Ex ante or Ex post? When the timing of merger assessment is up to the merging firms

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Abstract

We develop a framework in which the timing of the merger control is left to the merging firms’ discretion: before the completion of the merger (ex ante) or afterwards (ex post). We show that the choice of merger control timing by the firms always dominates the ex ante control in terms of expected consumer surplus. The choice of merger control timing also dominates the ex post control except if the expected merger outcome is very anti-competitive.

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1 Introduction

Information is crucial for the effectiveness of merger control. Merging firms are typically supposed to have better information beforehand than the Competition Authority on the true impact of the merger in terms of consumer surplus. In contrast, after the completion of the merger (ex post), many aspects of the merger are publicly observed so that it becomes easier to assess the impact of the merger (see for instance Ashenfelter and Hosken, 2010 or Blonigen and Pierce, 2016). This has led Ottaviani and Wickelgren, 2011, to examine the theoretical implications of an ex post merger control. Nevertheless, they show that the risk of an ex post costlier rejection may make the ex post merger control worse than the ex ante control by deterring some price-reducing mergers.

In this paper, we suggest a mix of both systems: we allow the merging firms to choose the timing of the assessment. We develop a model of merger control by a competition authority where ex ante (before the completion of the merger), both the merging firms and the authority have imperfect information on the true type of the merger. In case of ex post control (after the completion of the merger), the competition authority has better information but if the merger is rejected, the merging firms incur an additional cost due to this late rejection.

We first show that such a flexible merger control (ex post control or ex ante control according to the merging firms’ choice) increases the expected consumer surplus with respect to the ex ante control. Indeed, the flexible merger control preserves the possibility for the firms to be controlled ex ante and also allows the firms to be controlled ex post. In addition, we show that the flexible merger control does not always dominate the ex post control. The flexible merger control combines the informational benefit of the ex post control for the firms that actually choose the ex post control and the riskless benefit of the ex ante control but it also gives lower incentives to firms to adopt the ex post control. Eventually, we show that the flexible merger control is the most efficient control timing except if the firms are well informed ex ante or if the welfare cost of anticompetitive mergers is high. In that case it is preferable for the Competition Authority to impose the ex post control. As a policy implication of our
framework, we suggest that a Competition Agency (denoted CA henceforth) that wants to avoid anticompetitive mergers should adopt an ex post control. Instead, a CA that wants to avoid deterring pro-competitive mergers should adopt a flexible merger control.

Few papers examine the role of the timing on the effectiveness of the merger control. A recent debate on the possible tightening of US merger control led some scholars to advocate some sort of ex post review of mergers\(^1\) (see Salop, 2016 and Shapiro, 2018 and 2019), but without any formal analysis to assess the pros and cons of that ex post review. The possibility of a regime allowing the merging firms themselves to choose the timing of control of their merger is not considered either. Ottaviani and Wickelgren (2011) formally compare both timings. A distinct but related approach is taken by Choe and Shekhar (2010), who study the trade-off between voluntary and mandatory notification\(^2\). They show that the non mandatory notification acts as a screening device since the most efficient mergers will not be notified. However, their ex post merger assessment in case of voluntary notification involves no regulatory control by the agency but only costly litigation without information revelation.

The paper proceeds as follows: the model is presented next, and the main results derived in Section 3.

2 The model

We develop a merger control game between a competition authority (CA) and a merger (the merging firms). All merger projects enhance market power and thus have anticompetitive effects, but may also generate a pro-competitive effect\(^3\). We consider two types of merger that differ by the level of efficiency gains: one type with low efficiency gains \((i = L)\) and one

\(^1\)In France also, the Autorité de la Concurrence is considering the possibility to control some types of mergers ex-post (Autorité de la Concurrence, 2018).

\(^2\)Barros (2003) and Berges et al. (2008) study the opportunity of mandatory notifications for the agreement exemptions under Art.101 TFEU. They compare different timing of control of vertical restraints without allowing firms to choose their preferred timing of control.

