Cheap talk, monitoring and collusion

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Abstract

Many collusive agreements involve the exchange of self-reported sales data between competitors, which use them to monitor compliance with a target market share allocation. Such communication may facilitate collusion even if it is unverifiable cheap talk and the underlying information becomes publicly available with a delay. The exchange of sales information may allow firms to implement incentive-compatible market share reallocation mechanisms after unexpected swings, limiting the recourse to price wars. Such communication may allow firms to earn profits that could not be earned in any collusive, symmetric pure-strategy equilibrium without communication.

1 Introduction

This paper shows that in a market where demand is uncertain and data on other firms' sales become available with a delay, the early exchange of sales reports between firms can make collusion more efficient even if such communication is cheap talk. Our main finding is that - for some parameter values - collusion with near-monopoly pricing in all periods along the equilibrium path can occur *only* if communication is possible, even though such communication does not make sales data verifiable sooner.

The novel theoretical mechanism that is identified in this paper can contribute to the ongoing debate on the proper antitrust treatment of information exchanges between firms. The assumptions of our model and also the collusive equilibrium with communication that we derive exhibit features that are similar

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to those that are observed in many recent cartels:[1] Firms compete in prices; self-reported sales volumes are not instantaneously verifiable, but they can be compared to reliable data that become public with a lag;[2] the collusive scheme is based on a target market share allocation; firms exchange detailed sales reports before these data becomes verifiable; and when these reports point to a discrepancy between actual and target market shares, companies that sold above their quotas take steps to decrease their sales for a while: compensation takes the form of a temporary reallocation of market shares rather than inter-firm payments.

According to Harrington (2006), whereas in some recent cartels participants compensated market share swings by making payments to each other - often under the guise of interfirm sales; in other cases the compensation was through market share reallocations of the kind that is highlighted in this paper, as the continuous monitoring of sales volumes afforded colluding firms *"the opportunity to adjust their sales"*.

The equilibrium we derive is symmetric, which limits the need for pre-play coordination. It involves pure strategies along the equilibrium path, with no need for coordination on a public randomization device.

Overview of the mechanism that is involved The possibility for firms to collude more efficiently by exchanging reports on their own sales arises from the following difference between undercutting competitors and misreporting sales: Past misreporting can be spotted with certainty once verifiable sales information becomes public, whereas, if demand is noisy enough, undercutting cannot be distinguished from demand shocks.

In a collusive equilibrium with near-monopoly profits, price wars should be present (almost) only as an off-equilibrium threat. Since lying - unlike price undercutting - can be inferred with certainty once verifiable sales data become available, the threat of a price war can be used to deter lying at no cost to total profits, but not to deter undercutting.

The expectation that being caught lying will cause a price war induces firms to report their sales truthfully; in turn, their accurate reports allow firms to

[1] Harrington (2006), Levenstein and Suslow (2006). See in particular the developments in these papers on the lysine, copper plumbing, zinc phosphate, citric acid and vitamins cartels.

[2] Reliable information can come from companies' annual reports of from import statistics, or it can be collected and disseminated by a professional association - in some cases with the help of independent auditors. In several recent cases, market share information was found to become available with a delay of about one year.

correct market share imbalances sooner, without waiting until verifiable data become available. This quicker reallocation of market shares implies that firms have less to gain by secretly cutting their prices and pretending they were just lucky. This in turn facilitates the enforcement of collusive discipline through market share reallocations - without a need for costly price wars to take place along the equilibrium path.

Organization of the paper After a review of the related literature (section 2), we present the model (section 3) and derive a necessary condition for a pure-strategy symmetric equilibrium without communication to yield profits that are bounded away from some near-monopoly level (section 4). We then characterize a sufficient condition for such near-monopoly profits to be attainable with communication, by constructing a specific equilibrium (section 5). These results jointly yield a sufficient condition for communication to expand the set of attainable profit levels (section 6). The conclusion discusses the implications of these results for antitrust policy.

2 Relation to the literature

The theoretical literature on the role of communication in collusion comprises two main sets of contributions:

Several papers address pre-play communication in the presence of private information on costs or demand.[3] Another branch of the literature addresses the role of communication as a private monitoring tool in repeated games. Some papers derive folk theorems that show that infinitely patient players may achieve maximum profits if they can communicate to facilitate mutual monitoring.[4]

Aoyagi (2002), Harrington and Skrzypacz (2011, 'HS' hereafter), and Chan and Zhang (2015, 'CZ' hereafter) are closely related to this paper since they address the role of non-verifiable sales reports. They construct collusive equilibria in which companies monitor each other by exchanging such non-verifiable reports.

In contrast to these papers, ours provides a comparative statics result about the incremental impact on the feasibility of collusion of communication about past market outcomes.[5] Our assumptions depart from those in Aoyagi (2002),

[3] Athey and Bagwell (2001, 2008); see also Aoyagi (2007), Harrington (2017).

[4] Compte (1998), Kandori and Matsushima (1998), Obara (2009). See also Rahman (2014).

[5] Athey and Bagwell (2001, 2008) also derive comparative statics results on the role of

HS, and CZ. They assume that neither price nor sales data ever become public, whereas we assume that sales data become public with a delay.[6] Also, HS and CZ assume that firms can make direct payments to each other, whereas we rule out such payments.

None of these features is inherently more relevant than the others, since both types of information structures and both types of corrective mechanisms are observed. The corrective market share reallocation mechanism highlighted in this paper, however, is probably better suited to the analysis of the cases for which information exchanges are the main legal question - at least from the viewpoint of policy relevance. This is because frequent interfirm payments may provide evidence of unlawful coordination that makes the analysis of information exchanges superfluous for the legal assessment.

To our knowledge, Awaya and Krishna (2016, 'AK' hereafter, which is extended in Awaya and Krishna, 2019) is the only paper that derives conditions under which communication about past sales increases collusive profits relative to the 'best' equilibrium without communication. However, our model and AK's are relevant to different settings. AK assume that sales data remain private forever. Also, the nature of communication and the way that it affects firms' behavior in the collusive equilibrium are very different in their model and in ours.[7]

3 The model

The main result of this paper relies on the following ingredients: (i) sales information (at the firm level, or in aggregate form) becomes public after a lag; (ii) prices are private information; (iii) demand can be zero, which prevents a firm from inferring that a rival undercut the collusive price simply by observing its own zero sales; (iv) asymmetric demand shocks are possible, so that it is not possible to infer from asymmetric market shares that a firm undercut the

communication. But communication in their models is pre-play coordination meant to account for private information on costs.

[6] On the general analysis of repeated games in which information on other players' actions is revealed with lags, see Abreu, Milgrom, and Pearce (1991), and Fudenberg, Ishii, and Kominers (2014) on the impact of cheap-talk communication in such games. Igami and Sugaya (2021) estimate the impact of the lag on the possibility of collusion on the basis of a stylized model of the vitamins cartel.

[7] Another related paper is Mouraviev (2014), which shows that communication can increase collusive profits, albeit in an environment that is very different from ours since firms can communicate verifiable information and the only limit on communication comes from the risk of being caught.

collusive price; and (v) aggregate demand is stochastic, so that a firm cannot infer its market share from its own sales.

The model presented hereafter relies on an admittedly special, ad hoc demand function. This is the price to pay in order to obtain a tractable enough model that yields a comparative statics result on the impact of communication - rather than merely the characterization of a collusive equilibrium with communication - with an equilibrium that closely resembles what has been observed in several cartels.

3.1 Supply and demand

There are n ($n \geq 3$) identical firms that produce a heterogeneous good at constant marginal cost $c > 0$, and face the same discount factor δ. The assumption that there are at least three firms simplifies the informational structure because it makes it easier to identify which firm deviated from a hypothetical collusive equilibrium when market shares are asymmetric, as is explained in greater detail below (section 6.2).

There is a continuum of consumers. Its mass is normalized to 1.

The demand function depends on the state of the world, which is drawn from some (constant) probability distribution. The draws are independent across periods. A state of the world is characterized by two parameters: total demand (which is price-inelastic); and whether demand is 'normal' or 'biased' towards one firm. Total demand can take any value within an interval $S = [0, S_{max}]$, with $S_{max} > 0$.

Consumers have unit demand with valuation $V > c$. Demand is either 'normal' or 'biased', as defined hereafter.

Demand in the 'normal' states of the world In a normal state of the world such that total demand is Q, consumers consider all n goods as perfect substitutes. Sales are equally split among all of the firms that set the lowest price if that price is lower than or equal to V.

