introduce in a formal setting the concept of the zero concentration of two income sources. As anticipated in the introduction, we define the benchmark of zero concentration in the following way.

Definition 2.1. *We say that two income sources are zero-concentrated across a population when each individual has the same population shares of capital and labor. Formally, we have zero concentration of income sources when $\frac{W_i}{\Pi_i} = \frac{w}{\pi} \forall i$, or, equivalently, when $\alpha_i = \beta_i \forall i$.¹²*

Note that the previous definition is not related to the concept of income inequality: The population can exhibit zero concentration of income sources even with positive income inequality. Furthermore, note that only two elements are needed to determine the zero-concentration condition, notably the functional and personal distribution of income. Two populations characterized by different Lorenz curves, or by different shares of capital income, have two different conditions of zero-concentration. At this stage of the analysis we can define the zero-concentration curve, $\mathcal{L}^e(\mathbf{z}, p)$, corresponding to the distribution \mathbf{z} , which is the concentration curve for the income source z when the income sources are not concentrated as:

$$\mathcal{L}^e(\mathbf{z}, p) = \mathbf{z} \sum_{j=1}^i y_j \quad \forall i = 1, \dots, n, \quad (6)$$

with $\mathbf{z} = \pi, w$. The choice of \mathbf{z} depends on the particular source we analyze. If we were interested in the distribution of capital in the population, we would compare the *actual* concentration curve for capital with the concentration curve for capital in the case of zero concentration, $\mathcal{L}^e(\pi, p)$. It should be noted that the zero-concentration curve is a *scaled* version of the Lorenz curve for income, indeed we can write $\mathcal{L}^e(\mathbf{z}, p) = \mathbf{z} \mathcal{L}(\mathbf{y}, p) \forall p$. Let us now consider the following relationship:

$$\mathcal{L}(\mathbf{z}, p) = \mathcal{L}^e(\mathbf{z}, p) + \mathcal{R}(\mathbf{z}, p) \quad \forall i = 1, \dots, n, \quad (7)$$

¹²As $\frac{W_i}{\Pi_i} = \frac{w}{\pi} \iff \frac{W_i}{w} = \frac{\Pi_i}{\pi} \iff Y \times \frac{W_i}{W} = Y \times \frac{\Pi_i}{\Pi} \iff \alpha_i = \beta_i$.

where $\mathcal{R}(\mathbf{z}, p)$ is the residual-concentration curve corresponding to the distribution \mathbf{z} . When $\mathcal{L}(\mathbf{z}, p)$ is above $\mathcal{L}^e(\mathbf{z}, p)$ over all of the domain (i.e., $\mathcal{L}(\mathbf{z}, p) > \mathcal{L}^e(\mathbf{z}, p) \forall p$) then $\sum_{i=1}^n \mathcal{R}(\mathbf{z}, p) > 0$ and source \mathbf{z} is concentrated primarily at the bottom of the distribution; on the contrary, when $\mathcal{L}(\mathbf{z}, p)$ is below $\mathcal{L}^e(\mathbf{z}, p)$ over all of the domain then $\sum_{i=1}^n \mathcal{R}(\mathbf{z}, p) < 0$ and the opposite situation holds. In the case of zero concentration of income sources, equation 5 becomes:

$$\mathcal{G} = 1 - \frac{1}{n} \left(\sum_{i=1}^n \left(\sum_{j=1}^i \beta_j + \sum_{j=1}^{i-1} \beta_j \right) \right), \quad (8)$$

which is also equivalent to:

$$\mathcal{G} = 1 - \frac{1}{n} \left(\sum_{i=1}^n \left(\sum_{j=1}^i \alpha_j + \sum_{j=1}^{i-1} \alpha_j \right) \right). \quad (9)$$

The Gini coefficient in this particular case can thus be written as a function of individuals' relative shares of any one income source. Note that none of the two expressions above are functions of π or w , meaning that an increase in either the capital share or the labor share of income does not affect personal income inequality when income sources are not concentrated. Similarly, we can say that the “elasticity of inter-personal income Gini to changes in capital income share” is zero.¹³ This distribution of income sources represents the long-run distribution of factors across individuals in a neoclassical framework in which heterogeneity of both non-accumulated and accumulated factors are considered (Bertola et al. (2005)). It also represents the underlying distribution of factors in the *new capitalism 2* society defined by Milanovic (2017).

We conclude this Section with the following definition.

Definition 2.2. *We say that under zero-concentration of income sources, inequality in income composition is minimal.*

2.3 The Maximum-Concentration Curve

Let us focus our attention on the benchmark of maximum-concentration of two income sources, which we can define as follows.

¹³See Milanovic (2017) for further details.

Definition 2.3. We say that two income sources are maximum concentrated when the bottom $p\%$ of the income distribution has an income consisting only of the source z and the top $(1-p)\%$ of the income distribution has an income consisting only of the source z_- , where p s.t. $y_p = \mathcal{L}(\mathbf{y}, p) = z$, $1-p$ s.t. $y_{1-p} = 1 - \mathcal{L}(\mathbf{y}, p) = z_-$, $z_- = 1 - z$ and $z = \pi, w$.

As for the condition of zero-concentration, also the condition of maximum-concentration is already present in the literature. In his recent article, Milanovic defines the classical capitalism as a society in which “ownerships of capital and labor are totally separated, in the sense that workers draw their entire income from labor and have no income from the ownership of assets, while the situation for the capitalists is the reverse. Moreover, we shall assume that all workers are poorer than all capitalists. This gives us [...] two social groups, non-overlapping by income level” (Milanovic, 2017). We can therefore say that under the condition of maximum-concentration and specifically when the capital is owned by the top of the income distribution and the labor by the bottom, a society is a classical capitalism á la Milanovic.¹⁴

From a technical perspective, we can define the maximum-concentration curve, $\mathcal{L}^{max}(\mathbf{z}, p)$, corresponding to the distribution \mathbf{z} , as follows:

$$\mathcal{L}^{max}(\mathbf{z}, p) = \begin{cases} \mathcal{L}^M(\mathbf{z}, p) = \begin{cases} \mathcal{L}(\mathbf{y}, p) & \text{for } p \leq p' \\ z & \text{for } p > p' \end{cases} \\ \mathcal{L}^m(\mathbf{z}, p) = \begin{cases} 0 & \text{for } p \leq p'' \\ \mathcal{L}(\mathbf{y}, p) - z_- & \text{for } p > p'' \end{cases} \end{cases}, \quad (10)$$

with p' s.t. $\mathcal{L}(\mathbf{y}, p') = z$, p'' s.t. $\mathcal{L}(\mathbf{y}, p'') = 1 - z$ and $\mathbf{z} = \pi, w$. In addition, we have:

- (i) $\mathcal{L}^{max}(\mathbf{z}, p) = \mathcal{L}^M(\mathbf{z}, p)$ if $\mathcal{L}(\mathbf{z}, p) \geq \mathcal{L}^e(\mathbf{z}, p) \forall p$ and $\exists p^*$ s.t. $\mathcal{L}(\mathbf{z}, p^*) > \mathcal{L}^e(\mathbf{z}, p^*)$,

¹⁴This type of society can also be found in the works by Kaldor (1955), Pasinetti (1962) or more recently by Stiglitz (2015), in which a class of capitalists is counterposed to a class of workers. However, these authors do not necessarily assume that the former class is poorer than the latter in terms of total income.

- (ii) $\mathcal{L}^{max}(\mathbf{z}, p) = \mathcal{L}^m(\mathbf{z}, p)$ if $\mathcal{L}^e(\mathbf{z}, p) \leq \mathcal{L}(\mathbf{z}, p) \forall p$ and $\exists p^{**}$ s.t. $\mathcal{L}^e(\mathbf{z}, p^{**}) < \mathcal{L}(\mathbf{z}, p^{**})$.

To put it simply, $\mathcal{L}^{max}(\mathbf{z}, p) = \mathcal{L}^M(\mathbf{z}, p)$ when the actual concentration curve lies above the zero-concentration curve and that $\mathcal{L}^{max}(\mathbf{z}, p) = \mathcal{L}^m(\mathbf{z}, p)$ when the actual concentration curve lies below the zero-concentration curve.