\(^3\)Typically, cost savings or a quality increase due to synergies.
with high efficiency gains \((i = H)\). Each type has the same probability\(^4\). We denote by \(\pi(i)\) the profit increase of type \(i\) \((i = H, L)\) due to the merger. Efficiency gains improve the merger profitability and thus: \(\pi(H) > \pi(L) > 0\). The consumer surplus increase after the merger is equal to \(CS(i)\) with \(CS(L) < 0 < CS(H)\). In other words, only merger type \(H\) is beneficial to consumers\(^5\). We assume that the expected consumer surplus is negative: \(\frac{1}{2}CS(L) + \frac{1}{2}CS(H) < 0\). The objective of the merger control is then to block the anticompetitive merger (type \(L\)) without prohibiting the pro-competitive merger (type \(H\)).

Merger control, merging firms’ information and CA’s information

The merging firms.

Ex ante (before the completion of the merger), the merging firms observe an informative signal of quality \(\tilde{q}\). The merger of type \(H\) (type \(L\)) observes a signal \(h\) (signal \(l\)) with probability \(\tilde{q}\) and a signal \(l\) (signal \(h\)) with probability \(1 - \tilde{q}\) with \(\tilde{q}\) distributed on \([\frac{1}{2}, 1]\) according to the density function \(g(x)\). The quality of the signal, \(\tilde{q}\), is observed by the merging firms. This amounts to consider (see the appendix) that the merging firms observe a signal \(q\) equal to the probability of being of type \(H\) with \(q\) distributed on \([0, 1]\) according to the cdf \(G^i(x)\) for type \(i\) merging firms \((i = H, L)\) with \(G^H(x) < G^L(x)\). We derive the function \(G^i(x)\) in the appendix. Ex post (after the completion of the merger), the merging firms observe the true type of the merger.

The Competition Authority

The CA observes an imperfectly informative signal of the merger type. The signal may be either "good" (signal \(G\)) or "bad" (signal \(B\)).

Ex post, the CA is better informed because it observes some characteristics of the merger. For that reason, we assume that\(^6\):

\(^4\)Even though both types are equally likely, both types may have asymmetric impact on the consumer surplus.

\(^5\)These assumptions on the monotonicity of profit and consumer surplus functions are compatible with Cournot competition with homogenous goods (see Neven and Röller 2005, p.833-834), as well as with price competition for some specifications of product differentiation (see Neven 2001, p.432-433).

\(^6\)This assumption may be the result of endogenous investment in signal quality if ex post information is
Ex ante: \( \text{Probability}(G \mid H) = \mu^G > \frac{1}{2} \) and \( \text{Probability}(B \mid L) = \mu^B > \frac{1}{2} \).

Ex post: \( \text{Probability}(G \mid H) = 1 \) and \( \text{Probability}(B \mid L) = \mu^B > \frac{1}{2} \).

In other words, we assume that ex post the CA always detects efficiency gains when they exist but may also falsely consider that there are efficiency gains (ex post, a type L remains imperfectly detected). For simplicity, the bad signal informativeness remains unchanged after the completion of the merger\(^7\).

If the merger is blocked ex post, this leads to an extra cost \( K \) incurred by the merging firms\(^8\).

We contrast the outcome of the merger control with three possible timings: ex ante control, ex post control and a flexible merger control where the merging firms choose the period of control.

**The ex ante merger control game**

Stage 1. The merging firms observe the private signal \( q \).

Stage 2. The CA observes the signal \( j \ (j = G, B) \) and decides to clear or block the merger.

**The ex post merger control game**

Stage 1. The merging firms observe the private signal \( q \) and decide to merge or not (ex ante).

Stage 2. The CA observes the signal \( j \ (j = G, B) \) and decides to clear or to block the merger (ex post).

**The "flexible merger control" game where the merging firms choose between the ex ante merger control and the ex post merger control**

Stage 1. The merging firms observe the private signal \( q \) and decide to be controlled by the CA before or after completion of the merger.

Stage 2. If the firms decide to be controlled ex ante: the CA observes the signal \( j \)

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\(^7\) We may extend our results to the case where ex post the probability \( \mu^B \) is higher than ex ante.

\(^8\) This cost may also be viewed as the merger damage on consumers between the merger and its prohibition ex-post.
(j = G, B) and decides to clear or to block the merger ex ante.

Stage 3. If the firms decide to be controlled ex post: the CA observes the signal j (j = G, B) and decides to clear or to block the merger ex post.

We assume in the last game that if the merging firms choose the ex ante merger control, the CA commits not to control the merger ex post. This assumption allows us to contrast the three possible merger controls: the ex post merger control, the flexible merger control (endogenous timing) and the ex ante merger control. This assumption is also close to the British system where merger notification is voluntary\(^9\).