Demand in the 'biased' states of the world In these states of the world, demand is biased in favor of Firm i for some i: Consumers' willingness to pay for product i is drawn from some probability distribution ν over some interval (V, V') with $V' > V$, whereas their willingness to pay for other firms' products is V. For the sake of tractability, ν is assumed to be the uniform distribution.

The ratio $(V' - V)/(V - c)$ is denoted u. If consumers are indifferent between several firms' offers, demand is split evenly across them.

The probability distribution over demand functions The following assumptions about the states of the world and their probability are made throughout the paper.

Irrespective of whether demand is normal or biased, the probability distribution over total demand is absolutely continuous.

The demand function is symmetric across firms in the sense that (i) the distribution of total demand conditional on demand's being biased is independent of the identity of the firm in favor of which demand is biased; and (ii) the probability that demand is biased in favor of any particular firm is the same for all firms.

For simplicity, and without any loss of generality, we assume that total expected demand conditional on demand being biased is equal to total expected demand conditional on demand being normal. Let D denote this common expected value.

Demand is biased with some probability γ^B $(0 < \gamma^B < 1)$. Conditional on demand's being biased, the distribution of total demand is given by the probability density function $\mu^B(.)$ over $(0, S_{max})$, with $\mu^B(Q) > 0$ for all $Q \in (0, S_{max})$.

Conditional on demand being normal, the distribution of total demand is characterized by the probability distribution μ^N, which has an atom in zero and is characterized by a continuous density function $\mu^N(Q) > 0$ for $Q \in (0, S_{max})$. The overall probability that demand is zero is denoted γ^L (these notations imply that $\mu^N(\{0\}) = \frac{\gamma^L}{1-\gamma^B}$). $Max_Q \frac{\mu^N(Q)}{\mu^N(nQ)}$, $Max_Q \frac{\mu^N(Q)}{\mu^B(Q)}$ and $Max_Q \frac{\mu^B(Q)}{\mu^N(Q)}$ are assumed to be finite.

3.2 The game

The game is repeated for infinitely many periods, starting in period 1.

Each period is divided into the following stages.

Stage 1. Firms simultaneously set prices.

Stage 2. The state of the world is determined at random.

Stage 3. Each firm observes the demand addressed to it and serves it.

Stage 4 (when communication is possible). Firms simultaneously make a statement about their own sales.

At the end of period t, a firm observes only its own sales, the sales that have been made by the other firms in period $t-1$ (if $t \geq 2$) and other firms' sales reports (in the variant with communication).

We consider only pure strategies (see the discussion below). In the case when communication is not possible, a strategy is an infinite set of functions $(s_1, s_2, ..., s_t, ...)$, where s_t maps all the information that is known to a firm at the beginning of period t - its own past prices and sales and other firms' past observed sales until period $t-2$ - into the set of possible prices.

When communication is possible, a strategy is an infinite set of functions $(s_1^p, s_1^m, ..., s_t^p, s_t^m ...)$, where s_t^p maps all of the information that is known to a firm at the beginning of period t - its own past prices and sales and other firms' past observed sales and reports - into the set of possible prices, and s_t^m maps all of the information that is known to a firm at the end of period t - the information known at the beginning of the period plus its own period t price and sales - into the set of possible sales reports.

4 The scope for collusion without communication

In this section, we address the scope for collusion when communication is impossible. We restrict our attention to pure-strategy equilibria, which is in line with our focus on simple collusive behavior.[8] Also, we consider only equilibria that are symmetric in the sense that a firm's strategy is invariant to a permutation of other firms' identities, and all firms follow the same strategy.[9]

Proposition 1 provides an upper bound to the expected payoffs of pure-strategy, subgame-perfect symmetric equilibria (hereafter, 'PSSE') when communication is not possible. We start by introducing the following notation:

$$A = Max\left(\mu^N\left(\left(\frac{S_{max}}{n}, S_{max}\right)\right), \mu^B\left(\left(\frac{S_{max}}{n}, S_{max}\right)\right)\right).$$

A is an upper bound on the probability that aggregate demand is such that, if a firm captures the entire demand, it can infer its 100% market share by observing

[8]Several recent papers - such as Acemoglu, Bimpikis and Ozdaglar (2009) and Gentzkow and Kamenica (2017) - focus on equilibria that involve pure strategies along all equilibrium paths - but allow however mixed strategies off-equilibrium. It must be acknowledged however that collusive behavior that is based on mixed strategies along the equilibrium path seems to be observed in some industries (Wang, 2009) - in line with theoretical results that show that mixing can improve efficiency (Kandori and Obara, 1998).

[9]Models of collusion often make this assumption (see the discussion in Athey, Bagwell, and Sanchirico, 2004).

its own sales without any need for more information.

Proposition 1. *If the inequality*

$$1 + \delta + \delta^2 + (1 - A)\delta^3 > \frac{1}{n\,(1-\delta)} \tag{1}$$

holds, and γ^L, γ^B, $\frac{\gamma^B u}{\gamma^L}$ and $\frac{nu}{(1-\delta)\delta^3\gamma^B\gamma^L(1-A)}$ are close enough to zero, then total expected per-period profits in all PSSE are bounded away from $(V-c)D$: There exists a $\sigma > 0$ such that in any PSSE, the expected sum of all firms' future discounted profits is smaller than or equal to $\frac{(V-c)D}{1-\delta}(1-\sigma)$.

Condition (1) has a simple economic interpretation. Absent communication, in a collusive equilibrium with close-to-monopoly profits, a firm cannot be required to cut its sales (by setting a price above the monopoly level) until it knows (almost) surely that demand was biased in its favor: Otherwise, costly mistakes would happen after high-demand periods, as all firms -observing their own high sales - would mistakenly believe that they had a high share of a small demand pool when in fact each had $1/n$-th of a large one, and firms would simultaneously set a too-high price in an unjustified attempt to reduce their future sales.

This implies that if a firm undercuts the monopoly price in period 1, the market share imbalance is detected at the end of period 2, and the equilibrium strategies cannot mandate a market share reallocation to start sooner than in period 3. Failure to comply with the prescriptions of this market share reallocation would be detected at the end of period 4, and a price war would not be launched until period 5. A deviator could thus profitably undercut during at least four periods.

The left-hand side reflects the corresponding deviation profits, whereas the right-hand side reflects the profits that are yielded by compliance with a hypothetical collusive equilibrium. Condition (1) simply means that undercutting would be profitable, so that no collusive equilibrium with near-monopoly profits exists.[10]

[10]The above reasoning works unless total demand in period 1 is such that a firm would be able to infer that it had a 100% market share from observing its own sales, without any need for other firms' sales data. This is the case with probability less than or equal to A, hence the factor $(1-A)$ in (1).

5 The scope for collusion with communication

We consider now the same environment as before, with one addition: At the end of every period, each firm is required to report its sales. The key to the main result is that such communication can expose deviations sooner. To show this, we construct an efficient PSSE with communication, which has the property that a hypothetical deviator would be exposed after three periods.

Consider a firm that undercuts its competitors in period 1 and then enjoys a 100% market share. If this firm lies about its sales, its lie is exposed at the end of period 2. If it does not lie, then, since all firms know their market shares at the end of period 1, one can construct a collusive equilibrium that prescribes that a 'lucky' firm should withdraw from the market in period 2 in order to offset its period 1 luck. Failure to do so would be detected when period 2 sales are observed by all, at the end of period 3. In either case, a deviator cannot conceal its deviation for more than three periods (leaving aside the possibility of zero demand in some periods).

If sales reports are exchanged at the end of each period, a firm thus has room to deviate from a hypothetical collusive agreement and profitably undercut during three periods only, as opposed to four when communication is ruled out. This underlies the main result of this section - Proposition 2 - which establishes sufficient conditions for the existence of a PSSE with near-monopoly profits.

5.1 Description of the candidate equilibrium

We consider the following strategy profiles. There exist integers k and k' and a price $p^w < c$ such that, at the beginning of a period, the state of the game can be any of the following $(nk + k' + 1)$:

- normal collusion;
- j-th period of a price war $(1 \leq j \leq k')$;
- correction at the expense of some firm i $(1 \leq i \leq n)$ (the 'targeted' firm), with r remaining correction periods $(1 \leq r \leq k)$ (nk states of the game).

In a nutshell, normal collusion gives way to temporary correction phases whenever sales reports point to asymmetric sales that are compatible with equilibrium behavior: a distribution of sales that can be explained by biased demand. During a correction phase, the firm that sold more than the others in the last normal collusion period sells zero while others set a price V, after which firms return to normal collusion. Any unambiguous evidence of a deviation - such as price undercutting or sales misreporting - leads to a price war. [11]

The expected duration of a price war and the corresponding below-cost price are such that a firm's expected discounted sum of future profits at the start of a price war is zero. Price wars do not occur in equilibrium.