However, the two conditions above-mentioned ((i) and (ii)) are rather strong, since they need the two curves not to intersect along the distribution of income. By contrast, a weaker condition is the one which takes into consideration the area covered by each curve, as follows:¹⁵

- (i) $\mathcal{L}^{max}(\mathbf{z}, p) = \mathcal{L}^M(\mathbf{z}, p)$ if $\sum_{i=1}^n \sum_{j=1}^i \eta_j^k > \sum_{i=1}^n \sum_{j=1}^i y_j$,
- (ii) $\mathcal{L}^{max}(\mathbf{z}, p) = \mathcal{L}^m(\mathbf{z}, p)$ if $\sum_{i=1}^n \sum_{j=1}^i \eta_j^k < \sum_{i=1}^n \sum_{j=1}^i y_j$,

where $\eta_j^k = \alpha_j$ if $z = \pi$ and $\eta_j^k = \beta_j$ when $z = w$.

As is the case for the previous Section, we conclude this Section with the following definition.

Definition 2.4. *We say that under maximum-concentration of income sources, income composition inequality is maximized.*

2.4 Measuring Income Composition Inequality

In the previous Sections, we defined the two benchmarks of zero and maximum inequality in income composition, together with their corresponding concentration curves. When the actual concentration curve is relatively close to the zero-concentration curve, then income composition inequality is low. On the contrary, when the actual concentration curve is sufficiently close to the maximum-concentration curve, then income composition inequality is high.

Now, to precisely measure income composition inequality, we introduce an indicator which serves this purpose. We label this metric *income-factor concentration index*, \mathcal{I}_f ,

¹⁵Similarly, the first and second group of conditions can be regarded as first and second-order stochastic dominance conditions.

which we construct as follows.

Let us denote by $\mathcal{A}(\mathbf{z})$ the area between the zero-concentration curve and the concentration curve for income source z and by $\mathcal{B}(\mathbf{z})$ the area between the zero-concentration curve and the maximum-concentration curve.¹⁶ We can define the income-factor concentration index, $\mathcal{I}_f(\mathbf{z})$, corresponding to the distribution \mathbf{z} , as follows:

$$\mathcal{I}_f(\mathbf{z}) = \frac{\mathcal{A}(\mathbf{z})}{\mathcal{B}^{max}(\mathbf{z})}, \quad (11)$$

with $\mathbf{z} = \boldsymbol{\pi}, \mathbf{w}$.

This measure has considerable intuitive appeal: It is the area between the zero-concentration curve $\mathcal{L}^e(\mathbf{z}, p)$ and the concentration curve for income source $\mathcal{L}(\mathbf{z}, p)$, divided by the area between the zero-concentration curve $\mathcal{L}^e(\mathbf{z}, p)$ and the maximum-concentration curve $\mathcal{L}^{max}(\mathbf{z}, p)$.¹⁷ This measure lies therefore between -1 (when individuals at the bottom own source z and individuals at the top own source z_-) and 1 (when individuals at the bottom own source z_- and individuals at the top own source z). It is equal to zero when the area of the concentration curve is the same as that of the zero-concentration curve.¹⁸ Furthermore, note that $\mathcal{I}_f(\mathbf{z}) = -\mathcal{I}_f(\mathbf{z}_-)$ (see the Appendix for further details).

In light of the relationship previously discussed between the concentration curves and the ideal-typical social systems proposed by Milanovic, we can also interpret such indicator as a measure of the degree of capitalism of a given social system. Furthermore, the new type of capitalism can also be considered as a multiple sources of income society.

The income-factor concentration index is not a rank-based measure of association between labor and capital (Atkinson and Lakner, 2017). Indeed, a monotone transformation in the marginal distributions would affect the index by changing the ranking in the distribution of total income.¹⁹

¹⁶Formally, $\mathcal{A}(\mathbf{z}) = \frac{1}{2n} \sum_{i=1}^n [(\mathcal{L}^e(\mathbf{z}, \frac{i}{n}) + \mathcal{L}^e(\mathbf{z}, \frac{i-1}{n})) - (\mathcal{L}(\mathbf{z}, \frac{i}{n}) + \mathcal{L}(\mathbf{z}, \frac{i-1}{n}))]$ and $\mathcal{B}^{max}(\mathbf{z}) = \frac{1}{2n} \sum_{i=1}^n [(\mathcal{L}^e(\mathbf{z}, \frac{i}{n}) + \mathcal{L}^e(\mathbf{z}, \frac{i-1}{n})) - (\mathcal{L}^{max}(\mathbf{z}, \frac{i}{n}) + \mathcal{L}^{max}(\mathbf{z}, \frac{i-1}{n}))]$, with $max = m, M$.

¹⁷Note that the areas between the curves $\mathcal{L}^M(\mathbf{z}, p)$ and $\mathcal{L}^e(\mathbf{z}, p)$ and the curves $\mathcal{L}^e(\mathbf{z}, p)$ and $\mathcal{L}^m(\mathbf{z}, p)$ are the same for specific functional form of $\mathcal{L}(y, p)$ and for certain values of z (see the appendix for further details).

¹⁸The latter may happen without that the two curves coincide.

¹⁹For a full discussion on rank-based measures of association, see Dardanoni and Lambert (2001), Atkinson and Lakner (2017), Aaberge, Atkinson and Knigs (2018).

Although it may seem of little interest to consider negative values of the index, they have a powerful meaning in terms of income composition dynamics, as stated by the following Proposition.

Proposition 2.1. *Let $sign_{t,t+1}$ be the sign of $\mathcal{I}_f^t(\mathbf{z}) \cdot \mathcal{I}_f^{t+1}(\mathbf{z})$, where $\mathcal{I}_f^t(\mathbf{z})$ is the income-factor concentration index at time t , while $\mathcal{I}_f^{t+1}(\mathbf{z})$ the one at time $t+1$. We say that a change in the structure of income composition across the distribution of income occurs at time t if $sign_{t,t+1} < 0$.*

When a change in sign occurs at time $t+1$ (i.e., $sign_{t,t+1} < 0$), those individuals who mainly own source z at time t , earn mainly source z_- at time $t+1$ and vice versa. The normalization coefficient $\mathcal{B}^m(\mathbf{z})$ is a function of $\mathcal{L}(\mathbf{y}, p)$, z and p'' , while the coefficient $\mathcal{B}^M(\mathbf{z})$ is a function of $\mathcal{L}(\mathbf{y}, p)$, z and p' . To simplify the notation, let us generally denote by $\mathcal{B}(\mathbf{z})$ the denominator of the income-factor concentration index. A more compact expression for the index is, for $z = \pi$, as follows:

$$\mathcal{I}_f(\boldsymbol{\pi}) = \frac{w\pi(\tilde{\mu}_w - \tilde{\mu}_\pi)}{\mathcal{B}(\boldsymbol{\pi})}, \quad (12)$$

where $\tilde{\mu}_\pi = \frac{1}{2n} \sum_{i=0}^n \left(\sum_{j=0}^i \alpha_j + \sum_{j=0}^{i+1} \alpha_j \right)$ and $\tilde{\mu}_w = \frac{1}{2n} \sum_{i=0}^n \left(\sum_{j=0}^i \beta_j + \sum_{j=0}^{i+1} \beta_j \right)$ are the areas of the concentration curves for labor and capital multiplied by $\frac{1}{w}$ and $\frac{1}{\pi}$ respectively.²⁰ Similarly, for $z = w$, we have:

$$\mathcal{I}_f(\mathbf{w}) = \frac{w\pi(\tilde{\mu}_\pi - \tilde{\mu}_w)}{\mathcal{B}(\mathbf{w})}. \quad (13)$$

Equations 12 and 13 simply mean to illustrate the functional forms of this indicator once we separately analyze the concentration of capital and labor at the top, respectively. Specifically, when equation 12 is positive, then the capital is concentrated primarily at the top of the income distribution and the labor at the bottom. Conversely, when equation 13 is positive, then the labor is concentrated primarily at the top of the income distribution and the capital at the bottom. As we have previously discussed, the following relationship holds true: $\mathcal{I}_f(\boldsymbol{\pi}) = -\mathcal{I}_f(\mathbf{w})$.

The two functions $\tilde{\mu}_\pi$ and $\tilde{\mu}_w$ have a precise dynamics: They increase (decrease) when

²⁰Note that one minus twice $\tilde{\mu}_z$ gives the pseudo-Gini of income source z (see [Shorrocks, 1982](#)).

the source in question moves towards the bottom (top) of the distribution. These areas can thus be considered as approximate metrics of income-factor concentration.²¹ In a similar manner, the function $\tilde{\mu}_y$ is a measure of income inequality: When it rises so does the surface of the Lorenz curve, by therefore reducing its distance from the egalitarian line.