The objective of the CA is to clear a merger iff the expected consumer surplus is positive. We determine the Perfect Bayesian Equilibrium (PBE) of each game.

3 The optimal merger control timing

We first consider the ex ante merger control game. At stage 2, if the CA observes signal B, the expected consumer surplus is given by \((1 - \mu^G)CS(H) + \mu^BCS(L) < CS(H) + CS(L) < 0\) so that the CA blocks the merger. If the CA observes the signal G, the CA decides to clear the merger iff the expected consumer surplus given by \(\mu^GCS(H) + (1 - \mu^B)CS(L)\) is positive. As a result, if the CA adopts the ex ante control, the expected consumer surplus is equal to \(\frac{1}{2} \text{Max}(\mu^GCS(H) + (1 - \mu^B)CS(L), 0)\).

In the ex post merger control game, the CA and the merging firms’ decisions are interdependent. In particular, if a signal G is observed, the expected consumer surplus depends on the expected firms’ decision to merge. If the merging firms decide to merge when they observe a signal \(q \geq q^{EP}\), then, the CA clears the merger with a positive probability \(m\) if it observes a signal G iff\(^{10}\) \((1 - G^H(q^{EP}))CS(H) + (1 - \mu^B)(1 - G^L(q^{EP}))CS(L) \geq 0\). The CA blocks the merger if it observes a signal B since it is always sent by type L. The merging firms’ choice depends on the expected profit: if the CA clears a merger with probability \(m\) after observing a signal G, the firms decide to merge if the signal

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\(^9\)See for instance: https://www.gov.uk/guidance/mergers-how-to-notify-the-cma-of-a-merger

\(^{10}\)We ignore the consumer surplus during the interim period i.e. between the merger and its control.
\( q \) is high enough to ensure a positive expected profit. Formally, after observing a signal \( q \), the expected profit, \( m \left[ q\pi(H) + (1 - q)(1 - \mu^B)\pi(L) \right] - Km(1 - q)\mu^B \), is positive iff
\[
q \geq \frac{Km^B - m(1 - \mu^B)\pi(L)}{m(\pi(H) - (1 - \mu^B)\pi(L) + \mu^K)} = q^{EP}(m).
\]

We derive the equilibrium of the ex post merger control game in the following lemma.

**Lemma 1** In case of ex post control, there exists an equilibrium where the CA blocks the merger if it observes a signal \( B \) and clears the merger with probability \( m^*(0 < m^* \leq 1) \) if it observes a signal \( G \). The merging firms merge iff \( q \geq q^{EP}(m^*) \). The resulting expected consumer surplus is equal to \( \frac{1}{2} \max \left[ (1 - G^H(q^{EP}))CS(H) + (1 - \mu^B)(1 - G^L(q^{EP}))CS(L), 0 \right] \)
with \( q^{EP} = q^{EP}(1) \).

**Proof.** Appendix ■

At the equilibrium of the ex post control, the CA clears the merger with a positive probability \( m^* \) if it observes a signal \( G \). The merging firms decide to merge if they observe a signal above \( q^{EP}(m^*) \). If the consumer surplus loss due to the anticompetitive merger is high, the CA blocks the merger with positive probability even if it observes a signal \( G \).

We deduce (see the appendix) that the ex post control is more efficient than the ex ante control iff \( \Omega = \frac{-CS(L)}{CS(H)} \geq \frac{\max(0, \mu^G(1 - G^H(q^{EP}))}{(1 - \mu^B)(1 - G^L(q^{EP}))} \). That condition states that the ex post control is more efficient if the relative welfare cost of the anticompetitive merger \( (\Omega = \frac{-CS(L)}{CS(H)}) \) is high enough. Let us explain that condition.

Consider first the procompetitive merger type. The ex post control ensures the clearing of a type \( H \) but the risk incurred by the ex ante uniformed merger deters a type \( H \) to merge if the signal \( q \) is below \( q^{EP} \). Ex ante, the CA has less information and thus incurs the risk of blocking a pro-competitive merger. Eventually, the ex ante control may be more efficient for pro-competitive mergers only if the ex ante detection probability \( (\mu^G) \) is higher than the type \( H \) merging probability if controlled ex post: \( \max(0, \mu^G - (1 - G^H(q^{EP})) > 0 \). In addition, the ex post merger control better fights against a merger of type \( L \) by decreasing the incentive to merge. In the end, the ex post control must be preferred to the ex ante control in two configurations: (i) if the lack of ex ante information on merger of type \( H \) is high compared
to the lack of incentive \( \mu^G < (1 - G^H(q^{EP})) \)) or (ii) if the consumer surplus impact of the bad merger type deterrence is higher than the consumer surplus impact of the good merger disincentive. This is the case if the ratio \( \Omega \) is high enough.