Prices and messages in the candidate equilibrium Equilibrium actions at the beginning of period t depend only on the state of the game at the beginning of period t:

- If the state of the game at the beginning of period t is 'normal collusion', then all firms set a price equal to V;
- If the state of the game at the beginning of period t is 'collusion with a correction at the expense of Firm i', then Firm i sets a price equal to $V' + 1$, whereas all other firms set a price equal to V;
- If the state of the game at the beginning of period t is 'price war', then all firms set a price equal to p^w.

Also, in the candidate equilibrium, firms truthfully report their sales.

Transitions between states of the game in the candidate equilibrium The state of the game at the beginning of period 1 is 'normal collusion'. At the beginning of period t ($t \geq 2$), the state of the game is determined as follows.

- Deviations trigger a price war: If the reported period $(t-1)$ sales are incompatible with the prices that are prescribed by the candidate equilibrium in period $(t-1)$, or (if $t \geq 3$) the reports on period $(t-2)$ sales at the end of period $(t-2)$ do not match the actual period $(t-2)$ sales as observed at the end of period $(t-1)$, then the state of the world in the next period is the first period of a price war.

In all other cases - if the newly available information provides no evidence of a deviation - transition rules are as follows:

[11] As is explained below, transition rules are a bit more complex than this summary description, to account for the possibility that during a correction phase at the expense of some firm, another firm may benefit from biased demand and become the target of a new correction phase.

- Normal collusion is followed by normal collusion if period $(t-1)$ reported sales are equal; and otherwise a correction phase with k remaining periods at the expense of the only firm that reported non-zero sales.
- Price wars start over unless reported sales are equal and strictly positive. If reported sales are equal and strictly positive at the end of a price war period, the state of the game in the next period is a price war with one less period left, or normal collusion if the previous period was the last one (the k'-th one) of a price war.
- At the end of a correction period at the expense of Firm i with r remaining periods, if the reported period $(t-1)$ sales of all firms other than Firm i are equal, then at the beginning of period t the state of the game is 'normal collusion' if $r = 1$ and 'correction at the expense of Firm i with $(r-1)$ remaining periods' if $r > 1$. If only one firm (say Firm j, $j \neq i$) reported nonzero sales, then at the beginning of period t the state of the world is 'correction at the expense of Firm j, with k correction periods remaining'.

This candidate equilibrium is symmetric; it involves only pure strategies; and it yields expected total profits $(V-c)D$ in every period.

5.2 A sufficient condition for the candidate equilibrium to be an actual equilibrium

Proposition 2. *If γ^B and γ^L are close enough to zero and there exists an integer* k *such that Conditions (2) and (3)*

$$n(1-\delta)(1+\delta) < \delta^k \tag{2}$$

$$n(1-\delta) + \delta^{k+1} < 1 \tag{3}$$

hold, and γ^B, γ^L and u are close enough to zero, then there exist a positive integer k' and a price p^w such that the above strategy profiles correspond to a PSSE - with correction phases lasting k *periods and price wars lasting* k*' periods with price p^w. In this equilibrium, expected total profits are $(V-c)D$ in all periods.*

Conditions (2) and (3) have simple economic interpretations: (2) can be written $(1+\delta)(V-c)D < \frac{\delta^k}{1-\delta}\frac{(V-c)D}{n}$. This inequality means that it is optimal for a firm that had a 100% market share to comply with the equilibrium requirement that it should sell zero for k periods before a return to 'normal collusion', rather than: misreporting sales; undercutting its competitors; being detected after two periods; and then triggering a price war (with profits equal to the left-hand side side). (3) is equivalent to $(V-c)D + \frac{\delta^{k+1}}{1-\delta}\frac{(V-c)D}{n} < \frac{1}{1-\delta}\frac{(V-c)D}{n}$. This inequality implies that it is more profitable for a firm not to undercut in a 'normal collusion' period, rather than to: undercut; capture 100% of the market; and then go through a long period of market share compensation.

6 The marginal impact of communication

6.1 A comparative statics result

Combining Propositions 1 and 2 provides a sufficient condition for communication to expand the set of the profits that are attainable in a PSSE: This is the case if conditions (1), (2), and (3) are simultaneously satisfied. Proposition 3 - the main result of this paper - states that this may be the case for some parameter values. The economic interpretation is straightforward: With communication, a profitable deviation could yield near-monopoly profits during three periods, as opposed to four without communication. Enforcing collusive discipline is thus easier, and for some parameter values collusion at near-monopoly prices is an equilibrium only if communication is possible.

Proposition 3. *For each* n *between 3 and 10, there exist parameter values such that, if communication is possible, there exists a PSSE that leads to a discounted sum of total expected profits* $\frac{(V-c)D}{1-\delta}$*; whereas if communication is not possible, this sum, considered over all PSSE, is bounded away from* $\frac{(V-c)D}{1-\delta}$.

6.2 The limits of the above results

The granularity of sales data. The above results do not require an informational assumption as strong as ours. The strategies in the equilibrium with communication could be implemented under the weaker assumption that reliable data on total rather than individual market sales become available after a one-period lag. Such information would allow firms to detect that one of them lied, by comparing actual and reported total sales, and comparing total reported sales with its own individual sales would allow each firm to calculate its market share. Since, in the equilibrium with communication displayed above, the price war following a lie treats all firms symmetrically, the lack of information on a liar's identity would make no difference. Therefore, communication through a third party that collects firms' own-sales reports and communicates to all firms total sales data based on these reports would be sufficient for Proposition 2 to hold.[12]

At least three firms. With three firms or more, if a firm sets a price above consumers' valuation V in a 'normal collusion' period, the ensuing sales distribution is incompatible with any equilibrium path if demand is neither zero nor biased since all firms but one sell nonzero amounts, and there is no way for a firm to conceal such a deviation (it could not report the same sales as other firms because it would not know them), so that a price war would ensue. With only two firms, such a deviation cannot be ruled out as simply: If a firm sets a price above the monopoly level, this would lead to sales that looking similar to the result of demand that is biased in favor of its competitor, which could trigger a correction phase at the expense of this competitor. Having at least three firms eliminates the need to consider such deviations.

No interfirm payments. Our results would not hold if interfirm payments were possible. In that case, a sales imbalance in period 1 could trigger compensating payments at the end of period 2, just after it is publicly observed, even without communication.

Symmetric strategies. The restriction to symmetric strategies is crucial to our results. Absent this restriction, one could envision collusive equilibria without communication such that companies take turns and in each period, one serves the entire demand while the others set prohibitively high prices: This pattern is similar to what is observed when collusion takes place with respect to a series of periodic tenders. The informational problem that communication

[12] This remark is at odds with the oft-made claim that *"aggregating the data [on firms' historical and current prices, costs, and output] largely removes the value of information in facilitating collusion"* (Carlton, Gertner and Rosenfield, 1997). Awaya and Krishna (2020) present another mechanism through which the communication of credible aggregate sales data to individual firms can facilitate collusion: In their model (unlike in their 2016 and 2019 papers) firms have an incentive to misreport sales, hence the need for 'Swiss accountants'. It must be noted that in some models, more accurate information may decrease the set of attainable profits because it may help firms to identify more profitable deviations (Sugaya and Wolitzky, 2018a). However, such a mechanism cannot be present in our model, since the deviations considered here do not rely on the deviating firm's information on other firms' sales.

is meant to solve would not exist because deviations would be spotted as soon as sales data become observable.

Pure strategies. If mixed strategies were considered, one could envision candidate equilibria such that prices are drawn from an atomless distribution. A firm that makes nonzero sales would know that its market share is 100% right after observing its own sales. This would remove the informational problem that communication is meant to solve even without communication.

7 Conclusion

The antitrust handling of information exchanges between firms involves a difficult tradeoff: On the one hand, an outright ban on information exchanges would deny companies and consumers the procompetitive benefits that such exchanges may entail. Conversely, a too-lenient approach would allow companies to engage in practices that could facilitate collusion and harm consumers.[13]

Accordingly, in its guidelines on horizontal cooperation between undertakings,[14] the European Commission states that exchanges of information on past sales are not prohibited *per se* (unlike communication on future behavior) and that they should be assessed under a case-by-case approach. According to K.-U. Kühn, a former Chief Economist of the European Commission, this case-by-case approach should focus on the "marginal impact" of the information exchanges under scrutiny on the likelihood of collusion.[15]

This paper casts light on what the "marginal impact" can be: When sales data become public with a delay, the early exchange of sales reports may facilitate collusion even if it is pure cheap talk in the sense that it does not make any information verifiable sooner.