At a first glance, this indicator may bear resemblance to the *pseudo-Gini coefficient*, firstly proposed by Fei et al. (1978). However, these two metrics are very different from each other. Let us consider, for instance, the pseudo-Gini for capital income $\bar{\mathcal{G}}_\pi$, which can be written in the following way: $\bar{\mathcal{G}}_\pi = 1 - 2\tilde{\mu}_\pi$. This indicator is equal to zero when all individuals have the same *absolute level* of capital income, regardless of whether their total incomes may differ. Let me better illustrate this point with a simple example. Suppose we have a population of three individuals, whose relative income shares are described by the following vector $(y_1, y_2, y_3) = (\frac{1}{10}, \frac{3}{10}, \frac{6}{10})$. The pseudo-Gini coefficient is equal to zero when the vector of the relative shares of capital income is of the following form $(\alpha_1, \alpha_2, \alpha_3) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. Now, given that individual 1 has the same share of capital income of individual 3, this makes the former individual more of a capital owner than the latter individual. Therefore, in a society as such, an increase in the capital share of income would rise the income of individual 1 relatively more than the income of individual 3. For this reason, the pseudo-Gini coefficient cannot be regarded as a measure of the relationship between the functional and personal income distribution.

2.5 From Functional to Personal Distribution of Income

In this Section, we further investigate the relationship between functional income distribution and income inequality, in light of the novel metric previously illustrated. To this end, let us consider the well-known relationship between $\tilde{\mu}_y$ (the area of the Lorenz

²¹We can also observe that the term $\tilde{\mu}_\pi$ (and similarly $\tilde{\mu}_w$ and $\tilde{\mu}_y$) can be expressed as follows: $\tilde{\mu}_\pi = \sum_{i=1}^n \alpha_i \left(\frac{2n-2i+1}{2n}\right)$. It suffices to note that $\tilde{\mu}_\pi = \frac{1}{2n} \sum_{i=0}^n \left(\sum_{j=0}^i \alpha_j + \sum_{j=0}^{i+1} \alpha_j\right) = \frac{1}{2n} \sum_{i=1}^n \left(2 \sum_{j=1}^i \alpha_j + \alpha_i\right) = \frac{1}{n} \sum_{i=0}^n \sum_{j=0}^i \alpha_j + \frac{1}{2n} \sum_{i=0}^n \alpha_i$, from which we obtain the result.

curve) and the Gini coefficient:

$$\mathcal{G} = 1 - 2\tilde{\mu}_y. \quad (14)$$

The latter can be further developed, so to obtain:

$$\mathcal{G} = 1 - 2(z(\tilde{\mu}_z - \tilde{\mu}_{z_-}) + \tilde{\mu}_{z_-}). \quad (15)$$

The Gini coefficient can therefore be expressed as a function of the two approximate metrics of income-factor concentration $\tilde{\mu}_z$ and $\tilde{\mu}_{z_-}$ and of the factor share z . If we take the derivative of \mathcal{G} with respect to z , we obtain:

$$\frac{\partial \mathcal{G}}{\partial z} = 2(\tilde{\mu}_{z_-} - \tilde{\mu}_z). \quad (16)$$

The elasticity of personal income Gini to changes in the factor shares is (two times) the difference between the areas of the two concentration curves. Note that when $\tilde{\mu}_{z_-} - \tilde{\mu}_z < 0$, then an increase in the capital share reduces income inequality.

If we consider the standard decomposition of total income Gini into inequality contributed by each income source:²²

$$\mathcal{G} = z\mathcal{R}_z\mathcal{G}_z + z_-\mathcal{R}_{z_-}\mathcal{G}_{z_-}, \quad (17)$$

where $\mathcal{R}_z = \frac{\text{cov}(r(y), z)}{\text{cov}(r(z), z)}$ is the correlation ratio between the source z and total income, $r(y)$ and $r(z)$ are the individual's ranks according to total income and source z respectively and \mathcal{G}_z is the Gini coefficient of income source z , we can write:

$$\frac{\partial \mathcal{G}}{\partial z} = \mathcal{R}_z\mathcal{G}_z - \mathcal{R}_{z_-}\mathcal{G}_{z_-}, \quad (18)$$

and by combining both equations 16 and 18 we get:

$$2(\tilde{\mu}_{z_-} - \tilde{\mu}_z) = \mathcal{R}_z\mathcal{G}_z - \mathcal{R}_{z_-}\mathcal{G}_{z_-}. \quad (19)$$

According to Milanovic, “for the rising share of capital income to increase overall income Gini, we need therefore to have two ‘transmission’ tools, Gini coefficient of capital income and \mathcal{R}_π , positive and high” (Milanovic, 2017), or, from a more technical perspective, the

²²See Lerman and Yitzhaki (1985) for further details.

following condition must hold: $\mathcal{R}_\pi \mathcal{G}_\pi > \mathcal{R}_w \mathcal{G}_w$. It appears that such condition is well captured by the sign of the income-factor concentration index, which exclusively depends on $\tilde{\mu}_{z_-} - \tilde{\mu}_z$. Therefore, equation 16 shows that, for the analysis of the relationship between the functional and personal distribution of income, the indicator we propose can be considered a tool capable of linking these two concepts. For example, if the capital share of income was rising, then income inequality would grow only if the income-factor concentration index was greater than zero.

3 The Case of a Two-Person Economy

Let us now consider the scenario in which the population is divided into two groups (i.e., $n = 2$) of equal size. This exercise is of interest for two main reasons. The first reason is that, due to the lack of data, it may be difficult sometimes to compute the index previously illustrated, which requires information concerning the composition of individuals' income for the entire population.²³ The second reason is that the $n = 2$ version of the index has some interesting mathematical properties which deserve to be exposed.

Let us denote by y_p the income of the bottom $p\%$ of the income distribution and by y_{1-p} the income of the top $(1 - p)\%$, with $y_p \in]0, \frac{1}{2}[$. Figure 2 provides us with a graphical representation of the concentration curves for $n = 2$. The income-factor concentration index with $n = 2$ takes the following mathematical form:

$$\mathcal{I}_{f,2}(\mathbf{z}, p) = b_{z,p} w \pi (\eta_p^{z_-} - \eta_p^z) = t_{z,p} \rho I_f(\mathbf{z}, p), \quad (20)$$

where $\rho = w\pi$, $I_f(\mathbf{z}, p) = \eta_p^{z_-} - \eta_p^z$ and the normalization coefficient $b_{z,p}$ is defined as follows:

$$b_{z,p} = \begin{cases} \frac{1}{y_p^z} & \text{if } y_p > \eta_p^z \\ \frac{1}{[\min(y_p, z) - y_p^z]} & \text{if } y_p < \eta_p^z \end{cases}.$$

²³This information is generally provided by the surveys, which, however, tend to underestimate the income of individuals at the top of the distribution.

Concentration Curves for Income Source with $n = 2$

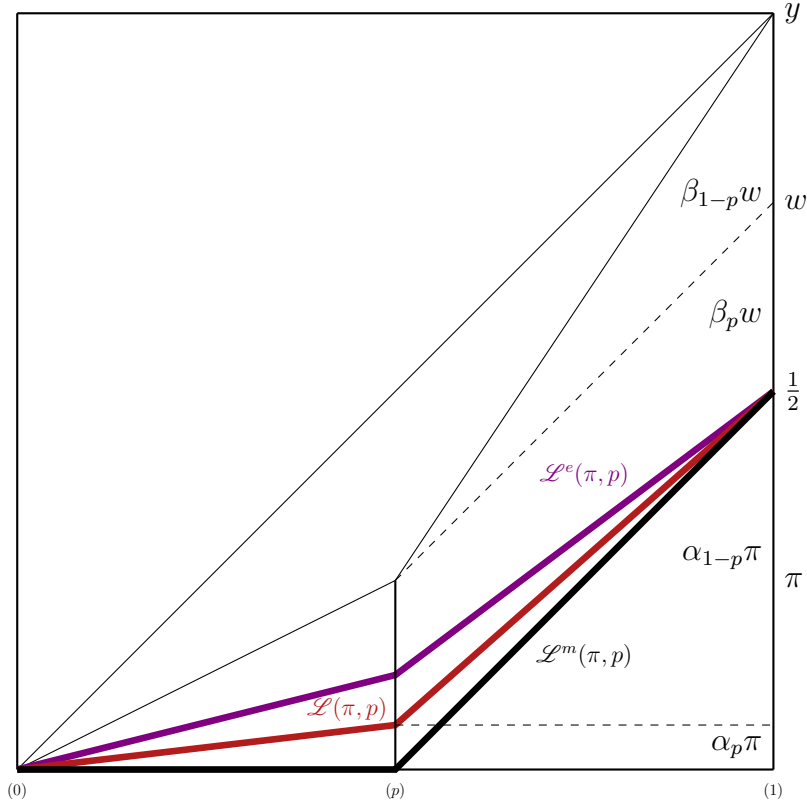


Figure 2: A graphical representation of the methodology in which two people (or groups) with different income ($y_p < y_{1-p}$, with $p = \frac{1}{2}$) and two sources of the same amount ($\pi = w$) are compared. The carnelian line $\mathcal{L}(\pi, p)$ is the concentration curve for capital, the violet line $\mathcal{L}^e(p)$ is the zero-concentration curve, while the black line $\mathcal{L}^m(\pi, p)$ is the maximum-concentration curve. The following values have been here assigned: $y_p = 0.25$, $\pi = w = \frac{1}{2}$, $\alpha_p = 0.12$ and $\beta_p = 0.38$.