We now investigate if allowing the merging firms to choose the merger control timing improves the efficiency of the control of mergers.

The following proposition gives the perfect Bayesian equilibrium of the game where the merging firms choose the period of control (ex post or ex ante). We then compare the outcome of each timing of control.

**Proposition 1**

(i) In case of flexible merger control (the firms choose the timing of control), there always exists an equilibrium where the merging firms choose the ex ante control iff they observe a signal \( q \leq q^F \) and choose the ex-post control otherwise with \( q^F \) a critical level of the observed signal.

(ii) The flexible merger control always leads to a higher expected consumer surplus than the ex ante control.

(iii) The ex post control leads to a higher expected consumer surplus than the flexible merger control iff \( \Omega = \frac{CS(L)}{CS(H)} \geq \min \left[ \frac{\max[\mu^G G^H(q^F) - (G^H(q^F) - G^H(q^{EP}))]}{(1 - \mu^H) G^{EP}(q^F)}, \frac{\mu^G G^H(q^F)}{(1 - \mu^H) G^{EP}(q^F)} \right] = \Omega^{EP/F} \).

**Proof.** Appendix

Let us first explain the merging firms’ decision under the flexible merger control. The flexible merger control leads the merging firms to trade-off the cost and the benefit of being controlled ex post or ex ante. The ex ante control allows the merging firms to avoid an extra cost if the merger is blocked while ex post, because the CA is better informed, there is no risk of being blocked if the merger is of type \( I \). In that trade off, the signal quality plays a critical role: the higher the signal quality, the lower the risk of incurring the extra cost in case of ex post control. Thus, the flexible merger control leads to a separation of the merging firms according to the signal quality: if it is high enough, the merging firms choose to be controlled ex post while if the merging firms observe a low signal quality, the merger chooses the ex
ante control. This separation process increases the expected consumer surplus with respect to the ex ante control. The reason is clear: the CA makes fewer errors ex post if the merger chooses the ex post control and if the merger chooses the ex ante control, the CA knows the merging firms have observed a lower signal and thus the CA takes a better decision than if it had faced the whole population of mergers.

The comparison between the flexible and the ex post merger control is less clear-cut and flexibility may be worse than the imposed ex post control. In the particular case where the consumer surplus cost of the anticompetitive merger is high, the CA will always block the merger ex ante and then the flexible control coincides with the ex post control. That is why if \( \Omega > \frac{\mu^G G^H(q^F)}{1 - \mu^H G^E(q^F)} \) the ex post control is as efficient as the flexible control. In other cases \( \Omega < \frac{\mu^G G^H(q^F)}{1 - \mu^H G^E(q^F)} \), two different forces explain which merger control maximizes the expected consumer surplus. First, the flexible control may improve the detection of type \( H \). Indeed, the flexible merger control allows the CA to control ex ante the merging firms of type \( H \) that do not take the risk to merge if the ex post control is imposed. This higher merger incentive under flexible control must be balanced with the CA ex ante imperfect information. Thus, the flexible control better detects type \( H \) if the ex ante lack of information is compensated by the higher merger incentive \( (\mu^G G^H(q^F) > (G^H(q^F) - G^H(q^{EP})) > 0) \). Second, the ex post control better fights against type \( L \) by reducing the incentive to merge (term \( (1 - \mu^L G^L(q^{EP})) \)).

Overall, the flexible control dominates if the consumer surplus benefit of better detected good merger type is higher than the consumer surplus cost of the ex ante poorly detected bad merger type; this is the case if \( \Omega \) is low enough.

We summarize the results as follows.

If the anti-competitive merger impact is high, the best control is the one that aims at blocking the bad merger type. For that purpose, the imposed ex post control that deters the merging firms that observe a bad signal is more efficient.

If the pro-competitive merger impact is high, the best control is the one that gives high incentives to merge. The flexible merger control allows the firms to avoid the risk of ex post extra cost if the merger is blocked and thus is the one that maximizes the incentives to merge.
except if the firms are sufficiently well informed ex ante.