This suggests that competition authorities should be suspicious of exchanges of information on past sales, even if they appear to be mere cheap talk and there is no evidence of other interfirm contacts. Another possible interpretation is that competition authorities should be concerned by exchanges of verifiable

[13]See Kühn (2001).

[14]Official Journal of the European Union, C 11/91, 14.1.2011, Communication from the Commission — Guidelines on the applicability of Article 101 of the Treaty on the Functioning of the European Union to horizontal co-operation agreements.

[15]Kühn (2011) advocates *"an analysis of the marginal impact of the information exchange on monitoring or the scope for coordination in the market. If the marginal impact appears small, the case should be closed."*

information on past sales even if such exchanges take place with a long lag, because they can facilitate collusion if they are supplemented with cheap talk that takes place with a shorter lag.

In particular, the exchange of sales reports should be considered suspicious if reports that reveal market share swings lead to prompt compensating movements to an extent that cannot be explained by individual firms' unilateral profit-maximizing behavior, given the intertemporal pattern of demand shocks.

Also, since our results continue to hold under the assumption that the data that become public after a lag are about total sales only, they imply that the dissemination of aggregated sales data by trade associations, based on firms' self-reported sales, may facilitate collusion.

Admittedly, our model relies on special and restrictive assumptions about demand. This is in our view the price to pay in order to estimate an upper bound on firms' profits in *all* (pure strategy, symmetric) equilibria without communication, which then allows us to state results on the marginal effect of communication. This is because little is known yet on bounds on equilibrium payoffs in infinitely repeated games under general assumptions (except at the limit when players are almost infinitely patient). However, recent advances on this topic may pave the way for additional results on the marginal impact of communication under less special assumptions.[16] This should be the focus of future research.

Appendix

Proof of Proposition 1. We introduce the following notations: λ_1 denotes $Max\left(\underset{Q\in(0,S_{max})}{Max}\frac{\mu^N(Q)}{\mu^B(Q)}, \underset{Q\in(0,S_{max})}{Max}\frac{\mu^B(Q)}{\mu^N(Q)}\right)$ and λ_2 denotes $\underset{Q\in\left(0,\frac{S_{max}}{n}\right)}{Max}\frac{\mu^N(Q)}{\mu^N(nQ)}$.

We consider a hypothetical PSSE $Eq*$ such that the expected sum of all firms' future discounted profits is greater than $\frac{(V-c)D}{1-\delta}(1-\sigma)$ for some small σ. The core of this proof is the calculation of a lower bound on the profits that some firms - say Firm 1 - could earn by following a certain strategy, which can be interpreted as undercutting in each of the first four periods. This requires intermediate results about the strategies that are followed by Firms 2 to n-conditional on the information available to them.

[16]See in particular Pai, Roth and Ullman (2016) and Sugaya and Wolitzky (2017, 2018b). Awaya and Krishna (2016, 2019) also overcome this difficulty by characterizing an upper bound on collusive profits in the no-communication case under quite general assumptions.

Step 1. Minimum combined profits after certain histories. We define the history of the game until period t as the prior sequence of prices and sales (including period t). We provide hereafter a lower bound on the expectation of total period t profits conditional on past history's belonging to some set H. Let $Prob(H)$ denote the probability, according to $Eq*$, that the history of the game until period $(t-1)$ belongs to H (and let it be equal to 1 if $t=1$). Since total expected profits in any period cannot exceed $(V-c)D(1+\gamma^B u)$, the expected discounted sum of total profits in all periods cannot exceed $(V-c)D(1+\gamma^B u)\left(\frac{1}{1-\delta}-\delta^{t-1}Prob(H)\right)+\delta^{t-1}Prob(H)\prod(H)$, where $\prod(H)$ denotes expected total period t profits conditional on past history belonging to H. This expected discounted sum is greater than $\frac{(V-c)D(1-\sigma)}{1-\delta}$, which implies that $\prod(H)>(V-c)D(1-\alpha_t(Prob(H),\sigma))$ with $\alpha_t(x,\sigma)=\frac{\sigma+\gamma^B u}{x(1-\delta)\delta^{t-1}}$.

Step 2. A quasi-lower bound on equilibrium prices. Assume that in period i, conditional on a certain event (a set of past histories), the expectation (according to $Eq*$) of total profits $\prod$ is greater than $(V-c)D(1-\alpha)$ for some α. Defining $g(\alpha)=\sqrt{\frac{\alpha+\gamma^B u}{1-\gamma^B}}$, the probability (conditional on that same event) that the lowest of all n prices in period i is greater than $c+(V-c)(1-g(\alpha))$ is greater than $1-g(\alpha)$. Proof: Total expected profits cannot exceed $D\left(Min\left(p_t^1,...,p_t^n,V\right)-c\right)$ if demand is normal and $(V-c)D(1+u)$ otherwise. If the probability that $Min\left(p_t^1,...,p_t^n\right)-c<(V-c)\left(1-g(\alpha)\right)$ is greater than $g(\alpha)$, then $\frac{\prod}{(V-c)D}<\gamma^B(1+u)+(1-\gamma^B)\left(1-g(\alpha)+g(\alpha)(1-g(\alpha))\right)=1-\alpha$.

Step 3. The profit from deviating in period 1. Since $Eq*$ is symmetric, it prescribes all firms to set the same price p_1^* in period 1. The total expected profit induced by $Eq*$ in period 1 is thus less than or equal to $D\left(p_1^*-c\right)$, which implies that $p_1^*-c>(V-c)\left(1-g_1\right)$, with $g_1=\alpha_1(1,\sigma)$. If Firm 1 deviates and sets a price $p_1^{dev}=c+(V-c)\left(1-g_1\right)$ in period 1, it serves the entire demand if demand is normal or biased in its favor, which happens with a probability that exceeds $1-\gamma^B$.

Step 4. Firm 1's possible deviation in period 2. Since $Eq*$ is symmetric, there exists $p_2^*(0)$ such that $Eq*$ prescribes a firm that sold zero in period 1 to set price $p_2^*(0)$ in period 2. The equilibrium probability that all firms sell zero in period 1 is γ^L. Step 1 implies that $p_2^*(0)>c+(V-c)\left(1-\alpha_2(\gamma^L,\sigma)\right)$. Let g_2 denote $\alpha_2(\gamma^L,\sigma)$. If Firm 1 deviates in period 2 by setting a price $p_2^{dev}=c+(V-c)\left(1-g_2\right)$ and all other firms made zero profits in period 1, its expected period 2 profit is greater than $(V-c)D\left(1-g_2\right)\left(1-\gamma^B\right)$.

Step 5. Firm 1's possible deviation in period 3. For each Q in $(0,S_{max}]$, let $p_3^*(Q)$ denote the price that is prescribed by $Eq*$ in period 3 for a firm that has observed that (i) it sold zero in period 2; and (ii) in period 1, one of the firms (not itself) sold Q while all others sold 0. We also define $g_3^*(Q)$ by the identity $p_3^*(Q) - c = (V - c)(1 - g_3^*(Q))$. The equilibrium probability that (i) Firms 2 to n had zero sales in periods 1 and 2 and (ii) Firm 1 had nonzero sales in period 1 is greater than $\frac{\gamma^B}{n}\gamma^L$. Steps 1 and 2 imply that conditional on (i) and (ii), with probability greater than $1 - g\left(\alpha_3(\frac{\gamma^B}{n}\gamma^L, \sigma)\right)$, the minimum of all prices in period 3 is above $c + (V - c)\left(1 - g\left(\alpha_3(\frac{\gamma^B}{n}\gamma^L, \sigma)\right)\right)$: There exists $S_3 \subset (0, S_{max})$ such that (i) $\mu^B(S_3) > 1 - g\left(\alpha_3(\frac{\gamma^B}{n}\gamma^L, \sigma)\right)$ and (ii) $\forall Q \in S_3$, $g_3^*(Q) < g_3$. Also, by the definition of λ_1, $\mu^N(S_3) > 1 - \lambda_1 g\left(\alpha_3(\frac{\gamma^B}{n}\gamma^L, \sigma)\right)$. Define $p_3^{dev} = c + (V - c)(1 - g_3)$ with $g_3 = g\left(\alpha_3(\frac{\gamma^B}{n}\gamma^L, \sigma)\right)$. Conditional on Firms 2 to n having sold zero in periods 1 and 2 and on Firm 1's having made nonzero sales in period 1 (which happens with probability greater than $\left(1 - \gamma^B\right)\left(1 - \gamma^B - \gamma^L\right)$ if Firm 1 deviated in the first two periods), if Firm 1 sets price p_3^{dev} in period 3, this price is lower than all other firms' prices with probability greater than $1 - \lambda_1 g_3$, which yields an expected profit greater than $(V - c)(1 - g_3)(1 - \lambda_1 g_3)\left(1 - \gamma^B\right)$.