This version can thus be regarded as the product of three elements, notably $t_{z,p}$, ρ and $I_f(\mathbf{z}, p)$. An interesting way to grasp their meaning is to rewrite the index as follows:

$$\mathcal{I}_{f,2}(\mathbf{z}, p) = b_{z,p} \begin{vmatrix} w & 0 \\ 0 & \pi \end{vmatrix} \begin{vmatrix} \eta_p^{z-} & \eta_p^z \\ \eta_{1-p}^{z-} & \eta_{1-p}^z \end{vmatrix},$$

where the product of the two determinants ρ and $I_f(\mathbf{z}, p)$ is simply the determinant of the following matrix A :

$$A = \begin{pmatrix} \eta_p^{z-} w & \eta_p^z \pi \\ \eta_{1-p}^{z-} w & \eta_{1-p}^z \pi \end{pmatrix}.$$

The $\mathcal{I}_{f,2}(\mathbf{z}, p)$ can hence be rewritten as the product between the determinants of two matrices and a normalizing coefficient. The first determinant, ρ , adjusts the degree of concentration for the level of income sources. The second determinant, $I_f(\mathbf{z}, p)$, is, instead, the channel through which the issue of income-source concentration is addressed. Interestingly, we can note that the following matrix A^* :

$$A^* = \begin{pmatrix} \beta_p & \alpha_p \\ \beta_{1-p} & \alpha_{1-p} \end{pmatrix},$$

whose determinant equals the component $I_f(\mathbf{z}, p)$, comes from the following relationship:

$$\bar{y} = A^* \bar{x},$$

where $\bar{y} = \begin{pmatrix} y_p \\ y_{1-p} \end{pmatrix}$ and $\bar{x} = \begin{pmatrix} w \\ \pi \end{pmatrix}$, which in turns is equivalent to the following system of equations:

$$\begin{cases} y_p = \beta_p w + \alpha_p \pi \\ y_{1-p} = \beta_{1-p} w + \alpha_{1-p} \pi \end{cases}.$$

When the matrix A^* is nonsingular (i.e., $\det A^* \neq 0$, thus $I_f(\mathbf{z}, p) \neq 0$), then we can write: $\bar{x} = (A^*)^{-1} \bar{y}$. It is of interest to observe that when $\det A^* = 1$, then ownerships of labor and capital are separated between individual 1 and 2. This explains why the coefficient $I_f(\mathbf{z}, p)$ can be considered as a proxy of $\mathcal{I}_f(\mathbf{z})$.

Another way of writing the $n = 2$ version of this indicator is the following. Assume that $y_1 < w$ and $\mathbf{z} = \boldsymbol{\pi}$, then we have:²⁴

$$\mathcal{I}_{f,2}(\boldsymbol{\pi}, p) = 1 - \frac{\alpha_1}{y_1}. \quad (21)$$

Equation 21 illustrates that the level of income composition inequality is in this very case determined by the ratio $\frac{\alpha_1}{y_1}$. This ratio combines individual 1's endowment of capital and overall income. When the ratio is greater than one, then individual 1 is more capital

²⁴This is a plausible assumption: In the contrary case, the labor share of income would have been lower than the capital share. In fact, if y_1 was greater than w , given that $y_1 < \frac{1}{2}$ by assumption, then we would have $w < \pi$. The latter is not supported by the empirical evidences concerning the developed countries (Stockhammer, 2013), with the exception of Mexico (Negrete, 2015).

poor than income poor. When it is equal to one, then she is as capital poor as income poor and when it is lower than one, then she is more income poor than capital poor. Therefore, income composition inequality is positive when the poorest part of the society is more capital poor than income poor and negative in the opposite case.

Let us now illustrate several properties of the $I_f(\mathbf{z}, p)$. First, the capital to labor ratio can be expressed as follows:

$$\frac{\pi}{w} = \frac{\frac{1}{1+\varphi} - \beta_{1-p}}{\frac{\varphi}{1+\varphi} - \alpha_p}, \quad (22)$$

where $\varphi = \frac{y_p}{y_{1-p}}$, from which we simply derive the following result.

Proposition 3.1. *A variation in φ has no effect on $\frac{\pi}{w}$ iff $I_f(\mathbf{z}, p) = 0$. Formally:*

$$\frac{\partial \frac{\pi}{w}}{\partial \varphi} = 0 \iff I_f(\mathbf{z}, p) = 0. \quad (23)$$

Proposition 23 sheds light on the relationship between income inequality, as measured by the ratio φ and factor shares of income ($\frac{\pi}{w}$).²⁵ A variation in φ does not affect the ratio $\frac{\pi}{w}$ when the determinant of the matrix A^* is equal to zero.

Let us now consider the relationship between the determinant $I_f(\mathbf{z}, p)$ and the between-group Gini coefficient G .²⁶ Precisely:

$$\frac{\partial G}{\partial z} = I_f(\mathbf{z}_-, p) p. \quad (24)$$

An increase in the factor share z reduces the between-group inequality G according to the degree of income-source concentration and the share of poor people, p . If we let p be equal to $\frac{1}{2}$ (thus we divide the population into two groups of equal size) and if we set $z = \pi$, then we get:

$$\frac{\partial G}{\partial \pi} = \frac{\alpha_{\frac{1}{2}} - \beta_{\frac{1}{2}}}{2}. \quad (25)$$

Equation 25 bears resemblance with equation 16. Specifically, in a two-person economy the condition for the rising share of capital income to increase income Gini is $I_f(\mathbf{z}, p) > 0$, or $\det A^* > 0$.

²⁵It is easy to note that $\frac{\partial \frac{\pi}{w}}{\partial \varphi} > 0$ when $I_f(\mathbf{z}, p) > 0$. In particular, when $I_f(\mathbf{z}, p) = 1$, an increase in φ raises the ratio $\frac{\pi}{w}$ of the same amount. Indeed, when $I_f(\mathbf{z}, p) = 1$ then $y_p = \pi$ and $y_{1-p} = w$, thus $\frac{\pi}{w} = \frac{y_p}{y_{1-p}}$.

²⁶See the appendix for further details.

4 Empirical Application

In this Section, we apply the method previously illustrated to the case of six European economies, namely Finland, France, Germany, Italy, Norway and The Netherlands. The data used come from the European Union Statistics of Income and Living Conditions (EU-SILC), which provide a representative sample of the European population. This data are firstly produced by the national statistical offices and later harmonized and released by Eurostat. In our analysis, we consider the period between 2007 and 2016. The country samples vary between 7000 and 19000 units and the unit of analysis is the household.

The choice of these specific countries is determined by their puzzling trends. Indeed, while income composition inequality follows a U-shaped trend for Germany, Norway and The Netherlands over the period considered, the reverse pattern is traced by Finland, France and Italy.

Our analysis relies on specific definitions of capital and labor incomes.²⁷ Precisely, we define capital income as the sum of household income from rental of a property or land, interests, dividends, capital from capital investments in unincorporated business, the capital component of gross cash benefits or losses from self-employment (including royalties) and pensions from individual private plans. The capital component of self-employment income, which is not directly provided by EU-SILC, is imputed by means of the procedure proposed by Glyn (2011). We attribute the average payroll income of the entire sample to represent the labor income component of the self-employed and “the margin of value added per head [...] is then regarded as accruing to the [self-employer] as property income” (Glyn, 2011, p. 8).²⁸ As stated by Glyn, none of the methods

²⁷The definitions of capital and labor income can be, to a certain extent, arbitrary. For instance, Cirillo, Corsi and D’Ippoliti (2017), who investigate the dynamics of the functional and personal distributions of income at the European level before and after the crisis, and who also rely on EU-SILC data, provide a slightly different definition of capital and labor income from that we propose. Indeed, their definition of income does not include self-employment remuneration.