We can endogenize the merger control timing considering that the CA must adopt the merger control timing before observing the merger impact on the Consumer Surplus. Then, the CA merger control timing decision will be based only on the distribution of $CS(H)$ and $CS(L)$. According to proposition 1, only the ratio $\Omega = \frac{-CS(L)}{CS(H)}$ matters for the merger control outcome. Let us assume that the CA observes the density distribution function of $\Omega$, $h(\Omega)$, on the interval $[1, +\infty]$. The flexible merger control dominates if:

$$\int_{\Omega^{E/F}} \Delta(\Omega) h(\Omega) d\Omega + \int_{\Omega^{E/F}} \Delta(\Omega) h(\Omega) d\Omega > 0,$$

with $\Omega = \frac{G^H(q^F)}{G^L(q^F)}$ and where $\Delta(\Omega)$ is the expected consumer surplus difference between the flexible control and the ex post control with

$$\Delta(\Omega) = \frac{1}{2} \left[ (\mu^G - (1 - G^H(q^F)) - \Omega(1 - \mu^B)G^L(q^F)) \right].$$

The flexible merger control tends to dominate if the probability of low levels of $\Omega$ is high. Indeed, if the merger is likely to be pro-competitive, the best merger control is the one that maximizes the incentive to merge. Instead, the CA should impose the ex post merger control if the probability of high levels of $\Omega$ is high. In the latter case, a very anticompetitive merger must be blocked. For that purpose, the CA controls the merger ex post with better information on the true impact of the merger and with higher deterrence effect on anticompetitive mergers\textsuperscript{11}.

4 Concluding remarks

We develop a model of merger control by an antitrust authority with different possible timings of control. We show that the introduction of a flexible merger control where the firms choose to be controlled after or before the merger completion increases the expected consumer surplus.

\textsuperscript{11}This result extends Ottaviani and Wickelgren (2011) who show that if the merger is pro-competitive, the ex ante merger control is likely to be more efficient than the ex post control that deters mergers that should be cleared. According to the present result, in the same configuration, the flexible merger control should be adopted.
with respect to the ex ante merger control. That flexible merger control is optimal if either the consumer surplus impact of pro-competitive mergers is high or if the firms are poorly informed ex ante. The imposed ex post control may remain the most efficient timing if either the merging firms are sufficiently well informed ex ante or if the consumer surplus impact of anti-competitive mergers is high.

5 Appendix

5.1 Merging firms’ information

Consider a merger that observes a signal of quality $\tilde{q}$. If the merger observes a signal $h$, the probability to be of type $H$ is equal to: $P(H \mid h) = \frac{\tilde{q} \cdot \frac{1}{2}}{\frac{1}{2}\tilde{q} + (1 - \tilde{q}) \cdot \frac{1}{2}} = \tilde{q}$ and if the merger observes a signal $l$, $P(H \mid l) = \frac{(1 - \tilde{q}) \cdot \frac{1}{2}}{\frac{1}{2}\tilde{q} + (1 - \tilde{q}) \cdot \frac{1}{2}} = 1 - \tilde{q}$.

In other terms, the merging firms observe a signal $q$ ($q \in [0, 1]$) that indicates the probability $q$ to be of type $H$ with $q = \tilde{q}$ if a signal $h$ is observed and with $q = 1 - \tilde{q}$ if a signal $l$ is observed.

We determine the cumulative distribution function of the signal $q$ according to the true type of the merger.

For type $H$, we have $G^H(x) = P(q \leq x \mid H)$ where $G^H(x)$ is the probability for type $H$ to observe a signal that indicates a probability to be of type $H$ lower than $x$.

If type $x < \frac{1}{2}$, $G^H(x)$ is thus the probability to observe a signal $l$ of quality $\tilde{q} > 1 - x$ equal to $\int_{1-x}^{1} (1 - q)g(q) dq$

If type $x > \frac{1}{2}$, $G^H(x)$ is the thus probability to observe a signal $l$ or to observe a signal $h$ of quality $\tilde{q} < x$ equal to $\int_{1/2}^{1} (1 - q)g(q) dq + \int_{1/2}^{x} qg(q) dq$.