Step 6. The period 2 prices set by a firm observing its period 1 sales belonged to $\left(0, \frac{S_{max}}{n}\right)$, according to $Eq*$.

For each $Q \in \left(0, \frac{S_{max}}{n}\right)$ let $p_2^*(Q)$ denote the equilibrium price set in period 2 by a firm that observes that it sold Q in period 1. We define the set S' as follows: $S' = \left\{Q \ s.t.\ Q \in \left(0, \frac{S_{max}}{n}\right) \ and\ p_2^*(Q) > V\right\}$. With probability $\left(1 - \gamma^B\right)\mu^N(nS')$ (with nS' denoting the set of all elements of S' multiplied by n), demand is normal and each firm sells some $Q \in S'$ in period 1, which implies that all firms set a price that is strictly above V in period 2, which leads to expected period 2 profits that are smaller than or equal to $\gamma^B(V - c)D(1+u)$. Step 1 implies therefore that $1 - \alpha_2\left(\left(1 - \gamma^B\right)\mu^N(nS'), \sigma\right) < \gamma^B(V - c)D(1 + u)$, or, after rearranging terms, $\mu^N(nS') < \frac{\sigma + \gamma^B u}{(1-\gamma^B)(1-\gamma^B(1+u))\delta}$, which implies $\mu^N(S') < \lambda_2 \frac{\sigma + \gamma^B u}{(1-\gamma^B)(1-\gamma^B(1+u))\delta}$.

Step 7. The probability, according to $Eq*$, that Firm 1's sales belong to $\left(0, \frac{S_{max}}{n}\right) \setminus S'$ in period 1 and to $(0, S_{max})$ in period 2 while all other firms sell zero in periods 1 and 2. According to $Eq*$, the probability that Firm 1's period 1 sales belong to $\left(0, \frac{S_{max}}{n}\right) \setminus S'$ whereas all other firms' period 1 sales are zero is $\frac{\gamma^B}{n}\left(1 - \mu^B(S') - \mu^B\left(\frac{S_{max}}{n}, S_{max}\right)\right)$. If Firm 1's period 1 sales belong to $\left(0, \frac{S_{max}}{n}\right) \setminus S'$ whereas all other firms' period 1 sales are zero, then according to $Eq*$, in period 2 Firm 1 sets a price $p_2^*(Q) \leq V$ and all other firms set a price $p_2^*(0) > (V-c)(1-g_2)+c$ (by Step 4), so that $p_2^*(Q) - p_2^*(0) < (V-c)g_2$, which implies that the entirety of period 2 demand goes to Firm 1 if demand is biased in Firm 1's favor and consumers' valuation v of Firm 1's product is such that $\frac{v-c}{V-c} - 1 > g_2$, which is the case with probability $\frac{\gamma^B}{n}\nu\left(((V-c)(1+g_2)+c, V')\right) = \frac{\gamma^B}{n}\left(1-\frac{g_2}{u}\right)$ if $g_2 < u$, or $\frac{\sigma+\gamma^B u}{\gamma^L(1-\delta)\delta} < u$. From here onwards, we assume that $\frac{\gamma^B}{\gamma^L}$ is small, and we consider values of σ that are small relative to u, so that $g_2 < u$. The probability according to $Eq*$ that firms other than Firm 1 sell zero in the first and the second period whereas Firm 1's sales in these periods belong respectively to $\left(0, \frac{S_{max}}{n}\right) \setminus S'$ and $(0, S_{max})$ is thus greater than $\left(\frac{\gamma^B}{n}\right)^2 \left(1-\mu^B(S') - \mu^B\left(\frac{S_{max}}{n}, S_{max}\right)\right)\left(1-\frac{g_2}{u}\right)$. $\mu^B(S') < \lambda_1\mu^N(S')$ so that (using the results of Step 6) the latter expression is greater than $\left(\frac{\gamma^B}{n}\right)^2\left(1-\lambda_1\lambda_2\frac{\sigma+\gamma^B u}{(1-\gamma^B)(1-\gamma^B(1+u))\delta} - A\right)\left(1-\frac{g_2}{u}\right)$.

Step 8. The period 4 prices set by all other firms after having sold zero in periods 1 to 3 and observing that Firm 1's sales belong to $\left(0, \frac{S_{max}}{n}\right) \setminus S'$ in period 1 and $(0, S_{max})$ in period 2. The probability, according to $Eq*$, that Firms 2 to n sell zero in periods 1, 2, and 3, while Firm 1 makes nonzero sales in periods 1 and 2, and its period 1 sales belong to $\left(0, \frac{S_{max}}{n}\right) \setminus S'$, is greater than the latter lower bound times γ^L. Define

$$g_4 = g\left(\alpha_4\left(\gamma_L\left(\frac{\gamma^B}{n}\right)^2\left(1-\lambda_1\lambda_2\frac{\sigma+\gamma^B u}{(1-\gamma^B)(1-\gamma^B(1+u))\delta} - A\right)\left(1-\frac{g_2}{u}\right), \sigma\right)\right).$$

Step 2 implies that conditional on Firm 1's having nonzero sales in periods 1 and 2 (belonging to $\left(0, \frac{S_{max}}{n}\right) \setminus S'$ in period 1) and other firms having zero sales in periods 1 to 3, then with probability greater than $1-g_4$, Firms 2 to n set a price that is greater than $p_4^{dev} = c + (V-c)(1-g_4)$. Notice that the probability that after Firm 1 deviated by setting prices p_i^{dev} in period i ($i = 1, 2, 3$), its sales belonged to $\left(0, \frac{S_{max}}{n}\right) \setminus S'$ in period 1 and $(0, S_{max})$ in period 2 while other firms sold zero in periods 1 to 3 is greater than $\left(1-\mu^N(S') - \mu^N\left(\frac{S_{max}}{n}, S_{max}\right) - \frac{\gamma^L}{1-\gamma^B}\right)\left(1-\gamma^B\right)^3$, which is greater than $\left(1-\lambda_2\frac{\sigma+\gamma^B u}{(1-\gamma^B)(1-\gamma^B(1+u))\delta} - A - \frac{\gamma^L}{1-\gamma^B}\right)\left(1-\gamma^B\right)^3$.

Step 9. Firm 1's possible deviations in periods 1 to 4 and the corresponding expected profits. Steps 1 to 8 imply that if Firm 1 sets price p_i^{dev} in period i ($i = 1, 2, 3, 4$) then its expected profit is at least $(V-c)D(1-g_1)\left(1-\gamma^B\right)$ in period 1, $(V-c)D(1-g_2)\left(1-\gamma^B\right)^2$ in period 2, $(V-c)D(1-\lambda_1 g_3)(1-g_3)\left(1-\gamma^B-\gamma^L\right)\left(1-\gamma^B\right)^2$ in period 3, and $(V-c)D\left(1-\lambda_2\frac{\sigma+\gamma^B u}{(1-\gamma^B)(1-\gamma^B(1+u))\delta} - A - \frac{\gamma^L}{1-\gamma^B}\right)\left(1-\gamma^B\right)^4(1-g_4)^2$

in period 4.

Step 10. (1) implies the existence of a profitable deviation. Assume that inequality (1) holds: $1+\delta+\delta^2+(1-A)\delta^3 > \frac{1}{n(1-\delta)}$. (1) is equivalent to $\delta^4 + A\delta^3 < 1-\frac{1}{n}$, which provides a strictly positive lower bound on the possible values of $\frac{1}{1-\delta}$. If γ^L, γ^B, $\frac{\gamma^B u}{\gamma^L}$, $\frac{nu}{(1-\delta)\delta^3\gamma^B\gamma^L(1-A)}$ and σ are close to zero, then: g_1, g_2, g_3, and g_4 are close to zero; each p_i^{dev} $(i=1,2,3,4)$ is close to V; and with a probability close to 1 in periods 1 to 3, and close to $1-A$ in period 4, a firm that sets price p_i^{dev} in period i $(i=1,2,3,4)$ serves the entire demand in each of the first four periods. Such a deviation yields an expected profit that is close to $(V-c)D\left(1+\delta+\delta^2+(1-A)\delta^3\right)$. If $1+\delta+\delta^2+(1-A)\delta^3 > \frac{1}{n(1-\delta)}$ and $\gamma^B u$ is close to zero (which follows from $\frac{\gamma^B}{\gamma^L}$ and $\frac{nu}{(1-\delta)\delta^3\gamma^B\gamma^L(1-A)}$ being close to zero), then this is greater than a firm's expected profit according to $Eq*$, which cannot exceed $\frac{(V-c)D(1+\gamma^B u)}{n(1-\delta)}$.