²⁸Precisely, we firstly compute the country average payroll income per household, $\mu_{payroll}$. Then, we decompose self-employment income into its capital and labor components in the following way. Let us denote by y_{se} the income from self-employment provided by EU-SILC. We can write that $y_{se} = y_{se}^{\pi} + y_{se}^w$, where $y_{se}^w = \begin{cases} y_{se} & \text{if } y_{se} \leq \mu_{payroll} \\ \mu_{payroll} & \text{if } y_{se} > \mu_{payroll} \end{cases}$, while $y_{se}^{\pi} = \begin{cases} 0 & \text{if } y_{se} \leq \mu_{payroll} \\ y_{se} - \mu_{payroll} & \text{if } y_{se} > \mu_{payroll} \end{cases}$.

adopted by the literature to decompose the self-employment income into its labor and capital components is wholly unproblematic. In this respect, also our method presents some issues. Namely, by considering the economy average payroll income as a threshold to determine the capital and labor components of self-employment income, we risk to underestimate the capital component for those sectors in which the sectorial average payroll income is lower than the economy average payroll income, and viceversa. However, we believe this decomposition to be more sophisticated than that which automatically attributes two thirds of the self-employment income in its labor component, and one third in its capital component. Furthermore, note that to the best of our knowledge, this is the first time such method is applied to decompose self-employment income at the micro level (as this method is generally adopted to decompose macroeconomic variables).

Labor income is defined as the difference between total household gross income minus capital income.²⁹

To overcome the issue of negative values, we replace the bottom part of the concentration curves for which such problem occurs with the horizontal line (i.e., the x -axis).³⁰

Let us begin the analysis with some descriptive statistics. Table 1 presents the income shares of four different income groups, defined as follows: *0-50%*, *50-90%*, *90-95%* and *95-100%*. These shares are computed for the six countries in 2007 and 2016, with the exception of Italy (2007 and 2015) due to missing information.

²⁹The sources of labor income that we consider are: gross employee cash or near cash income, gross non-cash employee income, employers' social insurance contributions, value of goods produced for own consumption, unemployment benefits, old-age benefits, survivor' benefits, sickness benefits, disability benefits, education-related allowances, family/children related allowances, social exclusion not elsewhere classified, housing allowances, regular inter-household cash transfers received and income received by people aged under 16.

³⁰When a given variable at stake displays negative values, the bottom part of the corresponding concentration curve lies below the horizontal axe.

Table 1: Income Shares

Income Group	Norway		Germany		Netherlands		Italy		France		Finland	
	2007	2016	2007	2016	2007	2016	2007	2015	2007	2016	2007	2016
<i>0-50%</i>	24%	24%	23%	23%	26%	22%	18%	19%	24%	24%	22%	23%
<i>50-90%</i>	50%	49%	48%	48%	47%	47%	48%	48%	48%	46%	49%	48%
<i>90-95%</i>	9%	9%	10%	10%	9%	10%	11%	11%	10%	9%	10%	10%
<i>95-100%</i>	15%	17%	17%	17%	16%	19%	19%	19%	16%	18%	17%	17%
<i>Gini</i>	0.36	0.37	0.38	0.39	0.34	0.39	0.45	0.45	0.37	0.38	0.39	0.39
<i>IFC</i>	0.51	0.59	0.48	0.50	0.54	0.63	0.53	0.49	0.38	0.46	0.46	0.45

The distribution of total income is almost the same in 2007 and 2016 for all countries, with the sole exception of The Netherlands, where the 3% of total income has moved from the bottom 50% to the top 5%, by hampering the Gini coefficient of 0.5 percentage points. Italy displays the higher level of total income inequality, with a Gini coefficient above 0.4, and a $\frac{\text{top } 5\%}{\text{bottom } 50\%}$ ratio lower than 1 in 2007, and equal to 1 in 2015.

Tables 2 and 3, instead, show the distributions of the capital and labor shares respectively, with individuals being indexed by their income ranking. Following [Shorrocks \(1982\)](#) and [Atkinson and Lakner \(2018\)](#), we may call these shares as “pseudo-shares”. Let us take a closer look at Table 2. A simple way to read this Table is the following: in 2007 in Norway the individuals at the bottom 50% of the total income distribution earn the 13% of total capital income in the economy. The share of the capital income earned by the same income group in 2016 is 4% points lower than the share in 2007.

Table 2: Capital Shares (Income Ranking)

Income Group	Norway		Germany		Netherlands		Italy		France		Finland	
	2007	2016	2007	2016	2007	2016	2007	2015	2007	2016	2007	2016
<i>0-50%</i>	13%	9%	11%	9%	11%	6%	5%	7%	14%	15%	10%	9%
<i>50-90%</i>	22%	20%	25%	30%	22%	23%	30%	33%	29%	29%	31%	36%
<i>90-95%</i>	7%	9%	9%	8%	8%	9%	13%	15%	12%	12%	12%	13%
<i>95-100%</i>	57%	61%	53%	52%	57%	60%	49%	44%	43%	46%	45%	39%
<i>Capital Share</i>	6%	8%	9%	6%	7%	7%	17%	16%	8%	12%	13%	10%
<i>Gini</i>	0.36	0.37	0.38	0.39	0.34	0.39	0.45	0.45	0.37	0.38	0.39	0.39
<i>IFC</i>	0.51	0.59	0.48	0.50	0.54	0.63	0.53	0.49	0.38	0.46	0.46	0.45

The first thing to notice is that in Norway, Germany and The Netherlands the individuals at the top 5% of the income distribution earn more than 50% of all capital income in the economy. On the contrary, in Italy, France and Finland the share of capital income earned by the same income group is lower than 50%. Moreover, differently from the total income distribution, the capital income distribution has changed over the period considered in almost all countries. However, it is hard to identify clear patterns between the six countries.

In Norway and The Netherlands the capital income has moved from the bottom 50% to the top 5% between 2007 and 2016. A similar scenario can be described for France, although less markedly than in the previous two countries. In Italy and Finland, instead, the top 5% has seen a reduction of its capital income share. However, while in Italy the capital income has shifted towards the bottom 50% and the middle class (50 – 90%), in Finland it has moved towards the middle class only. Finally, in Germany the middle class has gained capital income from both the top 10% and the bottom 50%.

Table 3: Labor Shares (Income Ranking)

Income Group	Norway		Germany		Netherlands		Italy		France		Finland	
	2007	2016	2007	2016	2007	2016	2007	2015	2007	2016	2007	2016
<i>0-50%</i>	25%	25%	25%	24%	27%	24%	21%	21%	25%	26%	24%	24%
<i>50-90%</i>	51%	51%	51%	50%	49%	49%	53%	52%	50%	49%	51%	50%
<i>90-95%</i>	9%	9%	10%	10%	9%	10%	10%	10%	9%	9%	9%	10%
<i>95-100%</i>	12%	12%	13%	14%	13%	15%	14%	15%	14%	14%	13%	14%
<i>Labor Share</i>	94%	92%	91%	94%	93%	93%	83%	84%	92%	88%	87%	90%
<i>Gini</i>	0.36	0.37	0.38	0.39	0.34	0.39	0.45	0.45	0.37	0.38	0.39	0.39
<i>IFC</i>	0.51	0.59	0.48	0.50	0.54	0.63	0.53	0.49	0.38	0.46	0.46	0.45

Table 3, instead, shows that very little changes have been registered in the labor income distribution for almost all countries between 2007 and 2016. It is only worth mentioning that in The Netherlands the labor income has mainly moved from the bottom 50% to the top 10%. Furthermore, in each country the middle class earns, on average, the 50% of all the labor income in the economy.

At this point of the analysis, we may address the following question: How to make sense

to all this information? As shown in Table 1, the dynamics of the four income shares is relatively well captured by the dynamics of the Gini coefficient. However, what can we say about the joint dynamics of the capital and labor shares? Are the capital and labor incomes better distributed across the populations or, rather, more concentrated at the top and at the bottom of the income distributions? Do these countries resemble more to a classical capitalism, characterized by the existence of social classes, or rather to a multiple sources of income new capitalism? To answer these questions, we apply the method we developed in the previous sections.