Thus, we have:

For type $H$, $G^H(x) = P(q \leq x \mid H) = \begin{cases} \int_{1-x}^{1} (1 - q)g(q) dq \text{ if } x \leq \frac{1}{2} \\ \int_{1/2}^{1} (1 - q)g(q) dq + \int_{1/2}^{x} qg(q) dq \text{ if } x \geq \frac{1}{2} \end{cases}$

For type $L$, $G^L(x) = P(q \leq x \mid L) = \begin{cases} \int_{1-x}^{1} qg(q) dq \text{ if } x \leq \frac{1}{2} \\ \int_{1/2}^{1} qg(q) dq + \int_{1/2}^{x}(1 - q)g(q) dq \text{ if } x \geq \frac{1}{2} \end{cases}$
We have $G^H(x) < G^L(x)$.

5.2 Proof of lemma 1: the BPE with ex post merger control

The CA strategy:

The CA clears or blocks the merger according to the signal observed. The decision is based on expected firms’ strategies since it affects the expected consumer surplus.

Let us denote by $\tilde{q}$ the critical level of signal above which a merger chooses to merge.

- if the signal $B$ is observed, it is a merger of type $L$ and thus the CA blocks the merger.
- if the signal $G$ is observed, the CA clears the merger if the expected consumer surplus is positive: $(1-G^H(\tilde{q}))CS(H)+(1-\mu^B)(1-G^L(\tilde{q}))CS(L) \geq 0$ iff $\Omega = \frac{-CS(L)}{CS(H)} \leq \frac{1-G^H(\tilde{q})}{(1-\mu^B)(1-G^L(\tilde{q}))}$.

The merging firms strategy:

If the merger is cleared ex post with probability $m > 0$ if the signal $G$ is observed, the merger decides to merge iff

$$m \left[q\pi(H) + (1-q)(1-\mu^B)\pi(L)\right] - Km(1-q)\mu^B \geq 0$$

iff $q \geq \frac{Km\mu^B-m(1-\mu^B)\pi(L)}{m(\pi(H)-(1-\mu^B)\pi(L)+\mu^BK)} = q^{EP}(m)$. We denote $q^{EP} = q^{EP}(1) = \frac{K\mu^B-(1-\mu^B)\pi(L)}{\pi(H)-(1-\mu^B)\pi(L)+\mu^BK}$.

We deduce the PBE of the ex post control game:

- if $\Omega \leq \frac{1-G^H(q^{EP}(1))}{(1-\mu^B)(1-G^L(q^{EP}(1)))}$, the merger is always cleared ex post iff a signal $G$ is observed ($m^* = 1$) and the merger decides to merge iff $q > q^{EP}$.
- if $\Omega > \frac{1-G^H(q^{EP}(1))}{(1-\mu^B)(1-G^L(q^{EP}(1)))}$, the CA clears the merger with probability $m^*$ iff the signal $G$ is observed with $m^*$ such that $\Omega = \frac{1-G^H(q^{EP}(m^*))}{(1-\mu^B)(1-G^L(q^{EP}(m^*)))}$. The merger chooses the ex post control iff $q \geq q^{EP}(m^*)$.

The expected consumer surplus is thus given by:

$$\frac{1}{2} \text{Max} \left[(1-G^H(q^{EP}))CS(H) + (1-\mu^B)(1-G^L(q^{EP}))CS(L),0 \right].$$

The ex post control leads to a higher consumer surplus than the ex ante control iff:

$$(1-G^H(q^{EP}))CS(H) + (1-\mu^B)(1-G^L(q^{EP}))CS(L) \geq \mu^GCS(H) + (1-\mu^B)CS(L)$$

iff $\Omega \geq \frac{\text{Max}(\mu^G - (1-G^h(q^{EP})),0)}{(1-\mu^B)G^L(q^{EP})}$.
5.3 Proof of proposition 1

(i) Existence of an equilibrium in case of flexible merger control.

The CA strategy:

The CA clears or blocks the merger according to the signal observed. The decision is based on expected firms’ strategies since it affects the expected consumer surplus.

Let us denote by $\tilde{q}$ the critical level of signal below which a merger chooses the ex ante control.

Ex ante:

- if the signal $B$ is observed, the expected consumer surplus is equal to:
  
  $$ (1 - \mu^G)G^H(\tilde{q})CS(H) + \mu^BG^L(\tilde{q})CS(L) = (1 - \mu^G)CS(H) + \mu^BG^L(\tilde{q})CS(L) < \frac{1}{2}CS(H) + \frac{1}{2}CS(L) < 0. $$
  
  Thus, the CA blocks the merger.