Proof of Proposition 2. Preamble: the parameters of the price war. We assume that condition (2) holds (which implies $\delta > \frac{1}{2}$), and that γ^B, γ^L and u are close to zero, and $c > 5(V'-V)$. We want to find price-war parameters such that: (i) any firm's expectation of the sum of its future discounted profits at the start of a price war is 0 if all firms are expected to follow the strategies that are described in Section 3.3 (that is, those described in Section 2.2 with the difference that in a correction period, the targeted firm sets price $V'+1$); and (ii) these strategies cannot be individually improved upon. Let $PW_r(p,k')$ denote a firm's expectation of the discounted sum of its future payoffs at the start of the r-th period of a price war lasting k' periods $(1 \le r \le k')$ with price p $(p < V$), on the assumption that all firms act according to the candidate equilibrium. At the beginning of the r-th period of a price war lasting k' periods, each firm's current period expected profit is $\frac{(p-c)D}{n}$, and with probability $\left(\gamma^B+\gamma^L\right)$, the state of the world in the next period is the first period of a price war; whereas with probability $1-\left(\gamma^B+\gamma^L\right)$, it is '$(r+1)$-th period of a price war' if $r<k'$ and 'normal collusion' if $r=k'$. This implies the following equalities:

$$PW_r(p,k') = \frac{(p-c)\,D}{n} + \delta(1-\gamma^B-\gamma^L)PW_{r+1}(p,k') + \delta(\gamma^B+\gamma^L)PW_1(p,k') \text{ if } r<k'$$
$$PW_{k'}(p,k') = \frac{(p-c)\,D}{n} + \delta(1-\gamma^B-\gamma^L)\frac{(V-c)\,D}{n(1-\delta)} + \delta(\gamma^B+\gamma^L)PW_1(p,k'),$$

which implies

$$PW_1(p,k')\left(1-\delta(\gamma^B+\gamma^L)\frac{\left(1-\left(\delta(1-\gamma^B-\gamma^L)\right)^{k'}\right)}{1-\delta(1-\gamma^B-\gamma^L)}\right)$$

$$=\frac{(p-c)\,D}{n}\frac{\left(1-\left(\delta(1-\gamma^B-\gamma^L)\right)^{k'}\right)}{1-\delta(1-\gamma^B-\gamma^L)}+\frac{(V-c)\,D}{n(1-\delta)}\left(\delta(1-\gamma^B-\gamma^L)\right)^{k'}.$$

Let $F(p,k')$ denote the right-hand side of this equation, multiplied by n/D:

$$F(p,\kappa)=\frac{(p-c)\left(1-\left(\delta(1-\gamma^B-\gamma^L)\right)^{\kappa}\right)}{1-\delta(1-\gamma^B-\gamma^L)}+\frac{(V-c)\left(\delta(1-\gamma^B-\gamma^L)\right)^{\kappa}}{(1-\delta)}.$$

If $\delta>\frac{1}{2}$ and γ^B, γ^L, and u are close enough to zero, then $F\left(c-(V'-V),1\right)>0$. Also, $\underset{\kappa\to\propto}{Lim}F\left(c-(V'-V),\kappa\right)<0$. Therefore, if $\delta>\frac{1}{2}$ and γ^B, γ^L, and u are close enough to zero, there exists an integer $k'\geq 1$ such that $F\left(c-(V'-V),k'+1\right)<0<F\left(c-(V'-V),k'\right)$. We show now that $F\left(c-5(V'-V),k'\right)<0$:

$$F\left(c-5(V'-V),k'\right)<0$$

$$\Longleftrightarrow\frac{5(V'-V)\left(1-\left(\delta(1-\gamma^B-\gamma^L)\right)^{k'}\right)}{1-\delta(1-\gamma^B-\gamma^L)}>\frac{(V-c)\left(\delta(1-\gamma^B-\gamma^L)\right)^{k'}}{(1-\delta)}.$$

But

$$F\left(c-(V'-V),k'+1\right)<0$$

$$\Longleftrightarrow\frac{(V'-V)\left(1-\left(\delta(1-\gamma^B-\gamma^L)\right)^{k'+1}\right)}{1-\delta(1-\gamma^B-\gamma^L)}>\frac{(V-c)\left(\delta(1-\gamma^B-\gamma^L)\right)^{k'+1}}{(1-\delta)}$$

$$\Rightarrow\frac{5(V'-V)\left(1-\left(\delta(1-\gamma^B-\gamma^L)\right)^{k'}\right)(1-\delta)}{(V-c)\left(\delta(1-\gamma^B-\gamma^L)\right)^{k'}\left(1-\delta(1-\gamma^B-\gamma^L)\right)}>\frac{5\delta(1-\gamma^B-\gamma^L)}{1+\left(\delta(1-\gamma^B-\gamma^L)\right)},$$

which is greater than 1 if $\delta>\frac{1}{2}$ and γ^B and γ^L are both close to zero, which imples that $F\left(c-5(V'-V),k'\right)<0$. By continuity, there exists some price p^w between $c-5(V'-V)$ and $c-(V'-V)$ such that $PW_1(p^w,k')=F(p^w,k')=0$.

We now prove that at the beginning of any stage of a price war, it is a best response for a firm to set price p^w and truthfully report its sales, given that all firms are expected to follow the strategies that are described above. First, the Bellman equation above implies that for all r such that $1<r\leq k'$,

$PW_r(p^w, k') > PW_1(p^w, k') = 0$.

Consider a firm at the start of the r-th period of a price war. Complying with the candidate equilibrium strategy yields an expectation of the sum of future discounted profits that is equal to $PW_r(p^w, k') \geq 0$. Let $BR_r(p^w, k') \geq 0$ denote the expectation of the sum of future discounted profits of a firm (say, Firm 1) at the beginning of the r-th period of a price war, on the assumption that the firm, from period 1 onwards, maximizes the expectation of the discounted sum of its future profits, and that all other firms act according to the candidate equilibrium.

First, at the beginning of any price war period, a firm cannot earn a strictly positive profit: This would require a price above c and therefore greater than $p^w + (V' - V)$, implying zero sales.

Second, if Firm 1 sets a price that is different from p^w, then with probability 1, the state of the world in the next period is 'first period of a price war'. This is because when not all firms set the same price, it is impossible that all firms have identical nonzero sales and whatever Firm 1's announcement, the probability that all firms report identical nonzero sales is zero.

This implies $BR_1(p^w, k') \leq \delta BR_1(p^w, k')$, which implies that $BR_1(p^w, k') \leq 0$ and therefore $BR_1(p^w, k') = 0$. It is therefore a best response in period 1 to set a price that is equal to p^w. If the best response in the r-th period of a price war ($r > 1$) involved a price different from p^w, then the above analysis implies $BR_r(p^w, k') \leq \delta BR_1(p^w, k') = 0 < BR_r(p^w, k')$, which is a contradiction. Therefore, the price war that is described above is indeed an equilibrium. The price war that is mentioned in the remainder of this proof is the one that is described above: with $PW_1(p^w, k') = 0$.

Step 1: the expected sum of future discounted profits for a firm according to the state of the game. Let $W_{c,r}$, $W_{c-,r}$ and W denote the expected sum of a firm's future discounted profits at the beginning of, respectively, a correction period at its own expense with r remaining periods ($1 \leq r \leq k$), a correction period at another firm's expense with r remaining periods, and a normal collusion period (on the assumption that all firms behave according to the candidate equilibrium). The assumptions of the model imply $W = \frac{(V-c)D}{n(1-\delta)}$. Also, since ahead of any period along any equilibrium path, n or $(n-1)$ firms are in a symmetric situation, $W_{c-,r} \leq \frac{(V-c)D}{(n-1)(1-\delta)}$.