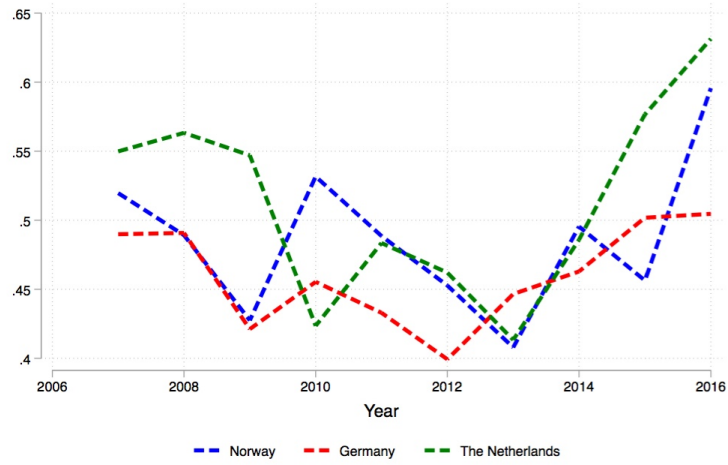
Figure 3 shows the overall dynamics of income composition inequality for the two groups of European countries, respectively. The first group is composed by Germany, Norway and The Netherlands, whilst the second by Italy, France and Finland. To begin with, note that the IFC index ranges between 0.4 and 0.6 in all countries. However, as we said at the beginning of this section, the two groups follow different trends over the time considered. Income composition inequality in the first group follows a U-shaped pattern, with its major peaks in 2007 and 2016, and its lowest peak in 2013-2014 (a).

On the contrary, income composition inequality in the second group follows an inverted U-shaped curve, with its peaks in 2011/2012 and its lowest levels at the beginning, and at the end of the period considered (b).

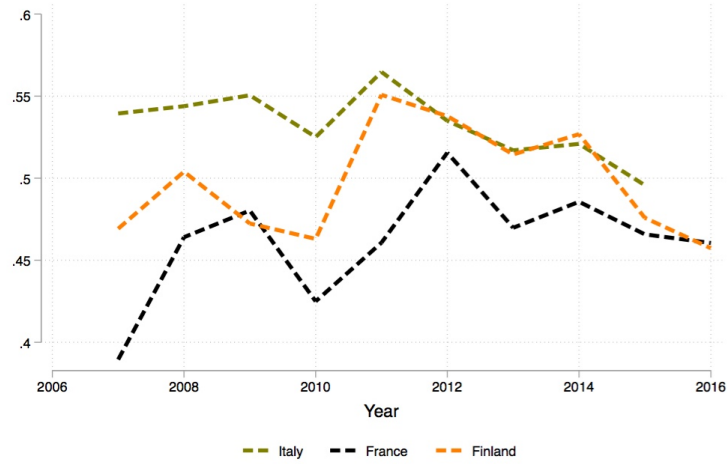
The patterns of Germany and The Netherlands almost coincide in both levels and trends, whilst that of Norway starts from a lower level in 2007, and ends with a higher level of income composition inequality in 2016.

Instead, all the countries of the second group exhibit a similar dynamic of income composition inequality. However, France and Italy show higher levels of the IFC index as compared to Finland. Following the framework previously discussed, we can say that the first three countries considered are moving towards becoming a classical capitalism, characterized by a group of rich people owning capital income and a group of poor people owning labor income. This type of society allows for a greater transmission of changes in the functional distribution of income into personal income inequality. Conversely, the second group of countries is moving towards becoming a new capitalism, in which both sources of income are better distributed across the entire population. In the latter society,

Income Composition Inequality



(a)



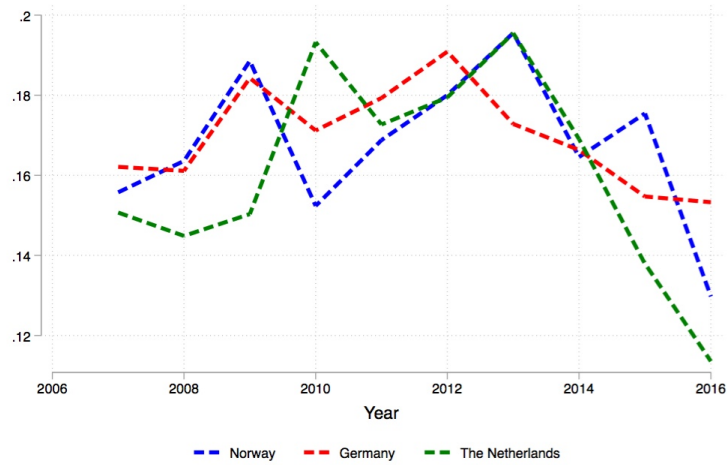
(b)

Figure 3: The series of income composition inequality 2007-2016. **Source:** Author's computation on basis of EU-SILC.

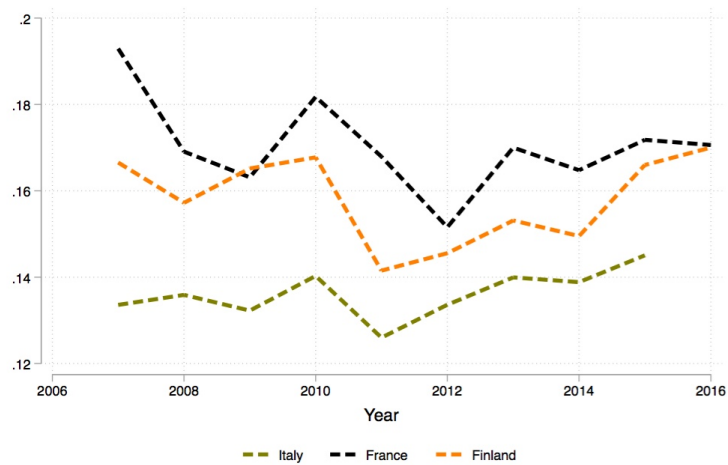
the relationship between functional and personal distribution of income is relatively weak, implying that fluctuations in both the capital and labor shares of income have a less severe impact on the dynamics of income inequality.

At this point of the analysis, let us analyze the role played by the two components of the iIFC index, notably $\tilde{\mu}_w$ and $\tilde{\mu}_\pi$, in shaping its overall dynamics. The evolution of the areas of the concentration curves for capital and labor are illustrated by Figures 4 and

Area of the Concentration Curve for Capital



(a)



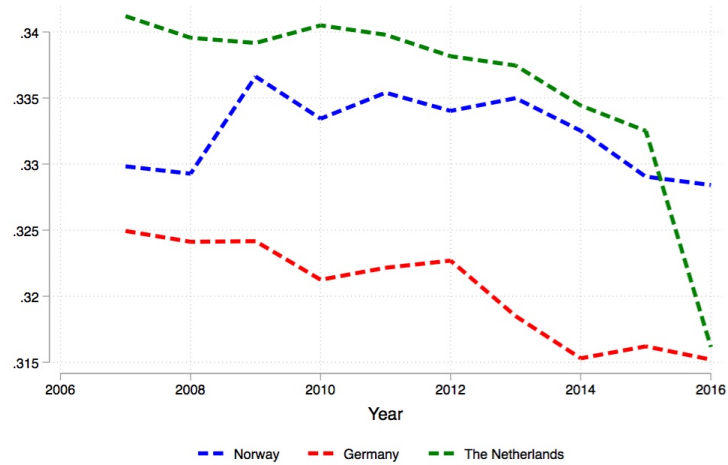
(b)

Figure 4: The series of the area of the concentration curves for capital income 2007-2016. **Source:** Author's computation on basis of EU-SILC.

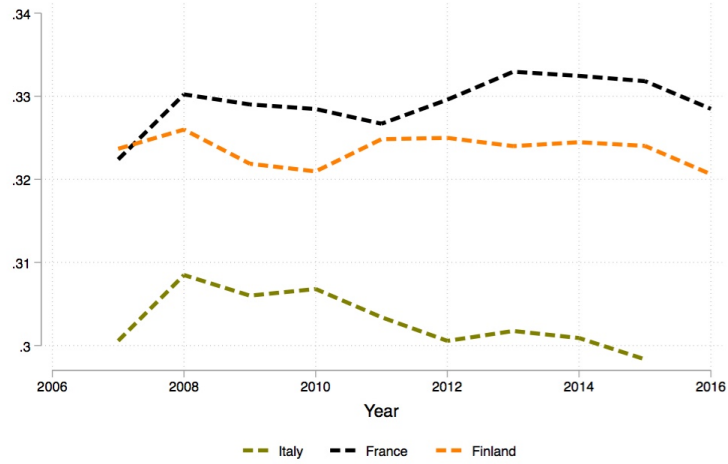
5. As already illustrated by Tables 2 and 3, the two metrics $\tilde{\mu}_w$ and $\tilde{\mu}_\pi$ follow completely independent patterns. Let us begin with the first group. For all countries the area of the concentration curve for capital rises up to 2013 and falls afterwards (Figure 4).