- if the signal $G$ is observed, the expected consumer surplus is equal to:
  
  $$ (1 - \mu^G)G^H(\tilde{q})CS(H) + (1 - \mu^B)G^L(\tilde{q})CS(L). $$
  
  The CA clears the merger iff $\Omega < \frac{\mu^GCS(H)}{(1 - \mu^B)G^L(\tilde{q})}.$

Ex post

- if the signal $B$ is observed, it is a merger of type $L.$ The CA blocks the merger.

- if the signal $G$ is observed, the expected consumer surplus is equal to:
  
  $$ (1 - G^H(\tilde{q}))CS(H) + (1 - \mu^B)(1 - G^L(\tilde{q}))CS(L). $$
  
  The CA clears the merger iff $\Omega < \frac{(1 - G^H(\tilde{q}))}{(1 - \mu^B)(1 - G^L(\tilde{q}))}.$

The merging firms strategy

- if the merger is always blocked ex ante and cleared ex post with probability $m$ if the signal $G$ is observed, the flexible control is equivalent to the ex post control and then the merger chooses the ex post control iff $q \geq q^{EP}(m).$

- if the merger is cleared both ex ante and ex post if the signal $G$ is observed, the merger chooses the ex post control iff

  $$ q\pi(H) + (1 - q)(1 - \mu^B)\pi(L) - K(1 - q)\mu^B \geq q\mu^G\pi(H) + (1 - q)(1 - \mu^B)\pi(L) $$
  
  iff $q \geq q^F = \frac{\mu^B \mu^K}{(1 - \mu^G)\pi(H) + \mu^B} > q^{EP}.$

We deduce the PBE of the flexible control game:

- if $\Omega > \frac{\mu^GCS(H)}{(1 - \mu^B)G^L(\tilde{q})^{EP}},$ the CA always blocks the merger ex ante for both signals. In this case the flexible control is equivalent to the ex post control. The merger chooses the ex post

control iff \( q \geq q^{EP}(m^*) \).

- if \( \Omega < \frac{\mu G H(q^{EP})}{(1-\mu_B) G^L(q^{EP})} \), the CA clears the merger ex ante and ex post iff the signal \( G \) is observed. The merger chooses the ex post control iff \( q \geq q^{F} \).

Thus two additional cases arise:

(a) if \( \frac{\mu G H(q^{EP})}{(1-\mu_B) G^L(q^{EP})} < \frac{\mu G H(q^{F})}{(1-\mu_B) G^L(q^{F})} \),

 the two equilibria described before coexist for \( \frac{\mu G H(q^{EP})}{(1-\mu_B) G^L(q^{EP})} < \Omega < \frac{\mu G H(q^{F})}{(1-\mu_B) G^L(q^{F})} \). We keep the equilibrium where the CA clears the merger ex ante and ex post iff the signal \( G \) is observed. This selection has no impact on the welfare comparison.

Thus, overall, in this case we have \( \tilde{q}^F = \begin{cases} q^{EP}(m^*) & \text{if } \Omega > \frac{\mu G H(q^{F})}{(1-\mu_B) G^L(q^{F})} \\ q^{F} & \text{if } \Omega < \frac{\mu G H(q^{F})}{(1-\mu_B) G^L(q^{F})} \end{cases} \)

(b) if \( \frac{\mu G H(q^{EP})}{(1-\mu_B) G^L(q^{EP})} > \frac{\mu G H(q^{F})}{(1-\mu_B) G^L(q^{F})} \),

for \( \frac{\mu G H(q^{EP})}{(1-\mu_B) G^L(q^{EP})} > \Omega > \frac{\mu G H(q^{F})}{(1-\mu_B) G^L(q^{F})} \), there exists only an equilibrium in mixed strategy where the CA clears the merger with probability \( n \) if it observes a signal \( G \) ex ante. The merger chooses the ex post control iff \( q \geq \frac{\mu B K -(1-n)(1-\mu_B) \pi(L)}{(1-n) \pi(H) + \mu B K -(1-n)(1-\mu_B) \pi(L)} = q^{F}(n) \). At the equilibrium, \( n^* \) is such that the CA is indifferent between clearing and blocking the merger if the signal \( G \) is observed ex ante:

\[ G^H(q^{F}(n^*)) CS(H) + (1 - \mu_B) G^L(q^{F}(n^*)) CS(L) = 0. \]