In any correction period, there is a probability $\frac{(n-1)\gamma^B}{n}$ that one of the non-targeted firms will benefit from biased demand. Therefore, for any r between

1 and k, $W_{c,r} = \delta \left[\left(1 - \frac{(n-1)\gamma^B}{n} \right) W_{c,r-1} + \frac{(n-1)\gamma^B}{n} W_{c-,k} \right]$ (with the notation $W_{c,0} = W$) and

$$W_{c,r} = \delta^r \left(1 - \frac{(n-1)\gamma^B}{n} \right)^r W + \frac{\frac{\delta(n-1)\gamma^B}{n}}{1 - \delta \left(1 - \frac{(n-1)\gamma^B}{n} \right)} \left[1 - \delta^r \left(1 - \frac{(n-1)\gamma^B}{n} \right)^r \right] W_{c-,k}.$$

The above results imply that for any number of firms $n \geq 3$, the following inequalities hold:

$$\frac{\delta^r (V-c) D}{(1-\delta) n} \left(1 - \frac{(n-1)\gamma^B}{n} \right)^r \leq W_{c,r} \leq \frac{\delta^r (V-c) D}{(1-\delta) n} \left(1 - \frac{(n-1)\gamma^B}{n} \right)^r + \frac{\delta \gamma^B (V-c) D}{n \left(1 - \delta \right)^2} \tag{4}$$

We prove now that for any r ($1 \leq r \leq k$), $W_{c,r} \leq W \leq W_{c-,r}$. First, we show by induction that either for all r, $W_{c,r} \leq W$, or for all r, $W_{c,r} \geq W$. Assume (by contradiction) that $W_{c,k} > W$. The equality $W_{c,k} + (n-1) W_{c-,k} = nW$ implies $W_{c-,k} < W$. This and the equation that characterizes $W_{c,1}$ implies $W_{c,1} < W$, which is a contradiction. Finally, the inequality $W_{c,r} \leq W$ implies that for any r ($2 \leq r \leq k$), $W_{c,r+1} \leq W_{c,r}$ and $W_{c-,r+1} \geq W_{c-,r}$.

Step 2. Whatever the state of the world at the beginning of period t, if Firm i complied with the strategies that are prescribed by the candidate equilibrium in all previous periods and it assumes that all other firms behave according to the candidate equilibrium, and Firm i did set a price at the beginning of period t in accordance with the candidate equilibrium, or the observation of its own sales combined with the knowledge of the price it set at the beginning of the period allows it to know that the distribution of current period sales will not reveal any deviation, then reporting sales truthfully is a best response for Firm i.

Proof. Since misreporting at the end of period t is detected at the latest at the end of period $(t+1)$, which leads to a price war from period $(t+2)$ onwards, misreporting would allow Firm i to earn at most $(V-c)D(1+\gamma^B u)$ in period $(t+1)$ and an expected discounted sum of subsequent profits equal to 0. Complying with the strategy prescribed by the candidate equilibrium would lead, at the beginning of period $(t+1)$, to an expected sum that is greater than or equal to $W_{c,k}$. Since (4) implies $W_{c,r} \geq (V-c)D(1+\gamma^B u)$ for all r if γ^B and u are close enough to zero (in which case it boils down to $\delta^k \geq n(1-\delta)$), truthful sales reporting is a best response .

Step 3. Notation: D_t denotes total demand in period t. At the beginning of a correction period, it is a best response for all firms to set the price that is prescribed by the candidate equilibrium.

Proof. Assume that the state of the game at the beginning of period t is 'correction at the expense of Firm 1 with r remaining periods'. We prove first that it is optimal for Firm 1 to behave as prescribed by the candidate equilibrium. We showed (Step 2) that conditional on setting a price that is equal to $V' + 1$ it is optimal for Firm 1 to report its zero sales truthfully. Assume that Firm 1 sets $p_t^1 \neq V' + 1$. Any price greater than V' yields the same payoff distribution for Firm 1 in period t and subsequent periods as $p_t^1 = V' + 1$. Consider now a price $p \in (V, V')$. Such a price leads to exactly the same outcome as $p_t^1 = V' + 1$ unless demand is biased in favor of Firm 1, with a valuation $v \geq p$. In this latter case, if Firm 1 sets $p_t^1 = V' + 1$, it earns zero in period t, and its expected sum of future profits is at least $\delta W_{c,r-1}$; whereas if it sets $p_t^1 = p$, its deviation will be detected at the end of the following period, which leads to profits that are below $(V - c)D\left[(1 + \delta)(1 + \gamma^B u)\right]$. (2) implies $\delta W_{c,r-1} > (V - c)D\left[(1 + \delta(1 + \gamma^B u)\right]$ if γ^B and u are close enough to zero, which makes such a deviation unprofitable.

Consider now a price $p \leq V$. Unless demand is zero or biased, setting such a price leads Firm 1 to being 'exposed' after two periods at most. The corresponding expected sum of future discounted profits is thus less than $(V - c)D(1 + \gamma^B u)\left[(1 + \delta) + \left(\gamma^L + \gamma^B\right)\right]$. If γ^B, γ^L and u are close enough to zero, (2) implies that this expression is less than $W_{c,k}$: less than $W_{c,r}$ for any $r \leq k$. Therefore, it is optimal for Firm 1 to follow the strategy that is prescribed by the candidate equilibrium.

Consider now a firm other than Firm 1 - say, Firm 2. Complying with the actions that are prescribed by the candidate equilibrium leads for Firm 2 to an expected sum of future discounted profits that is equal to $W_{c-,r}$. Assume now that Firm 2 deviates and sets a price $p_t^2 \neq V$. If $p_t^2 > V$, then Firm 2 earns zero in period t and whatever it reports at the end of period t, its deviation is detected at the end of period $(t + 1)$ unless demand in period t is zero, because a distribution of sales such that only two firms have zero sales is incompatible with equilibrium. Therefore the corresponding expected sum of future discounted profits is less than or equal to $(V - c)D(1 + \gamma^B u)\left[\delta + \frac{\gamma^L \delta^2}{1-\delta}\right]$. If γ^B, γ^L, and u are close enough to zero, (3) implies that this expression is less than $W_{c,k}$, and therefore less than $W_{c-,r}$. A price $p_t^2 > V$ therefore cannot improve upon the behavior that is prescribed by the candidate equilibrium for Firm 2.

Consider now the possibility of a price $p_t^2 < V$. Such a price would yield

Firm 2 at most $(V-c)D$ in expectation in period t. If demand in period t is zero, which happens with probability γ^L, period t sales are identical to what they would be absent a deviation by Firm 2. Therefore it would be optimal for Firm 2 in this case to report zero sales (Step 2), which leads in period $(t+1)$ to either 'normal collusion' (if $r=1$) or to 'correction at the expense of Firm 1, with $(r-1)$ remaining periods' (if $r \geq 2$). This would lead at the beginning of period $(t+1)$ to an expected sum of future discounted profits that is equal to $W_{c-,r-1}$ (with the notation $W_{c-,0}=W$). If demand is normal, a deviation leads to a sales profile that is compatible with equilibrium behavior (with demand biased in favor of Firm 2). It is then optimal for Firm 2 to reveal its sales truthfully (Step 2), which leads at the beginning of period $(t+1)$ to an expected sum of future discounted profits that is equal to $W_{c,k}$. Finally, if demand is biased, then with probability 1 one of the firms serves the entire demand, which leads to an expected sum of future discounted profits that is below $W_{c-,k}$. Therefore, a deviation with $p_t^2 < V$ would lead at the beginning of period t to an expected sum of Firm 2's future discounted profits that is less than or equal to $(V-c)D+\delta\left(\gamma^L W_{c-,r-1}+\left(\left(1-\gamma^B\right)-\gamma^L\right)W_{c,k}+\gamma^B W_{c-,k}\right)$, whereas in the absence of deviation this expected sum is equal to $W_{c-,r}$. The difference between this expected sum in the absence and in the presence of such a deviation is thus greater than or equal to

$$W_{c-,r}-(V-c)D-\delta\left(\gamma^L W_{c-,r-1}-\left(\left(1-\gamma^B\right)-\gamma^L\right)W_{c,k}+\gamma^B W_{c-,k}\right)$$

$$= \quad W+\delta\gamma^L\left(W_{c-,r}-W_{c,r-1}\right)+\left(1-\delta\gamma^L\right)\left(W_{c-,r}-W\right)- \\ -(V-c)D-\delta\left(\gamma^L W'-\left(\left(1-\gamma^B\right)-\gamma^L\right)W_{c,k}+\gamma^B W_{c-,k}\right)$$

$$\geq \quad W-(V-c)D-\delta\left(\gamma^L W+\left(\left(1-\gamma^B\right)-\gamma^L\right)W_{c,k}+\gamma^B W_{c-,k}\right)$$

$$\geq \quad \tfrac{(V-c)D}{n(1-\delta)}-(V-c)D\left[1+\delta\left(1-\gamma^B\right)\tfrac{\delta^k}{n(1-\delta)}\right] \\ -(V-c)D\left[\delta\left(\left(1-\gamma^B\right)\tfrac{\delta\gamma^B}{n(1-\delta)^2}+\tfrac{\gamma^B}{(n-1)(1-\delta)}\right)\right]$$

If γ^B is close enough to zero, (2) implies that this difference is positive, so that it is a best response for Firm 2 to follow the prescribed equilibrium strategy.