We remind that an increase (decrease) in $\tilde{\mu}_\pi$ implies that the capital income moves towards the bottom (top) of the income distribution. Therefore, we can state that Germany, Norway and The Netherlands saw their capital income flowing in the hands

Area of the Concentration Curve for Labor



(a)



(b)

Figure 5: The series of the area of the concentration curves for labor income 2007-2016. **Source:** Author's computation on basis of EU-SILC.

of the bottom part of the income distribution up to 2013 and afterwards coming back into possession of the rich part of the population. At the same time, the almost flat motion of the area of the concentration curve for labor $\tilde{\mu}_w$ for all the countries (Figure 5) clearly suggests that the principal driver of income composition inequality was the fluctuation in capital income. A different story can be told with regards to the second group of countries. The evolution of income composition inequality for Finland, France

and Italy, also here determined by the behavior of capital income, has been characterized by the latter moving firstly towards the top (up to 2013) and later towards the bottom of the income distribution (from 2013 onwards).

Nevertheless, to explain the major causes behind the dynamics of income composition inequality in the countries considered, we should further explore the same countries' political contexts. In addition to that, given the nature of the data we use, which indeed tend to underestimate the income of the individuals at the top of the distribution,³¹ our results should be taken with extreme caution.

5 Conclusion

One of the most important findings from Piketty's *Capital in the Twenty-First Century* (Piketty, 2014) is the rise in the capital share of income in many developed countries over the last decades (see also Piketty, 2015). Similar results were also found by Stockhammer (2013), who shows that the labour shares has fallen over the past 25 years in the OECD countries. The dynamics of the capital share of income (and, hence, of the labor share) is influenced by many macroeconomic phenomena, such as technical change, globalization, financialisation, bargaining power and market power of firms, among others (see Stockhammer et al. 2018). The rise in the capital share of income is generally considered to be one of the causes that led to the increase in personal income inequality (Piketty, 2014; Bengtsson and Waldenstrom, 2018). However, the study of the link between changes in the capital share of income and changes in personal income inequality needs to be further investigated. For this reason, the present paper proposed a methodology to examine the relationship between the functional and personal distribution of income. To this end, it introduced the concept of inequality in income composition. If we decompose total income into *two* factors, such as capital and labour income, then income composition inequality is the extent to which the income composition is distributed unevenly across the population. Inequality in income composition is *maximal* when individuals at the top and at the bottom of the income distribution separately earn

³¹See Jenkins (2017) for an overview of the issues which raise the survey data.

the two different types of income. On the contrary, it is *minimal* when each individual earns the same composition of the two factors. Under a high level of income composition inequality the link between the functional and personal distribution of income is strong, whereas under a low level of income composition inequality the link is weak. We then constructed a summary statistics to measure income composition inequality, and applied it to study the evolution of income composition inequality in six European economies. We showed that this summary statistics can be looked at in two ways. Firstly, from a technical perspective, it can be considered an elasticity of personal income inequality to fluctuations in the functional income distribution. In other words, it mathematically links the functional and personal distribution of income. Secondly, from a political economy perspective, it measures the “degree of capitalism” of a given social system. While we use capital and labor as income sources in this paper, it is important to underline that the study of income composition inequality can be applied to analyze the joint distribution of *any pair* of income (or wealth) components, such as net income and taxes, saving and consumption, and financial and non-financial assets, among others. Such flexible nature of the framework proposed paves therefore the way for further research on the topic.

References

- [1] Aaberge, R., Atkinson, A. B., and Knigs, S. 2018. *From Classes to Copulas: Labor, capital, and top incomes*. Discussion Papers, Statistics Norway Research department, No. 876.
- [2] Alacevich, M., Soci, A. 2017. *Inequality: A Short History*. Brookings Institution.
- [3] Araar, A., 2008. *On the decomposition of concentration indices: Illustrations with Chinese and Nigerian household surveys*. CIRPEE Working Paper No. 08-06.
- [4] Atkinson, A. B., 1997. *Bringing Income Distribution in From the Cold*. The Economic Journal, Vol. 107, No. 441, pp. 297-321.
- [5] Atkinson, A. B., Bourguignon, F., 2000. *Introduction: Income Distribution and Economics*. In A.B.Atkinson and F.Bourguignon (eds), Handbook of Income Distribution, Geneva, Elsevier Science.
- [6] Atkinson, A. B., 2009. *Factor shares: the principal problem of political economy?* Oxford Review of Economic Policy, Vol. 25, Number 1, 3-16.
- [7] Atkinson, A. B., Lakner, C., 2017. *Capital and Labor. The Factor Income Composition of Top Incomes in the United States, 1962-2006*. Policy Research Working Paper 8268. World Bank.
- [8] Bengtsson, E., Waldenstrom, D. 2015 *Capital Shares and Income inequality: Evidence from the Long Run*. IZA DP No. 9581.
- [9] Bertola, G., Foellmi, R., Zweimuller, J., 2005. *Income Distribution in Macroeconomic Models*. Princeton University Press.
- [10] Brandolini, A., 1992. *Nonlinear Dynamics, Entitlement Rules, and the Cyclical Behaviour of the Personal Income Distribution*. Centre for Economic Performance, Discussion Paper No. 84.

- [11] Cirillo, V., Corsi, M., D'Ippoliti, C., 2017. *European households' incomes since the crisis*. *Investigacion Economics*, vol. LXXVI, num. 301, pp. 57-85.
- [12] Dardanoni, V., Lambert, P., 2001. *Horizontal inequality comparisons*. *Social Choice Welfare*, 18: 799-817.
- [13] Deutsch, J., Silber, J., (2010). *Analyzing the Impact of Income Sources on Changes in Bi-Concentration*. in *The Measurement of Individual Well-Being and Group Inequalities: Essays in Memory of Z. M. Berrebi*, edited by Deutsch, J., and Silber, J., Routledge Economics, Taylor and Francis Group, pp. 127-152.
- [14] Deutsch, J., Fusco, A., Silber, J., (2013). *The BIP Trilogy (Bipolarization, Inequality and Concentration): One Saga but Three Different Stories*. *Economics: The Open-Access, Open-Assessment E- Journal*, Vol. 7, 2013-22.
- [15] Esteban, J., Ray, D., 1991. *On the Measurement of Concentration*. Boston University, Institute for Economic Development, Working Paper 18.
- [16] Esteban, J., Ray, D., 1994. *On the Measurement of Concentration*. *Econometrica*, 62(4): 819-851.
- [17] Fei, J., Ranis, G., Kuo, S., 1978. *Growth and the family distribution of income by factor components*. *Quarterly Journal of Economics*.
- [18] Foster, J. E., Wolfson, M. C., 1992. *Concentration and the Decline of the Middle Class: Canada and the US*. Ophi Working Paper No. 31.
- [19] Foster, J. E., Wolfson, M. C., 2010. *Concentration and the Decline of the Middle Class: Canada and the US*. *The Journal of Economic Inequality*, Volume 8, Issue 2, pp 247-273.
- [20] Francese, M., Mulas-Granados, C. 2015. *Functional Income Distribution and Its Role in Explaining Inequality*. IMF Working Paper, WP/15/244.
- [21] Gabbuti, G., forthcoming. *Labour Shares and Income Inequality: Insights from Italian Economic History, 1895-2015*.

- [22] Glyn, A., 2011. *Functional Distribution and Inequality*. The Oxford Handbook of Economic Inequality. Edited by Nolan, B., Salverda, W., and Smeeding, T. M.
- [23] Jenkins, S. P., 2017. *Pareto Models, Top Incomes and Recent Trends in UK Income Inequality*. *Economica*, Volume 84, Issue 334, pp 261-289.
- [24] Kakwani, N. C., 1977. *Applications of Lorenz Curves in Economic Analysis*. *Econometrica*, Vol. 45, No. 3, pp. 719-728
- [25] Kakwani, N. C., 1977. *Measurement of Tax Progressivity: An International Comparison*. *The Economic Journal*, Vol. 87, No. 345, pp. 71-8
- [26] Kaldor, N., 1955. *Alternative Theories of Distribution*. *The Review of Economic Studies*, Vol. 23, No. 2, pp. 83-100.
- [27] Lerman, R. I., Yitzhaki, S. 1985. *Income Inequality Effects by Income Source: A New Approach and Applications to the United States*. *The Review of Economics and Statistics*. Vol. 67, No. 1, pp. 151-156.
- [28] Milanovic, B., 2017. *Increasing Capital Income Share and its Effect on Personal Income Inequality*. In *After Piketty. The Agenda for Economics and Inequality*. Edited by Heather Boushey, J. Bradford DeLong, Marshall Steinbaum. Ch. 10.
- [29] Negrete, M. d. C. *La magnitud de la desigualdad en el ingreso y la riqueza en México*. CEPAL - Serie Estudios y Prospectivas - México - N 167, 2015.
- [30] Pasinetti, L., 1962. *Rate of Profit and Income Distribution in Relation to the Rate of Economic Growth*. *The Review of Economic Studies*, Vol. 29, No. 4, pp. 267-279.
- [31] Piketty, T. 2014. *Capital in the 21st century*. Harvard University Press.
- [32] Ranaldi, M., 2017. *Income-Factor Concentration: A Methodological Approach*. Documents de travail du Centre d'Economie de la Sorbonne 2017.54 - ISSN : 1955-611X.
- [33] Ricardo, D., 1911. *Principles of Political Economy*. London, Dent, first published.