Thus, we have \( \tilde{q}^{F} = \begin{cases} q^{EP}(m^*) & \text{if } \Omega > \frac{\mu G H(q^{F})}{(1-\mu_B) G^L(q^{F})} \\ q^{F}(n^*) & \text{if } \Omega < \frac{\mu G H(q^{F})}{(1-\mu_B) G^L(q^{F})} \end{cases} \)

In both cases (a) and (b), for \( \Omega > \frac{\mu G H(q^{F})}{(1-\mu_B) G^L(q^{F})} \) the expected consumer surplus ex ante is equal to 0.

(ii) **Comparison between the ex ante control and the flexible merger control**

*(endogenous timing control)*

The expected consumer surplus in case of ex ante control:

\[ \frac{1}{2} Max(\mu G CS(H) + (1 - \mu_B) CS(L), 0). \]

The expected welfare in case of flexible merger control:

\[ \frac{1}{2} Max \left[ \mu G H(\tilde{q}^{F}) CS(H) + (1 - \mu_B) G^L(\tilde{q}^{F}) CS(L), 0 \right] + \frac{1}{2} Max \left[ (1 - G^{H}(\tilde{q}^{F})) CS(H) + (1 - \mu_B)(1 - G^{L}(\tilde{q}^{F})) CS(L), 0 \right] \]
For $\Omega > \frac{\mu^G}{(1-\mu^B)}$ the ex ante expected consumer surplus is 0 and thus the flexible expected consumer surplus is higher.

For $\Omega < \frac{\mu^G}{(1-\mu^B)}$, we have:

Ex ante:
$$\frac{1}{2} \mu^G CS(H) + \frac{1}{2}(1-\mu^B)CS(L)$$

Flexible:
$$\frac{1}{2} [\mu^G G^H(\tilde{q}^F)CS(H) + (1-\mu^B)G^L(\tilde{q}^F)CS(L)]$$
$$+ \frac{1}{2} [(1-G^H(\tilde{q}^F))CS(H) + (1-\mu^B)(1-G^L(\tilde{q}^F))CS(L)]$$

We have:
$$\mu^G G^H(\tilde{q}^F)CS(H) + (1-\mu^B)G^L(\tilde{q}^F)CS(L) +$$
$$ (1-G^H(\tilde{q}^F))CS(H) + (1-\mu^B)(1-G^L(\tilde{q}^F))CS(L) >$$
$$\mu^G CS(H) + (1-\mu^B)CS(L).$$

Note that this result does not depend on $\tilde{q}^F$ and thus does not depend on the equilibrium selection.

(iii) Comparison between the ex post control and the flexible merger control (endogenous timing control).

If $\Omega > \frac{\mu^C G^H(\tilde{q}^E)G^L(\tilde{q}^F)}{(1-\mu^B)G^L(\tilde{q}^F)}$, the flexible control is equivalent to the ex post control.

If $\Omega < \frac{\mu^C G^H(\tilde{q}^E)}{(1-\mu^B)G^L(\tilde{q}^F)}$, we compare

Flexible:
$$\frac{1}{2} [\mu^G G^H(q^F)CS(H) + (1-\mu^B)G^L(q^F)CS(L)]$$
$$+ \frac{1}{2} [(1-G^H(q^F))CS(H) + (1-\mu^B)(1-G^L(q^F))CS(L)]$$

with

Ex post:
$$\frac{1}{2} [(1-G^H(q^{EP}))CS(H) + (1-\mu^B)(1-G^L(q^{EP}))CS(L)]$$

We have higher expected consumer surplus with flexible control iff:
$$\Omega \leq \frac{\text{Max}[G^H(q^{EP})-(1-\mu^C)G^H(q^F),0]}{(1-\mu^B)G^L(q^{EP})}.$$

Thus the flexible control leads to a higher expected consumer surplus iff
$$\Omega \leq \text{Min} \left[ \frac{\text{Max}[G^H(q^{EP})-(1-\mu^C)G^H(q^F),0]}{(1-\mu^B)G^L(q^{EP})}, \frac{\mu^C G^H(q^F)}{(1-\mu^B)G^L(q^F)} \right].$$
References