Step 4. If the state of the world at the beginning of period t is 'normal collusion' and all firms followed the strategy prescribed by the candidate equilibrium in previous periods, then setting $p_t^1 = V$ is a best response for Firm 1.

Proof. Consider a subgame that starts in period t, s. t. the state of the game at the beginning of period t is 'normal collusion'. Firm 1's expected sum of future discounted profits is W if it sets $p_t^1 = V$. We prove hereafter by contradiction that if (2)-(3) hold, then in a 'normal collusion' period, a price such that $p_t^1 < V$ or $p_t^1 > V$ leads to an expected sum of future discounted profits that is smaller than or equal to W.

We assume the existence of a best response such that with a positive probability Firm 1 sets a price $p_t^1 < V$ in some 'normal collusion' state. Let W' denote Firm 1's expected sum of future discounted profits at the beginning of any 'normal collusion' state, given this (or any other) best response.

In period t, setting a price $p_t^1 < V$ allows Firm 1 to serve the entire demand unless demand is biased in favor of some other firm. If $D_t = 0$, then the state of the world is 'normal collusion' again in period $(t+1)$, which leads to an expected sum of future discounted profits that is equal to W' at the beginning of period $(t+1)$. If demand is biased, then with probability 1 one of the firms (Firm 1 or another one) serves the entire demand, which leads to an expected sum of future discounted profits that is below $W_{c-,k}$. If demand is normal and nonzero, the state of the world at the beginning of period $(t+1)$ is 'correction at the expense of Firm 1, with k remaining periods', leading to an expected sum of future discounted profits $W_{c,k}$ (by Step 2). Therefore, a deviation with $p_t^1 < V$ would lead at the beginning of period t to an expected sum of future discounted profits that is less than or equal to $(V-c)D+\delta\left(\gamma^L W' + \gamma^B W_{c-,k} + \left(1-\gamma^B-\gamma^L\right) W_{c,k}\right)$:

$$W' \leq (V-c)D + \delta\left(\gamma^L W' + +\gamma^B W_{c-,k} + \left(1-\gamma^B-\gamma^L\right) W_{c,k}\right),$$

which implies

$$\begin{aligned} &\frac{\left(1-\gamma^L\delta\right) W'}{(V-c)D} \leq 1 + \delta\left(1-\gamma^B-\gamma^L\right)\frac{W_{c,k}}{(V-c)D} + \delta\gamma^B \frac{W_{c-,k}}{(V-c)D} \\ &\leq 1 + \delta\left(1-\gamma^B-\gamma^L\right)\left(\frac{\delta^k}{(1-\delta)n} + \frac{\delta\gamma^B}{n\,(1-\delta)^2}\right) + \frac{\delta\gamma^B}{(1-\delta)(n-1)} \end{aligned}$$

If γ^B and γ^L are close enough to zero, (3) implies that the right-hand term of this inequality is less than W, so that that $W' < W$.

We show now that there exists no best response such that, with a positive probability, $p_t^1 > V$. If $p_t^1 > V$ and demand is neither biased nor zero, then Firm 1 earns zero in period t. In this case its deviation is detected at the end of period $(t+1)$ because a distribution of sales such that only one firm has zero sales is incompatible with equilibrium. This, and the fact that a firm's per-period expected profit cannot exceed $(V-c)D(1+\gamma^B u)$, implies that the corresponding expected sum of future discounted profits is no greater than $(V-c)D(1+\gamma^B u)\left[\left(\frac{\gamma^B}{n}+\delta\right)+\gamma^L\frac{\delta^2}{1-\delta}\right]$. If γ^B, γ^L, and u are close enough to zero, (2) implies that this expression is less than W.

Step 5. The above steps imply that the strategy profile under consideration is an equilibrium strategy profile if γ^B, γ^L, and u are close enough to zero, and conditions (2)-(3) are satisfied. By construction, this equilibrium is symmetric, it involves only pure strategies,and the prevailing price is V in all periods along the equilibrium path with probability one.

Proof of Proposition 3. First, we prove that there exist triplets (n, δ, k) such that conditions (1)-(3) jointly hold when $A = 0$ and, by continuity, for some strictly positive $A > 0$. This can be checked numerically (Table 1).

Table 1. Examples of values of (n, δ, k) s.t. (1)-(3) jointly hold when $A = 0$

	$n = 3$,$k = 3$	$n = 4$,$k = 4$	$n = 5$,$k = 5$	$n = 6$,$k = 7$
Minimum value of δ	0.880	0.911	0.929	0.944
Maximum value of δ	0.903	0.930	0.945	0.955

Second, we consider a specific pair (n, k) such that (1)-(3) hold for some δ and some $A^* > 0$. Holding A^* constant, let δ_{min} and δ_{max} denote the lower and upper bound of the set of values of δ that satisfy (1)-(3) given k, n, and A^*. Notice that $\delta_{min} > 0$ and $\delta_{max} < 1$.

We show hereafter that it is possible to construct a set of stochastic demand functions that satisfy the assumptions of the model, with γ^L, γ^B, $\frac{\gamma^B u}{\gamma^L}$, and $\frac{nu}{(1-\delta_{max})\delta_{min}^3 \gamma^B \gamma^L}$ arbitrarily close to zero and such that $Max_Q \frac{\mu^N(Q)}{\mu^N(nQ)}$, $Max_Q \frac{\mu^N(Q)}{\mu^B(Q)}$ and $Max_Q \frac{\mu^B(Q)}{\mu^N(Q)}$ are bounded.

We choose some $D > 0$, $V > 0$ and $c \in (0, V)$. Let ε denote a small positive number. We define $\gamma^L(\varepsilon) = \varepsilon$ and $\gamma^B(\varepsilon) = \varepsilon^2$, and we consider a stochastic demand function that is defined as follows (using the definitions of 'normal' and 'biased' that are introduced in Section 2): With probability $\left(1 - \gamma^B(\varepsilon)\right)$ demand is normal; otherwise it is biased. We define some $R(\varepsilon)$, $p(\varepsilon)$, and $p^B(\varepsilon)$ such that, conditional on demand's being normal (resp. biased), demand is drawn from a distribution with: (i) an atom in zero with probability $\gamma^L(\varepsilon)$ (resp. probability zero); (ii) the uniform distribution over $(0, R(\varepsilon)D)$ with probability $1 - p(\varepsilon) - \gamma^L(\varepsilon)$ (resp. probability $1 - p^B(\varepsilon)$); and (iii) with probability $p(\varepsilon)$ (resp. $p^B(\varepsilon)$) the uniform distribution over $(R(\varepsilon)D, nR(\varepsilon)D)$. For expected demand to be D both in the case of normal and biased demand, and for the probability that demand exceeds $\frac{1}{n}$-th of its maximum value to be no greater than A^* both when demand is normal and when it is biased, it is sufficient that the following equalities hold: $\left(\left(1 - p(\varepsilon) - \gamma^L(\varepsilon)\right) + (n+1)p(\varepsilon)\right) R(\varepsilon) = 2$; $\left(\left(1 - p^B(\varepsilon)\right) + (n+1)p^B(\varepsilon)\right) R(\varepsilon) = 2$; and $p(\varepsilon) = A^*$. These three identities yield $R(\varepsilon)$, $p(\varepsilon)$, and $p^B(\varepsilon)$. One can check that as ε goes to zero, these distributions of demand - that are denoted μ_ε^N (for normal demand) and μ_ε^B (for biased demand) - are such that $Max_Q \frac{\mu_\varepsilon^N(Q)}{\mu_\varepsilon^N(nQ)}$, $Max_Q \frac{\mu_\varepsilon^N(Q)}{\mu_\varepsilon^B(Q)}$, and $Max_Q \frac{\mu_\varepsilon^B(Q)}{\mu_\varepsilon^N(Q)}$ remain bounded: they respectively converge towards $Max\left(\frac{1-A^*}{nA^*}, 1\right)$, 1, and 1 as ε converges towards zero.

Finally, we define $u(\varepsilon)$ by the identity $\frac{nu(\varepsilon)}{(1-\delta_{max})\delta_{min}^3\gamma^B\gamma^L(1-A^*)} = \varepsilon$. This demand function is such that γ^L, γ^B, $\frac{\gamma^B u}{\gamma^L}$, and $\frac{nu}{(1-\delta)\delta^3\gamma^B\gamma^L(1-A^*)}$ are all smaller than or equal to ε. Proposition 3 then implies that if there are n firms, then with any of these demand functions (if ε is small enough), the maximal attainable profits are greater with communication than without communication.

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