- [34] Shorrocks, A. F., 1982. *Inequality Decomposition by Factor Components*. *Econometrica*, Vol. 50, No. 1, pp. 193-211.
- [35] Shorrocks, A. F., 1983. *Ranking Income Distributions*. *Economica*, Vol. 50, No. 197, pp. 3-17.
- [36] Stiglitz, J., 2015. *Income and Wealth Among Individuals: Part III: Life Cycle Savings vs. Inherited Savings*. NBER.
- [37] Stockhammer, E. *Why have labor shares fallen? A panel analysis of the determinants of functional income distribution*. *Conditions of Work and Employment Series* No. 35. ILO. 2012.

A Sign of the Indicator

In order to show that $\mathcal{I}_f(\mathbf{z}) = -\mathcal{I}_f(\mathbf{z}_-)$, we need to proof that $\mathcal{B}^m(\mathbf{z}) = \mathcal{B}^M(\mathbf{z}_-)$. The latter relationship states that the denominator of $\mathcal{I}_f(\mathbf{z})$ equals that of $\mathcal{I}_f(\mathbf{z}_-)$. To this end, we should consider two different maximum-concentration curves.

From equations 12 and 13 we can simply notice that $\mathcal{A}(\mathbf{z}) = -\mathcal{A}(\mathbf{z}_-)$, thus that the numerator of the income-factor concentration index changes its sign (and not its absolute level) according to the source we analyze. Without loss of generality, if we assume that source z is mainly concentrated at the top, and that source z_- at the bottom, then the relationship $\mathcal{B}^m(\mathbf{z}) = \mathcal{B}^M(\mathbf{z}_-)$ can be written as follows:

$$\int_0^1 z \mathcal{L}(\mathbf{y}, p) dp - \int_{p^*}^1 [\mathcal{L}(\mathbf{z}, p) - z_-] dp = \int_0^{p^{**}} \mathcal{L}(\mathbf{y}, p) dp + (1 - p^{**}) z_- - \int_0^1 z_- \mathcal{L}(\mathbf{y}, p) dp.$$

Considering that p^* and p^{**} are such that $\mathcal{L}(\mathbf{y}, p^*) = \mathcal{L}(\mathbf{y}, p^{**}) = z_-$, then $p^* = p^{**}$, and the relationship holds true.

B Normalization Coefficient

As stated before in this article, we show that for specific functional forms of the Lorenz curve for income $\mathcal{L}(\mathbf{y}, p)$, and for specific values of z (and, thus, z_-), the following relationship holds true:

$$\mathcal{L}^M(\mathbf{z}) - \mathcal{L}^e(\mathbf{z}) = \mathcal{L}^e(\mathbf{z}) - \mathcal{L}^m(\mathbf{z}). \quad (26)$$

For simplicity, let us move to the continuous space. Suppose, therefore, that we have three continuous distribution functions: \mathbf{y} , $\boldsymbol{\pi}$, \mathbf{w} . The relationship 26 is equivalent to the following one:

$$z \int_0^1 \mathcal{L}(\mathbf{y}, p) dp - \int_{p''}^1 (\mathcal{L}(\mathbf{y}, p) - z_-) dp = \int_0^{p'} \mathcal{L}(\mathbf{y}, p) dp + (1 - p') z - z \int_0^1 \mathcal{L}(\mathbf{y}, p) dp. \quad (27)$$

We remember that p' s.t. $\mathcal{L}(\mathbf{y}, p') = z$, p'' s.t. $\mathcal{L}(\mathbf{y}, p'') = 1 - z$ and $\mathbf{z} = \boldsymbol{\pi}, \mathbf{w}$. From equation 27 we can write:

$$2z \int_0^1 \mathcal{L}(\mathbf{y}, p) dp = \int_{p''}^1 (\mathcal{L}(\mathbf{y}, p) - z_-) dp + \int_0^{p'} \mathcal{L}(\mathbf{y}, p) dp + (1 - p') z.$$

If we call $p' = f(z)$ and $p'' = f(z_-)$, where $f(y) = \mathcal{L}^{-1}(\mathbf{y}, p)$ then, after further manipulations, we get:

$$\int_0^1 \mathcal{L}(\mathbf{y}, p) dp = 1 + \frac{1}{z - z_-} \int_{f(z)}^{f(z_-)} \mathcal{L}(\mathbf{y}, p) dp + \frac{zf(z) - z_-f(z_-)}{z - z_-},$$

which is true only if the following relationship is satisfied:

$$(z - z_-) \int_0^1 f(y) dy = \int_z^{z_-} f(y) dy. \quad (28)$$

Note that equation 28 is true for $\pi = w$, $\pi = 1$, $w = 1$, regardless of the functional form of \mathcal{L} , and for the family of functions f of the form $f(x) = x^n$, for $n = 1, +\infty, -\infty$ only.

C Result 3.1

Provided that $y_p = \alpha_p \pi + \beta_p w$, and $y_{1-p} = \alpha_{1-p} \pi + \beta_{1-p} w$, where $y_p + y_{1-p} = y = \pi + w$, we can write:

$$\begin{aligned} \varphi &= \frac{\beta_p w + \alpha_p \pi}{\beta_{1-p} w + \alpha_{1-p} \pi}, & \iff \\ y_p(\beta_{1-p} w + \alpha_{1-p} \pi) &= y_{1-p}(\beta_p w + \alpha_p \pi), & \iff \\ \frac{\pi}{w} &= \frac{\beta_p y_{1-p} - \beta_{1-p} y_p}{\alpha_{1-p} y_p - \alpha_p y_{1-p}}, & \iff \\ \frac{\pi}{w} &= \frac{\beta_p - \varphi \beta_{1-p}}{-\alpha_p + \varphi \alpha_{1-p}}, & \iff \\ \frac{\pi}{w} &= \frac{1 - (1 - \varphi) \beta_{1-p}}{\varphi - (1 - \varphi) \alpha_p}, & \iff \\ \frac{\pi}{w} &= \frac{\frac{1}{1-\varphi} - \beta_{1-p}}{\frac{\varphi}{1+\varphi} - \alpha_p}. \end{aligned}$$

If we now take the first derivative of $\frac{\pi}{w}$ with respect to φ and we further manipulate, we obtain result 3.1.

D Relationship between Gini and IFC for $n = 2$

Let us rewrite y_1 (from equation 1) as follows:

$$y_1 = \beta_1 w \pm \alpha_1 w + \alpha_1 \pi.$$

After some algebraic manipulations, we get:

$$y_1 = I_f w + \alpha_1,$$

where I_f is the distribution's component of the index \mathcal{I}_f when $n = 2$. Let us now recall the expression of the Gini coefficient:

$$G = 1 - \sum_{k=1}^n (x_{k+1} - x_k)(y_{k+1} + y_k),$$

where the whole population is divided into n groups, and x_k, y_k represent the bottom $x_k\%$ of the population, and their cumulative income respectively. When $n = 2$ we can write:

$$G = 1 - py_p - (1 - p)y_p,$$

where p is the share of the poor group, and $(1 - p)$ the share of the rich. The following can be derived:

$$G = 1 - p(I_f w + \alpha_p) - (1 - p)$$

whence:³²

$$G = p(\alpha_{1-p} - I_f w)$$

from which, by taking the derivative with respect to w , we obtain equation 24.

³²It can be noticed that $G = p(\alpha_{1-p} - I_f w) = p(1 - y_p) = py_{1-p}$, which is a different way to express the two-groups Gini coefficient. Indeed, it clearly appears from the equation that inequality rises when either the share of poor people increases, or when the income share of the rich augments.